Name: ______________________________ Student Number: ________________

UNIVERSITEIT VAN PRETORIA
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Process Dynamics – CPN321

Exam

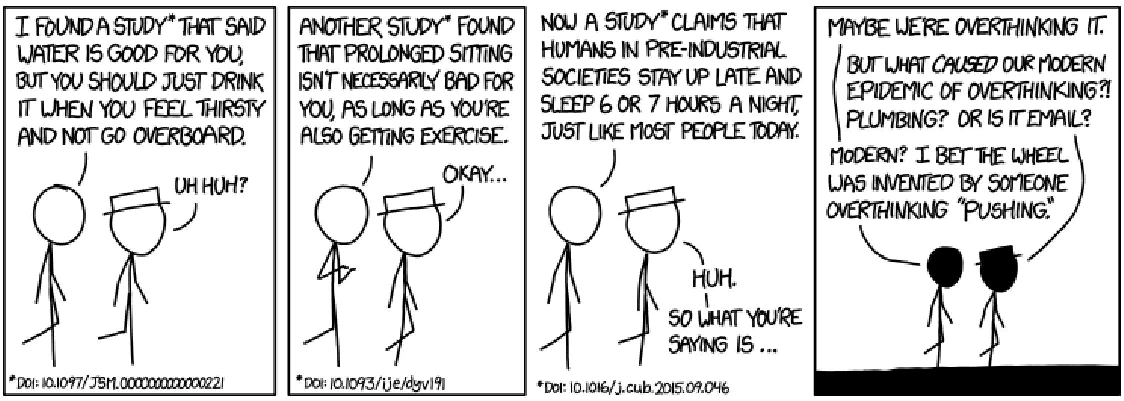
Chemical Engineering
Engineering and the Built Environment

Examiner: Carl Sandrock

November 2015 180 minutes

Instructions – Read carefully

• Answer all the questions on the paper on side 1 of the multiple choice form or in the blocks provided. • This is an open book test. You may bring any information you need into the exam venue. • Only the work written in the blocks provided will count toward your mark.

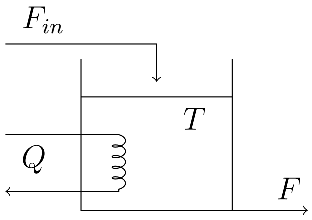


www.xkcd.org

1 Multiple choice

Answer this section on side 1 of the multiple choice form. Each question counts 5 marks.

1. In the following process (a well-mixed tank with a heating coil), choose a correct assignment of controlled, manipulated and disturbance variable:



	CV	MV	DV
a.	F_{in}	T	Q
b.	T	F_{in}	Q
c.	F_{in}	Q	T
d.	T	Q	F_{in}
e.	Q	F_{in}	T

2. Choose a correct statement about the numeric solution of differential equations:

a. All differential equations can be solved using Euler's method to arbitrary accuracy, given a small enough time step

b. Euler's method is the most reliable integration method

c. Euler's method may become unstable with large time steps

d. Euler's method has stability guaranteed, even with large time steps

e. Higher order methods like the Runge-Kutta method will always take longer to solve to a given accuracy than Euler's method

3. A flash drum with three components (and two phases) is being operated on a plant. Select the correct statement.

a. According to Gibbs' phase rule, there are 2 degrees of freedom.

b. According to the Ideal Gas Law, temperature and pressure cannot be controlled independently.

c. Due to phase equilibrium in the drum, temperature and pressure cannot be specified independently once the feed is fully specified.

d. The level in the tank can be controlled without affecting the equilibrium.

e. The degrees of freedom calculated by subtracting the number of equations from the number of variables must be the same as the number calculated by Gibbs' phase rule.

