Hence, $\frac{MD_1}{W} = \frac{\quad}{W_1} = \frac{\quad}{W_2}$

Previous Years' Questions

1997

Direction(Q. 1 and 2): Answer the questions based on the following information.

The Weirdo Holiday Resort follows a particular system of holidays for its employees. People are given holidays on the days where the first letter of the day of the week is the same as the first letter of their names. All employees work at the same rate.

1. Raja starts working on February 25, 1996, and finishes the job on March 2, 1996. How much time would T and J take to finish the same job if both start on the same day as Raja?
   (1) 4 days
   (2) 5 days
   (3) Either (1) or (2)
   (4) Cannot be determined
2. Starting on February 25, 1996, if Raja had finished his job on April 2, 1996, when would T and S together likely to have completed the job, had they started on the same day as Raja?

(1) March 15, 1996
(2) March 14, 1996
(3) March 22, 1996
(4) Data insufficient

2001

3. A can complete a piece of work in 4 days. B takes double the time taken by A, C takes double that of B, and D takes double that of C to complete the same task. They are paired in groups of two each. One pair takes two thirds the time needed by the second pair to complete the work. Which is the first pair?

(1) A, B (2) A, C
(3) B, C (4) A, D

4. There's a lot of work in preparing a birthday dinner. Even after the turkey is in oven, there are still the potatoes and gravy, yams, salad, and cranberries, not to mention setting the table. Three friends, Asit, Arnold, and Afzal, work together to get all of these chores done. The time it takes them to do the work together is six hours less than Asit would have taken working alone, one hour less than Arnold would have taken, and half the time Afzal would have taken working alone.

How long did it take them to do these chores working together?

(1) 20 minutes (2) 30 minutes
(3) 40 minutes (4) 50 minutes

2002

5. It takes 6 technicians a total of 10 hours to build a new server from Direct Computer, with each working at the same rate. If six technicians start to build the server at 11 a.m. and one technician per hour is added beginning at 5 p.m., at what time will the server be complete?

(1) 6:40 p.m. (2) 7:00 p.m.
(3) 7:20 p.m. (4) 8:00 p.m.

2003

6. A leather factory produces two kinds of bags, standard and deluxe. The profit margin is ₹20 on a standard bag and ₹30 on a deluxe bag. Every bag must be processed on machine A and on machine B. The processing times per bag on the two machines are as follows:

	Time required (Hours/bag)	
	Machine A	Machine B
Standard Bag	4	6
Deluxe Bag	5	10

The total time available on machine A is 700 hours and on machine B is 1250 hours. Among the following production plans, which one meets the machine availability constraints and maximizes the profit?

(1) Standard 75 bags, Deluxe 80 bags
(2) Standard 100 bags, Deluxe 60 bags
(3) Standard 50 bags, Deluxe 100 bags
(4) Standard 60 bags, Deluxe 90 bags

2004

7. In Nuts and Bolts factory, one machine produces only nuts at the rate of 100 nuts per minute and needs to be cleaned for 5 minutes after production of every 1000 nuts. Another machine produces only bolts at the rate of 75 bolts per minute and needs to be cleaned for 10 minutes after production of every 1500 bolts. If both the machines start production at the same time, what is the minimum duration required for producing 9000 pairs of nuts and bolts?

(1) 130 minutes (2) 135 minutes
(3) 170 minutes (4) 180 minutes

2017

8. A person can complete a job in 120 days. He works alone on Day 1. On Day 2, he is joined by another person who also can complete the job in exactly 120 days. On Day 3, they are joined by another person of equal efficiency. Like this, everyday a new person with the same efficiency joins the work. How many days are required to complete the job?

9. Amal can complete a job in 10 days and Bimal can complete it in 8 days. Amal, Bimal and Kamal together complete the job in 4 days and are paid a total amount of ₹1000 as remuneration. If this amount is shared by them in proportion to their work, then Kamal's share, in rupees, is

(1) 100 (2) 200
(3) 300 (4) 400

2018

9. Humans and robots can both perform a job but at different efficiencies. Fifteen humans and five robots working together take thirty days to finish the job, whereas five humans and fifteen robots working together take sixty days to finish it. How many days will fifteen humans working together (without any robot) take to finish it?
(1) 45 (2) 36
(3) 32 (4) 40

10. When they work alone, B needs 25% more time to finish a job than A does. They two finish the job in 13 days in the following manner: A works alone till half the job is done, then A and B work together for four days, and finally B works alone to complete the remaining 5% of the job. In how many days can B alone finish the entire job?
(1) 20 (2) 22
(3) 16 (4) 18

12. Ramesh and Ganesh can together complete a work in 16 days. After seven days of working together, Ramesh got sick and his efficiency fell by 30%. As a result, they completed the work in 17 days instead of 16 days. If Ganesh had worked alone after Ramesh got sick, in how many days would he have completed the remaining work?
(1) 14.5 (2) 11
(3) 13.5 (4) 12

2018

13. Three men and eight machines can finish a job in half the time taken by three machines and eight men to finish the same job. If two machines can finish the job in 13 days, then how many men can finish the job in 13 days?

14. John gets ₹57 per hour of regular work and ₹114 per hour of overtime work. He works altogether 172 hours and his income from overtime hours is 15% of his income from regular hours. Then, for how many hours did he work overtime?

15. Anil alone can do a job in 20 days while Sunil alone can do it in 40 days. Anil starts the job, and after 3 days, Sunil joins him. Again, after a few more days, Bimal joins them and they together finish the job. If Bimal has done 10% of the job, then in how many days was the job done?
(1) 12 (2) 13
(3) 15 (4) 14

2020

16. John takes twice as much time as Jack to finish a job. Jack and Jim together take one-thirds of the time to finish the job than John takes working alone. Moreover, in order to finish the job, John takes three days more than that taken by three of them working together. In how many days will Jim finish the job working alone?

Answer Key

1. (1)	2. (3)	3. (4)	4. (3)	5. (4)	6. (1)	7. (3)	8. 15	9. (1)	10. (3)
11. (1)	12. (3)	13. 13	14. 12	15. (2)	16. 4				

Answers with Explanations

1. **Option (1) is correct.**

As there is no day in the week whose first letter is R, it can be concluded that Raja does not have any holidays. Since 1996 is a leap year, we can figure out that Raja has totally worked for 7 days. Let his rate of doing the job be one unit per day. So, he would complete 7 units work in a week. J's situation is similar to Raja and does not have any holiday during the week. T will have two holidays in a week (Tuesday and Thursday). Since the rate of working for all the three of them is the same, the working pattern of J and T would be as follows.

Sun	2
Mon	2
Tue	1
Wed	2
Thu	1
Fri	2
Sat	2

We can find 25 Feb, 1996 was a Sunday. Hence, 7 unit can be completed in 4 days.

2. **Option (3) is correct.**

Raja has worked for (5 days in February + 31 days in March + 2 days in April) = 38 days. Let his rate of doing the work be one unit a day.

Hence, he completes 38 units totally.

In a week, T takes holiday on Tuesday and Thursday, while S takes holidays on Saturday and Sundays. We can see that their working pattern would be as follows.

Sun	1
Mon	2
Tue	1
Wed	2
Thu	1
Fri	2
Sat	1

So, in a week they work together 10 units work. Thus, in three weeks, they would complete 30 units work. It can be found out that February 25 is Sunday. So, the remaining 8 units of work can be completed only on Friday, i.e., March 22.

Option (4) is correct.

Assume the work to be 4 units

A's efficiency = 1 unit/day

B's efficiency = $\frac{1}{2}$ unit/day

C's efficiency = $\frac{1}{4}$ unit/day

D's efficiency = $\frac{1}{8}$ unit/day

Since one pair takes $\frac{2}{3}^{rd}$ five the other, we have to find that pair whose efficiency is $\frac{3}{2}$ of the efficiency of the other pair.

(A + D) efficiency = $1 + \frac{1}{8} = \frac{9}{8}$

(B + C) efficiency = $\frac{1}{2} + \frac{1}{4} = \frac{3}{4} = \frac{6}{8}$

$(A + D) = \frac{3}{2}(B + C)$

Hence, A, D are the pair.

4. **Option (3) is correct.**

Let 't' be the time taken by all three together, then

$$\frac{1}{t+6} + \frac{1}{t+1} + \frac{1}{2t} = \frac{1}{t}$$

Solving the above equation, we get

$3t^2 + 7t - 6 = 0$ or $t = \frac{2}{3}$ hr = 40 min

5. **Option (4) is correct.**

Assume the efficiency of each technician = 1 unit/hr.

Hence, the work = 6 × 10 = 60 units.

From 11 am till 5 p.m. *i.e.*, 6 hrs, 6 × 6 or 36 units work was done. Work left = 24 units.

From 5 pm to 6 p.m. : 7 units was done

6 pm to 7 p.m. : 8 units was done

7 pm to 8 p.m. : 9 units was done

Hence, (7 + 8 + 9) or 24 units which was remaining was completed at 8 p.m.

6. **Option (1) is correct.**

Total time available is 700 hrs on machine A and 1250 hrs on machine B.

Let, the number of Standard Bags be s and the number of Deluxe Bags be d.

Here, we have to maximize the profit margin *i.e.*, $20s + 30d$, subject to the constraints,

$4s + 5d \leq 700$ and $6s + 10d \leq 1250$

Now, consider options.

(1.) $s = 75$ and $d = 80$

$\therefore$ The profit = $75 \times 20 + 30 \times 80 = 3900$

$4s + 5d = 700$ and $6s + 10d = 1250$

$\therefore$ the constraints are satisfied.

(2.) $s = 100$ and $d = 60$

$\therefore$ The profit = $100 \times 20 + 60 \times 30 = 3800$

The profit is less than in option 1.

$\therefore$ Option 2 is not the answer.

(3.) $s = 50$ and $d = 100$

$\therefore$ The profit = $50 \times 20 + 100 \times 30 = 4000$

$4s + 5d = 700$ and $6s + 10d = 1300$

$\therefore$ The second constraint is not satisfied.

$\therefore$ Option 3 cannot be the answer.

(4.) $s = 60$ and $d = 90$

$\therefore$ The profit $= 60 \times 20 + 90 \times 30 = 3900$

$4s + 5d = 690$ and $6s + 10d = 1260$

$\therefore$ The second constraint is not satisfied.

$\therefore$ Option 4 cannot be the answer.

As only option (1) satisfies the constraints and also maximizes the profit.

Option (3) is correct.

Machine I:

Number of nuts produced in one minute = 100

To produce 1000 nuts time required = 10 min

Cleaning time for nuts = 5 min

Over all time to produce 1000 nuts = 15 min.

Over all time to produce 9000 = 135 min − 5 min = 130 min ...(i)

Machine II:

To produce 75 bolts time required = 1min

To produce 1500 bolts time required = 20 min

Cleaning time for bolts = 10 min.

Effective time to produce 1500 bolts = 30 min

Effective time to produce 9000 bolts = 30×6 − 10 = 170 min ... (ii)

From (i) and (ii),

Minimum time = 170 minutes

Correct answer is [15].

Let, the rate of work of a person be 1 unit/day.

Hence, the total work = 120.

It is given that on first day, one person works, on the second day two people work and so on.

The work done on day 1, day 2, ... will be 1, 2, 3, ..., respectively.

The sum should be equal to 120.

If n days are required

Then $\frac{n(n+1)}{2} = 120.$

Solving we get $n = 15$.

Hence, it takes 15 days to complete the work.

Option (1) is correct.

Let, the work be 40 units.

Hence, the efficiency of Amal $= \frac{40}{10}$

$= 4$ units/day

Efficiency of Bimal $= \frac{40}{8} = 5$ units/day

Efficiency of Amal + Kamal + Bimal $= \frac{40}{4}$

$= 10$ units/day

Hence, the efficiency of Kamal = 10 − 4 − 5 = 1

Kamal gets $\frac{1}{10}$ of 1000 = ₹ 100. Since, money will be divided in the ratio of their efficiencies which is 4 : 5 : 1.

10. **Option (3) is correct.**

Let, the efficiency of humans be 'h' and the efficiency of robots be 'r'.

