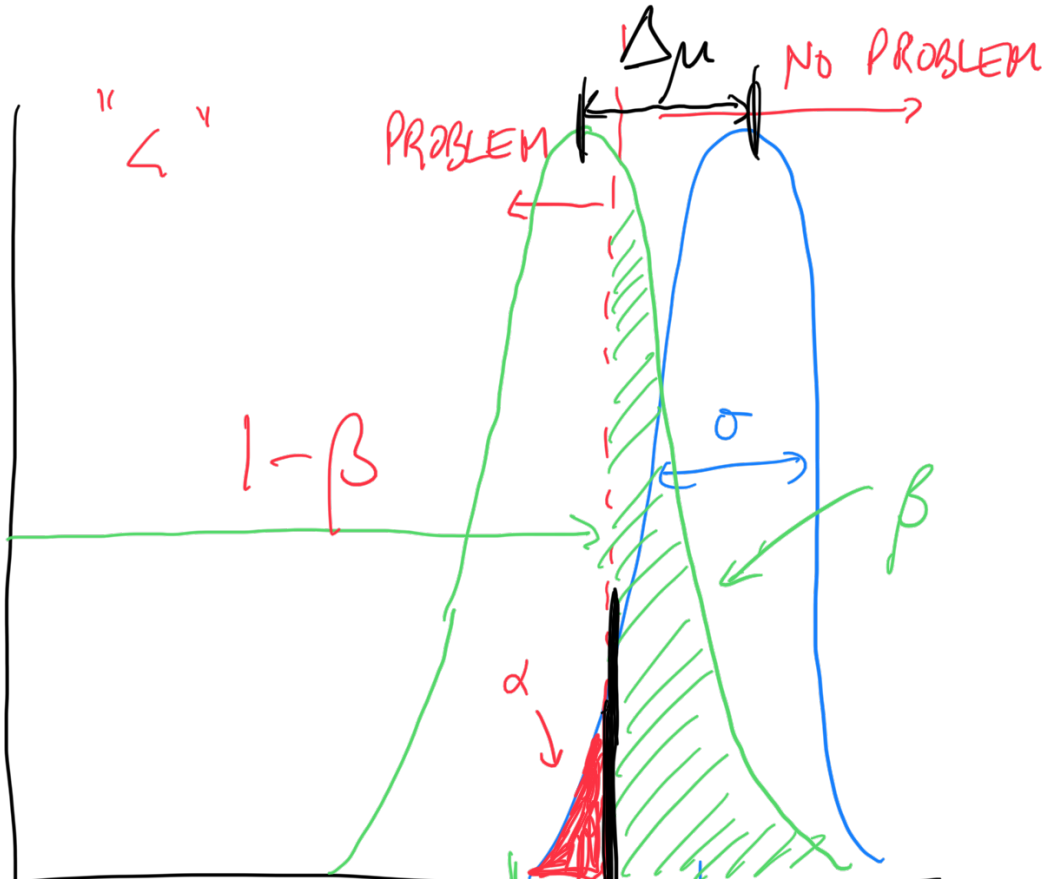
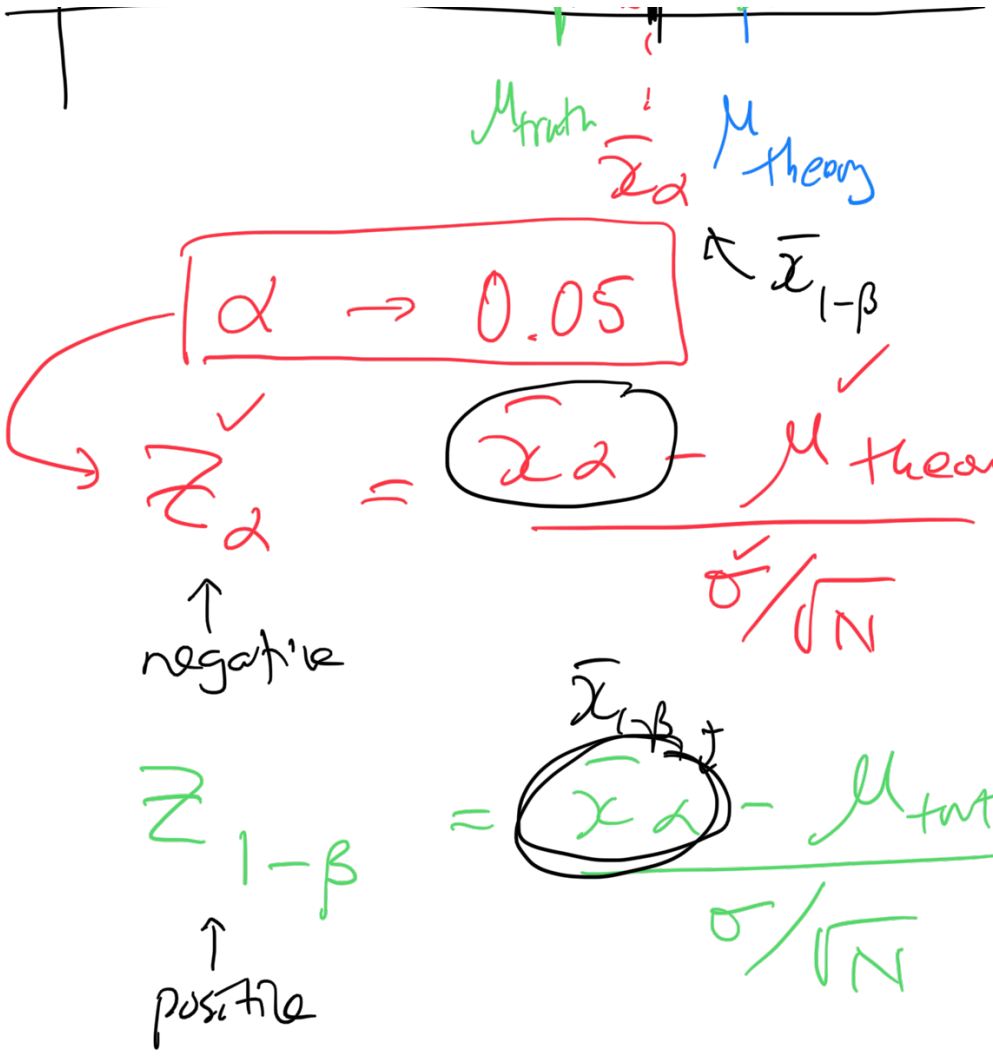
Physics 341 - Lecture 12

