MEMO

PROCESS DYNAMICS – CPD320

SEMESTER TEST 2

Chemical Engineering
Engineering and the Built Environment

Examiner: Carl Sandrock

October 2007 $\boxed{\text{90 minutes}}$

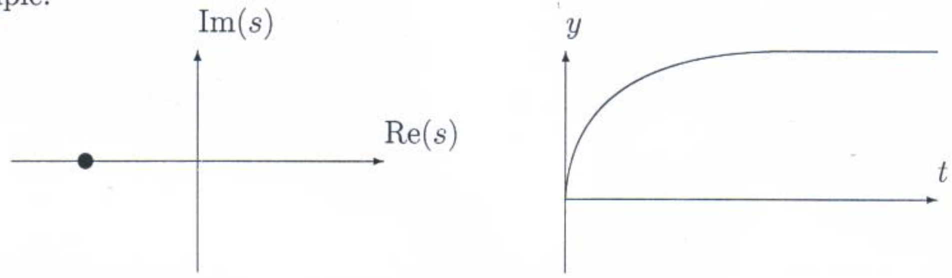
Instructions – Read carefully

• Answer all the questions on the paper in the blocks provided. • This is a closed book test. All the information you may use is contained in the paper and the attached formula sheet.

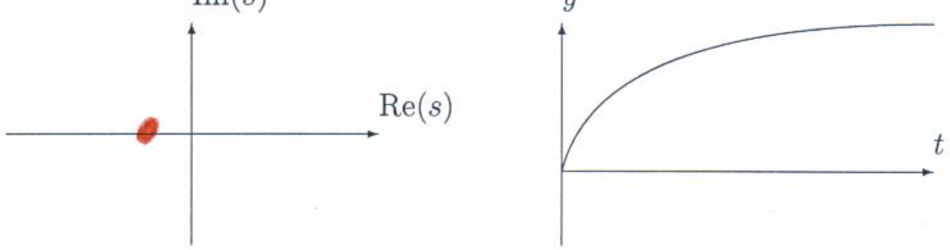
1 System Responses

In each of the following diagrams, a the system is described by $y = Gu$. Assume u is an ideal unit step. Draw a time response if it is missing, and draw poles (using filled circles) and zeros (using open circles) if they are missing. Assume that the steady state gain is positive.

Example:



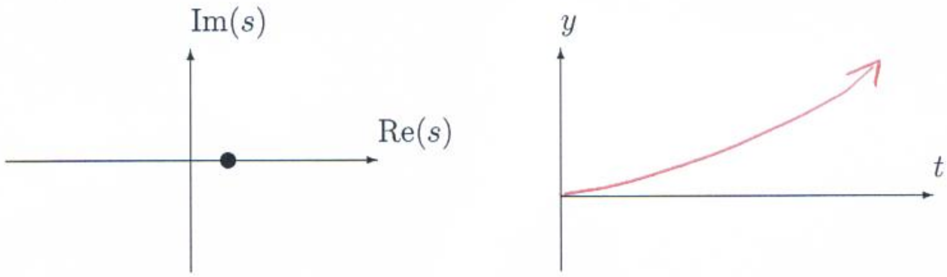
1. (This is different from the example)



$\boxed{2}$

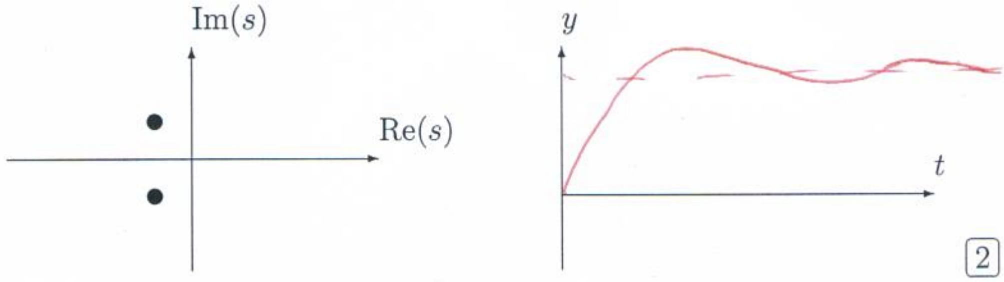
closer to 0

2.