In the first case,

Total work $= (15h + 5r) \times 30$ ...(i)

In the second case,

Total work $= (5h + 15r) \times 60$ ... (ii)

On equating (i) and (ii), we get

$(15h + 5r) \times 30 = (5h + 15r) \times 60$

Or, $15h + 5r = 10h + 30r$

Or, $5h = 25r$

Or, $h = 5r$

Total work $= (15h + 5r) \times 30$

$= (15h + h) \times 30 = 480h$

Time taken by 15 humans $= \frac{480h}{15h} = 32$ days.

11. **Option (1) is correct.**

Suppose A needs '$16a$' days to complete the job and B needs '$20a$'.

Number of days needed by A to complete half of the job = $8a$

Number of days needed by B to complete 5% of the job $= \frac{20a}{20} = a$

$8a + 4 + a = 13$

$a = 1$

$\therefore$ B alone can finish the entire job in 20 days.

12. **Option (3) is correct.**

Let 'R' and 'G' be the amount of work that Ramesh and Ganesh can complete in a day.

It is given that they can together complete a work in 16 days.

$\therefore$ Total amount of work = 16(R + G) ...(i)

For first 7 days both of them worked together. From 8th day, Ramesh worked at 70% of his original efficiency, whereas Ganesh worked at his original efficiency. It took them 17 days to finish the same work. *i.e.*, Ramesh worked at 70% of his original efficiency for 10 days.

The work that both can finish in 16 − 7 = 9 days got finished in 16 − 6 = 10 days, because Ramesh efficiency reduced to 70%.

$$9(R + G) = 10(0.7R + G)$$
$$9R + 9G = 7R + 10G$$
$$G = 2R$$

Total amount of work left when Ramesh got sick $= 16(R + G) - 7(R + G)$

$= 9(R + G) = 9(0.5G + G)$

$= 13.5G$

Therefore, time taken by Ganesh to complete the remaining work $= \frac{13.5G}{G} = 13.5$ days.

13. Correct answer is [13].

Consider the work done by a man in a day $= a$ and that by a machine $= b$

Since, three men and eight machines can finish a job in half the time taken by three machines and eight men to finish the same job,

Hence, the efficiency will be double.

$\Rightarrow \quad 3a + 8b = 2(3b + 8a)$

$\Rightarrow \quad 13a = 2b$

Hence, work done by 13 men in a day = work done by 2 machines in a day.

$\Rightarrow$ If two machines can finish the job in 13 days, then same work will be done by 13 men in 13 days.

Hence, the required number of men = 13

14. Correct answer is [12].

It is given that John works altogether 172 hours, *i.e.*, including regular and overtime hours.

Let a be the regular hours, $172 - a$ will be the overtime hours

John's income from regular hours $= 57 \times a$

John's income for working overtime hours $= (172 - a) \times 144$

It is given that his income from overtime hours is 15% of his income from regular hours

$a \times 57 \times 0.15 = (172 - a) \times 114$

$a = 160$

The number of hours for which he worked overtime $= 172 - 160 = 12$ hrs

15. Option (2) is correct.

Let, the total work be LCM of 20, 40 = 40 units

Efficiency of Anil and Sunil is 2 units and 1 unit per day, respectively.

Anil works alone for 3 days, so Anil must have completed 6 units.

Bimal completes 10% of the work while working along with Anil and Sunil.

Bimal must have completed 4 units.

The remaining 30 units of work is done by Anil and Sunil

Number of days taken by them $\frac{30}{3} = 10$

The total work is completed in $3 + 10 = 13$ days

16. Correct answer is [4].

Given, John : Jack

Time 2 : 1

So, efficiency 1 : 2 ...(i)

Again Jack + Jim : John

So, efficiency 3 : 1 ...(ii)

From (i) and (ii),

Efficiency of Jim $= 3 - 2$

$= 1$

Now, ratio of efficiency,

$\Rightarrow$ John : Jack : Jim : All three

1 : 2 : 1 : 4

Means John : All three

$\Rightarrow$ Efficiency 1 : 4

$\Rightarrow$ So time 4 : 1

Given, $4x - x = 3$

$x = 1$

Means time taken by John $= 4 \times 1$

$= 4$ day

• add/substract efficiency.

Date: __/__/____

Time and work.

* A → 10 days
B → 15 days

(A+B) = ? days.

1 day work
↳ per day work output
↓
efficiency

Method 1

A's 1 day work = $\frac{1}{10}$

B's 1 day work = $\frac{1}{15}$

A+B = t' days

(A+B)'s 1day work = $\frac{1}{t}$

$$\frac{1}{t} = \frac{1}{10} + \frac{1}{15} = \frac{3+2}{30} = \frac{1}{6}$$

$t = 6$ days

* A → 10 days
B → 15 days

Method 2

Step 1: take 'w' as LCM of given data, i.e no of days

LCM (10,15) = 30

10,15 = 5X2, 5X3
= 5X2X3

(LCM (48,56) = 6X8, 7X8
= 6X7X8
= 336)

total work = 30 units

efficiency of A and B

A → 10 days → 3 units/day
B → 15 days → 2 units/days

(A+B) = 5 units/day.

$\frac{30}{5} = 6$ days

→ A → 12 days → 3 units
B → 18 days → 2 units

A left after 2 days
B when complete another work.

total work = LCM(12,18) = 6X2, 6X3
= 6X2X3
= 36

(A+B) = 5 units

together 2 days = 5X2 = 10 units

26 | 13
2 | 1

B will finish the remaing work in 13 days

→ A = 12 days → 10 units
B ⇒ 20 days → 6 units
C ⇒ 40 days → 3 units

A+B+C = ?

~~total work = LCM(12,20,40)~~
~~= 4X3, 4X5, 4X10~~
~~= 4X3X5X10~~
~~= 12X50~~
~~= 600~~

total work = LCM(12,20,40)
= LCM(12,40)
= 120

4X3, 4X10
4X30
120

10+6+3
16+3 = 19 units

120 days
19

→ P → 15 days
Q → 21 days

Q started and after some days P joined him and both finished the work in 14 days. After how many days did P join Q.

total work = LCM (15, 21) = 3X5, 3X7
= 3X5X7
= 35X3
= 105

efficient of P = 7 units
Q = 5 units / days

Q worked throughout for 14 days

Pay
5
70

~~5X14 + 7X2 = 14~~
~~70 + 7*x = 14~~

5X14 = 70 ~~days~~ units works

105 - 70 = 35 units

$\frac{35}{7}$ = 5 days

P joined Q in the last 5 days
(i.e after 9 days)

→ A → 20 days → 6 units
B → 24 days → 5 units
C → 40 days → 3 units

A left after 6 days

B left befor 4 days of completion

total no of days to finish work?

1st method

total work

- LCM (20, 24, 40)
= LCM (24, 40) = 120
= ~~4×6, 4×10~~
= ~~4×6×10~~
= ~~60×4 = 240.~~

A+B+C = 120 units

$6 \times 6 = 36$ units

let w = x days

$6 \times 6 + (x-4)5 + 3x = 120$

$36 + 5x - 20 + 3x = 120$

$16 + 8x = 120$

$x = \frac{104}{8}$

$x = 13$

2nd method

A+B+C = 6+5+3 = 14 units/day.

14×6 + 24 + 3×4

84 B+C 12

A+B+C C

6 days 3 days 4 days

= 13 days

→ A → 10 days

B → 18 days

how many days, if they work alternatively

i) when A starts the work

total work =
LCM (10, 18) = 90

A → 9 units
B → 5 units

1st day = 9 units
2nd day = 5 units
] every 2 days 14 units work is done

3rd day = 9 units
4th day = 5 units

14 units → 2 days
×6 ×6
14×6 = 84 units → 12 days.

till this, it doesn't matter if A starts or B starts

12 days → 84 units w done

1st case

A starts, 13th day A will come

$A = \frac{6}{9} = \frac{2}{3}$ of that day

$\therefore 12 * \frac{2}{3}$ days

$\therefore 12\frac{2}{3}$ days

(ii) when B starts the wor

2nd case

when B starts, 13th day B will come

13th day B = 5 units

work = 84+5 = 89

14th day

$A = \frac{1}{9}$ units

$\therefore 13\frac{1}{9}$ days

3X6 3X10X6X5
3X5 30x

→ A+B = 30 days → 3 units
B+C = 18 days → 5 units
C+A = $22\frac{1}{2}$ day → 4 units

1) A+B+C = ?
2) A alone = ?
3) B alone = ?
4) C alone ?

$$\text{total} = \text{LCM}\left(\frac{30}{1}, \frac{18}{1}, \frac{45}{2}\right)$$

$$= \frac{90}{1} = 90$$

$\frac{\text{LCM of numerator}}{\text{HCF of denominator}}$

6X5 = 2X3X5
6X3 = 2X3X3
9X5 = 3X3X5

1)

~~A+B+C = 3+5+4 = 2(A+B+C)~~

$2(A+B+C) = 2(3+5+4) = 24$ units/day

3X5X2
9X10=90

~~A+B+C = 12 units/days~~

$2(A+B+C) = 12$ units/days

$A+B+C = 6$ units/day

$T_{(A+B+C)} = \frac{90}{6} = 15$ days

2) A alone

$A_{eff} = (A+B+C)_{eff} - (B+C)_{eff}$

$= 6-5 = 1$ unit/day

time = 90 days

3) B alone

$B_{eff} = 6-4$

$= 2$

$= \frac{90}{2} = 45$

4) C alone

$C_{eff} = 6-3 = 3$

$= \frac{90}{3} = 30$

→ A → 15 days → 6

B → 18 days → 5

A + B + C = 6 days → 15 units/day

C → _ units

total work = LCM(15, 18, 6)

= 90 units.

$15 = 3 \times 5$

$18 = 3 \times 6$

$= 3 \times 2 \times 3$

$6 = 2 \times 3$

$3^2 \times 2 \times 5$

$9 \times 2 \times 5$

90

$90 \div 18 = 5$, $90 \div 15 = 6$, $90 \div 6 = 15$

A + B = 11 units

C alone would do 15 − 11 units/day

= 4 units/day.

$\frac{90}{42} = 2\frac{1}{2}$ days

→ A → 10 days → 6 units

B → 15 days → 4 units/day

C → 20 days → 3 units/day

A + B + C + D → 4 ~~days~~ days

total work = LCM(10, 15, 20)

= 60 units

$60 \div 4 = 15$

60 units work → 4 days

15 units work/day

D must do remaining 2 units/day

payment is as per efficiency

A : B : C : D = 6 : 4 : 3 : 2

same ratio for payment.

15 units : 15000

∴ D = ₹2000

→ Efficiency is inversely proportional to time

$$E \propto \frac{1}{\text{time}}$$

$$\frac{E_1}{E_2} = \frac{T_2}{T_1}$$

more efficiency, less time to finish the work

If one quantity increases by $\frac{n}{d}$ times other decreases by $\frac{n}{d+n}$ times

→ Ratio of efficiency of A:B = 2:3
Ratio of time taken by A:B = $\frac{1}{2} : \frac{1}{3} = 3:2$

Eff → A : B : C
2 : 3 : 4

time ratio → $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

$= \frac{1}{2} \times 12 : \frac{1}{3} \times 12 : \frac{1}{4} \times 12$

$= 6:4:3$

→ A is 20% more efficient than B. B takes 30 days to finish a work. In how many days A will finish the work.

Method 1

	A	B
Efficiency	120	100
	6	5
time ratio:	5	6
	↓	↓
	25 days	30 days

Method 2

E	T	
$\frac{1}{5}\uparrow$	$\frac{1}{6}\downarrow$	A is 1/5 times more efficient than B
		B is 1/6 times less efficient than A

$A = 30 - \frac{1}{6}$ of 30
$= 25$ days.

→ A : B

150 : 100

3 : 2 ⇒ efficiency

↓

5 units 1 unit

↓ ↓

18 days 18 × 5 = 90 days

B : 2 units 45 days

A : 3 units 30 days

→ A : B

125 : 100

5 : 4

9 units

↳ 20 days

9 units → 20 days

1 units → 20 × 9 = 180 days

A : 5 units → $\frac{180}{5}$ = 36 days

B : 4 units → $\frac{180}{4}$ = 45 days

Multiplication shortcuts.

