

### PROCESS DYNAMICS - CPN321

## SEMESTER TEST 2

Chemical Engineering Engineering and the Built Environment

Examiner: Carl Sandrock

Date: 2017-10-09

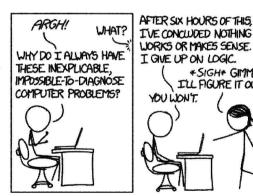
Duration: 90 minutes

Total: 90

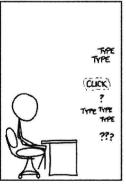
Total Pages: 6

#### Instructions - Read carefully

• Answer all the questions. • This is a closed book test. All the information you may use is contained in the paper. • You may use the computer • Make sure that you motivate all your answers and write legibly.









xkcd.com

## 1 Second order dynamics

Consider the following differential equation. u' and x' are deviation variables of u and x respectively. a, b and c are all positive.

$$a\frac{\mathrm{d}x'}{\mathrm{d}t^2} + b\frac{\mathrm{d}x'}{\mathrm{d}t} + cx'(t) = u'(t) \tag{1}$$

- 1. Find the transfer function between u and x.  $\boxed{5}$
- 2. Find the gain, time constant and damping coefficient of the system in terms of a, b and c.  $\boxed{10}$
- 3. When u is moved suddenly from  $2.3 \,\mathrm{kg/h}$  to  $4.0 \,\mathrm{kg/h}$ , x moves from  $12.1 \,\mathrm{^{\circ}C}$  and eventually ends up around  $15.8 \,\mathrm{^{\circ}C}$ . What is the gain of the process?  $\boxed{5}$
- 4. Plant personnel describe the step response as having a slight overshoot and a rise time of 10 minutes. Draw a sketch showing the response and indicate the rise time, overshoot and time constant. What does this information tell you about the damping coefficient of the system? 10

Total for question 1: (30)

# 2 Complex system dynamics

Due to limitations in software used on site, you have decided to approximate a model of a piece of equipment as a second order plus dead time system as follows:

$$\frac{(s-1)}{(3s+1)^3(s+2)} \approx \frac{Ke^{-\theta \xi}}{(\tau_1 s+1)(\tau_2 s+1)}$$
 (2)

- 1. Find K,  $\tau_1$  and  $\tau_2$  using Skogestad's half rule  $\boxed{20}$
- 2. Sketch a qualitative response of the original system to a step input. Indicate key features of the response on the graph. Mention the gain, poles and zeros. (10)
- 3. An engineer on site criticizes your approximation by saying "Your approximation no longer exhibits inverse response". Is she right? Is this a serious problem? [5]
- 4. Explain why Skogestad's half rule could not be used for the system in equation 1 with reference to the nature of the poles. 5

Total for question 2: (40)

#### 3 Multivariable system representations

Consider the following state space description of a system:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = Ax + Bu$$

$$y = Cx + Du$$
(3)

(4)

with

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & \frac{3}{2} & 0 \\ \frac{1}{2} & 0 & 1 \end{bmatrix} \quad \mathcal{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- 1. How many inputs, states and outputs does this model have? Motivate your answer. [5]
- 2. Obtain a transfer function matrix for this model. Derive any equations that you use in the process and write each matrix element as a simplified rational function of s. (10)
- 3. Determine whether the system is stable without referring to the transfer function model. Show your working. [5]

Total for question 3: (20)

Full Marks (90)

 $AT_{FX} 2_{\varepsilon}$ www.latex-project.org