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Chain rule

The chain rule tells us how to find the derivative of a composite function. Brush up on your knowledge of composite functions, and learn how to apply the chain rule correctly.

The chain rule says:

$$\frac{d}{dx}\left[f\Big(g(x)\Big)\right]=f'\Big(g(x)\Big)g'(x)$$

It tells us how to differentiate composite functions.

Quick review of composite functions

A function is *composite* if you can write it as $f\big(g(x)\big)$. In other words, it is a function within a function, or a function of a function.

For example, $\cos(x^2)$ is composite, because if we let $f(x)=\cos(x)$ and $g(x)=x^2$, then $\cos(x^2)=f\big(g(x)\big)$.

g is the function within f, so we call g the "inner" function and f the "outer" function.

$$\underbrace{\cos(\overbrace{x^2}^{\text{inner}})}_{\text{outer}}$$

On the other hand, $\cos(x) \cdot x^2$ is *not* a composite function. It is the *product* of $f(x) = \cos(x)$ and $g(x) = x^2$, but neither of the functions is within the other one.

PROBLEM 1

Is $g(x) = \ln(\sin(x))$ a composite function? If so, what are the "inner" and "outer" functions?

Choose 1 answer:

(A) g is composite. The "inner" function is $\ln(x)$ and the "outer" function is $\sin(x)$.

CORRECT (SELECTED)

( ) g is composite. The "inner" function is $\sin(x)$ and the "outer" function is $\ln(x)$.

(C) g is not a composite function.

Check

Explain

Common mistake: Not recognizing whether a function is composite or not

Usually, the only way to differentiate a composite function is using the chain rule. If we don't recognize that a function is composite and that the chain rule must be applied, we will not be able to differentiate correctly.

On the other hand, applying the chain rule on a function that isn't composite will also result in a wrong derivative.

Especially with transcendental functions (e.g., trigonometric and logarithmic functions), students often confuse compositions like $\ln(\sin(x))$ with products like $\ln(x)\sin(x)$.

PROBLEM 2

Is $h(x) = \cos^2(x)$ a composite function? If so, what are the "inner" and "outer" functions?

Choose 1 answer:

CORRECT (SELECTED)

h is composite. The "inner" function is $\cos(x)$ and the "outer" function is x^2.

INCORRECT

h is composite. The "inner" function is x^2 and the "outer" function is $\cos(x)$.

INCORRECT

h is not a composite function.

Check

Explain

Want more practice? Try this exercise.

Common mistake: Wrong identification of the inner and outer function

Even when a student recognized that a function is composite, they might get the inner and the outer functions wrong. This will surely end in a wrong derivative.

For example, in the composite function $\cos^2(x)$, the outer function is x^2 and the inner function is $\cos(x)$. Students are often confused by this sort of function and think that $\cos(x)$ is the outer function.

Worked example of applying the chain rule

Let's see how the chain rule is applied by differentiating $h(x) = (5 - 6x)^5$. Notice that h is a composite function:

$$h(x) = \underbrace{(\overbrace{5-6x}^{\text{inner}})^5}_{\text{outer}}$$

$$g(x) = 5 - 6x \qquad \text{inner function}$$

$$f(x) = x^5 \qquad \text{outer function}$$

Because h is composite, we can differentiate it using the chain rule:

$$\frac{d}{dx}\left[f\Big(g(x)\Big)\right] = f'\Big(g(x)\Big) \cdot g'(x)$$

Described verbally, the rule says that the derivative of the composite function is the inner function g within the derivative of the outer function f', multiplied by the derivative of the inner function g'.

Before applying the rule, let's find the derivatives of the inner and outer functions:

$$g'(x) = -6$$

$$f'(x) = 5x^4$$

Now let's apply the chain rule:

$$\frac{d}{dx}\left[f\Big(g(x)\Big)\right]$$

$$=f'\Big(g(x)\Big)\cdot g'(x)$$

$$=5(5-6x)^4\cdot -6$$

$$=-30(5-6x)^4$$

Practice applying the chain rule

PROBLEM 3.C

Problem set 3 will walk you through the steps of differentiating $\sin(2x^3-4x)$.

So we have the inner and outer functions and their derivatives:

$$\underbrace{\sin(\overbrace{2x^3-4x}^{\text{inner}})}_{\text{outer}}$$

$g(x)=2x^3-4x$ inner function

$f(x)=\sin(x)$ outer function

$f'(x)=\cos(x)$

$g'(x)=6x^2-4$

What is the derivative of $\sin(2x^3 - 4x)$?