- which are near to the power of 10

cross-addition (9-2) or (8-1)

9X -1 10^1

8 -2

7 | 2

= 72

97X -3 $10^2=100$

98 -2

95 | 06

↳ No of digits in this part is equal to the no of zero's in the base.

= 9506

96X -4 10^2

94 -6

90 | 24

= 9024

102X +2 10^2

104 +4

106 | 08

↳ base 2 zero's so add a zero

= 10608

112X +12 100

107 +7

119 | 84

= 11984

113X +13 $10^2=100$

109 +9

122 | 17

= 12317

82X -18

87 -13

69 | 234

= 7134

88X -12 $100=10^2$

87 -13

75 | 156

= 7656.

2

18X

13

54

18

234

(112-7) or (93+12)

(cross addition)

$10^2 = 100$

112 X	+12
93	−7
~~205~~ 105	−84
104 (carry 1)	16

to eliminate negative sign

1 → 100

100 − 84 = 64

= 10416

109 X	+9
96	−~~14~~ 4
105	−36
104 (carry 1)	64

= 10464

112 X	+12
89	−11
101	−132
109 (2 carry)	68

2 → 200

200 − 132 = 68

= 10968

114 X	+14
91	−9
105 (2 carry)	−126

200 − 126 = 74

10374

77 X	−23
89	−11
66	253

$10^2 = 100$

= 6853

116 x   +16
87   -13

103 | 208
100   92

= 10092

114 x   +14
86   -14

100 | -196

2 carry

98   4   = 9804

~~22 x   +28~~
~~89   +11~~

~~53 | 308~~

= ~~8608~~

72   -28
89   -11

61 | 308

= 6408

·/. base 10 → 1 carry → 10
base 100 → 1 carry → 100
base 1000 → 1 carry → 1000

1000

997 × −3
992 −8

989 | 024

= 989024

83×
62

46 |

1000

998× −2
989 −11

987 | 022

= 987022

1000

1006× +6
1013 +13

1019 | 078

= 1019078

1000

1012× +12
993 −7

1005 | −84

1 carry

1004 916

= 1004916

1000 − 84 = 916

1000

1008× +8
989 −11

997 | −88
996 912

~~1000612~~ = 996912

1000 − 88 = 912

→ If nos are close to 300, 200 or 400....

191x	-9	200
192	-8	100
183	72	
x 2		
366	72	

- 200 is not a base in powers of 10.
- take it as tempary base
- convert to any nearest proper base

= 36672

498x	-12	temp base = 200
193	-7	↓
181	84	100
x 2		
362		

= 36284

186x	-14	200
189	-11	↓
175	154	100
x 2		
350	154	

= 35154

200

212	+12	
217	+17	
229x	204	
2		
458		

= 46004

→ If nos are close to 300, 200 or 400.....

191X −9
192 −8
183 | 72
30 doubled → ×2
366 72

200 ÷2 → 100

- 200 is not a base in powers of 10.
- take it as temporary base
- convert to any nearest proper base

= 36672

688X −12
193 −7
181 | 84
×2
362

temp base = 200 ÷2 → 100

14
11
14
14
154

196
−186
14

= 36284

186X −14
189 −11
175 | 154
×2
350 154

= 35154

200 ÷2 → 100

1
12
17
84
12
204

200

212 +12
217 +17
229x | 204
2
458

= 46004

12x → 84

200

219x +19
212 +12
--- ---
231x 228
2

462

= 46428

200
÷990
52

48

213x +13
196 −4
--- ---
209x −52
2

418

417 48

= 41748

291 −9
308 +8
--- ---
299x −72
3
---
897 ← carry

= 89628

300

60 160

63x +3
71 +11
--- ---
274x 33
6

444

= 4473

apsara

↗ temp base - 80

80 | 8

→ base 10.

83x +3
62 −18
4 65x | −54
8
520 ~~54~~ 60-54
6 carry 6

514 ~~46~~ 6

= ~~5236~~ = 5146

5146

19x
12
38
19
228

119x +19 600
88 −12
107 | −2128
9 carry

104 72

= 10472

39
400
328
72

* Percentage Approximation.

$\frac{3437}{40211} = ?$

- 648
  - 10% = 64.8
  - 5% = 32.4
  - 1% = 6.48
  - 15% = 10% + 5% = 97.2%
  - 25% = $\frac{1}{4}$ of a number = 162
  - 75% = 25% × 3 (or) 100% − 25%
    = 162 × 3
    = 486

- 16% of 1234
  ↓
  10% + 5% + 1%

  10% of 1234 = 123.4
  5% = 61.7
  1% = 12.34 = 12.3

  123.4 + 61.7 + 12.3
  = 197.4

- $\frac{1}{1} = 100\%$
  $\frac{1}{2} = 50\%$
  $\frac{1}{3} = 33.33\%$
  $\frac{1}{4} = 25\%$
  $\frac{1}{5} = 20\%$
  $\frac{1}{6} = 16.66\%$
  $\frac{1}{7} = 14.28\%$
  $\frac{1}{8} = 12.5\%$

. 16% of 1234

$\frac{206}{1234}$

almost $\frac{1}{6}$ of 1234

$= 206$

(a) 187 (b) 197 (c) 207 (d) 217

. 27% of 864

↓

25% + 2%

$25\% = \frac{1}{4} \times 864 = 216$

1% = 8.64

2% = 17.28

$216 + 17.28 = 233.2$

. 166.66% of 3309

$66.66\% = \frac{2}{3}$

100% + 66.66%

$33.09 + \frac{2}{3} \times 3309$

~~33.09 + 2206~~

$1 + \frac{2}{3}$

$= \frac{5}{3} \times 3309 = 5515$

100% is the number

- 118% of 2346

100% + 18%

100% + 20% - 2%

2346 + 469.2 − 46.92

= 2346 + 469.2 − 46.9

= ~~2371~~ 2862.1

- 39% of 4662

50% − (18% + 1%)

2331 − (466.2 + 46.62)

= 1818.18

* $\frac{233}{864}$

(numerator is what percent of denominator.)

233 is what percent of 864?

→ around $\frac{1}{4}$th = 25%

$25\% = \frac{1}{4} \times 864 = 216 + 17$

↓ 2%

1% of 864 = 8.64

~~19% of 864~~

= 27%

= 0.27

$\frac{1}{1} = 100\%$

$\frac{1}{2} = 50\%$

$\frac{1}{3} = 33.33\%$

$\frac{1}{4} = 25\%$

$\frac{1}{5} = 20\%$

$\frac{1}{6} = 16.66\%$

apsara
Date

* $\frac{414}{869}$

414 is what percent of 869 ?

∴ it's around half.

(50%) (2-3·1.)

$\frac{869}{2} = 434·5 - 20 = 47·7\%$

↓ no is greater

so it will be less than 50%.

(a) 47·7%. (b) 51%. (c) 56%. (d) 59·3%

1% = 8·69 ≈ 8·7 | 0·1 = 0·87 ≈ 0·9

2% = 17·4

20 (or) be 17·4+2·6 — 2·6 = 0·3%.

17·4 + 2·6 = 20

6 13
373
324
49

2
373×
3
1119

2 1
373×4
1492

* $\frac{373}{1296}$

around $\frac{1}{4}$th of denominator.

(25%) 1296 | 324 = 324

324 + 49

↓

50% − 1%

648 − 12·96

635·04

around $\frac{1}{3}$rd

432
+1296
31

648
12·96
635·04

25% + 3·8%

= 28·8%

= 0·288

≈ 0·29

1% = 12·96 = 13 | 0·1% = 1·296 = 1·3

4% = 52

− 0·2% = −3

52 − 3 = 49

4% − 0·2%

3·8% = 49

Percentages → means per hundred.

* fraction to percentage conversion.

$\frac{1}{1} = 100\%$ $\quad \frac{1}{1} \times 100 = 100\%$

$\frac{1}{2} = 50\%$ $\quad \frac{1}{2} \times 100 = 50\%$

$\frac{1}{3} = 33.33\%$ $\quad \frac{1}{3} \times 100 = 33.33\%$

$\frac{1}{4} = 25\%$

$\frac{1}{5} = 20\%$

$\frac{1}{6} = 16.66\%$

$\frac{1}{7} = 14.28\%$ $\quad \frac{1}{7} \times 100 = 14 + \frac{2}{7} = 14\frac{2}{7}$

7×2 → 14×2

$\frac{1}{8} = 12.5\%$

$\frac{1}{9} = 11.11\%$

$\frac{1}{10} = 10\%$

$\frac{1}{11} = 9.09\%$ $\quad \frac{1}{11} \times 100 = 9 + \frac{1}{11} = 9\frac{1}{11}$

$\frac{1}{12} = 8.33\%$

$\frac{1}{13} = 7.7\%$

$\frac{1}{14} = 7.14\%$

$\frac{1}{15} = 6.66\%$ $\quad \frac{1}{15} = \frac{1}{3}$rd of $\frac{1}{5}$

$\frac{1}{16}$ = 6.25 %

$\frac{1}{17}$ = 5.88 %

$\frac{1}{18}$ = 5.55 %

$\frac{1}{19}$ = 5.26 %

1/2 of 1/18

$\frac{1}{19} \times 100 = 5.88\%$

half of 1/19

$\frac{1}{19} \times 100 = 5 + \frac{5}{19} = 0.26$

$\left(\frac{1}{2} = \frac{1}{4} = 25\%\right)$

* $\frac{1}{23} \times 100 = 4 + \frac{8}{23} = 0.34 + 4 = 4.34\%$

$\frac{8}{24} = \frac{1}{3} = 0.33$

* 37.5 % of 640

$\frac{3}{8} \times 640 = 240$

$\frac{1}{8} = 12.5\%$

$\frac{3}{8} = 37.5\%$ → 3 × 12.5

$\frac{5}{8} = 62.5\%$ → 5 × 12.5

$\frac{7}{8} = 87.5\%$

* 18.75 % of 176

18.75 % = $\frac{3}{16}$ | $\frac{1}{16} = 6.25\%$

× 3

* 83.33% of 606

$\frac{1}{12} = 8.33$

$\frac{10}{12} = 83.33\%$

$\therefore \frac{5}{6} = 83.33\%$

83.33%

50% + 33.33%

$\frac{1}{2} + \frac{1}{3}$

$= \frac{5}{6}$

$\frac{5}{6} \times 606 =$

* 43.75 % 352

$43.75\% = 50\% - 6.25\%$

$= \frac{1}{2} - \frac{1}{16}$

$= 7/16 \times 352 = 154$

* $56.25\% = 50\% + 6.25\%$

$= 1/2 + 1/16$

$= \frac{9}{16}$

* Multiplying factor:

↳ By what factor should we multiply?

* 840 decrease 87.5%

a number $x \xrightarrow{20\%\downarrow} x - 20\%$ of x

$= x - \frac{1}{5}$ of $x \Rightarrow x(1 - 1/5) = \frac{4}{5}x$

$= x \times (4/5)$ → multiplying factor

$\frac{100}{87}$
13

* $840 \xrightarrow{87.5\%\downarrow} ?$

$\therefore x \xrightarrow{87.5\%\downarrow} x - \frac{7}{8}$ of x

$= x \times \frac{1}{8}$

$= 840 \times \frac{1}{8} = 105$

$100\% - 12.5\%$

$12.5 \times 7 = 87.5$

* $x \xrightarrow{25\%\uparrow} x + \frac{1}{4}$ of x

$x\left(1 + \frac{1}{4}\right) = x \times \frac{5}{4}$

* Multiplying factor
  - increase x + fraction
  - decrease x − fraction

* $648 \xrightarrow{37.5\%\downarrow}$

$x - \frac{3}{8}x = \frac{5}{8}x$

$= \frac{5}{8} \times 648 = 405$

$12.5 \times 3 = 37.5$

$6.25 \times 3 = 18.75$

* $840 \xrightarrow{87.5\%\downarrow} ?$

$\therefore x - \frac{7}{8}x = \frac{1}{8}x$

$\frac{1}{8} \times 840 = 105$

$12.5 \times 7 = 87.5$

$\frac{1}{6} = 16.66\%$

$16\frac{2}{3}\%$

83.33%

$1212 \times \frac{11}{6}$

$1212 \times \frac{11}{6} = 2222.$

★ Successive ~~discount~~ Increase/Decrease :

- Successive discount → 60% + 20% ≠ 80%

discount of

$100\% \xrightarrow{-60} 40 \xrightarrow{-8} 32$

68%

- successive discount/decrease of 60% and 20% = 68%

- successive increase of 20%, 10%, 5% = ?

