UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

Denkleiers • Leading Minds • Dikgopolo tša Dihlalefi

Process Dynamics – CPN321

Semester Test 1

Chemical Engineering
Engineering and the Built Environment

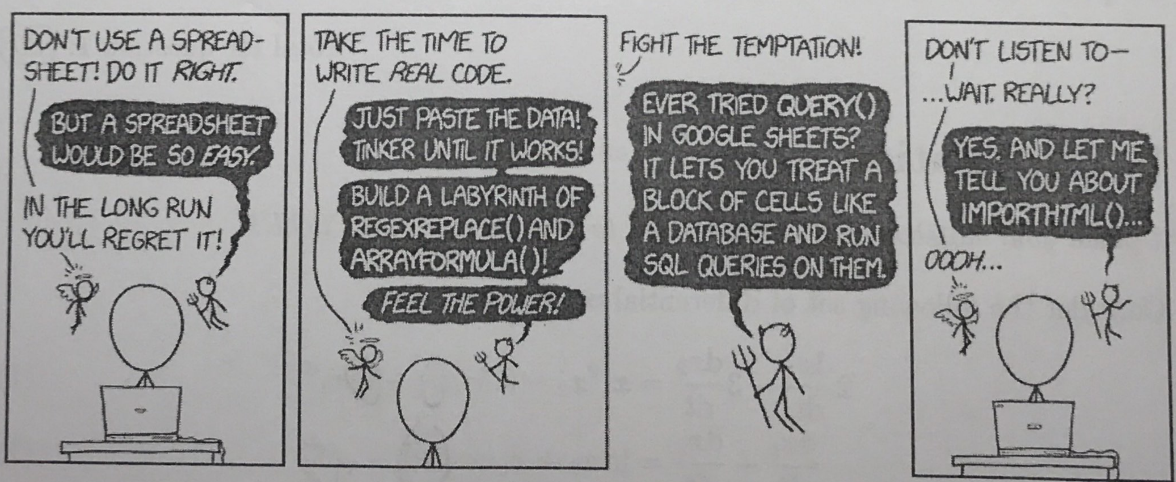
Examiner: Carl Sandrock

Date: 2019-08-19

Duration: 90 minutes   Total: 50   Total Pages: 10

Instructions – Read carefully

• Answer all the questions. • This is a closed book test. All the information you may use is contained in the paper. • You may use the computer • Make sure that you motivate all your answers and write legibly.



xkcd.com

1 Solar still

Figure 1 depicts a solar still. This device can be used to purify salt water using only solar energy. The device is a water-tight, insulated box with an open top. The top is then covered with a transparent glass cover, mounted at an angle. Salt water is fed into the bottom of the device. The salt water is heated and some of it evaporates, then condenses against the glass cover and rolls down to a collection point where it is removed from the system.

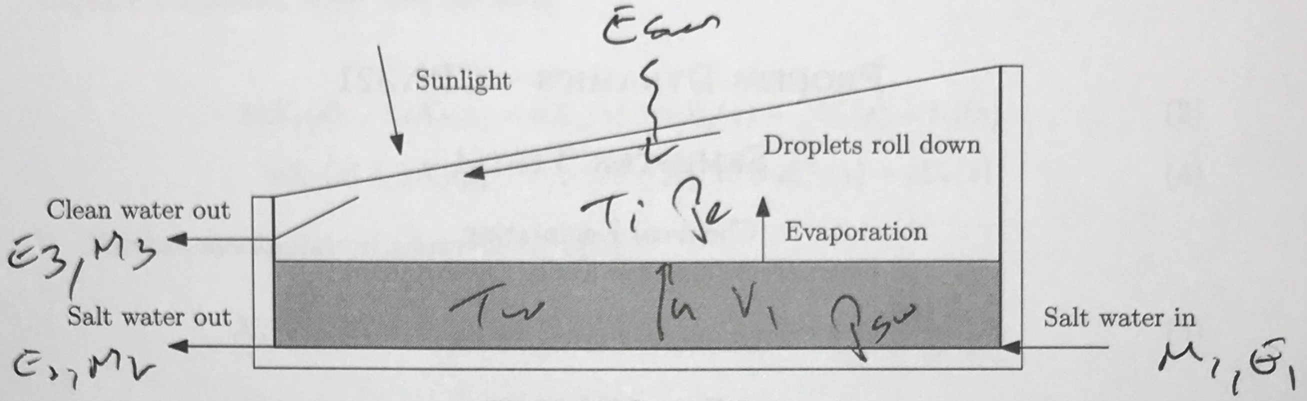


Figure 1: Solar still

Derive a dynamic model of the system and fill out the attached table. Use $f(x)$ liberally. You may assume

- that the droplets roll and collect instantaneously (ignore the dynamics of the clean water),
- that all the solar energy goes into the salt water

Make sure you classify all the symbols in your model as parameters, inputs and outputs and verify that the simulation problem is correctly specified for this system.

