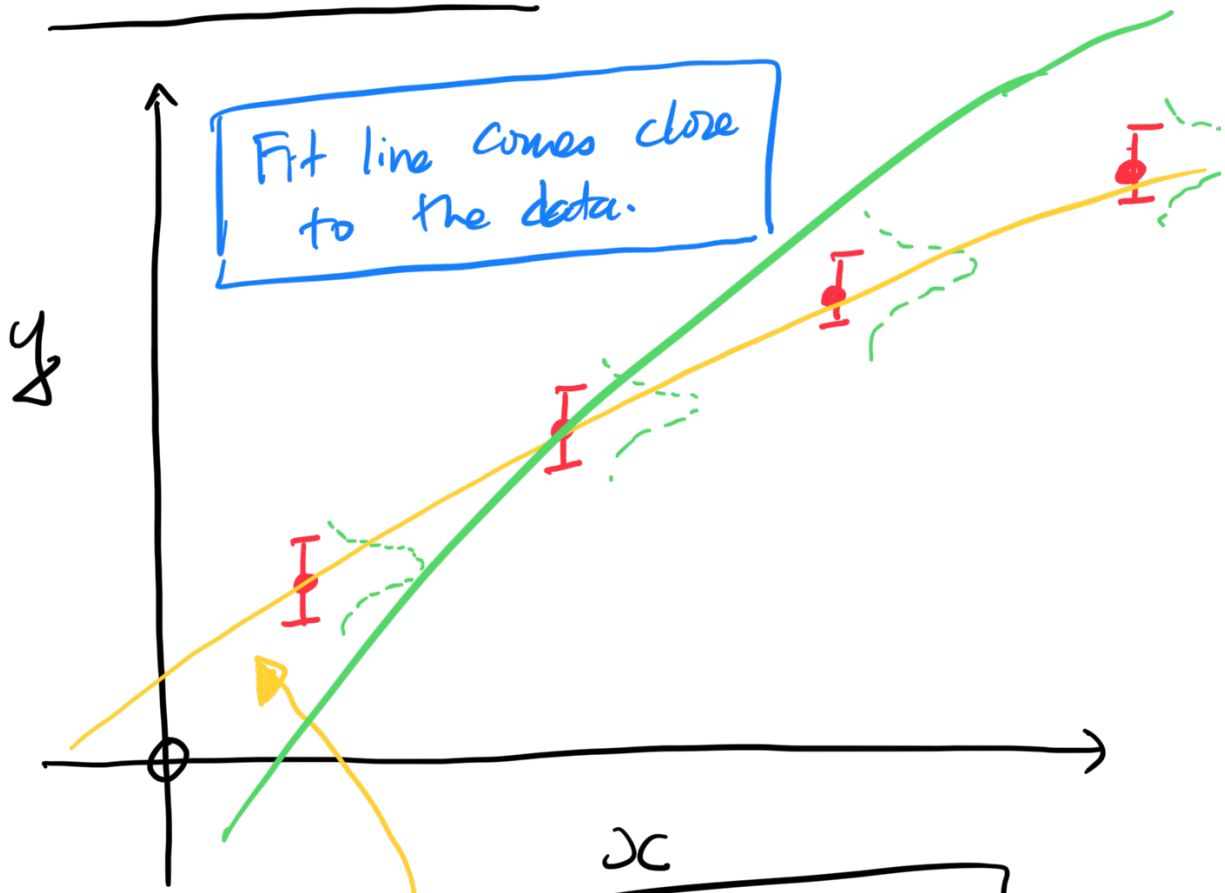
Physics 341 - Lecture 29

Fitting Data



Theory:

$$y = f(x)$$

$$y = ax^2 + bx + c$$

$$(x-c)^2$$

"Best fit"

$y =$ [illegible]

⋮

① Determine the values of a, b, c (fit parameters)

② $\delta a, \delta b, \delta c$

$$a \pm \delta a$$
$$b \pm \delta b$$
$$c \pm \delta c$$

results

No theory :

→ Choose the form of $f(x)$

Data — n data pts.

