2020

AP CollegeBoard

Please note: Some of the questions in this former practice exam may no longer perfectly align with the AP exam. Even though these questions do not fully represent the 2020 exam, teachers indicate that imperfectly aligned questions still provide instructional value. Teachers can consult the Question Bank to determine the degree to which these questions align to the 2020 Exam.

This exam may not be posted on school or personal websites, nor electronically redistributed for any reason. This exam is provided by the College Board for AP Exam preparation. Teachers are permitted to download the materials and make copies to use with their students in a classroom setting only. To maintain the security of this exam, teachers should collect all materials after their administration and keep them in a secure location.

Further distribution of these materials outside of the secure College Board site disadvantages teachers who rely on uncirculated questions for classroom testing. Any additional distribution is in violation of the College Board's copyright policies and may result in the termination of Practice Exam access for your school as well as the removal of access to other online services such as the AP Teacher Community and Online Score Reports.

SEATING POLICY FOR AP CALCULUS AB AND CALCULUS BC EXAMS

Testing Window	Exams Administered at Schools in the United States, Canada, Puerto Rico, and the U.S. Virgin Islands	Exams Administered at Schools Outside the United States, Canada, Puerto Rico, and the U.S. Virgin Islands
Regularly Scheduled Exams	Students must be seated no less than four feet apart.	Students must be seated no less than five feet apart.
Late-Testing Exams	Students must be seated no less than five feet apart.	

Graphing calculators are required to answer some of the questions on the AP Calculus Exams. Before starting the exam administration, make sure each student has a graphing calculator from the approved list on page 49 of the *2016-17 AP Coordinator's Manual*. If a student does not have a graphing calculator from the approved list, you may provide one from your supply. If the student does not want to use the calculator you provide or does not want to use a calculator at all, he or she must hand copy, date, and sign the release statement on page 47 of the *AP Coordinator's Manual*.

During the administration of Section I, Part B, and Section II, Part A, students may have no more than two graphing calculators on their desks. Calculators may not be shared. **Calculator memories do not need to be cleared before or after the exam.** Students with Hewlett-Packard 48–50 Series and Casio FX-9860 graphing calculators may use cards designed for use with these calculators. Proctors should make sure infrared ports (Hewlett-Packard) are not facing each other. **Since graphing calculators can be used to store data, including text, proctors should monitor that students are using their calculators appropriately. Attempts by students to use the calculator to remove exam questions and/or answers from the room may result in the cancellation of AP Exam scores.**

The AP Calculus AB Exam and the AP Calculus BC Exam should be administered simultaneously. They may be administered in separate rooms, or in the same room if it is more convenient.

SECTION I: Multiple Choice

Do not begin the exam instructions below until you have completed the appropriate General Instructions for your group.

These exams include survey questions. The time allowed for the survey questions is in addition to the actual test-taking time.

Make sure you begin the exams at the designated time. Remember, you must complete a seating chart for this exam. See pages 325–326 for a seating chart template and instructions. See the *2016-17 AP Coordinator's Manual* for exam seating requirements (pages 51–54).

If you are giving the regularly scheduled exam, say:

It is Tuesday morning, May 9, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.

If you are giving the alternate exam for late testing, say:

It is Thursday morning, May 18, and you will be taking either the AP Calculus AB Exam or the AP Calculus BC Exam.

In a moment, you will open the packet that contains your exam materials. By opening this packet, you agree to all of the AP Program's policies and procedures outlined in the *2016-17 Bulletin for AP Students and Parents*.

If you are giving the AP Calculus AB exam, say:

Look at your exam packet and confirm that the exam title is "AP Calculus AB." Raise your hand if your exam packet contains any title other than "AP Calculus AB" and I will help you.

If you are giving the AP Calculus BC exam, say:

Look at your exam packet and confirm that the exam title is "AP Calculus BC." Raise your hand if your exam packet contains any title other than "AP Calculus BC" and I will help you.

If you are giving both the AP Calculus AB and Calculus BC exams, say:

Look at your exam packet and confirm that the exam title is "AP Calculus AB" or "AP Calculus BC," depending upon which exam you are taking today. Raise your hand if your exam packet contains any other title and I will help you.

Once you confirm that all students have the correct exam, say:

You may now remove the shrinkwrap from your exam packet and take out the Section I booklet, but do not open the booklet or the shrinkwrapped Section II materials. Put the white seals aside. . . .

Carefully remove the AP Exam label found near the top left of your exam booklet cover. Now place it on page 1 of your answer sheet on the light blue box near the top right corner that reads "AP Exam Label."

If students accidentally place the exam label in the space for the number label or vice versa, advise them to leave the labels in place. They should not try to remove the label; their exam can still be processed correctly.

Read the statements on the front cover of Section I and look up when you have finished. . . .

Sign your name and write today's date. Look up when you have finished. . . .

Now print your full legal name where indicated. Are there any questions? . . .

Turn to the back cover of your exam booklet and read it completely. Look up when you have finished. . . .

Are there any questions? . . .