$100\% \xrightarrow[+20]{20\%\uparrow} 120 \xrightarrow[+12]{10\%\uparrow} 132 \xrightarrow[6.6]{5\%\uparrow} 138.6$

38.6% ↑

- $100\% \xrightarrow[+5]{5\%\uparrow} 105 \xrightarrow[+21]{20\%\uparrow} 126 \xrightarrow[+12.6]{10\%} 138.6$

38.6%

∴ order isn't important.

good to start in decending order for easier calculation.

* A number is increased by 18.75 % and decreased by 12.5 %, what is the percentage change?

$100 \xrightarrow{18.75\uparrow} 118.75 \xrightarrow{12.5\downarrow} \times$

$12.5\% = \frac{1}{8}$

$18.75 = \frac{3}{16}$

118.75×12.5

198

$118.75 \times \frac{1}{8}$

Multiplying factors help when there's fraction value.

Soln.

$1 \xrightarrow{3/16\uparrow} 1 \times \frac{19}{16} \xrightarrow{1/8\downarrow} \frac{19}{16} \times \frac{7}{8} = \frac{133}{128}$

$1 + \frac{3}{16} = \frac{19}{16}$

$\frac{19}{16} \quad \frac{7}{8}$

$\frac{19 - 14}{16} = \frac{15}{16}$

$\frac{9}{16} \quad \frac{1}{8} = \frac{9}{16}$

$1 - \frac{1}{8} = \frac{7}{8}$

a number from 1 changes to $\frac{133}{128}$

$1 \rightarrow \frac{133}{128}$

Diff-cat formula.

$$\% \text{ change} = \frac{\text{change}}{\text{initial value}} \times 100$$

Consider 1 as $\frac{128}{128}$

$\frac{133}{128} - \frac{128}{128} = \frac{5}{128}$

$$\frac{\frac{133}{128} - 1}{1} \times 100 = \frac{5}{128} \times 100$$

$0.1\% = 0.128$

$1\% = 1.28$

$4\% = 5.12$

$\frac{16}{\frac{48}{3}} = 16$ (33.33%) 2% $\frac{1}{3}$ rd of 48 is 17

$16 * 1 = 16$

$= 35.33\%$

* A number is first increased by 166.66%, decreased by 37.5% and then decreased by 18.75%. Find % change in the number.

$1 \xrightarrow{(5/3)\ 166.66\%\uparrow} 1 \times \frac{8}{3} \xrightarrow{(3/8)\ 37.5\downarrow} \frac{8}{3} \times \frac{5}{8} \xrightarrow{(13/16)\ 18.75\downarrow} \frac{8}{3} \times \frac{5}{8} \times \frac{13}{16}$

$= \frac{65}{48}$

$\frac{1}{3} = 33.33\%$

$\frac{2}{3} = 66.66\%$

% change

$\frac{65}{48} - \frac{48}{48} = \frac{17}{48} = 35.33\%$

$100 + 66.66\%$

$1 + \frac{2}{3}$

$\frac{5}{3}$

$1 - \frac{3}{8}$

$\frac{5}{8}$

* 4320 decreased successively 3 times by 16.66% (1/6)

$1 - \frac{1}{6} = \frac{5}{6}$

$1 + \frac{5}{3}$

$\frac{8}{3}$ $1 - \frac{3}{8}$

$4320 \xrightarrow{16.66\%\downarrow} \xrightarrow{16.66\%\downarrow} \xrightarrow{16.66\%\downarrow}$

$4320 \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = 2500.$

$12.5 \times 3 = 37.5$

$\frac{3}{8}$

$1 - \frac{3}{16}$

$\frac{13}{16}$.

* Percentage increase/decrease:

$\%\ \text{change} = \frac{\text{change}}{\text{initial value}} \times 100.$

* If a no is changed from 70 to 80

$\frac{10}{70} \times 100 = \frac{1}{7}$ increase.

$\frac{10}{70} \times 100 = \frac{1}{7}\uparrow = 14.28\%$

70 80

$\frac{10}{80} \times 100 = \frac{1}{8}\downarrow = 12.5\%$

$\frac{3}{13}$

$3 \times 4 = 12$

$\frac{13}{4} = 3.25$

(25%) (2%)

$3.25 - 0.25$

$= 28\%$

$1\% = 0.13$

$2\% 0.26$

$\frac{30}{130} = \frac{3}{13}\uparrow = 23\%$

130 160

$\frac{30}{160} = \frac{3}{16}\downarrow = 18.75\%$

$4.00 - 3.25 = 0.75$

$\frac{13}{4}$ 12

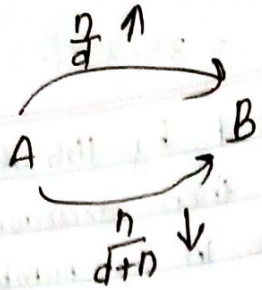
3.25

$1\% \quad 1.3 \quad 8.13$

$20\% +$

$2\% = 2.6$

※



ex: $\frac{3}{13}\uparrow$ $\frac{3}{16}\downarrow$ in previous qn.

* A is 40% more than B. By what percent B is less than A.

1st method

B = 100

B → A
100 → 140

$\frac{40}{140} \times 100 = 28.56\%$

2nd method

40% = 2/5

B → A

$\frac{2}{5+2} = \frac{2}{7}$

$\frac{2}{7} \times 100 = 28.56\%$

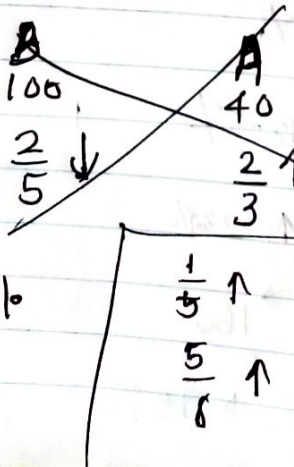
* A's salary is 40% less than B. By what percent B's more than A.

A 140 → 100 B

$\frac{2}{5}\downarrow$ (40%)

$\frac{2}{3}\uparrow$

$= 66.66\%$



$\frac{n}{d+n} = \frac{2}{5}$

$\frac{n}{d} = \frac{2}{5-2} = \frac{2}{3}$

* If A is $\frac{3}{8}$ less than B. By how much B is more than A.

A	B
$\frac{3}{8}\downarrow$	$\frac{3}{5}\uparrow$
$\frac{2}{9}\downarrow$	$\frac{2}{7}\uparrow$

* Quantity 1 Quantity 2

(applied in all the inverse ratios)

$\frac{n}{d}\uparrow$ $\frac{n}{d+n}\downarrow$

I) Expenditure = Price x consumption.

. Price of petrol is 60/ℓ, consuming 10 litres., increased by 25%

exp = 60x10 = 600.

$\frac{1}{4}\times 60^{15}$

₹75 petrol.

price increases by 25% ($\frac{1}{4}\uparrow$)
consumption decreased by 20% ($\frac{1}{5}\downarrow$)

₹600 = 75x x

$x = \frac{600}{75}$

= 8 litres.

* If expenditure is constant

K = price x consumption

$P = \frac{K}{C}$

$P \propto \frac{1}{C}$

* If price of sugar is increased by 30%. By how much %age, the consumption should be reduced to maintain constant expenditure.

exp = constant

$$p \propto \frac{1}{c}$$

P ↑ C ↓

$\frac{n}{d}$ ↑ $\frac{n}{d+n}$ ↓

$\frac{3}{10}$ ↑ $\frac{3}{13}$ ↓ = 23.1% decreased consumption

CAT 2020 Examination Paper

CAT 2020 SLOT 1

1. How many 3-digit numbers are there, for which the product of their digits is more than 2 but less than 7?

2. The mean of all 4-digit even natural numbers of the form 'aabb', where $a > 0$, is

   (a) 5544 (b) 4466

   (c) 4864 (d) 5050

3. If $f(5 + x) = f(5-x)$ for every real x, and $f(x) = 0$ has four distinct real roots, then the sum of these roots is

   (a) 20 (b) 0

   (c) 40 (d) 10

4. A circle is inscribed in a rhombus with diagonals 12 cm and 16 cm. The ratio of the area of circle to the area of rhombus is

   (a) $\frac{2\pi}{15}$ (b) $\frac{5\pi}{18}$

   (c) $\frac{3\pi}{25}$ (d) $\frac{6\pi}{25}$

5. The area of the region satisfying the inequalities $|x| - y \le 1, y \ge 0$ and $y \le 1$ is

6. If $x = (4096)^{7+4\sqrt{3}}$, then which of the following equals 64?

   (a) $\frac{\frac{7}{x^2}}{\frac{4}{x^{\sqrt{3}}}}$ (b) $\frac{\frac{7}{x^2}}{x^{2\sqrt{3}}}$

   (c) $\frac{x^7}{x^{4\sqrt{3}}}$ (d) $\frac{x^7}{x^{2\sqrt{3}}}$

7. Let A, B and C be three positive integers such that the sum of A and the mean of B and C is 5. In addition, the sum of B and the mean of A and C is 7. Then the sum of A and B is

   (a) 5 (b) 4

   (c) 7 (d) 6

8. A straight road connects points A and B. Car 1 travels from A to B and Car 2 travels from B to A, both leaving at the same time. After meeting each other, they take 45 minutes and 20 minutes, respectively, to complete their journeys. If Car 1 travels at the speed of 60 km/hr, then the speed of Car 2, in km/hr, is

   (a) 100 (b) 80

   (c) 70 (d) 90

9. Leaving home at the same time, Amal reaches office at 10:15 am if he travels at 8 km/hr, and at 9:40 am if he travels at 15 km/hr. Leaving home at 9:10 am, at what speed, in km/hr, must he travel so as to reach office exactly at 10 am?

   (a) 11 (b) 12

   (c) 14 (d) 13

10. A solid right circular cone of height 27 cm is cut into two piece along a plane parallel to its base a height of 18 cm from the base. If the difference in volume of the two pieces is 225 cc, the volume, in cc, of the original cone is

    (a) 256 (b) 232

    (c) 243 (d) 264

11. Among 100 students, x_1 have birthdays in January, x_2 have birthdays in February, and so on. If $x_0 = \max(x_1, x_2, \ldots x_{12})$, then the smallest possible value of x_0 is

    (a) 10 (b) 9

    (c) 12 (d) 8

12. An alloy is prepared by mixing three metals A, B and C in the proportion 3 : 4 : 7 by volume, Weights of the same volume of the metals A, B and C are in the ratio 5 : 2 : 6. In 130 kg of the alloy, the weight, in kg of the metal C is

    (a) 48 (b) 70

    (c) 96 (d) 84

13. On a rectangular metal sheet of area 135 sq in, a circle is painted such that the circle touches two opposite sides. If the area of the sheet left unpainted is two–thirds of the painted area then the perimeter of the rectangle in inches is

    (a) $3\sqrt{\pi}\left(5+\frac{12}{\pi}\right)$ (b) $5\sqrt{\pi}\left(3+\frac{9}{\pi}\right)$

    (c) $3\sqrt{\pi}\left(\frac{5}{2}+\frac{6}{\pi}\right)$ (d) $4\sqrt{\pi}\left(3+\frac{9}{\pi}\right)$

14. If a, b and c are positive integers such that $ab = 432$, $bc = 96$ and $c < 9$, then the smallest possible value of $a + b + c$ is
(a) 49 (b) 46
(c) 59 (d) 56

15. If y is a negative number such that $2^{y^2 \log_3 5} = 5^{\log_2 3}$, then y equals
(a) $-\log_2 (1/5)$ (b) $-\log_2 (1/3)$
(b) $\log_2 (1/3)$ (d) $\log_2 (1/5)$

16. Veeru invested ₹10000 at 5% simple annual interest, and exactly after two years, Joy invested ₹8000 at 10% simple annual interest. How many years after Veeru's investment, will their balances, i.e., principal plus accumulated interest, be equal?

