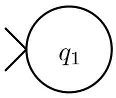
Theory of Computation, Fall 2023
Quiz 1&2 Solutions

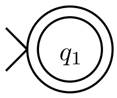
Q1. (a) True. (b) True. (c) True. (d) True.

Q2. The following DFAs meet the requirements, respectively.

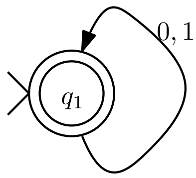
(a)



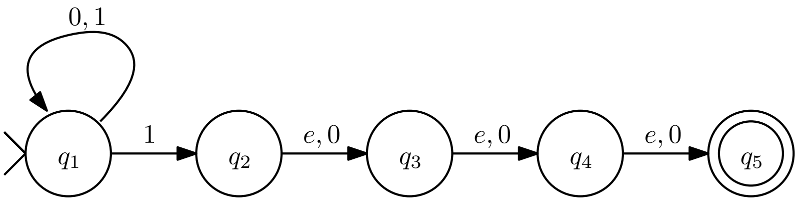
(b)



(c)



Q3. The following NFA meets the requirement.



Q4. Assume that NFA $M_A = (K_A, \Sigma, \delta_A, s_A, F_A)$ accepts A and NFA $M_B = (K_B, \Sigma, \delta_B, s_B, F_B)$ accpets B. Then, we construct the following NFA $M = (K, \Sigma, \delta, s, F)$ accepts L, thus L is regular. We find that L is composed of alternating elements in A and B, so we add a symbol $\mathcal{A}$ or $\mathcal{B}$ expanding the state to represent whether the currently string ends with a symbol in A or B, then δ can be constructed.

$$
\begin{aligned}
K &= K_A \times K_B \times \{\mathcal{A}, \mathcal{B}\} \\
s &= (s_A, s_B, \mathcal{A}) \\
F &= F_A \times F_B \times \{\mathcal{B}\} \cup \{s\}
\end{aligned}
$$

and

$$
\begin{aligned}
\delta((q_1, q_B, \mathcal{A}), a) &= (q_2, q_B, \mathcal{B}),\ \text{if } \delta_A(q_1, a) = q_2, \text{where } q_1, q_2 \in K_A, q_B \in K_B, a \in \Sigma \\
\delta((q_A, q_1, \mathcal{B}), b) &= (q_A, q_2, \mathcal{A}),\ \text{if } \delta_B(q_1, b) = q_2, \text{where } q_1, q_2 \in K_B, q_A \in K_A, b \in \Sigma
\end{aligned}
$$

Q5. L is not regular but context-free.

(a) Firstly, L is context-free because it can be generated by the following CFG.

$$
\begin{aligned}
S &\to ASA \\
A &\to a|b|c \\
S &\to a|b
\end{aligned}
$$

(b) Then, we prove L is not regular by the pumping theorem. Suppose, for the sake of contradiction, that this language is regular. Let p be the pumping length given by the pumping theorem. Consider the string $w = c^p a c^p \in L$. By pumping theorem, w can be written as $w = xyz$ such that the following holds.

1. $xy^i z \in L$ for any $i \geq 0$,
2. $|y| > 0$,
3. $|xy| \leq p$.

Since $|xy| \leq p$ implies that $y = c^k$ for some $k > 0$. Consider the string $xy^0 z = c^{p-k}ac^p$. Clearly, this string does not belong to L. Therefore, L is not regular.

Q6. As the PDA reads the input, we use the stack as a unary counter to record the value of $2A - B$, where A and B are the number of a's and b's that have been read so far. After all the input symbols are consumed, if the stack is not empty (i.e., $2A \neq B$), the PDA will accept the input. We use the symbol $+$ to denote $+1$, and $-$ to denote -1. The PDA $P = (K, \Sigma, \Gamma, \Delta, s, F)$ is as follows.

$$
\begin{aligned}
K &= \{q_1, q_2\}, \\
\Sigma &= \{a, b\}, \\
\Gamma &= \{+, -\}, \\
s &= q_1, \\
F &= \{q_2\},
\end{aligned}
$$

and Δ contains the following transitions.

(q, a, β)	(p, γ)
(q_1, a, e)	$(q_1, ++)$
$(q_1, a, -)$	$(q_1, +)$
$(q_1, a, --)$	(q_1, e)
(q_1, b, e)	$(q_1, -)$
$(q_1, b, +)$	(q_1, e)
$(q_1, e, +)$	(q_2, e)
$(q_1, e, -)$	(q_2, e)
$(q_2, e, +)$	(q_2, e)
$(q_2, e, -)$	(q_2, e)