Name: ______________________ Student Number: ______________

Process Dynamics – CPN321

Exam

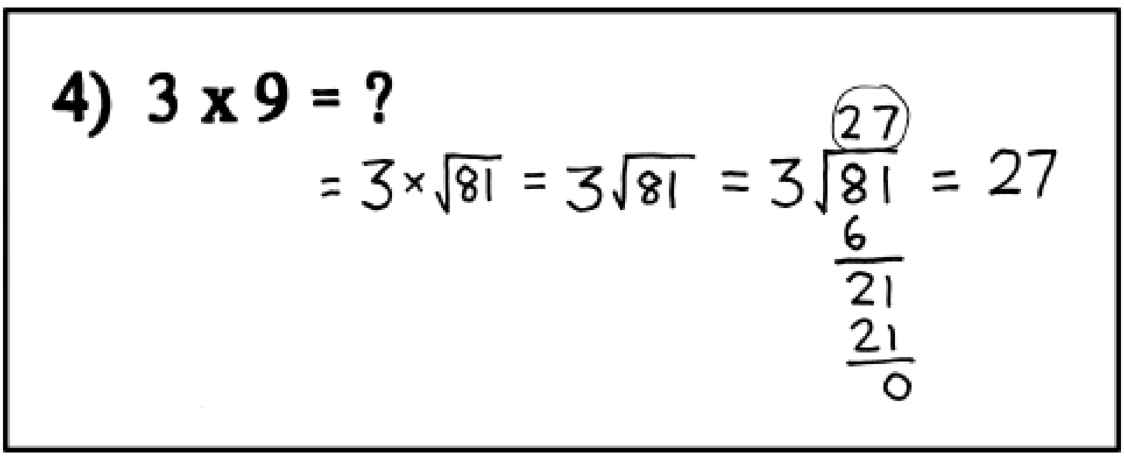
Chemical Engineering
Engineering and the Built Environment

Examiner: Carl Sandrock External examiner: PL de Vaal

November 2013 180 minutes

Instructions – Read carefully

• Answer all the questions on the paper on side 1 of the multiple choice form or in the blocks provided. • This is a closed book test. All the information you may use is contained in the paper and the attached formula sheet.



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1 Multiple choice

Answer this section on side 1 of the multiple choice form. Each question counts 5 marks.

1. Select the statement that is always true about the step response of a system with a pole with a positive real part

   a. The derivative at time 0 will be a different sign from the derivative as $t \to \infty$

   b. The response will oscillate a number of times

   c. There will be a time when the response is below the inital value and a time when the response is above it.

   d. The derivative of the response with respect to time will have more than one root

   e. The eventual response as $t \to \infty$ will be infinite.

2. Select a true statement about the approximation of dead time by a rational function if an analytic step response is to be obtained

   a. It is always necessary

   b. It is necessary if the dead time appears in the numerator of the transfer function

   c. It is necessary if the dead time appears in the denominator of the transfer function

   d. It is necessary if the dead time is added to terms without dead time

   e. It is never necessary

3. Select a true statement about state space representations. The state space represenation of a system ...

   a. ... is unique as long as it has a single input and a single output

   b. ... is unique as long as it is non-interacting.

   c. ... is unique if there is only one state

   d. ... is unique if $C = I$ and $D = 0$

   e. ... is not unique

4. Select a true statement about the relationship between transfer function models and state space models

   a. Any transfer function model can be realised in state space form.

   b. A transfer function model with dead time cannot be realised in state space form exactly

   c. A transfer function model with higher order derivatives cannot be converted to a transfer function model as it requires only first order derivatives.

   d. A transfer function model with inverse response cannot be realised in state space form as it requires negative real parts for all the poles

e. All realisations of transfer function models in state space form are approximations

5. What characteristic must the input posess for an exact discrete time model to predict the output exactly?

a. Continuous

b. Piecewise continuous

c. Piecewise differentiable

d. Smooth

e. Piecewise constant

6. Choose the true statement about discrete linear models:

a. Most discrete models have less parameters than their continuous counterparts

b. Most discrete models have more parameters than their continuous counterparts

c. Most convolution models have more parameters than the corresponding first-order difference equations

d. Most step-response models have more parameters than the corresponding impulse-response models

e. There is no general relationship between the number of parameters of continuous and discrete models.

