UNIVERSITY OF PRETORIA
DEPARTMENT OF CHEMICAL ENGINEERING
CPN 321 PROCESS DYNAMICS 321

TIME: 90 MINUTES | TEST 2 | 11th OCTOBER 2012

ANSWER ALL THE QUESTIONS
~~No programmable calculators may be used~~
Datasheet attached

QUESTION 1:

The flow rate F of a manipulated stream through a control valve with a linear trim is given by the following equation: $F(t) = C_v x(t)$, where F(t) is the flow in litres/minute and C_v is a constant set by the valve size. The control valve fractional valve opening, x(t),(fraction of wide open) is set by the output signal CO(t) of an analog electronic feedback controller of which the signal range is 4 – 20 milliamperes. The valve cannot be moved instantaneously and its dynamics can be represented by the following first order system:

$$\tau_v \frac{dx(t)}{dt} + [x(t) - 0] = K_v[CO(t) - 4]$$

The effect of flow rate of the manipulated variable on the process temperature T(t) is given by:

$$\tau_p \frac{dT(t)}{dt} + T(t) = K_p F(t)$$

Assume the following numerical values for the process parameters:

$\tau_v = 0{,}25$ (in minutes)
$\tau_p = 1{,}2$ (in minutes)
K_v = (1/16) (fraction open/mA)
K_p = 8 (°C/(litres/min))
C_v = 10
The fractional valve opening is 0,5 at steady-state.

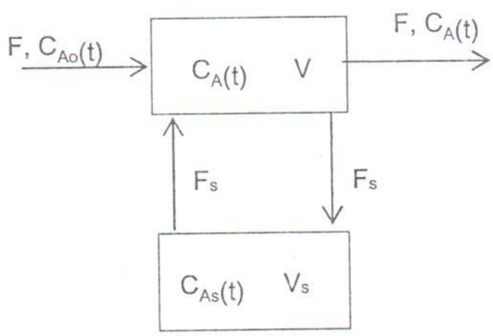
Now do the following:

(a) For the above system, rewrite into deviation variables & convert into the s-domain by means of the Laplace-transform. (7)
(b) Obtain the transfer function relating the temperature T to the controller output signal, CO. (3)
(c) Obtain an analytical expression (in the time domain) for the output T, after a step change of magnitude 4 mA, to the input CO (5)
(d) What is the temperature 1 minute after the step input? (5)

[20]

QUESTION 2:

The imperfect mixing in a chemical reactor can be modeled by splitting the total volume into two perfectly mixed reactors with circulation between them. Feed enters and leaves one section. The other section acts like a "side-capacity" element.



Assume holdups and flowrates are constant. The reaction is an irreversible first-order consumption of reactant A: $A \rightarrow B$. ($R_A=kC_A$) The system is isothermal.

Solve for the transfer function relating C_{Ao} and C_A. **[10]**

QUESTION 3:

A process is described by the following equations in the Laplace domain:

$$\frac{x(s)}{w(s)}=\frac{8(s+3)}{(2s+1)}$$
$$\frac{v(s)}{x(s)}=\frac{0{,}2}{(5s+1)}$$
$$\frac{y(s)}{x(s)}=\frac{2}{(4s+1)}$$
$$w(s)=r(s)-v(s)$$

