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Process Dynamics – CPB410

Semester Test 2

Chemical Engineering
Engineering and the Built Environment

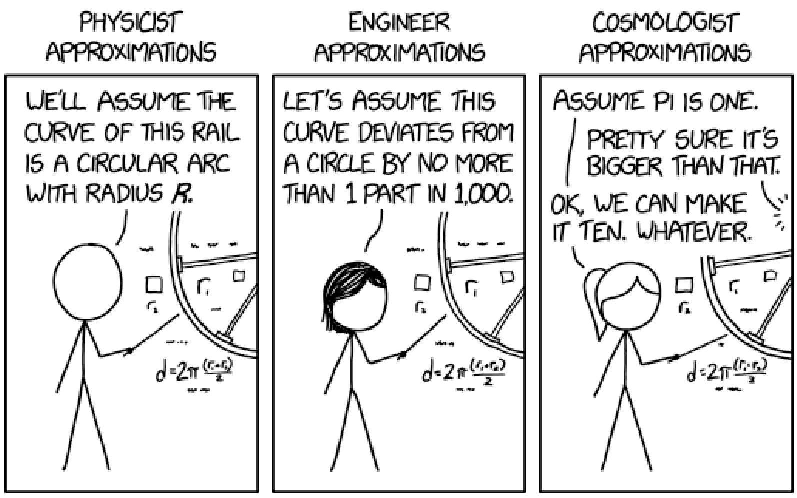
Examiner: Carl Sandrock

Date: 2019-09-30

Duration: 90 minutes   Total: 50   Total Pages: 8

Instructions – Read carefully

• Answer all the questions. • This is a closed book test. All the information you may use is contained in the paper. • You may use the computer. • Make sure that you motivate all your answers and write legibly.



xkcd.com

1 Response analysis

In the graph at the back of the paper, the function $u(t)$ has been drawn. Note that all the variables in this question are deviation variables.

1.1. *Sketch* (do not calculate) the integral and derivative of $u(t)$ on the blank charts provided. $\boxed{6}$

1.2. Express $u(t)$ as a combination of steps and ramps for $t < 8$. $\boxed{4}$

1.3. *Sketch* (do not calculate) $y(t)$ given $y(s) = \dfrac{2u(s)}{0.5s+1}$. Use the middle graph on the attached sheet. $\boxed{5}$

Total for question 1: $\boxed{15}$

2 Tank system

Consider the system shown in Figure 1. It consists of two identically shaped tanks connected by a long pipe.

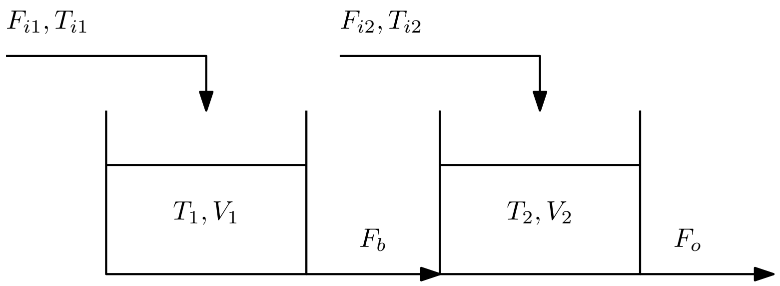


Figure 1: Tank system for question 2

A linearised model for the above system in terms of deviation variables and neglecting any transport delay is given below:

$$\frac{\mathrm{d}V_1}{\mathrm{d}t} = F_{i1} - F_b \tag{1}$$

$$\frac{\mathrm{d}V_2}{\mathrm{d}t} = F_b + F_{i2} - F_o \tag{2}$$

$$\overline{V_1}\frac{\mathrm{d}T_1}{\mathrm{d}t} + \overline{T_1}\frac{\mathrm{d}V_1}{\mathrm{d}t} = \overline{T_{i1}}F_{i1} + \overline{F_{i1}}T_{i1} - (\overline{T_1}F_b + \overline{F_b}T_1) \tag{3}$$

$$\overline{V_2}\frac{\mathrm{d}T_2}{\mathrm{d}t} + \overline{T_2}\frac{\mathrm{d}V_2}{\mathrm{d}t} = \overline{T_1}F_b + \overline{F_b}T_1 + \overline{T_{i2}}F_{i2} + \overline{F_{i2}}T_{i2} - (\overline{T_2}F_o + \overline{F_o}T_2) \tag{4}$$

$$F_b = k_1(V_1 - V_2) \tag{5}$$

$$F_o = k_2 V_2 \tag{6}$$

The inputs of the system are $F_{i1}, T_{i1}, F_{i2}, T_{i2}$. The barred symbols as well as k_1 and k_2 are constant parameters. V_1, V_2, T_1, T_2, F_b and F_o are outputs.