7. Select a true statement:

a. The output of any system subjected to a sinusoidal input will be exactly sinusoidal with the same frequency

b. The output of a linear system subjected to a sinusoidal input will be exactly sinusoidal with the same frequency

c. The output of a linear system subjected to a sinusoidal input will be approximately sinusoidal with the same frequency

d. If the output of a system is exactly sinusoidal, the input had to have been sinusoidal

e. If the output of a system is exactly sinusoidal, the system had to have been linear

8. If you had only the gain part of a Bode diagram, what property of the transfer function of a system would stop you from being able to draw the phase part?

a. Positive zero

b. Complex poles

c. Time delay

d. (a) and (b)

e. (a) and (c)

9. Select a true statement about *aliasing*. Due to aliasing . . .

   a. . . . digital signals are always less accurate than analog ones

   b. . . . good digital filters must be used to distinguish between high and low frequency sinusoids after sampling

   c. . . . analog filters must be used to remove high frequency components before sampling

   d. . . . process noise can appear as real process data in sampled data

   e. . . . sampling must be done at approximately 100 times the dominant time constant

10. Select a true statement about reconstructing analog signals from sampled versions

   a. Perfect reconstruction is never possible

   b. Sampling always discards information

   c. Reconstruction always discards information

   d. Sampling always introduces a delay

   e. Reconstruction always introduces a delay

50

2 Higher order dynamics

2.1 Inverse

Consider the following system:

$$\frac{Y_1}{U_1} = G_1 = \frac{K(s+a)}{(s+b)(s+c)}, \quad 0 < b < c \tag{1}$$

Find the conditions (in terms of inequalities concerning K, a, b and c) under which $y_1(t)$ will exhibit a minimum or maximum value which is different from the value as $t \to \infty$ if $u(t)$ is a unit step. 15

2.2 Poles and zeros

Sketch the step response of the system for $K > 0$, $a < 0$. Explain your sketch with reference to the poles and zeros of the process. $\boxed{5}$

$\boxed{20}$

3 Multivariable system description

Consider the following state space realisation:

$$A = \begin{bmatrix} 1 & 1 \\ \alpha & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{2}$$

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix} \qquad D = 1 \tag{3}$$

3.1 Conversion

Find the (SISO) transfer function representation of this process $\boxed{5}$

3.2 Analysis

Under what conditions will the system exhibit oscillatory response? Use the state-space version of the model to answer this question. $\boxed{15}$

20

4 Higher order system

Consider the transfer function below:

$$\frac{Y_2}{U_2} = G_2(s) = \frac{1 - s}{(20s + 1)(5s + 1)(s + 1)^2)} \tag{4}$$

4.1 Padé approximation

Assuming that G_2 was the result of a Padé approximation of dead time, find the dead time. 5

4.2 Higher-order systems

Approximate G_2 by a first-order-plus-deadtime model using Skogestad's half rule. 15

20

4.3 Bode diagram

Sketch an asymptotic bode diagram of G_2 on the graph paper provided. Make sure you annotate your graph with your major construction points. 20

20

5 Identification

Use the step response of G_2 in the attached figure to answer these questions.

Assume that the process was sampled once every five seconds ($T = 5$). Consider a model of the form

$$y_2(kT) = (1 - e^{-T/\tau})Ku_2[(k-2)T] + e^{-T/\tau}y_2[(k-2)T] \tag{5}$$

5.1 Least squares

Assume that the model predictions can be written in the form

$$Y = X\beta + \epsilon \tag{6}$$

where β is a vector of coefficients

Write down the equation for the parameters which minimise the sum of the square of the residuals. 5

5.2 Matrix construction

Find the matrices Y and X which would be used in linear regression to find β. Make notes on the graph you are reading to explain. 15

5.3 Application

Find the value of β. $\boxed{10}$

$\boxed{\mathit{30}}$

6 Discrete systems

6.1 z-transform

Rewrite equation 5 in discrete transfer function form (z domain). $\boxed{5}$

6.2 Bucket brigade

Consider a batch process consisting of N tanks of equal volume V. At time $t = 0$ tank 1 contains $V/2$ of black ink and the rest of the tanks contain $V/2$ of pure water. During a single shift, workers empty tank 1 into tank 2, then transfer $V/2$ of the mixed liquid then in tank 2 into the tank 3 and so on to the end of the chain. They then add $V/2$ of pure water back into tank 1. The liquid in the last tank is taken for analysis. Let $x_{s,n}$ indicate the fraction of ink in tank n at a particular shift s. This process is shown graphically in figure 1.