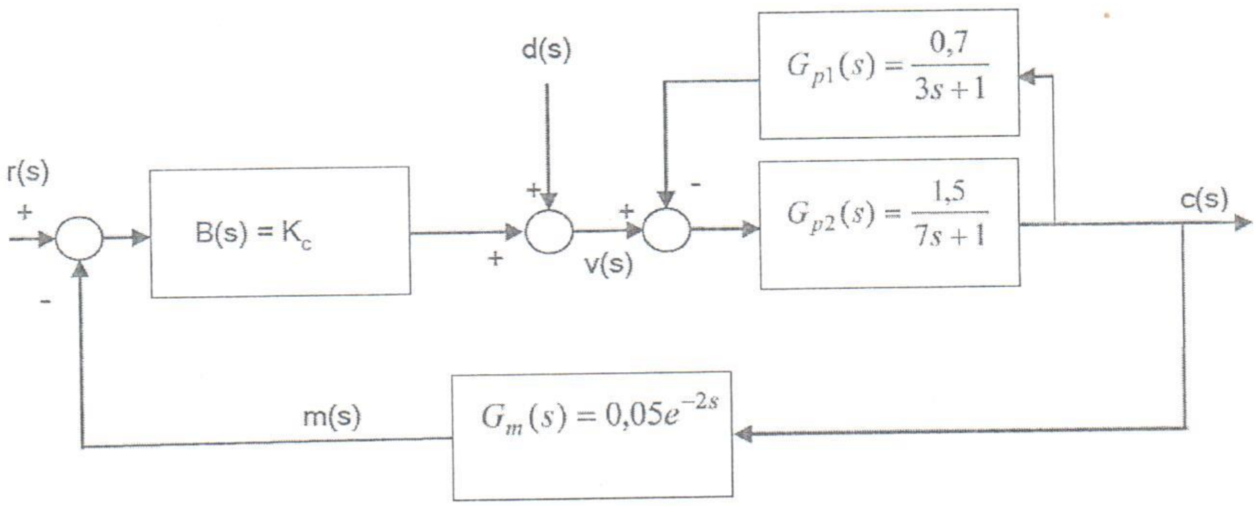
(a) Make a fully annotated block-diagrammatic representation of this process. (5)

(b) Give the values of the zero's and poles of this system. (5)

(c) Is this system stable/unstable and underdamped/overdamped? Motivate your answer. (5)

[15]

QUESTION 4:



For the system represented by the above block diagram, answer the following:

(a) To simplify the system, obtain a single transfer function relating c(s) and v(s). (4)

(b) Now obtain the closed-loop transfer function relating the measuring signal, m(s), to the input signal d(s) (6)

[10]

TOTAL: {55}

Aanvaar dat volumes en vloeitempo's deurgaans konstant is. Die reaksie is 'n onomkeerbare eerste-orde reaksie waar reagens A gebruik word: $A \rightarrow B$. (R_A=kC_A). Die sisteem is isotermies.

Verkry die oordragsfunksie wat die verband gee tussen C_{Ao}en C_A. **[10]**

VRAAG 3:

'n Proses word deur die volgende funksies in die Laplace-stelsel beskryf:

$$\frac{x(s)}{w(s)} = \frac{8(s+3)}{(2s+1)}$$
$$\frac{v(s)}{x(s)} = \frac{0{,}2}{(5s+1)}$$
$$\frac{y(s)}{x(s)} = \frac{2}{(4s+1)}$$
$$w(s) = r(s) - v(s)$$

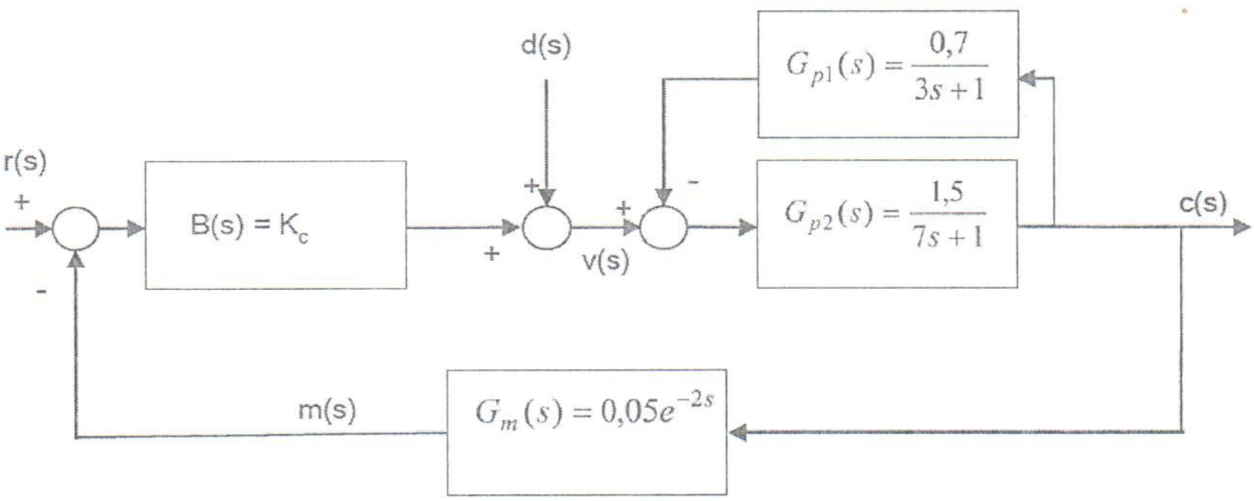
(a) Maak 'n volledig-gedokumenteerde blokdiagrammatiese voorstelling van die proses (5)

