Process Dynamics – CPN321

Semester Test 2

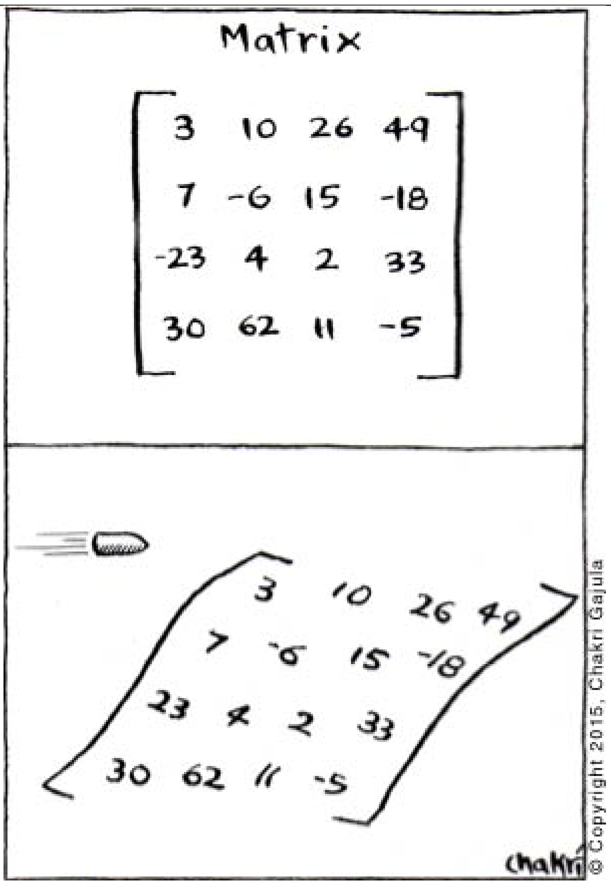
Chemical Engineering
Engineering and the Built Environment

Examiner: Carl Sandrock

October 2015 90 minutes

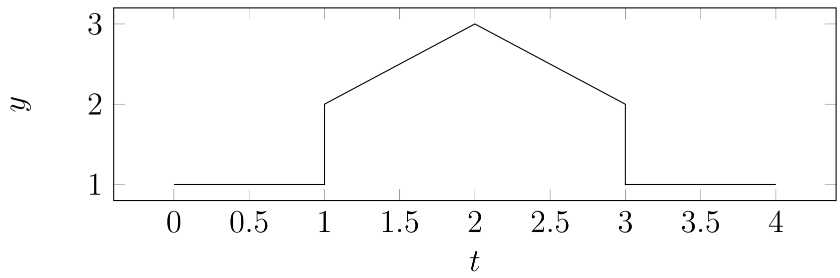
Instructions – Read carefully

• Answer all the questions. • This is a closed book test. All the information you may use is contained in the paper. • You may use the computer • Make sure that you motivate all your answers and write legibly.



1 Input modelling

Calculate the Laplace transform of the following function:



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2 Multivariable representation

The following system of equations have been linearised and are written in terms of deviation variables:

$$\frac{\mathrm{d}^2(p(t)+q(t))}{\mathrm{d}t^2} = p(t) + 3q(t) + m(t) \tag{1}$$

$$\frac{\mathrm{d}q(t)}{\mathrm{d}t} = q(t) + m(t) + u(t) \tag{2}$$

$$m(t) = p(t - D) \tag{3}$$

1. Transform these equations to the Laplace domain and determine the transfer function between u and m. $\boxed{10}$
2. Write the relationship between u and p in state space form (assume $D = 0$) $\boxed{10}$

20

3 Second order system

Consider the following transfer function describing the influence of u on y:

$$\frac{y}{u} = \frac{(5s+2)\,e^{-7s}}{5s^3 + 12s^2 + 24s + 8} \tag{4}$$

Use the attached graph paper (page 5) to answer this question.

