Process Dynamics – CPN321

Semester Test 1

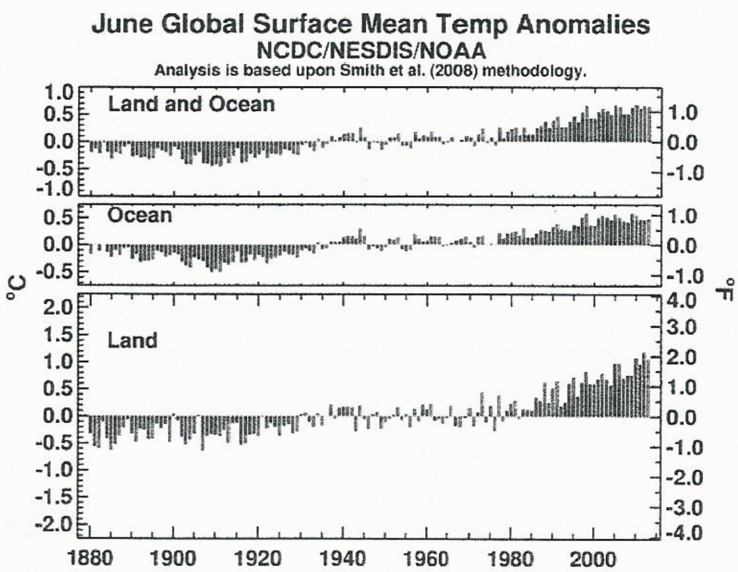
Chemical Engineering
Engineering and the Built Environment

Examiner: Carl Sandrock

August 2013 $\boxed{\text{90 minutes}}$

Instructions – Read carefully

• Answer all the questions. • This is a closed book test. All the information you may use is contained in the paper. • You may use the computer • Make sure that you motivate all your answers and write legibly.



1 Flotation cell

A *flotation cell* as shown in figure 1 operates by introducing air to a feed F containing water and two kinds of suspended mineral solids: valuable *product* and unwanted *gangue*. A small amount of a special surfactant

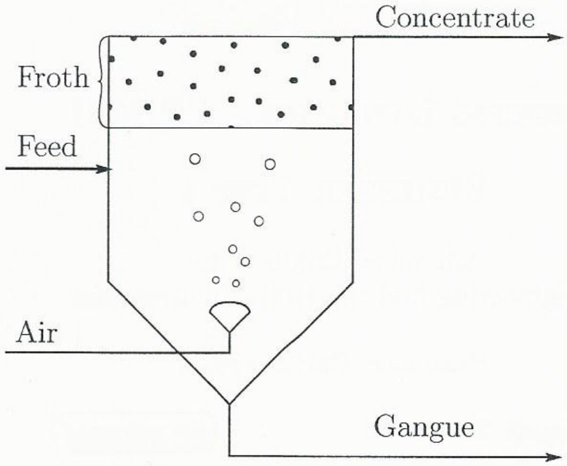


Figure 1: Flotation cell

causes the product to cling to the bubbles. The product-rich froth flows over the weir for further processing, while the remaining liquid and gangue flows from the bottom due to gravity. Note that there is some valuable product in the bottom and some gangue in the top concentrate. The liquid level needs to be controlled.

You may assume that

- the rate at which the valuable product attaches to the bubbles is proportional to the air fed, and
- the rate at which air is released from popping bubbles in the froth to atmosphere is proportional to the height of the froth.

1. Draw a sketch showing your choice of variables clearly. $\boxed{5}$
2. Develop a model for this process. Be sure to explain your model choices clearly using words rather then only listing formulae. $\boxed{20}$
3. Show that specifying the inputs and the parameters of your model completely specifies the problem. $\boxed{15}$

$\boxed{40}$

2 Beer drone

At the recent OppiKoppi Bewilderbeast festival, a drone could be ordered to deliver a beer near your location. A similar drone is shown in figure 2.

Figure 2: Octocopter drone similar to the Oppikoppi Beer Drone

There are eight independent propellers mounted at a slight angle (θ) to the vertical. You may assume that they exert a force perpendicular to the plane of the propeller proportional to some control signal when activated.

- Draw a diagram showing the forces acting on a drone hovering in the air $\boxed{8}$
- Remembering that $\mathbf{F} = m\mathbf{a}$, develop a model describing the motion of the center of mass of the drone as a function of the propeller control signals in three dimensions (x, y and z). $\boxed{15}$
- List the manipulated, controlled and disturbance variables for this situation. $\boxed{7}$

$\boxed{30}$

3 Simulation

Consider the following differential equation:

$$\frac{\mathrm{d}^3 y}{\mathrm{d}t^3} + \frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + \frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{y}{2}$$

The system is started from the steady state with $y = 1$.

3.1 Matrix differential equation

First, rewrite the equation as a system of linear first order differential equations of the form $\boxed{5}$

$$\dot{\mathrm{x}} = A\mathrm{x}$$

Show your working in your answer book.

