UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA
Denkleiers • Leading Minds • Dikgopolo tša Dihlalefi

Process Dynamics – CPN321

Semester Test 2

Chemical Engineering
Engineering and the Built Environment

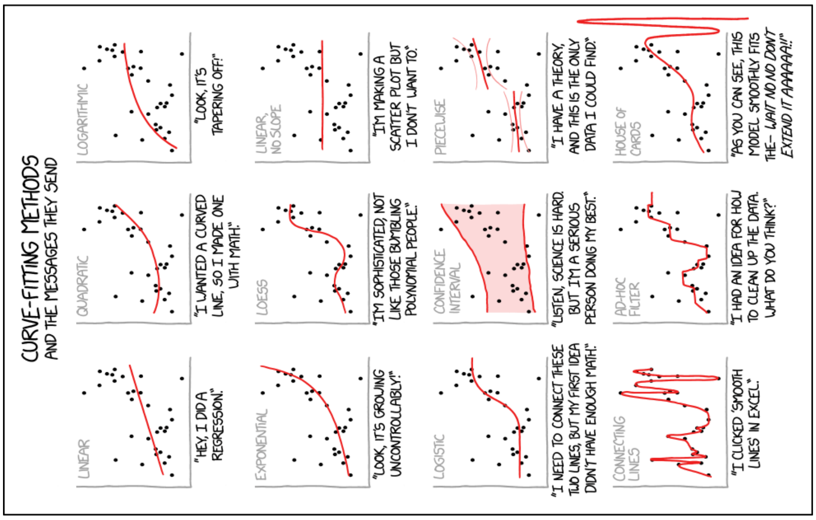
Examiner: Carl Sandrock

Date: 2018-10-08

Duration: 90 minutes   Total: 50   Total Pages: 6

Instructions – Read carefully

• Answer all the questions. • This is a closed book test. All the information you may use is contained in the paper. • You may use the computer. • Make sure that you motivate all your answers and write legibly.



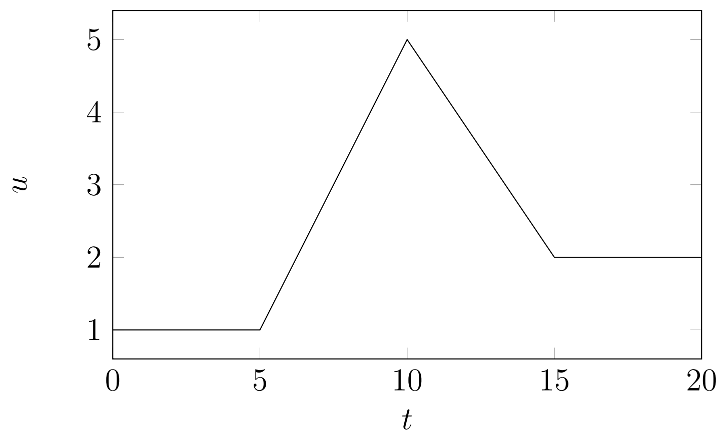
xkcd.com

1 Comparing ramps

In this question we will consider the responses of two real systems described by the following transfer functions:

$$G_1 = \frac{y_1}{u} = \frac{2}{3s+4} \quad \text{and} \quad G_2 = \frac{y_2}{u} = \frac{2}{2s^2+3s+4}$$

to the input shown below



You may assume u, y_1 and y_2 were all 1 for a long time before the experiment started.

1.1. Find the Laplace transform of u. $\boxed{5}$

1.2. Calculate the gain, time constant and damping coefficient of the second order system. $\boxed{3}$

1.3. Sketch (do not calculate) the response of both systems on the paper provided. Indicate clearly which curve is which, as well as main features of the response. $\boxed{7}$

Total for question 1: $\boxed{15}$

2 Temperature response

Consider the following transfer function showing the response of a temperature measurement (T) on a plant to a steam flow rate (Q_s).