1. Determine the time-domain function representing the response of y to a step of height 1 in u starting at time $t = 1$. $\boxed{10}$
2. Determine the steady state gain, time constant and damping coefficient of this system $\boxed{5}$

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4 Higher order functions

Consider the following transfer function:

$$G = \frac{y}{u} = \frac{Ke^{-2Ds}}{s(\tau^2 s^2 + 2\tau\zeta s + 1)} \tag{5}$$

Assume $K > 0$, $\tau > 0$, $D > 0$ and $0 < \zeta < 1$.

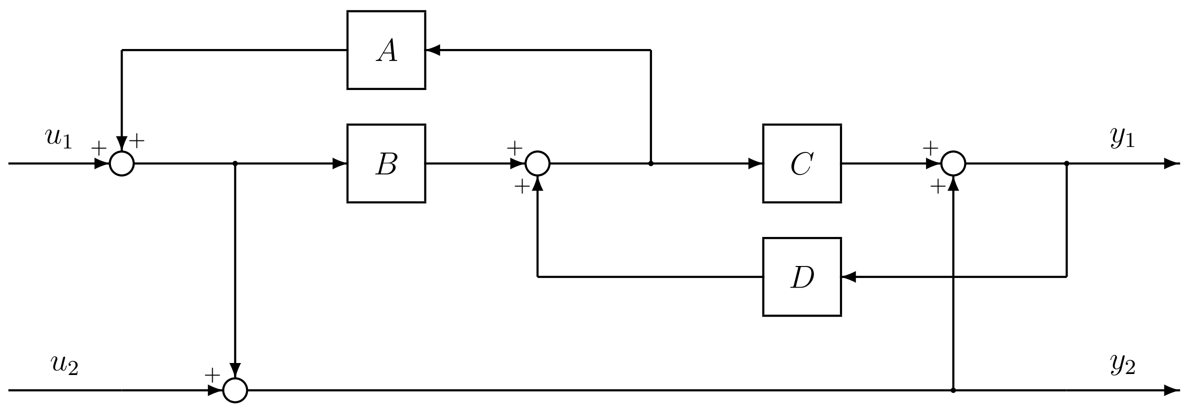
1. Sketch the unit step response of this system, next to a plot on the complex plane showing the poles with $\times$ and zeros with $\circ$ of the system. Indicate clearly how the variables in the expression relate to the response and to the position of the poles. $\boxed{10}$

2. Dead time is often approximated using Padé approximation. Assume a (1,1) Padé approximation is used and indicate the new poles with $+$ and new zeros with $\square$ of the new system on the same plot. Also draw the new step response using dashed lines (—). Indicate clearly on the graph how the new poles and zeros are related to the behaviour of the system. $\boxed{10}$

$\boxed{20}$

5 Block diagrams

Write down the (scalar) transfer function from u_1 to y_2 for the following block diagram:



$\boxed{20}$

Full Marks $\boxed{90}$

LaTeX 2_ε
www.latex-project.org

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Use this page to answer question 4. Name: ______________________ Student number: __________

