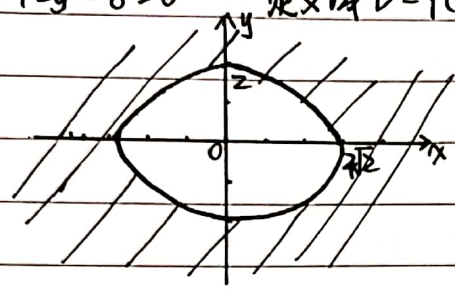
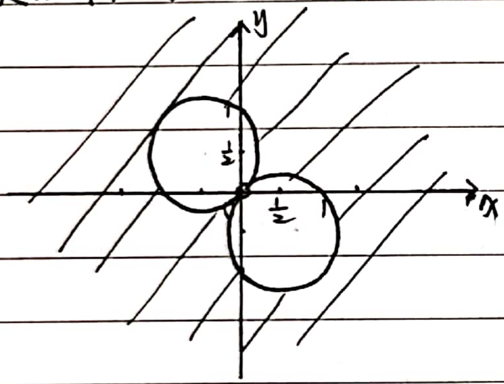
2.(3) $x^2+2y^2-8>0$ 定义域 $D=\{(x,y)\mid x^2+2y^2-8>0\}$



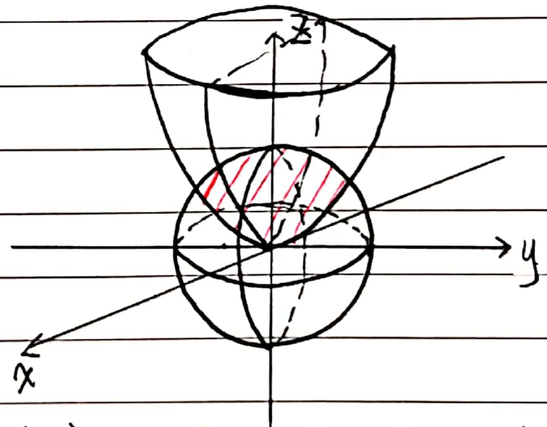
(5) $\begin{cases}\left|\dfrac{x-y}{x^2+y^2}\right|\le 1\\ x^2+y^2\neq 0\end{cases}\Rightarrow\begin{cases}x^2+y^2-x+y\ge 0 \text{ 且 } x^2+y^2+x-y\ge 0\\ (x,y)\neq(0,0)\end{cases}$

定义域 $D=\{(x,y)\mid x^2+y^2-x+y\ge 0 \text{ 且 } x^2+y^2+x-y\ge 0,\ (x,y)\neq(0,0)\}$



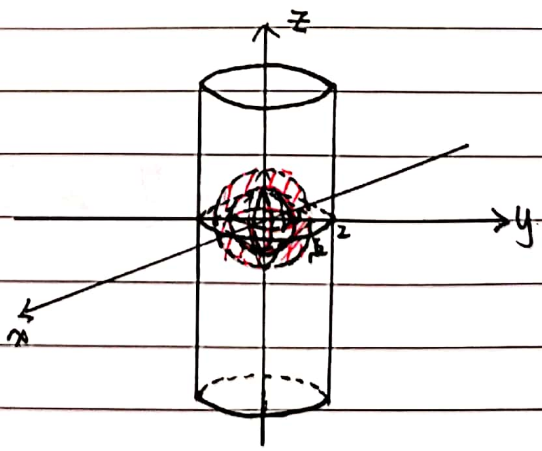
3.(1) $\begin{cases}4-x^2-y^2-z^2\ge 0\\ z-x^2-y^2>0\end{cases}\Rightarrow\begin{cases}x^2+y^2+z^2\le 4\\ x^2+y^2>z\end{cases}$

定义域 $D=\{(x,y,z)\mid x^2+y^2+z^2\le 4 \text{ 且 } x^2+y^2>z\}$



以原点为球心，以2为半径的球与一抛物面相交的空间

(4) $\begin{cases}4-x^2-y^2\ge 0\\ x^2+y^2+z^2-1>0\\ x^2+y^2+z^2-1\neq 1\end{cases}\Rightarrow\begin{cases}x^2+y^2\le 4\\ x^2+y^2+z^2>1\\ x^2+y^2+z^2\neq 2\end{cases}$



扫描全能王 创建

6.(2) $\forall \varepsilon, \exists \delta = \min\{\frac{2}{3}, \frac{2\varepsilon}{6+3\varepsilon}\}$

