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The Theory of Conditionals
2439 Words (sorry)

Background
A conditional is any "if… then…" statement. They are common in almost every form of writing. For example, the conditional (1) is adapted from Shakespeare, and (2) is a Roman adage.

(1) If there were a rose by any other name, then it would smell just as sweet.
(2) If you want peace, then prepare for war.

Conditionals are expressed symbolically as A→B (if A, then B).

Example (1) is a counterfactual conditional, and (2) is a command conditional. This paper will focus on indicative conditionals.

Indicative conditionals are a type of conditional where the condition parts A and B are factual or in the indicative mood. For example, (3) is an indicative conditional:

(3) If the store is out of blackberries, then the store didn't order enough.

Several theories seek to evaluate conditionals. All of them answer two questions: When is a conditional valid? And what "good" inferences can a conditional give us?

Introduction
It is my position that the No-Truth-Value theory (NTV theory), also called the Expressivist Theory, is correct when evaluating conditionals. I believe the NTV theory is correct because it can reliably make good inferences on simple conditionals.

In this paper, I will explain what the Ramsey Test is, and its counterargument the triviality result. Then, I will elaborate on two major theories: the Material Conditional theory (MC theory), and Stalnaker's possible worlds theory. I will explain how the MC theory contains too many counterexamples to be consistent, and how Stalnaker's theory, while consistent, can't make predictions as easily as the NTV theory.

Afterwards, I will present the No-Truth-Value theory (NTV theory), and its close connection to the Ramsey Test. Finally, I will evaluate the problem of embeddings with the NTV theory, and some approaches that can possibly help remedy the issue.

The Ramsey Test
The Ramsey Test comes from the idea that conditionals are verbal representations of supposing. Also, how much someone believes a conditional, can be shown with probability.

It states that P(A→B) (probability of if A then B) is given by first adding A to their beliefs, then evaluating B based on A, or P(B|A) (The probability of B given A). Using probability theory, the probability of a conditional $P(B|A)$ is equal to $P(A \wedge B) \div P(A)$ (P(A and B) divided by P(A)).

For instance, in (3), if $P(A \wedge B)$ is 5% and P(B) is 20%, the conditional probability is 25%. There is a 25% chance that if the store is out of blackberries, then it will be because they didn't order enough.

The Triviality Result

The Ramsey Test runs into a problem of *Triviality Results*, which show that $P(A \rightarrow B)=P(B|A) =P(B)$. Since it's a formal proof, it's beyond the scope of this paper, but the problem posed is easy to spot.

If the conditional $P(A \rightarrow B)$ is only reliant on the consequence, then it leads to incorrect assertions. Consider:

(4) If the dice lands on an even number, then it will land on an odd number.

The probability should be 0%, but with the triviality result it gives 50%. Clearly a different theory is necessary.

The Material Conditional Theory

According to the Material Conditional theory (MC theory), conditionals are truth-functional, meaning that for a given conditional $A \rightarrow B$ (if A then B), the truth value of the conditional can be found with the truth value of the antecedent, A, and the truth value of the consequent, B.

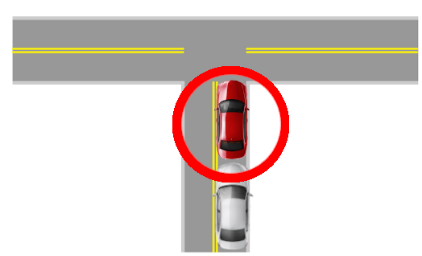
The conditional truth value follows the formula $\neg A \vee B$ (not A or B), meaning if the antecedent is not true ($\neg A$), or the consequent (B) is true, then the conditional is also true. It is typically represented by the "$\supset$" operator as $A \supset B$ (A horseshoe B).

One of the criticisms of the MC theory is that it is too "weak", because 3/4 possibilities are true. The only way to make a conditional false is to have a true antecedent and a false consequent. This also means that it's easy for $A \rightarrow B$ to be true in every situation where $A \supset B$ is true. This is called entailment ($\vDash$).

One of the strongest arguments for the MC theory is the or-to-if inference. Consider the behaviour of the car in front of you at a T intersection. Does (6) come from (5)?

(5) Either the car will turn right or the car will turn left ($A \vee B$)

(6) If the car didn't turn right, then it turned left ($\neg A \rightarrow B$)



T-Intersection

These two statements make it seem like $A \vee B \vDash$(entails) $\neg A \rightarrow B$, which if we remember the definition of the MC theory ($\neg A \vee B$), can be reduced to $A \supset B \vDash A \rightarrow B$.