Total for question 1: 30

2 Simulation

Upload your notebook with the solution to this question on ClickUP

Consider the following set of differential equations:

$$2\frac{\mathrm{d}x_1}{\mathrm{d}t} - 3\frac{\mathrm{d}x_2}{\mathrm{d}t} = {x_1}^2 x_2 - 0.5 x_2 u_1 + u_2 {x_1}^3 \tag{1}$$

$$\frac{\mathrm{d}x_1}{\mathrm{d}t} + \frac{\mathrm{d}x_2}{\mathrm{d}t} = \ln x_2 + x_1 \cos(2u_1) - \sqrt{u_2} \tag{2}$$

Find values for u_1 and u_2 such that $x_1 = 1, x_2 = 1$ is a steady state of this system. It is known that the values $u_1 = 2.6$ and $u_2 = 0.3$ are close to the solution.

Total for question 2: 10

3 Transfer functions

Answer this question on ClickUP. Full marks will be awarded for correct answers, 75% for correct method without correct numbers.

For this question, assume $\bar{x}_1 = \bar{x}_2 = 1, \bar{u}_1 = 3.4, \bar{u}_2 = 0.71$ regardless of your answer for the previous question.

If we linearise equations 1 and 2, express them in deviation variable form and Laplace transform, they take the form:

$$2sX_1(s) - 3sX_2(s) = aX_1(s) + bX_2(s) - \frac{1}{2}U_1(s) + U_2(s) \tag{3}$$

$$sX_1(s) + sX_2(s) = cX_1(s) + X_2(s) + dU_1(s) + eU_2(s) \tag{4}$$

Further manipulation allows us to write

$$X_1(s) = G_1(s)U_1(s) + G_2(s)U_2(s) \quad G_1(s) = \frac{\alpha s + \beta}{\gamma s^2 + \delta s + 1} \tag{5}$$

3.1. Calculate the numeric values of a and b. $\boxed{4}$

3.2. Calculate the numeric value of α, β, γ and δ $\boxed{6}$

Total for question 3: $\boxed{10}$

Full Marks $\boxed{50}$

LaTeX 2ε
www.latex-project.org

Datasheet: CPN321/CPB410

Compiled on May 5, 2018

General solution of 1st order DE:

$$\dot{x} + P(t)x = Q(t) \quad \Rightarrow \quad x = \frac{1}{F_I}\int Q(t)F_I \mathrm{d}t + c_1 \quad \text{with} \quad F_I = \exp\left(\int P(t)\mathrm{d}t\right)$$

Taylor Series expansion near point $x = a$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$

$$f(x_1, \ldots, x_d) = \sum_{n_1=0}^{\infty} \cdots \sum_{n_d=0}^{\infty} \frac{(x_1 - a_1)^{n_1} \cdots (x_d - a_d)^{n_d}}{n_1! \cdots n_d!} \left(\frac{\partial^{n_1+\cdots+n_d} f}{\partial x_1^{n_1} \cdots \partial x_d^{n_d}}\right)(a_1, \ldots, a_d)$$

Linear approximation around point $\mathbf{x} = \mathbf{0}$, where $f(\mathbf{x}) = \mathbf{0}$:

$$f(\mathbf{x}) \approx \nabla f(\mathbf{0}) \cdot \mathbf{x}$$

$$f(x_1, x_2, \cdots, x_d) \approx \frac{\partial f}{\partial x_1}(0)x_1 + \frac{\partial f}{\partial x_2}(0)x_2 + \cdots + \frac{\partial f}{\partial x_d}(0)x_d$$

Partial fraction expansion (for strictly proper rational functions of s)

$$F(s) = \frac{(s-z_1)(s-z_2)(s-z_3)\cdots}{(s-p_1)^n(s-p_2)(s-p_3)\cdots} = \underbrace{\sum_{m=0}^{n-1} \frac{A_m}{(s-p_1)^{n-m}}}_{\text{repeated roots}} + \frac{B}{s-p_2} + \frac{C}{s-p_3} \cdots$$

$$A_m = \lim_{s \to p_1} \left\{ \frac{\mathrm{d}^m}{\mathrm{d}s^m}\left[(s-p_1)^n F(s)\right]\right\} \frac{1}{m!}$$

$$B = \lim_{s \to p_2} \left[(s-p_2)F(s)\right]$$

Euler identity:

$$e^{i\theta} = \cos\theta + i\sin\theta \quad \therefore e^{-i\theta} = \cos\theta - i\sin\theta \quad \text{and} \quad e^{i\pi} - 1 = 0$$

