AP® CALCULUS AB
2018 SCORING GUIDELINES

Question 1

(a) $g'(3) \approx \dfrac{g(5) - g(1)}{5 - 1} = \dfrac{20.5 - 15.1}{4} = 1.35$

At time $t = 3$ minutes, the rate at which grain is being added to the silo is increasing at a rate of 1.35 cubic feet per minute per minute.

$2 : \begin{cases} 1 : \text{approximation} \\ 1 : \text{interpretation with units} \end{cases}$

(b) The total amount of grain added to the silo from time $t = 0$ to time $t = 8$ is $\int_0^8 g(t)\, dt$ cubic feet.

$$\int_0^8 g(t)\, dt \approx g(1)\cdot(1-0) + g(5)\cdot(5-1) + g(6)\cdot(6-5) + g(8)\cdot(8-6)$$
$$= 15.1\cdot 1 + 20.5\cdot 4 + 18.3\cdot 1 + 22.7\cdot 2 = 160.8$$

$3 : \begin{cases} 1 : \text{integral expression} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

(c) $\int_0^8 w(t)\, dt = 99.051497$

The approximate amount of unspoiled grain remaining in the silo at time $t = 8$ is $160.8 - \int_0^8 w(t)\, dt = 61.749$ (or 61.748) cubic feet.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $g(6) - w(6) = 18.3 - 16.063173 = 2.236827 > 0$

Because $g(6) - w(6) > 0,$ the amount of unspoiled grain is increasing at time $t = 6$.

$2 : \begin{cases} 1 : \text{considers } g(6) - w(6) \\ 1 : \text{answer} \end{cases}$

Question 2

(a) $v'(5) = 0.538462$ The acceleration of the snail at time $t = 5$ minutes is 0.538 inches per minute per minute.	1 : answer
(b) $\int_0^{15} v(t)\,dt = 76.043074$ The displacement of the snail over the interval $0 \le t \le 15$ minutes is 76.043 inches.	2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(c) $\frac{1}{15}\int_0^{15} v(t)\,dt = 5.069538$ $1.4\ln\left(1 + t^2\right) = 5.069538 \Rightarrow t = 6.031$ minutes	2 : $\begin{cases} 1 : \text{average value expression} \\ 1 : \text{answer} \end{cases}$
(d) The velocity of the ant at time t, $12 \le t \le 15$, is $\int 2\,dt = 2t + c$ inches per minute for some constant c. For $12 \le t \le 15$, the displacement of the ant is $\int_{12}^{15} (2t + c)\,dt = \left(t^2 + ct\right)\Big\vert_{t=12}^{t=15} = 81 + 3c$ inches. Thus, $81 + 3c = 76.043074 \Rightarrow c = -1.652309$. The velocity of the ant at time $t = 12$ is $B = 2 \cdot 12 - 1.652309 = 22.348$ (or 22.347) inches per minute. — OR — The velocity of the ant at time t, $12 \le t \le 15$, is $2(t - 12) + B$ inches per minute. For $12 \le t \le 15$, the displacement of the ant is $\int_{12}^{15} (2(t - 12) + B)\,dt = \left((t - 12)^2 + Bt\right)\Big\vert_{t=12}^{t=15} = 9 + 3B$ inches. $9 + 3B = 76.043074 \Rightarrow B = 22.348$ (or 22.347) inches per minute	4 : $\begin{cases} 1 : \text{ant's velocity} \\ 1 : \text{ant's displacement} \\ 1 : \text{equation} \\ 1 : \text{answer} \end{cases}$