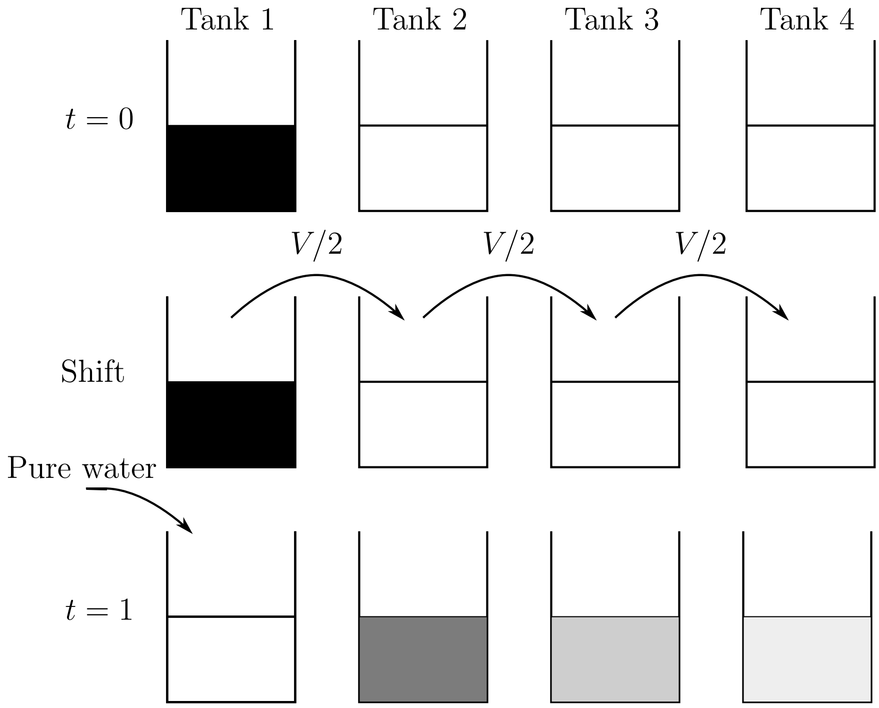

Figure 1: Tank system for question 6.2

a. Write an expression for the concentration in the last tank for the specific case explained above $\boxed{5}$

b. Write a general z-domain transfer function for the process relating the concentration in the last tank to the concentration in the first. $\boxed{10}$

