Theory of Computation, Fall 2022
Assignment 4 (Due October 25 Wednesday 10:00 am)

Only part I will be graded.

1 Part I

Q1. What is the language generated by the following context-free grammar?

$$S \to 0S1 \mid A$$
$$A \to S$$

Q2. Design a context-free grammar that generates the following language.

$$\{ww^R : w \in \{0,1\}^*\}$$

Q3. Design a pushdown automaton that accepts the language in Q2.

Q4. Consider the following language. Either design a context-free grammar that generates it, or construct a pushdown automaton that accepts it.

$$\{w \in \{0,1\}^* : \text{no prefix of } w \text{ has more 0's than 1's}\}$$

Note that $u \in \Sigma^*$ is a prefix of $w \in \Sigma^*$ if $w = uv$ for $v \in \Sigma^*$.

2 Part II

Q5. Construct a pushdown automaton that accepts the following language.

$$\{w \in \{a,b\}^* : \#a\text{'s} = 2\#b\text{'s in } w\}$$

Q6. Let A and B be two regular languages. Prove that the following language is context-free.

$$A \diamond B = \{xy : x \in A, y \in B, \text{ and } |x| = |y|\}$$

More precisely, let $M_A = (K_A, \Sigma, \Delta_A, s_A, F_A)$ and $M_B = (K_B, \Sigma, \Delta_B, s_B, F_B)$ be FAs that accepts A and B, respectively. You should use them to construct a PDA that accepts $A \diamond B$. (Hint: $A \diamond B$ is quite similar to $A \circ B$. The only difference is that $A \diamond B$ requires $|x| = |y|$. Luckily, this can be verified using the stack.)