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Question 3

(a) $f(7) = 3 \cdot 7 + \int_0^7 g(t)\,dt = 21 - \frac{9\pi}{2} + 3 = 24 - \frac{9\pi}{2}$ $f'(7) = 3 + g(7) = 3 + 3 = 6$	$2 : \begin{cases} 1 : f(7) \\ 1 : f'(7) \end{cases}$
(b) On the interval $-4 \le x \le 3$, $f'(x) = 3 + g(x)$. Because $f'(x) \ge 0$ for $-4 \le x \le 3$, f is nondecreasing over the entire interval, and the maximum must occur when $x = 3$.	2 : answer with justification
(c) $\lim\limits_{x \to 0^-} g'(x) = -\frac{1}{2}$ $\lim\limits_{x \to 0^+} g'(x)$ does not exist.	$2 : \begin{cases} 1 : \text{left-hand limit} \\ 1 : \text{right-hand limit} \end{cases}$
(d) $\lim\limits_{x \to -2} (f(x) + 7) = -6 + \int_0^{-2} g(t)\,dt + 7 = 0$ $\lim\limits_{x \to -2} \left(e^{3x+6} - 1\right) = 0$ Using L'Hospital's Rule, $\lim\limits_{x \to -2} \frac{f(x) + 7}{e^{3x+6} - 1} = \lim\limits_{x \to -2} \frac{f'(x)}{3e^{3x+6}} = \frac{3 + g(-2)}{3} = \frac{3 + 1}{3} = \frac{4}{3}.$	$3 : \begin{cases} 1 : \text{limits equal 0} \\ 1 : \text{applies L'Hospital's Rule} \\ 1 : \text{answer} \end{cases}$ Note: max $1/3$ [1-0-0] if no limit notation attached to a ratio of derivatives

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Question 4

(a) $f(x) = 3 \Rightarrow x = -1$ and $x = 1$

$$\int_{-1}^{1} (f(x) - 3)\, dx = \int_{-1}^{1} \left(\frac{6}{1 + x^2} - 3 \right) dx$$
$$= \left(6\tan^{-1} x - 3x\right)\Big|_{-1}^{1}$$
$$= \left(6\tan^{-1} 1 - 3\right) - \left(6\tan^{-1}(-1) + 3\right)$$
$$= \left(6 \cdot \frac{\pi}{4} - 3\right) - \left(6 \cdot \left(-\frac{\pi}{4}\right) + 3\right)$$
$$= 3\pi - 6$$

The area of R is $3\pi - 6$.

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_{-1}^{1} \left((7 - 3)^2 - (7 - f(x))^2 \right) dx$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

(c) $h(x) = 3 - f(x) = 3 - \dfrac{6}{1 + x^2}$ for $x > 1$

$h'(x) = \dfrac{12x}{\left(1 + x^2\right)^2}$ for $x > 1$

$h'(2) = \dfrac{12 \cdot 2}{5^2} = \dfrac{24}{25}$

$3 : \begin{cases} 1 : h(x) \\ 1 : h'(x) \\ 1 : h'(2) \end{cases}$

Question 5

Solution	Scoring
(a) $y'(0) = -0.02\left(10^2\right) = -2$ An equation for the line tangent to the graph of $y = f(t)$ at $t = 0$ is $y = 10 - 2t$. $y(2) \approx 10 - 2(2) = 6$ grams	$2 : \begin{cases} 1 : y'(0) \\ 1 : \text{approximation} \end{cases}$
(b) $\dfrac{dy}{dt} = -0.02y^2 \le 0,$ so the graph of f is nonincreasing. The graph of f cannot resemble the given graph because the given graph is increasing on a portion of its domain.	1 : answer with reason
(c) $\displaystyle\int\left(-\frac{1}{y^2}\right)dy = \int 0.02\, dt$ $\dfrac{1}{y} = 0.02t + C$ $\dfrac{1}{10} = 0.02(0) + C \Rightarrow C = 0.1$ $\dfrac{1}{y} = 0.02t + 0.1 \Rightarrow y = \dfrac{1}{0.02t + 0.1} = \dfrac{50}{t + 5}$ Note: this solution is valid for $t > -5$.	$4 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ \quad \text{and uses initial condition} \\ 1 : \text{answer} \end{cases}$ Note: max $2/4$ [1-1-0-0] if no constant of integration Note: $0/4$ if no separation of variables
(d) $\dfrac{d^2y}{dt^2} = -0.04y\dfrac{dy}{dt}$ $= -0.04y\left(-0.02y^2\right)$ $= 0.0008y^3$ Because $y > 0$, $0.0008y^3 > 0$. The amount of substance is changing at an increasing rate. — OR — From part (c), $f(t) = \dfrac{50}{t + 5}$, and from context, $t \ge 0$. $f'(t) = \dfrac{-50}{(t + 5)^2}$ and $f''(t) = \dfrac{100}{(t + 5)^3} > 0$ for $t \ge 0$. The amount of substance is changing at an increasing rate.	$2 : \begin{cases} 1 : \dfrac{d^2y}{dt^2} \\ 1 : \text{answer with reason} \end{cases}$

