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MAE 119

Homework # 6

1] Darrieus H Turbine

$u = 6$ m/s @ 10m

~~[illegible]~~

$P = 1.01$ atm $= 102338.3$ Pa

$T = 16°C = 289$ K

$z_1 = 20$m (center)

$h_R = 8$m

$D_E = 6$ m

a) Air density ρ

$$\rho = \frac{pM}{RT} = \frac{(102338.3 \text{ Pa})(29 \text{ kg/kmol})}{(8314 \text{ Nm/kmol K})(289 \text{ K})} = \boxed{1.235 \text{ kg/m3}}$$

b) wind speeds @ $z = 16, z = 20, z = 24$ m w/ $\alpha = 1/7$

$$\frac{u(z_1)}{u(z_2)} = \left(\frac{z_1}{z_2}\right)^{\alpha}$$ let $z_2 = 10$m, $u(z_2) = 6$ m/s

@ $z_1 = 16$ m

$$u(z_1) = 6 \text{ m/s} \left(\frac{16\text{m}}{10\text{m}}\right)^{1/7}$$

$$\boxed{u(16\text{m}) = 6.417 \text{ m/s}}$$

@ $z_1 = 20$m

$$u(20\text{m}) = 6 \text{ m/s} \left(\frac{20\text{m}}{10\text{m}}\right)^{1/7}$$

$$\boxed{u(20\text{m}) = 6.625 \text{ m/s}}$$

@ $z_1 = 24$ m

$$u(24\text{m}) = 6 \text{ m/s} \left(\frac{24\text{m}}{10\text{m}}\right)^{1/7}$$

$$\boxed{u(24\text{m}) = 6.799 \text{ m/s}}$$

c) Wind power density @ $u(20m)$

$z = 20m$

$u(20m) = 6.625\ m/s$

$$\frac{P_W}{A} = \frac{\rho u^3}{2} = \frac{(1.235\ kg/m^3)(6.625\ m/s)^3}{2}$$

$$\boxed{\frac{P_W}{A} = 179.55\ W/m^2}$$

d) Max turbine power output

Betz limit, $C_{P,betz} = 0.593$

$A_R = \pi\left(\frac{D}{2}\right)^2$

@ $z_1 = 24m$

$u(z_1) = 6.799\ m/s$

$$P_{Betz} = C_{P,Betz}\, \rho \frac{Au^3}{2}$$

$$= 0.593\left(1.235 \frac{kg}{m^3}\right) \frac{\pi(3m)^2}{2} (6.799\ m/s)^3$$

$$\boxed{P_{Betz} = 3.254\ MW}$$

e) Actual $C_p = 0.31$ (Taken from: "Net Power Coeffient of Vertical and Horizontal Wind Turbines with crossflow runners" MDPI.com)

$$P_{real} = 0.31\left(1.235\ kg/m^3\right) \frac{\pi(3m)^2}{2} (6.799\ m/s)^3$$

$$\boxed{P_{real} = 1.701\ MW}$$

$\boxed{TSR = 2.6}$

f)

$$TSR = \frac{\omega R}{u} \Rightarrow \omega = \frac{TSR \cdot u}{R} = \frac{2.6\,(6.799\,m/s)}{3m} = 5.89\ \frac{rad}{s}$$

$$= \boxed{56.24\ RPM}$$

$$t_{cycle} = \frac{60s}{56.24\ RPM} = \boxed{1.067\ s}$$

2| Weibull parameters c, K

$\bar{u}_{avg} = 8\ m/s$ 　　　　　　　　　　Find: c, K, P_{Betz}

$SD_u = 3.7\ m/s$: $\sigma_u = 3.7\ m/s$

$z = 20\ m$ 　　　$\sigma^2 = 13.69$ 　　　K: shape factor

HAWT - 2 Blade 　　　　　　　　　　c: scale factor

$D_R = 100m$

$z_{center} = 100\ m$

$\rho = 1.225\ kg/m^3$ @ $z_{centr} = 100\ m$

$\alpha = 1/7$

c and K solved numerically via Matlab

$$\boxed{\begin{array}{l} K = 2.2921 \\ C = 9.0306 \end{array}} \quad @ \quad \bar{u} = 8\,m/s \quad @ \quad z = 20m$$
$$\sigma = 3.7\,m/s$$

$\bar{u}(20m) = 8\ m/s$

$\sigma(20m) = 3.7\ m/s$

$$\bar{P}_{Betz} = \frac{A\bar{\rho}\eta C_{p,Betz}\, c^3\, \Gamma(1 + 3/k)}{2}$$

$$\frac{\bar{u}(z_1)}{\bar{u}(z_2)} = \left(\frac{z_1}{z_2}\right)^{\alpha}$$

$$\bar{u}(100m) = \bar{u}(20m)\left(\frac{20m}{100m}\right)^{1/7}$$

$$= 8\ m/s\left(\frac{20}{100}\right)^{1/7}$$

$$\boxed{\bar{u}(100m) = 6.357\ m/s}$$

$$\frac{\sigma(z_1)}{\sigma(z_2)} = \left(\frac{z_1}{z_2}\right)^{\alpha}$$

$$\sigma(100m) = 3.7\ m/s\left(\frac{20}{100}\right)^{1/7}$$

$$\boxed{\sigma = 2.94\ m/s}$$

$$\boxed{\begin{array}{l} K_{new} = 2.2922 \\ C_{new} = 7.1759 \end{array}}$$

$$\bar{P}_{Betz} = \frac{A \bar{\rho} \eta C_{p,Betz} c^3 \Gamma(1 + 3/k)}{2}$$

assume: $\bar{\rho} = \rho = 1.225 \ kg/m3$

$C_{p,Betz} = 0.593$

$\eta = 0.9$

$c = c_{new} = 7.1759$

$k = k_{new} = 2.2922$

$\boxed{\bar{P}_{Betz} = 1.1127 \ MW}$ @ $z = 100m$

3) $\frac{u(z_1)}{u(z_2)} = \left(\frac{z_1}{z_2}\right)^\alpha$

let $z_1 = 30\,m$
$z_2 = 10\,m$

$y = mx + b$

$u(z_1) = u(z_2)\left(\frac{30}{10}\right)^\alpha$

$u(z_1) = u(z_2)\, 3^\alpha$

from matlab : scatter(u_2, u_1)

linear fit eqn: $y = 1.2x + 0.0031$

$y = 3^\alpha x \quad : \quad y = 1.2x + 0.0031$

$3^\alpha x = 1.2x + 0.0031$

$.0031 = x(3^\alpha - 1.2)$

$\frac{.0031}{x} = 3^\alpha - 1.2$

$\boxed{\alpha = 0.16669}$