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Process Dynamics – CPN321

Semester Test 1

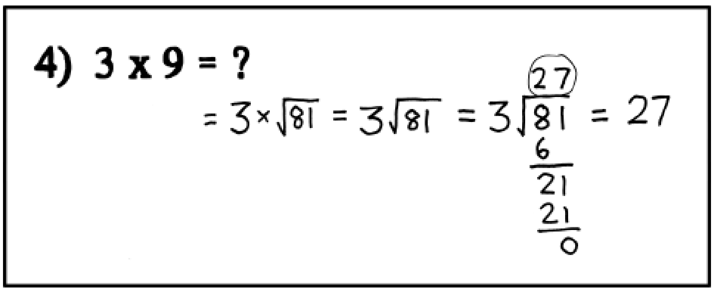
Chemical Engineering
Engineering and the Built Environment

Examiner: Carl Sandrock

2015-08-28 | 90 minutes

Instructions – Read carefully

• Answer all the questions. • This is a closed book test. All the information you may use is contained in the paper. • You may use the computer • Make sure that you motivate all your answers and write legibly.



xkcd.com

1 Modelling

Consider a liquid flow system consisting of a sealed tank with noncondensible gas above the liquid as shown in Figure 1. The walls of the tank are well-insulated. The incoming flow contains only liquid and the gas does not dissolve into the liquid. The pressure at the end of the outgoing line is P_a.

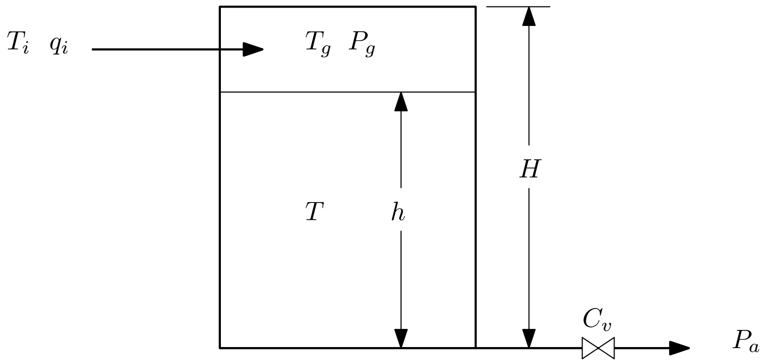


Figure 1: Closed tank system for question 1

You may further assume:

- the gas obeys the ideal gas law,
- the liquid density is constant,
- flow through the valve is proportional to the square root of the pressure difference over it,
- Newton's law of cooling describes the heat transfer between bulk gas and liquid temperatures.

1. Derive a dynamic model of the system which will allow you to predict the temperature of the liquid and the level in the tank. $\boxed{40}$
2. Indicate each symbol in your model as a parameter, an input or an output. $\boxed{5}$
3. Show that specifying the parameters and inputs in the model completely specifies it. $\boxed{5}$
4. It is desired to control the temperature and the level in the tank. Comment on how feedback control could be used to achieve this (think carefully about realistic manipulated variables). $\boxed{5}$

$\boxed{55}$

2 Simulation and Linear analysis

The following equations describe a reactor in a water bath at constant temperature T_c:

				P	I	O
1	$\frac{\mathrm{d}V}{\mathrm{d}t}$	$=$	$q_i - q_o$		q_i	q_o, V
2	$\frac{\mathrm{d}C_A V}{\mathrm{d}t}$	$=$	$q_i C_{Ai} - q_o C_A + Vr$		C_{Ai}	C_A, r
3	$C\rho\frac{\mathrm{d}VT}{\mathrm{d}t}$	$=$	$C\rho q_i T_i - C\rho q_o T - \Delta H_R V r - Q$	C, ρ, ΔH_R, T_C	T_i	T
4	Q	$=$	$UA(T_C - T)$	U, A, T_C		Q
5	k	$=$	$k_0 \exp\left(-\frac{E}{RT}\right)$	k_0, E, R		k
6	r	$=$	kC_A			
7	A	$=$	$k_1 V$	k_1		A
8	q_o	$=$	$k_2\sqrt{V}$	k_2		
			DOF $= 0$	12	3	8