You will now take the multiple-choice portion of the exam. You should have in front of you the multiple-choice booklet and your answer sheet. You may never discuss the multiple-choice exam content at any time in any form with anyone, including your teacher and other students. If you disclose the multiple-choice exam content through any means, your AP Exam score will be canceled.

Open your answer sheet to page 2. You must complete the answer sheet using a No. 2 pencil only. Mark all of your responses beginning on page 2 of your answer sheet, one response per question. Completely fill in the circles. If you need to erase, do so carefully and completely. No credit will be given for anything written in the exam booklet. Scratch paper is not allowed, but you may use the margins or any blank space in the exam booklet for scratch work.

Section I is divided into two parts. Each part is timed separately, and you may work on each part only during the time allotted for it. Calculators are not allowed in Part A. Please put your calculators under your chair. Are there any questions? . . .

You have 1 hour for Part A. Part A questions are numbered 1 through 30. Mark your responses for these questions on page 2 of your answer sheet. Open your Section I booklet and begin.

Note Start Time here _____. Note Stop Time here _____. Check that students are marking their answers in pencil on page 2 of their answer sheets and that they are not looking beyond Part A. The line of A's at the top of each page will assist you in monitoring students' work. After 50 minutes, say:

There are 10 minutes remaining.

After 10 minutes, say:

Stop working on Part A and turn to page 24 in your Section I booklet. . . .

On that page, you should see an area marked "PLACE SEAL HERE." Making sure all of your other exam materials, including your answer sheet, are out of the way, take one of your seals and press it on that area and then fold the seal over the open edge to the front cover. Be sure you don't seal the Part B section of the booklet or let the seal touch anything except the marked areas. . . .

After all students have sealed Part A, say:

Graphing calculators are required for Part B. You may get your calculators from under your chair and place them on your desk. Part B questions are numbered 76 through 90. Fold your answer sheet so only page 3 is showing and mark your responses for these questions on that page. You have 45 minutes for Part B. You may begin.

Note Start Time here _____. Note Stop Time here _____. Check that students have sealed their booklets properly and are now working on Part B. The large B's in an alternating shaded pattern at the top of each page will assist you in monitoring their work. Proctors should make sure that students are using their calculators appropriately. Proctors should also make sure Hewlett-Packard calculators' infrared ports are not facing each other. After 35 minutes, say:

There are 10 minutes remaining.

After 10 minutes, say:

Stop working and turn to page 38. You have 3 minutes to answer Questions 91–94. These are survey questions and will not affect your score. Note that each survey question has five answer options. You may not go back to work on any of the exam questions. . . .

Give students approximately 3 minutes to answer the survey questions. Then say:

Close your booklet and put your answer sheet on your desk, face up. Make sure you have your AP number label and an AP Exam label on page 1 of your answer sheet. Sit quietly while I collect your answer sheets.

Collect an answer sheet from each student. Check that each answer sheet has an AP number label and an AP Exam label. After all answer sheets have been collected, say:

Now you must seal your Section I booklet. Remove the remaining white seals from the backing and press one on each area of your exam booklet cover marked "PLACE SEAL HERE." Fold each seal over the back cover. When you have finished, place the booklet on your desk, face up. I will now collect your Section I booklet. . . .

Collect a Section I booklet from each student. Check that each student has signed the front cover of the sealed Section I booklet.

There is a 10-minute break between Sections I and II. When all Section I materials have been collected and accounted for and you are ready for the break, say:

Please listen carefully to these instructions before we take a 10-minute break. All items you placed under your chair at the beginning of this exam must stay there, and you are not permitted to open or access them in any way. Leave your shrinkwrapped Section II packet on top of your desk during the break. You are not allowed to consult teachers, other students, notes, or textbooks during the break. You may not make phone calls, send text messages, use your calculators, check email, use a social networking site, or access any electronic or communication device. Remember, you may never discuss the multiple-choice exam content at any time in any form with anyone, including your teacher and other students. If you disclose the multiple-choice exam content through any means, your AP Exam score will be canceled. Are there any questions? . . .

You may begin your break. Testing will resume at _____.

COMPLETE THIS AREA AT EACH EXAM (IF APPLICABLE).

P. SURVEY QUESTIONS — Answer the survey questions in the AP Student Pack. Do not put responses to exam questions in this section.

1	A B C D E F G H I	4	A B C D E F G H I	7	A B C D E F G H I
2	A B C D E F G H I	5	A B C D E F G H I	8	A B C D E F G H I
3	A B C D E F G H I	6	A B C D E F G H I	9	A B C D E F G H I

Q. LANGUAGE — Do not complete this section unless instructed to do so.

If this answer sheet is for the French Language and Culture, German Language and Culture, Italian Language and Culture, Spanish Language and Culture, or Spanish Literature and Culture Exam, please answer the following questions. Your responses will not affect your score.

1. Have you lived or studied for one month or more in a country where the language of the exam you are now taking is spoken?

○ Yes ○ No

2. Do you regularly speak or hear the language at home?

○ Yes ○ No

QUESTIONS 1–75

Indicate your answers to the exam questions in this section (pages 2 and 3). Mark only one response per question for Questions 1 through 120. If a question has only four answer options, do not mark option E. Answers written in the multiple-choice booklet will not be scored.