2.1. Rewrite the (time domain) equations to accommodate the situation where liquid takes D seconds to move from the left hand tank to the right hand one. You only need to rewrite the equations which will change. Motivate your thinking briefly. $\boxed{4}$

2.2. Show that the transfer function relating F_{i1} to V_2 is given by $\boxed{6}$

$$G(s) = \frac{k_1}{k_1k_2 + 2k_1s + k_2s + s^2}$$

Note: Read 2.8 before attempting this question. It can be done separately but it is probably more efficient to do 2.8 first and use that answer here.

2.3. Find the gain, time constant and damping coefficient of $G(s)$ given in 2.2. $\boxed{6}$

2.4. Is $G(s)$ from 2.2 underdamped, critically damped, or overdamped? Motivate qualitatively $\boxed{4}$

2.5. Defend your answer to 2.4 numerically or prove symbolically $\boxed{2}$.

2.6. If this system were to be written in state space form, what would be the dimensions of the A, B, C and D matrices? $\boxed{4}$

2.7. Does this system exhibit interaction? Explain. $\boxed{3}$

2.8. Write the complete system in transfer function matrix form. It is recommended that you use sympy to do this calculation and you may upload your Jupyter Notebook on ClickUP to be graded. $\boxed{6}$

Total for question 2: $\boxed{35}$

Full Marks $\boxed{50}$

LaTeX 2_ε
www.latex-project.org

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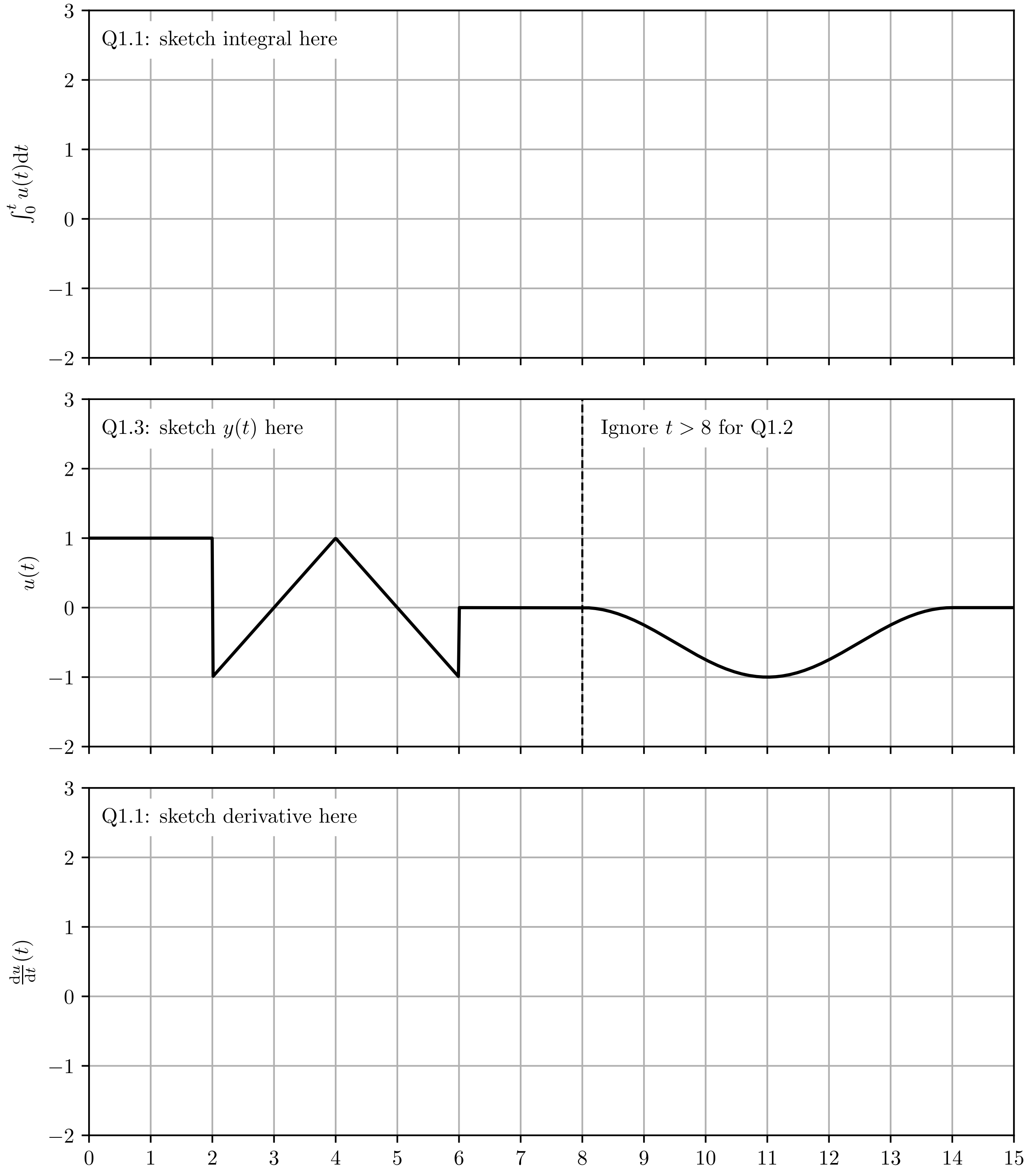
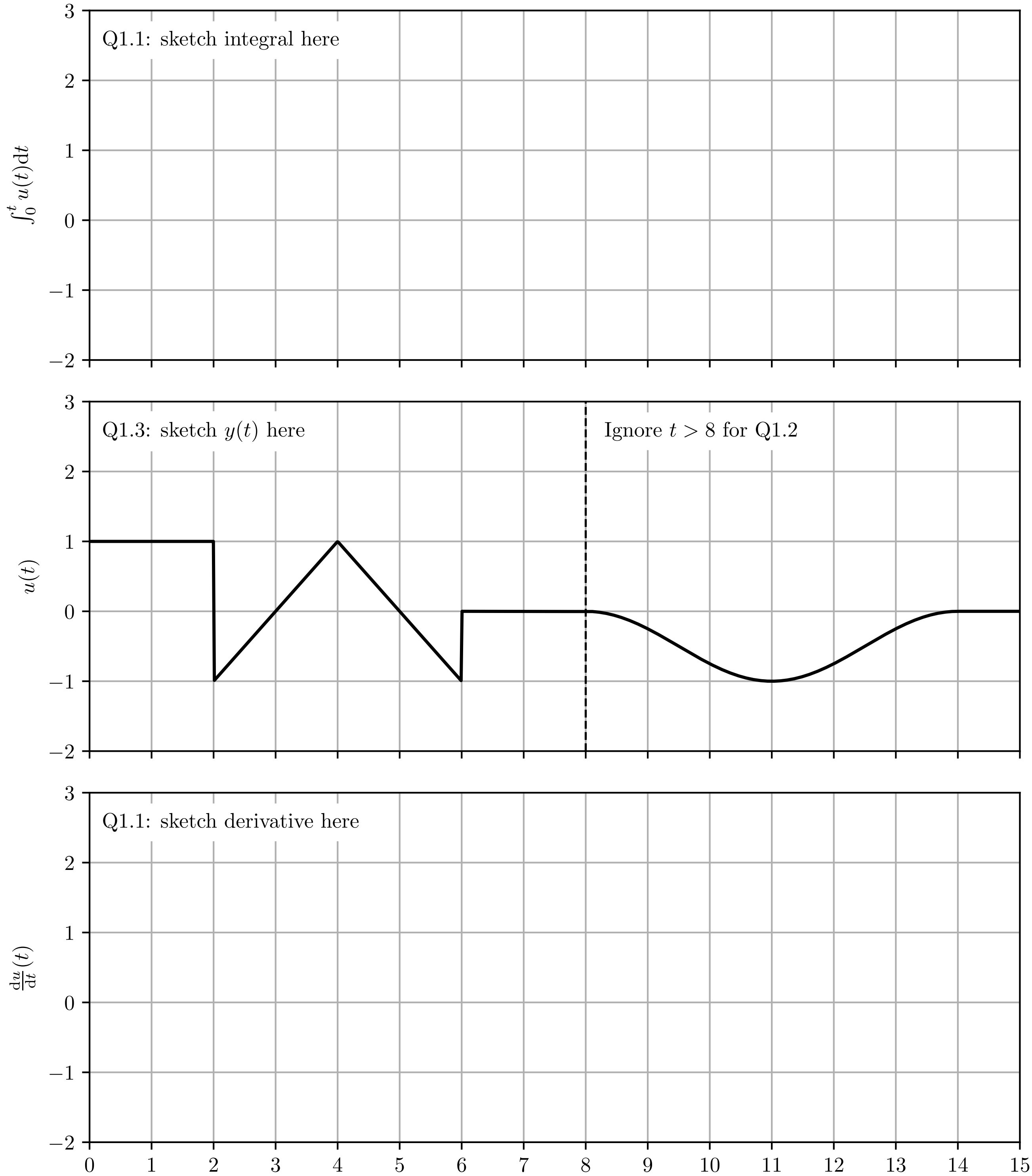
Name: ______________________ Student Number: ______________

