Question 1

(a) $E'(7) = 6.164924$ The rate of change of $E(t)$ at time $t = 7$ is 6.165 (or 6.164) cars per hour per hour.	1 : answer with units
(b) $\int_0^{12} E(t)\,dt = 520.070489$ To the nearest whole number, 520 cars enter the parking lot from time $t = 0$ to time $t = 12$.	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(c) $\int_2^{12} L(t)\,dt \approx (5-2)\cdot\frac{L(2)+L(5)}{2} + (9-5)\cdot\frac{L(5)+L(9)}{2} + (11-9)\cdot\frac{L(9)+L(11)}{2} + (12-11)\cdot\frac{L(11)+L(12)}{2}$ $= 3\cdot\frac{15+40}{2} + 4\cdot\frac{40+24}{2} + 2\cdot\frac{24+68}{2} + 1\cdot\frac{68+18}{2}$ $= 345.5$ $\int_2^{12} L(t)\,dt$ is the number of cars that leave the parking lot in the 10 hours between 7 A.M. ($t = 2$) and 5 P.M. ($t = 12$).	$3 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{approximation} \\ 1 : \text{explanation} \end{cases}$
(d) $5\int_0^{6} E(t)\,dt + 8\int_6^{12} E(t)\,dt = 3530.1396$ To the nearest dollar, 3530 dollars are collected from time $t = 0$ to time $t = 12$.	$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constants} \\ 1 : \text{answer} \end{cases}$

Question 2

(a) $\int_0^3 (f(x) - g(x))\,dx = \int_0^3 f(x)\,dx - \int_0^3 g(x)\,dx$

$= \int_0^3 f(x)\,dx - 3.24125 = 4.919585$

The area of region S is 4.920 (or 4.919).

$3 : \begin{cases} 1 : \text{definite integral of } f \\ 1 : \text{uses area of } R \\ 1 : \text{answer} \end{cases}$

(b) $\pi \int_0^3 \left((f(x) + 3)^2 - (g(x) + 3)^2 \right) dx$

$= \pi \left(\int_0^3 (f(x) + 3)^2\,dx - \int_0^3 \left((g(x))^2 + 6g(x) + 9 \right) dx \right)$

$= \pi \left(\int_0^3 (f(x) + 3)^2\,dx - \int_0^3 (g(x))^2\,dx - 6\int_0^3 g(x)\,dx - \int_0^3 9\,dx \right)$

$= \pi \left(\int_0^3 (f(x) + 3)^2\,dx - 5.32021 - 6 \cdot 3.24125 - 9 \cdot 3 \right)$

$= 156.263709$

The volume of the solid is 156.264 (or 156.263).

$4 : \begin{cases} 1 : \text{form of integrand} \\ 1 : \text{integrand} \\ 1 : \text{uses areas of } R \text{ and } T \\ 1 : \text{limits, constant, and answer} \end{cases}$

(c) Volume $= \int_0^3 7(f(x) - g(x))^2\,dx$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{expression} \end{cases}$

