Theory of Computation, Fall 2023
Assignment 1 (Due September 27 Wednesday 10:00 am)

Only the problems in part I will be graded.

1 Part I

Q1. Are the following statements true of false? No explanation is required.

(a) Let w be a string. The number of distinct symbols that appear in w must be finite.

(b) Let L be a language. The number of distinct symbols that appear in L must be finite.

(c) Every DFA accepts one and only one string.

(d) Every DFA accepts one and only one language.

(e) If $(q_1, w_1) \vdash_M^* (q_2, w_2)$ and $(q_2, w_2) \vdash_M (q_3, w_3)$, then $(q_1, w_1) \vdash_M^* (q_3, w_3)$.

(f) If a language is finite, it must be regular.

(g) Let L be a language over some alphabet Σ. L is also a language over any alphabet that is a superset of Σ.

Q2. Prove that the following languages are regular.

$$L = \{w \in \{0,1\}^* : w \text{ ends with } 00.\}$$

Q3. Let A and B be two regular languages over some alphabet Σ. Define

$$A \cap B = \{w : w \in A \wedge w \in B\}.$$

Show that $A \cap B$ is also regular. (Hint: think about how we proved $A \cup B$ is regular in class.)

Q4. Let A be a regular language over Σ. Consider the following language.

$$\overline{A} = \{w \in \Sigma^* : w \notin A\}$$

Show that $\overline{A}$ is regular.

2 Part II

Q5. Prove that the following language is regular.

$$L = \{w \in \{0,1\}^* : w \text{ starts with 1 and is a multiple of 3 when interpreted as a binary integer}\}$$

(Hint: use states to maintain the remainder of number read so far, when divided by 3.)

Q6. Let $M = (K, \Sigma, \delta, s, F)$ be a DFA. Let Σ' be some superset of Σ. Extend the input alphabet of M to Σ' without changing the language it accepts. In other words, you need to construct a DFA $M' = (K', \Sigma', \delta', s', F')$ such that $L(M') = L(M)$.

Q7. Let A and B be two regular languages over some alphabet Σ. Let $\#$ be a symbol not in Σ. Consider the following language.

$$L = \{w_1 \# w_2 : w_1 \in A \wedge w_2 \in B\}$$

Show that L is also regular.