# CPN321 – T2

2019

## 1 Dead time

Consider the system shown in Figure 1 that has two stirred chemical reactors separated by a pipe which is so long that liquid exiting the first tank takes $D$ seconds to reach the second tank. There is plug flow through the pipe (no mixing, no diffusion). Assume constant holdups ($V_1$ and $V_2$), constant throughput ($F$), constant density, isothermal operation at temperatures $T_1$ and $T_2$, and first-order kinetics with simultaneous reactions:

$$\mathrm{A} \xrightarrow{\mathrm{k_1}} \mathrm{B} \qquad \mathrm{A} \xrightarrow{\mathrm{k_2}} \mathrm{C}$$

No reaction occurs in the pipe. Write the equations describing the system.

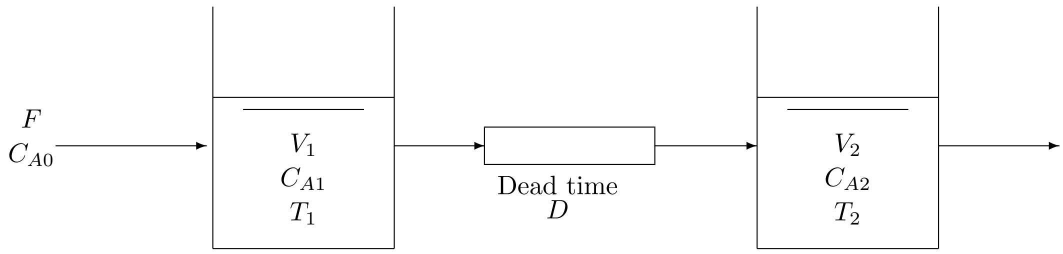


Figure 1: Reaction system with dead time plug flow

## 2 Stirred tank heaters

Consider the two stirred tank heaters in the sketch below.

1. Identify the state variables of the system.
2. Determine what balances you should perform.
3. Develop the dynamic models that describe the time-dependant behaviour of the system. Ensure that the degrees of freedom of the final system are zero by specifying some of the input variables.

For tank 1, the stream is injected directly into the liquid water. Water vapour is produced in the second tank. $A_1$ and $A_2$ are the cross-sectional areas of the two tanks. Assume that the effluent flowrates are proportional to the liquid static pressure that causes their flow. $A_t$ is the heat transfer area of the steam coil.

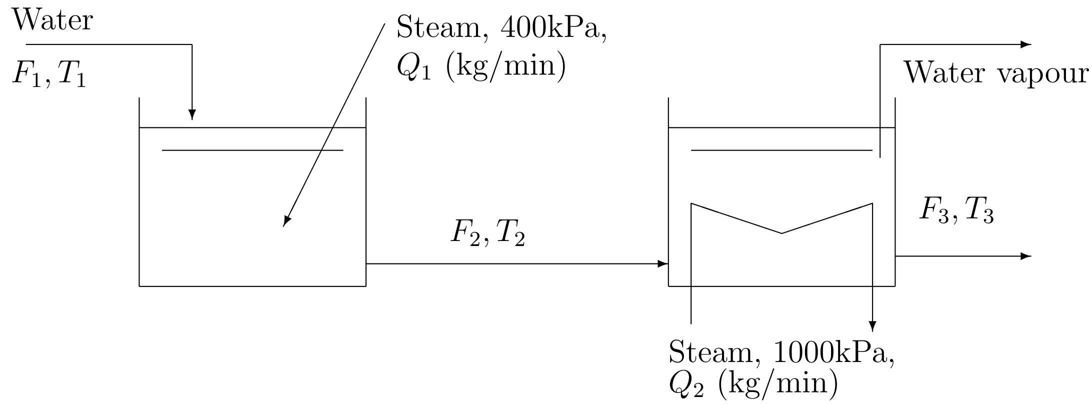


Figure 2: Stirred tank heaters with direct and indirect steam heating

# 3 Irreversible reaction, imperfect mixing

*See St1 Q1, 2011* An isothermal, irreversible reaction

$$\mathrm{A} \xrightarrow{k} \mathrm{B}$$

takes place in the liquid phase in a constant-volume reactor. The mixing is *not* perfect. Observation of flow patterns indicate that a two-tank system with back mixing, as shown in the sketch below, should approximate the imperfect mixing.

Assuming $F$ and $F_R$ are constant, write the equations describing the system.

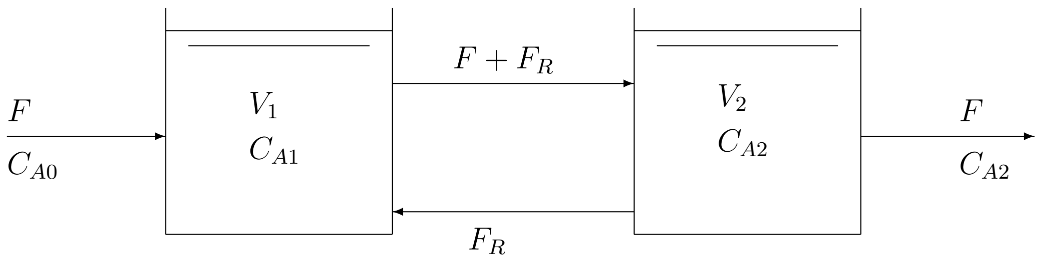


Figure 3: Back mixing system to approximate imperfect mixing

# 4 Control valve

The flow rate $F$ of a manipulated stream through a control valve with equal-percentage trim is given by the following equation:

$$F = C_v \alpha^{x-1}$$

where $F$ is the flow in gallons per minute and $C_v$ and $\alpha$ are constants set by the valve size and type. The control valve stem position $x$ (fraction of wide open) is set by the output signal CO of an analog electronic feedback controller whose signal range is 4 mA to 20 mA. The valve cannot be moved instantaneously. It is approximately a first-order system:

$$\tau_p \frac{dx}{dt} + x = \frac{CO - 4}{16}$$

The effect of the flow rate of the manipulated variable on the process temperature $T$ is given by

$$\tau_p \frac{dT}{dt} + T = K_p F$$

Derive one second order ordinary differential equation that gives the dynamic dependence of process temperature on controller output signal CO.