(b) Wat is die zero's en pole van hierdie sisteem? (5)

(c) Is die sisteem stabie/onstabiel ondergedemp/oorgedemp? Motiveer u antwoord. (5)

[15]

VRAAG 4:



Vir die sisteem wat deur die blokdiagram hierbo aangetoon word, beantwoord die volgende:

(a) Om die sisteem te vereenvoudig, verkry 'n enkele oordragsfunksie om die verband tussen c(s) en v(s) aan te toon. (4)

(b) Verkry nou die geslotelus oordragsfunksie wat die verband gee tussen die sein uit die meetelement, m(s) en die insetsein, d(s) (6)

[10]

TOTAAL: {55}

CHN 321 Test 2 2012

Question 1

$$F(t) = C_v x(t) \quad -(1)$$

$$\tau_v \frac{dx(t)}{dt} + [x(t) - 0] = K_v [CO(t) - 4] \quad -(2)$$

$$\tau_p \frac{dT(t)}{dt} + T(t) = K_p F(t) \quad -(3)$$

(a) (1) at SS: $\bar{F} = C_v \bar{x} \quad -(4)$

(2) at SS: $\tau_v \frac{d\bar{x}}{dt} + \bar{x} - 0 = K_v[\overline{CO} - 4] \quad -(5)$

(3) at SS: $\tau_p \frac{d\bar{T}}{dt} + \bar{T} = K_p \bar{F} \quad -(6)$

(1) − (4): $F(t) - \bar{F} = C_v (x(t) - \bar{x})$

(2) − (5): $\tau_v \frac{d(x(t) - \bar{x})}{dt} + (x(t) - \bar{x}) = K_v (CO(t) - \overline{CO})$

(3) − (6): $\tau_p \frac{d(T(t) - \bar{T})}{dt} + (T(t) - \bar{T}) = K_p (F(t) - \bar{F})$

Define deviation variables

$$F'(t) = F(t) - \bar{F}$$
$$X(t) = x(t) - \bar{x}$$
$$CO'(t) = CO(t) - \overline{CO}$$
$$T'(t) = T(t) - \bar{T}$$
$$F'(t) = F(t) - \bar{F}$$

$\Rightarrow$ $F'(t) = C_v X(t)$

$$\tau_v \frac{dX(t)}{dt} + X(t) = K_v CO'(t)$$

$$\tau_p \frac{dT'(t)}{dt} + T'(t) = K_p F'(t)$$

$\Rightarrow$ $F'(s) = C_v X(s) \Rightarrow F'(s)/X(s) = C_v$

$(\tau_v s + 1) X(s) = K_v CO'(s) \Rightarrow X(s)/CO'(s) = K_v/(\tau_v s + 1)$

$(\tau_p s + 1) T'(s) = K_p F'(s) \Rightarrow T'(s)/F'(s) = K_p/(\tau_p s + 1)$

(b)

$$CO'(s) \rightarrow \boxed{\frac{K_v}{\tau_v s + 1}} \xrightarrow{X(s)} \boxed{C_v} \xrightarrow{F'(s)} \boxed{\frac{K_p}{\tau_p s + 1}} \rightarrow T'(s)$$

$$\Rightarrow \frac{T'(s)}{CO'(s)} = \frac{T'(s)}{F'(s)} \cdot \frac{F'(s)}{X(s)} \cdot \frac{X(s)}{CO'(s)} = \frac{K_p}{\tau_p s + 1} \cdot C_v \cdot \frac{K_v}{\tau_v s + 1}$$

$$= \frac{8 \cdot 10 \cdot 1/16}{(1{,}2s + 1)(0{,}25s + 1)} = \frac{5}{(1{,}2s + 1)(0{,}25s + 1)}$$