COMPLETE MARK ● EXAMPLES OF INCOMPLETE MARKS 

You must use a No. 2 pencil and marks must be complete. Do not use a mechanical pencil. It is very important that you fill in the entire circle darkly and completely. If you change your response, erase as completely as possible. Incomplete marks or erasures may affect your score.

1	A B C D E	26	A B C D E	51	A B C D E
2	A B C D E	27	A B C D E	52	A B C D E
3	A B C D E	28	A B C D E	53	A B C D E
4	A B C D E	29	A B C D E	54	A B C D E
5	A B C D E	30	A B C D E	55	A B C D E
6	A B C D E	31	A B C D E	56	A B C D E
7	A B C D E	32	A B C D E	57	A B C D E
8	A B C D E	33	A B C D E	58	A B C D E
9	A B C D E	34	A B C D E	59	A B C D E
10	A B C D E	35	A B C D E	60	A B C D E
11	A B C D E	36	A B C D E	61	A B C D E
12	A B C D E	37	A B C D E	62	A B C D E
13	A B C D E	38	A B C D E	63	A B C D E
14	A B C D E	39	A B C D E	64	A B C D E
15	A B C D E	40	A B C D E	65	A B C D E
16	A B C D E	41	A B C D E	66	A B C D E
17	A B C D E	42	A B C D E	67	A B C D E
18	A B C D E	43	A B C D E	68	A B C D E
19	A B C D E	44	A B C D E	69	A B C D E
20	A B C D E	45	A B C D E	70	A B C D E
21	A B C D E	46	A B C D E	71	A B C D E
22	A B C D E	47	A B C D E	72	A B C D E
23	A B C D E	48	A B C D E	73	A B C D E
24	A B C D E	49	A B C D E	74	A B C D E
25	A B C D E	50	A B C D E	75	A B C D E

ETS USE ONLY

Exam		0 1 2 3 4 5 6 7 8 9
		0 1 2 3 4 5 6 7 8 9
Exam		0 1 2 3 4 5 6 7 8 9
		0 1 2 3 4 5 6 7 8 9

SELECTED MEDIA EXAMS	R	W	O	OTHER EXAMS	R	W	O
PT02				TOTAL			
PT03				Subscore (if applicable)			
PT04				Subscore (if applicable)			

DO NOT WRITE IN THIS AREA

QUESTIONS 76–120

Be sure each mark is dark and completely fills the circle. If a question has only four answer options, do not mark option E.

76	A B C D E	91	A B C D E	106	A B C D E
77	A B C D E	92	A B C D E	107	A B C D E
78	A B C D E	93	A B C D E	108	A B C D E
79	A B C D E	94	A B C D E	109	A B C D E
80	A B C D E	95	A B C D E	110	A B C D E
81	A B C D E	96	A B C D E	111	A B C D E
82	A B C D E	97	A B C D E	112	A B C D E
83	A B C D E	98	A B C D E	113	A B C D E
84	A B C D E	99	A B C D E	114	A B C D E
85	A B C D E	100	A B C D E	115	A B C D E
86	A B C D E	101	A B C D E	116	A B C D E
87	A B C D E	102	A B C D E	117	A B C D E
88	A B C D E	103	A B C D E	118	A B C D E
89	A B C D E	104	A B C D E	119	A B C D E
90	A B C D E	105	A B C D E	120	A B C D E

QUESTIONS 121–126

For Students Taking AP Biology

Write your answer in the boxes at the top of the griddable area and fill in the corresponding circles. Mark only one circle in any column. You will receive credit only if the circles are filled in correctly.

121 122 123 124 125 126

QUESTIONS 131–142

For Students Taking AP Computer Science Principles, AP Physics 1, or AP Physics 2

Mark two responses per question. You will receive credit only if both correct responses are selected.

131	A B C D	135	A B C D	139	A B C D
132	A B C D	136	A B C D	140	A B C D
133	A B C D	137	A B C D	141	A B C D
134	A B C D	138	A B C D	142	A B C D

DO NOT WRITE IN THIS AREA

AP® Calculus AB Exam

SECTION I: Multiple Choice

2017

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
30
Time
1 hour
Electronic Device
None allowed

Part B

Number of Questions
15
Time
45 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 30 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 90 on page 3 of the answer sheet. Because Part A and Part B offer only four answer options for each question, do not mark the (E) answer circle for any question. The survey questions are numbers 91 through 94.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Form I
Form Code 4NBP4-S

66

CALCULUS AB

SECTION I, Part A

Time—1 hour

Number of questions—30

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

1. If $f(x) = \left(2x^2 + 5\right)^7$, then $f'(x) =$

   (A) $7(4x)^6$

   (B) $7\left(2x^2 + 5\right)^6$

   (C) $14x^2\left(2x^2 + 5\right)^6$

   (D) $28x\left(2x^2 + 5\right)^6$

---

2. $\displaystyle\int \frac{1}{3x + 12}\, dx =$

   (A) $-3 \ln |x + 4| + C$

   (B) $\dfrac{1}{3} \ln |x + 4| + C$

   (C) $\ln |x + 4| + C$

   (D) $3 \ln |x + 4| + C$

GO ON TO THE NEXT PAGE.

