1 Second Order Dynamics

$$a\frac{d^2x'}{dt^2} + b\frac{dx'}{dt} + cx'(t) = u'(t)$$

1.) Laplace: $a\left(s^2X(s) - s\cancel{x'(0)}^{0} - \cancel{x''(0)}^{0}\right) + bsX(s) + cX(s) = U(s)$ $\boxed{3}$

$$X(s)(as^2 + bs + c) = U(s)$$

$$G(s) = \frac{X(s)}{U(s)} = \left(\frac{1}{as^2+bs+c}\right) \qquad \boxed{2}$$

2) Standard form for 2^{nd} order: $G(s) = \dfrac{K}{\tau^2 s^2 + 2\tau\zeta s + 1}$ $\boxed{3}$

$$G(s) = \frac{1/c}{\frac{a}{c}s^2 + \frac{b}{c}s + 1}$$

$\therefore \quad K = 1/c \quad \boxed{2}$

$\tau^2 = a/c \Rightarrow \tau = \sqrt{a/c} \quad \boxed{2}$

$$2\tau\zeta = b/c \Rightarrow \zeta = \frac{b}{2\sqrt{a/c}}$$

$$\text{or} \quad \zeta = \frac{b}{2\tau c} = \frac{b\tau}{2\tau^2 c} = \frac{b\sqrt{a/c}}{2a} \qquad \boxed{3}$$