1. Rewrite these equations in a form that will be suitable for computer solution by an ODE solver. $\boxed{5}$
2. Assume that $q = q_i = q_o$ and that T_i and q are not changing over time. Find the transfer function for the relationship between $C'_{Ai}(t)$ and $C'_A(t)$ $\boxed{25}$
3. If $C'_A(s) \approx \frac{1}{s+1}C'_{Ai}(s)$ and $c'_{Ai}(t) = \sin(2t)$, find $C_A(t)$. Show your working. If you are using the computer for this, please write down what you have done to obtain the answer. $\boxed{5}$

$\boxed{35}$

Full Marks $\boxed{90}$

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www.latex-project.org

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Datasheet: CPN321/CPB410

Compiled on October 31, 2013

General solution of 1$^{\text{st}}$ order DE:

$$\dot{x} + P(t)x = Q(t) \quad \Rightarrow \quad x = \frac{1}{F_I}\int Q(t)F_I \mathrm{d}t + c_1 \quad \text{with} \quad F_I = \exp\left(\int P(t)\mathrm{d}t\right)$$

Taylor Series expansion near point $x = a$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$
$$f(x_1, \dots, x_d) = \sum_{n_1=0}^{\infty} \cdots \sum_{n_d=0}^{\infty} \frac{(x_1 - a_1)^{n_1} \cdots (x_d - a_d)^{n_d}}{n_1! \cdots n_d!} \left(\frac{\partial^{n_1+\cdots+n_d} f}{\partial x_1^{n_1} \cdots \partial x_d^{n_d}}\right)(a_1, \dots, a_d)$$

Linear approximation around point $\mathbf{x} = \mathbf{0}$, where $f(\mathbf{x}) = \mathbf{0}$:

$$f(\mathbf{x}) \approx \nabla f(\mathbf{0}) \cdot \mathbf{x}$$
$$f(x_1, x_2, \cdots, x_d) \approx \frac{\partial f}{\partial x_1}(0)x_1 + \frac{\partial f}{\partial x_2}(0)x_2 + \cdots + \frac{\partial f}{\partial x_d}(0)x_d$$

Partial fraction expansion (for strictly proper rational functions of s)

$$F(s) = \frac{(s-z_1)(s-z_2)(s-z_3)\cdots}{(s-p_1)^n(s-p_2)(s-p_3)\cdots} = \underbrace{\sum_{m=0}^{n-1} \frac{A_m}{(s-p_1)^{n-m}}}_{\text{repeated roots}} + \frac{B}{s-p_2} + \frac{C}{s-p_3} \cdots$$

$$A_m = \lim_{s \to p_1} \left\{ \frac{\mathrm{d}^m}{\mathrm{d}s^m}\Big[(s-p_1)^n F(s)\Big] \right\} \frac{1}{m!}$$
$$B = \lim_{s \to p_2} \big[(s-p_2)F(s)\big]$$

Euler identity:

$$e^{i\theta} = \cos\theta + i\sin\theta \quad \therefore e^{-i\theta} = \cos\theta - i\sin\theta \quad \text{and} \quad e^{i\pi} - 1 = 0$$

(1,1) Padé approximation of dead time:

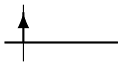
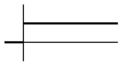
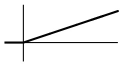
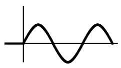
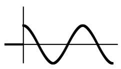
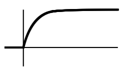
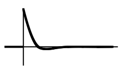
$$e^{-Ds} \approx \frac{1 - \frac{D}{2}s}{1 + \frac{D}{2}s}$$