3. If $f(x) = \dfrac{5 - x}{x^3 + 2}$, then $f'(x) =$

(A) $\dfrac{-4x^3 + 15x^2 - 2}{\left(x^3 + 2\right)^2}$

(B) $\dfrac{-2x^3 + 15x^2 + 2}{\left(x^3 + 2\right)^2}$

(C) $\dfrac{2x^3 - 15x^2 - 2}{\left(x^3 + 2\right)^2}$

(D) $\dfrac{4x^3 - 15x^2 + 2}{\left(x^3 + 2\right)^2}$

GO ON TO THE NEXT PAGE.

t	0	0.5	2	3
$v(t)$	20	60	40	30

4. The table above gives the velocity $v(t)$, in miles per hour, of a truck at selected times t, in hours. Using a trapezoidal sum with the three subintervals indicated by the table, what is the approximate distance, in miles, the truck traveled from time $t = 0$ to $t = 3$?

   (A) 140 (B) 130 (C) 125 (D) 120

---

5. If $f(x) = \sin(x^2 + \pi)$, then $f'(\sqrt{2\pi}) =$

   (A) $-2\sqrt{2\pi}$ (B) -2 (C) -1 (D) $\cos(2\sqrt{2\pi})$

GO ON TO THE NEXT PAGE.

6. If f is the function given by $f(x) = 3x^2 - x^3$, then the average rate of change of f on the closed interval $[1, 5]$ is

(A) -21 (B) -13 (C) -12 (D) -9

---

7. If $\int_4^{-10} g(x)\,dx = -3$ and $\int_4^6 g(x)\,dx = 5$, then $\int_{-10}^6 g(x)\,dx =$

(A) -8 (B) -2 (C) 2 (D) 8

GO ON TO THE NEXT PAGE.

8. If f is the function given by $f(x) = e^{x/3}$, which of the following is an equation of the line tangent to the graph of f at the point $(3 \ln 4, 4)$?

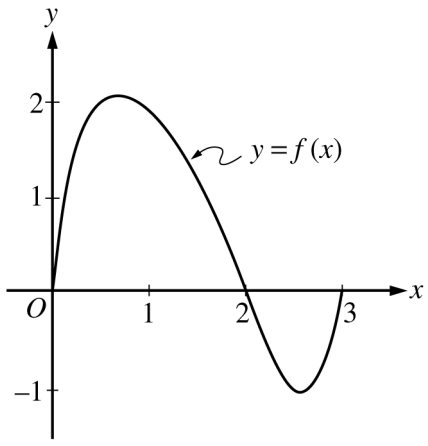
(A) $y - 4 = \frac{4}{3}(x - 3 \ln 4)$

(B) $y - 4 = 4(x - 3 \ln 4)$

(C) $y - 4 = 12(x - 3 \ln 4)$

(D) $y - 3 \ln 4 = 4(x - 4)$

GO ON TO THE NEXT PAGE.



9. The graph of a function f is shown above. Which of the following expresses the relationship between $\int_0^2 f(x)\,dx$, $\int_0^3 f(x)\,dx$, and $\int_2^3 f(x)\,dx$?

(A) $\int_0^2 f(x)\,dx < \int_0^3 f(x)\,dx < \int_2^3 f(x)\,dx$

(B) $\int_0^3 f(x)\,dx < \int_0^2 f(x)\,dx < \int_2^3 f(x)\,dx$

(C) $\int_2^3 f(x)\,dx < \int_0^2 f(x)\,dx < \int_0^3 f(x)\,dx$

(D) $\int_2^3 f(x)\,dx < \int_0^3 f(x)\,dx < \int_0^2 f(x)\,dx$

GO ON TO THE NEXT PAGE.

