EE 430 Term Project, Part 2

In this part of the project, you are going to study a range and speed estimation problem by using the short-time Fourier transform (STFT) and matched filtering as your mathematical tools. Range of the source will be determined by time-of-arrival estimation and speed of the source will be determined by frequency estimation. A signal emitted by a moving source is received in a modified form due to Doppler effect. The form of modification depends on the velocity vector of the source with respect to the receiver.

You are going to implement signal processing methods in a computer environment. Guidelines are given below.

Prepare a descriptive and clearly written (in language and format) report. You are going to upload this report and the MATLAB code to ODTÜClass.

1. Signals emitted by a moving source

Consider the scenario given in Figure 1.

Figure 1: An emitting source moving towards a sensor.

The source emits a short pulse with a known starting time (although starting time will be unknown in practical circumstances). The sensor observes the delayed and warped (due to Doppler effect) version of the transmitted signal with additive "noise". Assume that the clocks of the sensor and the source are synchronized, i.e. sampling instants are the same on both sides. Assume also that the signal travels at a constant speed of $c = 340$ m/s in the medium.

If the source is moving directly towards the sensor at a constant speed, v, and emitting the signal $s_e(t)$, the sensor will receive $s_r(t) = s_e\left(\frac{c}{c-v}(t-t_0)\right)$ where t_0 is the time delay the signal incurs. Note that if the signal is a sinusoidal pulse of frequency Ω_0 and duration Δ, $s_e(t) = A\sin(\Omega_0 t)\left(u(t) - u(t-\Delta)\right)$, the observed signal will again be sinusoidal pulse, however, of a different frequency and a different duration.

In this work you will basically experiment with two kinds of pulses, a sinusoidal pulse and a linear chirp (linear FM) pulse,

$$s_e(t) = \begin{cases} A\cos\left(2\pi\left(f_0 t + \frac{m}{2\Delta}t^2\right)\right), & 0 \le t \le \Delta \\ 0, & \text{otherwise} \end{cases}.$$

2. Generating the Signals

The signal received by the sensor will be

$$s_r(t) = s_e\left(\frac{c}{c-v}(t-t_0)\right) + e(t)$$

where $e(t)$ is the noise component. You will test the performance of your method based on multiple trials (Monte Carlo trials). Let N denote the number of trials for each specific task. In each trial $e(t)$ will be different. $e(t)$ will be generated as a zero mean, unit variance, Gaussian, white process.

Determine your own ranges for the values of Δ, Ω_0, m, t_0, v. You will choose the values of the parameters from these sets. Determine N. Determine your sampling frequency, $f_s = \frac{1}{T_s}$. You may need to do some research and trials to determine these values.

Use t_0 values like $t_0 = (n + k0.1)T_s, n > 0, k = 0,1,2,3,4,5$.

In your report, describe your study to determine the specific values of your parameters.

3. Spectrogram Based Method

Sinusoidal Pulse as the Emitted Signal

In this part, you are going to use the spectrogram to detect the presence of a transmitted pulse and then estimate the range and the speed of the source. You must be able to do this both by visual inspection of the spectrogram and by automatic calculations using the STFT values.

For your tests, consider varying the window size of your spectrogram implementation and the amount of overlap between adjoining sections. How do the frequency resolution and the frequency estimates change with these parameters? How about the time resolution and the time of arrival estimates?

Specify (write) your method.

Specify (write) your test procedure; parameter values chosen, number of trials, signal-to- noise ratio values you choose.

Specify how you are going to present the performance of your method; what you are going to measure, what the figures of performance are, what tables/figures will be presented.

Can you suggest ways to improve the accuracy of your frequency estimates?

Perform your tests.

Discuss your results.

State your conclusions about the performance with respect to different choices of parameter values.

Linear Chirp Pulse as the Emitted Signal

Repeat the work for the linear chirp pulse following a path similar to that for the sinusoidal pulse.

4. Matched filter based method (This part is not compulsory to work out, however, you may get 30% bonus credit.)

In this part, by using the matched filter, you are going to detect the presence, and then estimate the arrival time of a signal under additive noise. For the detection, we consider two cases which are called H_0 and H_1 hypotheses. The H_0 hypothesis corresponds to the *no signal* case in which the observed sequence is given as

$$s_r(t) = e(t) \qquad (H_0)\,.$$

The other hypothesis H_1 corresponds to the case in which a *known* signal $s_e(t)$ is transmitted and $s_r(t) = s_e\left(\frac{c}{c-v}(t-t_0)\right) + e(t)$ is received.

