CPN 321 Project Part 2

The temperature control lab development

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1. Introduction

A mathematical model, of the Temperature control Lab (TCLab) developed by the BYU Prism group, was developed. It was done in order to be able to simulate the dynamic model. The dynamic model can then be used to determine the parameters, when compared to experimental data and readings. This will ensure that the model is a reasonable representation of the real system at hand. The TCLab unit was critically evaluated. Mathematical equations were determined for each unit in the TCLab and combined in order to develop the final mathematical model. The roles as well as the units of each symbol in the model were determined. A degree of freedom analysis was performed, in order to determine whether the system is: over specified; under specified; or fully specified. The mathematical model, that was created, is fully specified.

2. Literature Review

Energy exists in various different forms. Some of these forms include: thermal; mechanical; chemical; and magnetic (Çengel & Ghajar, 2011: 7). In this assignment the emphasise will be on thermal energy and its movement from and between different units in a TCLab unit.

Thermal energy, also known as heat, transfers from one system to another, due to a temperature gradient (Çengel & Ghajar, 2011: 17). The temperature gradient is the driving force for the heat transfer. The transfer of energy occurs between an object with a high temperature and an object with a low temperature (Çengel & Ghajar, 2011: 17). Heat transfer from an object to the environment or between two or more objects can be classified in three different modes, namely: conduction, convection and radiation (Çengel & Ghajar, 2011: 17). All three of the modes will be used in order to determine a mathematical representation of the TCLab unit.

Heat transfer in the form of conduction is between more energetic particles and adjacent less energetic particles (Çengel & Ghajar, 2011: 17). Convection occurs due to the interaction between the particles (Çengel & Ghajar, 2011: 17). Fourier's law of heat conduction is used in order to determine the heat transfer through conduction (Çengel & Ghajar, 2011: 18). The thermal conductivity of stainless steel was determined as 14.4 W m^{-1} K^{-1} (The Engineering Toolbox, 2019)[h]. Conduction is represented as

$$\dot{Q}_{cond} = -kA\frac{dT}{dt} \quad (1)$$

The second mode of heat transfer, convection, is defined as heat transfer between a solid surface and an adjacent gas or liquid that is in motion (Çengel & Ghajar, 2011: 24). When the fluid moves faster, the heat transfer between the different mediums increase (Çengel & Ghajar, 2011: 24). Newton's law of cooling is used to determine the convection. It is described as (Çengel & Ghajar, 2011: 26).

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \quad (2)$$

The convective heat transfer coefficient (h_∞) for the ambient air was calculated by using Equation 3 (The Engineering Toolbox, 2019). The parameter, used in the model, for the heat transfer coefficient of air, was calculated by assuming the air surrounding the system to be stagnant, thus its velocity is zero. When modelling the air, it is assumed not to be stagnant.

$$h_c = 10.45 - v + 10v^{\frac{1}{2}} \quad (3)$$

The heat transfer of the ambient air was calculated to be 10.45 W m^{-2} K^{-1}[a]. The ambient air temperature (T_∞), in Pretoria, was determined by analysing the temperature over the past year (World Weather Online, 2019). A temperature of 297.15 K (24 ˚C) is used in the model.

Due to changes in the electron configurations of molecules and atoms, energy is being emitted in the form of electromagnetic waves (Çengel & Ghajar, 2011: 27). The transfer of energy is referred to as radiation. Radiation is represented by Equation 4 (Çengel & Ghajar, 2011: 28). The emissivity (ε) of stainless steel, used in the model equations, is 0.85 (The Engineering Toolbox, 2019)[d]. The Stefan-Boltzmann constant is $5.67\text{x}10^{-8}$ W m^{-2} K^{-4} (Çengel & Ghajar, 2011: 28)[e].

$$\dot{Q}_{emit} = \varepsilon\sigma A_s T_s^4 \quad (4)$$

The area as well as the volume of the fin banks, heaters and sensors were determined by measuring the dimensions, of the physical TCLab unit, with a ruler[b]. The mass of a unit was calculated by using Equation 5 (Çengel & Ghajar, 2011: 13). The density of stainless steel is 7900 kg m^{-3} (The Physics Factbook, 2019)[g].

$$\dot{m} = \rho V \quad (5)$$

The heat capacity of stainless steel is 502.42 J kg^{-1} ˚C^{-1} (Engineers Edge, 2019)[f]. A different amount of energy is being supplied to the heaters[j]. Each heater has its own heater factor (BYU, 2019)[i].