Ic) If $CO'(s) = 4/s$:

$$T'(s) = \frac{20}{s(1{,}2s + 1)(0{,}25s + 1)} = \frac{20}{1{,}2 \times 0{,}25\, s(s + 1/1{,}2)(s + 1/0{,}25)}$$

$$= \frac{66{,}67}{s(s + 0{,}833)(s + 4)} = \frac{A}{s} + \frac{B}{s + 0{,}833} + \frac{C}{s + 4}$$

$$A = \lim_{s \to 0} \left[\frac{66{,}67}{(s + 0{,}833)(s + 4)} \right] = \frac{66{,}67}{0{,}833 \times 4} = 20$$

$$B = \lim_{s \to -0{,}833} \left[\frac{66{,}67}{s(s + 4)} \right] = \frac{66{,}67}{-0{,}833(-0{,}833 + 4)} = -25{,}27$$

$$C = \lim_{s \to -4} \left[\frac{66{,}67}{s(s + 0{,}833)} \right] = \frac{66{,}67}{-4(-4 + 0{,}833)} = 5{,}263$$

$$\Rightarrow T'(s) = \frac{20}{s} - \frac{25{,}27}{s + 0{,}833} + \frac{5{,}263}{s + 4}$$

$$T'(t) = 20u(t) - 25{,}27 e^{-0{,}833t} + 5{,}263 e^{-4t}$$

$$T(t) = \bar{T} + 20 - 25{,}27 e^{-0{,}833t} + 5{,}263 e^{-4t}$$

If $\bar{x} = 0{,}5$:

$\Rightarrow \bar{x} - 0 = \frac{1}{16}(\overline{CO} - 4)$

$\Rightarrow 0{,}5 \times 16 + 4 = \overline{CO} = 12 \text{ mA} \rightarrow$

and $\bar{F} = C_v \bar{x} = 10 \times 0{,}5 = 5 \text{ l/min} \rightarrow$

and $\bar{T} = K_p \bar{F} \Rightarrow \bar{T} = 8 \times 5 = 40^\circ C \rightarrow$

$\Rightarrow T(t) = 40 + 20 - 25{,}27 e^{-0{,}833t} + 5{,}263 e^{-4t}$

$T(1) = 60 - 25{,}27 e^{-0{,}833} + 5{,}263 e^{-4}$

$= 60 - 10{,}986 + 0{,}096$

$= 49{,}11^\circ C \rightarrow$

(3)

Question 2:

A-balance over tank 1

$$F C_{A0}(t) - F C_A(t) + F_s C_{As}(t) - F_s C_A(t) - kV C_A(t) = V \frac{d C_A(t)}{dt}$$

A-balance over tank 2:

$$F_s C_A(t) - F_s C_{As}(t) - kV_s C_{As}(t) = V_s \frac{d C_{As}(t)}{dt}$$

No nonlinear terms $\Rightarrow$ deviation variables can be implemented

$$\Rightarrow F C_{A0}(t) - F C_A(t) + F_s C_{As}(t) - F_s C_A(t) - kV C_A(t) = V \frac{d C_A(t)}{dt}$$

and $F_s C_A(t) - F_s C_{As}(t) - kV_s C_{As}(t) = V_s \frac{d C_{As}(t)}{dt}$

$$\Rightarrow F C_{A0}(s) - F C_A(s) + F_s C_{As}(s) - F_s C_A(s) - kV C_A(s) = Vs\, C_A(s)$$

and $F_s C_A(s) - F_s C_{As}(s) - kV_s C_{As}(s) = V_s s\, C_{As}(s)$

$$\Rightarrow F C_{A0}(s) + F_s C_{As}(s) = [Vs + F + F_s + kV] C_A(s) \quad - (1)$$

and $F_s C_A(s) = [V_s s + F_s + kV_s] C_{As}(s)$

$$\Rightarrow C_{As}(s) = \frac{F_s C_A(s)}{[V_s s + F_s + kV_s]} \quad - (2)$$

(2) into (1): $F C_{A0}(s) + \frac{F_s^2 C_A(s)}{[V_s s + F_s + kV_s]} = [Vs + F + F_s + kV] C_A($