10. $\int_0^2 \left(x^3 + 1\right)^{1/2} x^2 \, dx =$

(A) $\frac{52}{9}$ (B) 6 (C) $\frac{26}{3}$ (D) $\frac{52}{3}$

---

11. If $x^2 + xy - 3y = 3$, then at the point $(2, 1)$, $\frac{dy}{dx} =$

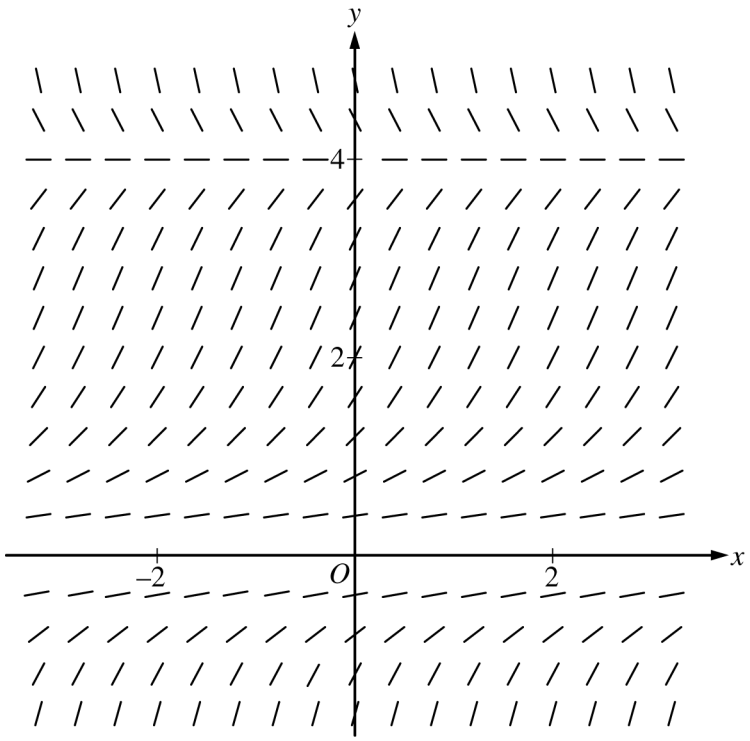
(A) 5 (B) 4 (C) $\frac{7}{3}$ (D) 2

GO ON TO THE NEXT PAGE.

12. The number of gallons of water in a storage tank at time t, in minutes, is modeled by $w(t) = 25 - t^2$ for $0 \le t \le 5$. At what rate, in gallons per minute, is the amount of water in the tank changing at time $t = 3$ minutes?

(A) 66 (B) 16 (C) -3 (D) -6

GO ON TO THE NEXT PAGE.



13. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x(4-y)}{4}$

(B) $\frac{dy}{dx} = \frac{y(4-y)}{4}$

(C) $\frac{dy}{dx} = \frac{xy(4-y)}{4}$

(D) $\frac{dy}{dx} = \frac{y^2(4-y)}{4}$

GO ON TO THE NEXT PAGE.

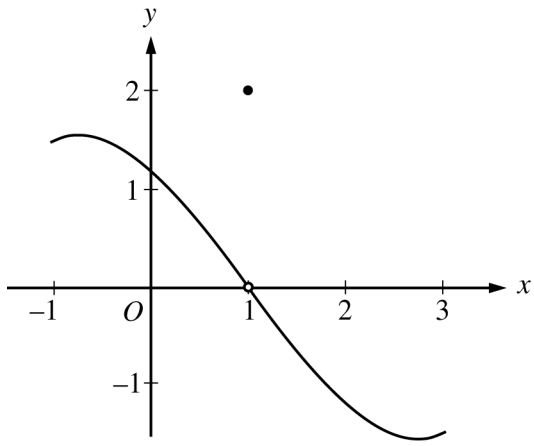
14. The weight of a population of yeast is given by a differentiable function y, where $y(t)$ is measured in grams and t is measured in days. The weight of the yeast population increases according to the equation $\frac{dy}{dt} = ky$, where k is a constant. At time $t = 0$, the weight of the yeast population is 120 grams and is increasing at the rate of 24 grams per day. Which of the following is an expression for $y(t)$?

(A) $120e^{24t}$

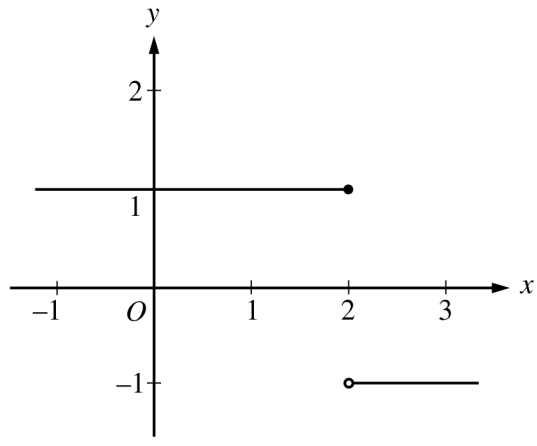
(B) $120e^{t/5}$

(C) $e^{t/5} + 119$

(D) $24t + 120$



Graph of f



Graph of g

15. The graphs of the functions f and g are shown in the figures above. Which of the following statements is false?

(A) $\lim_{x \to 1} f(x) = 0$

(B) $\lim_{x \to 2} g(x)$ does not exist.

(C) $\lim_{x \to 1} (f(x)\, g(x+1))$ does not exist.

(D) $\lim_{x \to 1} (f(x+1)\, g(x))$ exists.

16. Let f be the function defined by $f(x) = -3 + 6x^2 - 2x^3$. What is the largest open interval on which the graph of f is both concave up and increasing?

(A) $(0, 1)$ (B) $(1, 2)$ (C) $(0, 2)$ (D) $(2, \infty)$

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17. A particle moves along the x-axis so that at time $t > 0$ its position is given by $x(t) = 12e^{-t}\sin t$. What is the first time t at which the velocity of the particle is zero?

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) π

GO ON TO THE NEXT PAGE.

18. Let F be the function given by $F(x) = \int_3^x (\tan(5t)\sec(5t) - 1)\, dt$. Which of the following is an expression for $F'(x)$?