$$G_3 = \frac{T}{Q_s} = \frac{5e^{-10s}}{(20s+1)(5s+1)^2}$$

2.1. Approximate G_3 by a first order plus dead time model using Skogestad's method. $\boxed{10}$

2.2. A colleague who is investigating this system converted the dead time to a rational function using a 1,1 Padé approximation, obtaining $G_4 \approx G_3$. They are puzzled because the step response of G_4 appears to show that the temperature will go down shortly after the steam flow rate is increased. Explain this observation by referring to the poles and zeros of G_4. $\boxed{10}$

2.3. Explain the benefits and disadvantages of the above methods of approximation. $\boxed{5}$

Total for question 2: $\boxed{\textit{25}}$

3 Electrically heated stirred tank system

Consider the following equations which represent an electrically heated stirred tank system, written in terms of deviation variables.

$$mC\frac{\mathrm{d}T}{\mathrm{d}t} = wC(T_i - T) + h_eA_e(T_e - T) \tag{1}$$

$$m_eC_e\frac{\mathrm{d}T_e}{\mathrm{d}t} = Q - h_eA_e(T_e - T) \tag{2}$$

It has been determined that $m_eC_e = wC = 20\,\mathrm{kcal/°C}$, $mC = 200\,\mathrm{kcal/°C}$ and $h_eA_e = 20\,\mathrm{kcal/(°C \cdot min)}$.

Without using the Laplace domain, show that bounded changes in Q and T_i will result in bounded changes in T_e and T.

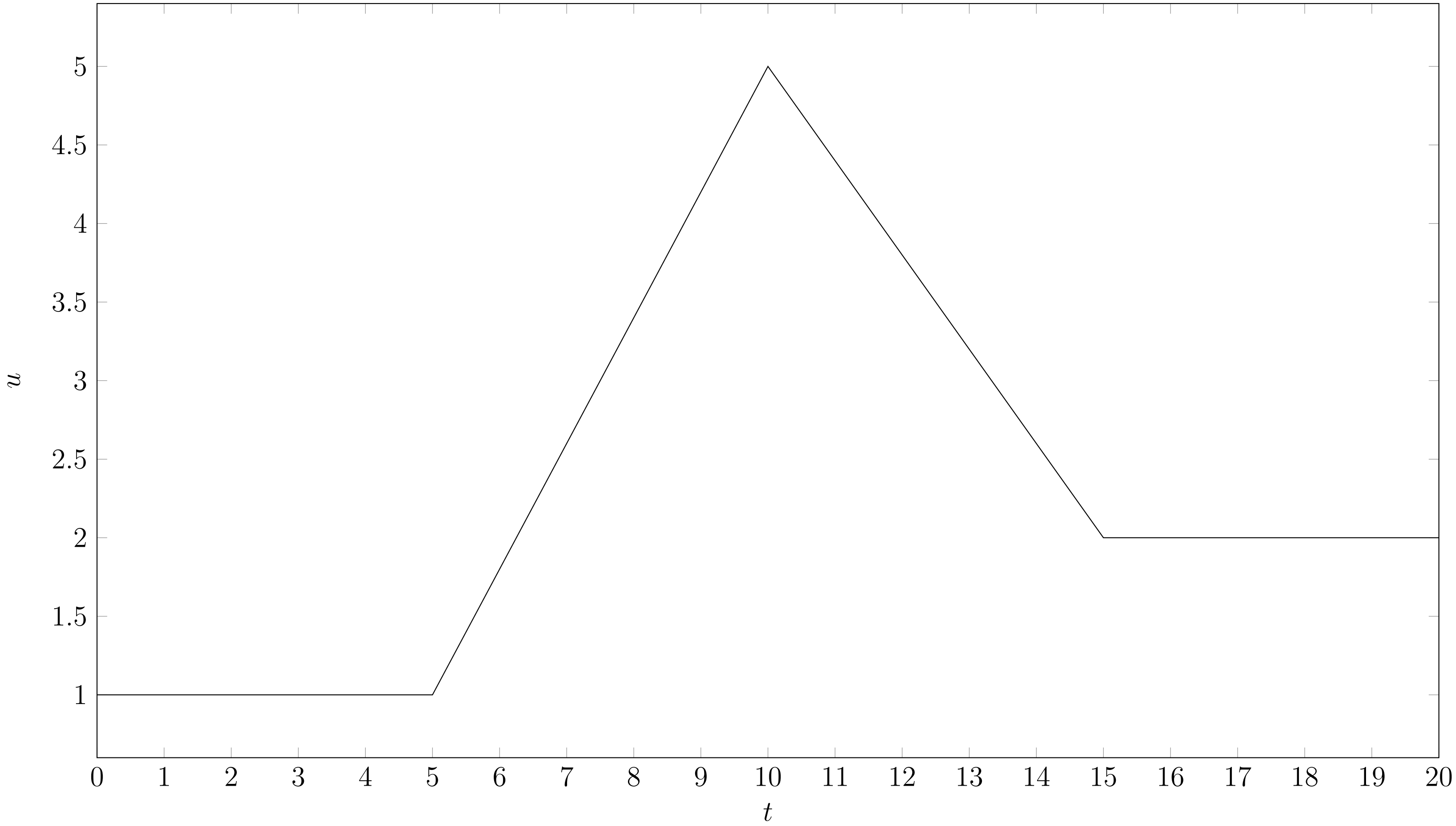
Total for question 3: $\boxed{\textit{10}}$

