Process Dynamics – CPN321

Exam

Chemical Engineering
Engineering and the Built Environment

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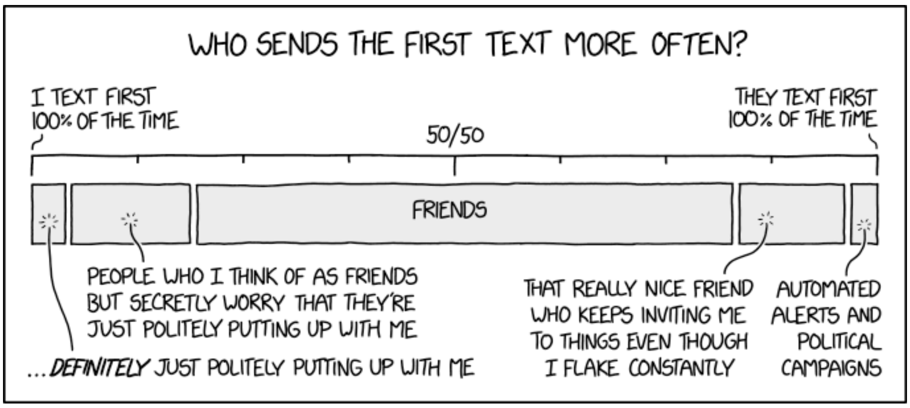
Date: 2018-11-12 — Date of perusal: 2018-11-21

Duration: 180 minutes — Total: 100 — Total Pages: 8

Instructions – Read carefully

• This is an open book test. You may bring any information you need into the exam venue. You may access the Internet. • Unmotivated answers will receive zero marks.



www.xkcd.com

Stirred-tank reaction process

All the questions in this paper will be related to the the stirred-tank reaction process in Figure 1. I recommend that you take apart the paper and keep this sheet separate as a reference.

The reaction

$$\mathrm{A} \longrightarrow \mathrm{B}$$

takes place with a reaction rate given by $r = kC_A$.

The system has a single feed, two tanks, and a single product. The volumes in the tanks are kept constant by weirs on the outflows. The tanks are well mixed and the density is constant.

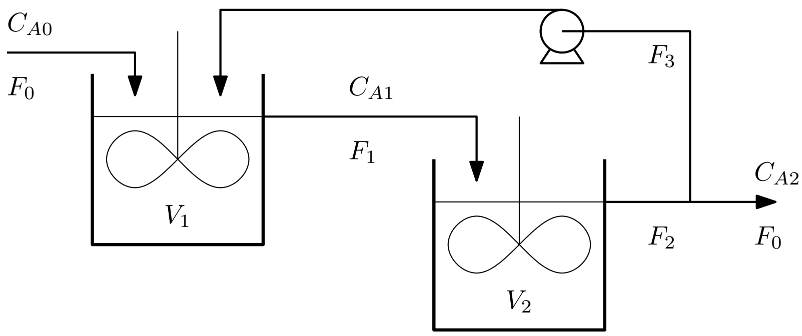


Figure 1: Stirred-tank reaction process

1 Multiple choice

Answer these questions on ClickUP.

1.1. Whenever material is moved in a plant there is a time delay associated with the movement. Select the true statement about transport delay below:

a. Transport delay is independent of pipe length

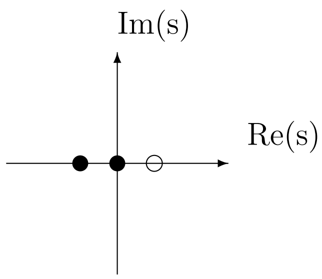
b. Transport delay is independent of the flow rate in the pipe.

c. When a fluid is incompressible, flow rate changes propagate without transport delay.

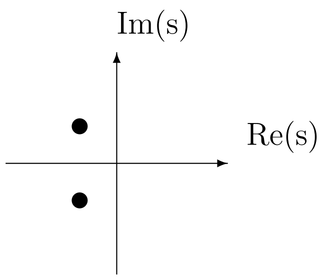
d. Transport delay cannot be accurately represented in the Laplace domain.

e. Transport delay must be approximated before analysing linear systems

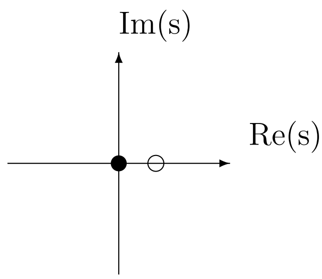
1.2. The following diagram shows a qualitative system response of a system described by $y = Gu$. Assume u is an ideal unit step. Select the plot of the poles (indicated by $\bullet$) and zeros (indicated by $\circ$) on the s-plane that best matches this response.