PID controller:

$$m = K_C\left(\varepsilon + \frac{1}{\tau_I}\int_0^t \varepsilon \mathrm{d}t + \tau_D \frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\right) \qquad \frac{m}{\varepsilon}(s) = K_C\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Tuning rules:

	Ziegler-Nichols			Cohen-Coon (with $\phi = \frac{t_D}{\tau_P}$)		
	K_C	τ_I	τ_D	K_C	τ_I	τ_D
P	$\frac{K_u}{2}$			$\frac{\phi+3}{3K_P\phi}$		
PI	$\frac{K_u}{2.2}$	$\frac{P_u}{1.2}$		$\frac{5\phi+54}{60K_P\phi}$	$t_D\frac{30+3\phi}{9+20\phi}$	
PID	$\frac{K_u}{1.7}$	$\frac{P_u}{2}$	$\frac{P_u}{8}$	$\frac{3\phi+16}{12K_P\phi}$	$t_D\frac{32+6\phi}{13+8\phi}$	$\frac{4}{11+2\phi}$

Time domain		Laplace-transform	z-transform ($b = e^{-aT}$)
Impulse: $\delta(t)$		1	1
Unit step: $u(t)$		$\frac{1}{s}$	$\frac{1}{1-z^{-1}}$
Ramp: t		$\frac{1}{s^2}$	$\frac{Tz^{-1}}{\left(1-z^{-1}\right)^2}$
t^n		$\frac{n!}{s^{n+1}}$	$\lim\limits_{a\to 0}(-1)^n \frac{\partial^n}{\partial a^n}\frac{1}{1-bz^{-1}}$
e^{-at}		$\frac{1}{s+a}$	$\frac{1}{1-bz^{-1}}$
te^{-at}		$\frac{1}{(s+a)^2}$	$\frac{Tbz^{-1}}{\left(1-bz^{-1}\right)^2}$
t^2e^{-at}		$\frac{2}{(s+a)^3}$	$\frac{T^2bz^{-1}(1+bz^{-1})}{\left(1-bz^{-1}\right)^3}$
$\sin(\omega t)$		$\frac{\omega}{s^2+\omega^2}$	$\frac{z^{-1}\sin(\omega T)}{1-2z^{-1}\cos(\omega T)+z^{-2}}$
$\cos(\omega t)$		$\frac{s}{s^2+\omega^2}$	$\frac{1-z^{-1}\cos(\omega T)}{1-2z^{-1}\cos(\omega T)+z^{-2}}$
$1-e^{-at}$		$\frac{a}{s(s+a)}$	$\frac{(1-b)z^{-1}}{\left(1-z^{-1}\right)\left(1-bz^{-1}\right)}$
$e^{-at}\sin(\omega t)$		$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{z^{-1}b\sin(\omega T)}{1-2z^{-1}b\cos(\omega T)+b^2z^{-2}}$
$e^{-at}\cos(\omega t)$		$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{1-z^{-1}b\cos(\omega T)}{1-2z^{-1}b\cos(\omega T)+b^2z^{-2}}$
Initial value theorem: $\lim\limits_{t\to 0} f(t)$		$\lim\limits_{s\to\infty} sF(s)$	$\lim\limits_{z\to\infty} F(z)$
Final value theorem: $\lim\limits_{s\to\infty} f(t)$		$\lim\limits_{s\to 0} sF(s)$	$\lim\limits_{z\to 1}\left[\left(1-z^{-1}\right)F(z)\right]$
Translation: $f(t-D)u(t-D)$		$e^{-Ds}F(s)$	$F(z)z^{-n}$ where $D = nT$
Derivative: $\frac{\mathrm{d}^n f(t)}{\mathrm{d}t^n} = f^n(t)$		$s^nF(s) - \sum\limits_{k=1}^{n} s^{k-1}f^{n-k}(0)$	
Integral: $\int_0^t f(t)\mathrm{d}t$		$\frac{1}{s}F(s)$	
Zero$^{\text{th}}$ order hold		$H(s) = \frac{1-e^{-Ts}}{s}$	