Use this page to answer Question 1. There is a copy on the other side.



Name: ______________________ Student Number: ______________

Use this page to answer Question 1. There is a copy on the other side.



Datasheet: CPN321/CPB410

Compiled on May 5, 2018

General solution of 1st order DE:

$$\dot{x} + P(t)x = Q(t) \quad \Rightarrow \quad x = \frac{1}{F_I}\int Q(t)F_I \mathrm{d}t + c_1 \quad \text{with} \quad F_I = \exp\left(\int P(t)\mathrm{d}t\right)$$

Taylor Series expansion near point $x = a$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$
$$f(x_1,\dots,x_d) = \sum_{n_1=0}^{\infty}\cdots\sum_{n_d=0}^{\infty} \frac{(x_1-a_1)^{n_1}\cdots(x_d-a_d)^{n_d}}{n_1!\cdots n_d!}\left(\frac{\partial^{n_1+\cdots+n_d}f}{\partial x_1^{n_1}\cdots\partial x_d^{n_d}}\right)(a_1,\dots,a_d)$$

Linear approximation around point $\mathbf{x} = \mathbf{0}$, where $f(\mathbf{x}) = \mathbf{0}$:

$$f(\mathbf{x}) \approx \nabla f(\mathbf{0}) \cdot \mathbf{x}$$
$$f(x_1, x_2, \cdots, x_d) \approx \frac{\partial f}{\partial x_1}(0)x_1 + \frac{\partial f}{\partial x_2}(0)x_2 + \cdots + \frac{\partial f}{\partial x_d}(0)x_d$$

Partial fraction expansion (for strictly proper rational functions of s)

$$F(s) = \frac{(s-z_1)(s-z_2)(s-z_3)\cdots}{(s-p_1)^n(s-p_2)(s-p_3)\cdots} = \underbrace{\sum_{m=0}^{n-1}\frac{A_m}{(s-p_1)^{n-m}}}_{\text{repeated roots}} + \frac{B}{s-p_2} + \frac{C}{s-p_3}\cdots$$

$$A_m = \lim_{s\to p_1}\left\{\frac{\mathrm{d}^m}{\mathrm{d}s^m}\Big[(s-p_1)^n F(s)\Big]\right\}\frac{1}{m!}$$
$$B = \lim_{s\to p_2}\big[(s-p_2)F(s)\big]$$

Euler identity:

$$e^{i\theta} = \cos\theta + i\sin\theta \quad \therefore e^{-i\theta} = \cos\theta - i\sin\theta \quad \text{and} \quad e^{i\pi} - 1 = 0$$

(1,1) Padé approximation of dead time:

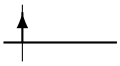
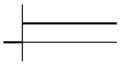
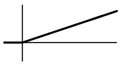
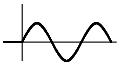
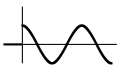
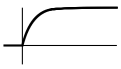
$$e^{-Ds} \approx \frac{1-\frac{D}{2}s}{1+\frac{D}{2}s}$$