Choose 1 answer:

(A) $\cos(2x^3 - 4x)$

(B) $\cos(6x^2 - 4)$

CORRECT (SELECTED)
$\cos(2x^3 - 4x)(6x^2 - 4)$

(D) $\cos(6x^2 - 4)(2x^3 - 4x)$

Check

Explain

PROBLEM 4

$$\frac{d}{dx}[\sqrt{\cos(x)}] = ?$$

Choose 1 answer:

CORRECT (SELECTED)
$-\dfrac{\sin(x)}{2\sqrt{\cos(x)}}$

(B) $\dfrac{\cos(x)}{2\sqrt{\sin(x)}}$

(C) $\dfrac{\sin(x)}{2\sqrt{\cos(x)}}$

(D) $-\dfrac{\cos(x)}{2\sqrt{\sin(x)}}$

Check Next question

Explain

Want more practice? Try this exercise.

PROBLEM 5

x	$f(x)$	$h(x)$	$f'(x)$	$h'(x)$
-1	9	-1	-5	-6
2	3	-1	1	6

$$G(x) = f\Big(h(x)\Big)$$

$G'(2) =$ -30

Check Next question

Hide explanation

$$G'(x) = f'\Big(h(x)\Big) \cdot h'(x).$$

We can now plug $x = 2$ into this expression and evaluate:

$$G'(2) = f'\Big(h(2)\Big) \cdot h'(2)$$

$$= f'(-1) \cdot 6 \qquad h(2) = -1\ ,\ h'(2) = 6$$

$$= -5 \cdot 6 \qquad f'(-1) = -5$$

$$= -30$$

Want more practice? Try this exercise.

PROBLEM 6

Katy tried to find the derivative of $(2x^2 - 4)^3$. Here is her work:

Step 1: Let $f(x) = x^3$ and $g(x) = 2x^2 - 4$, then $(2x^2 - 4)^3 = f\big(g(x)\big)$.

Step 2: $f'(x) = 3x^2$

Step 3: The derivative is $f'\big(g(x)\big)$:

$$\frac{d}{dx}[(2x^2 - 4)^3] = 3(2x^2 - 4)^2$$

Is Katy's work correct? If not, what's her mistake?

Choose 1 answer:

(A) Katy's work is correct.

Katy actually made a mistake. Try going

MATH › AP® CALCULUS AB › DIFFERENTIATION: COMPOSITE, IMPLICIT, AND INVERSE FUNCTIONS › THE CHAIN RULE: INTRODUCTION

The chain rule: introduction

Chain rule

Identifying composite functions

Practice: Identify composite functions

composite functions

Worked example: Derivative of $\cos^3(x)$ using the chain rule

Worked example: Derivative of $\sqrt{(3x^2-x)}$ using the chain rule

Worked example: Derivative of $\ln(\sqrt{x})$ using the chain rule

closely over her solution one more time.

(B) **Step 1 is incorrect.** $(2x^2-4)^3$ is equal to $g(f(x))$, not $f(g(x))$.

Katy identified the inner and the outer functions correctly.

(C) **Step 2 is incorrect**. The derivative of f isn't $3x^2$.

Derivative of f is indeed $3x^2$.

CORRECT (SELECTED)

Step 3 is incorrect. The derivative isn't $f'(g(x))$.

Check

Common mistake: Forgetting to multiply by the derivative of the inner function

A common mistake is for students to only differentiate the outer function, which results in $f'(g(x))$, while the correct derivative is $f'(g(x))g'(x)$.

Another common mistake: Computing $f'(g'(x))$

Another common mistake is to differentiate $f(g(x))$ as the composition of the derivatives, $f'(g'(x))$.

This is also incorrect. The function that should be inside of $f'(x)$ is $g(x)$, not $g'(x)$.

Remember: *The derivative of* $f(g(x))$ *is* $f'(g(x))g'(x)$. *Not* $f'(g(x))$ *and not* $f'(g'(x))$.

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Questions Tips & Thanks

Question

Ask a question...

itan-ola eniitan 2 years ago

What id the derivative of f(x)=3sin2x,

Reply • 3 comments (4 votes) Upvote Downvote F

Angell, J 2 years ago

you would treat it as u = 2x, and v = 3sin(u). Then u' = 2, and v' = 3cos(u).