$\boxed{20}$

Full Marks $\boxed{180}$

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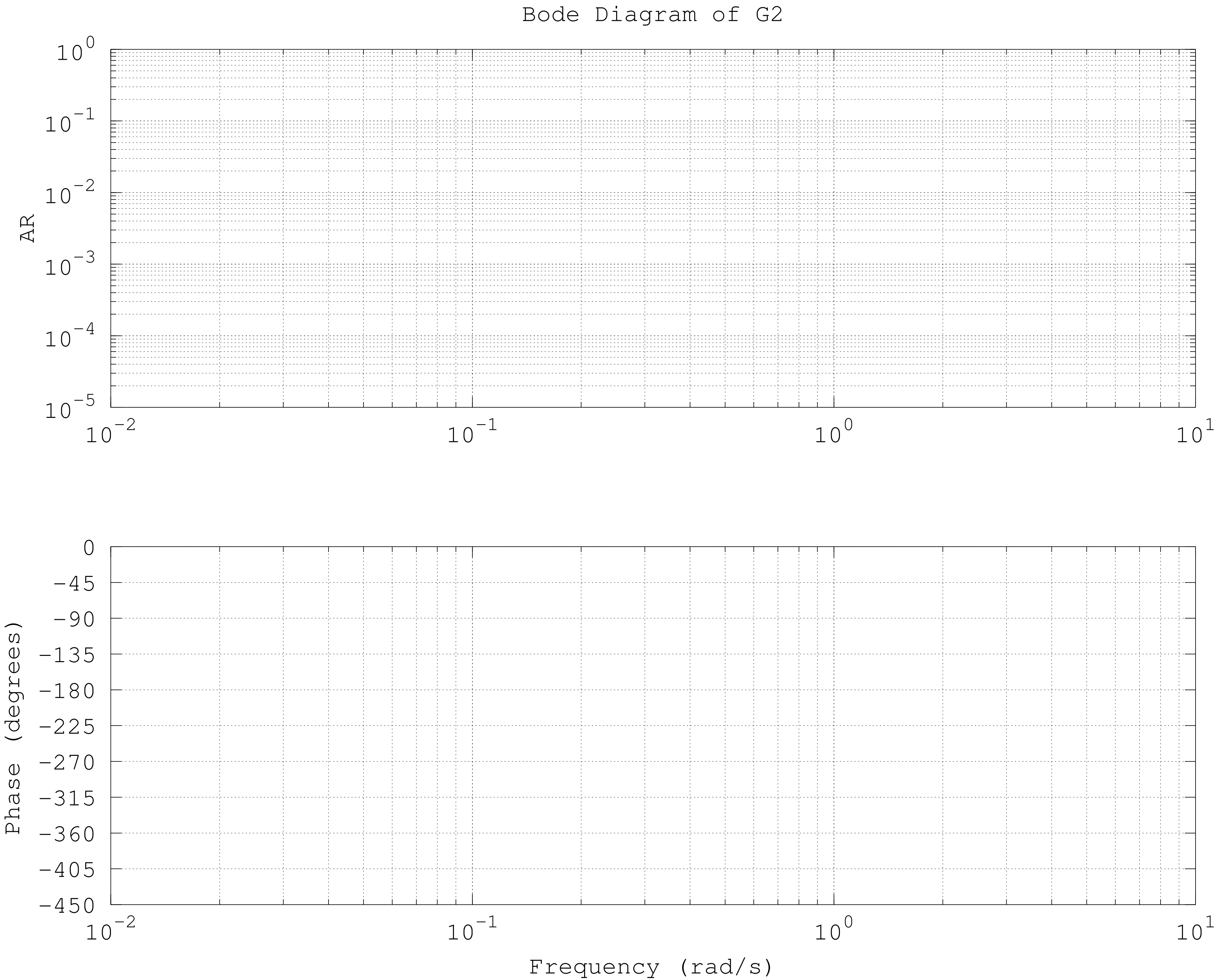
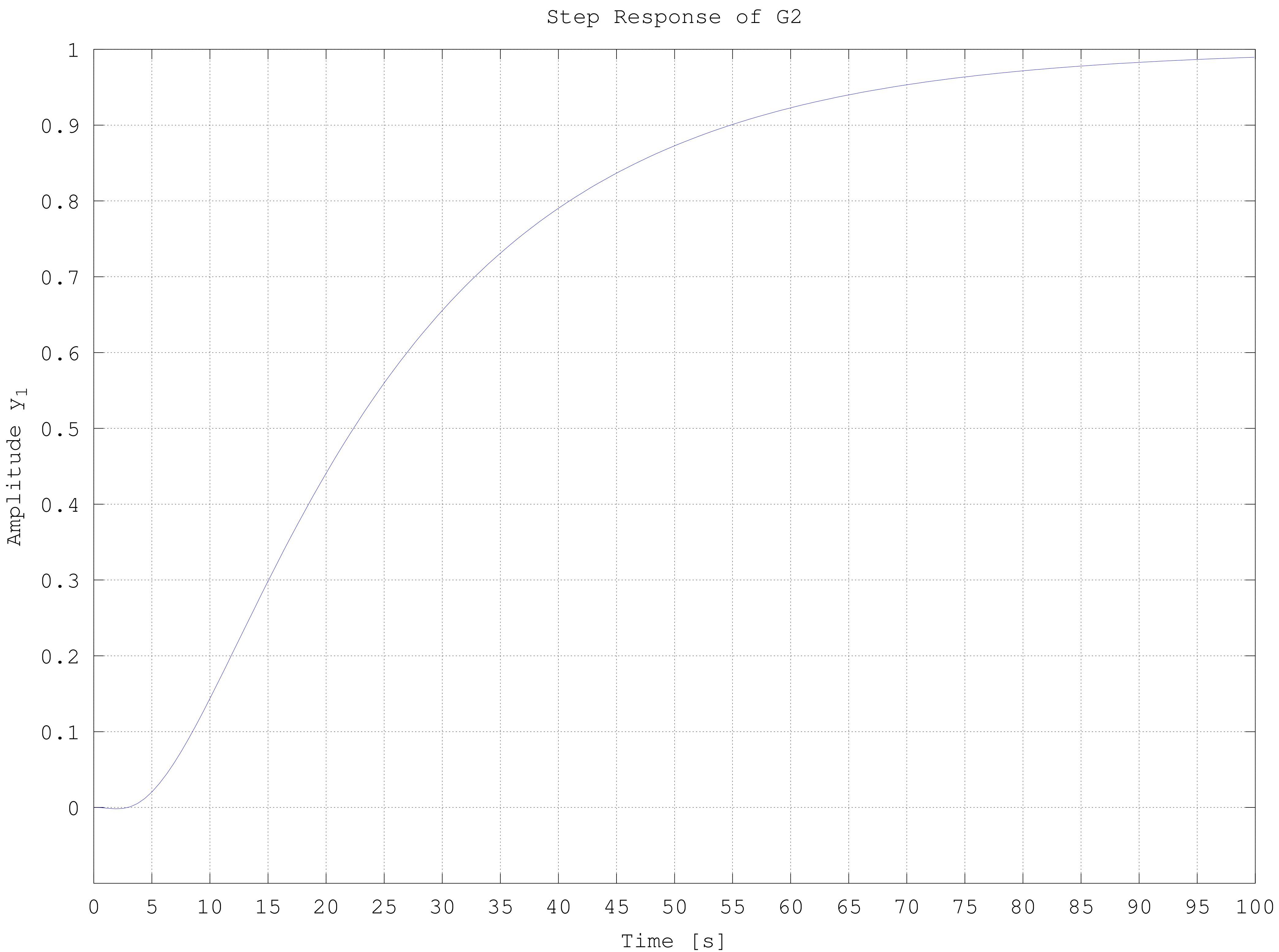
www.latex-project.org