PID controller:

$$m = K_C\left(\varepsilon + \frac{1}{\tau_I}\int_0^t \varepsilon \mathrm{d}t + \tau_D\frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\right) \qquad \frac{m}{\varepsilon}(s) = K_C\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Tuning rules:

	Ziegler-Nichols			Cohen-Coon (with $\phi = \frac{t_D}{\tau_P}$)		
	K_C	τ_I	τ_D	K_C	τ_I	τ_D
P	$\frac{K_u}{2}$			$\frac{\phi+3}{3K_P\phi}$		
PI	$\frac{K_u}{2.2}$	$\frac{P_u}{1.2}$		$\frac{5\phi+54}{60K_P\phi}$	$t_D\frac{30+3\phi}{9+20\phi}$	
PID	$\frac{K_u}{1.7}$	$\frac{P_u}{2}$	$\frac{P_u}{8}$	$\frac{3\phi+16}{12K_P\phi}$	$t_D\frac{32+6\phi}{13+8\phi}$	$\frac{4t_D}{11+2\phi}$

Time domain	Laplace-transform	z-transform ($b = e^{-aT}$)
Impulse: $\delta(t)$ 	1	1
Unit step: $u(t)$ 	$\frac{1}{s}$	$\frac{1}{1-z^{-1}}$
Ramp: t 	$\frac{1}{s^2}$	$\frac{Tz^{-1}}{\left(1-z^{-1}\right)^2}$
t^n 	$\frac{n!}{s^{n+1}}$	$\lim_{a\to 0}(-1)^n \frac{\partial^n}{\partial a^n}\frac{1}{1-bz^{-1}}$
e^{-at} 	$\frac{1}{s+a}$	$\frac{1}{1-bz^{-1}}$
te^{-at} 	$\frac{1}{(s+a)^2}$	$\frac{Tbz^{-1}}{\left(1-bz^{-1}\right)^2}$
t^2e^{-at} 	$\frac{2}{(s+a)^3}$	$\frac{T^2bz^{-1}(1+bz^{-1})}{\left(1-bz^{-1}\right)^3}$
$\sin(\omega t)$ 	$\frac{\omega}{s^2+\omega^2}$	$\frac{z^{-1}\sin(\omega T)}{1-2z^{-1}\cos(\omega T)+z^{-2}}$
$\cos(\omega t)$ 	$\frac{s}{s^2+\omega^2}$	$\frac{1-z^{-1}\cos(\omega T)}{1-2z^{-1}\cos(\omega T)+z^{-2}}$
$1-e^{-at}$ 	$\frac{a}{s(s+a)}$	$\frac{(1-b)z^{-1}}{\left(1-z^{-1}\right)\left(1-bz^{-1}\right)}$
$e^{-at}\sin(\omega t)$ 	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{z^{-1}b\sin(\omega T)}{1-2z^{-1}b\cos(\omega T)+b^2z^{-2}}$
$e^{-at}\cos(\omega t)$ 	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{1-z^{-1}b\cos(\omega T)}{1-2z^{-1}b\cos(\omega T)+b^2z^{-2}}$
Initial value theorem: $\lim_{t\to 0} f(t)$	$\lim_{s\to\infty} sF(s)$	$\lim_{z\to\infty} F(z)$
Final value theorem: $\lim_{s\to\infty} f(t)$	$\lim_{s\to 0} sF(s)$	$\lim_{z\to 1}\left[\left(1-z^{-1}\right)F(z)\right]$
Translation: $f(t-D)u(t-D)$	$e^{-Ds}F(s)$	$F(z)z^{-n}$ where $D = nT$
Derivative: $\frac{\mathrm{d}^n f(t)}{\mathrm{d}t^n} = f^n(t)$	$s^nF(s) - \sum_{k=1}^{n} s^{k-1}f^{n-k}(0)$	
Integral: $\int_0^t f(t)\mathrm{d}t$	$\frac{1}{s}F(s)$	
Zero$^{\text{th}}$ order hold	$H(s) = \frac{1-e^{-Ts}}{s}$	