- More on β - probability
- Hypothesis testing
- Student's T-test





$$\bar{x}_\alpha = \mu_{theory} + z_\alpha \cdot \frac{\sigma}{\sqrt{N}}$$

$$\bar{x}_\alpha = \mu_{truth} + z_{1-\beta} \cdot \frac{\sigma}{\sqrt{N}}$$

$$\mu_{truth} + z_{1-\beta} \cdot \frac{\sigma}{\sqrt{N}} = \mu_{theory} + z_\alpha \cdot \frac{\sigma}{\sqrt{N}}$$

$$(z_{1-\beta} - z_\alpha)\frac{\sigma}{\sqrt{N}} = \underbrace{\mu_{theory} - \mu_{true}}_{\Delta\mu}$$

$$N = \left(\frac{z_{1-\beta} - z_\alpha}{\Delta\mu}\right)^2 \sigma^2$$

Experiment Design.

① As α gets smaller,
z_α gets more negative.

z_α = stats.norm.ppf(α)

Increase N ✓

② As β gets smaller
$z_{1-\beta}$ gets more positive

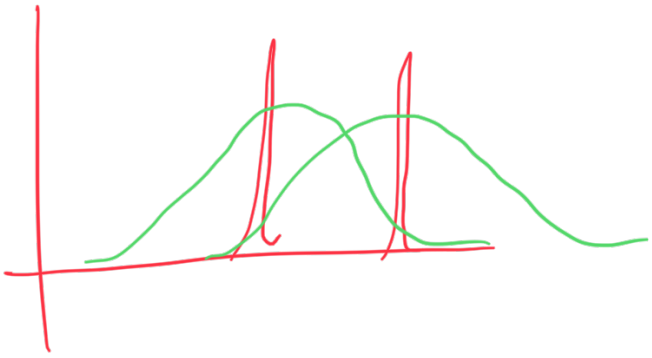
→ Increase N ✓

③ As $\Delta\mu$ gets smaller,

N increases. ✓

④ As σ gets larger

N increases. ✓



Nature is a bitch, then you suffer!

