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EE230 TERM PROJECT

2.1.

a) $R = E[R_t] = 20P_G + 5P_M + 0P_B = 20P_G + 5P_M$

$Var(R_t) = E[R_t^2] - (E[R_t])^2$

$E[R_t^2] = 400P_G + 25P_M$

$(E[R_t])^2 = 400P_G^2 + 200P_GP_M + 25P_M^2$

$Var(R_t) = 400P_G + 25P_M - 400P_G^2 - 200P_GP_M - 25P_M^2$

b) $E[R^T] = E\left[\frac{1}{T}\sum_{t=1}^{T} R_t\right] = \frac{1}{T}E\left[\sum_{t=1}^{T} R_t\right]$

$= \frac{1}{T}E[R_1 + R_2 + \dots + R_T] = \frac{1}{T}\left(E[R_1] + E[R_2] + \dots + E[R_T]\right)$

expectation is linear

$= \frac{1}{T}(20P_G + 5P_M) \cdot T = 20P_G + 5P_M$

$Var(R^T) = Var\left(\frac{R_1 + R_2 + \dots + R_T}{T}\right) = Var\left(\frac{R_1}{T}\right) + Var\left(\frac{R_2}{T}\right) \dots + Var\left(\frac{R_T}{T}\right)$

Variance is linear since $R_1, R_2, \dots R_T$ are independent

$Var\left(\frac{R_t}{T}\right) = \frac{1}{T^2}Var(R_t) = \frac{1}{T^2}\left(400P_G + 25P_M - 400P_G^2 - 200P_GP_M - 25P_M^2\right)$

$Var(R^T) = \sum_{t=1}^{T}\frac{1}{T^2}Var(R_t) = \frac{1}{T^2}Var(R_t) \cdot T = \frac{1}{T}Var(R_t)$

$= \frac{1}{T}\left(400P_G + 25P_M - 400P_G^2 - 200P_GP_M - 25P_M^2\right)$

c) If x takes only nonnegative values, then $P(X \geq a) \leq \frac{\langle x \rangle}{a}$. To prove,

$\langle x \rangle = \int_0^\infty x P(x)\,dx = \int_0^a x P(x)\,dx + \int_a^\infty x P(x)\,dx$

$\langle x \rangle = \int_0^a x P(x)\,dx + \int_a^\infty x P(x)\,dx \geq \int_a^\infty x P(x)\,dx \geq \int_a^\infty a P(x)\,dx$

$\int_a^\infty P(x)\,dx = a\,P(X \geq a) = \langle x \rangle \Rightarrow P(X \geq a) = \frac{\langle x \rangle}{a}$

Let $a = k^2$. $P[(x-\mu)^2 \geq k^2] \leq \dfrac{\langle (x-\mu)^2 \rangle}{k^2} = \dfrac{\sigma^2}{k^2}$ → expectation → variance

$P(|x-\mu| \geq k) \leq \dfrac{\sigma^2}{k^2}$ ⇒ Putting $k\sigma$ into k ⇒

$P(|x-\mu| \geq k\sigma) \leq \dfrac{1}{k^2}$ Let $R^T = \dfrac{1}{T}\sum_{t=1}^{T} R_t$

$$R^T = \frac{R_1 + R_2 + \dots + R_T}{T}$$

Since P_G, P_M, P_B are independent and identically distributed random variables, each $R_1, R_2, \dots, R_T$ have a mean μ and a standard deviation σ.

So, $\langle R^T \rangle = \dfrac{1}{T}\left(\langle R_1 \rangle + \langle R_2 \rangle + \dots + \langle R_T \rangle \right)$

$= \dfrac{1}{T} \cdot (\mu \cdot T) = \mu$

$Var(R^T) = Var\left(\dfrac{R_1 + R_2 + \dots + R_T}{T}\right) \overset{\text{independence}}{=} Var\left(\dfrac{R_1}{T}\right) + Var\left(\dfrac{R_2}{T}\right) + \dots + Var\left(\dfrac{R_T}{T}\right)$

$= \dfrac{\sigma^2}{T^2} \cdot T = \dfrac{\sigma^2}{T}$

f) Since are outage event lasts longer than k frames for $k \geq 1$, where $(1-P_B)$ term denotes the exist from outage event,

$$\sum_{m=k+1}^{\infty} P_B^m (1-P_B)$$

$$= \sum_{m=0}^{\infty} P_B^{m+k+1} (1-P_B) = P_B^{k+1} \cdot (1-P_B) \sum_{m=0}^{\infty} P_B^m$$