Bode Diagram of G2
AR
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-2}
10^{-1}
10^{0}
10^{1}
Phase (degrees)
0
-45
-90
-135
-180
-225
-270
-315
-360
-405
-450
Frequency (rad/s)


Step Response of G2
Amplitude y_1
Time [s]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100

Datasheet: CPN321/CPB410

Compiled on October 31, 2013

General solution of 1st order DE:

$$\dot{x} + P(t)x = Q(t) \quad \Rightarrow \quad x = \frac{1}{F_I}\int Q(t)F_I \mathrm{d}t + c_1 \quad \text{with} \quad F_I = \exp\left(\int P(t)\mathrm{d}t\right)$$

Taylor Series expansion near point $x = a$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$

$$f(x_1, \dots, x_d) = \sum_{n_1=0}^{\infty} \cdots \sum_{n_d=0}^{\infty} \frac{(x_1 - a_1)^{n_1} \cdots (x_d - a_d)^{n_d}}{n_1! \cdots n_d!} \left(\frac{\partial^{n_1+\cdots+n_d} f}{\partial x_1^{n_1} \cdots \partial x_d^{n_d}}\right)(a_1, \dots, a_d)$$

Linear approximation around point $\mathbf{x} = \mathbf{0}$, where $f(\mathbf{x}) = \mathbf{0}$:

$$f(\mathbf{x}) \approx \nabla f(\mathbf{0}) \cdot \mathbf{x}$$

$$f(x_1, x_2, \cdots, x_d) \approx \frac{\partial f}{\partial x_1}(0)x_1 + \frac{\partial f}{\partial x_2}(0)x_2 + \cdots + \frac{\partial f}{\partial x_d}(0)x_d$$

Partial fraction expansion (for strictly proper rational functions of s)

$$F(s) = \frac{(s-z_1)(s-z_2)(s-z_3)\cdots}{(s-p_1)^n(s-p_2)(s-p_3)\cdots} = \underbrace{\sum_{m=0}^{n-1} \frac{A_m}{(s-p_1)^{n-m}}}_{\text{repeated roots}} + \frac{B}{s-p_2} + \frac{C}{s-p_3} \cdots$$

$$A_m = \lim_{s \to p_1} \left\{ \frac{\mathrm{d}^m}{\mathrm{d}s^m} \Big[(s-p_1)^n F(s) \Big] \right\} \frac{1}{m!}$$

$$B = \lim_{s \to p_2} \big[(s-p_2)F(s) \big]$$

Euler identity:

$$e^{i\theta} = \cos\theta + i\sin\theta \quad \therefore e^{-i\theta} = \cos\theta - i\sin\theta \quad \text{and} \quad e^{i\pi} - 1 = 0$$

(1,1) Padé approximation of dead time:

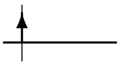
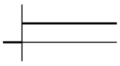
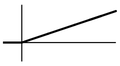
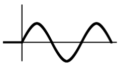
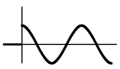
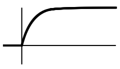
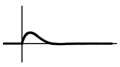
$$e^{-Ds} \approx \frac{1 - \frac{D}{2}s}{1 + \frac{D}{2}s}$$