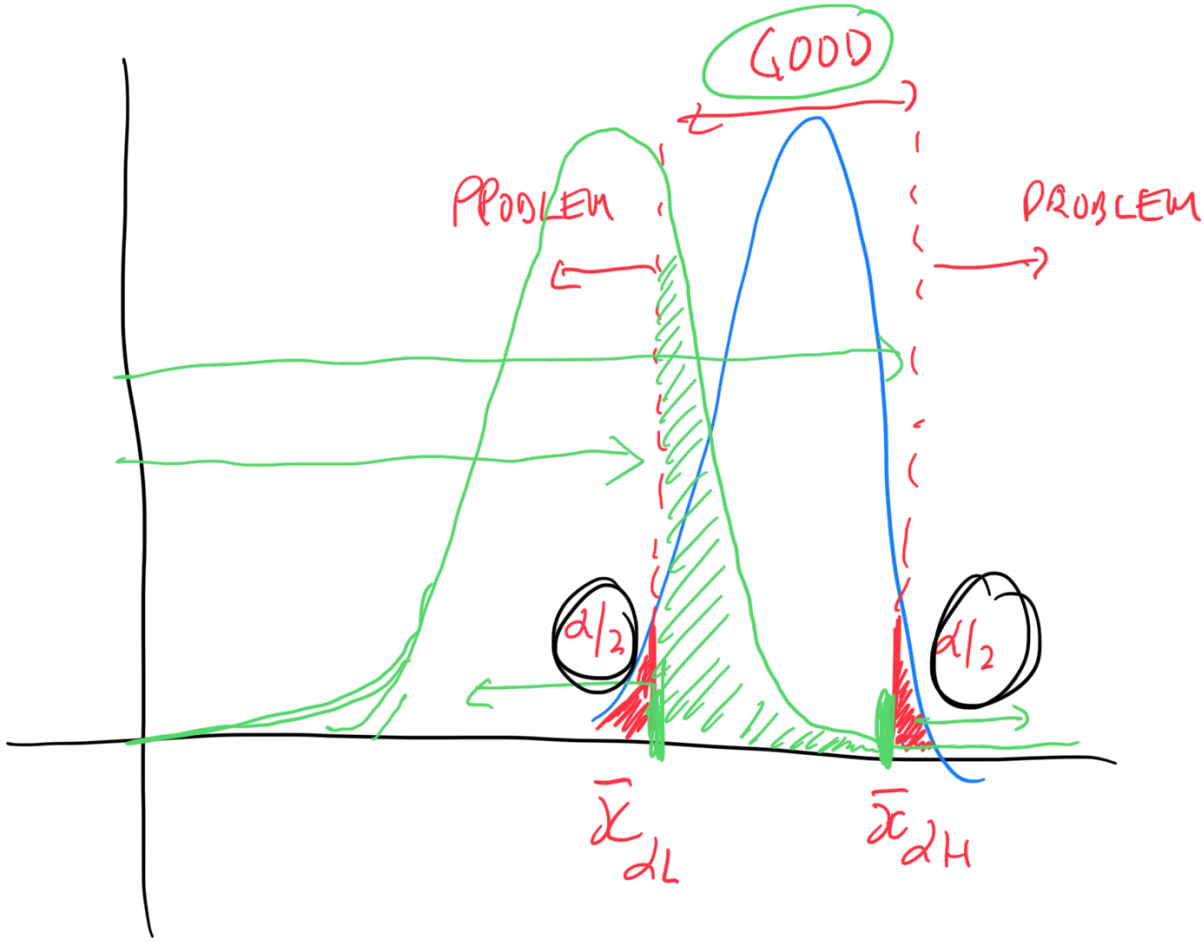
$\mu = 100.0$ $\Delta\mu = 0.3$

$\sigma = 10.0$

if $\mu < 99.7$ ←

$A \rightarrow 87/100$ ← $\Delta\mu = 13$

$11 = 11$



1) $z_{\dots} = \dfrac{\bar{x}_{\alpha L} - \mu_{th}}{\dots}$

(1) $\alpha_L = \dfrac{\bar{x}_{\alpha_L} - \mu_{H_0}}{\sigma/\sqrt{N}}$

(2) $z_{\alpha_H} = \dfrac{\bar{x}_{\alpha_H} - \mu_{H_0}}{\sigma/\sqrt{N}}$

(3) $\boxed{z_{1-\beta_L}} = \dfrac{\bar{x}_{\alpha_L} - \mu_{true}}{\sigma/\sqrt{N}}$

