Solution to Quiz #1

2024 年 3 月 14 日

Problem 1: Construct the NFA for the following regular expression, then convert it to the minimum-state DFA.

(a b)* b a (a | b)*

致谢： 本解答来自于 TonyCrane.

根据正则表达式转化为 NFA.

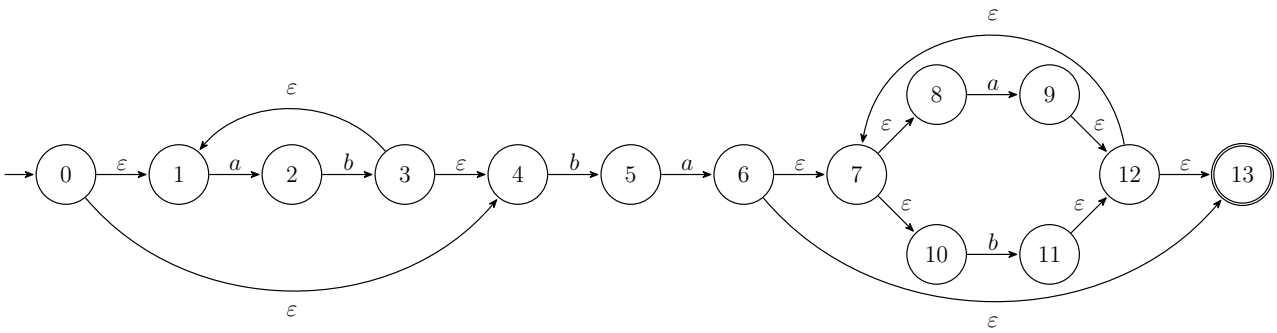

Figure 1: Constructed NFA

子集构造法将 NFA 转化为 DFA. 记 $\overline{X}$ 为状态集合 X 的 ε-closure.

- $S_0 := \overline{\{0\}} = \{0, 1, 4\}$
- S_0 接收 a 转移到状态 $S_1 := \overline{\{2\}} = \{2\}$
- S_0 接收 b 转移到状态 $S_2 := \overline{\{5\}} = \{5\}$
- S_1 接收 b 转移到状态 $S_3 := \overline{\{3\}} = \{1, 3, 4\}$
- S_2 接收 a 转移到状态 $S_4 := \overline{\{6\}} = \{6, 7, 8, 10, 13\}$
- S_4 接收 a 转移到状态 $S_5 := \overline{\{9\}} = \{7, 8, 9, 10, 12, 13\}$
- S_5 接收 b 转移到状态 $S_6 =:= \overline{\{11\}} = \{7, 8, 10, 11, 12, 13\}$

- S_3 接收 a 转移到 S_1，接收 b 转移到 S_2，S_5 与 S_6 接收 a 或者 b 均转移到 S_5 和 S_6，构造结束。

得到状态转移表：

Table 1: transition table of constructed DFA

state	接收 a	接收 b
S_0	S_1	S_2
S_1	/	S_3
S_2	S_4	/
S_3	S_1	S_2
S_4	S_5	S_6
S_5	S_5	S_6
S_6	S_5	S_6

根据状态转移表画出 DFA（dead state 未画出）：

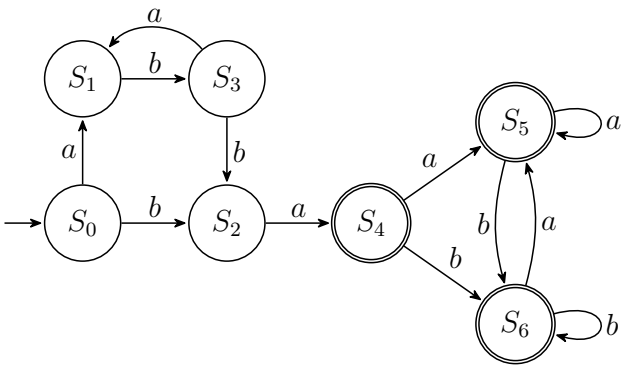

Figure 2: Constructed DFA

最小化 DFA，初始划分为 $\{S_0, S_1, S_2, S_3\}$ 与 $\{S_4, S_5, S_6\}$. 根据状态转移表，可以分割出 S_1 和 S_2，最终划分为 $\{S_0, S_3\}, \{S_1\}, \{S_2\}, \{S_4, S_5, S_6\}$. 从而，我们得到最小化 DFA:

当然，严格来说，按照 DFA 的定义，你得加上 dead state:

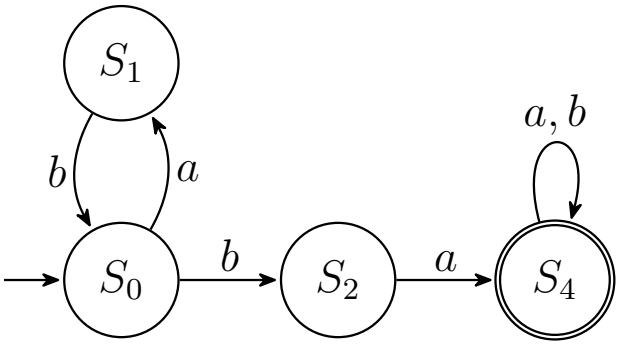

Figure 3: minimalized DFA

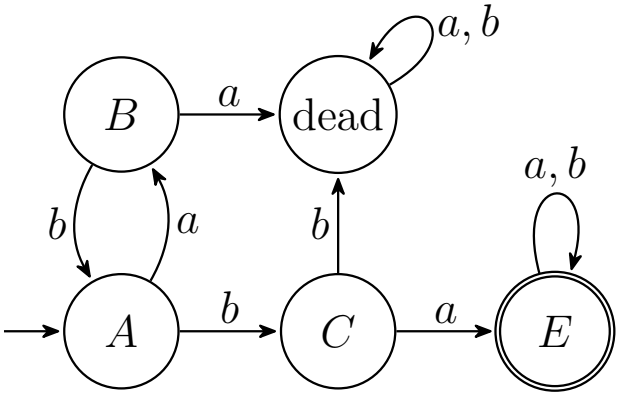

Figure 4: minimalized DFA with dead state