$\boxed{2}$

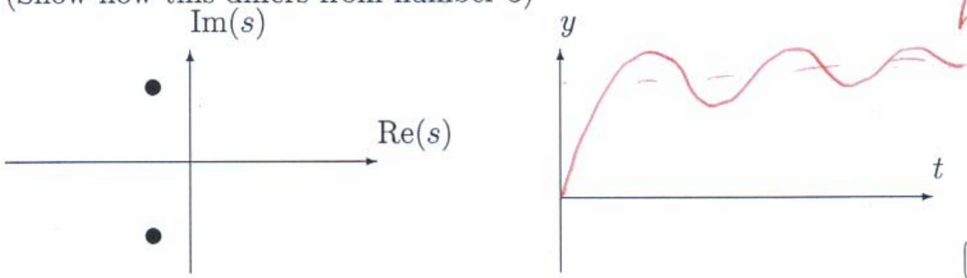
3.



$\boxed{2}$

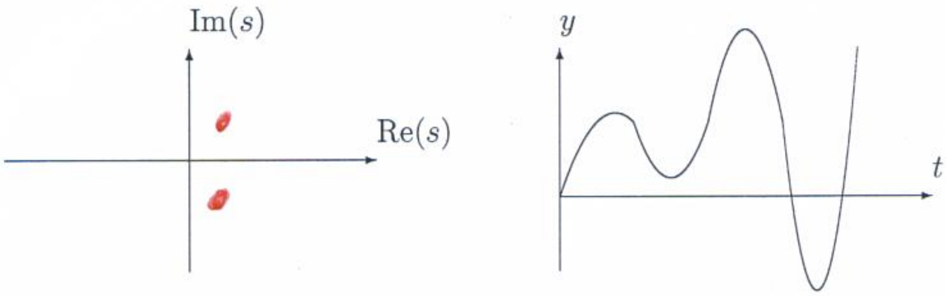
4. (Show how this differs from number 3)

higer freq.



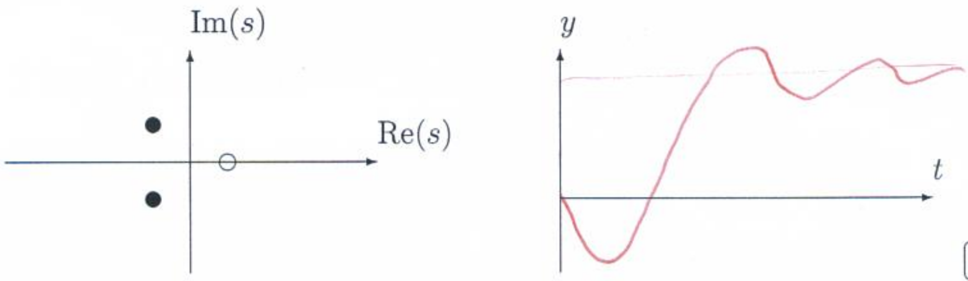
$\boxed{2}$

5.



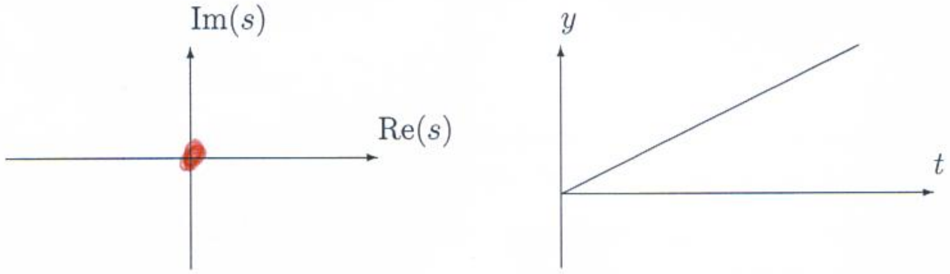
$\boxed{2}$

6.



$\boxed{2}$

7.



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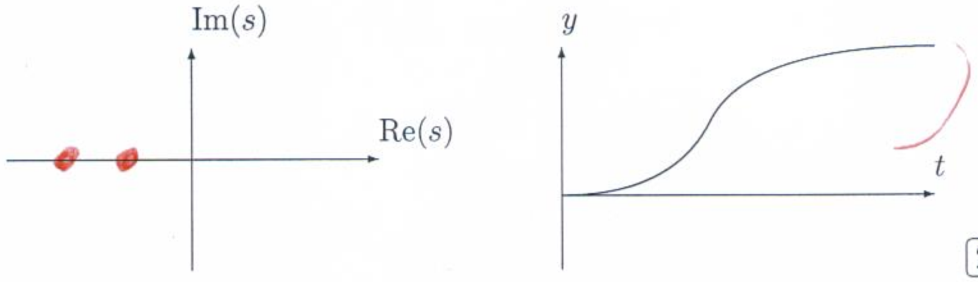
8.