17. A solution, of volume 40 litres, has dye and water in the proportion 2 : 3. Water is added to the solution to change this proportion to 2 : 5. If one-fourths of this diluted solution is taken out, how many litres of dye must be added to the remaining solution to bring the proportion back to 2 : 3?

18. A gentleman decided to treat a few children in the following manner. He gives half of his total stock of toffees and one extra to the first child, and then the half of the remaining stock along with one extra to the second and continues giving away in this fashion.
His total stock exhausts after he takes care of 5 children. How many toffees were there in his stock initially?

19. Two persons are walking beside a railway track at respective speeds of 2 and 4 km per hour in the same direction. A train came from behind them and crossed them in 90 and 100 seconds, respectively. The time, in seconds, taken by the train to cross an electric post is nearest to
(a) 82 (b) 87
(c) 78 (d) 75

20. In a group of people, 28% of the members are young while the rest are old. If 65% of the members are literates, and 25% of the literates are young, then the percentage of old people among the illiterates is nearest to
(a) 66 (b) 55
(c) 59 (d) 62

21. The number of real-valued solutions of the equation $2^x + 2^{-x} = 2 - (x - 2)^2$ is
(a) 2 (b) infinite
(c) 0 (d) 1

22. If $\log_4 5 = (\log_4 y)(\log_6 \sqrt{5})$, then y equals

23. A train travelled at one-thirds of its usual speed, and hence reached the destination 30 minutes after the scheduled time. On its return journey, the train initially travelled at its usual speed for 5 minutes but then stopped for 4 minutes for an emergency. The percentage by which the train must now increase its usual speed so as to reach the destination at the scheduled time, is nearest to
(a) 67 (b) 61
(c) 50 (d) 58

24. The number of distinct real roots of the equation $\left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right) + 2 = 0$ equals

25. A person spent ₹50000 to purchase a desktop computer and a laptop computer. He sold the desktop at 20% profit and the laptop at 10% loss. If overall he made a 2% profit then the purchase price, in rupees, of the desktop is

26. How many distinct positive integer-valued solutions exist to the equation $(x^2 - 7x + 11)^{(x^2 - 13x + 42)} = 1$?
(a) 6 (b) 2
(c) 8 (d) 4

CAT 2020 SLOT 2

1. Let C be a circle of radius 5 meters having center at O. Let PQ be a chord of C that passes through points A and B where A is located 4 meters north of O and B is located 3 meters east of O. Then, the length of PQ, in meters, is nearest to
(a) 7.8 (b) 7.2
(c) 8.8 (d) 6.6

2. In a car race, car A beats car B by 45 km, Car B beats Car C by 50 kms and car A beats car C by 90 km. The distance (in km) over which the race has been conducted is
(a) 500 (b) 450
(c) 475 (d) 550

3. The distance from B to C is thrice that from A to B. Two trains travel from A to C via B. The speed of train 2 is double that of train 1 while traveling from A to B and their speeds are interchanged while traveling from B to C. The ratio of the time taken by train 1 to that taken by train 2 in travelling from A to C is
(a) 4:1 (b) 5:7
(c) 7:5 (d) 1:4

4. The number of integers that satisfy the equality $(x^2 - 5x + 7)^{x+1} = 1$ is

(a) 3 (b) 5
(c) 2 (d) 4

5. John takes twice as much time as Jack to finish a job. Jack and Jim together take one-thirds of the time to finish the job than John takes working alone. Moreover, in order to finish the job, John takes three days more than that taken by three of them working together. In how many days will Jim finish the job working alone?

6. Students in a college have to choose at least two subjects from chemistry, mathematics and physics. The number of students choosing all three subjects is 18, choosing mathematics as one of their subjects is 23 and choosing physics as one of their subjects is 25. The smallest possible number of students who could choose chemistry as one of their subjects is
(a) 19 (b) 21
(c) 20 (d) 22

7. If x and y are positive real number satisfying $x + y = 102$, then the minimum possible value of $\left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)$ is

8. The sum of the perimeters of an equilateral triangle and a rectangle is 90 cm. The area, T, of the triangle and the area, R, of the rectangle, both in sq cm, satisfy the relationship $R = T^2$. If the sides of the rectangle are in the ratio 1:3, then the length, in cm, of the longer side of the rectangle is
(a) 18 (b) 24
(c) 21 (d) 27

9. A and B are two points on a straight line. Ram runs from A to B while Rahim runs from B to A. After crossing each other, Ram and Rahim reach their destinations in one minute and four minutes, respectively. If they start at the same time, then the ratio of Ram's speed to Rahim's speed is
(a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) $\frac{1}{2}$ (d) 2

10. In a group of 10 students, the mean of the lowest 9 scores is 42 while the mean of the highest 9 scores is 47. For the entire group of 10 students, the maximum possible mean exceeds the minimum possible mean by
(a) 3 (b) 6
(c) 4 (d) 5

11. Anil buys 12 toys and labels each with the same selling price. He sells 8 toys initially at 20% discount on the labeled price. Then he sells the remaining 4 toys at an additional 25% discount on the discounted price. Thus, he gets a total of ₹2112, and makes a 10% profit. With no discounts, his percentage of profit would have been
(a) 60 (b) 55
(c) 50 (d) 54

12. Let the mth and nth terms of a geometric progression be $\frac{3}{4}$ and 12, respectively, where $m < n$. If the common ratio of the progression is an integer r, then smallest possible value of $r + n - m$ is
(a) 2 (b) –2
(c) –4 (d) 6

13. For the same principal amount, the compound interest for two years at 5% per annum exceeds the simple interest for three years at 3% per annum by ₹1125. Then the principal amount in rupees is

14. For real x, the maximum possible value of $\frac{x}{\sqrt{1+x^4}}$ is
(a) $\frac{1}{\sqrt{2}}$ (b) 1
(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$

15. Let C1 and C2 be concentric circles such that the diameter of C1 is 2 cm longer than that of C2. If a chord of C1 has length 6cm and is tangent to C2, then the diameter, in cm, of C1 is

16. A sum of money is split among Amal, Sunil and Mita so that the ratio of the shares of Amal and Sunil is 3 : 2, while the ratio of the shares of Sunil and Mita is 4 : 5. If the difference between the largest and the smallest of these three shares is ₹400, then Sunil's share, in rupees, is

17. The value of $\log_a\left(\frac{a}{b}\right)+\log_b\left(\frac{b}{a}\right)$, for $1 < a \le b$ cannot be equal to
(a) –0.5 (b) 1
(c) 0 (d) –1

18. From an interior point of an equilateral, triangle, perpendiculars are drawn on all three sides. The sum of the lengths of the three perpendiculars is s. Then the area of the triangle is
(a) $\frac{2s^2}{\sqrt{3}}$ (b) $\frac{s^2}{\sqrt{3}}$
(c) $\frac{s^2}{2\sqrt{3}}$ (d) $\frac{\sqrt{3}s^2}{2}$

19. If x and y are non-negative integers such that $x + 9 = z$, $y + 1 = z$ and $x + y < z + 5$, then the maximum possible value of $2x + y$ equals

20. In how many ways can a pair of integers (x, a) be chosen such that $x^2 - 2|x| + |a - 2| = 0$?

(a) 4 (b) 6
(c) 7 (d) 5

21. How many 4-digit numbers, each greater than 1000 and each having all four digits distinct, are there with 7 coming before 3?

22. Aron bought some pencils and sharpeners. Spending the same amount of money as Aron, Aditya bought twice as many pencils and 10 less sharpeners. If the cost of one sharpener is ₹2 more than the cost of a pencil, then the minimum possible number of pencils bought by Aron and Aditya together is

(a) 27 (b) 36
(c) 33 (d) 30

23. Two circular tracks T1 and T2 of radii 100 m and 20 m, respectively touch at a point A. Starting from A at the same time, Ram and Rahim are walking on track T1 and track T2 at speeds 15 km/hr and 5 km/hr respectively. The number of full rounds that Ram will make before he meets Rahim again for the first time is

(a) 2 (b) 5
(c) 3 (d) 4

24. The number of pairs of integers (x, y) satisfying $x \geq y \geq -20$ and $2x + 5y = 99$ is

25. In May, John bought the same amount of rice and the same amount of wheat as he had bought in April, but spent ₹150 more due to price increase of rice and wheat by 20% and 12%, respectively. If John had spent ₹450 on rice in April, then how much did he spend on wheat in May?

(a) ₹580 (b) ₹590
(c) ₹560 (d) ₹570

26. Let $f(x) = x^2 + ax + b$ and $g(x) = f(x + 1) - f(x - 1)$. If $f(x) \geq 0$ for all real x, and $g(20) = 72$ then the smallest possible value of b is

(a) 1 (b) 16
(c) 4 (d) 0

CAT 2020 SLOT 3

1. Let k be a constant. The equations $kx + y = 3$ and $4x + ky = 4$ have a unique solution if and only if

(a) $|k| = 2$ (b) $k \neq 2$
(c) $k = 2$ (d) $|k| \neq 2$

2. Anil, Sunil, and Ravi run along a circular path of length 3 km, starting from the same point at the same time, and going in the clockwise direction. If they run at speeds of 15 km/hr, 10 km/hr, and 8 km/hr, respectively, how much distance in km will Ravi have run when Anil and Sunil meet again for the first time at the starting point?

(a) 4.6 (b) 5.2
(c) 4.8 (d) 4.2

3. How many of the integers 1, 2, ..., 120 are divisible by none of 2, 5 and 7?

(a) 43 (b) 40
(c) 41 (d) 42

4. A and B are two railway stations 90 km apart. A train leaves A at 9:00 am, heading towards B at a speed of 40 km/hr. Another train leaves B at 10:30 am, heading towards A at a speed of 20 km/hr. The trains meet each other at

(a) 11:20 am (b) 11:00 am
(c) 10:45 am (d) 11:45 am

5. A man buys 35 kg of sugar and sets a marked price in order to make a 20% profit. He sells 5 kg at this price, and 15 kg at a 10% discount. Accidentally, 3 kg of sugar is wasted. He sells the remaining sugar by raising the marked price by p percent so as to make an overall profit of 15%. Then p is nearest to

(a) 35 (b) 25
(c) 31 (d) 22

6. In a trapezium ABCD, AB is parallel to DC, BC is perpendicular to DC and $\angle BAD = 45°$. If DC = 5 cm, BC = 4 cm, the area of the trapezium in sq cm is

7. If $x_1 = -1$ and $x_m = x_{m-1} + (m + 1)$ for every positive integer m, then x_{100} equals

(a) –5150 (b) –5051
(c) –5050 (d) –5151

8. $\dfrac{2\times4\times8\times16}{(\log_2 4)^2(\log_4 8)^3(\log_8 16)^4}$ equals

9. The points (2, 1) and (–3, –4) are opposite vertices of a parallelogram. If the other two vertices lie on the line $x + 9y + c = 0$ then c is

(a) 13 (b) 14
(c) 12 (d) 15

10. If a, b, c are non-zero and $14^a = 36^b = 84^c$, then $6b\left(\frac{1}{c} - \frac{1}{a}\right)$ is equal to

11. Let N, x and y be positive integers such that $N = x + y$, $2 < x < 10$ and $14 < y < 23$. If $N > 25$, then how many distinct values are possible for N?