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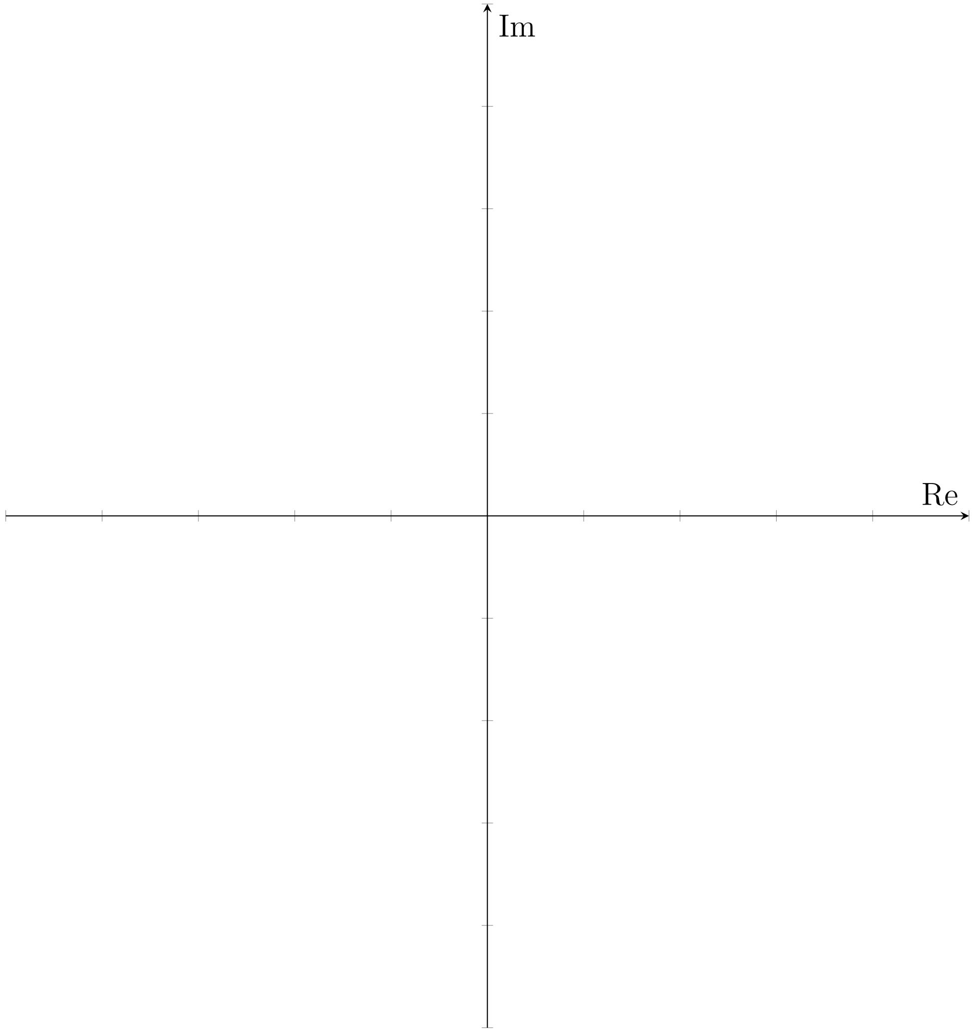