Full Marks $\boxed{50}$

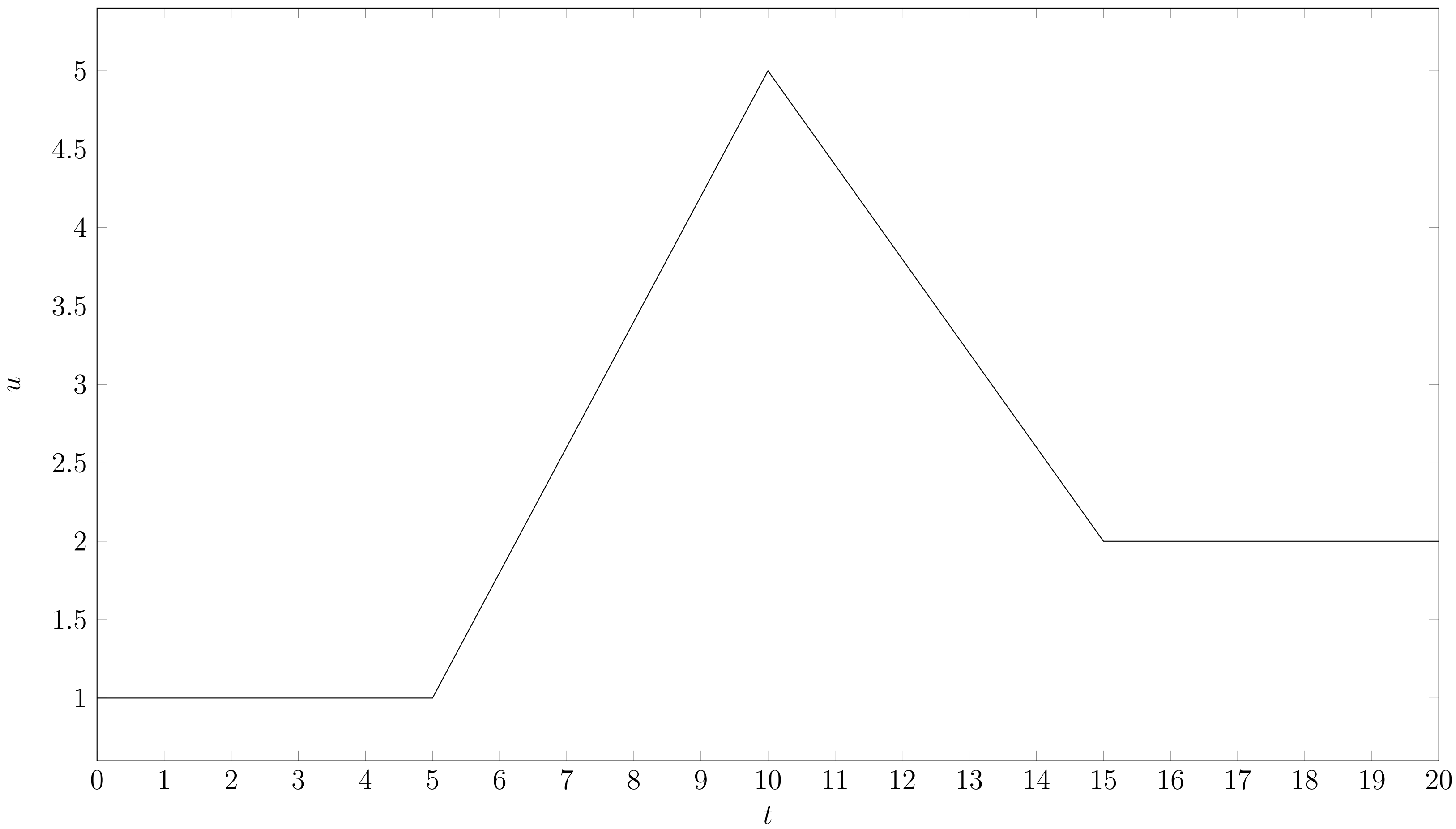
LaTeX 2ε
www.latex-project.org

This page is blank

Name: ______________________________ Student Number: ________________



Name: ______________________ Student Number: ______________



Datasheet: CPN321/CPB410

Compiled on May 5, 2018

General solution of 1st order DE:

$$\dot{x} + P(t)x = Q(t) \quad \Rightarrow \quad x = \frac{1}{F_I}\int Q(t)F_I\mathrm{d}t + c_1 \quad \text{with} \quad F_I = \exp\left(\int P(t)\mathrm{d}t\right)$$

Taylor Series expansion near point $x = a$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$

$$f(x_1, \ldots, x_d) = \sum_{n_1=0}^{\infty} \cdots \sum_{n_d=0}^{\infty} \frac{(x_1-a_1)^{n_1} \cdots (x_d-a_d)^{n_d}}{n_1! \cdots n_d!} \left(\frac{\partial^{n_1+\cdots+n_d} f}{\partial x_1^{n_1} \cdots \partial x_d^{n_d}}\right)(a_1, \ldots, a_d)$$

Linear approximation around point $\mathbf{x} = \mathbf{0}$, where $f(\mathbf{x}) = \mathbf{0}$:

$$f(\mathbf{x}) \approx \nabla f(\mathbf{0}) \cdot \mathbf{x}$$

$$f(x_1, x_2, \cdots, x_d) \approx \frac{\partial f}{\partial x_1}(0)x_1 + \frac{\partial f}{\partial x_2}(0)x_2 + \cdots + \frac{\partial f}{\partial x_d}(0)x_d$$

Partial fraction expansion (for strictly proper rational functions of s)