12. A person invested a certain amount of money at 10% annual interest, compounded half-yearly. After one and a half years, the interest and principal together became ₹18522. The amount, in rupees, that the person had invested is

13. Dick is thrice as old as Tom and Harry is twice as old as Dick. If Dick's age is 1 year less than the average age of all three, then Harry's age, in years, is

14. The vertices of a triangle are (0, 0), (4, 0) and (3, 9). The area of the circle passing through these three points is

(a) $\frac{12\pi}{5}$ (b) $\frac{205\pi}{9}$

(c) $\frac{123\pi}{7}$ (d) $\frac{14\pi}{3}$

15. Vimla starts for office every day at 9 am and reaches exactly on time if she drives at her usual speed of 40 km/hr. She is late by 6 minutes if she drives at 35 km/hr. One day, she covers two-thirds of her distance to office in one-thirds of her usual time to reach office, and then stops for 8 minutes. The speed, in km/hr, at which she should drive the remaining distance to reach office exactly on time is

(a) 26 (b) 27

(c) 28 (d) 29

16. How many integers in the set {100, 101, 102, ..., 999} have at least one digit repeated?

17. The area, in sq. units, enclosed by the lines $x = 2$, $y = |x - 2| + 4$ the x-axis and the y-axis is equal to

(a) 8 (b) 6

(c) 12 (d) 10

18. In the final examination, Bishnu scored 52% and Asha scored 64%. The marks obtained by Bishnu is 23 less, and that by Asha is 34 more than the marks obtained by Ramesh. The marks obtained by Geeta, who scored 84%, is

(a) 417 (b) 399

(c) 439 (d) 357

19. A contractor agreed to construct a 6 km road in 200 days. He employed 140 persons for the work. After 60 days, he realized that only 1.5 km road has been completed. How many additional people would he need to employ in order to finish the work exactly on time?

20. Let m and n be positive integers, If $x^2 + mx + 2n = 0$ and $x^2 + 2nx + m = 0$ have real roots, then the smallest possible value of $m + n$ is

(a) 8 (b) 7

(c) 6 (d) 5

21. If $f(x + y) = f(x) f(y)$ and $f(5) = 4$, then $f(10) - f(-10)$ is equal to

(a) 0 (b) 3

(c) 15.9375 (d) 14.0625

22. How many pairs (a, b) of positive integers are there such that $a \le b$ and $ab = 4^{2017}$?

(a) 2020 (b) 2019

(c) 2017 (d) 2018

23. Two alcohol solutions, A and B, are mixed in the proportion 1 : 3 by volume. The volume of the mixture is then doubled by adding solution A such that the resulting mixture has 72% alcohol. If solution A has 60% alcohol, then the percentage of alcohol in solution B is

(a) 92% (b) 94%

(c) 90% (d) 89%

24. Let $\log_a 30 = A, \log_a \frac{5}{3} = -B$ and $\log_2 a = \frac{1}{3}$, then $\log_3 a$ equals

(a) $\frac{2}{A+B-3}$ (b) $\frac{A+B-3}{2}$

(c) $\frac{A+B}{2} - 3$ (d) $\frac{2}{A+B} - 3$

25. Let m and n be natural numbers such that n is even and $0.2 < \frac{m}{20}, \frac{n}{m}, \frac{n}{11} < 0.5$. Then $m - 2n$ equals

(a) 3 (b) 4

(c) 1 (d) 2

26. A batsman played $n + 2$ innings and got out on all occasions. His average score in these $n + 2$ innings was 29 runs and he scored 38 and 15 runs in the last two innings. The batsman scored less than 38 runs in each of the first n innings. In these n innings, his average score was 30 runs and lowest score was x runs. The smallest possible value of x is

(a) 1 (b) 4

(c) 2 (d) 3

ANSWER KEY

CAT 2020 SLOT 1

1. 21	2. (a)	3. (a)	4. (d)
5. 3	6. (b)	7. (d)	8. (d)
9. (b)	10. (c)	11. (b)	12. (d)
13. (a)	14. (b)	15. (c)	16. 12 Years
17. 8	18. 62	19. (a)	20. (a)
21. (c)	22. 36	23. (a)	24. 1
25. 20000	26. (a)		

CAT 2020 SLOT 2

1. (c)	2. (b)	3. (b)	4. (a)
5. 4	6. (c)	7. 2704	8. (d)
9. (d)	10. (c)	11. (c)	12. (b)
13. 90000	14. (a)	15. 10	16. 800
17. (b)	18. (b)	19. 23	20. (c)
21. 315	22. (c)	23. (c)	24. 17
25. (c)	26. (c)		

CAT 2020 SLOT 3

1. (d)	2. (c)	3. (c)	4. (b)
5. (b)	6. 28	7. (c)	8. 24
9. (b)	10. 3	11. 6	12. 16000
13. 18 Years	14. (b)	15. (c)	16. 252
17. (d)	18. (b)	19. 40 Men	20. (c)
21. (c)	22. (d)	23. (a)	24. (a)
25. (c)	26. (c)		

SOLUTIONS AND SHORTCUTS

CAT 2020 SLOT 1

1. We are looking for 3 digit numbers, whose product of digits is either 3,4,5 or 6. For product to be 3, the number has to be formed using the digits 1,1,3 $\Rightarrow$ 3 numbers (113,131 and 311); For product = 4, the digits used should be 2,2,1 or 4,1,1. There will be a total of 3 + 3 = 6 such numbers. For product to be 5, the number has to be formed using the digits 1,1,5 $\Rightarrow$ 3 numbers (115,151 and 511). For product = 6, the digits used should be 6,1,1 or 1,2,3. There will be a total of 3 + 6 = 9 such numbers. Total 3 + 6 + 3 + 9 = 21 numbers.
2. The numbers will be 1100, 1122, 1144, 1166, 1188, 2200, 2222, ...9988. There will be a total of 100 numbers of this type. The average value of the last two digits will be the average of 00, 22, 44, 66 and 88, which is 44 (since these numbers will appear an equal number of times in the last two digits of the numbers formed). The average of the first two digits of the number would be the average of 11,22,33,44,55,..99 $\Rightarrow$ which is 55. Hence, the correct answer is 5544. Option (a) is correct.
3. Since the value of $f(5 + x)$ and $f(5 - x)$ are equal, it means that if a root exists at $f(6)$, it will also exist at $f(4)$. This is because the function can be thought of as $f(6) = f(4)$ when $x = 1$; $f(7) = f(3)$ when $x = 2$; $f(8) = f(4)$ when $x = 3$. Further, the function will also equate itself for decimal values of x. For instance, at $x = 1.5$, we can see that $f(6.5) = f(3.5)$. Thus, whenever we discover a root at $f(5 + x)$, there is also a root at $f(5 - x)$. This means that each pair of roots will have their arguments adding to 10. Since, there are a total of 4 roots, it means that there are two pairs of roots. Thus, the sum of the roots is 10 + 10 = 20
4. In the figure, AB = 10, since it is the hypotenuse of the right triangle AOB, with AO = 6 and OB = 8. (Note: AO = 6, since AC = 12, while OB = 8, since BD = 16). Let, OT = r = radius of the inscribed circle.

   In right triangle OTB, $r^2 + (10 - x)^2 = 8^2$, while in right triangle AOT, $x^2 + r^2 = 36$. Subtracting the second equation from the first we get $20x = 72$ and hence, $x = 3.6$. Consequently, from the second equation, we have $r^2 = 36 - x^2 = 36 - (3.6)^2$ $= 23.04$. Also, the area of the rhombus is given by:

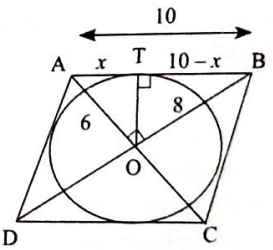
   $\frac{1}{2}\times$ product of diagonals

   $= \frac{1}{2} \times 192 = 96.$

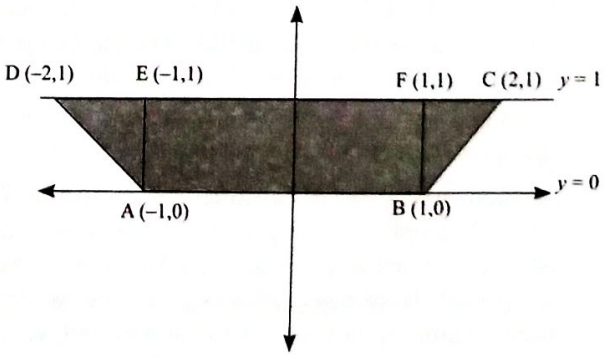
   Required ratio $= \frac{23.04\pi}{96} = \frac{23.04\pi \times 25}{96 \times 25}$

   $= \frac{576\pi}{96 \times 25} = \frac{6\pi}{25}$

   Option (d) is correct.



5. The area formed is the area shown by the shaded region ABCD in the figure. It can be calculated using Required area = (Area of triangle ADE + Area of triangle BCF + Area of rectangle ABFE) = 0.5 + 0.5 + 2 = 3. The correct answer is 3.



6. $x = 4096^{7+4\sqrt{3}} = (2^{12})^{7+4+\sqrt{3}} = 2^{84+48\sqrt{3}}$

   Checking the options we see that

   $x^{7/2} = 2^{294+168\sqrt{3}}$ Also $x^{2\sqrt{3}} = 2^{168\sqrt{3}+288}$

   Option (b) gives us : $\frac{2^{294+168\sqrt{3}}}{2^{168\sqrt{3}+288}} = 64$

7. It is given to us that:

$$A+\frac{B+C}{2}=5 \Rightarrow 2A+B+C=10$$

Also, $2B + A + C = 14$

Thus, we get: $B - A = 4$ and also $3A + 3B + 2C = 24$. Checking the options, if we choose $A + B = 6$ from option (d), we get: $2C = 6 \Rightarrow C = 3$. Also, since $B - A = 4$, we get $B = 5$ and $A = 1$. These values satisfy all the equations. Hence, this is the correct option. If we try to use a wrong option (say $A + B = 7$ from option (c)), we get: $2C = 3 \Rightarrow C = 1.5$, which is not possible, since A,B,C are integers. Option (a) also gets rejected, since we get C as a decimal value, which is not possible. If we tried option (b), $A + B = 4$, we get $C = 6$. This gives us $B = 4$ and $A = 0$, which is again not possible, since A,B,C are positive integers. Hence, option (d) is correct.

8. Since the speeds of the cars are constant we have from the A to M journey: $\frac{\text{Speed of car 1}}{\text{Speed of car 2}}=\frac{20}{t}$

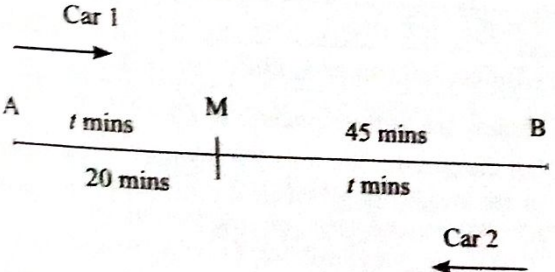
From the M to B journey, we have:

$$\frac{\text{Speed of car 1}}{\text{Speed of car 2}}=\frac{t}{45}$$

Hence, $\frac{\text{Speed of car 1}}{\text{Speed of car 2}}=\frac{20}{t}=\frac{t}{45}$

$\Rightarrow t = 30$ mins.

Hence, the ratio of $\frac{\text{Speed of car 1}}{\text{Speed of car 2}}=\frac{2}{3}$. Hence, if Car 1 travels at 60 kmph, Car 2 travels at 90 kmph. Option (d) is correct.



9. From a speed of 8 kmph, to 15 kmph, Amal, has increased his speed by $\frac{7}{8}$. This means that his time will go down by $\frac{7}{15}$ (since the distance is constant). The time actually has gone down by 35 minutes. Hence, if the original time required is t mins, then $\frac{7}{15}\times t=35 \Rightarrow t = 75$ mins. Hence, since he reaches office at 10:15, it follows that he must have left home at 9 AM. Thus, the distance = 8 × 1.25 = 10 kms. If he leaves at 9:10 and has to reach at 10 AM, he has to reach in 50 minutes or $\frac{5}{6}$ of an hour. The speed required should be such that: $s\times\frac{5}{6}=10 \Rightarrow s = 12$ kmph.