3. Modelling Approach

3.1.1. Assumptions

In order to be able to define the mathematical equations, that describe the dynamic model of a system, certain assumptions must be made. The following assumptions were made in order to be able to define the equations of the TCLab unit:

- The system is operating under unsteady state conditions (the differential equations are not equal to zero) and the air surrounding the system is considered not to be stagnant.
- The environment is at a constant temperature of 24 ˚C and a pressure of 87 kPa (World Weather Online, 2019)[c].
- The air has a constant heat transfer coefficient of 10.45 W m^{-2} K^{-1} (The Engineering Toolbox, 2019). The heat transfer coefficient for the units are the same as that of air.
- The fins, heaters and sensors are constructed of the same material (Dynamics and Control, 2019). The material of construction of the units is stainless steel 306 (Dynamics and Control, 2019).
- The units have a constant thermal conductivity of 14.4 W m^{-1} K^{-1} (The Engineering Toolbox, 2019).
- The units have the same emissivity (ε).
- The thermal epoxy between the sensors and the heaters are of negligible thickness (the units are seen to be in direct contact with one another).
- Uniform conduction occurs in the fins. The nuts and bolts are excluded in the modelling of the system.
- All the heat being transferred to the fins, through conduction, is being transferred to the environment by means of convective as well as radiation.
- The air between the fins (surrounding heater one) and the fins (surrounding heater two), are the same ambient air that surrounds the system, with the same characteristics.
- The conduction occurring between the heater and the sensor and the conduction occurring between the heater and the fins are over the same distance.

- More energy is supplied to heater one than to heater two. This will cause a temperature gradient between the two fin banks.
- The energy supplied to the heaters will not be the same energy being conducted from the heaters to the fins and the sensors. Both heaters have a heater factor (α), because the heaters are not ideal elements (Dynamics and Control, 2019).
- The heat mechanisms between the fin banks are convection as well as radiation.
- All the heat transferred to the sensors, through conduction, is being transferred to the environment by means of convection as well as radiation.
- The only inputs to the system are energy to the heating units. The temperature of the ambient air as well as the heat transfer coefficient are assumed to be known variables (parameters). No heat transfer occurs back to the basis of the TCLab (mother board).
- The outputs of the system are heat being transferred from and between the units, as well as the temperature of the different units.
- The dimensions of the units were determined by physical measurements (with a ruler) and calculations, using the TCLab unit, as well as literature sources.
- The temperature sensors are thermistors (Dynamics and Control, 2019).

3.1.2. Changes made to the model

Python is used for comparing the model, as described in this document, with the experimental model created by the TCLab unit. No changes were made to the assumptions as stated in the document. All of the assumptions, for the model, were used for the simulations done in Python.

No parameters were changed when creating the model in Python. The dimensions as calculated in Part 1 of the project were used. When simulating the model in Python, it created extremely large values for $Q_{H1,S1}$ and $Q_{H2,S2}$. As a result, the differential equations did not converge to a solution. After evaluation, it was determined that the value of the thermal conduction terms was too large. After dividing it by a factor of a thousand, the differential equations converged to a solution. These parameters changes will be discussed and analysed in Part 3 of the project.

After revising the model, it was noticed that Equation 4 was applied incorrectly. Changes were made to some of the equations listed in Table 3. Table 1 summarises the changes that were made to the model equations. The new equations were used in Python, in order to create a comparison between the theoretical model and the experimental model.

Table 1: Changes made to the model equations.