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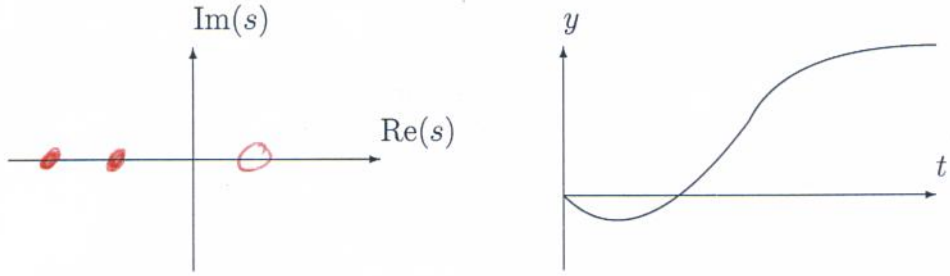
9.



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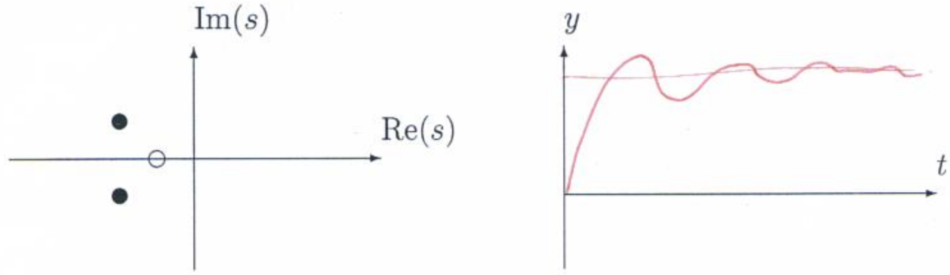
10.



2

20

2 Time domain

Determine the steady state gain, time constant and damping coefficient of the following second order differential equation.

$$5\ddot{x}(t) = x(t) + \frac{x(t)}{1+k} + \theta\dot{x}(t) + u(t) \tag{1}$$

$$\boxed{\tau_p = \sqrt{\frac{-5}{1+\frac{1}{1+k}}} \qquad \xi = \frac{-\theta\left(1+\frac{1}{1+k}\right)}{2\tau_p} \qquad K_p = \frac{-1}{1+\frac{1}{1+k}}}$$

$$= \sqrt{\frac{-5(1+k)}{2+k}} \qquad = \frac{-\theta(2+k)}{2\tau_p(1+k)} \qquad = -\frac{1+k}{2+k}$$

5

3 Laplace Domain

3.1 Random noise

A colleague suggests that the Laplace transform of a function generating uniform random values between −1 and 1 for every t should be zero for all s. Do you agree? Give a brief explanation of your opinion. 5

YES. Random signals are per definition not correlated to any specific frequency

3.2 Transform

The following linearised differential equations represent the behaviour of a chemical system in terms of deviation variables.

$$2\frac{\mathrm{d}x(t)}{\mathrm{d}t} = kx(t) + C_A(t) \tag{2}$$

$$\frac{\mathrm{d}C_A(t)}{\mathrm{d}t} = 2C_{A0}x(t) + x_0C_A(t) + F(t) \tag{3}$$

Transform to the Laplace domain and determine the transfer function between F and x. 10

$2sx(s) = kx(s) + C_A(s)$ (1)

$s\,C_A(s) = 2C_{A0}x(s) + x_0C_A(s) + f(s)$

from (1): $C_A(s) = 2sx(s) - kx(s)$

$\therefore\ s\,x(s)(2s-k) = 2C_{A0}x(s) + x_0x(s)(2s-k) + f(s)$

$F(s) = x(s)\left[(2s-k) - 2C_{A0} - x_0(2s-k)\right]$

$\frac{x(s)}{F(s)} = \frac{1}{(2s-k)(1-x_0) - 2C_{A0}}$

3.3 Inverse

The following equations describe a system in the Laplace domain

$$(s+1)y(s) = e^{-2s}x(s) \tag{4}$$
$$(s^2+2s+2)x(s) = (s-1)u(s) \tag{5}$$

Determine the time-domain function representing the response of y to a pulse of height 1 in u starting at time $t=1$ and ending at time $t=2$. (20)

notice that y is a function of delayed x without delay, and with $u(s) = \frac{1}{s}$

