Name: ______________________ Student Number: ______________

Process Dynamics – CPN321

Exam

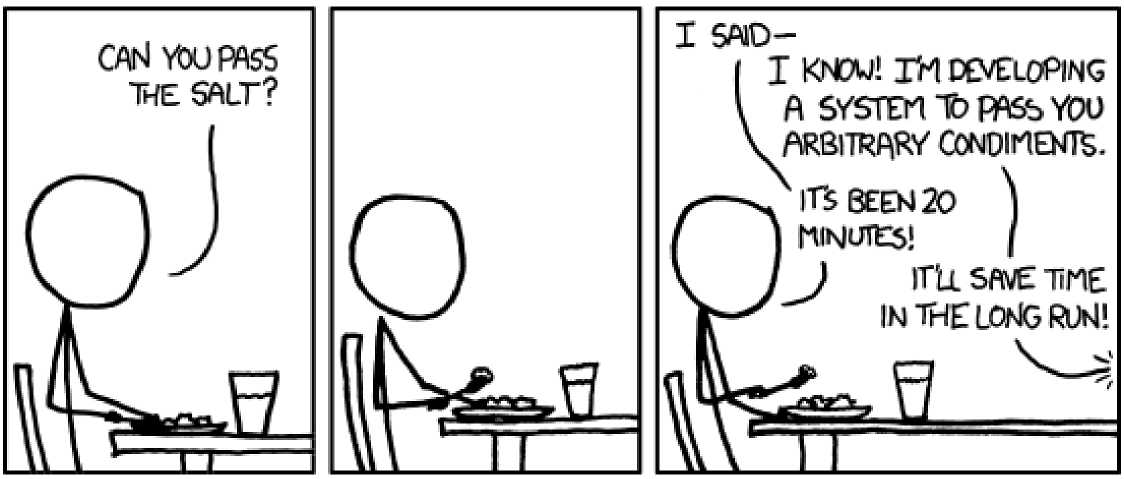
Chemical Engineering
Engineering and the Built Environment

Examiner: Carl Sandrock External examiner: W Nicol

November 2011 180 minutes

Instructions – Read carefully

• Answer all the questions on the paper on side 1 of the multiple choice form or in the blocks provided. • This is a closed book test. All the information you may use is contained in the paper and the attached formula sheet.



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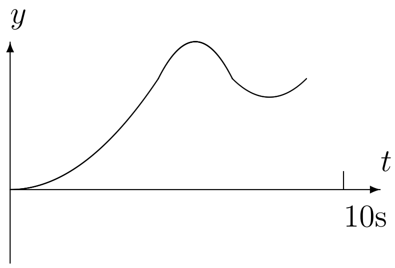
1 Multiple choice

Answer this section on side 1 of the multiple choice form. Each question counts 5 marks.

1. The following response was obtained from a linear system after a unit step was applied. Select the parameter values that best correspond to this output.



a. First order, $\tau = 3$

b. First order, $\tau = 10$

c. Second order, $\tau = 4$, $\zeta = 1$

d. Second order, $\tau = 4$, $\zeta = 0.5$

e. First order plus dead time, $\tau_d = 2$, $\tau = 2$

2. Select the correct equation for the parameters θ used in identifying discrete model parameters. The outputs x can be described by a model of the form $x = A\theta$. A contains the process values, and θ the process parameters.
a. $\theta = A^{-1}x$ b. $\theta = A \setminus x$ c. $\theta = x/A$ d. $\theta = [A^T A]^{-1}A^T x$ e. $\theta = [A^{-1}A^T]^{-1}$

3. Given that the human ear is capable of perceiving frequencies up to 22 kHz, what frequency do you expect audio CDs to be sampled at? Think of the Nyquist sampling theorem.
a. 11 kHz b. 22 kHz c. 33 kHz d. 44 kHz e. 192 kHz

4. Which of the following statements is correct regarding aliasing in sampled systems:

a. Modern computers and digital systems have made analog filtering obsolete as aliasing can be avoided by faster sampling

b. Signals must be filtered using an analog filter before sampling to avoid aliasing errors

c. Aliasing errors appear because we cannot accurately produce an ideal Dirac delta

d. Aliasing is not a problem for real-world systems as we seldom see ideal sinusoids as inputs

e. Aliasing means that analog storage is always superior to digital storage, as sampling causes losses.