Equation number	Original symbol	New symbol	New equation
5.	h_∞	A_{F1}	$Q_{rad,1} = \varepsilon\sigma A_{F1}(T_{F1}^4 - T_\infty^4)$
6.	h_∞	A_{F2}	$Q_{rad,2} = \varepsilon\sigma A_{F2}(T_{F2}^4 - T_\infty^4)$
7.	h_∞	A_{FIN1}	$Q_{conv1,2} = h_\infty A_{FIN1}(T_{F1} - T_{F2})$
8.	$A_{FIN1} + A_{FIN2}$	A_{FIN1}	$Q_{rad1,2} = \varepsilon\sigma A_{FIN1}(T_{F1}^4 - T_{F2}^4)$
17.	h_{H1}	A_{H1}	$Q_{H1,rad} = \varepsilon\sigma A_{H1}(T_{H1}^4 - T_\infty^4)$
18.	h_{H2}	A_{H2}	$Q_{H2,rad} = \varepsilon\sigma A_{H2}(T_{H2}^4 - T_\infty^4)$
25.	h_{S1}	A_{S1}	$Q_{S1,rad} = \varepsilon\sigma A_{S1}(T_{S1}^4 - T_\infty^4)$
26.	h_{S2}	A_{S2}	$Q_{S2,rad} = \varepsilon\sigma A_{S2}(T_{S2}^4 - T_\infty^4)$

3.1.3. List of symbols

Table 2 summarises the symbols used in the dynamic model equations, as well as their role. The role of a symbol can either be: an input (I); an output (O); or a parameter (P).

Table 2: Symbols used in the mathematical equations.

Symbol	Description	Role	Units	Numeric Value
$Q_{conv,1}$	Convection from fin bank 1 to ambient air.	O	W	
h_{∞}	Heat transfer coefficient of ambient air.	P	$W\ m^{-2}\ K^{-1}$	10.45[a]
A_{F1}	Area of fin bank 1.	P	m^2	0.0004[b]
T_{F1}	Temperature of fin bank 1.	O	K	
T_{∞}	Temperature of ambient air.	I	K	297.15[c]
$Q_{conv,2}$	Convection from fin bank 1 to environment.	O	W	
A_{F2}	Area of fin bank 2.	P	m^2	0.0004[b]
T_{F2}	Temperature of fin bank 2.	O	K	
$Q_{cond,1}$	Conduction to fin bank 1 from heater 1.	O	W	
$Q_{cond,2}$	Conduction to fin bank 2 from heater 2.	O	W	
$Q_{rad,1}$	Radiation from fin bank 1 to environment.	O	W	
ε	Emissivity.	P	-	0.85[d]
σ	Stefan-Boltzmann constant.	P	$W\ m^{-2}\ K^{-4}$	$5.67\text{x}10^{-8}$[e]
$Q_{rad,2}$	Radiation from fin bank 2 to environment.	O	W	

Symbol	Description	Role	Units	Numeric Value
$Q_{conv1,2}$	Convection between the two heaters.	O	W	
A_{FIN1}	Side area of fins on fin bank 1.	P	m²	0.00009[b]
$Q_{rad1,2}$	Radiation between the two heaters.	O	W	
m_{F1}	Mass of fin bank 1.	P	kg	0.0029[b]
Cp_{F1}	Heat capacity of fin bank 1.	P	J kg⁻¹ K⁻¹	502.42[f]
m_{F2}	Mass of fin bank 2.	P	kg	0.0029[b]
Cp_{F2}	Heat capacity of fin bank 2.	P	J kg⁻¹ K⁻¹	502.42[f]
ρ_{F1}	Density of fin bank 1.	P	kg m⁻³	7900[g]
ρ_{F2}	Density of fin bank 2.	P	kg m⁻³	7900[g]
V_{F1}	Volume of fin bank 1.	P	m³	0.00036[b]
V_{F2}	Volume of fin bank 2.	P	m³	0.00036[b]
$Q_{H1,F1}$	Conduction from heater 1 to fin bank 1.	O	W	
k_{H1}	Thermal conductivity.	P	W m⁻¹ K⁻¹	14.40[h]
A_{H1}	Area of heater 1.	P	m²	0.00008[b]
L_{H1}	Path length that heat travels in heater 1.	P	m	0.0015[b]
T_{H1}	Temperature of heater 1.	O	K	
$Q_{H1,S1}$	Heat transfer from heater 1 to sensor 1, through conduction.	O	W	
T_{S1}	Temperature of sensor 1.	O	K	