4. Select a true statement about the approximation of dead time by a rational function if an analytic step response is to be obtained

a. It is always necessary

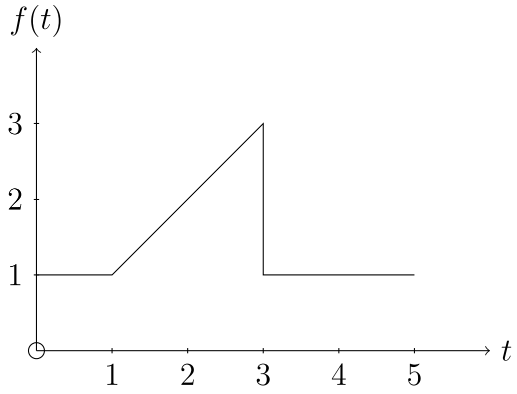
b. It is necessary if the dead time appears in the numerator of the transfer function

c. It is necessary if the dead time appears in the denominator of the transfer function

d. It is necessary if the dead time is added to terms without dead time

e. It is never necessary

5. Choose the correct expression for the signal shown below:



a. $f(t) = u(t-1) \cdot (t-1) - u(t-3) \cdot (t+1)$

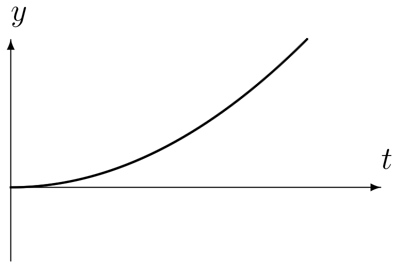
b. $f(t) = 1 + u(t-1) - u(t-3) \cdot (t+1)$

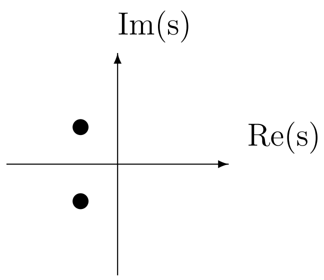
c. $f(t) = 1 + u(t-1) \cdot (t-1) - u(t-3) \cdot (t-1)$

d. $F(s) = \dfrac{e^{-s}}{s} - \dfrac{e^{-3s}}{s^2}$

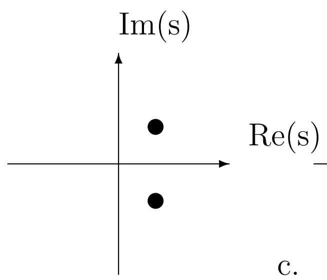
e. $F(s) = \dfrac{1}{s}\left(1 + e^{-s} - \dfrac{e^{-3s}}{s}\right)$

6. The following diagram shows a qualitative system response of a the system described by $y = Gu$. Assume u is an ideal unit step. Select the plot of the poles on the s-plane that best matches this response.

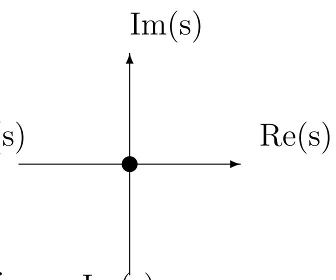




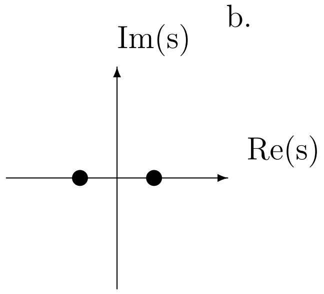
a.



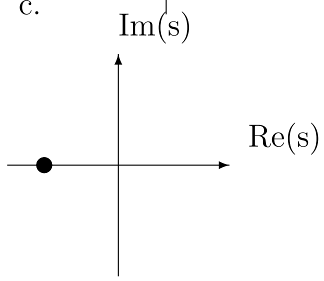
b.



c.



d)



e.