3) $\Delta u = (6.0 - 2{,}3)\ kg/h$

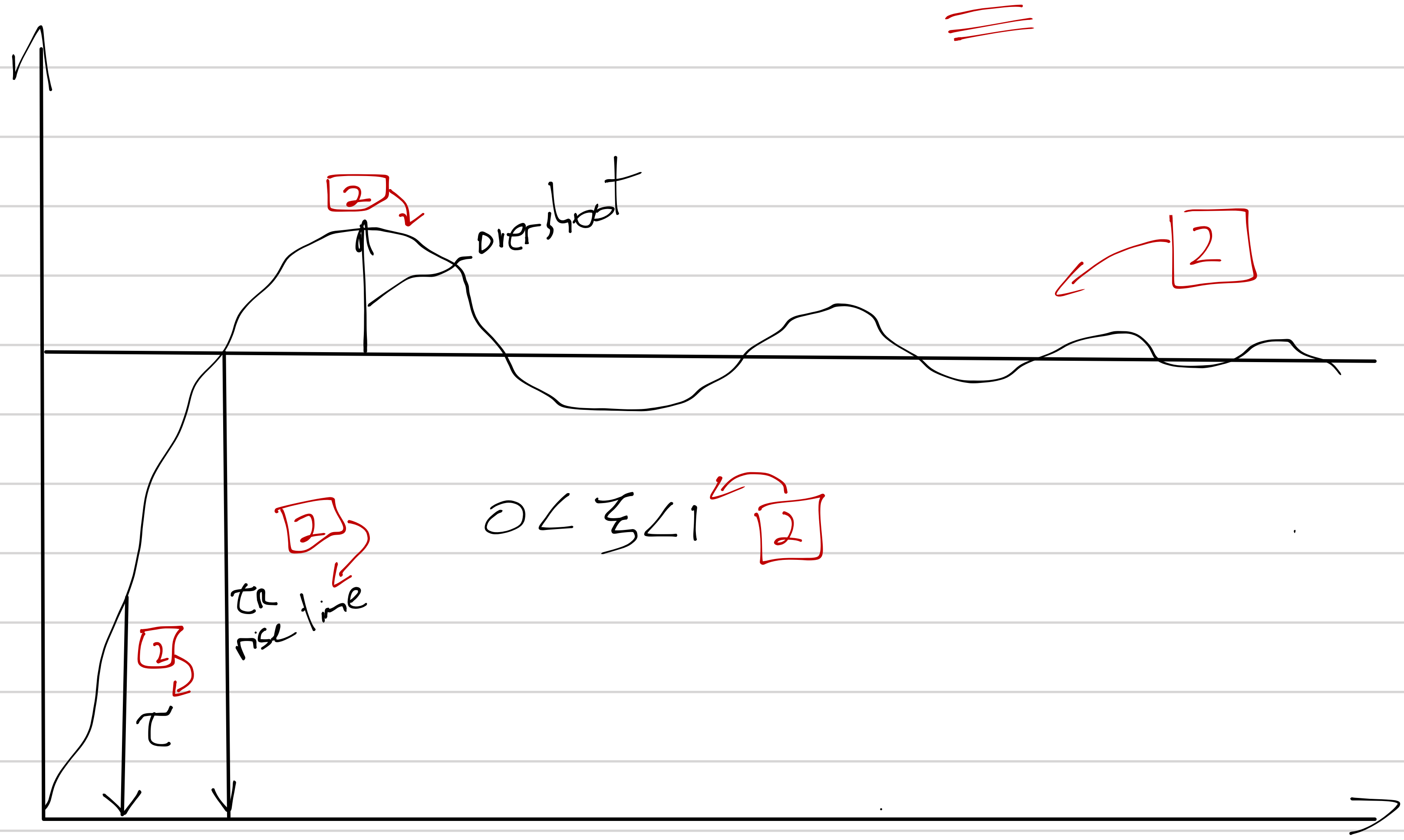
$\Delta x = (15.8 - 12{,}1)\ °C$

$$\text{Gain} = \frac{\Delta x}{\Delta u} \qquad \boxed{2}$$

$$= \frac{3{,}7\ °C}{1{,}7\ kg/h} = 2{,}2\ °C.h/kg \qquad \boxed{1} \qquad \boxed{2}$$

↳ NB!!! Units!

4)



② Complex System Dynamics

1)

$$\frac{(s-1)}{(3s+1)^3(s+2)} \approx \frac{K e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$$

$$\frac{(s-1)}{(3s+1)^3(s+2)} = \frac{-\frac{1}{2}(-s+1)}{(3s+1)(3s+1)(3s+1)(\frac{1}{2}s+1)} \qquad \boxed{2}$$

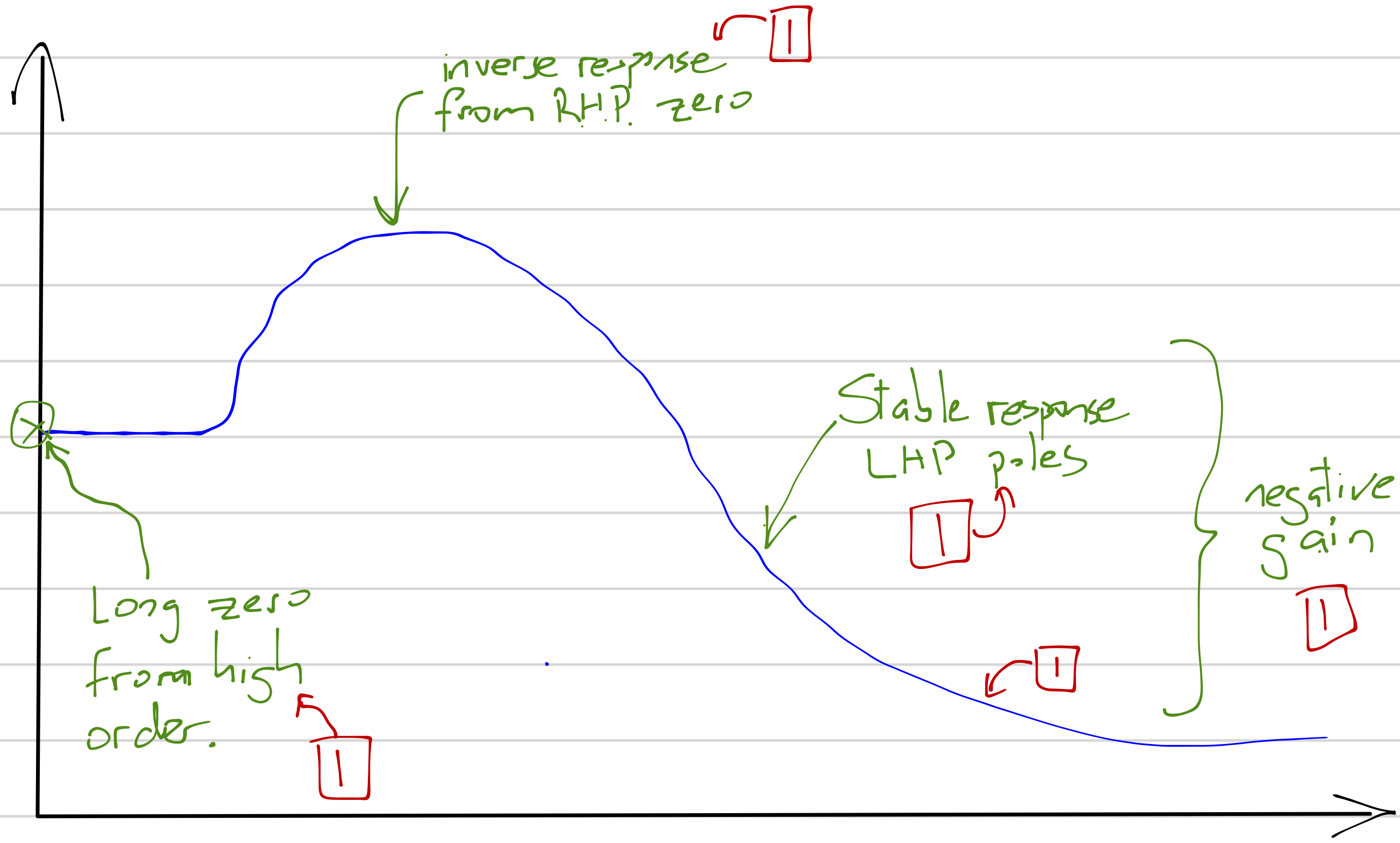
$$\Rightarrow K = -\frac{1}{2} \qquad \boxed{2}$$

