Memo

# PROCESS DYNAMICS – CPD320

## SEMESTER TEST 2

Chemical Engineering
Engineering and the Built Environment

Examiner: Carl Sandrock

**October 2009** | 90 minutes

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*Instructions – Read carefully*

- Answer all the questions on the paper in the blocks provided.
- This is a closed book test. All the information you may use is contained in the paper and the attached formula sheet.

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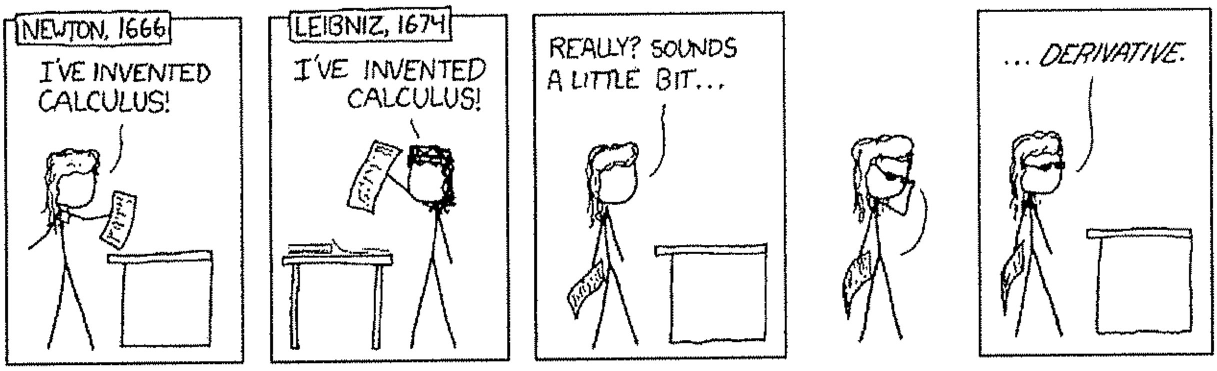



www.xkcd.org

# 1 Stelselresponse

Die volgende vergelykings verteenwoordig 'n chemiese stelsel.

$$\frac{\mathrm{d}}{\mathrm{d}t}[x(t)y(t)] = 2x(t)^2 + 12\frac{y(t)}{x(t)}$$
$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t) + \frac{\mathrm{d}x(t)}{\mathrm{d}t}$$

## 1.1 Nieliniariteit

Identifiseer nie liniêre terme deur om hulle te omkring. 3

## 1.2 Lineariseer

Herskryf die stelsel as 'n gelineariseerde model in terme van afwykingsveranderlikes. 12

$$\bar{x}\frac{dy^P}{dt} + \bar{y}\frac{dx^P}{dt} = \left(4\bar{x} - \frac{12\bar{y}}{\bar{x}}\right)x^P + \frac{12}{\bar{x}}y$$

$$-\frac{dx^P}{dt} + \frac{dy^P}{dt} = y^P$$

$x^P, y^P$

$\bar{x}, \bar{y}$

## 1.3 Matriksvorm

Herskryf die gelineariseerde stelsel in matriksvorm. 5

$$\begin{bmatrix} \bar{y} & \bar{x} \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \frac{dy^P}{dt} \\ \frac{dx^P}{dt} \end{bmatrix} = \begin{bmatrix} 4\bar{x} - \frac{12\bar{y}}{\bar{x}} & \frac{12}{\bar{x}} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x^P \\ y^P \end{bmatrix}$$

form

20

# 2 Laplace Domain

## 2.1 Transform

The following linearised differential equations represent the behaviour of a chemical system in terms of deviation variables.

$$3\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = 2x(t-d) + 10C_A(t) \tag{1}$$
$$4\frac{\mathrm{d}C_A(t)}{\mathrm{d}t} = 4x(t) + 6C_A(t) + 8F(t) \tag{2}$$