This results in a logical proof of the MC theory.
$A \to B \vDash A \supset B$, (If A then B entails A horseshoe B) From the MC theory being weak
$A \supset B \vDash A \to B$, (If A horseshoe B entails A then B) From the or-to-if inference
$A \to B \Leftrightarrow A \supset B$, (If A then B is logically equivalent to A horseshoe B) Conclusion

But to many this is unsatisfying. What if both A and B are false? What if the car crashes into the barrier in front? Breaks down while trying to shift gears? These are extremely unlikely from a single intersection, so it's not a bad inference to say (6) given the information that cars typically don't break down and drivers tend to follow the road, but because false-false situations exist, $A \vee B$ cannot entail $\neg A \to B$.

Another problem to emerge with the MC theory is the case of conditionals which don't sound correct. Consider the following:

(7) If Julius Caesar crossed the Rubicon, then the Senate declared him an enemy of the state. ($A \to B$) (If A then B)
(8) If Julius Caesar did not cross the Rubicon, then the Senate declared him an enemy of the state. ($\neg A \to B$)
(9) If Julius Caesar crossed the Rubicon, then Alexander the Great conquered Persia ($A \to C$)
(10) If Julius Caesar did not cross the Rubicon, then the Earth was destroyed by an asteroid in the year of Sulla and Pompeius. ($\neg A \to D$)

Under the MC theory, all these conditionals are true. Yet only (7) sounds correct. (8) sounds incorrect, and (9) and (10) don't make sense.

The main counter to these examples is that although valid, they are inassertable because the form of a conditional adds nothing to the statement. Consider the allegory of the Oracle and the Acolyte. The Oracle receives truths from the gods, and it's the job of the Acolyte to record them.

Should the Oracle state A, Julius Caesar crossed the Rubicon, and the Acolyte could validly record $\neg A \to \_$ (fill "_" with any conclusion) as in (8) and (10), but it's not a useful inference. The Acolyte already knows A, so ¬A cannot occur. Since ¬A can never occur, the inference in (11) called *modus ponens* can also never occur, rendering "_" a useless statement.

The Oracle can also say C, Alexander the Great conquered Persia, and the Acolyte could write $\_ \to C$ as a valid inference (7), but he wouldn't since he knows C, so he could never make the inference in (12) called *modus tollens*.

(11)	$A \to B$	(ex. If the water is boiling, then I can start cooking)
	A	(The water is boiling)
	$\therefore B$	(Therefore I can start cooking)

(12)	$A \rightarrow B$	(ex. If I am low on gas, then the low fuel light will turn on)
	$\neg B$	(The low fuel light is not on)
	$\therefore \neg A$	(Therefore I am not low on gas)

This explanation doesn't always work. It doesn't work in situations where assertability doesn't matter. Imagine a Top Gear opinion survey with yes or no questions, and a fan who is consistent in his beliefs:

(13)	You drive a manual (A)	No
(14)	You drive an automatic transmission ($\neg A$)	Yes
(15)	If you drive a manual, your gearbox shifts automatically ($A \rightarrow B$)	No

According to the MC theory, the consistent citizen has inconsistent beliefs by answering "No" to (13), and "No" again to (15). The only response a MC Theorist has is that the word "if" changes the meaning of the antecedent, which is plausible, but as of right now an undeveloped theory.

Another issue with the MC theory is the same age example. Consider (16)

(16) If I were born in 1989, then I was born in the same year as Taylor Swift.

Although it seems like $A \rightarrow B$ (if A then B) is true or false in the MC Theory independent of the conditions, I believe this conditional doesn't express enough information to be a valid argument. In reality, it is expressing:

(17) If I were born in 1989, then if Taylor Swift was born in 1989, then I was born in the same year as Taylor Swift. ($A \rightarrow (B \rightarrow C)$) (If A then (If B then C))

The conditional has been shortened for pragmatic necessity rather than logical prudence. B is only implied in this sentence, but critical to evaluate it. These "ghost" conditionals can help evaluate some conditionals previously thought to not be evaluable with the MC theory.

Another issue with the MC theory is how it handles probability. Consider:

(18) If a coin is tossed, then it will land on heads. ($A \rightarrow B$)

The way the MC theory evaluates probability is the probability of not A or B. This is problematic, because not A is how likely the coin is not going to be tossed in the first place, and B is the probability of the coin. This means that if you walk away from a coin, the probability of the conditional goes up and approaches 0.75 (1 * 0.5) over time. There's no real way to account for this effect. The MC theory just doesn't do probability well.

