1. Question 1

a)

We can translate this problem into a net flow model. In the net graph, there are a start node S and a end node E.

* From end node E, we have Y edges to each year node. According to the harvest limit of each year, each edge has a weight of uj.
* From the year node, we have k edges to each forest node. According to the mature condition of each forest, each edge has a weight of Wij.
* Also, we need add the ecosystem constrains for each forst. So we have Y edges for each forest in each year to a eco-forst node. The edge's weight is Wij for forest i and year j.
* Meanwhile, the tree in forest can be sold in later years, so there is edges between forest and next year nodes. The edge weight is Wij.
* At last, there are k edges from eco-forest node to the start node E. Each edge has a weight of Ti for the environment conscious.



b)

We can see that the above net graph meet all the constrains of the problem.

The start edge limit the every year's tree sold number; the forest node edge

limit the mature tree number; the end edge limit the single forest's tree sold

number. By this way, the max flow of the net can be equal to the maximum sold trees.

2. Question 2

a)

According to our strategy in question 1-a), we can reuse it for given Y year and

k forest. Still one start node S and one end node E. And Y edges to different

Y years and each year k edge to different forest. Also, Y\*k edges for forest

collects and k\*(k-1) edges for forest's relation.

b)

The proof can also be referred as question 1-b).

c)

The time complexity is compound of two parts: one is net build and the other

is max flow algorithm.

Building net contains 2+Y+Y\*k+k nodes and Y+Y\*k+k\*(k-1)+k edges. So building

time is O(Y+k), where Y is year count and k is forest count.

Max flow algorithm using Ford-Fulkerson will cost O(m^2 log C), where C is the

maximum flow and m is the edge count.

So the upper time complexity is O(m^2 log C).

d)

a4.py