$$F(s) = \frac{(s-z_1)(s-z_2)(s-z_3)\cdots}{(s-p_1)^n(s-p_2)(s-p_3)\cdots} = \underbrace{\sum_{m=0}^{n-1} \frac{A_m}{(s-p_1)^{n-m}}}_{\text{repeated roots}} + \frac{B}{s-p_2} + \frac{C}{s-p_3} \cdots$$

$$A_m = \lim_{s \to p_1} \left\{ \frac{\mathrm{d}^m}{\mathrm{d}s^m} \Big[(s-p_1)^n F(s) \Big] \right\} \frac{1}{m!}$$

$$B = \lim_{s \to p_2} \big[(s-p_2)F(s) \big]$$

Euler identity:

$$e^{i\theta} = \cos\theta + i\sin\theta \quad \therefore e^{-i\theta} = \cos\theta - i\sin\theta \quad \text{and} \quad e^{i\pi} - 1 = 0$$

(1,1) Padé approximation of dead time:

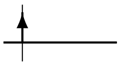
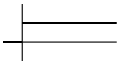
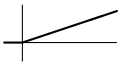
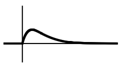
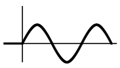
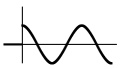
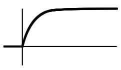
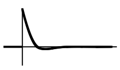
$$e^{-Ds} \approx \frac{1 - \frac{D}{2}s}{1 + \frac{D}{2}s}$$

PID controller:

$$m = K_C\left(\varepsilon + \frac{1}{\tau_I}\int_0^t \varepsilon \mathrm{d}t + \tau_D \frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\right) \qquad \frac{m}{\varepsilon}(s) = K_C\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Tuning rules:

	Ziegler-Nichols			Cohen-Coon (with $\phi = \frac{t_D}{\tau_P}$)		
	K_C	τ_I	τ_D	K_C	τ_I	τ_D
P	$\frac{K_u}{2}$			$\frac{\phi+3}{3K_P\phi}$		
PI	$\frac{K_u}{2.2}$	$\frac{P_u}{1.2}$		$\frac{5\phi+54}{60K_P\phi}$	$t_D\frac{30+3\phi}{9+20\phi}$	
PID	$\frac{K_u}{1.7}$	$\frac{P_u}{2}$	$\frac{P_u}{8}$	$\frac{3\phi+16}{12K_P\phi}$	$t_D\frac{32+6\phi}{13+8\phi}$	$\frac{4t_D}{11+2\phi}$

Time domain		Laplace-transform	z-transform ($b = e^{-aT}$)
Impulse: $\delta(t)$		1	1
Unit step: $u(t)$		$\dfrac{1}{s}$	$\dfrac{1}{1-z^{-1}}$
Ramp: t		$\dfrac{1}{s^2}$	$\dfrac{Tz^{-1}}{\left(1-z^{-1}\right)^2}$
t^n		$\dfrac{n!}{s^{n+1}}$	$\lim\limits_{a\to 0}(-1)^n \dfrac{\partial^n}{\partial a^n}\dfrac{1}{1-bz^{-1}}$
e^{-at}		$\dfrac{1}{s+a}$	$\dfrac{1}{1-bz^{-1}}$
te^{-at}		$\dfrac{1}{(s+a)^2}$	$\dfrac{Tbz^{-1}}{\left(1-bz^{-1}\right)^2}$
t^2e^{-at}		$\dfrac{2}{(s+a)^3}$	$\dfrac{T^2bz^{-1}(1+bz^{-1})}{\left(1-bz^{-1}\right)^3}$
$\sin(\omega t)$		$\dfrac{\omega}{s^2+\omega^2}$	$\dfrac{z^{-1}\sin(\omega T)}{1-2z^{-1}\cos(\omega T)+z^{-2}}$
$\cos(\omega t)$		$\dfrac{s}{s^2+\omega^2}$	$\dfrac{1-z^{-1}\cos(\omega T)}{1-2z^{-1}\cos(\omega T)+z^{-2}}$
$1-e^{-at}$		$\dfrac{a}{s(s+a)}$	$\dfrac{(1-b)z^{-1}}{\left(1-z^{-1}\right)\left(1-bz^{-1}\right)}$
$e^{-at}\sin(\omega t)$		$\dfrac{\omega}{(s+a)^2+\omega^2}$	$\dfrac{z^{-1}b\sin(\omega T)}{1-2z^{-1}b\cos(\omega T)+b^2z^{-2}}$
$e^{-at}\cos(\omega t)$		$\dfrac{s+a}{(s+a)^2+\omega^2}$	$\dfrac{1-z^{-1}b\cos(\omega T)}{1-2z^{-1}b\cos(\omega T)+b^2z^{-2}}$
Initial value theorem: $\lim\limits_{t\to 0} f(t)$		$\lim\limits_{s\to\infty} sF(s)$	$\lim\limits_{z\to\infty} F(z)$
Final value theorem: $\lim\limits_{s\to\infty} f(t)$		$\lim\limits_{s\to 0} sF(s)$	$\lim\limits_{z\to 1}\left[\left(1-z^{-1}\right)F(z)\right]$
Translation: $f(t-D)u(t-D)$		$e^{-Ds}F(s)$	$F(z)z^{-n}$ where $D=nT$
Derivative: $\dfrac{\mathrm{d}^n f(t)}{\mathrm{d}t^n} = f^n(t)$		$s^nF(s)-\sum\limits_{k=1}^{n} s^{k-1}f^{n-k}(0)$	
Integral: $\int_0^t f(t)\mathrm{d}t$		$\dfrac{1}{s}F(s)$	
Zero$^{\text{th}}$ order hold		$H(s)=\dfrac{1-e^{-Ts}}{s}$	