(1,1) Padé approximation of dead time:

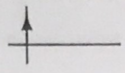
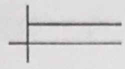
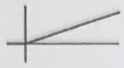
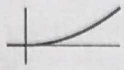
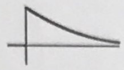
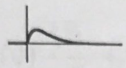
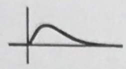
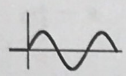
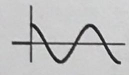
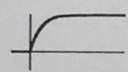
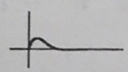
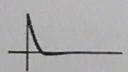
$$e^{-Ds} \approx \frac{1 - \frac{D}{2}s}{1 + \frac{D}{2}s}$$

PID controller:

$$m = K_C\left(\varepsilon + \frac{1}{\tau_I}\int_0^t \varepsilon \mathrm{d}t + \tau_D \frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\right) \qquad \frac{m}{\varepsilon}(s) = K_C\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Tuning rules:

	Ziegler-Nichols			Cohen-Coon (with $\phi = \frac{t_D}{\tau_P}$)		
	K_C	τ_I	τ_D	K_C	τ_I	τ_D
P	$\frac{K_u}{2}$			$\frac{\phi+3}{3K_P\phi}$		
PI	$\frac{K_u}{2.2}$	$\frac{P_u}{1.2}$		$\frac{5\phi+54}{60K_P\phi}$	$t_D\frac{30+3\phi}{9+20\phi}$	
PID	$\frac{K_u}{1.7}$	$\frac{P_u}{2}$	$\frac{P_u}{8}$	$\frac{3\phi+16}{12K_P\phi}$	$t_D\frac{32+6\phi}{13+8\phi}$	$\frac{4t_D}{11+2\phi}$

Time domain	Laplace-transform	z-transform ($b = e^{-aT}$)
Impulse: $\delta(t)$ 	1	1
Unit step: $u(t)$ 	$\frac{1}{s}$	$\frac{1}{1-z^{-1}}$
Ramp: t 	$\frac{1}{s^2}$	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
t^n 	$\frac{n!}{s^{n+1}}$	$\lim_{a\to 0}(-1)^n \frac{\partial^n}{\partial a^n}\frac{1}{1-bz^{-1}}$
e^{-at} 	$\frac{1}{s+a}$	$\frac{1}{1-bz^{-1}}$
te^{-at} 	$\frac{1}{(s+a)^2}$	$\frac{Tbz^{-1}}{(1-bz^{-1})^2}$
t^2e^{-at} 	$\frac{2}{(s+a)^3}$	$\frac{T^2bz^{-1}(1+bz^{-1})}{(1-bz^{-1})^3}$
$\sin(\omega t)$ 	$\frac{\omega}{s^2+\omega^2}$	$\frac{z^{-1}\sin(\omega T)}{1-2z^{-1}\cos(\omega T)+z^{-2}}$
$\cos(\omega t)$ 	$\frac{s}{s^2+\omega^2}$	$\frac{1-z^{-1}\cos(\omega T)}{1-2z^{-1}\cos(\omega T)+z^{-2}}$
$1-e^{-at}$ 	$\frac{a}{s(s+a)}$	$\frac{(1-b)z^{-1}}{(1-z^{-1})(1-bz^{-1})}$
$e^{-at}\sin(\omega t)$ 	$\frac{\omega}{(s+a)^2+\omega^2}$	$\frac{z^{-1}b\sin(\omega T)}{1-2z^{-1}b\cos(\omega T)+b^2z^{-2}}$
$e^{-at}\cos(\omega t)$ 	$\frac{s+a}{(s+a)^2+\omega^2}$	$\frac{1-z^{-1}b\cos(\omega T)}{1-2z^{-1}b\cos(\omega T)+b^2z^{-2}}$
Initial value theorem: $\lim_{t\to 0} f(t)$	$\lim_{s\to\infty} sF(s)$	$\lim_{z\to\infty} F(z)$
Final value theorem: $\lim_{s\to\infty} f(t)$	$\lim_{s\to 0} sF(s)$	$\lim_{z\to 1}\left[\left(1-z^{-1}\right)F(z)\right]$
Translation: $f(t-D)u(t-D)$	$e^{-Ds}F(s)$	$F(z)z^{-n}$ where $D = nT$
Derivative: $\frac{\mathrm{d}^n f(t)}{\mathrm{d}t^n} = f^n(t)$	$s^nF(s) - \sum_{k=1}^{n} s^{k-1}f^{n-k}(0)$	
Integral: $\int_0^t f(t)\mathrm{d}t$	$\frac{1}{s}F(s)$	
Zero$^{\text{th}}$ order hold	$H(s) = \frac{1-e^{-Ts}}{s}$	