The detection system that you are going to implement takes the observed sequence $s_r(t)$ as input and decides the signal model of the observed signal (H_0 or H_1). We say that a signal is detected if we decide H_1. The detection system may detect a signal when the actual hypothesis is H_1. The probability of doing so is called the probability of detection (P_{d}). Also, it may detect a signal, when the actual hypothesis is H_0. The probability of doing so is called the probability of false alarm (P_{fa}). It is desired that P_{d} is high while at the same time P_{fa} is low.

When the signal is detected, the arrival time can be estimated. The estimation system takes the observed sequence $s_r(t)$ as input and returns an estimated value $\hat{t}_0$ for t_0. "Matched filtering" is a method of -optimal detection in some mathematical sense. The derivation and the criteria to show the optimality of the approach is not in the scope of this work.

Now, let $x[n] = s_r(nT_s)$. The detector to be used in such a scenario is of the form given in Figure 2.

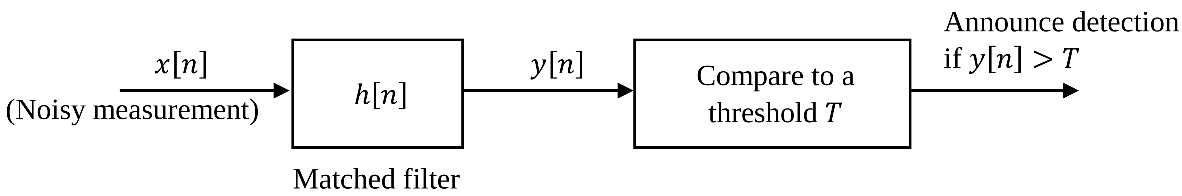

Figure 2: Detecting the presence of a signal under additive white noise using a matched filter.

In a continuous-time setting, for a known signal $s_e(t)$ transmitted, the matched filter's impulse response is $h(t) = s_e(-t)$. However, note that

a) you are going to implement the matched filter in discrete-time,
b) the received signal is not simply a delayed replica of the transmitted signal.

Specify (write) your method. Specify how your methods copes with the "deviation" of the received signal relative to the transmitted signal.

Specify (write) your test procedure; parameter values chosen, number of trials, range of signal-to- noise ratio values.

Specify how you are going to present the performance your method; what you are going to measure, what the figures of performance are, what tables/figures will be presented.

Perform your tests.

Specify how you can "optimally" choose the threshold.

Discuss your results.

State your conclusions about the performance with respect to different choices of parameter values.

5. Some Basic Information on How a Matched Filter Works

For a given positive scalar β, and for a known signal $s[n]$ transmitted, if we set the impulse response of the LTI filter in this figure as

$$h[n] = \beta s[-n] \quad \text{(anticausal version)}$$

then,

$$y[n] = \sum_{k\in\mathbb{Z}} h[k]x[n-k] = \beta \sum_{k=0}^{N-1} s[k]x[n+k] = \underbrace{\beta \sum_{k=0}^{N-1} s[k]s[k+n-n_0]}_{\text{Term 1}} + \underbrace{\beta \sum_{k=0}^{N-1} s[k]e[n+k]}_{\text{Term 2}}.$$

Term 1 is maximum when $n = n_0$. Also note that the statistics of Term 2 does not change with n. The signal-to-noise ratio at the n_0'th sample of the matched filter output is defined as

$$\text{SNR}_{(\text{out})} \triangleq \frac{(\text{Mean of } y[n_0])^2}{\text{Variance of } y[n_0]} = \frac{\beta^2\left(\sum_{k=0}^{N-1} s^2[k]\right)^2}{\beta^2\sigma^2 \sum_{k=0}^{N-1} s^2[k]} = \frac{E}{\sigma^2}.$$

This ratio is directly related to the detection performance.

Note that the filter defined above is anti-causal. In practice, we can use the causal version

$$h[n] = \beta s[N-1-n] \quad \text{(causal version)}$$

in which case the maximum of Term 1 will be at $n = n_0 + N - 1$. The LTI system defined by $h[n]$ is called the matched filter. In order for $h[n]$ to have unit total energy, we set

$$\beta = \frac{1}{\sqrt{E}}$$

where $E \triangleq \sum_{k=0}^{N-1} s^2[k]$ is the total energy of the signal.