Tutorial of Derivation of Inverter Transfer Function

1. Filter Analysis

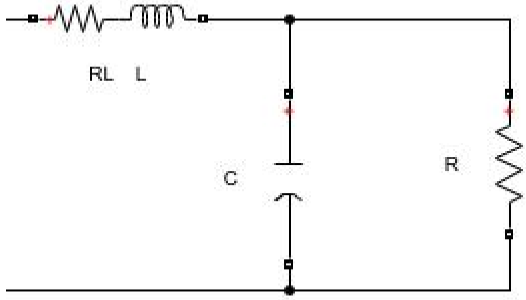

Fig.1 Equivalent circuit of inverter filter

KVL Law's:

(1) $V_{in} = L\frac{di}{dt} + i_L R_L + V_o$

(2) $C\frac{dV_C}{dt} = i_L - i_{out}$

1.1 Output Voltage Version

After Laplace Transfer:

(3) $V_{in} = Li_L s + i_L R_L + V_o$

(4) $i_L = \frac{V_o}{R} + CV_O s\ (V_C = V_O)$

By substituting the value of i_L in (3)

$$V_{in} = (Ls + R_L)\left(\frac{V_O}{R} + CV_O s\right) + V_O$$

So

$$V_{in} = V_O[CLs^2 + \left(\frac{L}{R} + CR_L\right)s + \frac{R + R_L}{R}]$$

1.2 Output Current Version

After Laplace Transfer:

(5) $V_{in} = Li_L s + i_L R_L + i_o R$

(6) $i_L = i_O(CRs + 1)$

By substituting the value of i_L in (5)

$$V_{in} = Li_O(CRs + 1)s + i_O(CRs + 1)R_L + i_O R$$

So

$$V_{in} = i_O[LCRs^2 + (L + CRR_L)s + R + R_L]$$

1.3 Inverter Current Version

After Laplace Transfer:

(7) $V_{in} = Li_L s + i_L R_L + i_o R$

(8) $i_O = \frac{i_L}{(CRs+1)}$

By substituting the value of i_O in (7)

$$V_{in} = Li_L s + i_L R_L + \frac{i_L R}{CRs + 1}$$

So

$$V_{in} = i_L(Ls + R_L + \frac{R}{CRs + 1})$$

2. PWM Analysis

$$G_{PWM}(s) = \frac{V_x(s)}{m(s)} [m(s)\ is\ Modulation\ signal\ ]$$

$$G_{PWM}(s) = \frac{1}{C_{pk}} * \frac{1 - s\frac{T_s}{4}}{1 + s\frac{T_s}{4}} 2V_{DC}$$

The duty ratio, which is the fraction of time in a cycle that the single update modulating signal (m(s)) is higher than the triangle carrier, can be expressed by the value of a single update modulating signal in a cycle to the peak of triangle carrier (C_{pk}).

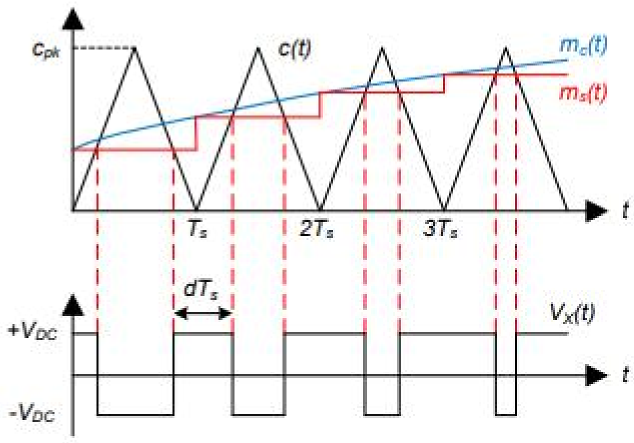

Fig.2 Single update PWM with triangle carrier

3. Summary

3.1 Output Voltage Version

$$G_{PWM}(s)m(s) = V_O[CLs^2 + \left(\frac{L}{R} + CR_L\right)s + \frac{R+R_L}{R}] (V_X=V_{in})$$

So the transfer function of modulation signal and output voltage

$$\frac{V_{\boldsymbol{O}}}{m}=\frac{G_{PWM}(s)}{LCs^2+\left(\frac{L}{R}+CR_L\right)s+\frac{R+R_L}{R}}$$

3.2 Output Current Version

$$G_{PWM}(s)m(s)=i_O[LCRs^2+(L+CRR_L)s+R+R_L]$$

So the transfer function of modulation signal and output current

$$\frac{i_O}{m}=\frac{G_{PWM}(s)}{LCs^2+\left(\frac{L}{R}+CR_L\right)s+\frac{R+R_L}{R}}$$

3.3 Inverter Current Version

$$G_{PWM}(s)m(s)=i_L(Ls+R_L+\frac{R}{CRs+1})$$

So the transfer function of modulation signal and inverter current

$$\frac{i_L}{m}=\frac{Cs+\frac{1}{R}}{LCs^2+\left(\frac{L}{R}+CR_L\right)s+\frac{R+R_L}{R}}G_{PWM}(s)$$