$$\Rightarrow F C_{A0}(s) = [Vs + F + F_s + kV] C_A(s) - \frac{F_s^2}{[V_s s + F_s + kV_s]} C_A(s)$$

$$= \left[Vs + F + F_s + kV - \frac{F_s^2}{V_s s + F_s + kV_s}\right] C_A(s)$$

$$G_{A_0}(s) = \left\{ \frac{(Vs + F + F_s + kV)(V_s s + F_s + kV_s) - F_s^2}{V_s s + F_s + kV_s} \right\} G_A(s)$$

$$\Rightarrow \frac{G_A(s)}{G_{A_0}(s)} = \frac{F(V_s s + F_s + kV_s)}{(Vs + F + F_s + kV)(V_s s + F_s + kV_s) - F_s^2}$$

$$= \frac{\frac{F}{F_s + kV_s}\left[\frac{V_s}{F_s + kV_s} s + 1\right]}{(F + F_s + kV)\left[\frac{V}{F + F_s + kV} s + 1\right](F_s + kV_s)\left[\frac{V_s}{F_s + kV_s} s + 1\right] - F_s^2}$$

$$= \frac{K_1[\tau_1 s + 1]}{K_2[\tau_2 s + 1]K_3[\tau_1 s + 1] - F_s^2}$$

$$= \frac{K_1(\tau_1 s + 1)}{K_2 K_3(\tau_1 s + 1)(\tau_2 s + 1) - F_s^2}$$

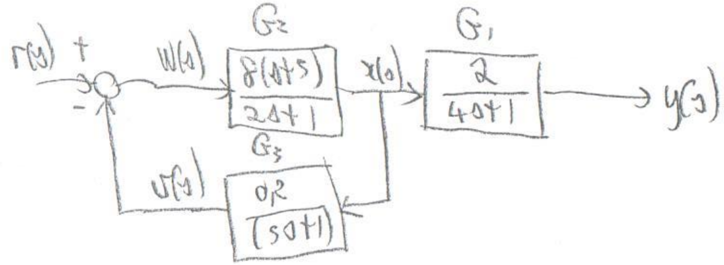
$$= \frac{\frac{K_1}{K_2K_3}(\tau_1 s + 1)}{\tau_1\tau_2 s^2 + (\tau_1 + \tau_2)s + (1 - F_s^2)}$$

$$= \frac{\frac{K_1}{K_2K_3}(\tau_1 s + 1)}{(1 - F_s^2)\left[\frac{\tau_1\tau_2}{(1 - F_s^2)} s^2 + \frac{(\tau_1 + \tau_2)}{(1 - F_s^2)} s + 1\right]}$$

$$= \frac{K_P(\tau_1 s + 1)}{\tau_p^2 s^2 + 2\zeta\tau_p s + 1} \Rightarrow \left(\frac{1^{st}\ \text{order}}{2^{nd}\ \text{order}}\right).$$

Question 3:

(a)



(b)

$$\Rightarrow \frac{X(s)}{r(s)} = \frac{G_2}{1 + G_2G_3} = G_4$$

$$\Rightarrow \frac{y(s)}{r(s)} = G_1G_4 = \frac{G_1G_2}{1 + G_2G_3}$$

$$= \frac{\frac{2}{4s+1} \cdot \frac{8(s+3)}{2s+1}}{1 + \frac{8(s+3)}{(2s+1)} \cdot \frac{0{,}2}{(5s+1)}}$$

$$= \frac{\frac{16(s+3)}{(4s+1)(2s+1)}}{\frac{(2s+1)(5s+1) + 1{,}6(s+3)}{(2s+1)(5s+1)}}$$

$$= \frac{16(s+3)}{(4s+1)\cancel{(2s+1)}} \cdot \frac{\cancel{(2s+1)}(5s+1)}{(2s+1)(5s+1) + 1{,}6(s+3)}$$