Symbol	Description	Role	Units	Numeric Value
$Q_{H2,F2}$	Conduction from heater 2 to fin bank 2.	O	W	
k_{H2}	Thermal conductivity.	P	W m^{-1} K^{-1}	14.40[h]
A_{H2}	Area of heater 2.	P	m^{2}	0.00008[b]
L_{H2}	Path length that heat travels in heater 2.	P	m	0.0015[b]
T_{H2}	Temperature of heater 2.	O	K	
$Q_{H2,S2}$,	Heat transfer from heater 2 to sensor 2, through conduction.	O	W	
T_{S2}	Temperature of sensor 2.	O	K	
$Q_{H1,conv}$	Convection from heater 1 to environment.	O	W	
h_{H1}	Heat transfer coefficient of heater 1.	P	W m^{-2} K^{-1}	10.45[a]
$Q_{H2,conv}$	Convection from heater 2 to environment.	O	W	
h_{H2}	Heat transfer coefficient of heater 2.	P	W m^{-2} K^{-1}	10.45[a]
$Q_{H1,rad}$	Radiation from heater 1.	O	W	
$Q_{H2,rad}$	Radiation from heater 2.	O	W	
m_{H1}	Mass of heater 1.	P	kg	0.00019[b]
Cp_{H1}	Heat capacity of heater 1.	P	J kg^{-1} K^{-1}	502.42[f]
m_{H2}	Mass of heater 2.	P	kg	0.00019[b]
Cp_{H2}	Heat capacity of heater 2.	P	J kg^{-1} K^{-1}	502.42[f]
α_1	Heat factor of heater 1.	P	W	0.01[i]
Q_{IN1}	Heat supplied to heater 1.	I	W	1.00[j]

Symbol	Description	Role	Units	Numeric Value
α_2	Heat factor of heater 2.	P	W	0.0075[i]
Q_{IN2}	Heat supplied to heater 2.	I	W	0.75[j]
$Q_{S1,cond}$	Heat conducted from heater 1 to sensor 1.	O	W	
$Q_{S2,cond}$	Heat conducted from heater 2 to sensor 2.	O	W	
$Q_{S1,conv}$	Heat transfer through convection from sensor 1 to the environment.	O	W	
$Q_{S2,conv}$	Heat transfer through convection from sensor 2 to the environment.	O	W	
h_{S1}	Heat transfer coefficient of sensor 1.	P	$W\ m^{-2}\ K^{-1}$	10.45[a]
A_{S1}	Area of sensor 1.	P	m^2	0.000016[b]
h_{S2}	Heat transfer coefficient of sensor 2.	P	$W\ m^{-2}\ K^{-1}$	10.45[a]
A_{S2}	Area of sensor 2.	P	m^2	0.000016[b]
$Q_{S1,rad}$	Heat transfer in the form of radiation from sensor 1.	O	W	
$Q_{S2,rad}$	Heat transfer in the form of radiation from sensor 2.	O	W	
m_{S1}	Mass of sensor 1.	P	kg	0.00025[b]
Cp_{S1}	Heat capacity of sensor 1.	P	$J\ kg^{-1}\ K^{-1}$	502.42[f]
m_{S2}	Mass of sensor 2.	P	kg	0.00025[b]
Cp_{S2}	Heat capacity of sensor 2.	P	$J\ kg^{-1}\ K^{-1}$	502.42[f]

3.1.4. Equations describing the system

Table 3 summarises the mathematical equations that were obtained in order to develop the mathematical model. Each symbol in an equation was categorised as: an input (I); an output (O); or a parameter (P).

