Theory of Computation, Fall 2023
Assignment 6 (Due November 22 Wednesday 10:00 am)

Only part I will be graded.

1 Part I

Q1. Let $\Sigma = \{a, b, c, \triangleright, \sqcup\}$. Let $M_i = (K_i, \Sigma, \delta_i, s_i, H_i)$ for $i = 1, 2, 3$ be three Turing machines. Give the definition of the following Turing machine in terms of M_1, M_2, M_3.

$$\begin{tikzcd} > M_1 \arrow[r, "a"] \arrow[d, "b"'] & M_2 \\ M_3 & \end{tikzcd}$$

Q2. Design a right-shifting machine $S_{\rightarrow}$ that transforms $\triangleright \sqcup w \underline{\sqcup}$ into $\triangleright \sqcup \sqcup w \underline{\sqcup}$, where w is a string that contains no blank symbol. You may use the machines and the diagrams we presented in class.

Q3. Are the following statements true or false? Briefly explain your answer.

(a) Every standard Turing machine semidecides some language.

(b) Every standard Turing machine decides some language.

Q4. Let M be a Turing machine that decides some language. What is $L(M)$? (Recall that $L(M)$ is the language semidecided by M.)

Q5. Let L be a recursive language. Prove that $\overline{L}$ is also recursive.

2 Part II

Q6. Let D be a DFA. Consider the following decision problem.

Given a string w, does D accept w?

(a) What is the language corresponding to the following problem?

(b) Is this language recursive?

(c) Prove that every regular language is recursive.

Q7. Let $L = \{w \in \{0,1\}^* : w \text{ contains an odd number of 1's}\}$. Define

$$A_L = \{\text{"}D\text{"} : D \text{ is a DFA that accepts } L\}.$$

Show that A_L is recursive. (Hint: you may reduce A_L to EQ_{DFA}.)