7. Select a true statement regarding the relationship between state space and transfer functions

a. Any transfer function can be written in state space form

b. Any state space model can be written as a transfer function

c. Any rational transfer function can be written in state space form

d. Only state space models with single outputs can be written in transfer function form

e. None of the above

8. Choose a correct statement regarding discrete time models. Discrete time models ...

a. ... can exactly predict the output of a continuous system at the sampling intervals only if the input is piecewise constant

b. ... can only approximate the step response of continuous time models at the sampling intervals

c. ... must be simulated with small time steps to avoid errors

d. ... can exactly predict the output of a continuous system at the sampling intervals only if the input is smooth

e. None of the above

9. Select a true statement:

a. The output of any system subjected to a sinusoidal input will be exactly sinusoidal with the same frequency

b. The output of a linear system subjected to a sinusoidal input will be exactly sinusoidal with the same frequency

c. The output of a linear system subjected to a sinusoidal input will be approximately sinusoidal with the same frequency

d. If the output of a system is exactly sinusoidal, the input had to have been sinusoidal

e. If the output of a system is exactly sinusoidal, the system had to have been linear

10. Select a true statement about *aliasing*. Due to aliasing ...

a. ... digital signals are always less accurate than analog ones

b. ... good digital filters must be used to distinguish between high and low frequency sinusoids after sampling

c. ... analog filters must be used to remove high frequency components before sampling

d. ... process noise can appear as real process data in sampled data

e. ... sampling must be done at approximately 100 times the dominant time constant

50

2 Higher order dynamics

2.1 Inverse

Consider the following system:

$$\frac{Y_1}{U_1} = G_1 = \frac{K(s+a)}{(s+b)(s+c)}, \quad 0 < b < c \tag{1}$$

Find the conditions (in terms of inequalities concerning K, a, b and c) under which $y_1(t)$ will exhibit a minimum or maximum value which is different from the value as $t \to \infty$ if $u(t)$ is a unit step. $\boxed{15}$

2.2 Poles and zeros

Sketch the impulse response of the system for $K > 0$, $a < 0$. Explain your sketch with reference to the poles and zeros of the process. $\boxed{5}$

$\boxed{20}$

3 State space

Consider the following state space realisation:

$$A = \begin{bmatrix} 1 & \alpha \\ 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} \beta \\ 0 \end{bmatrix} \tag{2}$$

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix} \qquad D = 1 \tag{3}$$

3.1 Conversion

Find the (SISO) transfer function representation of this process $\boxed{5}$

3.2 Analysis

Under what conditions will the system exhibit oscillatory response? Use the state-space version of the model to answer this question. $\boxed{15}$

$\boxed{20}$

4 Higher order system

Consider the transfer function below:

$$\frac{Y_2}{U_2} = G_2(s) = \frac{1-s}{(10s+1)(5s+1)(s+1)^2} \tag{4}$$

4.1 Padé approximation

Find a 1/1 Padé approximation of G_2. $\boxed{10}$

4.2 Higher-order systems

Approximate G_2 by a first-order-plus-deadtime model using Skogestad's half rule. $\boxed{5}$

4.3 Comparison

Sketch the step response of the two approximations above (Padé and half-rule) on the attached graph showing the step response of G_2. Indicate which is which clearly. $\boxed{5}$

$\boxed{\mathit{20}}$

4.4 Bode diagram

Sketch an asymptotic Bode diagram of G_2 on the graph paper provided. Make sure you annotate your graph with your major construction points. Note: A sketch showing only the Bode diagram without the asymptote lines will gain no marks. $\boxed{20}$

Show your working in the box provided below:

20

5 Identification

Use the step response of G_2 in the attached figure to answer these questions.

Assume that the process was sampled once every five seconds ($T = 5$). Consider a model of the form

$$y_2(kT) = (1 - e^{-T/\tau})Ku_2[(k-2)T] + e^{-T/\tau}y_2[(k-2)T] \tag{5}$$

5.1 Least squares

Assume that the model predictions can be written in the form

$$Y = X\beta + \epsilon \tag{6}$$

where β is a vector of coefficients

5.2 Matrix construction

Find the matrices Y and X which would be used in linear regression to find β. You may write only the first 4 rows of each matrix, but you must specify the full size.

