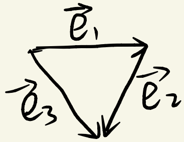
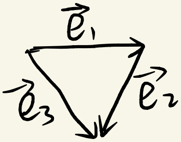
2. (1)



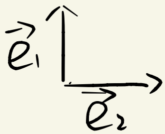
$\vec{e_1}+\vec{e_2}=\vec{e_3}$ $\quad\langle\vec{e_1},\vec{e_2}\rangle=120^\circ$

(2)



$\vec{e_3}-\vec{e_1}=\vec{e_2}$ $\quad\langle\vec{e_1},\vec{e_3}\rangle=60^\circ$

(3)



$|\vec{e_1}+\vec{e_2}|=|\vec{e_1}-\vec{e_2}|$ $\quad\langle\vec{e_1},\vec{e_2}\rangle=90^\circ$

4. 证：$\because \vec{PA}=\vec{PG}+\vec{GA}$，$\vec{PB}=\vec{PG}+\vec{GB}$，$\vec{PC}=\vec{PG}+\vec{GC}$

$\therefore \vec{PA}+\vec{PB}+\vec{PC}=3\vec{PG}+(\vec{GA}+\vec{GB}+\vec{GC})$

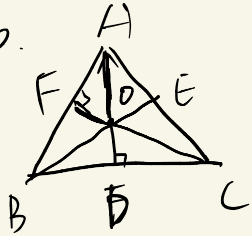
即证 $\vec{GA}+\vec{GB}+\vec{GC}=0$

$\because$ G为△ABC的重心

$\therefore$ 由平行四边形法则可知 $\vec{GA}+\vec{GC}=2\vec{GD}=\vec{BG}$

$\therefore \vec{GA}+\vec{GC}+\vec{GB}=\vec{BG}+\vec{GB}=0$

综上可证 $\vec{PG}=\frac{1}{3}(\vec{PA}+\vec{PB}+\vec{PC})$

6. 

作AD⊥BC、CF⊥AB、BE⊥AC

AD与BE交于O点，只需证明OC⊥AB.

即可说明O、C、F三点共线.

令 $\vec{OA}=\vec{a}$．$\vec{OB}=\vec{b}$．$\vec{OC}=\vec{c}$

$\vec{a}\cdot\vec{BC}=0$ $\quad \vec{BC}=\vec{c}-\vec{b}$

$\vec{b}\cdot\vec{AC}=0$ $\quad \vec{AC}=\vec{c}-\vec{a}$

需证明 $\vec{c}\cdot\vec{AB}=0$

即 $\begin{cases}\vec{a}\cdot(\vec{c}-\vec{b})=0\\ \vec{b}\cdot(\vec{c}-\vec{a})=0\end{cases} \Rightarrow \begin{matrix}\vec{a}\cdot\vec{c}-\vec{a}\cdot\vec{b}=0\\ \vec{b}\cdot\vec{c}-\vec{b}\cdot\vec{a}=0\end{matrix}$

$\vec{a}\cdot\vec{c}-\vec{b}\cdot\vec{c}=0$

$(\vec{a}-\vec{b})\cdot\vec{c}=0$

即 $\vec{BA}\cdot\vec{OC}=0$ 即OC⊥AB.

得证.

8. ∵ $|\vec{a}+\vec{b}|=\sqrt{19}$ 且 $|\vec{a}|=2$，$|\vec{b}|=3$

∴ $|\vec{a}+\vec{b}|^2=\vec{a}^2+\vec{b}^2+2\vec{a}\cdot\vec{b}$

∴ $2\vec{a}\cdot\vec{b}=6$

∴ $|\vec{a}-\vec{b}|^2=\vec{a}^2+\vec{b}^2-2\vec{a}\cdot\vec{b}=7$

∴ $|\vec{a}-\vec{b}|=\sqrt{7}$

9. $[(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})]\cdot(\vec{c}+\vec{a})$

$= (\vec{a}\times\vec{b}+\vec{a}\times\vec{c}+\vec{b}\times\vec{b}+\vec{b}\times\vec{c})\cdot(\vec{c}+\vec{a})$

$= (\vec{a}\times\vec{b})\cdot\vec{c}+(\vec{a}\times\vec{c})\cdot\vec{c}+(\vec{b}\times\vec{c})\cdot\vec{c}+(\vec{a}\times\vec{b})\cdot\vec{a}+(\vec{a}\times\vec{c})\cdot\vec{a}$
$+(\vec{b}\times\vec{c})\cdot\vec{a}$

$= (\vec{a}\times\vec{b})\cdot\vec{c}+(\vec{b}\times\vec{c})\cdot\vec{a}$

$= 6$

11. 由题可知 $\begin{cases}(\vec{a}+3\vec{b})\cdot(7\vec{a}-5\vec{b})=0\\(\vec{a}-4\vec{b})\cdot(7\vec{a}-2\vec{b})=0\end{cases}$

