Theory of Computation, Fall 2023
Assignment 7 (Due November 29 Wednesday 10:00 am)

Only part I will be graded.

1 Part I

Q1. Prove that every language is countable. You can use any theorem that we have proved in class.

Q2. Prove that there is an undecidable subset of $\{1\}^*$.

Q3. Let A be an arbitrary recursively enumerable langauge. Show that there is a reduction from A to H.

Q4. Prove that the following language is not recursive.

$$\{\text{``}M_1\text{''}\text{``}M_2\text{''}\text{``}k\text{''} : M_1 \text{ and } M_2 \text{ are two Turing machines with } |L(M_1) \cap L(M_2)| \geq k\}$$

You can reduce from any non-recursive language we present in class.

2 Part II

Q5. Let A and B be two languages. Suppose that we have a reduction f from A to B. If B is recursively enumerable, what conclusion can you draw about A? Prove your conclusion.

Q6. Let A and B be two languages. Prove that if $A \leq B$, then $\overline{A} \leq \overline{B}$.

Q7. Let A be a language. Prove that if A and $\overline{A}$ are recursively enumerable, then A must be recursive.

Q8. Let A be a recursively enumerable language. Prove that if $A \leq \overline{A}$, then A is recursive. (Hint: you may use the conclusion of the above three questions)