(x_i, y_i) $\quad i = 1, \ldots, n$

δy_i ← uncertainty.

How close?

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_i - y_{fit}(x_i)\right)^2}$$

ROOT — mean — square. — small as possible

How good is the fit..

$$y_{fit} = ax^2 + bx + c$$

$$y_{fit}(x_i) = ax_i^2 + bx_i + c$$

$$\sum_{i=1}^{n}\left(y_i - y_{fit}(x_i)\right)^2 = \chi^2$$

squared deviations.

minimize → differentiate = 0 → solve.

$$\frac{\partial \chi^2}{\partial a} = 0 \qquad \frac{\partial \chi^2}{\partial b} = 0 \qquad \frac{\partial \chi^2}{\partial c} = 0$$

→ 3 equations, in 3 unknowns!

Linear

(a, b, c)

Ordinary Least Squares.

polynomial fit function.

sm.ols(' ——— ',

Alcohol = a(Tobacco) + b

↓

' Alcohol ~ Tobacco '

11 data pts.

$\nu_{TOTAL} = 10$

$\nu_{\quad} = 2 - 1$

$\nu_{TREATMENT}$ → $\nu_{FIT/MODEL} = 1$

$\nu_{ERROR} = 9$

Alcohol = (4.3512 ± 1.607)

$+ \boxed{(0.3019 \pm .4390)}$ Tobacco

consistent with zero

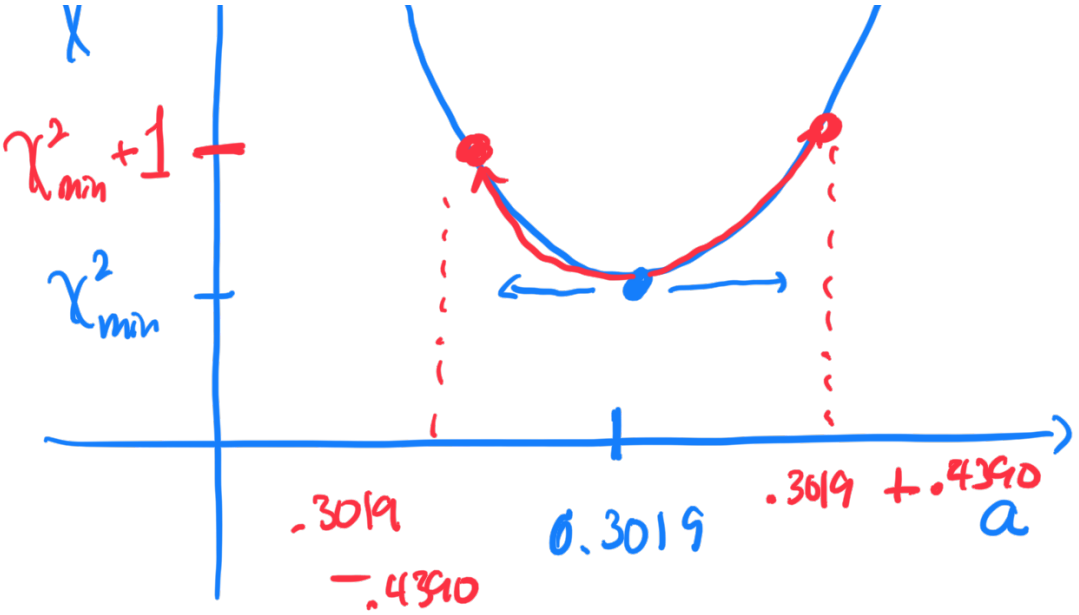
$$\chi^2 = \sum_{i=1}^{n} \left| y_i - y_{fit}(x_i) \right|^2 = \chi^2_{min}$$

$$y_{fit} = \underset{\uparrow}{a}x + \underset{\uparrow}{b}$$

0.3019 4.3512

allow a, b to vary ...

ν_2 ↑ ↓


χ²
χ²min +1
χ²min
.3019
−.4340
0.3019
.3019 +.4340
a