Question 3

(a) Average rate of change $= \dfrac{f(4)-f(-3)}{4-(-3)} = \dfrac{-1-0}{7} = -\dfrac{1}{7}$	1 : answer
(b) $f(3) = -3 + 3\cos\left(\dfrac{3\pi}{2}\right) = -3$ For $0 < x < 4$, $f'(x) = -1 + \left(-3\sin\left(\dfrac{\pi x}{2}\right)\right)\cdot\dfrac{\pi}{2}$ $f'(3) = -1 + \left(-3\sin\left(\dfrac{3\pi}{2}\right)\right)\cdot\dfrac{\pi}{2} = -1 + \dfrac{3\pi}{2}$ An equation for the tangent line is $y = -3 + \left(-1 + \dfrac{3\pi}{2}\right)(x-3)$.	$2 : \begin{cases} 1 : f'(3) \\ 1 : \text{equation} \end{cases}$
(c) The average value of f on the interval $-3 \le x \le 4$ is $\dfrac{1}{4-(-3)}\displaystyle\int_{-3}^{4} f(x)\,dx$. $\displaystyle\int_{-3}^{4} f(x)\,dx = \int_{-3}^{0} f(x)\,dx + \int_{0}^{4} f(x)\,dx$ $\displaystyle\int_{-3}^{0} f(x)\,dx = \int_{-3}^{0} \sqrt{9-x^2}\,dx = \frac{9\pi}{4}$ $\displaystyle\int_{0}^{4} f(x)\,dx = \int_{0}^{4}\left(-x + 3\cos\left(\frac{\pi x}{2}\right)\right)dx = \left[-\frac{1}{2}x^2 + \frac{6}{\pi}\sin\left(\frac{\pi x}{2}\right)\right]_0^4 = -8$ $\displaystyle\frac{1}{4-(-3)}\int_{-3}^{4} f(x)\,dx = \frac{1}{7}\left(\frac{9\pi}{4} - 8\right)$	$4 : \begin{cases} 1 : \text{integrals of } f \text{ over} \\ \quad -3 \le x \le 0 \text{ and } 0 \le x \le 4 \\ 1 : \text{value of } \int_{-3}^{0}\sqrt{9-x^2}\,dx \\ 1 : \text{antiderivative of} \\ \quad -x + 3\cos\left(\frac{\pi x}{2}\right) \\ 1 : \text{answer} \end{cases}$
(d) $\displaystyle\lim_{x\to 0^-} f(x) = f(0) = 3$ and $\displaystyle\lim_{x\to 0^+} f(x) = 3$, so f is continuous at $x = 0$. Because f is continuous on $[-3, 4]$, the Extreme Value Theorem guarantees that f attains an absolute maximum on $[-3, 4]$.	$2 : \begin{cases} 1 : \text{continuity at } x = 0 \\ 1 : \text{answer with justification} \end{cases}$

Question 4

(a) $g(0) = \int_{-4}^{0} f(t)\,dt = \frac{9}{2} - 3 = \frac{3}{2}$

$$\begin{aligned} g(4) &= \int_{-4}^{4} f(t)\,dt \\ &= \int_{-4}^{0} f(t)\,dt + \int_{0}^{1} f(t)\,dt + \int_{1}^{4} f(t)\,dt \\ &= \frac{3}{2} + 5 + \int_{1}^{4} \left(-t^2 + 5t - 4\right) dt \\ &= \frac{3}{2} + 5 + \left[-\frac{1}{3}t^3 + \frac{5}{2}t^2 - 4t\right]_1^4 \\ &= \frac{3}{2} + 5 + \left[\left(-\frac{1}{3}\cdot 4^3 + \frac{5}{2}\cdot 4^2 - 4\cdot 4\right) - \left(-\frac{1}{3}\cdot 1^3 + \frac{5}{2}\cdot 1^2 - 4\cdot 1\right)\right] \\ &= \frac{3}{2} + 5 + \left(\frac{8}{3} - \left(-\frac{11}{6}\right)\right) = 11 \end{aligned}$$

$4 : \begin{cases} 1 : g(0) \\ 1 : \text{integral of } f \text{ over } 1 \le t \le 4 \\ 1 : \text{antiderivative} \\ 1 : g(4) \end{cases}$

(b) $g'(x) = f(x)$ is negative for $-1 < x < 0$, and nonnegative elsewhere. Thus, the absolute minimum value of g on $[-4, 4]$ can only occur at $x = -4$ or $x = 0$.

$g(-4) = 0$
$g(0) = \frac{3}{2}$

The absolute minimum value of g on $[-4, 4]$ is $g(-4) = 0$.