5. Which description best describes the equations below?

$$\begin{aligned}\frac{\mathrm{d}T}{\mathrm{d}t} &= aT + bX\\ \frac{\mathrm{d}X}{\mathrm{d}t} + \frac{\mathrm{d}T}{\mathrm{d}t} &= cT + dX\end{aligned}$$

a. System of linear differential equations

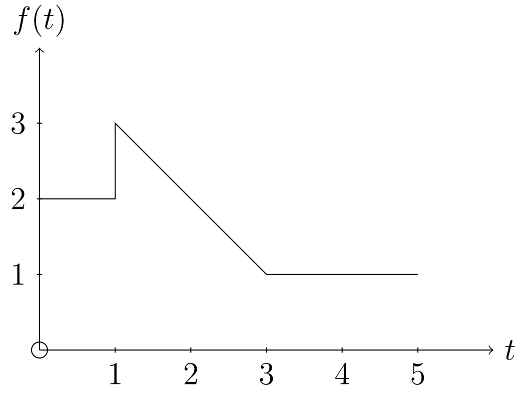
b. Partial differential equations

c. Differential-Algebraic equations

d. A second order system

e. A state-space system

6. Choose the correct expression for the signal shown below, given that $u(t)$ is the unit step and $r(t)$ is the unit ramp (both are zero for $t < 0$).



a. $f(t) = 3 - u(t-1) - r(t-1) + r(t-3)$

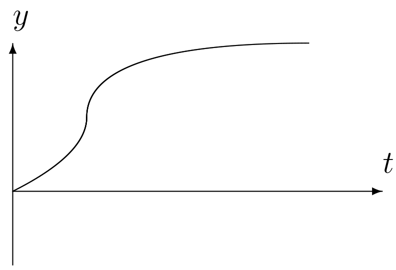
b. $f(t) = 2 + u(t-1)(2-t) + u(t-3)(t-3)$

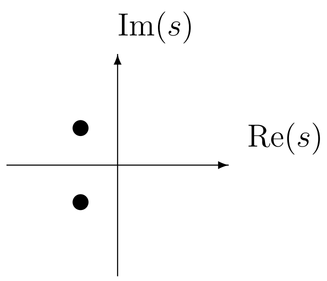
c. $f(t) = 1 + u(t-1) - r(t-1) + r(t-3)$

d. $f(t) = r(t-1) + r(t-3)$

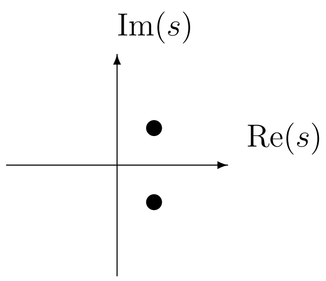
e. $f(t) = -u(t-1) + r(t-3)$

7. The following diagram shows a qualitative system response of a the system described by $y = Gu$. Assume u is an ideal unit step. Select the plot of the poles on the s-plane that best matches this response.

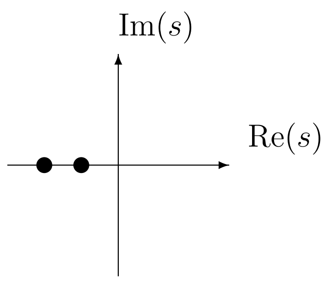




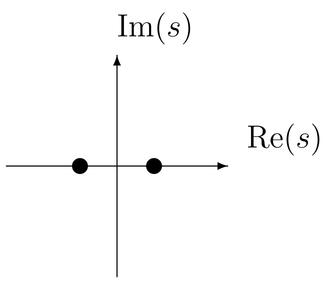
a.



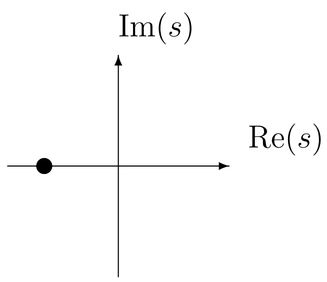
b.



c.