即 $\begin{cases}7\vec{a}^2-15\vec{b}^2+16\vec{a}\cdot\vec{b}=0 & ①\\7\vec{a}^2+8\vec{b}^2-30\vec{a}\cdot\vec{b}=0 & ②\end{cases}$

②$-$①得 $23\vec{b}^2=46\vec{a}\cdot\vec{b}$

$\therefore |\vec{b}|=2|\vec{a}|\cos\theta$ 代入①式得

$7\vec{a}^2-15\times4\vec{a}^2\cos^2\theta+16\cdot2\cdot\vec{a}^2\cos^2\theta=0$

$28\cos^2\theta=7$

$\therefore \cos\theta=\pm\frac{1}{2}$

又 $\because |\vec{b}|=2|\vec{a}|\cos\theta$

$\therefore \cos\theta=\frac{1}{2}$，即 $\theta=\frac{\pi}{3}$

12 解：设 $M(x,y,z)$

$$\overrightarrow{M_1M}=(x-x_1,\ y-y_1,\ z-z_1)$$

$$\overrightarrow{MM_2}=(x_2-x,\ y_2-y,\ z_2-z)$$

又∵ $\overrightarrow{M_1M}=\frac{1}{5}\overrightarrow{MM_2}$ 即 $\begin{cases} x-x_1=\frac{1}{5}(x_2-x) \\ y-y_1=\frac{1}{5}(y_2-y) \\ z-z_1=\frac{1}{5}(z_2-z) \end{cases} \Rightarrow \begin{cases} x=\frac{5x_1+x_2}{6} \\ y=\frac{5y_1+y_2}{6} \\ z=\frac{5z_1+z_2}{6} \end{cases}$

$$M\left(\frac{5x_1+x_2}{6},\ \frac{5y_1+y_2}{6},\ \frac{5z_1+z_2}{6}\right)$$

14. ∵ $\vec{c}$ 与 $\vec{a},\vec{b}$ 共面

∴ $\vec{c}=k_1\vec{a}+k_2\vec{b}=(2k_1+k_2,\ k_1+3k_2,\ 2k_1+5k_2)$

又∵ $\vec{a}\perp\vec{c}$

∴ $\vec{a}\cdot\vec{c}=2(2k_1+k_2)+(k_1+3k_2)+2(2k_1+5k_2)=0$

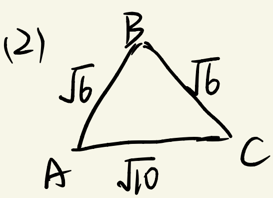
∴ $k_2=-\frac{3}{5}k_1$

∴ $\vec{c}=k(7,-4,-5)\ (k\neq 0)$

16. (1) $\overrightarrow{AB}=(2,1,-1)$，$\overrightarrow{AC}=(1,3,0)$，$\overrightarrow{BC}=(-1,2,1)$

∴ $|\overrightarrow{AB}|=\sqrt{6}$，$|\overrightarrow{AC}|=\sqrt{10}$，$|\overrightarrow{BC}|=\sqrt{6}$

(2)



$\cos A=\frac{6+10-6}{2\sqrt{6}\cdot\sqrt{10}}=\frac{\sqrt{15}}{6}$

∴ $\sin A=\frac{\sqrt{21}}{6}$

∴ $S_{\triangle ABC}=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{\sqrt{35}}{2}$

(3) 设单位向量 $\vec{e}=(x,y,z)$

$$\begin{cases}\vec{e}\cdot\overrightarrow{AB}=0\\ \vec{e}\cdot\overrightarrow{AC}=0\\ |\vec{e}|=1\end{cases}$$ 即 $$\begin{cases}2x+y-z=0\\ x+3y=0\\ \sqrt{x^2+y^2+z^2}=1\end{cases}$$ 得 $$\begin{cases}x=\frac{-3}{\sqrt{35}}\\ y=\frac{1}{\sqrt{35}}\\ z=\frac{-5}{\sqrt{35}}\end{cases}$$ 或 $$\begin{cases}x=\frac{3}{\sqrt{35}}\\ y=-\frac{1}{\sqrt{35}}\\ z=\frac{5}{\sqrt{35}}\end{cases}$$

$\therefore \vec{e}=\left(\frac{-3}{\sqrt{35}},\frac{1}{\sqrt{35}},\frac{-5}{\sqrt{35}}\right)$ 或 $\vec{e}=\left(\frac{3}{\sqrt{35}},-\frac{1}{\sqrt{35}},\frac{5}{\sqrt{35}}\right)$

(4) 由题意得

$$\begin{cases}(\overrightarrow{AB}\times\overrightarrow{AC})\cdot\overrightarrow{AD}=0\\ \overrightarrow{AD}=(0,1,k-1)\end{cases}$$ 即 $$\begin{vmatrix}2&1&-1\\1&3&0\\0&1&k-1\end{vmatrix}=0$$ 得 $k=\frac{6}{5}$