# 5 Thermocouple

Common temperature measurement devices rely on heat transfer between the measurement device and the area being measured. The temperature measurement device many people are most familiar with is the liquid filled glass thermometer. Figure 4a shows how the surrounding fluid can be at temperature $T$ while the mercury can be at a different temperature $T_m$ due to heat transfer across the glass wall.

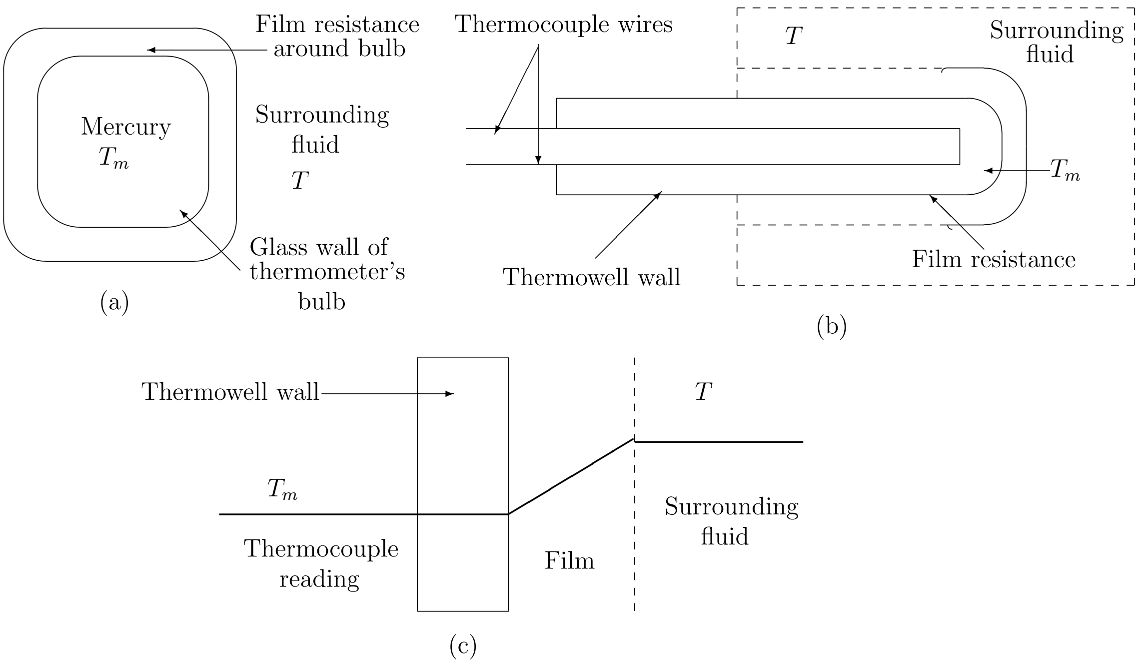


Figure 4: Cross sections showing thermocouple measurements

The thermocouple is a more widely used temperature sensing device used in industry. Because thermocouples are very fragile, they are often introduced into the temperature measurement area inside a steel tube known as a thermowell. This situation is shown in Figure 4b. It is often assumed that there is thermal resistance to heat transfer in a film around the thermowell, but that there is good thermal contact between the thermowell and the thermocouple so that the temperature profile looks like the one shown in Figure 4c.

For this question, assume that the resistance to heat transfer does not come only from the external film between the surrounding fluid and the thermowell wall but also from the internal film between the thermowell wall and the thermocouple as shown in Figure 5.

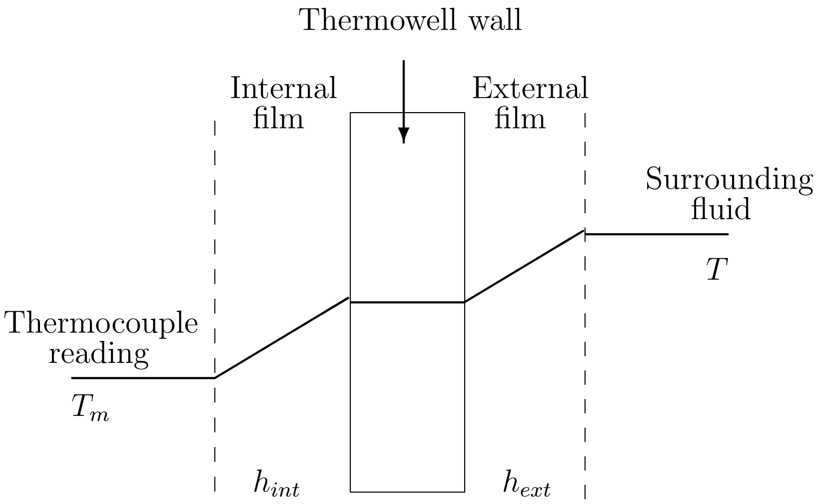


Figure 5: Cross section of thermocouple wall

Let $h_{ext}$ and $h_{int}$ be the heat transfer coefficients for these two films.

1. Show that the thermocouple reading $T_m$ follows second-order dynamic behavior with respect to any changes in the surrounding fluid temperature.

2. Will this system be over-damped, critically damped or under-damped? Why?

3. Design the thermocouple in such a way that it exhibits a slightly over-damped behavior (e.g. $\zeta = 1.2$).