AP® CollegeBoard

Please note: Some of the questions in this former practice exam may no longer perfectly align with the AP exam. Even though these questions do not fully represent the 2020 exam, teachers indicate that imperfectly aligned questions still provide instructional value. Teachers can consult the Question Bank to determine the degree to which these questions align to the 2020 Exam.

This exam may not be posted on school or personal websites, nor electronically redistributed for any reason. This exam is provided by the College Board for AP Exam preparation. Teachers are permitted to download the materials and make copies to use with their students in a classroom setting only. To maintain the security of this exam, teachers should collect all materials after their administration and keep them in a secure location.

Further distribution of these materials outside of the secure College Board site disadvantages teachers who rely on uncirculated questions for classroom testing. Any additional distribution is in violation of the College Board's copyright policies and may result in the termination of Practice Exam access for your school as well as the removal of access to other online services such as the AP Teacher Community and Online Score Reports.

Collect an answer sheet from each student. Check that each answer sheet has an AP number label and an AP Exam label.

After all answer sheets have been collected, say:

> Now you must seal your Section I booklet. Remove the remaining white seals from the backing and press one on each area of your exam booklet cover marked "PLACE SEAL HERE." Fold each seal over the back cover. When you have finished, place the booklet on your desk, faceup. I will now collect your Section I booklet. . . .

Collect a Section I booklet from each student. Check that each student has signed the front cover of the sealed Section I booklet.

There is a 10-minute break between Sections I and II.

When all Section I materials have been collected and accounted for and you are ready for the break, say:

> Please listen carefully to these instructions before we take a 10-minute break. All items you placed under your chair at the beginning of this exam must stay there, and you are not permitted to open or access them in any way. Leave your shrinkwrapped Section II packet on top of your desk during the break. You are not allowed to consult teachers, other students, notes, or textbooks during the break. You may not make phone calls, send text messages, use your calculators, check email, use a social networking site, or access any electronic or communication device. Remember, you may never discuss the multiple-choice exam content with anyone, and if you disclose the content through any means, your AP Exam score will be canceled. Are there any questions? . . .

> You may begin your break. Testing will resume at ———————— .

SECTION II: Free Response

After the break, say:

> May I have everyone's attention? Place your Student Pack on your desk. . . .

> You may now remove the shrinkwrap from the Section II packet, but do not open the Section II exam booklet until you are told to do so. . . .

> Read the bulleted statements on the front cover of the exam booklet. Look up when you have finished. . . .

> Now take an AP number label from your Student Pack and place it on the shaded box. If you don't have any AP number labels, write your AP number in the box. Look up when you have finished. . . .

> Read the last statement. . . .

> Using your pen, print the first, middle, and last initials of your legal name in the boxes and print today's date where indicated. This constitutes your signature and your agreement to the statements on the front cover. . . .

> Turn to the back cover and, using your pen, complete Item 1 under "Important Identification Information." Print the first two letters of your last name and the first letter of your first name in the boxes. Look up when you have finished. . . .

> In Item 2, print your date of birth in the boxes. . . .

> **In Item 3, write the school code you printed on the front of your Student Pack in the boxes. . . .**
>
> **Read Item 4. . . .**
>
> **Are there any questions? . . .**
>
> **If this is your last AP Exam, you may keep your Student Pack. Place it under your chair for now. Otherwise I will collect all Student Packs. . . .**
>
> **Read the information on the back cover of the exam booklet, paying careful attention to the bulleted statements in the instructions. Do not open the exam booklet or break the seals in the exam booklet until you are told to do so. Look up when you have finished. . . .**

Collect the Student Packs.