(A) $\frac{1}{5}\sec(5x) - 1$

(B) $\frac{1}{5}\sec(5x) - x$

(C) $\tan(5x)\sec(5x)$

(D) $\tan(5x)\sec(5x) - 1$

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19. Let f be the function given by $f(x) = 2\cos x + 1$. What is the approximation for $f(1.5)$ found by using the line tangent to the graph of f at $x = \frac{\pi}{2}$?

(A) -2 (B) 1 (C) $\pi - 2$ (D) $4 - \pi$

GO ON TO THE NEXT PAGE.

20. Let f be the function given by $f(x) = \dfrac{x-2}{2|x-2|}$. Which of the following is true?

(A) $\lim\limits_{x \to 2} f(x) = \dfrac{1}{2}$

(B) f has a removable discontinuity at $x = 2$.

(C) f has a jump discontinuity at $x = 2$.

(D) f has a discontinuity due to a vertical asymptote at $x = 2$.

---

21. If $f(x) = \ln x$, then $\lim\limits_{x \to 3} \dfrac{f(x) - f(3)}{x - 3}$ is

(A) $\dfrac{1}{3}$ (B) e^3 (C) $\ln 3$ (D) nonexistent

GO ON TO THE NEXT PAGE.

22. Which of the following is the solution to the differential equation $\frac{dy}{dx} = \frac{2y}{2x+1}$ with the initial condition $y(0) = e$ for $x > -\frac{1}{2}$?

(A) $y = e^{2x^2+2x+1}$

(B) $y = 2ex + e$

(C) $y = \sqrt{x^2 + x + e^2}$

(D) $y = \sqrt{\frac{1}{2}\ln(2x+1) + e^2}$

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23. The region enclosed by the graphs of $y = x^2$ and $y = 2x$ is the base of a solid. For the solid, each cross section perpendicular to the y-axis is a rectangle whose height is 3 times its base in the xy-plane. Which of the following expressions gives the volume of the solid?

(A) $3\displaystyle\int_0^4 \left(\sqrt{y} - \frac{y}{2}\right)^2 dy$

(B) $3\displaystyle\int_0^4 \left(\sqrt{y} + \frac{y}{2}\right)^2 dy$

(C) $3\displaystyle\int_0^2 (2x - x^2)^2 dx$

(D) $3\displaystyle\int_0^2 (2x + x^2)^2 dx$

---

24. If the average value of a continuous function f on the interval $[-2, 4]$ is 12, what is $\displaystyle\int_{-2}^{4} \frac{f(x)}{8} dx$?

(A) $\frac{3}{2}$ (B) 3 (C) 9 (D) 72

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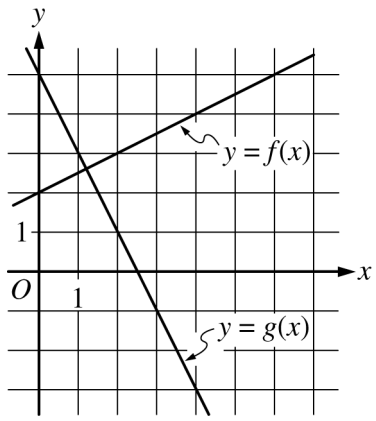

25. The figure above shows the graphs of the functions f and g. If $h(x) = f(x)g(x)$, then $h'(2) =$

(A) $\frac{13}{2}$ (B) $\frac{1}{2}$ (C) -1 (D) $-\frac{11}{2}$

---

26. $\lim_{x \to \infty} \frac{\ln\left(e^{3x} + x\right)}{x} =$

(A) 0 (B) 1 (C) 3 (D) ∞

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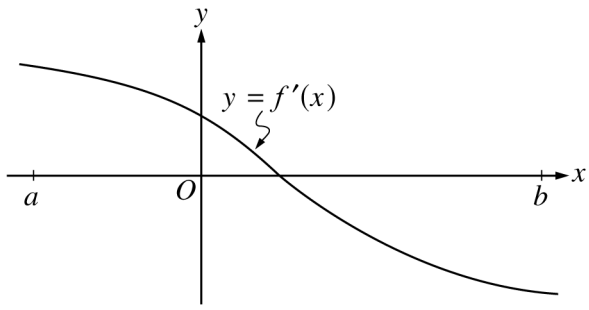

27. The graph of f', the derivative of the function f, is shown in the figure above. Which of the following statements must be true?

I. f is continuous on the open interval (a, b).

II. f is decreasing on the open interval (a, b).

III. The graph of f is concave down on the open interval (a, b).

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

GO ON TO THE NEXT PAGE.

28. An isosceles right triangle with legs of length s has area $A = \frac{1}{2}s^2$. At the instant when $s = \sqrt{32}$ centimeters, the area of the triangle is increasing at a rate of 12 square centimeters per second. At what rate is the length of the hypotenuse of the triangle increasing, in centimeters per second, at that instant?