5.3 Application

Find the value of β. $\boxed{15}$

$\boxed{15}$

6 Discrete systems

6.1 z-transform

Rewrite equation 5 in discrete transfer function form (z domain). $\boxed{5}$

6.2 Sampling

Suppose we have a discrete transfer function given by $G_6(z) = \frac{y(z)}{u(z)} = \frac{z^{-1}}{1-0.3z^{-1}}$. Suppose a continuous step signal $u(t)$ is sampled with $\delta t = 1$, processed by G_6 and then sent to a zero order hold device (H) to yield a signal $w(t)$. This signal is then filtered by a first order filter with a time constant of 2 seconds to yield a filtered signal $f(t)$.

6.2.1 Block diagram

Draw a block diagram showing this setup. $\boxed{5}$

6.2.2 Response

Sketch $w(t)$ and $f(t)$ on the same graph. Use 10 time steps. $\boxed{15}$

6.3 Analytic result

If $f(t)$ were now sampled at the same time instances as $u(t)$, find a z domain expression for $f(z)$ $\boxed{10}$

$\boxed{35}$

Full Marks $\boxed{180}$

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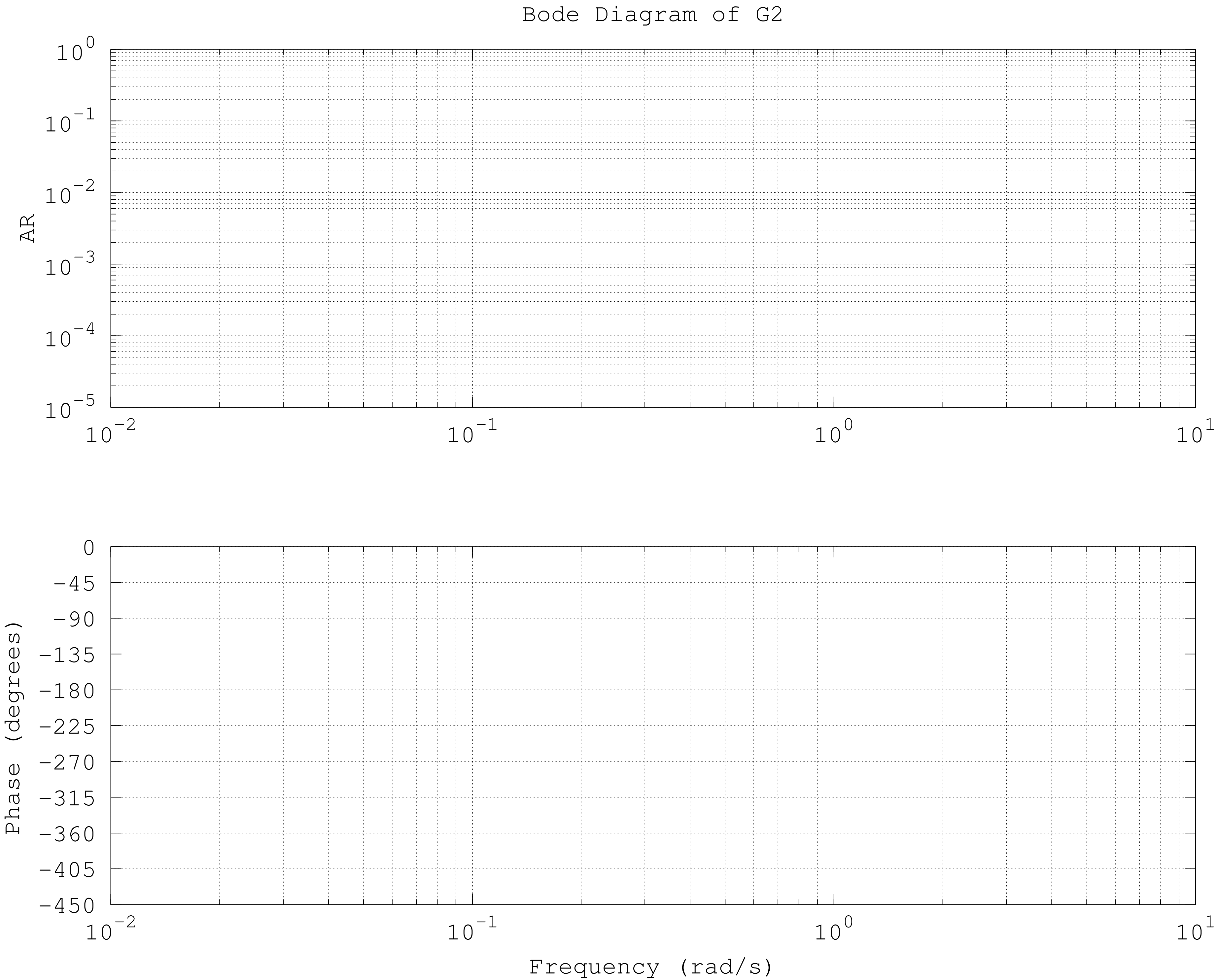
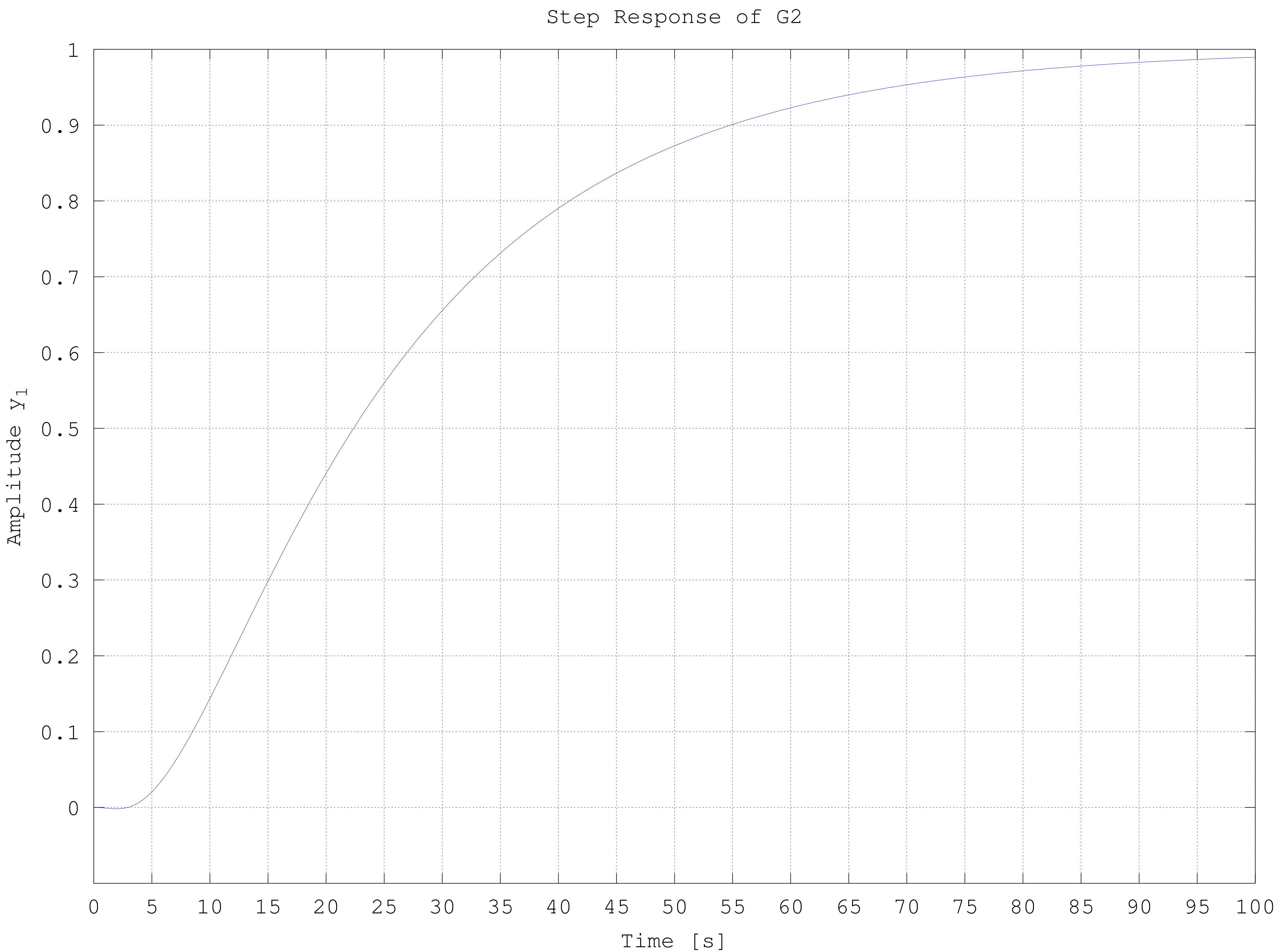