Then say:

> **Are there any questions? . . .**
>
> **Section II also has two parts that are timed separately. You are responsible for pacing yourself and may proceed freely from one question to the next within each part. Graphing calculators are required for Part A, so you may keep your calculators on your desk. You must write your answers in the appropriate space in the exam booklet using a No. 2 pencil or a pen with black or dark blue ink. Do not break the seals for Part B at this time. Are there any questions? . . .**
>
> **You have 30 minutes to answer the questions in Part A. If you need more paper during the exam, raise your hand. At the top of each extra sheet of paper you use, write only your AP number and the question number you are working on. Do not write your name. Open your exam booklet and begin.**

🕐 **Note Start Time _____ . Note Stop Time _____ .**

Check that students are working on Part A only and writing their answers in their exam booklets using pencils or pens with black or dark blue ink. The pages for the Part A questions are marked with large 1's or 2's at the top of each page to assist you in monitoring their work.

After 20 minutes, say:

> **There are 10 minutes remaining in Part A.**

After 10 minutes, say:

> **Stop working on Part A. Calculators are not allowed for Part B. Please put all of your calculators under your chair. . . .**
>
> **Turn to page 13. You have 1 hour for Part B. During this time you may go back to Part A, but you may <u>not</u> use your calculator. Remember to show your work and write your answer to each part of each problem in the appropriate space in the exam booklet. Are there any questions? . . .**
>
> **Using your finger, break open the seals on Part B. Do not peel the seals away from the booklet. You may go on to the next page and begin Part B.**

🕐 **Note Start Time _____ . Note Stop Time _____ .**

After 50 minutes, say:

> **There are 10 minutes remaining in Part B.**

After 10 minutes, say:

> **Stop working and close your exam booklet. Place it on your desk, faceup. . . .**

If any students used extra paper for a question in the free-response section, have those students staple the extra sheet(s) to the first page corresponding to that question in their exam booklets. Complete an Incident Report after the exam (see After-Exam Tasks below).

Then say:

> **Remain in your seat, without talking, while the exam materials are collected. . . .**

Collect a Section II exam booklet from each student. Check for the following:

- Exam booklet front cover: The student placed an AP number label on the shaded box and printed their initials and today's date.
- Exam booklet back cover: The student completed the "Important Identification Information" area.

When all exam materials have been collected and accounted for, return to students any electronic devices you may have collected before the start of the exam.

> **If you are giving the regularly scheduled exam, say:**
> **You may not discuss or share the free-response exam content with anyone unless it is released on the College Board website in about two days. Your AP Exam score results will be available online in July.**
>
> **If you are giving the alternate exam for late testing, say:**
> **None of the content in this exam may ever be discussed or shared in any way at any time. Your AP Exam score results will be available online in July.**

If any students completed the AP number card at the beginning of this exam, say:

> **Please remember to take your AP number card with you. You will need the information on this card to view your scores and order AP score reporting services online.**

Then say:

> **You are now dismissed.**

After-Exam Tasks

Be sure to give the completed seating chart to the AP coordinator. Schools must retain seating charts for at least six months (unless the state or district requires that they be retained for a longer period of time). Schools should not return any seating charts in their exam shipments unless they are required as part of an Incident Report.

NOTE: If you administered exams to students with accommodations, review the *2017-18 AP Coordinator's Manual* and the *2017-18 AP SSD Guidelines* for information about completing the NAR form, and returning these exams.

The exam proctor should complete the following tasks if asked to do so by the AP coordinator. Otherwise, the AP coordinator must complete these tasks:

- Complete an Incident Report for any students who used extra paper for the free-response section. (Incident Report forms are provided in the coordinator packets sent with the exam shipments.) **These forms must be completed with a No. 2 pencil.** It is best to complete a single Incident Report for multiple students per exam subject, per administration (regular or late testing), as long as all required information is provided. Include all exam booklets with extra sheets of paper in an Incident Report return envelope (see page 67 of the *2017-18 AP Coordinator's Manual* for complete details).

AP® Calculus AB Exam

SECTION II: Free Response

2018

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.33%

Part B

Number of Questions
4
Time
1 hour
Electronic Device
None allowed
Percent of Section II Score
66.67%

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During Part B, you may continue to work on the questions in Part A without the use of a calculator.