(A) $\frac{3}{4}$ (B) 3 (C) $\sqrt{32}$ (D) 48

---

29. The graph of which of the following functions has exactly one horizontal asymptote and no vertical asymptotes?

(A) $y = \frac{1}{x^2 + 1}$

(B) $y = \frac{1}{x^3 + 1}$

(C) $y = \frac{1}{e^x - 1}$

(D) $y = \frac{1}{e^x + 1}$

GO ON TO THE NEXT PAGE.

30. For a certain continuous function f, the right Riemann sum approximation of $\int_0^2 f(x)\,dx$ with n subintervals of equal length is $\dfrac{2(n+1)(3n+2)}{n^2}$ for all n. What is the value of $\int_0^2 f(x)\,dx$?

(A) 2 (B) 6 (C) 12 (D) 20

END OF PART A

IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

PART B STARTS ON PAGE 26.

CALCULUS AB

SECTION I, Part B

Time—45 minutes

Number of questions—15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–90.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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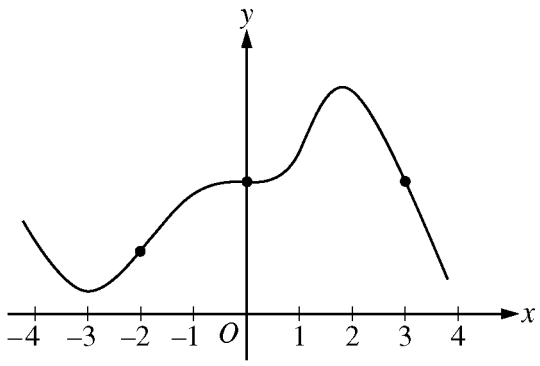

Graph of f

76. The graph of a differentiable function f is shown in the figure above. Which of the following is true?

(A) $f'(-2) < f'(0) < f'(3)$

(B) $f'(-2) < f'(3) < f'(0)$

(C) $f'(3) < f'(-2) < f'(0)$

(D) $f'(3) < f'(0) < f'(-2)$

GO ON TO THE NEXT PAGE.

77. Let $H(x)$ be an antiderivative of $\dfrac{x^3 + \sin x}{x^2 + 2}$. If $H(5) = \pi$, then $H(2) =$

(A) -9.008 (B) -5.867 (C) 4.626 (D) 12.150

---

78. The continuous function f is positive and has domain $x > 0$. If the asymptotes of the graph of f are $x = 0$ and $y = 2$, which of the following statements must be true?

(A) $\lim\limits_{x \to 0^+} f(x) = \infty$ and $\lim\limits_{x \to 2} f(x) = \infty$

(B) $\lim\limits_{x \to 0^+} f(x) = 2$ and $\lim\limits_{x \to \infty} f(x) = 0$

(C) $\lim\limits_{x \to 0^+} f(x) = \infty$ and $\lim\limits_{x \to \infty} f(x) = 2$

(D) $\lim\limits_{x \to 2} f(x) = \infty$ and $\lim\limits_{x \to \infty} f(x) = 2$

GO ON TO THE NEXT PAGE.

79. A file is downloaded to a computer at a rate modeled by the differentiable function $f(t)$, where t is the time in seconds since the start of the download and $f(t)$ is measured in megabits per second. Which of the following is the best interpretation of $f'(5) = 2.8$?

(A) At time $t = 5$ seconds, the rate at which the file is downloaded to the computer is 2.8 megabits per second.

(B) At time $t = 5$ seconds, the rate at which the file is downloaded to the computer is increasing at a rate of 2.8 megabits per second per second.

(C) Over the time interval $0 \le t \le 5$ seconds, 2.8 megabits of the file are downloaded to the computer.

(D) Over the time interval $0 \le t \le 5$ seconds, the average rate at which the file is downloaded to the computer is 2.8 megabits per second.

---

80. The function f has first derivative given by $f'(x) = x^4 - 6x^2 - 8x - 3$. On what intervals is the graph of f concave up?

(A) $(2, \infty)$ only

(B) $(0, \infty)$

(C) $(-1, 2)$

(D) $(-\infty, -1)$ and $(3, \infty)$

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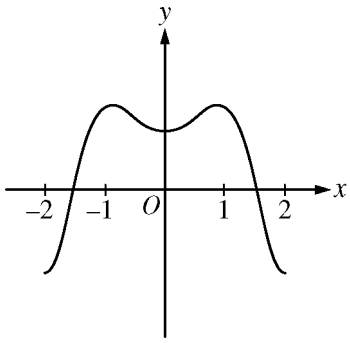
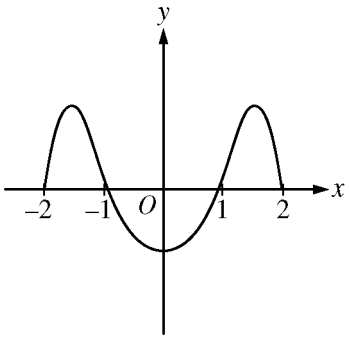


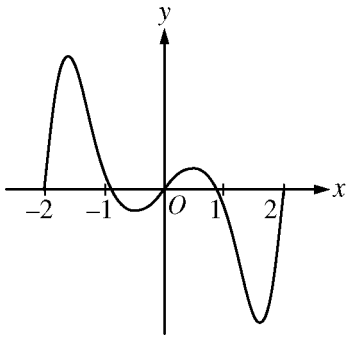
Graph of f

81. The graph of the function f is shown above for $-2 \le x \le 2$. Which of the following could be the graph of an antiderivative of f ?