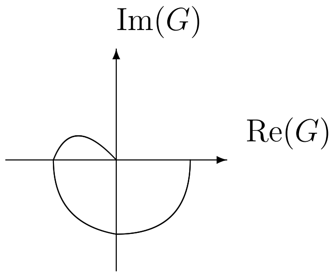


d.



e.

8. Select the model which best fits with the Nyquist diagram shown below:



a. $\dfrac{1}{s+1}$ b. $\dfrac{1}{(s+1)^2}$ c. $\dfrac{1}{(s+1)^3}$

d. $\dfrac{e^{-s}}{s+1}$ e. $\dfrac{1}{(s+1)(s+2)}$

9. Choose the correct statement regarding the frequency domain:

a. It is possible to obtain the Laplace-domain expressions from frequency domain expressions, as long as an analytical equation is available

b. It is possible to approximate the Laplace-domain expressions from frequency domain graphs such as Bode diagrams

c. The Laplace domain and frequency domain tell us nothing about one another

d. Poles in the Laplace domain have no effect on the frequency domain except when they are on the imaginary axis

e. Zeros in the Laplace domain have no effect on the frequency domain except when they are on the imaginary axis

10. Choose an incorrect statement about sampled systems:

a. The act of sampling always throws away information

b. The act of reconstruction via a hold device always introduces lag

c. The mathematical representation of the sampled signal cannot be exactly reproduced

d. A hold device is required to interface sampled systems with continuous ones

e. A sampler is required to interface continuous systems with discrete ones.

11. Which of the following system properties will mean that a system model cannot be represented in state space?

a. Dead time

b. Higher order derivatives

c. Multiple integrators

d. Non-physically realisable system

e. Unstable

55

2 Thermal model

In the following model for a system losing heat via radiative cooling, F_i is the input and T is the output. The parameters of the system are Q_0, T_∞, k_1, k_2, k_3, k_4. Q_i is the internal energy of the system, T is the temperature, F_i is the inflow rate.

$$
\begin{aligned}
\frac{\mathrm{d}N}{\mathrm{d}t} &= k_1(F_i - F_o) \\
\frac{\mathrm{d}Q_i}{\mathrm{d}t} &= k_2(T - T_\infty)^4 \\
Q_i &= k_3 T N + Q_0 \\
F_o &= k_4\sqrt{N}
\end{aligned}
$$

2.1 Linearisation

Linearise the system and express it in terms of deviation variables. $\boxed{10}$

2.2 Transfer function

Find the transfer function relating F_i to T in the linear model $\boxed{10}$

2.3 Block diagram

Draw a block diagram of the system, showing all of the variables in the original system as distinct lines. 10

2.4 State Space

Convert the linearised system to a state space representation 5

35

3 Combined system

Consider the following system:

$$G = \frac{3(s+1)}{4s^2 + s + 1}$$

3.1 Second order

Determine the time constant τ and damping coefficient ζ of the second order system. Is the system over, under or critically damped? 5

3.2 Frequency response

Calculate the gain and phase of the frequency response of the system at $\omega = 0.5$ $\boxed{5}$

3.3 Bode diagram

Sketch a Bode diagram of the system. Make sure you indicate the corner frequencies and asymptotes you have used. $\boxed{10}$

Calculations:

Graph:

$\boxed{\mathit{20}}$

4 Sampled systems

4.1 Sampling theory

Use diagrams to show the difference between a signal ($f(t)$), the mathematical representation of the sampled version of the signal ($f^*(t)$) and this signal after being sent through a zero-order hold. $\boxed{12}$

$f(t)$ $f^*(t)$ Held signal

4.2 Moving average

A certain discrete process calculates its output by averaging the last three values of its inputs. Determine the z-domain transfer function that relates the sampled inputs to the sampled outputs for this process. $\boxed{8}$

4.3 Response

Calculate the first three values of the response of the process given by $G(z) = 0.5(1 + z^{-1})$ to a unit ramp signal input with $T = 1$. $\boxed{20}$

$\boxed{40}$

5 Process identification

The graph in the attached graph sheet shows the result of a pulse test. The input was pulsed as shown in the first part of the figure, and the system response was as shown. Use this result along with the attached graph paper to find the frequency response of the system at ω =1 Hz.

Remember the definition of the Laplace transform:

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\mathrm{d}t$$

30

Full Marks 180

LaTeX 2_ε
www.latex-project.org