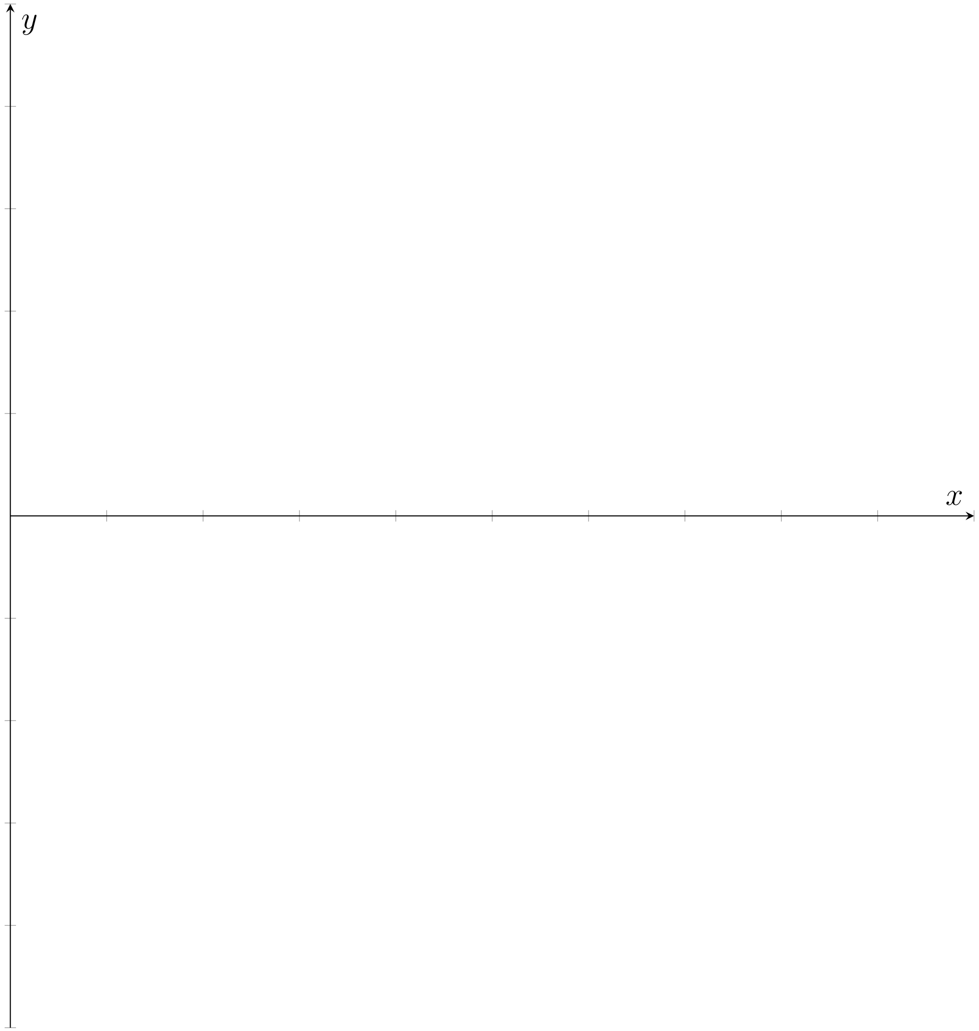

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