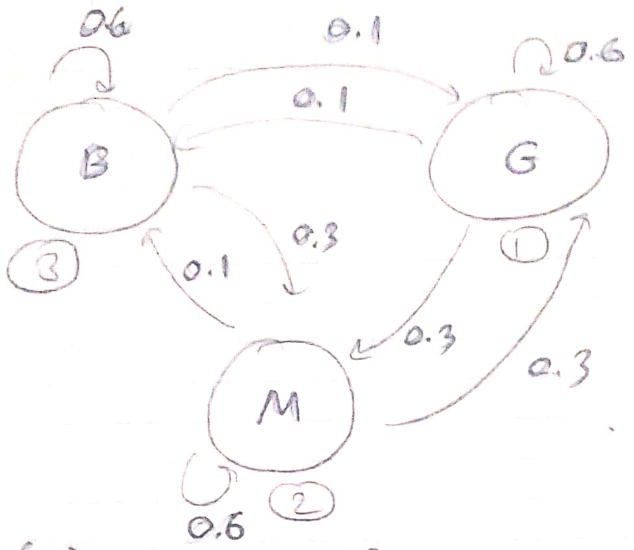
$$= P_B^{k+1} (1-P_B) \left(\frac{1}{1-P_B} \right)$$

$$\boxed{\sum_{m=k+1}^{\infty} P_B^m (1-P_B) = P_B^{k+1}}$$

Scanned by CamScanner

2.2

a)



$P_{BB} = 0.6$ $P_{GB} = 0.1$

$P_{BM} = 0.3$ $P_{GM} = 0.3$

$P_{BG} = 0.1$ $P_{GG} = 0.6$

$P_{MG} = 0.3$ $P_{MM} = 0.6$

$P_{MB} = 0.1$

b)
$$P = \begin{array}{c} \begin{array}{ccc} (G) & (M) & (B) \end{array} \\ \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.6 & 0.1 \\ 0.1 & 0.3 & 0.6 \end{bmatrix} \end{array} \qquad P^2 = \begin{bmatrix} 0.46 & 0.39 & 0.15 \\ 0.37 & 0.48 & 0.15 \\ 0.21 & 0.39 & 0.4 \end{bmatrix}$$

c)
$$P^{10} = \begin{bmatrix} 0.3716 & 0.4286 & 0.1998 \\ 0.3716 & 0.4286 & 0.1998 \\ 0.3706 & 0.4286 & 0.2008 \end{bmatrix}$$

$$P^{20} = \begin{bmatrix} 0.3714 & 0.4286 & 0.2 \\ 0.3714 & 0.4286 & 0.2 \\ 0.3714 & 0.4286 & 0.2 \end{bmatrix}$$

$$P^{25} = \begin{bmatrix} 0.3714 & 0.4286 & 0.2 \\ 0.3714 & 0.4286 & 0.2 \\ 0.3714 & 0.4286 & 0.2 \end{bmatrix}$$

Difference between P^{20} and P^{25} can be seen in long format, but since we want to find what they are converging it is not necessary.

They are converging to $\boxed{0.3714,\ 0.4286,\ 0.2}$

d) $\pi = \pi . P$ let $\pi = [a \quad b \quad c]$ then, we have to use two linearly independent vectors from P and $a+b+c=1$ equation

$$\pi = [a \ b \ c] \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.6 & 0.1 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$$

$$\rightarrow \pi = [\underbrace{0.6a + 0.3b + 0.3c}_{a} \quad \underbrace{0.3a + 0.6b + 0.3c}_{b} \quad \underbrace{0.1a + 0.1b + 0.6c}_{c}]$$

$$\left.\begin{array}{l} -0.4a + 0.3b + 0.1c = 0 \\ 0.1a + 0.1b - 0.4c = 0 \\ a + b + c - 1 = 0 \end{array}\right\} \rightarrow \boxed{\begin{array}{l} a = 0.3714 \\ b = 0.4286 \\ c = 0.2 \end{array}}$$