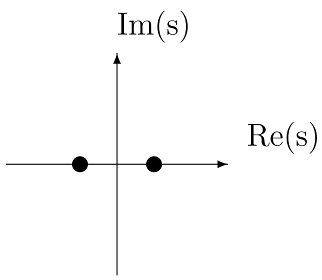
a.



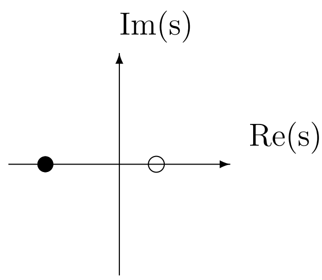
b.



c.



d.



e.

1.3. How many states will a state-space model of the system shown in Figure 1 have if there is no transport delay in the pipes?
a. 1 b. 2 c. 3 d. 4 e. 5

1.4. Step tests are often used to determine process characteristics. Select a true statement about step tests:

a. If the process measurement is contaminated by measurement noise, step test results are useless

b. Step tests are unnecessary because all chemical processes can be modelled using first principles.

c. Step tests can only be used if the process is linear

d. Most processes exhibit step responses which are very close to ideal first order responses

e. Undetected disturbances can affect the parameters identified by step tests

1.5. Select a true statement about Bode diagrams

a. The gain of the system is always found at the intersection of the AR graph with the y axis

b. Because dead time has a fixed delay, the phase lag of a dead time system is unbounded

c. The AR graph is always sufficient to find the transfer function associate with the frequency response

d. Positive phase lags can be seen in frequency response tests on real systems

e. It is always possible to find the phase lag of a system at a single frequency if I have $G(i\omega)$ at that single frequency.

1.6. Select a true statement about sampled signals

a. Reconstruction of a sampled signal is always perfect as long as the sampling rate is high enough

b. Digital filters can be used to recover the original signal values given noisy measurements

c. Analog filters can be used to recover the original signal values given noisy measurements

d. When measurements are noisy, it is better to sample less frequently

e. When measurements are noisy, it is better to sample more frequently

Total for question 1: $\boxed{30}$

2 Constant flow rates

If all the flow rates and volumes are constant and the transport delays in the pipes are negligible, the following linear equations describe this system. Note that primes indicate deviation variables:

$$
\begin{aligned}
V_1\frac{\mathrm{d}C'_{A1}}{\mathrm{d}t} &= F_0C'_{A0} + F_3C'_{A2} - F_1C'_{A1} - V_1kC'_{A1} && (1)\\
V_2\frac{\mathrm{d}C'_{A2}}{\mathrm{d}t} &= F_1C'_{A1} - F_2C'_{A2} - V_2kC'_{A2} && (2)\\
F_1 &= F_2 && (3)\\
F_2 &= F_0 + F_3 && (4)
\end{aligned}
$$

2.1. Derive all individual transfer functions which describe this system. $\boxed{5}$

2.2. Construct a block diagram of this system, showing the individual transfer functions. The diagram should show all the variables in Figure 1 $\boxed{5}$

2.3. Derive the overall transfer functions which indicates the effect of changes in C_{A0} on C_{A2}. $\boxed{5}$

2.4. Calculate the eventual value of C_{A2} after a step change of size ΔC_{A0} in C_{A0}. $\boxed{5}$

Total for question 2: $\boxed{20}$

3 Variable recycle

On the real plant the recycle flow F_3 is manipulated in order to control C_{A2} and F_0 is not constant (but the volumes still are).