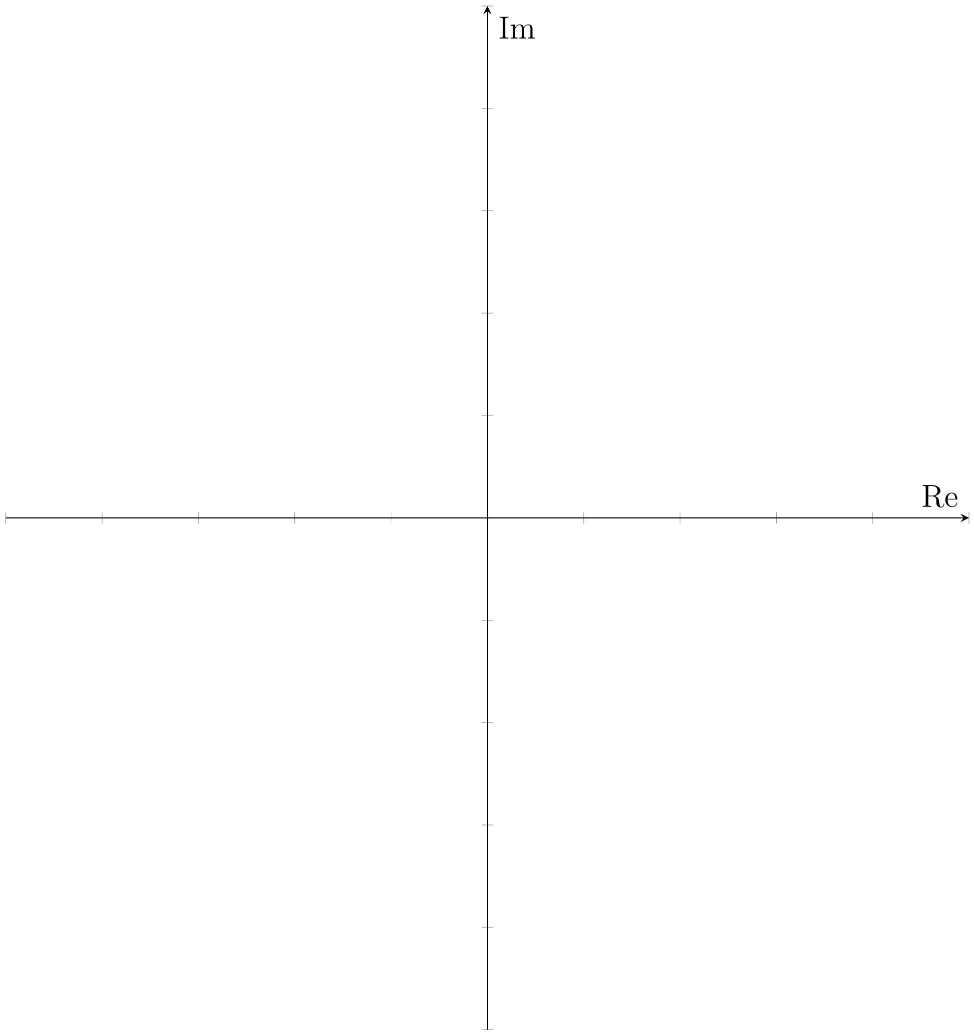
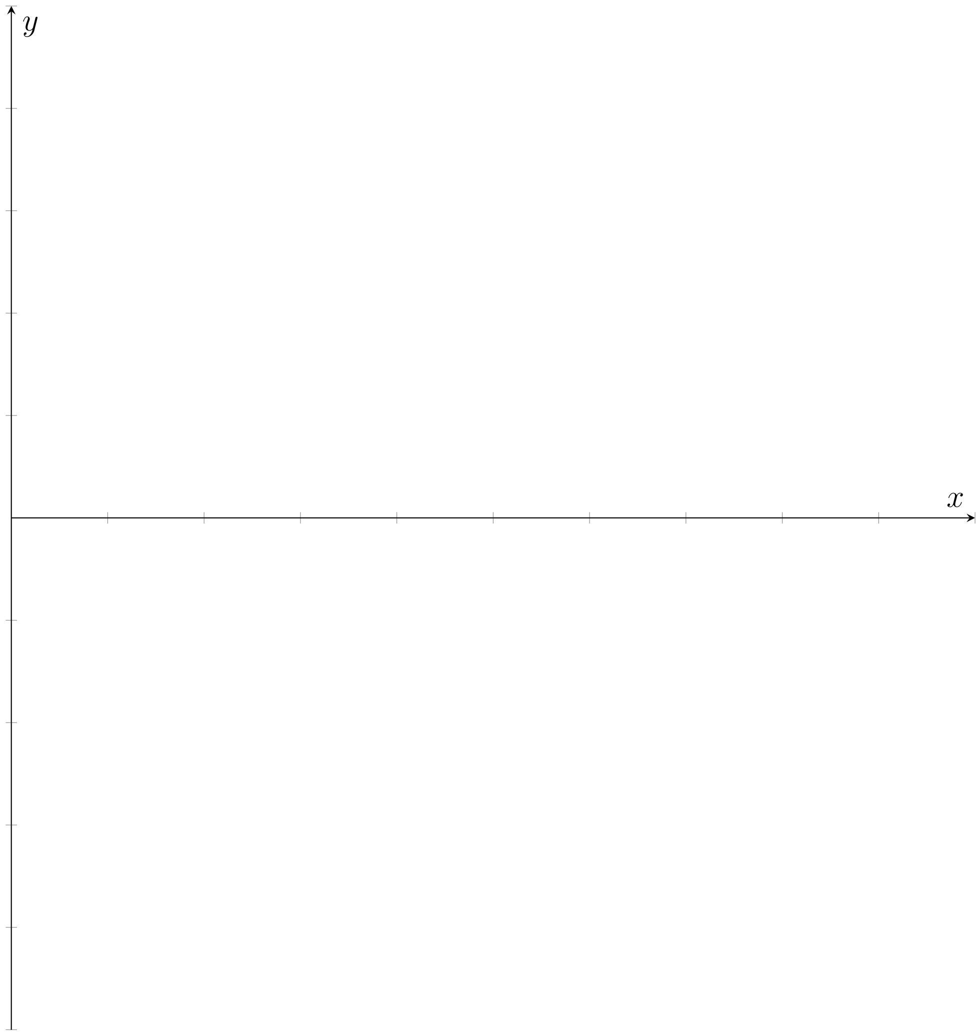