$3 : \begin{cases} 1 : g'(x) = f(x) \\ 1 : \text{identifies } x = -4 \text{ and } x = 0 \text{ as candidates} \\ 1 : \text{answer with justification} \end{cases}$

(c) The graph of g is concave down on the intervals $-2 < x < -\frac{1}{2}$, $\frac{1}{2} < x < 1$, and $\frac{5}{2} < x < 4$ because $g'(x) = f(x)$ is decreasing on these intervals.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

Question 5

(a) $\int_0^2 te^{4-t^2}\,dt = -\frac{1}{2}e^{4-t^2}\Big|_{t=0}^{t=2} = -\frac{1}{2} + \frac{1}{2}e^4$

Chloe traveled $-\frac{1}{2} + \frac{1}{2}e^4$ miles from time $t = 0$ to time $t = 2$.

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(b) $C(3) = -6 < 0$
$C'(3) = -9 < 0$

Chloe's speed is increasing at time $t = 3$ because her velocity and acceleration have the same sign.

$2 : \begin{cases} 1 : C(3) < 0 \text{ and } C'(3) < 0 \\ 1 : \text{answer with reason} \end{cases}$

(c) B is differentiable $\Rightarrow$ B is continuous on $[0, 4]$.

$\frac{B(4) - B(0)}{4 - 0} = \frac{11 - 1}{4 - 0} = 2.5$

By the Mean Value Theorem, there is a time t, for $0 < t < 4$, such that $B'(t) = 2.5$ miles per hour per hour.

$2 : \begin{cases} 1 : \frac{B(4) - B(0)}{4 - 0} \\ 1 : \text{answer with justification} \end{cases}$

(d) B and C are continuous on $[0, 2]$, therefore $B - C$ is continuous on $[0, 2]$.

$B(0) - C(0) = 1 - 0 > 0$
$B(2) - C(2) = 1.5 - 2 < 0$

By the Intermediate Value Theorem, there is a time t, for $0 < t < 2$, such that $B(t) - C(t) = 0$, or $B(t) = C(t)$.

$2 : \begin{cases} 1 : \text{considers } B(t) - C(t) \\ 1 : \text{answer with justification} \end{cases}$

Question 6

(a) $\frac{d}{dx}\left(2x^2 + 3y^2 - 4xy\right) = \frac{d}{dx}(36) \Rightarrow 4x + 6y\frac{dy}{dx} - 4y - 4x\frac{dy}{dx} = 0$

$\Rightarrow (6y - 4x)\frac{dy}{dx} = 4y - 4x \Rightarrow \frac{dy}{dx} = \frac{4y - 4x}{6y - 4x} = \frac{2y - 2x}{3y - 2x}$

$2 : \begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{verification} \end{cases}$

(b) $x = 6 \Rightarrow 2 \cdot 6^2 + 3y^2 - 4 \cdot 6 \cdot y = 36$

$\Rightarrow y^2 - 8y + 12 = 0 \Rightarrow y = 2$ or $y = 6$

$\left.\frac{dy}{dx}\right|_{(x,\, y) = (6,\, 2)} = \frac{4 - 12}{6 - 12} = \frac{4}{3}$

The slope of the line tangent to the curve at $(6, 2)$ is $\frac{4}{3}$.

$\left.\frac{dy}{dx}\right|_{(x,\, y) = (6,\, 6)} = \frac{12 - 12}{18 - 12} = 0$

The slope of the line tangent to the curve at $(6, 6)$ is 0.

$2 : \begin{cases} 1 : \text{slope at } (6, 2) \\ 1 : \text{slope at } (6, 6) \end{cases}$

(c) The curve has vertical tangent lines where $3y - 2x = 0$ and $2y - 2x \neq 0$.

$3y - 2x = 0 \Rightarrow y = \frac{2}{3}x \Rightarrow 2x^2 + 3\left(\frac{2}{3}x\right)^2 - 4x \cdot \frac{2}{3}x = 36$

$\Rightarrow \left(2 + \frac{4}{3} - \frac{8}{3}\right)x^2 = \frac{2}{3}x^2 = 36 \Rightarrow x^2 = 54$

Because x is positive, $x = 3\sqrt{6}$.