Bode Diagram of G2
AR
Phase (degrees)
Frequency (rad/s)


Step Response of G2
Amplitude y_1
Time [s]
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100

Datasheet: CPN321/CPB410

Compiled on October 31, 2013

General solution of 1st order DE:

$$\dot{x} + P(t)x = Q(t) \quad \Rightarrow \quad x = \frac{1}{F_I}\int Q(t)F_I \mathrm{d}t + c_1 \quad \text{with} \quad F_I = \exp\left(\int P(t)\mathrm{d}t\right)$$

Taylor Series expansion near point $x = a$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$

$$f(x_1, \ldots, x_d) = \sum_{n_1=0}^{\infty} \cdots \sum_{n_d=0}^{\infty} \frac{(x_1 - a_1)^{n_1} \cdots (x_d - a_d)^{n_d}}{n_1! \cdots n_d!} \left(\frac{\partial^{n_1+\cdots+n_d} f}{\partial x_1^{n_1} \cdots \partial x_d^{n_d}}\right)(a_1, \ldots, a_d)$$

Linear approximation around point $\mathbf{x} = \mathbf{0}$, where $f(\mathbf{x}) = \mathbf{0}$:

$$f(\mathbf{x}) \approx \nabla f(\mathbf{0}) \cdot \mathbf{x}$$

$$f(x_1, x_2, \cdots, x_d) \approx \frac{\partial f}{\partial x_1}(0)x_1 + \frac{\partial f}{\partial x_2}(0)x_2 + \cdots + \frac{\partial f}{\partial x_d}(0)x_d$$

Partial fraction expansion (for strictly proper rational functions of s)

$$F(s) = \frac{(s-z_1)(s-z_2)(s-z_3)\cdots}{(s-p_1)^n(s-p_2)(s-p_3)\cdots} = \underbrace{\sum_{m=0}^{n-1} \frac{A_m}{(s-p_1)^{n-m}}}_{\text{repeated roots}} + \frac{B}{s-p_2} + \frac{C}{s-p_3} \cdots$$

$$A_m = \lim_{s \to p_1} \left\{ \frac{\mathrm{d}^m}{\mathrm{d}s^m} \Big[(s-p_1)^n F(s) \Big] \right\} \frac{1}{m!}$$

$$B = \lim_{s \to p_2} \big[(s-p_2)F(s) \big]$$

Euler identity:

$$e^{i\theta} = \cos\theta + i\sin\theta \quad \therefore e^{-i\theta} = \cos\theta - i\sin\theta \quad \text{and} \quad e^{i\pi} - 1 = 0$$

(1,1) Padé approximation of dead time:

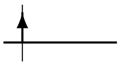
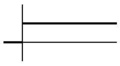
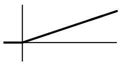
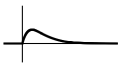
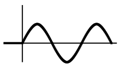
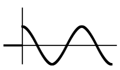
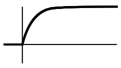
$$e^{-Ds} \approx \frac{1 - \frac{D}{2}s}{1 + \frac{D}{2}s}$$

PID controller:

$$m = K_C\left(\varepsilon + \frac{1}{\tau_I}\int_0^t \varepsilon \mathrm{d}t + \tau_D \frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\right) \qquad \frac{m}{\varepsilon}(s) = K_C\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Tuning rules:

	Ziegler-Nichols			Cohen-Coon (with $\phi = \frac{t_D}{\tau_P}$)		
	K_C	τ_I	τ_D	K_C	τ_I	τ_D
P	$\frac{K_u}{2}$			$\frac{\phi+3}{3K_P\phi}$		
PI	$\frac{K_u}{2.2}$	$\frac{P_u}{1.2}$		$\frac{5\phi+54}{60K_P\phi}$	$t_D\frac{30+3\phi}{9+20\phi}$	
PID	$\frac{K_u}{1.7}$	$\frac{P_u}{2}$	$\frac{P_u}{8}$	$\frac{3\phi+16}{12K_P\phi}$	$t_D\frac{32+6\phi}{13+8\phi}$	$\frac{4}{11+2\phi}$

Time domain	Laplace-transform	z-transform ($b = e^{-aT}$)
Impulse: $\delta(t)$ 	1	1
Unit step: $u(t)$ 	$\frac{1}{s}$	$\frac{1}{1 - z^{-1}}$
Ramp: t 	$\frac{1}{s^2}$	$\frac{Tz^{-1}}{(1 - z^{-1})^2}$
t^n 	$\frac{n!}{s^{n+1}}$	$\lim_{a \to 0} (-1)^n \frac{\partial^n}{\partial a^n} \frac{1}{1 - bz^{-1}}$
e^{-at} 	$\frac{1}{s + a}$	$\frac{1}{1 - bz^{-1}}$
te^{-at} 	$\frac{1}{(s + a)^2}$	$\frac{Tbz^{-1}}{(1 - bz^{-1})^2}$
$t^2 e^{-at}$ 	$\frac{2}{(s + a)^3}$	$\frac{T^2 bz^{-1}(1 + bz^{-1})}{(1 - bz^{-1})^3}$
$\sin(\omega t)$ 	$\frac{\omega}{s^2 + \omega^2}$	$\frac{z^{-1} \sin(\omega T)}{1 - 2z^{-1} \cos(\omega T) + z^{-2}}$
$\cos(\omega t)$ 	$\frac{s}{s^2 + \omega^2}$	$\frac{1 - z^{-1} \cos(\omega T)}{1 - 2z^{-1} \cos(\omega T) + z^{-2}}$
$1 - e^{-at}$ 	$\frac{a}{s(s + a)}$	$\frac{(1 - b)z^{-1}}{(1 - z^{-1})(1 - bz^{-1})}$
$e^{-at} \sin(\omega t)$ 	$\frac{\omega}{(s + a)^2 + \omega^2}$	$\frac{z^{-1} b \sin(\omega T)}{1 - 2z^{-1} b \cos(\omega T) + b^2 z^{-2}}$
$e^{-at} \cos(\omega t)$ 	$\frac{s + a}{(s + a)^2 + \omega^2}$	$\frac{1 - z^{-1} b \cos(\omega T)}{1 - 2z^{-1} b \cos(\omega T) + b^2 z^{-2}}$
Initial value theorem: $\lim_{t \to 0} f(t)$	$\lim_{s \to \infty} sF(s)$	$\lim_{z \to \infty} F(z)$
Final value theorem: $\lim_{s \to \infty} f(t)$	$\lim_{s \to 0} sF(s)$	$\lim_{z \to 1} \left[\left(1 - z^{-1}\right) F(z) \right]$
Translation: $f(t - D)u(t - D)$	$e^{-Ds} F(s)$	$F(z) z^{-n}$ where $D = nT$
Derivative: $\frac{\mathrm{d}^n f(t)}{\mathrm{d}t^n} = f^n(t)$	$s^n F(s) - \sum_{k=1}^{n} s^{k-1} f^{n-k}(0)$	
Integral: $\int_0^t f(t) \mathrm{d}t$	$\frac{1}{s} F(s)$	
Zero$^{\text{th}}$ order hold	$H(s) = \frac{1 - e^{-Ts}}{s}$	