The behaviour of the system with these assumptions system can be expressed as a multivariable transfer function like this:

$$
\begin{bmatrix} C_{A1} \\ C_{A2} \\ F_1 \\ F_2 \end{bmatrix} = G_3 \begin{bmatrix} C_{A0} \\ F_0 \\ F_3 \end{bmatrix}
$$

3.1. Sketch the qualitative response of C_{A2} to a step increase in F_3 $\boxed{7}$

3.2. Write down a simplified version of G_3 using the notation $O(n, m)$ to indicate a transfer function with n zeros and m poles. For example $\frac{K}{\tau s+1}$ is written as $O(0, 1)$, a pure gain is written as $O(0, 0)$. Write a brief motivation of each choice. $\boxed{13}$

Total for question 3: $\boxed{20}$

4 Parameter estimation

The file `response.csv` contains values for C_{A0}, C_{A1}, and C_{A2} recorded when tests were done to find the effect of C_{A0} on the system. Note that the values recorded here for C_{A1} and C_{A2} were obtained using a gas chromatograph which takes 5 seconds to analyse a sample.The results from a sample are only available when the next sample is taken.

4.1. Find the transfer function relating C_{A0} and C_{A1} using the data on file using a method of your choice. Explain your method and why you have chosen this method. Plot your predictions on the same axes as a plot of C_{A1} $\boxed{10}$

4.2. Estimate a continuous transfer function relating C_{A0} and C_{A2} comparing frequency responses. Plot the frequency response of your transfer function on the same axes as a numeric frequency response calculated via the FFT. Comment on the similarities and differences between the responses, paying special attention to the higher frequencies. $\boxed{10}$

4.3. Find the discrete transfer function relating the sampled values of C_{A0} to the sampled values of C_{A2}. $\boxed{10}$

Total for question 4: $\boxed{\mathit{30}}$

Full Marks $\boxed{100}$

Datasheet: CPN321/CPB410

Compiled on May 5, 2018

General solution of 1st order DE:

$$\dot{x} + P(t)x = Q(t) \quad \Rightarrow \quad x = \frac{1}{F_I}\int Q(t)F_I \mathrm{d}t + c_1 \quad \text{with} \quad F_I = \exp\left(\int P(t)\mathrm{d}t\right)$$

Taylor Series expansion near point $x = a$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$

$$f(x_1, \ldots, x_d) = \sum_{n_1=0}^{\infty} \cdots \sum_{n_d=0}^{\infty} \frac{(x_1 - a_1)^{n_1} \cdots (x_d - a_d)^{n_d}}{n_1! \cdots n_d!} \left(\frac{\partial^{n_1+\cdots+n_d} f}{\partial x_1^{n_1} \cdots \partial x_d^{n_d}}\right)(a_1, \ldots, a_d)$$

Linear approximation around point $\mathbf{x} = \mathbf{0}$, where $f(\mathbf{x}) = \mathbf{0}$:

$$f(\mathbf{x}) \approx \nabla f(\mathbf{0}) \cdot \mathbf{x}$$

$$f(x_1, x_2, \cdots, x_d) \approx \frac{\partial f}{\partial x_1}(0)x_1 + \frac{\partial f}{\partial x_2}(0)x_2 + \cdots + \frac{\partial f}{\partial x_d}(0)x_d$$

Partial fraction expansion (for strictly proper rational functions of s)

$$F(s) = \frac{(s-z_1)(s-z_2)(s-z_3)\cdots}{(s-p_1)^n(s-p_2)(s-p_3)\cdots} = \underbrace{\sum_{m=0}^{n-1} \frac{A_m}{(s-p_1)^{n-m}}}_{\text{repeated roots}} + \frac{B}{s-p_2} + \frac{C}{s-p_3}\cdots$$

$$A_m = \lim_{s \to p_1} \left\{ \frac{\mathrm{d}^m}{\mathrm{d}s^m}\left[(s-p_1)^n F(s)\right] \right\} \frac{1}{m!}$$

$$B = \lim_{s \to p_2} \left[(s-p_2)F(s)\right]$$

Euler identity:

$$e^{i\theta} = \cos\theta + i\sin\theta \quad \therefore e^{-i\theta} = \cos\theta - i\sin\theta \quad \text{and} \quad e^{i\pi} - 1 = 0$$

(1,1) Padé approximation of dead time:

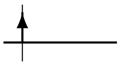
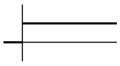
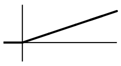
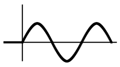
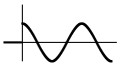
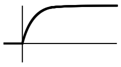
$$e^{-Ds} \approx \frac{1 - \frac{D}{2}s}{1 + \frac{D}{2}s}$$