$a = P_G \quad b = P_M \quad c = P_B$

$\pi = [0.3714 \ 0.4286 \ 0.2]$ Steady State Probability Vector

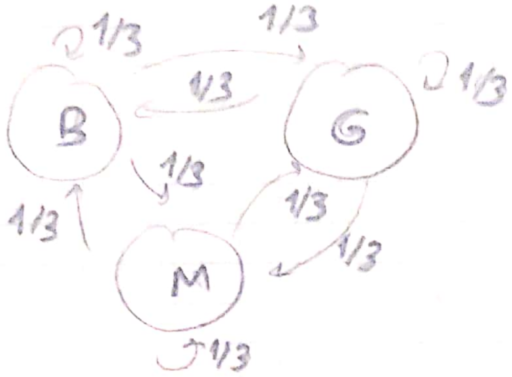
e) $\sum_{k=1}^{\infty} k \, P_{BB}^{(k-1)} (1 - P_{BB}) = E[T]$ T: Duration of an outage

Since it is a geometric series $E[T] = \frac{1}{P_{BB}} = \frac{10}{6}$

$$Var(T) = \frac{1-(P_{BB})}{(P_{BB})^2} = \frac{0.4}{0.36} = \frac{10}{9}$$

f)

2.2 - g



$$P = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

All values are same for all steps and they are 1/3.

$\pi = \pi . P$ Let $\pi = [a \quad b \quad c]$

$$\left.\begin{aligned} \frac{a}{3} + \frac{b}{3} + \frac{c}{3} &= a \\ \frac{a}{3} + \frac{b}{3} + \frac{c}{3} &= b \\ a + b + c &= 1 \end{aligned}\right\} \quad a = b = c = 1/3$$