Datasheet: CPN321/CPB410

Compiled on October 31, 2013

General solution of 1$^{\text{st}}$ order DE:

$$\dot{x} + P(t)x = Q(t) \quad \Rightarrow \quad x = \frac{1}{F_I}\int Q(t)F_I \mathrm{d}t + c_1 \quad \text{with} \quad F_I = \exp\left(\int P(t)\mathrm{d}t\right)$$

Taylor Series expansion near point $x = a$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$

$$f(x_1,\ldots,x_d) = \sum_{n_1=0}^{\infty}\cdots\sum_{n_d=0}^{\infty} \frac{(x_1-a_1)^{n_1}\cdots(x_d-a_d)^{n_d}}{n_1!\cdots n_d!}\left(\frac{\partial^{n_1+\cdots+n_d} f}{\partial x_1^{n_1}\cdots\partial x_d^{n_d}}\right)(a_1,\ldots,a_d)$$

Linear approximation around point $\mathbf{x} = \mathbf{0}$, where $f(\mathbf{x}) = \mathbf{0}$:

$$f(\mathbf{x}) \approx \nabla f(\mathbf{0}) \cdot \mathbf{x}$$

$$f(x_1, x_2, \cdots, x_d) \approx \frac{\partial f}{\partial x_1}(0)x_1 + \frac{\partial f}{\partial x_2}(0)x_2 + \cdots + \frac{\partial f}{\partial x_d}(0)x_d$$

Partial fraction expansion (for strictly proper rational functions of s)

$$F(s) = \frac{(s-z_1)(s-z_2)(s-z_3)\cdots}{(s-p_1)^n(s-p_2)(s-p_3)\cdots} = \underbrace{\sum_{m=0}^{n-1}\frac{A_m}{(s-p_1)^{n-m}}}_{\text{repeated roots}} + \frac{B}{s-p_2} + \frac{C}{s-p_3}\cdots$$

$$A_m = \lim_{s\to p_1}\left\{\frac{\mathrm{d}^m}{\mathrm{d}s^m}\Big[(s-p_1)^n F(s)\Big]\right\}\frac{1}{m!}$$

$$B = \lim_{s\to p_2}\big[(s-p_2)F(s)\big]$$

Euler identity:

$$e^{i\theta} = \cos\theta + i\sin\theta \quad \therefore e^{-i\theta} = \cos\theta - i\sin\theta \quad \text{and} \quad e^{i\pi} - 1 = 0$$

(1,1) Padé approximation of dead time:

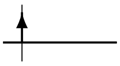
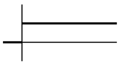
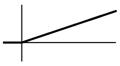
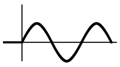
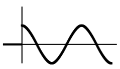
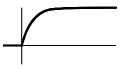
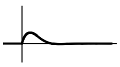
$$e^{-Ds} \approx \frac{1 - \frac{D}{2}s}{1 + \frac{D}{2}s}$$

PID controller:

$$m = K_C\left(\varepsilon + \frac{1}{\tau_I}\int_0^t \varepsilon \mathrm{d}t + \tau_D\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\right) \qquad \frac{m}{\varepsilon}(s) = K_C\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Tuning rules:

