2020

AP CollegeBoard

Please note: Some of the questions in this former practice exam may no longer perfectly align with the AP exam. Even though these questions do not fully represent the 2020 exam, teachers indicate that imperfectly aligned questions still provide instructional value. Teachers can consult the Question Bank to determine the degree to which these questions align to the 2020 Exam.

This exam may not be posted on school or personal websites, nor electronically redistributed for any reason. This exam is provided by the College Board for AP Exam preparation. Teachers are permitted to download the materials and make copies to use with their students in a classroom setting only. To maintain the security of this exam, teachers should collect all materials after their administration and keep them in a secure location.

Further distribution of these materials outside of the secure College Board site disadvantages teachers who rely on uncirculated questions for classroom testing. Any additional distribution is in violation of the College Board's copyright policies and may result in the termination of Practice Exam access for your school as well as the removal of access to other online services such as the AP Teacher Community and Online Score Reports.

Then say:

Close your booklet and put your answer sheet on your desk, faceup. Make sure you have your AP number label and an AP Exam label on page 1 of your answer sheet. Sit quietly while I collect your answer sheets.

Collect an answer sheet from each student. Check that each answer sheet has an AP number label and an AP Exam label.

After all answer sheets have been collected, say:

Now you must seal your Section I booklet. Remove the remaining white seals from the backing and press one on each area of your exam booklet cover marked "PLACE SEAL HERE." Fold each seal over the back cover. When you have finished, place the booklet on your desk, faceup. I will now collect your Section I booklet. . . .

Collect a Section I booklet from each student. Check that each student has signed the front cover of the sealed Section I booklet.

There is a 10-minute break between Sections I and II.

When all Section I materials have been collected and accounted for and you are ready for the break, say:

Please listen carefully to these instructions before we take a 10-minute break. All items you placed under your chair at the beginning of this exam, including your Student Pack, must stay there, and you are not permitted to open or access them in any way. Leave your shrinkwrapped Section II packet on top of your desk during the break. You are not allowed to consult teachers, other students, notes, textbooks, or any other resources during the break. You may not make phone calls, send text messages, use your calculators, check email, use a social networking site, or access any electronic or communication device. You may not leave the designated break area. Remember, you may never discuss the multiple-choice exam content with anyone, and if you disclose the content through any means, your AP Exam score will be canceled. Are there any questions? . . .

You may begin your break. Testing will resume at _____ .

SECTION II: Free Response

After the break, say:

May I have everyone's attention? Place your Student Pack on your desk. . . .

You may now remove the shrinkwrap from the Section II packet, but do not open the Section II exam booklet until you are told to do so. . . .

Read the bulleted statements on the front cover of the exam booklet. Look up when you have finished. . . .

Now take an AP number label from your Student Pack and place it on the shaded box. If you don't have any AP number labels, write your AP number in the box. Look up when you have finished. . . .

Read the last statement. . . .

Using your pen, print the first, middle, and last initials of your legal name in the boxes and print today's date where indicated. This constitutes your signature and your agreement to the statements on the front cover. . . .

Now turn to the back cover. Using your pen, complete Items 1 through 3 under "Important Identification Information."...

Read Item 4....

Are there any questions?...

If this is your last AP Exam, you may keep your Student Pack. Place it under your chair for now. Otherwise if you are taking any other AP Exams this year, leave your Student Pack on your desk and I will collect it now....

Read the remaining information on the back cover of the exam booklet, paying careful attention to the bulleted statements in the instructions. Do not open the exam booklet or break the seals in the exam booklet until you are told to do so. Look up when you have finished....

Collect the Student Packs from students who are taking any other AP Exams this year.

Then say:

Are there any questions?...

Section II also has two parts that are timed separately. You are responsible for pacing yourself and may proceed freely from one question to the next within each part. Graphing calculators are required for Part A, so you may keep your calculators on your desk. You must write your answers in the appropriate space in the exam booklet using a No. 2 pencil or a pen with black or dark blue ink. Do not break the seals for Part B at this time. Are there any questions?...