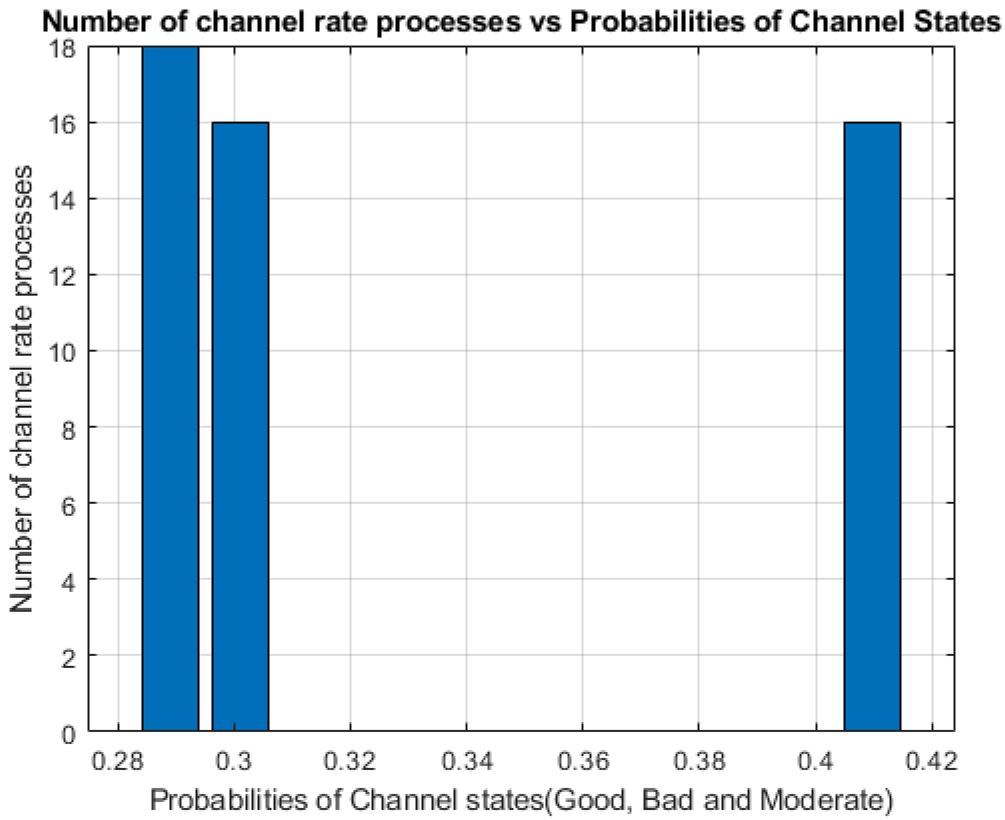
$\pi = [1/3 \quad 1/3 \quad 1/3]$

Steady state probability vector

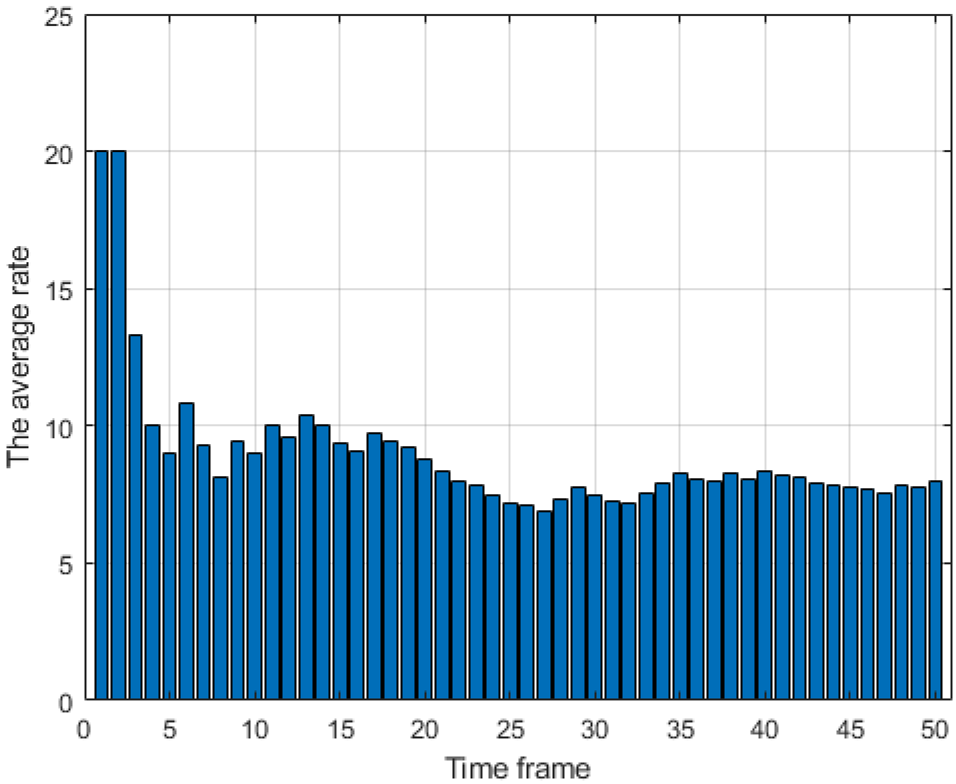
$$E[T] = \sum_{k=1}^{\infty} (P_{BB})^{k-1} . (1 - P_{BB}) = \frac{1}{P_{BB}} = 3$$