	Ziegler-Nichols			Cohen-Coon (with $\phi = \frac{t_D}{\tau_P}$)		
	K_C	τ_I	τ_D	K_C	τ_I	τ_D
P	$\frac{K_u}{2}$			$\frac{\phi+3}{3K_P\phi}$		
PI	$\frac{K_u}{2.2}$	$\frac{P_u}{1.2}$		$\frac{5\phi+54}{60K_P\phi}$	$t_D\frac{30+3\phi}{9+20\phi}$	
PID	$\frac{K_u}{1.7}$	$\frac{P_u}{2}$	$\frac{P_u}{8}$	$\frac{3\phi+16}{12K_P\phi}$	$t_D\frac{32+6\phi}{13+8\phi}$	$\frac{4}{11+2\phi}$

Time domain	Laplace-transform	z-transform ($b = e^{-aT}$)
Impulse: $\delta(t)$ 	1	1
Unit step: $u(t)$ 	$\dfrac{1}{s}$	$\dfrac{1}{1-z^{-1}}$
Ramp: t 	$\dfrac{1}{s^2}$	$\dfrac{Tz^{-1}}{(1-z^{-1})^2}$
t^n 	$\dfrac{n!}{s^{n+1}}$	$\lim\limits_{a\to 0}(-1)^n \dfrac{\partial^n}{\partial a^n}\dfrac{1}{1-bz^{-1}}$
e^{-at} 	$\dfrac{1}{s+a}$	$\dfrac{1}{1-bz^{-1}}$
te^{-at} 	$\dfrac{1}{(s+a)^2}$	$\dfrac{Tbz^{-1}}{(1-bz^{-1})^2}$
t^2e^{-at} 	$\dfrac{2}{(s+a)^3}$	$\dfrac{T^2bz^{-1}(1+bz^{-1})}{(1-bz^{-1})^3}$
$\sin(\omega t)$ 	$\dfrac{\omega}{s^2+\omega^2}$	$\dfrac{z^{-1}\sin(\omega T)}{1-2z^{-1}\cos(\omega T)+z^{-2}}$
$\cos(\omega t)$ 	$\dfrac{s}{s^2+\omega^2}$	$\dfrac{1-z^{-1}\cos(\omega T)}{1-2z^{-1}\cos(\omega T)+z^{-2}}$
$1-e^{-at}$ 	$\dfrac{a}{s(s+a)}$	$\dfrac{(1-b)z^{-1}}{(1-z^{-1})(1-bz^{-1})}$
$e^{-at}\sin(\omega t)$ 	$\dfrac{\omega}{(s+a)^2+\omega^2}$	$\dfrac{z^{-1}b\sin(\omega T)}{1-2z^{-1}b\cos(\omega T)+b^2z^{-2}}$
$e^{-at}\cos(\omega t)$ 	$\dfrac{s+a}{(s+a)^2+\omega^2}$	$\dfrac{1-z^{-1}b\cos(\omega T)}{1-2z^{-1}b\cos(\omega T)+b^2z^{-2}}$
Initial value theorem: $\lim\limits_{t\to 0} f(t)$	$\lim\limits_{s\to\infty} sF(s)$	$\lim\limits_{z\to\infty} F(z)$
Final value theorem: $\lim\limits_{s\to\infty} f(t)$	$\lim\limits_{s\to 0} sF(s)$	$\lim\limits_{z\to 1}\left[\left(1-z^{-1}\right)F(z)\right]$
Translation: $f(t-D)u(t-D)$	$e^{-Ds}F(s)$	$F(z)z^{-n}$ where $D=nT$
Derivative: $\dfrac{\mathrm{d}^n f(t)}{\mathrm{d}t^n} = f^n(t)$	$s^nF(s) - \sum\limits_{k=1}^{n} s^{k-1}f^{n-k}(0)$	
Integral: $\int_0^t f(t)\mathrm{d}t$	$\dfrac{1}{s}F(s)$	
Zero$^{\text{th}}$ order hold	$H(s) = \dfrac{1-e^{-Ts}}{s}$	