As you begin each part, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\, dx$ may not be written as fnInt(X², X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 40BP4-S

66

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (minutes)	0	1	5	6	8
$g(t)$ (cubic feet per minute)	12.8	15.1	20.5	18.3	22.7

1. Grain is being added to a silo. At time $t = 0$, the silo is empty. The rate at which grain is being added is modeled by the differentiable function g, where $g(t)$ is measured in cubic feet per minute for $0 \leq t \leq 8$ minutes. Selected values of $g(t)$ are given in the table above.

   (a) Using the data in the table, approximate $g'(3)$. Using correct units, interpret the meaning of $g'(3)$ in the context of the problem.

   (b) Write an integral expression that represents the total amount of grain added to the silo from time $t = 0$ to time $t = 8$. Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate the integral.

Do not write beyond this border.

Do not write beyond this border.

(c) The grain in the silo is spoiling at a rate modeled by $w(t) = 32 \cdot \sqrt{\sin\left(\dfrac{\pi t}{74}\right)}$, where $w(t)$ is measured in cubic feet per minute for $0 \leq t \leq 8$ minutes. Using the result from part (b), approximate the amount of unspoiled grain remaining in the silo at time $t = 8$.

(d) Based on the model in part (c), is the amount of unspoiled grain in the silo increasing or decreasing at time $t = 6$? Show the work that leads to your answer.

GO ON TO THE NEXT PAGE.

2. A snail is traveling along a straight path. The snail's velocity can be modeled by $v(t) = 1.4 \ln\left(1 + t^2\right)$ inches per minute for $0 \leq t \leq 15$ minutes.

(a) Find the acceleration of the snail at time $t = 5$ minutes.

(b) What is the displacement of the snail over the interval $0 \leq t \leq 15$ minutes?

Do not write beyond this border.

Do not write beyond this border.

(c) At what time t, $0 \leq t \leq 15$, is the snail's instantaneous velocity equal to its average velocity over the interval $0 \leq t \leq 15$?

(d) An ant arrives at the snail's starting position at time $t = 12$ minutes and follows the snail's path. During the interval $12 \leq t \leq 15$ minutes, the ant travels in the same direction as the snail with a constant acceleration of 2 inches per minute per minute. The ant catches up to the snail at time $t = 15$ minutes. The ant's velocity at time $t = 12$ is B inches per minute. Find the value of B.

Unauthorized copying or reuse of any part of this page is illegal.

-7-

GO ON TO THE NEXT PAGE.

END OF PART A

**IF YOU FINISH BEFORE TIME IS CALLED,
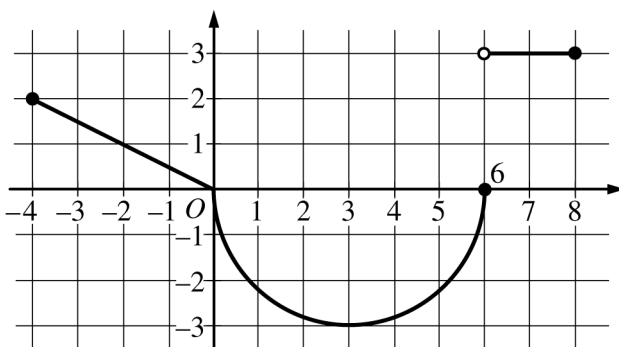YOU MAY CHECK YOUR WORK ON PART A ONLY.**

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

Graph of g

3. The function g is defined on the closed interval $[-4, 8]$. The graph of g consists of two linear pieces and a semicircle, as shown in the figure above. Let f be the function defined by $f(x) = 3x + \int_0^x g(t) \, dt$.

(a) Find $f(7)$ and $f'(7)$.