neglected terms: $(3s+1)(\frac{1}{2}s+1)$ $\boxed{2}$

	θ	τ_1	τ_2	
	0	3	3	2
Largest neglected τ	3/2 [2]		3/2	2
Smallest neglected τ	1/2 [1]			
RHP zero	1 [2]			
	3	3	9/2	4

$$\frac{(s-1)}{(3s+1)^3(s+2)} \approx \frac{-\frac{1}{2}e^{-3s}}{(3s+1)(\frac{9}{2}s+1)}$$

2.)



3.) Yes. The approximation lacks the RHP [2] zero required for inverse response. This could cause incorrect conclusions to be drawn when predicting [3] outputs because we may for instance assume the value will remain below a certain level.

4) Skogestad's half rule only works for systems with real zeros $\boxed{1}$ and poles. For complex poles $\boxed{2}$ there is oscillatory response which cannot be captured by systems with real poles. $\boxed{2}$

③ Multivariable System Representations

$$\frac{dx}{dt} = Ax + Bu \qquad A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

$$y = Cx + Du$$

$$B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 0 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & \frac{3}{2} & 0 \\ \frac{1}{2} & 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

1)

$$\begin{matrix} B & u \\ [3 \times 2] & [2 \times 1] \end{matrix} \Rightarrow 2 \text{ inputs} \quad \boxed{1}$$

$$\begin{matrix} A & x \\ [3 \times 3] & [3 \times 1] \end{matrix} \Rightarrow 3 \text{ states} \quad \boxed{1}$$

$$\begin{matrix} y & = & C & x \\ [2 \times 1] & = & [2 \times 3] & [3 \times 1] \end{matrix} \Rightarrow 2 \text{ outputs} \quad \boxed{1}$$

motivation $\boxed{2}$

2) (*) $\frac{dy}{dt} = Ax + Bu$

$$sX(s) = AX(s) + BU(s) \qquad \boxed{2}$$
$$(Is - A)X(s) = BU(s)$$
$$X(s) = (Is - A)^{-1} BU(s), \qquad \boxed{2}$$

(*) $y = Cx + Du$

$$Y(s) = CX(s) + DU(s)$$
$$Y(s) = C\left[(Is-A)^{-1} BU(s)\right] + DU(s) \qquad \boxed{2}$$
$$= \left[C(Is-A)^{-1}B + D\right] U(s)$$

$$Y(s) = \begin{bmatrix} \left(\frac{2}{s+1}\right) \boxed{1} & \left(\frac{3}{s}\right) \boxed{1} \\ \left(\frac{1}{s+1}\right) \boxed{1} & \left(\frac{2}{2s+1}\right) \boxed{1} \end{bmatrix} U(s)$$

3) The poles of the system are the eigenvalues of A: $\boxed{2}$

numpy.linalg.eigvals(A): $\boxed{1}$

$(-1;\ 0;\ -0.5)$: $\boxed{1}$

$\left.\begin{array}{l}\text{Pole at 0 means unstable.} \\ \text{Rest are in LHP, therefore stable}\end{array}\right\} \boxed{1}$