$$Var(T) = \frac{1 - P_{BB}}{(P_{BB})^2} = \frac{2/3}{1/9} = 6$$

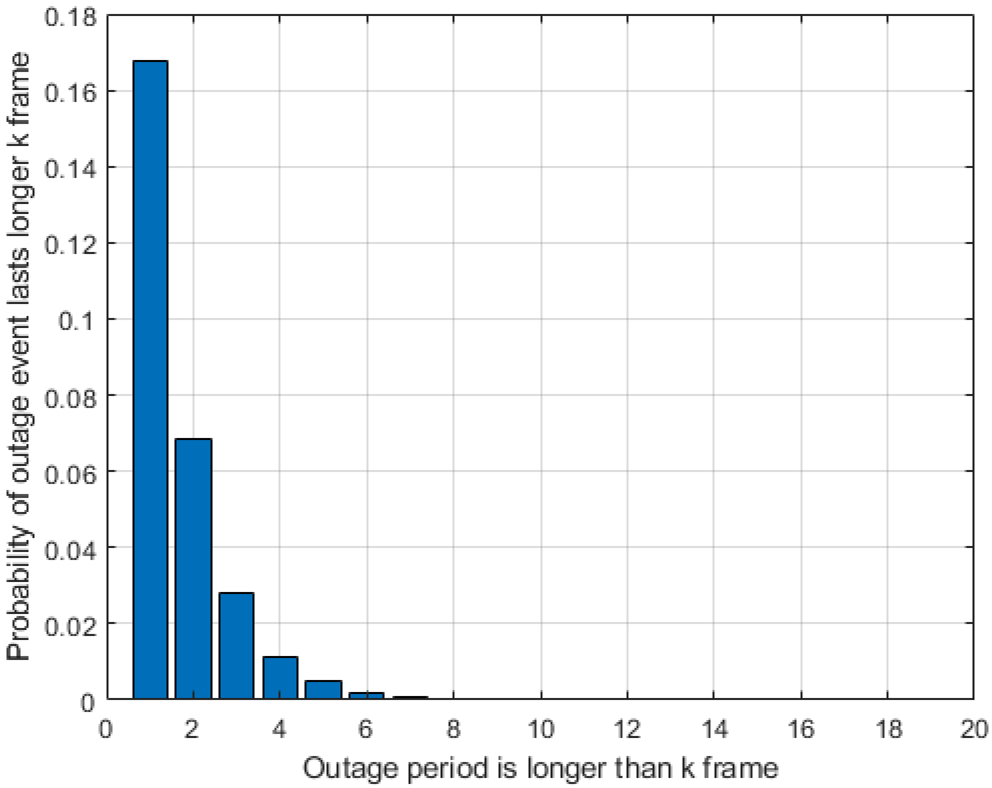
```
%2.1-part-c
x=rand(2,1);
a=x(1,1);
b=x(2,1);



while (a+b) > 1%%we generate two variable until their sum is lower
 than 1

    x=rand(2,1);
    a=x(1,1);
    b=x(2,1);




    continue
end
c=1-a-b;%Then by using these two variables we get 3 random
 probabilities.
p=zeros(3,1);
p=[a;b;c];%Probability vector
Y1=rand(1,50);%Frame vector
R=zeros(3,1);%Number of Good, Bad and Moderate vector(R(1,1)=#Good,
 R(2,1)=#Bad R(3,1)=#Moderate)
Y1=Y1.';
Y2=zeros(50,1);
for i=1:50
    if 0<Y1(i,1) && Y1(i,1)<=a
     R(1,1)=R(1,1)+1;
     Y2(i,1)=20;
    else if a<Y1(i,1) && Y1(i,1)<=b+a
            R(2,1)=R(2,1)+1;
            Y2(i,1)=5;
        else

          R(3,1)=R(3,1)+1;
          Y2(i,1)=0;
        end
    end
end
figure(1)
bar(p,R);
grid on;
ylabel('Number of channel rate processes');
xlabel('Probabilities of Channel states(Good, Bad and Moderate)');
title('Number of channel rate processes vs Probabilities of Channel
 States')
```




```
%2.1-part-d
Y3=zeros(50,1);%Running time average vector
for T=1:50
    for i=1:T
        Y3(T,1)=Y3(T,1)+Y2(i,1);
    end
    Y3(T,1)=Y3(T,1)/T;


end
T1=[1:50];
figure(2)
bar(T1,Y3);
grid on
ylim([0 25]);
xlim([0 51]);
xlabel('Time frame');
ylabel('The average rate');
```




```
%part 2.1-f
B=zeros(50,1);%outage vector

for k=1:50
    for m=k:50
        B(k,1)=B(k,1)+((b^(m+1))*(1-b)); %b=outage probability
    end
end

u=[1:50];
figure(3)
bar(u, B);
grid on;
xlim([0,20]);
xlabel('Outage period is longer than k frame');
ylabel('Probability of outage event lasts longer k frame');
```




```
%Analysis of the Indoor Link
%2.2-part c
P1=[.6, .3, .1; .3, .6, .1; .1, .3, .6];%Transition probability matrix
n=25;
X1=P1^n;%X1=P1^25.
A=[-0.4, 0.3, 0.1; 0.1, 0.1, -0.4; 1 1 1];
Y=[0; 0; 1];
K=inv(A)*Y %K steady state probability vector
```

```
K =

    0.3714
    0.4286
    0.2000
```

```
%2.2 part-f
F=NaN(500,1);
k=rand();%We define the first state
if 0<k<=1/3
    F(1,1)=0;%G=0
end
if 1/3<k && k<2/3
    F(1,1)=1;%B=1
else
    F(1,1)=2;%M=2
```

```
end

for i=2:500
    if F(i-1,1)==0%We are stating ith state according to i-1th state
        k=rand();

        if 0<=k && k<=0.6
            F(i,1)=0;%G
        end
    elseif 0.6<k && k<=0.7

            F(i,1)=1;

    else F(i,1)=2;


    end

    if F(i-1,1)==1%We are stating ith state according to i-1th state
        k=rand();

        if 0<=k && k<=0.6
            F(i,1)=1;

    elseif 0.6<k &&  k<=0.7

            F(i,1)=0;
        else
            F(i,1)=2;

        end
    end



    if F(i-1,1)==2%We are stating ith state according to i-1th state
        k=rand();

        if 0<=k &&  k<=0.6
            F(i,1)=2;

    elseif  0.6<k && k<=0.7

            F(i,1)=1;

    else F(i,1)=0;
        end
    end



end
for i=1:160%In F vector I got NaN values so I randomly assign values
 for specific NaN's.
```

```
    if F(i,1)~= 1 && F(i,1)~=0 && F(i,1)~=2
        F(i,1)=1;
    end
end
for i=161:320
    if F(i,1)~= 1 && F(i,1)~=0 && F(i,1)~=2
        F(i,1)=0;
    end
end
for i=321:500
    if F(i,1)~= 1 && F(i,1)~=0 && F(i,1)~=2
        F(i,1)=2;
    end
end








    q=0;
    for i=1:500
        if F(i,1)==1
           q=i;%First index of B
        break;
        end


    end
    k3=0;
for i=1:500

    if F(501-i,1)==1
        k3=501-i;%last index of B

    break;
    end

end
q3=0;
for i=1:500
    if F(i,1)==1
        q3=q3+1;%number of B
    end

end
j=(k3-q)/(q3-1)%Expected duration of the period between two outages.


j =

    5.1263
```

