Physics 341
Lecture 7

1. Assignment 1 Q6
2. The normal distribution - understanding the statistics of sampling

"Expected"

Expectation value

$$\langle x \rangle = \int_{-\infty}^{\infty} x \, P(x) \, dx$$

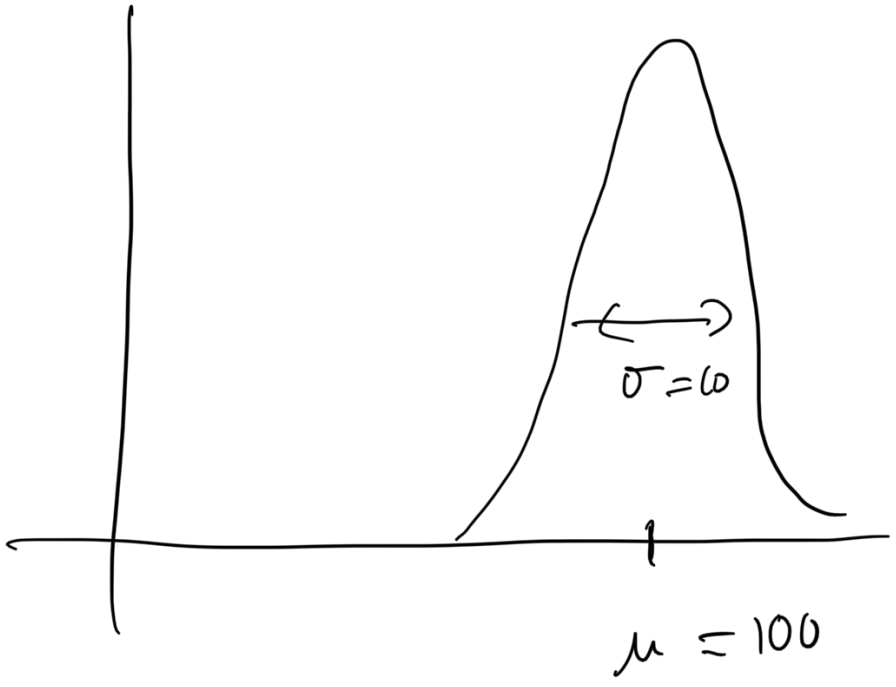
$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 P(x) \, dx$$

$$\langle x^{-} \rangle = \int_{-\infty}^{\infty} \quad a$$

$$\langle \underbrace{\sin(x)}_{?} \rangle = \int_{-\infty}^{\infty} \underbrace{\sin(x)}\ \underbrace{P(x)}\, dx$$

$$\langle \text{"available places"} \rangle = \int_{-\infty}^{\infty} (\text{available places})\, P(x)\, dx$$

$$\text{nspaces}[i] * X.\text{pmf}(i)$$



x
100.2
99.6
110.4
⋮

Data Sample

$x_i \qquad i = 1, \ldots, N$

$$\bar{x} \equiv \frac{1}{N} \sum_{i=1}^{N} x_i$$ ← sample mean

↑

np.mean(x)

standard deviation

$$\sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2} = S$$

+ve

sample
std. dev s

$$\langle x^2 \rangle = \int x^2 P_G(x)\, dx$$

$$= \sigma^2$$

$$\langle |x| \rangle = \int |x| P_G(x)\, dx$$

$$\neq \sigma$$

"Best estimator"