Option (b) is correct.

10. When you cut the solid circular cone at a height of 18 from the base, it means you are cutting it at a height of 9 from the top. Since, the height of the new cone thus formed is $1/3^{rd}$ the height of the original cone, the volume of the new cone will be $\frac{1}{27}$ of the volume of the original cone and the volume of the frustum of the cone formed, would be $\frac{26}{27}$ of the original cone. Assume the volume of the original cone is 27V, then the volume of the frustum would be 26V and the volume of the new cone (smaller one with height 9), would be V. The difference between 26V and V = 25V is given to us as 225.

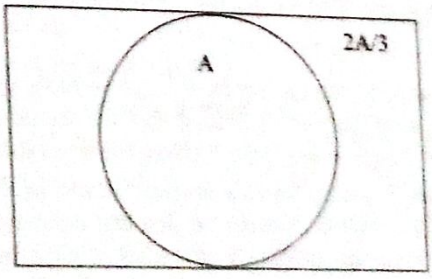
Hence, $V = \frac{225}{25} = 9$. Thus, the volume of the original cone = 27V = 279 = 243.

Option (c) is correct.

11. This question is based on the pigeon hole principle. The required answer will be derived if we equally place 8 students having birthdays in each month from January to December. This would account for the birthday of 96 children. The birthday of the remaining 4 children can be distributed as 1 each to 4 different months amongst January to December. Thus, there will be a pattern that will look like: x_0 = Max (8, 8, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9). The required maximum is 9. Hence, option (b) is correct.

12. The weights of the three alloys would be in the ratio: 3 × 4 : 4 × 2 : 7 × 6 = 15:8:42. Since, the total weight is 130 kg, it means that the weight of the heaviest piece would be 42 × 2 = 84 kg. Option (d) is correct.

13. The given sheet of metal will look as shown in the figure.



Since, the total area of the rectangular sheet is

135, we get that the area of the circle A = 81 and $\frac{2A}{3} = 54$. We can calculate the radius of the circle and consequently find the height of the rectangle by doubling the radius (since the height of the rectangular sheet would be equal to the diameter of the circle). $\pi \times r^2 = 81$

$\Rightarrow r = \frac{9}{\sqrt{\pi}}$

Hence, the height of the rectangle

= diameter of the circle = $\frac{18}{\sqrt{\pi}}$.

We are also given the area of the rectangular sheet as 135. Using:

Area of a rectangle = $b \times h$

$\Rightarrow 135 = \frac{18}{\sqrt{\pi}} \times b$

$\Rightarrow b = \frac{15\sqrt{\pi}}{2}$.

The perimeter of the rectangle would be:

$2 \times \frac{18}{\sqrt{\pi}} + 2 \times \frac{15\sqrt{\pi}}{2}$

$= \frac{36}{\sqrt{\pi}} + 15\sqrt{\pi} = 3\sqrt{\pi}\left(5 + \frac{12}{\pi}\right)$

Hence, option (a) is correct.

14. We can form the following possibilities for a,b,c. Note: We start with $bc = 96$ to find pairs of values of b and c, using $c < 9$ as a constraint and then try to calculate the value of a using $ab = 432$

a	b	c	$a+b+c$
Not integral	96	1	NA
9	48	2	59
Not integral	32	3	NA
18	24	4	46
27	16	6	49
36	12	8	56

Hence, option (b) is correct.

15. Taking logs to base 3 on both sides, we get: $y^2 \log_3 5 \times \log_3 2 = \log_3 3 \times \log_3 5$

$\Rightarrow y^2 = (\log_3 3)^2$.

Thus, $y = \pm \log_3 3$. Since, it is given to us that y is a negative number, it follows that we should consider $y = -\log_3 3 = \log_3 (1/3)$. Option (c) is correct.

16. After 2 years, Veeru's amount would be 10000 + 1000 = 11000. Further his interest earned per year will continue to be 5% of 10000 = 500 (since, it is simple interest). Joy, invests 8000 and 10% pa simple interest. This means that he would earn 800 per year simple interest. Thus, Joy would catch up with Veeru's amount at the rate of 300 rupees a year. He will take 10 years to catch up. Hence, the total time required to catch up after Veeru's investment would be 2 + 10 = 12 years.

17. Initially the solution has 16 dye and 24 water. To this we have to add 16 water to make the dye to water equal a ratio of 2:5. At this stage there will be 16 dye and 40 water, a total of 56 liters. When one fourth of this diluted mixture is withdrawn, 14 liters would be withdrawn out of the 56 liters of mixture. Dye and water will come out in the same proportion (i.e.2:5). Thus, dye withdrawn = 4 and water withdrawn = 10. Dye left = 16 – 4 = 12, water left = 40 – 10 = 30. We would need to add 8 liters of water to make it's dye and water ratio as 20:30 = 2:3. Hence, the correct answer is 8.

18. Before the 5th child is given toffees, the gentleman would have 2 toffees. Only then, would he be left with 0, if he gives half of his stock and one extra. Prior to the 4th child, the gentleman would be left with 6 toffees – from 6 if you were to give half the stock + one extra you would give out 4 toffees and be left with 2. Prior to the 3rd child the gentleman would have 14 toffees (14/2 = 7 plus 1 = 8 and 14 – 8 = 6). The series of numbers in reverse order for toffees left would be:

Toffees at the end	0
Toffees left after 4th child	2
Toffees left after 3rd child	6
Toffees left after 2nd child	14
Toffees left after 1st child	30
Toffees left before 1st child	62

Hence, the correct answer is 62.

19. Let the speed of the train be S kmph. Then, according to the condition given in the question we have:

$(S - 2) \times 90 = (S - 4) \times 100$

$\Rightarrow S = 22$ kmph

Total length of train = $(22 - 2) \times \frac{5}{18} \times 90$

= 500 meters.

Hence, the time required to cross an electric post would be: $\frac{500}{22 \times \frac{5}{18}} \approx 82$ seconds.

20. You can think of the numbers as shown here:

Young (28)		Old (72)	
Literate	Illiterate	Literate	Illiterate
16.25 (= 25% of 65)	11.75 (=28 – 16.25)		23.25 (= 35 – 11.75)

The required percentage of old people among the illiterates is $\frac{23.25}{35} \approx 66$ percent

21. The given expression on the LHS is of the form:

$2^x + \frac{1}{2^x}$

The minimum value of the LHS is 2 and occurs at $x = 0$. The RHS, at $x = 0$ is $2 - (4) = -2$. The RHS becomes equal to 2 only at $x = 2$, but at that value the LHS is not equal to 2. Hence, since the LHS ≥ 2 and the RHS ≤ 2, and they are not equal to 2 at the same time, it means that there is no solution where the LHS and RHS of the given equation are equal. Hence, option (c) is correct.

22. $\log_4 5 = (\log_4 y)(\log_6 \sqrt{5})$

$$\Rightarrow \log_4 5 = \frac{(\log_4 y)(\log_4 \sqrt{5})}{\log_4 6}$$

$$= \frac{1}{2} \times \frac{(\log_4 y)(\log_4 5)}{(\log_4 6)}$$

Canceling $\log_4 5$ from both sides, we get:

$\log_4 y = 2 \log_4 6$

$\Rightarrow \log_4 y = \log_4 36$.

Hence, the value of $y = 36$.

23. From the first statement, we get that the train must have taken thrice the time it normally takes, since it traveled at 1/3rd of its speed for the same distance. Thus, if 't' is the original time it takes, on this day it took time '$3t$'. Extra time taken = $3t - t = 2t$ = 30 minutes

$\Rightarrow$ Normal time = 15 minutes.

On the return journey, in 5 minutes at normal speed, the train would cover 1/3rd the distance. It would be left with a journey of 10 minutes at normal speed. However, it has to cover this distance in 6 minutes $\Rightarrow$ since it stopped for 4 minutes. This entails a 40% decrease in time taken for the same distance – which, means a 66.66% increase in speed would be required. Option (a) is correct.

Note: Alternately, we could have thought of this as:

drop in time: $\frac{2}{5}$;

Hence, increase in speed = $\frac{2}{3}$.

24. Think of the expression as $A^2 - 3A + 2 = 0$.

The value of $A = \frac{3 \pm \sqrt{9-8}}{2} = 1$ or 2.

However, an expression like $x + \frac{1}{x}$ cannot be equal to 1, since it's minimum value is 2. Hence, there is only one real root of the equation, when $x + \frac{1}{x} = 2$, viz: $x = 1$. The correct answer is that there is only one distinct real root for the equation.

25. Using the concept of alligation, you can work out that the ratio of the price of the desktop to the laptop is 2:3. Note: In the case of profit and loss situations, the average profit percentage created on the sale of two items of different costs, is the weighted average of the individual profits on the two items. The weights to be used in such a case would be the ratio of the costs of the two items. In this question, we are given that he makes a profit of 20% and a loss of 10% on the desktop and laptop respectively. Also that he makes an overall profit of 2% $\Rightarrow$ which is what gives us that the ratio of the price of the desktop to the laptop is 2:3. Hence, the price of the desktop is 20000.

26. An expression of the form A^B equals a value of 1, in three cases: Case 1: B = 0, A can take any value except 0, as then $A^0 = 1$; Case 2: A = 1, B can take any value; Case 3: A = – 1, B can take any even value.

Checking the given expression for each of the three cases.

Checking for Case 1

B = 0: $x^2 - 13x + 42 = 0$

$\Rightarrow$ $x = 6,7$. Two solutions.

Checking for Case 2

A = 1 and

B $\neq$ 0: $x^2 - 7x + 11 = 1$

$\Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow$ $x = 2,5$ and

we already know that B = 0 is occurring at $x = 6,7$. Hence, we can count both these values as solutions.

Checking for Case 3

A = – 1 and B is even: $x^2 - 7x + 11 = -1$

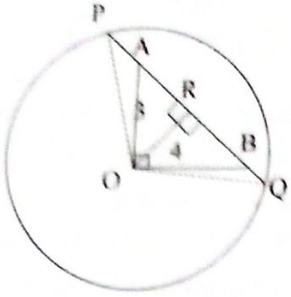
$\Rightarrow x^2 - 7x + 12 = 0$

$\Rightarrow x = 3,4$ and

we can check at $x = 3$ and $x = 4$, the value of $x^2 - 13x + 42$ is even. Hence, we can count both these values as solutions.

Thus, there are a total of 6 solutions in this case.

CAT 2020 SLOT 2

1. 

O is the center of the circle and the radius of the circle = 5

In triangle AOB, AB = 5 (Pythagoras theorem).

Drop a perpendicular from O to R as shown. Using the property of right angled triangles,

$OR = \frac{3 \times 4}{5} = 2.4$. OR being dropped from the centre as a perpendicular to the chord, means that it bisects the chord. Thus, we can find the length of PR and double it to get PQ.

In triangle ORP, $OP^2 = OR^2 + PR^2$, we get: $OR^2 = 19.24$, $OR = \sqrt{19.24} \approx 4.4$.

Hence, PQ ≈ 8.8. Option (c) is correct.

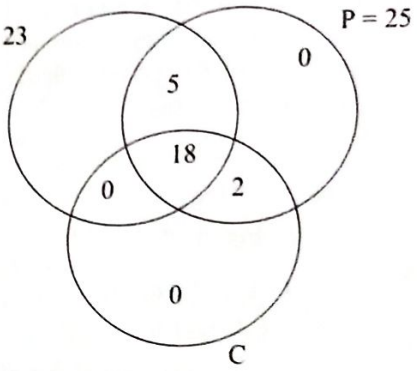
2. Let the race be for a distance of 'd'. Then according to the question, we have $\frac{d}{d-50} = \frac{d-45}{d-90}$

Inserting values for d from the options, we see that $d = 450$, satisfies this equation. Hence, option (b) is correct.

3. Let the distance from A to B be 50 km and from B to C be 150 km. Let the initial speeds of the two trains be 25 and 50 kmph. Train 1 will cover the distance in: $\frac{50}{25} + \frac{150}{50} = 5$ hours; Train 2 will cover the distance in: $\frac{50}{50} + \frac{150}{25} = 7$ hours. Required ratio of time taken is 5:7. Option (b) is correct.