Transform to the Laplace domain and determine the transfer function between $F$ and $x$. $\boxed{10}$

$$3s^2X = 2Xe^{-ds} + 10C_A \quad ①$$
$$4sC_A = 4X + 6C_A + 8F \quad ②$$
$$C_A = \frac{4X + 8F}{4s - 6}$$

in ①:

$$3s^2X = 2Xe^{-ds} + \frac{40X}{4s-6} + \frac{80F}{4s-6}$$
$$\frac{X}{F} = \frac{80}{(4s-6)\left(3s^2 - 2e^{-ds} - \frac{40}{4s-6}\right)}$$
$$= \frac{80}{12s^3 - 18s^2 - 4se^{-ds} + 12e^{-ds} - 40}$$

## 2.2 Inverse

The following equations describe a system in the Laplace domain

$$(s+2)y(s) = x(s)$$
$$(s^2+3s+2)x(s) = (s-2)u(s)e^{-2s}$$
$$z(s) = u(s) + y(s)$$

Determine the time-domain function representing the response of $z$ to a unit step in $u$ starting at time $t = 1$. $\boxed{30}$

$$y = \frac{(s-2)\,u(s)\,e^{-2s}}{(s+2)(s^2+3s+2)} = \frac{(s-2)\,u(s)\,e^{-2s}}{(s+2)^2(s+1)}$$

$$z = u(s)\left[1 + \frac{(s-2)e^{-2s}}{(s+2)^2(s+1)}\right] \qquad 5$$

$$u(s) = e^{-s}\frac{1}{s} \qquad 3$$

$$z(s) = \underbrace{e^{-s}}_{1\,\text{sec delay}}\left[\frac{1}{s} + \underbrace{\left\{\frac{s-2}{s(s+2)^2(s+1)}\right\}}_{\text{expand.}}\underbrace{e^{-2s}}_{2\,\text{sec delay}}\right]$$

$g(s) =$

$$\frac{s-2}{s(s+2)^2(s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} + \frac{D}{(s+2)^2} \qquad 5$$

$$A = \lim_{s\to 0} s.g(s) = -\tfrac{1}{2}$$

$$B = \lim_{s\to -1} (s+1)g(s) = 3$$

$$C = \lim_{s\to -2} \frac{d}{ds}\left[(s+2)^2 g(s)\right] = -\tfrac{5}{2} \qquad 5$$

$$D = \lim_{s\to -2} (s+2)^2 g(s) = -2.$$

$$g(t) = -\frac{1}{2}e^{-2t}\left(4t - 6e^{t} + e^{2t} + 5\right) \qquad 5$$

Now

$$z(t) = \begin{cases} 0 & t < 1 \\ 1 & 1 < t \le 3 \\ 1 + g(\underbrace{t-3}_{3}) & 3 < t \end{cases} \qquad 4$$