You have 30 minutes to answer the questions in Part A. If you need more paper to complete your responses, raise your hand. At the top of each extra sheet of paper you use, write only:

- **your AP number,**
- **the exam title, and**
- **the question number you are working on.**

Do not write your name. Open your exam booklet and begin.

Note Start Time _____. Note Stop Time _____.

Check that students are working on Part A only and writing their answers in their exam booklets using pencils or pens with black or dark blue ink. The pages for the Part A questions are marked with large 1's or 2's at the top of each page to assist you in monitoring their work.

After 20 minutes, say:

There are 10 minutes remaining in Part A.

After 10 minutes, say:

Stop working on Part A. Calculators are not allowed for Part B. Please put all of your calculators under your chair....

Turn to page 13. You have 1 hour for Part B. During this time you may go back to Part A, but you may not use your calculator. Remember to show your work and write your answer to each part of each problem in the appropriate space in the exam booklet. Are there any questions?...

Using your finger, break open the seals on Part B. Do not peel the seals away from the booklet. You may go on to the next page and begin Part B.

Note Start Time _____. Note Stop Time _____.

After 50 minutes, say:

There are 10 minutes remaining in Part B.

After 10 minutes, say:

Stop working and close your exam booklet. Place it on your desk, faceup. . . .

If any students used extra paper for a question in the free-response section, have those students staple the extra sheet(s) to the first page corresponding to that question in their free-response exam booklets. Complete an Incident Report after the exam and return these free-response booklets with the extra sheets attached in the Incident Report return envelope (see page 68 of the *2018-19 AP Coordinator's Manual* for complete details).

Then say:

Remain in your seat, without talking, while the exam materials are collected. . . .

Collect a Section II exam booklet from each student. Check for the following:

- Exam booklet front cover: The student placed an AP number label on the shaded box and printed their initials and today's date.
- Exam booklet back cover: The student completed the "Important Identification Information" area.

When all exam materials have been collected and accounted for, return to students any electronic devices you may have collected before the start of the exam.

If you are giving the regularly scheduled exam, say:
You may not discuss or share the free-response exam content with anyone unless it is released on the College Board website in about two days. Your AP Exam score results will be available online in July.

If you are giving the alternate exam for late testing, say:
None of the content in this exam may ever be discussed or shared in any way at any time. Your AP Exam score results will be available online in July.

If any students completed the AP number card at the beginning of this exam, say:

Please remember to take your AP number card with you. You will need the information on this card to view your scores and order AP score reporting services online.

Then say:

You are now dismissed.

After-Exam Tasks

Be sure to give the completed seating chart to the AP coordinator. Schools must retain seating charts for at least six months (unless the state or district requires that they be retained for a longer period of time). Schools should not return any seating charts in their exam shipments unless they are required as part of an Incident Report.

NOTE: If you administered exams to students with accommodations, review the *2018-19 AP Coordinator's Manual* and the *2018-19 AP SSD Guidelines* for information about completing the Nonstandard Administration Report (NAR) form, and returning these exams.

AP® Calculus AB Exam

SECTION II: Free Response

2019

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour and 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.33%

Part B

Number of Questions
4
Time
1 hour
Electronic Device
None allowed
Percent of Section II Score
66.67%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐
   First letter of your first name ☐
2. Date of birth
   ☐☐ Month ☐☐ Day ☐☐☐☐ Year
3. Six-digit school code
   ☐☐☐☐☐☐

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.
   No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During Part B, you may continue to work on the questions in Part A without the use of a calculator.

As you begin each part, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 4PBP4-S

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

GO ON TO THE NEXT PAGE.

t (hours)	2	5	9	11	12
$L(t)$ (cars per hour)	15	40	24	68	18

1. The rate at which cars enter a parking lot is modeled by $E(t) = 30 + 5(t - 2)(t - 5)e^{-0.2t}$. The rate at which cars leave the parking lot is modeled by the differentiable function L. Selected values of $L(t)$ are given in the table above. Both $E(t)$ and $L(t)$ are measured in cars per hour, and time t is measured in hours after 5 A.M. ($t = 0$). Both functions are defined for $0 \le t \le 12$.

