Question 1

(a) $v'(5.1) = 6.491816$ The acceleration of the particle at time $t = 5.1$ is 6.492 (or 6.491).	1 : answer
(b) $\lvert v(t) \rvert = 1$ for $0 \le t \le 2 \Rightarrow t = 0.771829, t = 1.400556$ The speed of the particle is 1 at times $t = 0.772$ (or 0.771) and $t = 1.401$ (or 1.400).	$2 : \begin{cases} 1 : \text{considers } \lvert v(t) \rvert = 1 \\ 1 : \text{answer} \end{cases}$
(c) Let $x(t)$ be the position of the particle at time t. $x(4) = 7 + \int_0^4 v(t)\,dt = 6.711558$ The position of the particle at time $t = 4$ is 6.712 (or 6.711). The particle is moving away from the origin because $x(4) > 0$ and $v(4) = 1.213064 > 0$.	$4 : \begin{cases} 1 : \text{integral} \\ 1 : \text{uses initial condition} \\ 1 : \text{position} \\ 1 : \text{movement relative to the} \\ \quad\;\; \text{origin, with justification} \end{cases}$
(d) $x'(t) = v(t) \Rightarrow x(t)$ is continuous on $0 < t \le 4$. $x(1) = 7 + \int_0^1 v(t)\,dt = 9.403593$ From part (c), $x(4) = 6.712$. From time $t = 1$ to $t = 4$, the particle moves from $x = 9.404$ to $x = 6.712$. By the Intermediate Value Theorem, the particle must return to $x = 7$ during the time interval.	$2 : \begin{cases} 1 : \text{sandwiches initial position} \\ 1 : \text{answer with reason} \end{cases}$

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Question 2

(a) $\int_0^{4.5} a(t)\,dt = 66.532128$

At time $t = 4.5$, tank A contains 66.532 liters of water.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $a(k) = 20.5 \Rightarrow k = 0.892040$

$\int_0^{k} (20.5 - a(t))\,dt = 10.599191$

At time $t = k$, the difference in the amounts of water in the tanks is 10.599 liters.

$3 : \begin{cases} 1 : \text{sets } a(k) = 20.5 \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^{2.416} b(t)\,dt = \int_0^{k} b(t)\,dt + \int_k^{2.416} b(t)\,dt$

$\int_0^{k} b(t)\,dt = 20.5 \cdot k = 18.286826$

On $k < t < 2.416$, tank A receives $\int_k^{2.416} a(t)\,dt = 44.497051$ liters of water, which is 14.470 more liters of water than tank B.

Therefore, $\int_k^{2.416} b(t)\,dt = \int_k^{2.416} a(t)\,dt - 14.470 = 30.027051.$

$\int_0^{k} b(t)\,dt + \int_k^{2.416} b(t)\,dt = 48.313876$

At time $t = 2.416$, tank B contains 48.314 (or 48.313) liters of water.

$2 : \begin{cases} 1 : \int_k^{2.416} a(t)\,dt \\ 1 : \text{answer} \end{cases}$

(d) $w'(3.5) - a'(3.5) = -1.14298 < 0$

The difference $w(t) - a(t)$ is decreasing at $t = 3.5$.

$2 : \begin{cases} 1 : w'(3.5) - a'(3.5) < 0 \\ 1 : \text{conclusion} \end{cases}$

Question 3

(a) $\dfrac{g(5)-g(-5)}{5-(-5)} = \dfrac{12-(\pi+7)}{10} = \dfrac{5-\pi}{10}$	$3 : \begin{cases} 1 : \text{difference quotient} \\ 2 : \text{answer} \end{cases}$
(b) $g'(x) = f(x)$ $g'(3) = f(3) = 4$ The instantaneous rate of change of g at $x = 3$ is 4.	1 : answer
(c) The graph of g is concave up on $-5 < x < -2$ and $0 < x < 3$, because $g'(x) = f(x)$ is increasing on these intervals.	2 : intervals with justification
(d) $g'(x) = f(x)$ is defined at all x with $-5 < x < 5$. $g'(x) = f(x) = 0$ at $x = -2$ and $x = 1$. Therefore, g has critical points at $x = -2$ and $x = 1$. g has neither a local maximum nor a local minimum at $x = -2$ because g' does not change sign there. g has a local minimum at $x = 1$ because g' changes from negative to positive there.	$3 : \begin{cases} 1 : \text{considers } f(x) = 0 \\ 1 : \text{critical points at} \\ \quad x = -2 \text{ and } x = 1 \\ 1 : \text{answers with justifications} \end{cases}$

Question 4

(a) $\int_0^6 f'(x)\,dx \approx 2 \cdot 3.5 + 2 \cdot 0.8 + 2 \cdot 5.8 = 20.2$

$f(6) - f(0) = \int_0^6 f'(x)\,dx$

$f(6) = f(0) + \int_0^6 f'(x)\,dx \approx 20 + 20.2 = 40.2$

$3 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$

(b) Since $f'(x) \le 7$, $\int_0^6 f'(x)\,dx \le 6 \cdot 7 = 42$.