Table 3: Equations used to create a mathematical model.

Number	Equations	I	O	P
1.	$Q_{conv,1} = h_\infty A_{F1}(T_{F1} - T_\infty)$	T_∞	T_{F1}, $Q_{conv,1}$	h_∞, A_{F1}
2.	$Q_{conv,2} = h_\infty A_{F2}(T_{F2} - T_\infty)$		$Q_{conv,2}$, T_{F2}	A_{F2}
3.	$Q_{cond,1} = Q_{H1,F1}$		$Q_{cond,1}$, $Q_{H1,F1}$	
4.	$Q_{cond,2} = Q_{H2,F2}$		$Q_{cond,2}$, $Q_{H2,F2}$	
5.	$Q_{rad,1} = \varepsilon\sigma A_{F1}(T_{F1}^4 - T_\infty^4)$		$Q_{rad,1}$	ε, σ
6.	$Q_{rad,2} = \varepsilon\sigma A_{F2}(T_{F2}^4 - T_\infty^4)$		$Q_{rad,2}$	
7.	$Q_{conv1,2} = h_\infty A_{FIN1}(T_{F1} - T_{F2})$		$Q_{conv1,2}$	A_{FIN1}
8.	$Q_{rad1,2} = \varepsilon\sigma A_{FIN1}(T_{F1}^4 - T_{F2}^4)$		$Q_{rad1,2}$	

Number	Equations	I	O	P
9.	$m_{F1}Cp_{F1}\frac{d(T_{F1})}{dt} = -Q_{conv,1} - Q_{rad,1} + Q_{cond,1} - (Q_{conv1,2} + Q_{rad1,2})$			m_{F1}, Cp_{F1}
10.	$m_{F2}Cp_{F2}\frac{d(T_{F2})}{dt} = -Q_{conv,2} - Q_{rad,2} + Q_{cond,2} + (Q_{conv1,2} + Q_{rad1,2})$			m_{F2}, Cp_{F2}
11.	$Q_{H1,F1} = \frac{k_{H1}A_{H1}}{L_{H1}}(T_{H1} - T_{F1})$		T_{H1}	k_{H1}, A_{H1}, L_{H1}
12.	$Q_{H2,F2} = \frac{k_{H2}A_{H2}}{L_{H2}}(T_{H2} - T_{F2})$		T_{H2}	k_{H2}, A_{H2}, L_{H2}
13.	$Q_{H1,S1} = \frac{k_{H1}A_{H1}}{L_{H1}}(T_{H1} - T_{S1})$		$Q_{H1,S1}, T_{S1}$	
14.	$Q_{H2,S2} = \frac{k_{H2}A_{H2}}{L_{H2}}(T_{H2} - T_{S2})$		$Q_{H2,S2}, T_{S2}$	
15.	$Q_{H1,conv} = h_{H1}A_{H1}(T_{H1} - T_{\infty})$		$Q_{H1,conv}$	h_{H1}
16.	$Q_{H2,conv} = h_{H2}A_{H2}(T_{H2} - T_{\infty})$		$Q_{H2,conv}$	h_{H2}
17.	$Q_{H1,rad} = \varepsilon\sigma A_{H1}(T_{H1}^4 - T_{\infty}^4)$		$Q_{H1,rad}$	