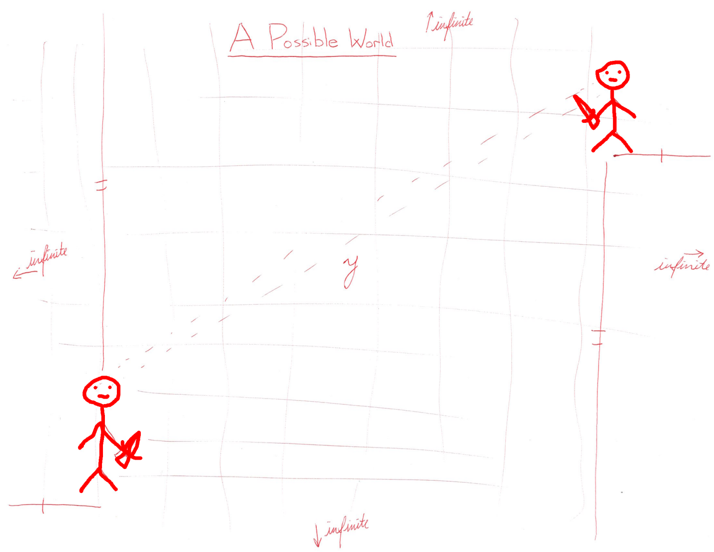
I believe that I have presented more than enough to the inadequacy of the MC theory, which leaves the question of if the MC theory is so inaccurate, then what other theories are there?

Stalnaker's Possible Worlds
Stalnaker's possible worlds theory says that conditionals describe possible worlds. These worlds can be the actual world or an alternate world. All possible worlds are ordered with respect to closeness to the actual world, and a world of nonsense which he calls λ (lambda), where even contradictions are true. Stalnaker believes there is a fixed ordering of worlds, and there are always closer or further from each other, such that there cannot be two worlds closest from the actual world.

Which world a conditional chooses is decided by what he calls the "selection function", which selects the world closest to the actual world where the antecedent is true. Should the antecedent be true in the actual world, then the function will choose the actual world, since the actual world is closest to itself. With Stalnaker's theory, a conditional is then only true if in the selected world where the antecedent A is true, the consequent B is also true.

My main issue with Stalnaker's theory is an epistemic issue of the selection function. Although it is the same in both TT and TF worlds as the MC theory, since they are the actual world, when it comes to selecting a possible world, how to know which world is truly closest is impossible. Consider the following:

(19) You are facing an infinite plane with 2 people identical in every way standing still facing each other diagonally on each end of your field of vision.
(20) If one walks to the other, then the other will die.



With the NTV theory, we do not need to figure out the epistemic problem of which one will kill the other, rather, we only need to evaluate whether or not we think that one will die if one walks towards the other. Afterall, both have brandished a knife.

The Theory of Inquiry
The No-Truth-Value Theory says that conditionals do not express propositions and thus do not have truth values. It states that conditionals are devices for expressing high conditional credence or belief in the consequent, given the antecedent.

To evaluate a conditional, one first assume the antecedent A, then, ignoring ¬A, evaluate B on A.

It's important to note that conditional credence is not the same thing as probability, which is where the NTV theory is most different from the Ramsey Test. Under this theory, a conditional does not express propositions, so it does not have truth or probability. And since conditionals don't express probability, it cannot be used to make a triviality result.

The most common objection to the NTV theory is that it doesn't deal with embeddings well. Consider:

(21) If Amanda leaves the party if Amy leaves the party, then it means that Amanda has a crush on Amy.
(22) If the U.S continues the Space Race, then the U.S.S.R will construct a doomsday device, and if the U.S doesn't continue the Space Race, then the U.S.S.R will not construct a doomsday device.
(23) No student passed the class if the paper was hard.

A potential solution is to evaluate (21) in the same way as normally in the NTV theory, only we assume that Amanda leaves the party if Amy leaves the party, and evaluate whether or not Amanda has a crush on Amy based on that antecedent. There's the issue of how to add "If... If... " statements, and I would like to suggest that in "if A if B", B seems to come first in a hypothetical chronological arrangement. Evaluating "if B then A" first, then adding that to your beliefs and evaluating the rest of the conditionals as "if D then C", where D is "if B then A"

(22) is a conjunction and I can't think of an argument for how to evaluate this. The NTV theory doesn't deal well with conjunctions and disjunctions.

Going back to Ramsey, you cannot apply the Ramsey test if the conclusion is certain, and the same rules apply for the NTV theory in (23). Gricean arguments still have a place as well. It would be more pragmatic to say "The paper was hard, so no student passed the class". Why make it a conditional if it being a conditional doesn't add anything to the sentence.

Some people see the NTV theory of conditionals as an "anti-realist" argument, since it creates a void, and, to many people, doesn't fill it well. While this is true to some extent, the theory is functional if not truth functional, and simple, while withstanding counterexamples and criticism. As a bonus, the Ramsey Test gets to become "vindicated". I think it's absurd to get emotionally attached to a theory of conditionals, but I'd say that it's "just" that the Ramsey Test get vindication, because it seems like how someone with no knowledge of logic would act, in contrast to a more complex theory like Stalnaker's or even the MC theory.