CPN321 T4
2019

1 Heated CSTR

Consider the CSTR shown in Figure 1. Stream 1 is a mixture of A and B with composition C_{A1} and C_{B1} (moles/volume) and has a volumetric flowrate F_1 and a temperature T_1. Stream 2 is pure R.

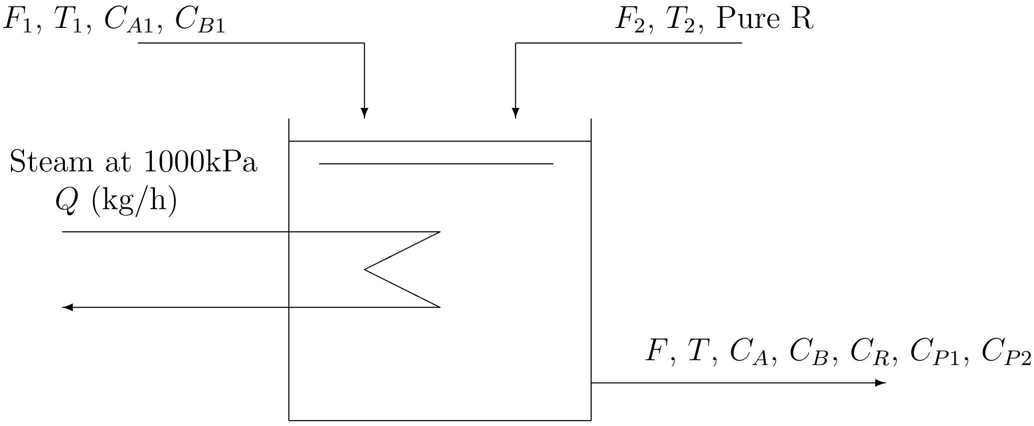

Figure 1: CSTR with steam heating

The reactions taking place are:

$$\mathrm{A} + \mathrm{R} \xrightarrow{k_1} \mathrm{P}_1 \tag{1}$$

$$\mathrm{B} + 2\mathrm{R} \xrightarrow{k_2} \mathrm{P}_2 \tag{2}$$

Both reactions are endothermic and the reaction rates follow the law of mass action and Arrhenius's law. Heat is supplied to the reaction mixture by saturated steam which flows through a coil, immersed in the reactor's content, with a heat transfer area A_t. The steam condenses fully.

You may assume constant C_p for the liquids involved. You may also assume that the flow out of the tank is proportional to the square root of the level in the tank.

1. What are the state variables describing the natural state of the system?
2. What are the continuity balances that you should consider? *hint: the continuity balances and the natural state should correspond.*
3. Derive the stoicheometric matrix for these reactions.
4. Draw a causal diagram for the variables, showing each variable in a circle and arrows indicating which variable it "listens to".
5. Develop a state model (time-dependent) of the CSTR system.
6. Identify all the non-linear terms (simply highlighting them is sufficient).
7. Linearise this model using the first-order Taylor series expansion and rewrite it in terms of deviation variables.

2 Linearization 1

For the following multiple-input, multiple-output (MIMO) non-linear dynamic models, linearise and introduce deviation variables, then rewrite in matrix form ($\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$)

1.

$$\begin{aligned}\frac{\mathrm{d}x_1}{\mathrm{d}t} &= 2{x_1}^2 + 3x_1x_2 + u_1 - {u_2}^3 \\ \frac{\mathrm{d}x_1x_2}{\mathrm{d}t} &= \sqrt{x_2} - 4u_1u_2\end{aligned}$$

2.

$$\begin{aligned}2\frac{\mathrm{d}x_1}{\mathrm{d}t} - 3\frac{\mathrm{d}x_2}{\mathrm{d}t} &= {x_1}^2x_2 - 0.5x_2u_1 + u_2{x_1}^3 \\ \frac{\mathrm{d}x_1}{\mathrm{d}t} + \frac{\mathrm{d}x_2}{\mathrm{d}t} &= \ln x_2 + x_1\cos 2u_1 - \sqrt{u_2}\end{aligned}$$

Remember that you can use `sympy.series` or `tbcontrol.symbolic.linearise` to help you with the linearisation of difficult functions.