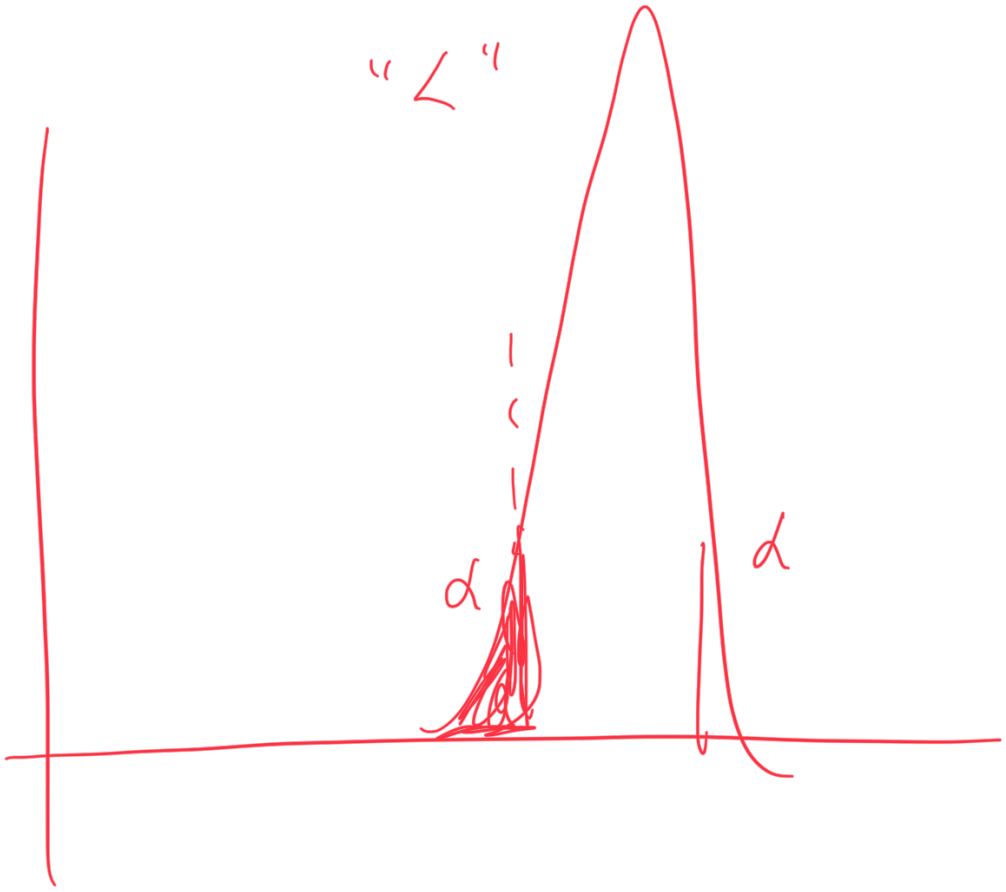
(4) $\boxed{z_{1-\beta_H}} = \dfrac{\bar{x}_{\alpha_H} - \mu_{true}}{\sigma/\sqrt{N}}$ ←

(5) $\beta = cdf(z_{(1-\beta_H)}) - cdf(z_{1-\beta_L})$

→ HARD PROBLEM

$\Delta\mu$ → sensitivity.

$\frac{\Delta\mu}{\sigma}$ → sens. → # of error bar.