$$= \frac{16(s+3)(5s+1)}{(4s+1)[10s^2 + 7s + 1 + 1{,}6s + 4{,}8]}$$

$$= \frac{16(s+3)(5s+1)}{(4s+1)[10s^2 + 8{,}6s + 5{,}8]}$$

roots of $10s^2 + 8{,}6s + 5{,}8$

$$\frac{-8{,}6 \pm \sqrt{8{,}6^2 - 4\times10\times5{,}8}}{20}$$

$= -0{,}43 \pm j0{,}629 \qquad (s + 0{,}43 + 0{,}629j)(s + 0{,}43 - 0{,}629j)$

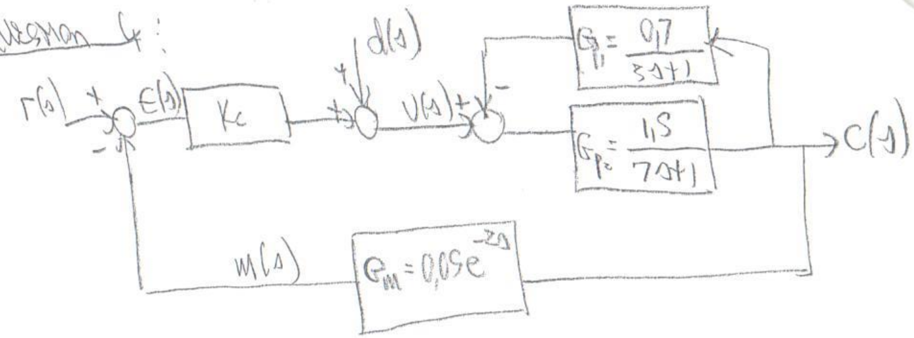
$$\Rightarrow \frac{1{,}6(s+3)(5s+1)}{(4s+1)(s+0{,}43+0{,}629j)(s+0{,}43-0{,}629j)}$$

$\Rightarrow$ zero's $\quad s = -3 \;;\; s = -1/5$

poles . $\quad s = -1/4 \;;\; s = -0{,}43 + 0{,}629j \;;\; s = -0{,}43 - 0{,}629j$

(c) · stable : all poles have negative real values
· underdamped due to complex conjugate pair of roots.

Question 4 :



(a) $\dfrac{c(s)}{v(s)} = \dfrac{G_{p_2}}{1+G_{p_1}G_{p_2}} = G_{p_3}$

(b)

$\Rightarrow m(s) = G_m\, c(s)$

$= G_m G_{p_3} v(s)$

$= G_m G_{p_3} (d(s) + K_c E(s))$

$= G_m G_{p_3} [d(s) + K_c (r(s) - m(s))]$

$\Rightarrow m(s) = G_m G_{p_3} d(s) + G_m G_{p_3} K_c r(s) - K_c m(s) G_m G_{p_3}$

$\Rightarrow m(s)[1 + K_c G_{p_3} G_m] = G_m G_{p_3} d(s) + G_m G_{p_3} K_c r(s)$

$$\Rightarrow \frac{m(s)}{d(s)} = \frac{G_m G_{p_3}}{1 + K_c G_{p_3} G_m}$$

$$= \frac{0{,}05e^{-2s} \cdot \left(\frac{G_{p_2}}{1+G_{p_1}G_{p_2}}\right)}{1 + K_c \cdot 0{,}05e^{-2s}\left(\frac{G_{p_2}}{1+G_{p_1}G_{p_2}}\right)}$$

$$= \frac{0{,}05e^{-2s} G_{p_2}}{(1+G_{p_1}G_{p_2}) + 0{,}05e^{-2s} K_c G_{p_2}}$$

(7)

$$\frac{(s)}{d(s)} = \frac{0{,}05e^{-2s} \cdot \frac{1{,}5}{7s+1}}{1+\frac{0{,}7}{3s+1}\cdot\frac{1{,}5}{7s+1} + 0{,}05e^{-2s}\,K_c\,\frac{1{,}5}{7s+1}}$$

$$= \frac{\frac{0{,}075\,e^{-2s}}{7s+1}}{\frac{(3s+1)(7s+1)+1{,}05+0{,}05e^{-2s}K_c(3s+1)1{,}5}{(3s+1)(7s+1)}}$$

$$= \frac{0{,}075e^{-2s}(3s+1)}{(3s+1)(7s+1)+1{,}05+0{,}05e^{-2s}K_c(3s+1)1{,}5}$$

$$= \frac{0{,}075(3s+1)e^{-2s}}{(21s^2+10s+2{,}05)+0{,}05e^{-2s}K_c(3s+1)1{,}5}$$