Number	Equations	I	O	P
18.	$Q_{H2,rad} = \varepsilon\sigma A_{H2}(T_{H2}^4 - T_\infty^4)$		$Q_{H2,rad}$	
19.	$m_{H1}Cp_{H1}\dfrac{d(T_{H1})}{dt} = -Q_{H1,F1} - Q_{H1,S1} - Q_{H1,conv} - Q_{H1,rad} + \alpha_1 Q_{IN1}$	Q_{IN1}		$m_{H1}, Cp_{H1}, \alpha_1$
20.	$m_{H2}Cp_{H2}\dfrac{d(T_{H2})}{dt} = -Q_{H2,F2} - Q_{H2,S2} - Q_{H2,conv} - Q_{H2,rad} + \alpha_2 Q_{IN2}$	Q_{IN2}		$m_{H2}, Cp_{H2}, \alpha_2$
21.	$Q_{S1,cond} = Q_{H1,S1}$		$Q_{S1,cond}$	
22.	$Q_{S2,cond} = Q_{H2,S2}$		$Q_{S2,cond}$	
23.	$Q_{S1,conv} = h_{S1}A_{S1}(T_{S1} - T_\infty)$		$Q_{S1,conv}$	h_{S1}, A_{S1}
24.	$Q_{S2,conv} = h_{S2}A_{S2}(T_{S2} - T_\infty)$		$Q_{S2,conv}$	$h_{S2},, A_{S2}$
25.	$Q_{S1,rad} = \varepsilon\sigma A_{S1}(T_{S1}^4 - T_\infty^4)$		$Q_{S1,rad}$	
26.	$Q_{S2,rad} = \varepsilon\sigma A_{S2}(T_{S2}^4 - T_\infty^4)$		$Q_{S2,rad}$	
27.	$m_{S1}Cp_{S1}\dfrac{d(T_{S1})}{dt} = Q_{S1,cond} - Q_{S1,rad} - Q_{S1,conv}$			m_{S1}, Cp_{S1}
28.	$m_{S2}Cp_{S2}\dfrac{d(T_{S2})}{dt} = Q_{S2,cond} - Q_{S2,rad} - Q_{S2,conv}$			m_{S2}, Cp_{S2}

Table 4 summarises the number of equations and variables obtained in the model. These results were used in order to determine the degrees of freedom of the model.

Table 4: Results obtained from the mathematical equations.

Description	Number
Equations	28
Variables	67
Inputs	3
Outputs	28
Parameters	36

3.1.5. Degree of Freedom (DOF)

The degree of freedom analysis is performed in order to determine whether or not a system is fully specified. Equation 6 is used in order to determine the DOF of the dynamic model (Seborg *et al*, 2011: 21).

$$N_F = N_V - N_E \quad (6)$$

After the dynamic model was set up the degrees of freedom was calculated in order to determine whether the model is fully specified. The degree of freedom was calculated as 0.

4. References

BYU (2019), ‘’Dynamics and Control’’, https://apmonitor.com/pdc/index.php/Main/ArduinoModeling2, [2019, 24 August].

Çengel, YA and Ghajar, AJ (2011), *Heat and Mass Transfer: Fundamentals & Applications*, 5th edition, McGraw-Hill Education, New York.

Engineers Edge (2019), ‘’Specific heat capacity of metals table chart.’’, https://www.engineersedge.com/materials/specific_heat_capacity_of_metals_13259.htm, [2019, 23 August].

Sandrock, C (2019), *Process Control CPN 321*, University of Pretoria, Pretoria.

Seborg, DE, Edgar, TF, Mellichamp, DA and Doyle III, FJ (2011), *Process Dynamics and Control*, 3rd edition, John Wiley and Sons, Asia.

The Engineering Toolbox (2019), ‘’Thermal Conductivity.’’, https://www.engineeringtoolbox.com/thermal-conductivity-metals-d_858.html, [2019, 23 August].

The Engineering Toolbox (2019), ‘’Convective Heat Transfer.’’, https://www.engineeringtoolbox.com/convective-heat-transfer-d_430.html, [2019, 23 August].

The Engineering Toolbox (2019), ‘’Emissivity.’’, https://www.engineeringtoolbox.com/emissivity-coefficients-d_447.html, [2019, 23 August].

The Physics Factbook (2019), ‘’Density of stainless steel.’’, https://hypertextbook.com/facts/2004/KarenSutherland.shtml, [2019, 23 August].

World Weather Online (2019), ‘’Pretoria monthly climate average.’’, https://www.worldweatheronline.com/pretoria-weather-averages/gauteng/za.aspx, [2019, 23 August].