PID controller:

$$m = K_C\left(\varepsilon + \frac{1}{\tau_I}\int_0^t \varepsilon \mathrm{d}t + \tau_D \frac{\mathrm{d}\varepsilon}{\mathrm{d}t}\right) \qquad \frac{m}{\varepsilon}(s) = K_C\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Tuning rules:

	Ziegler-Nichols			Cohen-Coon (with $\phi = \frac{t_D}{\tau_P}$)		
	K_C	τ_I	τ_D	K_C	τ_I	τ_D
P	$\frac{K_u}{2}$			$\frac{\phi+3}{3K_P\phi}$		
PI	$\frac{K_u}{2.2}$	$\frac{P_u}{1.2}$		$\frac{5\phi+54}{60K_P\phi}$	$t_D\frac{30+3\phi}{9+20\phi}$	
PID	$\frac{K_u}{1.7}$	$\frac{P_u}{2}$	$\frac{P_u}{8}$	$\frac{3\phi+16}{12K_P\phi}$	$t_D\frac{32+6\phi}{13+8\phi}$	$\frac{4}{11+2\phi}$

Time domain	Laplace-transform	z-transform ($b = e^{-aT}$)
Impulse: $\delta(t)$ 	1	1
Unit step: $u(t)$ 	$\dfrac{1}{s}$	$\dfrac{1}{1-z^{-1}}$
Ramp: t 	$\dfrac{1}{s^2}$	$\dfrac{Tz^{-1}}{(1-z^{-1})^2}$
t^n 	$\dfrac{n!}{s^{n+1}}$	$\lim\limits_{a\to 0}(-1)^n \dfrac{\partial^n}{\partial a^n}\dfrac{1}{1-bz^{-1}}$
e^{-at} 	$\dfrac{1}{s+a}$	$\dfrac{1}{1-bz^{-1}}$
te^{-at} 	$\dfrac{1}{(s+a)^2}$	$\dfrac{Tbz^{-1}}{(1-bz^{-1})^2}$
t^2e^{-at} 	$\dfrac{2}{(s+a)^3}$	$\dfrac{T^2bz^{-1}(1+bz^{-1})}{(1-bz^{-1})^3}$
$\sin(\omega t)$ 	$\dfrac{\omega}{s^2+\omega^2}$	$\dfrac{z^{-1}\sin(\omega T)}{1-2z^{-1}\cos(\omega T)+z^{-2}}$
$\cos(\omega t)$ 	$\dfrac{s}{s^2+\omega^2}$	$\dfrac{1-z^{-1}\cos(\omega T)}{1-2z^{-1}\cos(\omega T)+z^{-2}}$
$1-e^{-at}$ 	$\dfrac{a}{s(s+a)}$	$\dfrac{(1-b)z^{-1}}{(1-z^{-1})(1-bz^{-1})}$
$e^{-at}\sin(\omega t)$ 	$\dfrac{\omega}{(s+a)^2+\omega^2}$	$\dfrac{z^{-1}b\sin(\omega T)}{1-2z^{-1}b\cos(\omega T)+b^2z^{-2}}$
$e^{-at}\cos(\omega t)$ 	$\dfrac{s+a}{(s+a)^2+\omega^2}$	$\dfrac{1-z^{-1}b\cos(\omega T)}{1-2z^{-1}b\cos(\omega T)+b^2z^{-2}}$
Initial value theorem: $\lim\limits_{t\to 0} f(t)$	$\lim\limits_{s\to\infty} sF(s)$	$\lim\limits_{z\to\infty} F(z)$
Final value theorem: $\lim\limits_{s\to\infty} f(t)$	$\lim\limits_{s\to 0} sF(s)$	$\lim\limits_{z\to 1}\left[\left(1-z^{-1}\right)F(z)\right]$
Translation: $f(t-D)u(t-D)$	$e^{-Ds}F(s)$	$F(z)z^{-n}$ where $D = nT$
Derivative: $\dfrac{\mathrm{d}^n f(t)}{\mathrm{d}t^n} = f^n(t)$	$s^nF(s) - \sum\limits_{k=1}^{n} s^{k-1}f^{n-k}(0)$	
Integral: $\int_0^t f(t)\mathrm{d}t$	$\dfrac{1}{s}F(s)$	
Zero$^{\text{th}}$ order hold	$H(s) = \dfrac{1-e^{-Ts}}{s}$	