4. In general the value of an expression like A^B can be equal to 1 for 3 cases. First: If B = 0, the value of A does not matter, since A^0 is always equal to 1; Second: If A = 1, the value of B does not matter since $1^B = 1$, no matter what value B takes. Also, for A = –1, B should be even for A^B to be 1. In this situation, we have: for $x = -1$, the first case takes place. For the second case to happen, $x^2 - 5x + 7 = 1 \rightarrow x^2 - 5x + 6 = 0 \rightarrow x = 2$ and 3. If we look for the third case, we want to get integer values of x, for $x^2 - 5x + 7 = -1 \rightarrow x^2 - 5x + 8 = 0$. But this equation has imaginary roots and hence the third case would not occur here. Thus there are a total of 3 integer values of x. Option (a) is correct.

5. If John takes time $2t$, then Jack takes time t for the same work. This means that the work ratio of John and Jack are in the ratio 1:2. Also, it is given to us that Jack and Jim finish the work in one third the time that John takes to finish the work. This means that the work rate of Jack and Jim (combined) is thrice the work rate of John alone. Hence, the ratio of work rates of John, Jack and Jim is 1:2:1. If the total work is taken as 4 units, and their individual rates of work are taken as 1 unit, 2 units and 1 unit per day, we get that together they will take 1 day, while John will take 4 days (i.e. 3 days more than they take together). This satisfies the given condition and hence, Jim will finish the work in $\frac{4}{1} = 4$ days. The correct answer is 4.

6. 

Since, we are given the value of all 3 as 18 and that to keep choosing Chemistry as minimum, it means that we need to maximise the number of people who chose Physics and Mathematics. This can be maximised at 5. Hence, the minimum number of people who chose Chemistry is 18 + 2 = 20.

7. To find the minimum possible value keep x and y as close as possible

So, $x = y = 51$

$$= 2601\left(1+\frac{1}{51}\right)\left(1+\frac{1}{51}\right) = 52 \times 52 = 2704.$$

8. Let the lengths of the sides of the rectangle be 'l' and '$3l$' respectively. The area of the rectangle $R = 3l^2 = T^2 \Rightarrow T = l\sqrt{3}$ = area of the triangle.

Also, area of an equilateral triangle having area 'a' is given by: $\frac{\sqrt{3}}{4}a^2 = l\sqrt{3} \Rightarrow a = 2\sqrt{l}$

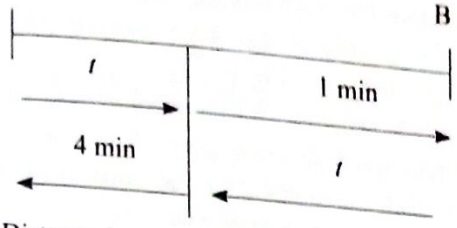
The sum of the perimeters

$= 6\sqrt{l} + 8l = 90 \Rightarrow l = 9$

Hence, the length of the longer side of the rectangle = 27.

Option (d) is correct.

9. A



Distance is constant, So ratio of their time taken is same

$\frac{t}{4} = \frac{1}{t}$.

$t^2 = 4$

$t = 2$

Ram takes 3 minutes and Rahim takes 6 minutes

So Ratio of Ram's speed : Rahim's speed

= 2 : 1 = 2

Option (d) is correct.

10. Let the numbers be a, b, c, d, e, f, g, h, i and j, such that a is the smallest number and j the largest. The average of the first 9 numbers is given as 42, hence the sum of the first 9 numbers (a to i) would be 42 × 9 = 378. Likewise, the sum of the last 9 numbers (b to j) would be 47 × 9 = 423. Thus, $j - a = 45$. The instance of the maximum average would be if we take a to i as 42 each and j as 87. In such a case, the sum of the 10 numbers is 378 + 87 = 465 and their average is 46.5. The other extreme is the minimum average, this would occur when the values of a = 2 and b to j is 47 each. In this case, the total is 423 + 2 = 425 and the minimum average is 42.5.

(Note: we cannot take $a = 0$ and $i = 45$ as in such a case, there will be at least 1 number greater than j to achieve an average of 47 for the last 9 numbers).

The required difference of averages is

46.5 – 42.5 = 4

Option (c) is correct.

11. Assume, he buys each toy and labels at ₹ 1 each. Thus, the labeled price of 12 toys would be a total of rupees 12. He sells, 8 toys at 0.8 and 4 toys at 0.6. Hence, he recovers a total of 6.4 + 2.4 = ₹ 8.8. This represents a profit of 10% on his cost, which means that his cost is $\frac{8.8}{1.1} = 8$ rupees. If he were able to sell all his toys at the labeled price, he would have recovered ₹ 12 on a cost of 8 rupees. This would mean a profit percentage of 50%.

Option (c) is correct.

12. The GP to minimise $r + n - m$, can be visualised as $\frac{3}{4}$, –3, 12. In this case, $r = -4$, $m = 1$ and $n = 3$.

$r + n - m = -4 + 3 - 1 = -2$.

Option (b) is correct.

13. $P(1.05)^2 - P = CI$

$P = \frac{3 \times 3}{100} = SI$

$CI = 0.1025\ P$

$SI = 0.09P$

$CI - SI = 0.0125P = \left(\frac{125}{10000}\right) P = 1125$

$P = 90000$

14. Dividing both numerator and denominator by x,

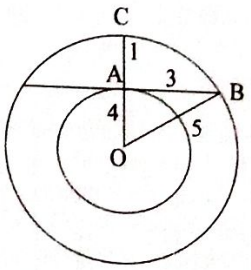
$$\frac{1}{\sqrt{\frac{1+x^4}{x^2}}} = \frac{1}{\sqrt{\frac{1}{x^2}+x^2}}$$

$$\frac{1}{x^2}+x^2 \geq 2$$

$$\sqrt{\left[\frac{1}{x^2}+x^2\right]} \geq \sqrt{2}$$

$$\frac{1}{\sqrt{\frac{1}{x^2}+x^2}} \leq \frac{1}{\sqrt{2}}$$

15. We can visualise the 3-4-5 Pythagoras triplet as shown in the figure. Hence, the radius of $C_1 = 5$ and it's diameter is 10.



16. The ratio of money split between Amal, Sunil and Mita will be 6:4:5 or $6x$, $4x$ and $5x$. The difference between the largest and the smallest, i.e. $6x$ and $4x$ is given as 400.

Hence, $2x = 400$

$\Rightarrow x = 200$ and hence, Sunil will get ₹ 800.

17. The given expression can be written as:

$(\log_a a - \log_a b) + (\log_b b - \log_b a)$.

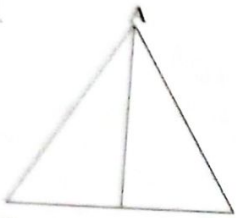
This simplifies to: $2 - \left\{\log_a b + \frac{1}{\log_a b}\right\}$.

maintain constant expenditure.

The value of $\left(x+\frac{1}{x}\right)$ is always ≥ 2. Hence, the expression $2-\left\{\log_a b+\frac{1}{\log_a b}\right\}$ can never be equal to 1. It can be 0 or negative.

Hence, option (b) is correct.

18.



From any point on an equilateral triangle, if we draw three perpendiculars on other sides those three perpendiculars will add up to the Altitude of the equilateral triangle. We know that the value of the altitude of an equilateral triangle with side 'a' is $\frac{\sqrt{3}}{2}a$

Hence, $s=\frac{\sqrt{3}}{2}a$

$\Rightarrow \quad a=\frac{2s}{\sqrt{3}}$..

Thus, the area of the triangle

$=\frac{\sqrt{3}}{4}a\times a=\frac{\sqrt{3}}{4}\times\frac{2s}{\sqrt{3}}\times\frac{2s}{\sqrt{3}}\rightarrow\frac{s^2}{\sqrt{3}}$.

Hence, option (b) is correct.

19. If we assume, $x = 1$, z will equal 10 and $y = 9$. In this case checking for $x + y < z + 5 \Rightarrow 10 < 15$. We are asked to find the maximum value of $2x + y$. Hence, we should look for larger values of x and y.

Value of x	Value of z	Value of y	Value of $x+y$	Value of $z+5$	Is $x+y<z+5$
2	11	10	12	16	Yes
3	12	11	14	17	Yes
4	13	12	16	18	Yes
5	14	13	18	19	Yes
6	15	14	20	20	No

Hence, the maximum value of $2x + y$
$= 10 + 13 = 23$.

20. $x^2 - 2 | x | + |a - 2| = 0$

For the given expression to be equal to 0, we see that for $x = 1, -2$ $| x | = -2$ and hence $x^2 - 2 | x | = -1$.

At $x = 2$, the value of $x^2 - 2 | x | = 0$. Values of x above 2 will not work, since the expression cannot be equal to 0, if $x^2 - 2 | x |$ is greater than 0. Also, the same conditions will be met for $x = -1$ and $x = -2$. Besides, at $x = 0$, we will get $x^2 - 2 | x | = 0$ and there can be a solution of this expression for that value of x too.

For $x = 1$, since $x^2 - 2 | x |$ is equal to -1, we get $|a - 2| = 1 \Rightarrow a = 3$ or $a = 1$;

For $x = 2$, since $x^2 - 2 | x |$ is equal to 0, we get $|a - 2| = 0 \Rightarrow a = 2$;

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For $x = 0$, since $x^2 - 2 | x |$ is equal to 0, we get $|a - 2| = 0 \Rightarrow a = 2$

We can thus see a total of 7 solutions for the integer pair (x, a).

Hence, option (c) is correct.

21. Each of these numbers would have 7 and 3 in them. Also, the order of using 7 and 3 is fixed. Thus, there will be 6 kinds of numbers having 3 after 7 in a 4 digit number. We can think of these as:

	Thousands Place	Hundreds Place	Tens Place	Units Place	Number of cases for a and b
Case 1	7	3	a	b	8×7
Case 2	7	a	3	b	8×7
Case 3	7	a	b	3	8×7
Case 4	a	7	3	b	7×7
Case 5	a	7	b	3	7×7
Case 6	a	b	7	3	7×7
				Total cases	315

22. Let Aron buy P pencils and S sharpeners at prices of x and $(x + 2)$. Then, we have Aditya would buy 2P pencils and S – 10 sharpeners at the same prices. It is also given to us that the amount spent by both are equal. Hence,

$Px + S(x + 2) = 2Px + (S - 10)(x + 2)$

$\Rightarrow Px + Sx + 2S = 2Px + Sx + 2S - 10x - 20$

$\Rightarrow Px = 10x + 20$

$x(P - 10) = 20$

The, minimum value of P for this expression to hold would be $P = 11$. Hence, the combined number of pencils they would have bought would be $3P = 33$. Hence, option (c) is correct.

maintain constant expenditure.

The value of $\left(x+\frac{1}{x}\right)$ is always ≥ 2. Hence, the expression $2-\left\{\log_a b+\frac{1}{\log_a b}\right\}$ can never be equal to 1. It can be 0 or negative.

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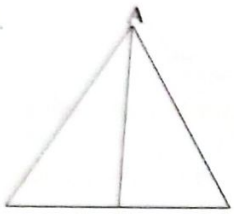

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Hence, the maximum value of $2x + y$
$= 10 + 13 = 23$.

20. $x^2 - 2 \mid x \mid + \mid a - 2 \mid = 0$

For the given expression to be equal to 0, we see that for $x = 1$, $-2 \mid x \mid = -2$ and hence $x^2 - 2 \mid x \mid = -1$.

At $x = 2$, the value of $x^2 - 2 \mid x \mid = 0$. Values of x above 2 will not work, since the expression cannot be equal to 0, if $x^2 - 2 \mid x \mid$ is greater than 0. Also, the same conditions will be met for $x = -1$ and $x = -2$. Besides, at $x = 0$, we will get $x^2 - 2 \mid x \mid = 0$ and there can be a solution of this expression for that value of x too.

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We can thus see a total of 7 solutions for the integer pair (x, a).

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