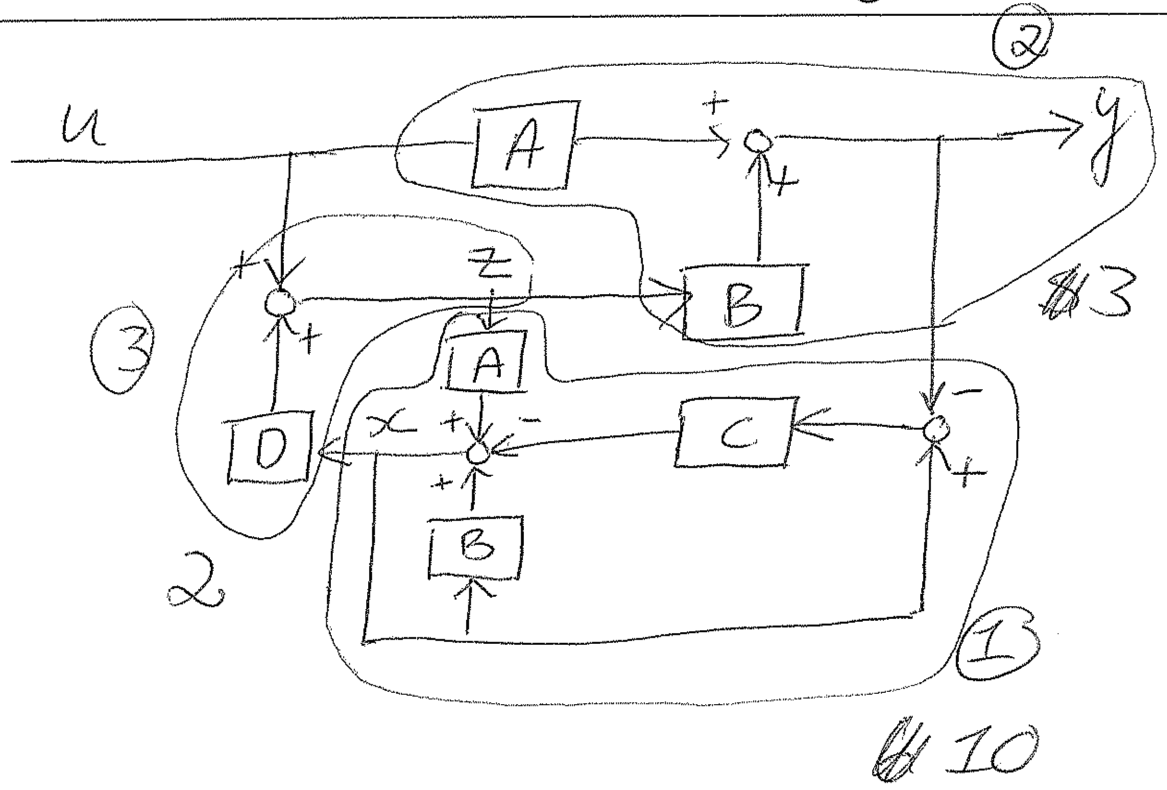
40

# 3 Blokdiagramme

## 3.1 Teken

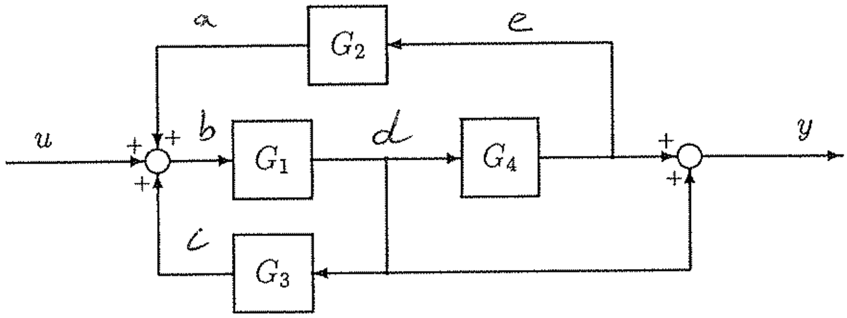
Teken 'n blokdiagram wat die volgende stelsel vergelykings verteenwoordig. Die inset vir die stelsel is $u$ en die uitset is $y$. (15)

$$\begin{aligned} x(s) &= Az(s) + Bx(s) - C(x(s) - y(s)) \quad 1 \\ y(s) &= Bz(s) + Au(s) \quad 2 \\ z(s) &= u(s) + Dx(s) \quad 3 \end{aligned}$$



## 3.2 Vergelykings

Skryf die oordragsfunksie tussen $u_1$ en $y_3$ neer uit die volgende blokdiagram. (15)




$$b = u + G_2 G_4 G_1 b + G_1 G_3 b$$ 4

$$b/u = \frac{1}{1 - G_2 G_4 G_1 - G_1 G_3}$$ 3

$$y = G_1 b (1 + G_4)$$ 3

$$y/b = G_1(1 + G_4)$$

$$y/u = \frac{G_1(1+G_4)}{1 - G_2 G_4 G_1 - G_1 G_3}$$ 5

(30)

Volpunte (90)

$\LaTeX\ 2_\varepsilon$
*www.latex-project.org*