当 $|x|<\delta, |y|<\delta$ 时

$\left|\frac{2x+y+2}{2-x-2y} - 1\right| = \left|\frac{3x+3y}{2-x-2y}\right| \le \frac{6\delta}{2-3\delta} \le \varepsilon$

4. $f(x+\frac{1}{x}, y-1) = x^2+y^2+2xy-\frac{1}{x^2}+\frac{2y}{x}-2(x+y)-\frac{2}{x}+4$

$= (x+\frac{1}{x})^2+2(x+\frac{1}{x})(y-1)+(y-1)^2+1$

$\Rightarrow f(x,y) = x^2+2xy+y^2+1 = (x+y)^2+1$

6.(1) 证: $\forall \varepsilon>0$, $\exists \delta = 2\varepsilon$, 当 $|x-0|<\delta$, $|y-0|<\delta$ 且 $(x,y)\neq(0,0)$ 时,

$\left|\frac{xy}{|x|+|y|}-0\right| \le \frac{1}{2}\frac{x^2+y^2}{x+y} < \frac{\delta}{2} = \varepsilon$

$\left|\frac{xy}{|x|+|y|}\right| \le \frac{1}{2}\frac{x^2+y^2}{|x|+|y|} \le \frac{1}{2}\frac{(|x|+|y|)^2}{|x|+|y|} \le \frac{1}{2}(|x|+|y|) < \delta$

$\therefore \lim\limits_{(x,y)\to(0,0)} \frac{xy}{|x|+|y|} = 0$

7. (2) 点P沿x轴方向趋向于原点时, $\lim\limits_{\substack{x\to0\\y=0}} \frac{x^2y^2}{x+y} = 0$

点P沿曲线 $y=-x+x^4$ 趋向于原点时, $\lim\limits_{\substack{y=-x+x^4\\x\to0}} \frac{x^2y^2}{x+y} = \lim\limits_{x\to0} \frac{x^2(-x+x^4)^2}{x^4} = 1$

$\therefore \lim\limits_{(x,y)\to(0,0)} \frac{x^2y^2}{x+y}$ 不存在 对比 P101 例9.2.8

(5) 令 $x = r\cdot\cos\theta$, $y = r\cdot\sin\theta$ $(r>0)$, 则: $(x,y)\to(0,0)$ 等价于 $r\to0^+$.

$\because 0 \le \left|\frac{x^2+y^2}{|x|+|y|}\right| = \frac{r^2}{r(|\cos\theta|+|\sin\theta|)} = \frac{r}{|\cos\theta|+|\sin\theta|} < r$, $\lim\limits_{r\to0^+} \frac{r}{|\cos\theta|+|\sin\theta|} = 0$

$\therefore \lim\limits_{(x,y)\to(0,0)} \frac{x^2+y^2}{|x|+|y|} = 0$

(7) 点P沿直线 $y=0$ 趋向于原点时, $\lim\limits_{\substack{y=0\\x\to0}} \frac{x^2y^2}{x^2y^2+(x-y)^2} = 0$

点P沿直线 $y=x$ 趋向于原点时, $\lim\limits_{\substack{y=x\\x\to0}} \frac{x^2y^2}{x^2y^2+(x-y)^2} = 1$

$\therefore \lim\limits_{(x,y)\to(0,0)} \frac{x^2y^2}{x^2y^2+(x-y)^2}$ 不存在

9. (1) $\lim\limits_{\substack{x\to0\\y\to2}} \frac{1-\cos(xy)}{\ln(1-2x^2)} = \lim\limits_{(x,y)\to(0,2)} \frac{2\sin^2(\frac{xy}{2})}{-2x^2} = \lim\limits_{y\to2} \frac{y^2}{-4} = -1$

(2) $\lim\limits_{\substack{x\to0\\y\to2}} \frac{\tan x - x}{\sqrt{1+yx^3}-1} = \lim\limits_{x\to0} \frac{\tan x - x}{\sqrt{1+2x^3}-1} = \lim\limits_{x\to0} \frac{\frac{x^3}{3}(\sqrt{1+2x^3}+1)}{(\sqrt{1+2x^3}-1)(\sqrt{1+2x^3}+1)} = \lim\limits_{x\to0} \frac{\sqrt{1+2x^3}+1}{6} = \frac{2}{6} = \frac{1}{3}$