   (a) What is the rate of change of $E(t)$ at time $t = 7$? Indicate units of measure.

   (b) How many cars enter the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole number.

Do not write beyond this border.

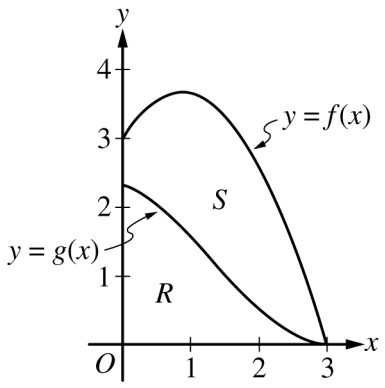
Continue question 1 on page 5.

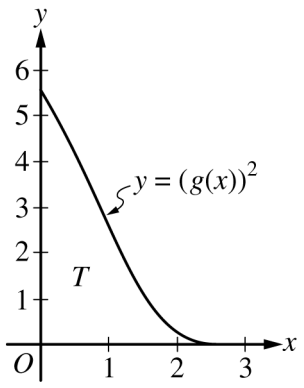
(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate $\int_2^{12} L(t)\,dt$. Using correct units, explain the meaning of $\int_2^{12} L(t)\,dt$ in the context of this problem.

(d) For $0 \le t < 6$, 5 dollars are collected from each car entering the parking lot. For $6 \le t \le 12$, 8 dollars are collected from each car entering the parking lot. How many dollars are collected from the cars entering the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole dollar.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.





2. The function f is defined by $f(x) = 3(1+x)^{0.5}\cos\left(\frac{\pi x}{6}\right)$ for $0 \le x \le 3$. The function g is continuous and decreasing for $0 \le x \le 3$ with $g(3) = 0$.

The figure above on the left shows the graphs of f and g and the regions R and S. R is the region bounded by the graph of g and the x- and y-axes. Region R has area 3.24125. S is the region bounded by the y-axis and the graphs of f and g.

The figure above on the right shows the graph of $y = (g(x))^2$ and the region T. T is the region bounded by the graph of $y = (g(x))^2$ and the x- and y-axes. Region T has area 5.32021.

(a) Find the area of region S.

Do not write beyond this border.

Continue question 2 on page 7.

(b) Find the volume of the solid generated when region S is revolved about the horizontal line $y = -3$.

(c) Region S is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 7 times the length of its base in region S. Write, but do not evaluate, an integral expression for the volume of this solid.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

END OF PART A

**IF YOU FINISH BEFORE TIME IS CALLED,
YOU MAY CHECK YOUR WORK ON PART A ONLY.**

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED

$$f(x) = \begin{cases} \sqrt{9 - x^2} & \text{for } -3 \le x \le 0 \\ -x + 3\cos\left(\dfrac{\pi x}{2}\right) & \text{for } 0 < x \le 4 \end{cases}$$

3. Let f be the function defined above.

   (a) Find the average rate of change of f on the interval $-3 \le x \le 4$.

   (b) Write an equation for the line tangent to the graph of f at $x = 3$.