$y(s) = \frac{s-1}{(s+1)(s^2+2s+2)s}$

now, $y = \frac{A}{s+1} + \frac{B}{s+1+i} + \frac{C}{s+1-i} + \frac{D}{s}$

$A = \lim_{s\to -1} \frac{s-1}{(s^2+2s+2)s} = 2 \quad B = -\frac{3-i}{4}$

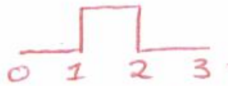
$C = -\frac{3+i}{4} \quad D = -1/2$

$\therefore$ from table, $f(t) = 2e^{-2t} - \frac{1}{2} - \left[\frac{3-i}{4} e^{(1+i)t} + \frac{3+i}{4} e^{(1-i)t}\right]$

$f(t) = 2e^{-2t} - \frac{1}{2} - \frac{3-i}{4} e^{-t}(i\sin(t) + \cos(t))$

$\qquad - \frac{3+i}{4} e^{-t}(-i\sin(t) + \cos(t))$

$= 2e^{-2t} + \frac{1}{2} + \frac{e^{-t}}{2}(\sin(t) \mp 3\cos(t))$

now, using info about times 

, delayed 2sec.

$$y(t) = \begin{cases} 0 & \text{for } t < 3 \; (1+2) \\ f(t-3) & \text{for } 3 < t < 4 \\ f(t-3) - f(t-4) & \text{for } t \geq 4. \end{cases}$$

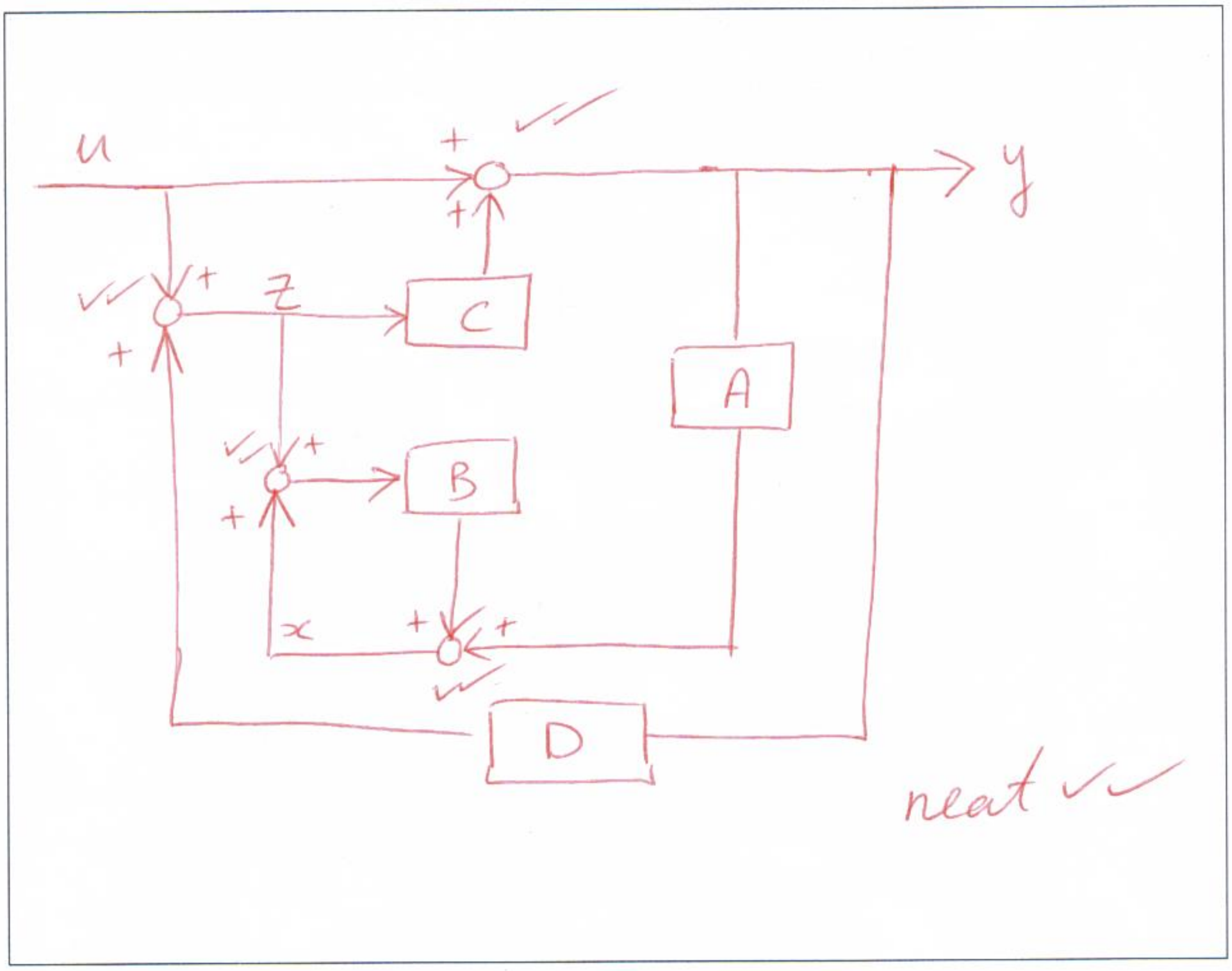
(35)