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```
P1=[1/3, 1/3, 1/3; 1/3, 1/3, 1/3; 1/3, 1/3, 1/3];%Transition
 probability matrix
n=25;
X1=P1^n;%X1=P1^25.

K=[1/3; 1/3; 1/3] %K steady state probability vector


K =

    0.3333
    0.3333
    0.3333


%2.2 part-f
F=NaN(500,1);
k=rand();%We define the first state
if 0<k<=1/3
    F(1,1)=0;%G=0
end
if 1/3<k && k<2/3
    F(1,1)=1;%B=1
else
    F(1,1)=2;%M=2
end

for i=2:500
    if F(i-1,1)==0%We are stating ith state according to i-1th state
        k=rand();

        if 0<=k && k<=1/3
            F(i,1)=0;%G
        end
    elseif 1/3<k && k<=2/3

            F(i,1)=1;

    else F(i,1)=2;


    end

    if F(i-1,1)==1%We are stating ith state according to i-1th state
        k=rand();

        if 0<=k && k<=1/3
            F(i,1)=1;

    elseif 1/3<k&&  k<=2/3

            F(i,1)=0;
        else
```

```
            F(i,1)=2;

        end
    end



    if F(i-1,1)==2%We are stating ith state according to i-1th state
        k=rand();

        if 0<=k &&  k<=1/3
            F(i,1)=2;

    elseif  1/3<k && k<=2/3

            F(i,1)=1;

    else F(i,1)=0;
        end
    end



end
for i=1:160%In F vector I got NaN values so I randomly assign values
 for specific NaN's.
    if F(i,1)~= 1 && F(i,1)~=0 && F(i,1)~=2
        F(i,1)=1;
    end
end
for i=161:320
    if F(i,1)~= 1 && F(i,1)~=0 && F(i,1)~=2
        F(i,1)=0;
    end
end
for i=321:500
    if F(i,1)~= 1 && F(i,1)~=0 && F(i,1)~=2
        F(i,1)=2;
    end
end





    q=0;
    for i=1:500
        if F(i,1)==1
           q=i;%First index of B
        break;
        end
```

```
    end
    k3=0;
for i=1:500

    if F(501-i,1)==1
        k3=501-i;%last index of B

    break;
    end

end
q3=0;
for i=1:500
    if F(i,1)==1
        q3=q3+1;%number of B
    end

end
j=(k3-q)/(q3-1)%Expected duration of the period between two outages.
```

```
j =

    3.1392
```

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3. Conclusions

In the "Outdoor Link" part, the system is memoryless, since the probabilies of getting a Good (G), Moderate (M) or Bad (B) state are constant, hence independent of the previous states.

On the other hand, in the "Indoor Link" part, the probability of getting a state is dependent of the previous state, which indicates that the system is with memory. For example, probabilities of entering a Bad (B) state changes with previous state: $P_{BB}=0.6$, $P_{GB}=0.1$, $P_{MB}=0.1$.

Increasing the memory of a system (in this case, entering the Indoor Link situation from Outdoor Link situation) lowers the variance of the outage duration.

The system with increased memory is better, since its variance is lower, it is more stable.