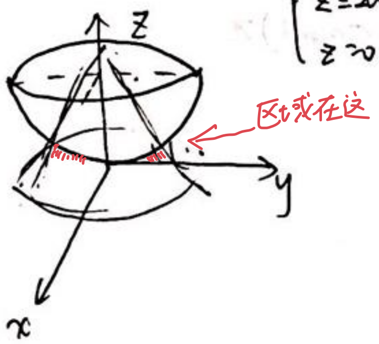
24. (4).



$$\begin{cases} x^2+y^2=az \\ z=2a-\sqrt{x^2+y^2} \\ z\geq 0 \end{cases} \Rightarrow 0\leq z\leq a$$

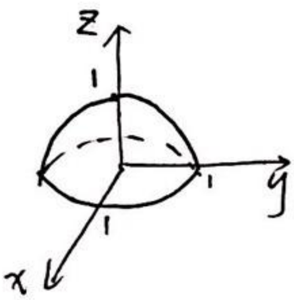
$$V=\int_0^a dz\iint_{D_z} dxdy$$

$$A(D_z)=\pi(2a-z)^2-\pi az=\pi(z^2-5az+4a^2)$$

$$V=\pi\int_0^a z^2-5az+4a^2\,dz=\pi\left(\frac{1}{3}z^3-\frac{5}{2}az^2+4a^2z\right)\Big|_0^a$$

$$=\frac{11}{6}a^3\pi$$

25. (3).



$$\bar{z}=\frac{\iiint z\,dxdydz}{\iiint dxdydz}=\frac{\int_0^1 z\cdot\pi(1-z)dz}{\int_0^1\pi(1-z)dz}$$

$$=\frac{\left(\frac{1}{2}\pi z^2-\frac{1}{3}\pi z^3\right)\Big|_0^1}{\left(\pi z-\frac{1}{2}\pi z^2\right)\Big|_0^1}=\frac{1}{3}$$

由对称性，$\bar{x}=\bar{y}=0$.

形心 $(0, 0, \frac{1}{3})$.

27. $\rho(x,y,z)=\frac{1}{x^2+y^2}$

$$I_z=\iiint(x^2+y^2)\rho(x,y,z)\,dxdydz=\iiint dxdydz$$

$$\cancel{\iint_{z^2\tan^2\alpha\leq x^2+y^2\leq 2Rz-z^2}(x^2+y^2)dxdy=\iint_{z^2\tan^2\alpha\leq r^2\leq 2Rz-z^2}r^2\,rdrd\theta=\int_0^{2\pi}d\theta\int_{z\tan\alpha}^{\sqrt{2Rz-z^2}}r^3dr}$$

$$\cancel{=\int_0^{2\pi}\frac{1}{4}r^4\Big|_{z\tan\alpha}^{\sqrt{2Rz-z^2}}d\theta}$$

$$\iint_{z^2\tan^2\alpha\leq x^2+y^2\leq 2Rz-z^2}dxdy=\iint_{z^2\tan^2\alpha\leq r^2\leq 2Rz-z^2}rdrd\theta$$

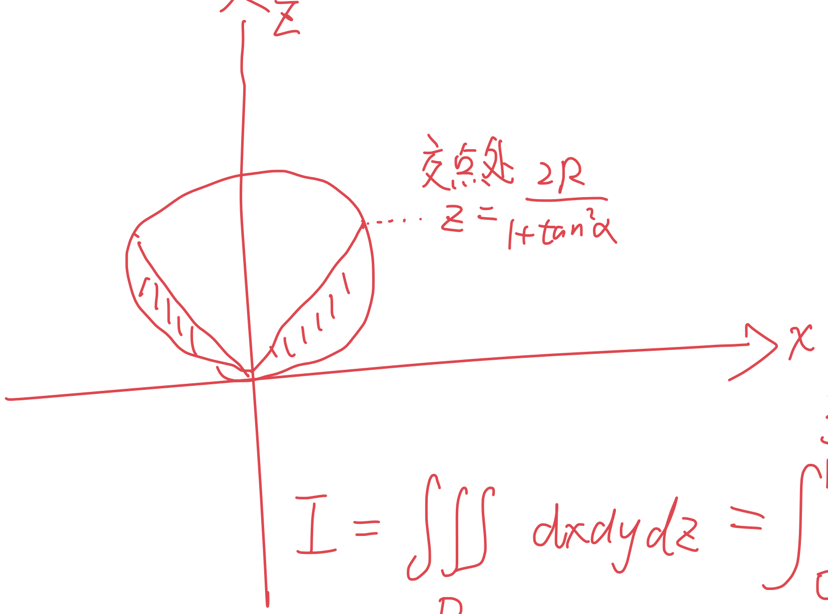
$$=\int_0^{2\pi}d\theta\int_{\sqrt{z^2\tan^2\alpha}}^{\sqrt{2Rz-z^2}}rdr=\int_0^{2\pi}\frac{1}{2}r^2\Big|_{z\tan\alpha}^{\sqrt{2Rz-z^2}}d\theta$$

$$=(2Rz-z^2-z^2\tan^2\alpha)\pi$$

$$I_z=\pi\int_0^{2R\cos^2\alpha}(2Rz-z^2-z^2\tan^2\alpha)\,dz$$

$$=\pi\left(Rz^2-\frac{z^3}{3\cos^2\alpha}\right)\Big|_0^{2R\cos^2\alpha}=\frac{4\pi}{3}R^3\cos^4\alpha$$

这实际上是截面法



$$I=\iiint\limits_{D} dxdydz=\int_0^{\frac{2R}{1+\tan^2\alpha}} dz \iint\limits_{D_z} dxdy$$

注意截面 D_z 是一个圆环，$\iint\limits_{D_z} dxdy=$ 圆环面积

$$=\pi(2Rz-z^2)-\pi(z^2\tan^2\alpha)$$

所以 $I=\int_0^{\frac{2R}{1+\tan^2\alpha}}\pi(2Rz-(1+\tan^2\alpha)z^2)dz$

$$=\frac{4\pi R^3}{3}\frac{1}{(1+\tan^2\alpha)^2}=\frac{4\pi R^3}{3}\cos^4\alpha$$