PID controller:

$$m = K_C\left(\varepsilon + \frac{1}{\tau_I}\int_0^t \varepsilon \mathrm{d}t + \tau_D \frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\right) \qquad \frac{m}{\varepsilon}(s) = K_C\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Tuning rules:

	Ziegler-Nichols			Cohen-Coon (with $\phi = \frac{t_D}{\tau_P}$)		
	K_C	τ_I	τ_D	K_C	τ_I	τ_D
P	$\frac{K_u}{2}$			$\frac{\phi+3}{3K_P\phi}$		
PI	$\frac{K_u}{2.2}$	$\frac{P_u}{1.2}$		$\frac{5\phi+54}{60K_P\phi}$	$t_D\frac{30+3\phi}{9+20\phi}$	
PID	$\frac{K_u}{1.7}$	$\frac{P_u}{2}$	$\frac{P_u}{8}$	$\frac{3\phi+16}{12K_P\phi}$	$t_D\frac{32+6\phi}{13+8\phi}$	$\frac{4t_D}{11+2\phi}$

Time domain		Laplace-transform	z-transform ($b = e^{-aT}$)
Impulse: $\delta(t)$		1	1
Unit step: $u(t)$		$\frac{1}{s}$	$\frac{1}{1 - z^{-1}}$
Ramp: t		$\frac{1}{s^2}$	$\frac{Tz^{-1}}{(1 - z^{-1})^2}$
t^n		$\frac{n!}{s^{n+1}}$	$\lim_{a \to 0} (-1)^n \frac{\partial^n}{\partial a^n} \frac{1}{1 - bz^{-1}}$
e^{-at}		$\frac{1}{s+a}$	$\frac{1}{1 - bz^{-1}}$
te^{-at}		$\frac{1}{(s+a)^2}$	$\frac{Tbz^{-1}}{(1 - bz^{-1})^2}$
$t^2 e^{-at}$		$\frac{2}{(s+a)^3}$	$\frac{T^2 bz^{-1}(1 + bz^{-1})}{(1 - bz^{-1})^3}$
$\sin(\omega t)$		$\frac{\omega}{s^2 + \omega^2}$	$\frac{z^{-1}\sin(\omega T)}{1 - 2z^{-1}\cos(\omega T) + z^{-2}}$
$\cos(\omega t)$		$\frac{s}{s^2 + \omega^2}$	$\frac{1 - z^{-1}\cos(\omega T)}{1 - 2z^{-1}\cos(\omega T) + z^{-2}}$
$1 - e^{-at}$		$\frac{a}{s(s+a)}$	$\frac{(1-b)z^{-1}}{(1 - z^{-1})(1 - bz^{-1})}$
$e^{-at}\sin(\omega t)$		$\frac{\omega}{(s+a)^2 + \omega^2}$	$\frac{z^{-1}b\sin(\omega T)}{1 - 2z^{-1}b\cos(\omega T) + b^2 z^{-2}}$
$e^{-at}\cos(\omega t)$		$\frac{s+a}{(s+a)^2 + \omega^2}$	$\frac{1 - z^{-1}b\cos(\omega T)}{1 - 2z^{-1}b\cos(\omega T) + b^2 z^{-2}}$
Initial value theorem: $\lim_{t \to 0} f(t)$		$\lim_{s \to \infty} sF(s)$	$\lim_{z \to \infty} F(z)$
Final value theorem: $\lim_{s \to \infty} f(t)$		$\lim_{s \to 0} sF(s)$	$\lim_{z \to 1} \left[\left(1 - z^{-1}\right) F(z)\right]$
Translation: $f(t-D)u(t-D)$		$e^{-Ds}F(s)$	$F(z)z^{-n}$ where $D = nT$
Derivative: $\frac{\mathrm{d}^n f(t)}{\mathrm{d}t^n} = f^n(t)$		$s^n F(s) - \sum_{k=1}^{n} s^{k-1} f^{n-k}(0)$	
Integral: $\int_0^t f(t)\mathrm{d}t$		$\frac{1}{s}F(s)$	
Zero$^{\text{th}}$ order hold		$H(s) = \frac{1 - e^{-Ts}}{s}$	