$$S = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2}$$

RMS

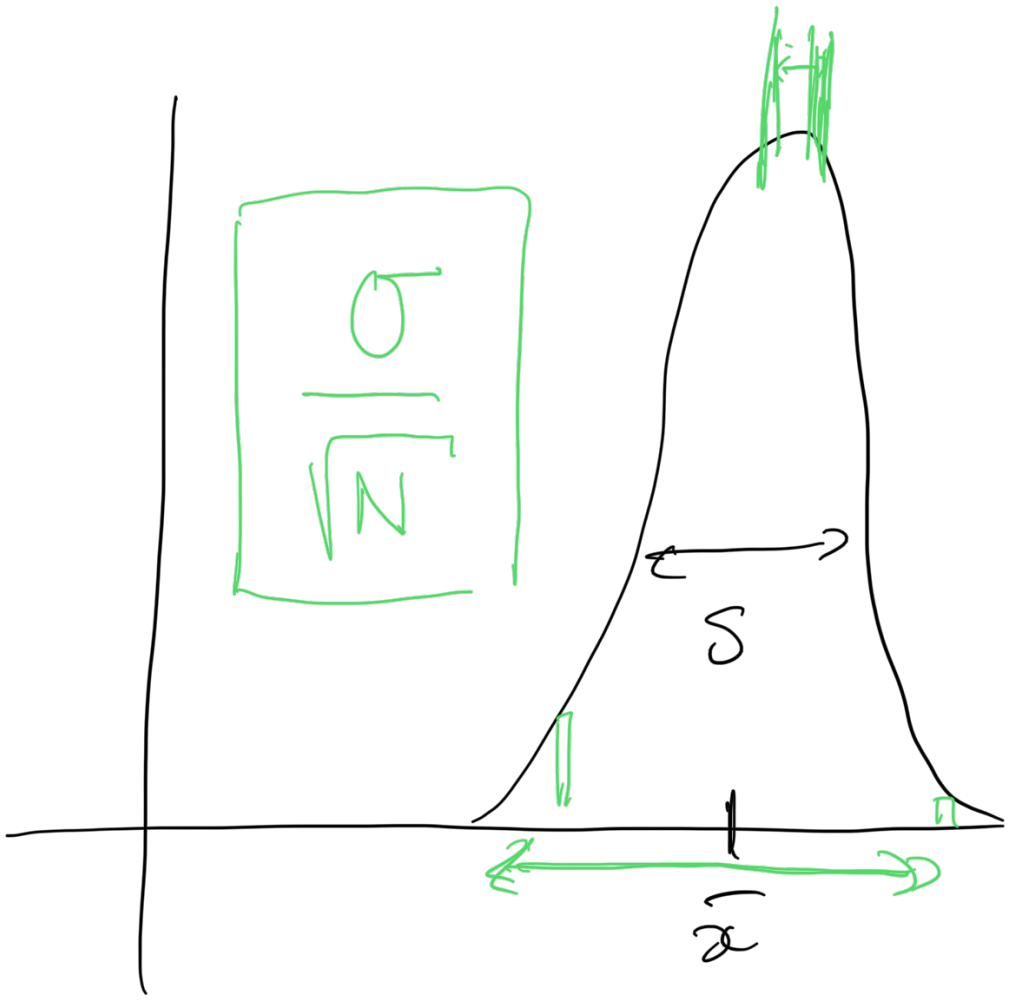
$$S = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2}$$

$s = \sqrt{\frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N-1}}$

best estimator

$np.std(x)$

$np.std(x, ddof=1)$



How well do we know the centroid of this distribution?

3000 → 0.18 $\frac{\sigma}{\sqrt{N}}$

30 → 1.8 ↑

↑ ↑

factor of 100 — factor of 10

Population Mean → μ ← truth / theory

Sample Mean → $\bar{x}$

Population std. dev. → σ ← truth

Sample std. dev. → s

① $\mu = 100.0$

$\bar{x} = \_\_ \pm \_\_$

constant

② unlucky

$\mu = 100.0$

→ $\bar{x} = 46.5 \pm 1.8$

→ Type I-error → α

EXACT

③ $\mu \neq 100.0$ There $\mu = 100$

truth

☺ $\bar{x} = 103.05 \pm 0.18$ $\mu = 103$

④ $\mu \neq 100.00$ $\mu_{truth} = 103.0$

$\bar{x} = 100.05 - 2.10$

no problem.

Type II - Error β