$\sqrt{2} \cdot \frac{\Delta\mu}{\sigma}$

| Hypothesis Testing. | ← MATH 125

→ NULL HYPOTHESIS

assume no problem

Is $\mu = 100$? Assume $\mu = 100$

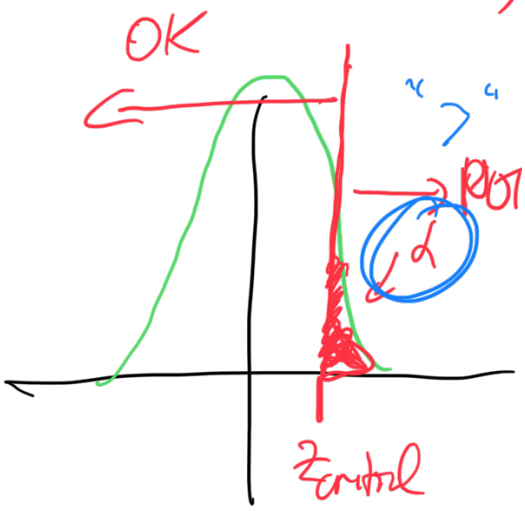
REJECT NULL HYPOTHESIS

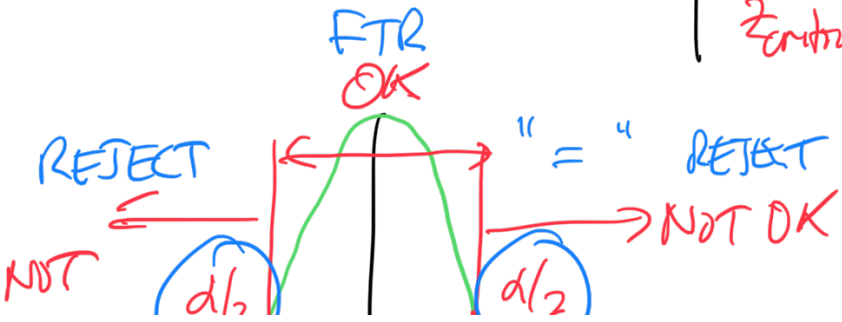
($\bar{x}$ in the "bad" region)

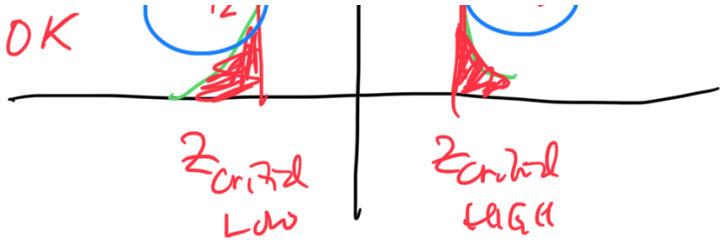
FAIL TO REJECT NULL HYPOTHESIS

($\bar{x}$ in the good region)

$$Z_\alpha = \frac{\bar{x} - \mu_{Hyp}}{\sigma/\sqrt{n}}$$





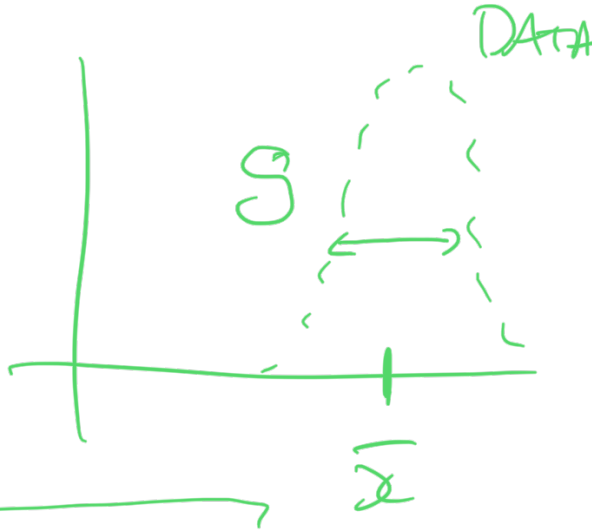


What if we don't know σ?

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{N}}$$

STATISTICS → If we don't know it, measure it.

N data pts.



$$S = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

ESTIMATE of σ

$$t = \frac{\bar{x} - \mu_{theory}}{S/\sqrt{N}}$$

$z \neq t$!!!

Math of small samples.

1. Water ← ?
2. Barley ← ?
3. Yeast ← ?

④ flops ← !