Theory of Computation, Fall 2023
Quiz 1&2

Q1. Let A and B be two regular languages over some alphabet Σ. Show that the following language is also regular.

$$L = \{a_1b_1a_2b_2 \cdots a_kb_k : (a_i, b_i \in \Sigma \cup \{e\}) \wedge (a_1a_2 \cdots a_k \in A) \wedge (b_1b_2 \cdots b_k \in B) \wedge (k \geq 0)\}.$$

Q2. (a) Let L be an infinite regular language. Show that L can be divided into two disjoint subsets A and B such that A and B are infinite regular languages.

(b) Let A and C be two regular languages. We say $A \Subset C$ if $A \subset C$ and C contains infinitely many strings that are not in A. Prove that if $A \Subset C$, then there is a regular language B such that $A \Subset B \Subset C$.

Q3. We say a context-free grammar $G = (V, \Sigma, S, R)$ is a regular grammar if its rules are of the following three forms.

(i) $A \to e$ where $A \in V - \Sigma$

(ii) $A \to a$ where $A \in V - \Sigma$ and $a \in \Sigma$

(iii) $A \to aB$ where $A, B \in V - \Sigma$ and $a \in \Sigma$

Prove that a language is regular if and only if some regular grammar generates it.

Q4. For $i \geq 0$, we define the set $\mathcal{T}_i$ of languages as follows. A language L belongs to $\mathcal{T}_i$ if L is accepted by some pushdown automaton with no more than i states.

(a) Show that there is some context-free language that does not belong to $\mathcal{T}_1$.

(b) Prove that every context-free language belongs to $\mathcal{T}_2$.

Q5. For any language A, define $PREFIX(A) = \{u : uv \in A \text{ for some } v\}$. Prove that if A is context-free, so is $PREFIX(A)$.