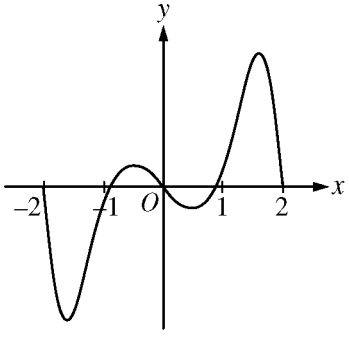
(A)



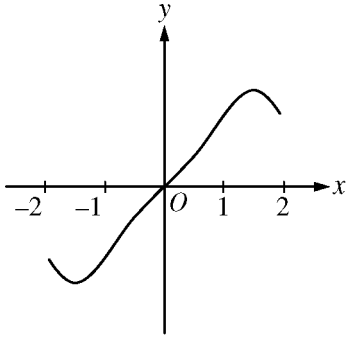
(B)



(C)



(D)



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82. A particle travels along a straight line with velocity $v(t) = 3e^{-t/2}\sin(2t)$ meters per second. What is the total distance, in meters, traveled by the particle during the time interval $0 \le t \le 2$ seconds?

(A) 0.835 (B) 1.850 (C) 2.055 (D) 2.261

---

83. Let f be a function with derivative given by $f'(x) = \dfrac{x^3 - 8x^2 + 3}{\sqrt{x^3 + 1}}$ for $-1 < x < 9$. At what value of x does f attain a relative maximum?

(A) -0.591 (B) 0 (C) 0.638 (D) 7.953

GO ON TO THE NEXT PAGE.

84. The number of bacteria in a container increases at the rate of $R(t)$ bacteria per hour. If there are 1000 bacteria at time $t = 0$, which of the following expressions gives the number of bacteria in the container at time $t = 3$ hours?

(A) $R(3)$ (B) $1000 + R(3)$ (C) $\int_0^3 R(t)\,dt$ (D) $1000 + \int_0^3 R(t)\,dt$

---

85. The function g is continuous on the closed interval $[1, 4]$ with $g(1) = 5$ and $g(4) = 8$. Of the following conditions, which would guarantee that there is a number c in the open interval $(1, 4)$ where $g'(c) = 1$?

(A) g is increasing on the closed interval $[1, 4]$.

(B) g is differentiable on the open interval $(1, 4)$.

(C) g has a maximum value on the closed interval $[1, 4]$.

(D) The graph of g has at least one horizontal tangent in the open interval $(1, 4)$.

GO ON TO THE NEXT PAGE.

$$f''(x) = x(x-1)^2(x+2)^3$$
$$g''(x) = x(x-1)^2(x+2)^3 + 1$$
$$h''(x) = x(x-1)^2(x+2)^3 - 1$$

86. The twice-differentiable functions f, g, and h have second derivatives given above. Which of the functions f, g, and h have a graph with exactly two points of inflection?

(A) g only

(B) h only

(C) f and g only

(D) f, g, and h

x	1	2	3	4	5
$f(x)$	9	4	0	–3	–5

87. The table above gives values of a function f at selected values of x. If f is twice-differentiable on the interval $1 \le x \le 5$, which of the following statements could be true?

(A) f' is negative and decreasing for $1 \le x \le 5$.

(B) f' is negative and increasing for $1 \le x \le 5$.

(C) f' is positive and decreasing for $1 \le x \le 5$.

(D) f' is positive and increasing for $1 \le x \le 5$.

GO ON TO THE NEXT PAGE.

88. Let f be the function defined by $f(x) = \ln(x^2 + 1)$, and let g be the function defined by $g(x) = x^5 + x^3$. The line tangent to the graph of f at $x = 2$ is parallel to the line tangent to the graph of g at $x = a$, where a is a positive constant. What is the value of a ?

(A) 0.246 (B) 0.430 (C) 0.447 (D) 0.790

GO ON TO THE NEXT PAGE.

89. Let f be a continuous function for all real numbers. Let g be the function defined by $g(x) = \int_1^x f(t)\,dt$. If the average rate of change of g on the interval $2 \le x \le 5$ is 6, which of the following statements must be true?

(A) The average value of f on the interval $2 \le x \le 5$ is 6.

(B) $g'(2) = 6$

(C) $\dfrac{g'(5) + g'(2)}{2} = 6$

(D) $\int_2^5 g(x)\,dx = 6$

GO ON TO THE NEXT PAGE.

90. For any function f, which of the following statements must be true?

I. If f is defined at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

II. If f is continuous at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

III. If f is differentiable at $x = a$, then $\lim_{x \to a} f(x) = f(a)$.

(A) III only

(B) I and II only

(C) II and III only

(D) I, II, and III

GO ON TO THE NEXT PAGE.