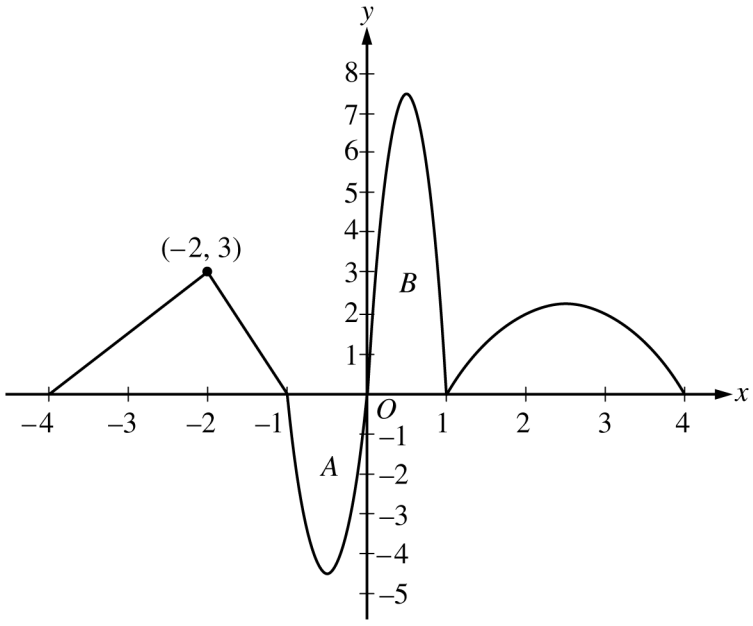
Do not write beyond this border.

Continue question 3 on page 15.

(c) Find the average value of f on the interval $-3 \le x \le 4$.

(d) Must there be a value of x at which $f(x)$ attains an absolute maximum on the closed interval $-3 \le x \le 4$? Justify your answer.

NO CALCULATOR ALLOWED



Graph of f

4. The continuous function f is defined for $-4 \le x \le 4$. The graph of f, shown above, consists of two line segments and portions of three parabolas. The graph has horizontal tangents at $x = -\frac{1}{2}$, $x = \frac{1}{2}$, and $x = \frac{5}{2}$. It is known that $f(x) = -x^2 + 5x - 4$ for $1 \le x \le 4$. The areas of regions A and B bounded by the graph of f and the x-axis are 3 and 5, respectively. Let g be the function defined by $g(x) = \int_{-4}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g(4)$.

Do not write beyond this border.

Continue question 4 on page 17.

(b) Find the absolute minimum value of g on the closed interval $[-4, 4]$. Justify your answer.

(c) Find all intervals on which the graph of g is concave down. Give a reason for your answer.

NO CALCULATOR ALLOWED

t (hours)	0	1	2	3	4
$B(t)$ (miles per hour)	1	8	1.5	–5	11

5. Brandon and Chloe ride their bikes for 4 hours along a flat, straight road. Brandon's velocity, in miles per hour, at time t hours is given by a differentiable function B for $0 \le t \le 4$. Values of $B(t)$ for selected times t are given in the table above. Chloe's velocity, in miles per hour, at time t hours is given by the piecewise function C defined by

$$C(t) = \begin{cases} te^{4-t^2} & \text{for } 0 \le t \le 2 \\ 12 - 3t - t^2 & \text{for } 2 < t \le 4. \end{cases}$$

(a) How many miles did Chloe travel from time $t = 0$ to time $t = 2$?

(b) At time $t = 3$, is Chloe's speed increasing or decreasing? Give a reason for your answer.

Do not write beyond this border.

Continue question 5 on page 19.

NO CALCULATOR ALLOWED

(c) Is there a time t, for $0 \le t \le 4$, at which Brandon's acceleration is equal to 2.5 miles per hour per hour? Justify your answer.

(d) Is there a time t, for $0 \le t \le 2$, at which Brandon's velocity is equal to Chloe's velocity? Justify your answer.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

6. Consider the curve defined by $2x^2 + 3y^2 - 4xy = 36$.

(a) Show that $\frac{dy}{dx} = \frac{2y - 2x}{3y - 2x}$.

(b) Find the slope of the line tangent to the curve at each point on the curve where $x = 6$.

Do not write beyond this border.

Continue question 6 on page 21.

(c) Find the positive value of x at which the curve has a vertical tangent line. Show the work that leads to your answer.

(d) Let x and y be functions of time t that are related by the equation $2x^2 + 3y^2 - 4xy = 36$. At time $t = 1$, the value of x is 2, the value of y is -2, and the value of $\frac{dy}{dt}$ is 4. Find the value of $\frac{dx}{dt}$ at time $t = 1$.