$2 : \begin{cases} 1 : \text{sets } 3y - 2x = 0 \\ 1 : \text{answer} \end{cases}$

(d) $\frac{d}{dt}\left(2x^2 + 3y^2 - 4xy\right) = \frac{d}{dt}(36)$

$\Rightarrow 4x\frac{dx}{dt} + 6y\frac{dy}{dt} - 4\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right) = 0$

$\Rightarrow (4x - 4y)\frac{dx}{dt} + (6y - 4x)\frac{dy}{dt} = 0$

$(4 \cdot 2 - 4 \cdot (-2))\left.\frac{dx}{dt}\right|_{t=1} + (6 \cdot (-2) - 4 \cdot 2) \cdot 4 = 0 \Rightarrow \left.\frac{dx}{dt}\right|_{t=1} = 5$

— OR —

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{2y - 2x}{3y - 2x} \cdot \frac{dx}{dt}$

$4 = \frac{2 \cdot (-2) - 2 \cdot 2}{3 \cdot (-2) - 2 \cdot 2} \cdot \left.\frac{dx}{dt}\right|_{t=1} = \frac{4}{5} \cdot \left.\frac{dx}{dt}\right|_{t=1} \Rightarrow \left.\frac{dx}{dt}\right|_{t=1} = 5$

$3 : \begin{cases} 1 : \text{chain rules or product rule} \\ 1 : \text{derivative with respect to } t \\ 1 : \text{answer} \end{cases}$

2019 AP Calculus AB
Question Descriptors and Performance Data

Question	Skill	Learning Objective	Topic	Key	% Correct
76	2.E	FUN-4.A	Determining Intervals on Which a Function Is Increasing or Decreasing	D	82
77	1.E	CHA-3.B	Straight-Line Motion: Connecting Position, Velocity, and Acceleration	B	86
78	1.E	CHA-3.F	Approximating Values of a Function Using Local Linearity and Linearization	A	66
79	3.F	CHA-4.B	Finding the Average Value of a Function on an Interval	B	61
80	1.E	FUN-6.B	The Fundamental Theorem of Calculus and Definite Integrals	C	62
81	2.B	LIM-2.A	Exploring Types of Discontinuities	C	81
82	1.E	FUN-6.A	Applying Properties of Definite Integrals	A	71
83	3.D	LIM-2.C	Removing Discontinuities	A	53
84	3.D	FUN-1.A	Working with the Intermediate Value Theorem	C	48
85	3.D	FUN-4.A	Using the First Derivative Test to Find Relative (Local) Extrema	D	51
86	2.B	FUN-4.A	Connecting a Function, Its First Derivative, and Its Second Derivative	A	76
87	1.E	CHA-4.C	Connecting Position, Velocity, and Acceleration Functions Using Integrals	C	32
88	1.E	CHA-3.A	Interpreting the Meaning of the Derivative in Context	C	33
89	3.E	FUN-4.A	Sketching Graphs of Functions and Their Derivatives	B	33
90	1.E	FUN-3.E	Differentiating Inverse Functions	C	37

Free-Response Questions

Question	Skill	Learning Objective	Topic	Mean Score
1	1.E|3.D|3.F|4.D|4.A|4.B|4.C|4.E	CHA-2.D|CHA-4.E|LIM-5.A	2.3|8.3|6.2|8.3	6.03
2	1.D|1.E|2.B|4.E	CHA-5.A|CHA-5.C|CHA-5.B	8.4|8.12|8.7	3.46
3	1.D|1.E|3.C|3.D|4.A	CHA-2.A|CHA-2.C|CHA-4.B|FUN-1.C	2.1|2.2|8.1|5.2	1.71
4	1.D|1.E|2.B|2.E|4.A	FUN-6.A|FUN-4.A	6.6|5.5|5.6	2.98
5	1.D|1.E|3.D|3.E|4.A	CHA-4.C|CHA-3.B|FUN-1.B|FUN-1.A	8.2|4.2|5.1|1.16	2.71
6	1.C|1.E|3.G|4.C	FUN-3.D|FUN-4.E|CHA-3.E	3.2|5.12|4.5	2.98