3 Flash drum

A liquid stream is a mixture of two components A and B and has a volumetric (volume/time) flow rate F, temperature T_f, and pressure p_f. Let c_A and c_B be the mole fractions of A and B in the liquid stream. It is assumed that the pressure p_f is larger than the bubble point pressure of the mixture A and B, so that there is no vapor present.

The liquid passes through an isenthalpic expansion valve and is "flashed" into a flash drum (see Figure 2). The pressure p in the drum is assumed to be lower than the bubble

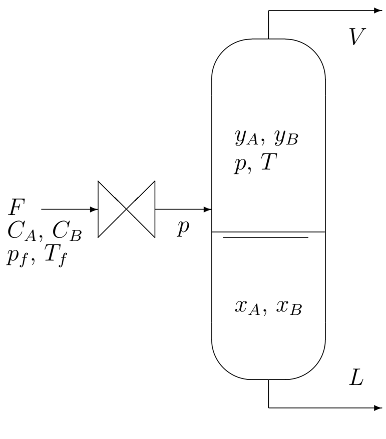

Figure 2: Flash drum with isenthalpic expansion valve

point pressure of the liquid mixture at T_f. As a result, two phases at equilibrium with each other appear in the flash drum; a vapour phase with a composition y_A and y_B (molar fractions) which is drawn with a volumetric flowrate V, and a liquid phase with a composition x_A and x_B (molar fractions) drawn with a volumetric flowrate L. Let T be the temperature of the two phases at equilibrium in the flash drum. L and V are determined by downstream processes.

1. What are the fundamental dependent quantities whose values describe the natural state of the flash drum?

2. What are the boundaries of the system around which you will perform the various balances?

3. What are the relevant balances?

4. Besides the balance equations, what additional relationships do you need to complete the state model for the flash drum?

5. Identify the state variables and the input variables (manipulations, disturbances) of the system.

6. Draw a causal diagram for the variables, showing each variable in a circle and arrows indicating which variable it "listens to".

7. Is it possible to manipulate F, V and L independently? Hint: remember the name of the subject.

8. Develop the complete state model of the system.

4 Control valve

Refer back to T2 Problem 4. These two problems work together to show you how much easier linear systems are to manipulate than nonlinear ones.

The flow rate F of a manipulated stream through a control valve with equal-percentage trim is given by the following equation:

$$F = C_v \alpha^{x-1}$$

where F is the flow in gallons per minute and C_v and α are constants set by the valve size and type. The control valve stem position x (fraction of wide open) is set by the output signal CO of an analog electronic feedback controller whose signal range is 4 mA to 20 mA. The valve cannot be moved instantaneously. It is approximately a first-order system:

$$\tau_p \frac{\mathrm{d}x}{\mathrm{d}t} + x = \frac{CO - 4}{16}$$

The effect of the flow rate of the manipulated variable on the process temperature T is given by

$$\tau_p \frac{\mathrm{d}T}{\mathrm{d}t} + T = K_p F$$

In the previous tut you derived one non-linear ordinary differential equation that gives the dynamic dependence of process temperature on controller output signal CO.

Now do the following:

1. For the above system, linearize the non-linear terms, rewrite into deviation variables and transform into the s-domain by means of the Laplace-transform.

2. Obtain the transfer function relating the process temperature T to the controller output signal, CO.

3. Keeping in mind that $\mathcal{L}\left\{\frac{\mathrm{d}x(t)}{\mathrm{d}t}\right\} = sX(s)$ for deviation variables, find the single second order differential equation that gives the dynamic dependence of the process temperature on controller output signal CO.

   Was this easier or harder than finding the previous non-linear relationship?

4. Draw a block diagram for this process, clearly indicating the relevant transfer functions and the relationships between the various blocks.

5. Obtain an analytical expression (in the time domain) for the output T, after the following disturbances to the input CO:

   (a) a step disturbance of magnitude k

   (b) a pulse input with a magnitude k and duration equal to the time constant of the system.

   Hint: You can use sympy to calculate the inverse laplace for you.

6. *Sketch* what these responses will look like by hand.