Question 6

(a) $\frac{d}{dx}(2(x-y)) = \frac{d}{dx}(3+\cos y)$ $2 - 2\frac{dy}{dx} = (-\sin y)\frac{dy}{dx}$ $2 = (2-\sin y)\frac{dy}{dx}$ $\frac{dy}{dx} = \frac{2}{2-\sin y}$	$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{verification} \end{cases}$
(b) $\frac{dy}{dx} = \frac{2}{2-\sin y} = 1 \Rightarrow \sin y = 0 \Rightarrow y = 0$ $2(x-0) = 3+\cos 0 \Rightarrow 2x = 4 \Rightarrow x = 2$ Point P has coordinates $(2, 0)$.	$2 : \begin{cases} 1 : \frac{dy}{dx} = 1 \\ 1 : \text{answer} \end{cases}$
(c) $\frac{d^2y}{dx^2} = \frac{-2}{(2-\sin y)^2}(-\cos y)\frac{dy}{dx} = \frac{4\cos y}{(2-\sin y)^3}$ $\frac{d^2y}{dx^2} > 0$ for all y in the interval $-\frac{\pi}{2} < y < \frac{\pi}{2}$. Therefore, the curve is concave up for $-\frac{\pi}{2} < y < \frac{\pi}{2}$.	$3 : \begin{cases} 2 : \frac{d^2y}{dx^2} \\ 1 : \text{answer with reason} \end{cases}$
(d) By the Mean Value Theorem, for some value c in the interval $(2, 2.1)$, $f'(c) = \frac{f(2.1)-f(2)}{0.1}$. For all points on the curve, $\frac{2}{3} \le f'(x) \le 2$. Thus, $\frac{2}{3} \le \frac{f(2.1)-f(2)}{0.1} \le 2 \Rightarrow \frac{1}{15} \le f(2.1)-f(2) \le \frac{1}{5}$.	$2 : \begin{cases} 1 : \text{applies Mean Value Theorem} \\ 1 : \text{verification} \end{cases}$

2018 AP Calculus AB
Question Descriptors and Performance Data

Free-Response Questions

Question	Learning Objective	Essential Knowledge	Mathematical Practice for AP Calculus	Mean
1	2.1B|2.3A|2.3A|3.2B|3.3B(b)|3.4A	2.1B1|2.3A1|2.3A2|3.2B2|3.3B2|3.4A2	Reasoning with definitions and theorems|Connecting concepts|Implementing algebraic/computational processes|Connecting multiple representations|Building notational fluency|Communicating	4.98
2	2.3C|3.3B(b)|3.4B|3.4C	2.3C1|3.3B2|3.4B1|3.4C1	Reasoning with definitions and theorems|Connecting concepts|Implementing algebraic/computational processes|Building notational fluency|Communicating	3.31
3	1.1A(b)|1.1B|1.1C|2.2A|3.2C|3.2C|3.3A	1.1A3|1.1B1|1.1C3|2.2A1|3.2C1|3.2C3|3.3A2	Reasoning with definitions and theorems|Connecting concepts|Implementing algebraic/computational processes|Connecting multiple representations|Building notational fluency|Communicating	2.34
4	2.1C|2.3A|3.3B(b)|3.4D|3.4D	2.1C4|2.3A2|3.3B2|3.4D1|3.4D2	Reasoning with definitions and theorems|Connecting concepts|Implementing algebraic/computational processes|Connecting multiple representations|Building notational fluency|Communicating	2.63
5	2.1C|2.1D|2.2A|2.3B|3.5A	2.1C4|2.1D1|2.2A1|2.3B2|3.5A2	Reasoning with definitions and theorems|Connecting concepts|Implementing algebraic/computational processes|Connecting multiple representations|Building notational fluency|Communicating	2.34
6	1.2B|2.1C|2.1D|2.2A|2.3B|2.4A	1.2B1|2.1C5|2.1D1|2.2A1|2.3B1|2.4A1	Reasoning with definitions and theorems|Connecting concepts|Implementing algebraic/computational processes|Building notational fluency|Communicating	2.78