4 Block diagrams

4.1 Drawing

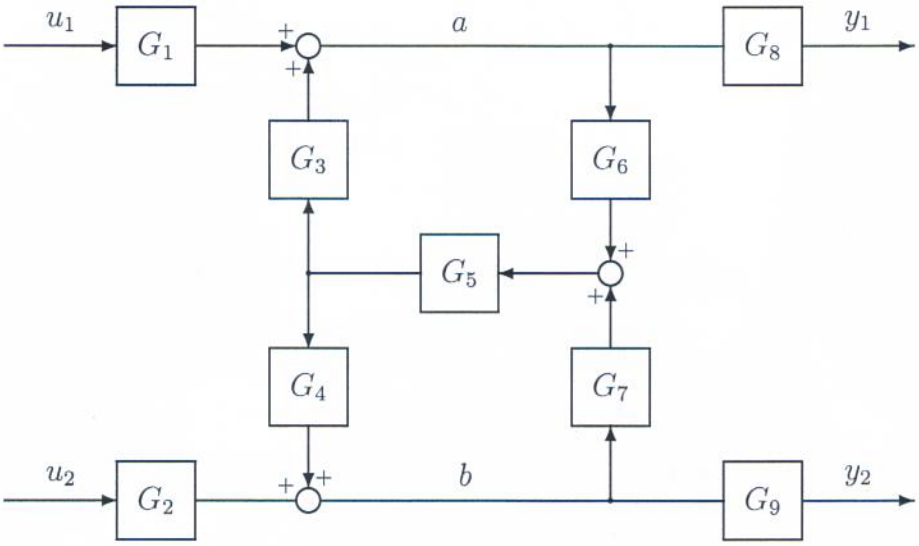
Draw a block diagram representing the following set of equations. The input to the system is u and the output is y. (10)

$$
\begin{aligned}
x(s) &= \overset{A}{\boxed{\frac{1}{s+2}}}\,y(s) + \overset{B}{\boxed{\frac{5(s+1)}{s-3}}}\,(z(s)+x(s)) && (6)\\
y(s) &= \overset{C}{\boxed{\frac{s+1}{s-1}}}\,z(s) + u(s) && (7)\\
z(s) &= \underset{D}{\boxed{\frac{1}{(s+1)(s+2)}}}\,y(s) + u(s) && (8)
\end{aligned}
$$



4.2 Equations

Write down the transfer function from u_1 to y_2 from the following block diagram. (20)



$$y_2 = g_9 b$$

$$b = g_4 g_5 (g_6 a + g_7 b)$$

$$a = g_1 u_1 + g_3 g_5 (g_6 a + g_7 b)$$

$$= \frac{g_1 u_1 + g_3 g_5 g_7 b}{1 - g_3 g_5 g_6}$$

$$b = g_4 g_5 \left[g_6 \left(\frac{g_1 u_1 + g_3 g_5 g_7 b}{1 - g_3 g_5 g_6} \right) + g_7 b \right]$$

$$b = \frac{g_4 g_5 g_6 g_1 u_1}{1 - g_3 g_5 g_6 - g_4 g_5 g_7}$$

$$\boxed{\frac{y_2}{u_1} = \frac{g_9 g_4 g_5 g_6 g_1}{1 - g_3 g_5 g_6 - g_4 g_5 g_7}}$$

(30)

Full Marks (90)

$\LaTeX\ 2_\varepsilon$
www.latex-project.org