(b) Find the value of x in the closed interval $[-4, 3]$ at which f attains its maximum value. Justify your answer.
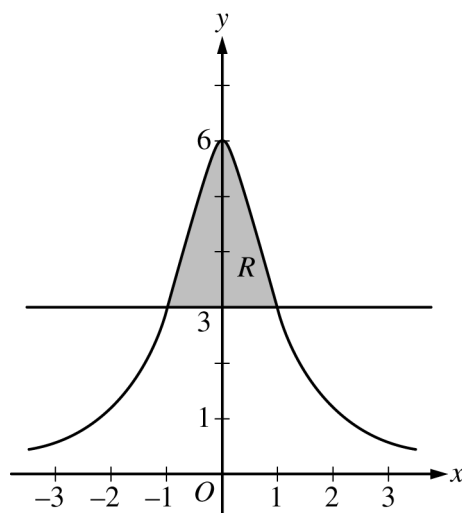
NO CALCULATOR ALLOWED

(c) For each of $\lim\limits_{x \to 0^-} g'(x)$ and $\lim\limits_{x \to 0^+} g'(x)$, find the value or state that it does not exist.

(d) Find $\lim\limits_{x \to -2} \dfrac{f(x) + 7}{e^{3x+6} - 1}$.

-15-

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

4. Let f be the function defined by $f(x) = \dfrac{6}{1 + x^2}$. Let R be the shaded region bounded by the graph of f and

the horizontal line $y = 3$, as shown in the figure above.

(a) Find the area of R.

-16-

Do not write beyond this border.

(b) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Let $h(x)$ be the vertical distance between the point $(x, f(x))$ and the horizontal line $y = 3$. Find the rate of change of $h(x)$ with respect to x at $x = 2$.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

5. During a chemical reaction, the function $y = f(t)$ models the amount of a substance present, in grams, at

time t seconds. At the start of the reaction $(t = 0)$, there are 10 grams of the substance present. The function

$y = f(t)$ satisfies the differential equation $\dfrac{dy}{dt} = -0.02y^2$.

(a) Use the line tangent to the graph of $y = f(t)$ at $t = 0$ to approximate the amount of the substance
remaining at time $t = 2$ seconds.

(b) Using the given differential equation, determine whether the graph of f could resemble the following
graph. Give a reason for your answer.

-18-
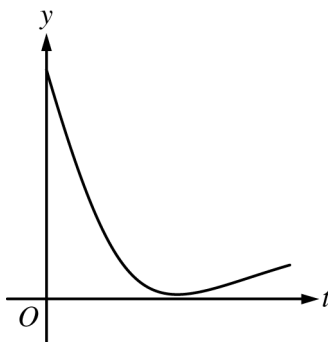
NO CALCULATOR ALLOWED

(c) Find an expression for $y = f(t)$ by solving the differential equation $\dfrac{dy}{dt} = -0.02y^2$ with the initial

condition $f(0) = 10$.

<div style="writing-mode: vertical-rl">Do not write beyond this border.</div>

<div style="writing-mode: vertical-rl">Do not write beyond this border.</div>

(d) Determine whether the amount of the substance is changing at an increasing or a decreasing rate. Explain your reasoning.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

6. Consider the curve given by the equation $2(x - y) = 3 + \cos y$. For all points on the curve, $\dfrac{2}{3} \le \dfrac{dy}{dx} \le 2$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{2}{2 - \sin y}$.

(b) For $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$, there is a point P on the curve through which the line tangent to the curve has slope 1.

Find the coordinates of the point P.

NO CALCULATOR ALLOWED

(c) Determine the concavity of the curve at points for which $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$. Give a reason for your answer.

(d) Let $y = f(x)$ be a function, defined implicitly by $2(x - y) = 3 + \cos y$, that is continuous on the closed

interval $[2, 2.1]$ and differentiable on the open interval $(2, 2.1)$. Use the Mean Value Theorem on the

interval $[2, 2.1]$ to show that $\dfrac{1}{15} \le f(2.1) - f(2) \le \dfrac{1}{5}$.

GO ON TO THE NEXT PAGE.

Do not write beyond this border.