Theory of Computation, Fall 2023
Quiz 1&2

Q1. (20 pts) Are the following statements true or false? No explanation is required.

(a) Let A and B be two languages. If A is context-free while $A \cap B$ is not, then B may be context-free but must not be regular.

(b) If L is a non-empty finite language, then the minimum pumping length that works for L is 1+ the length of the longest string in L.

(c) $(a \cup b)^* = a^*(ba^*)^*$.

(d) Let $G = (V, \Sigma, S, R)$ be some context-free grammar in Chomsky norm form. For any string $w \in L(G)$, the number of distinct derivations from S to w is finite.

Q2. (15 pts) Construct DFAs to accept the following languages.

(a) $\emptyset$

(b) $\{e\}$

(c) $\{0, 1\}^*$

Q3. (15 pts) Construct a NFA to accept the following language. Your NFA should have no more than 5 states, and at most one of them can be a final state.

$$L = \{w \in \{0, 1\}^* : \text{at least one of the last four symbols of } w \text{ is } 1\}$$

Note that when $|w| < 4$, the term "the last four symbols of w" means all the symbols of w.

Q4. (15 pts) Let A and B be two regular languages over some alphabet Σ. Show that the following language is also regular.

$$L = \{a_1 b_1 a_2 b_2 \cdots a_k b_k : (a_i, b_i \in \Sigma) \wedge (a_1 a_2 \cdots a_k \in A) \wedge (b_1 b_2 \cdots b_k \in B) \wedge (k \geq 0)\}.$$

Q5. (20 pts) Are the following language regular? or not regular but context-free? or not context-free? Prove your conclusion.

$$L = \{w \in \{a, b, c\}^* : |w| \text{ is odd, and the symbol in the middle of } w \text{ is not } c\}$$

Q6. (15 pts)Prove that the following language is context-free.

$$L = \{w \in \{a, b\}^* : \text{in } w,\ 2 \cdot \#a \neq \#b\}$$

where $\#a$ and $\#b$ are the numbers of a's and b's, respectively.