$f(6) - f(0) \le 42 \Rightarrow f(6) \le 20 + 42 = 62$

Therefore, the actual value of $f(6)$ could not be 70.

$2 : \begin{cases} 1 : \text{integral bound} \\ 1 : \text{answer with reasoning} \end{cases}$

(c) $\int_2^4 f''(x)\,dx = f'(4) - f'(2) = 1.7 - 2 = -0.3$

$2 : \begin{cases} 1 : \text{Fundamental Theorem of Calculus} \\ 1 : \text{answer} \end{cases}$

(d) $\lim_{x\to 0}\left(f(x) - 20e^x \right) = 0$

$\lim_{x\to 0}(0.5 f(x) - 10) = 0$

Using L'Hospital's Rule,

$$\lim_{x\to 0}\frac{f(x) - 20e^x}{0.5 f(x) - 10} = \lim_{x\to 0}\frac{f'(x) - 20e^x}{0.5 f'(x)} = \frac{4 - 20}{0.5(4)} = -8$$

$2 : \begin{cases} 1 : \text{L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$

Question 5

(a) Area $= \int_0^{\pi/2} (g(x) - f(x))\, dx$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \end{cases}$

(b)
$$\begin{aligned} \text{Volume} &= \pi \int_0^{\pi/2} \left((g(x))^2 - (f(x))^2 \right) dx \\ &= \pi \int_0^{\pi/2} \left(\left(e^x\right)^2 - \left(\sqrt{\cos x}\right)^2 \right) dx \\ &= \pi \int_0^{\pi/2} \left(e^{2x} - \cos x \right) dx = \pi \left[\frac{1}{2} e^{2x} - \sin x \right]_{x=0}^{x=\pi/2} \\ &= \pi \left(\frac{1}{2} e^{\pi} - \sin \frac{\pi}{2} - \left(\frac{1}{2} - 0 \right) \right) = \pi \left(\frac{1}{2} e^{\pi} - \frac{3}{2} \right) \end{aligned}$$

$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivatives} \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \dfrac{1}{2} \displaystyle\int_0^{\pi/2} \pi \left(\frac{g(x) - f(x)}{2} \right)^2 dx$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{constant} \end{cases}$

Question 6

(a) $A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\left.\frac{dA}{dt}\right|_{r=9} = 2\pi \cdot 9 \cdot \frac{3}{2} = 27\pi$$

When the radius is 9 centimeters, the area is changing at a rate of $27\pi\ \text{cm}^2/\text{sec}$.

$$3 : \begin{cases} 1 : \dfrac{dA}{dt} \\ 1 : \text{answer} \\ 1 : \text{units} \end{cases}$$

(b) $w'(t) = (-12)(-0.5)e^{-0.5t} = 6e^{-0.5t}$

$$6e^{-0.5t} = 3 \Rightarrow e^{-0.5t} = \frac{1}{2} \Rightarrow -0.5t = \ln\left(\frac{1}{2}\right) \Rightarrow t = 2\ln 2$$

The radius is increasing at a rate of 3 centimeters per second at time $t = 2\ln 2$ seconds.

$$3 : \begin{cases} 1 : w'(t) \\ 1 : \text{sets } w'(t) = 3 \\ 1 : \text{answer} \end{cases}$$

(c) $\displaystyle\int_0^3 \left(t^2 - 4t + 4\right) dt = \left[\frac{1}{3}t^3 - 2t^2 + 4t\right]_{t=0}^{t=3}$

$$= \left(\frac{1}{3}\cdot 3^3 - 2\cdot 3^2 + 4\cdot 3\right) - \left(\frac{1}{3}\cdot 0^3 - 2\cdot 0^2 + 4\cdot 0\right)$$

$$= 9 - 18 + 12 = 3$$

The radius increases by 3 centimeters from time $t = 0$ to time $t = 3$ seconds.

$$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$