3.2 Programming

Regardless of what you obtained above, assume

$$A = \begin{bmatrix} \frac{1}{20} & 1 & -\frac{1}{10} \\ \frac{1}{10} & \frac{1}{10} & 1 \\ -\frac{2}{3} & -\frac{3}{2} & -1 \end{bmatrix}$$

and compare the solutions obtained using

1. the matrix exponential $\boxed{5}$
2. numeric solution using Euler integration with a step size of 0,1. $\boxed{5}$
3. numeric solution using lsode $\boxed{5}$

Your program should create a single plot with three curves for the value of $y(t)$, indicating clearly which one is which. Name your program `st1simulation.m`.

$\boxed{20}$

Full Marks $\boxed{90}$

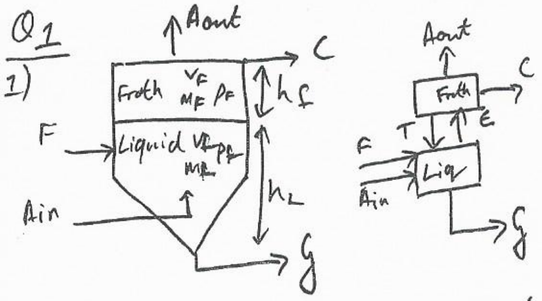
LaTeX 2_ε
www.latex-project.org

CPN 321 ST1 2013

Name Endres, JC

Q1

1)



all streams: x_v valuable, Feed
non air: C concentrate, I concentrate
air, gangue

Diagram: ☑3
All vars. defined: ☑2

1.1 ✓/5

2) Assumptions: froth + liq. well-mixed ☐3

from prob. statement: $E, x_{v,E} = k_1 A_{in}$ ☑2
$A_{out} = k_2 h_f$ ☑1

MB over froth, liquid ☐2

CB froth, liquid
v ■ ■
g ■ ■
A ■ ■ ☑4

$m_i = \rho_i v_i$

$\rho_f = f(x)$
$\rho_L = f(x)$ ☑2 $h_i = f_i(v_i)$ ☐2

fractions the same in exiting streams ☑2

flow out α h ☐1

1.2 ✓/20

3) 5 liq. streams f, x_v, x_g, x_A } 20

2 air flowrates 2

2 parts $m, v, \rho, h, x_v, x_g, x_A$ 14
36

spec inputs: 10

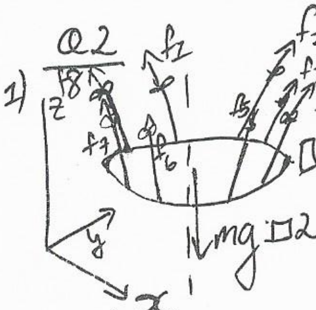
DOF = 36 − 26 = 10

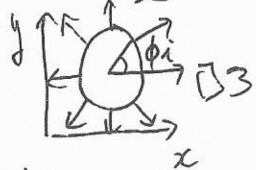
Counting
DOF ☐5 pars ☐2
inputs ☐5
☐3

1.3 ✓/15

Q2

1)





2.1 ✓/8

2) $\underline{F} = m\underline{a}$

$\underline{a} = \frac{d^2}{dt^2}\underline{x}$ ☐2

$\underline{F} = \sum_{i=1}^{8} F_i + mg$ ☑2

$g = -\begin{bmatrix}0\\0\\-9.8\end{bmatrix}$ ☐

$F_i = kc_i \begin{bmatrix}\cos\theta \\ \sin\theta\sin\phi_i \\ \sin\theta\cos\phi_i\end{bmatrix}$ ☐2 ☐4 ☐4

2.2 ✓/15

3) manip. c_i ☐ ✓ 1

control x, y, z + deriv. 2☐ ☐2

disturbances eq wind 2☐

2.3 5/7

Q3

1) $y = x_1$
$\frac{dy}{dt} = x_2$
$\frac{d^2y}{dt^2} = x_3$ ☐2

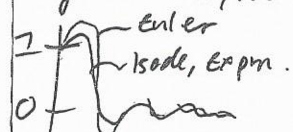
$-2\dot{x}_1 + 2\dot{x}_2 - 2\dot{x}_3$
$1\dot{x}_1 + 0 + 0$
$0 + 1\dot{x}_2 + 0$

☐

$A = \begin{bmatrix}-2 & -2 & -2\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}^{-1}$ 2☐

3.1

```
A = [1/20   1    -1/10
     1/10   1/10  1
     -2/3   -3/2  -1];
                ☐1
tspan = 0:0.1:40
dt    = 0.1;
x0    = [1; 0; 0]  ☐1
xe    = x0;
yplot = zeros(size(tspan),3);
for i = 1:length(tspan)  2☐
    t = tspan(i);
    x = expm(A*t)*x0;  3☐
    xe = xe + dt*A*xe;  3☐
    yplot(i, [1,2]) = [x(1), xe(1)];
end
         2☐
xx = lsode(@(x) A*x, x0, tspan)  3☐
plot(tspan, yplot)
legend('Analytic', 'Euler', 'lsode')
```



✗/15

Q1: $\boxed{\frac{40}{40}}$ + Q2 $\boxed{\frac{28}{30}}$ + Q3 $\boxed{\frac{0}{20}}$ = $\frac{68}{90}$ = 76%