Modeling and Numerical Solution of Thermo-Hydraulic Networks

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October 9, 2020

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1 Configuration of Thermo-Hydraulic Networks

- Network is composed of nodes and elements
- Pressure drop is formulated only along elements
- elements can be pipes, pumps, heat exchanges, valves etc.
- Each *element* has exactly one inlet (left) and one outlet (right) in the schematics
- Nodes connect elements and hold a pressure and temperature state (no flow, no pressure drop)
- Mass flux through an element is conserved, i.e. the same at either side.

1.1 Model Generation Process / Transformation of GIS based Topologies

The transformation process of GIS-based network data to model data is a three-step process:

1. Determine geometry of all active components ("nodes" in GIS terminology), e.g. houses, pumps, storage systems
2. Semi-automatic routing of pipes ("edges" in GIS terminology); Based on some predefined pipe layout the remaining pipe connections are determined using a "shortest path" algorithm. In the result all active component nodes are connected with the network. Afterwards all redundant edges are merged, when they have the same property, yet keeping their total length.
3. Idealization and creation of the Thermo-Hydraulic network. Each of the "nodes" and "edges" from the GIS import process are transformed into "nodes" and "elements" in terms of the thermo-hydraulic network model. Hereby all elements of the network are expressed in terms of elements with exactly two flow connections and a finite (though maybe extremely small) pressure drop across the element. All elements are connected in nodes.

TODO : Hauke, add figs from presentation and add specific example of step 3

1.2 Abstract Governing Equations

1.2.1 Elements

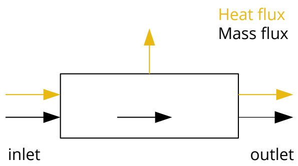

Figure 1: Schematics of an abstract flow element and sign definition of fluxes across model boundary

For each network element the following equations are defined. The system equation for each element (1) defines the relationship between mass flow across the element and the pressures at inlet (in) and outlet (out). The mass flux direction and sign definition is shown in Figure 1. For a typical pipe, the pressure drop would be formulated as $\Delta p = p_{in} - p_{out}$.

$$0 = f\left(\dot{m}, p_{in}, p_{out}\right) \tag{1}$$

$$0 = \dot{Q}_{in} - \dot{Q}_{out} - \sum \dot{Q} \tag{2}$$

Equation (2) is the energy balance across the flow element, which typically involves the energy quantities/enthalpies transported with the mass flow across the inlet and outlet. Optionally, there may be other energy fluxes between the flow element and its environment, for example through heat conduction over its surface, denoted by the $\sum \dot{Q}$ term in the equation.

Note that the actual pressure drop equation may be a function of temperature as well. Since a flow element is connected to two nodes, it is hence also in contact with two node temperatures. With respect to numerical stability of the solution procedure, the temperature at the inflow node should be used when evaluating the media properties in the element's system equation, a method called *upwinding*.

1.2.2 Nodes

For each node the following conservation equations are defined:

$$\sum \dot{m}_i = 0 \tag{3}$$
$$\sum \dot{Q}_i = 0 \tag{4}$$

Also, the pressure p_i at some node i is equal to all inlet/outlet pressures of the connected flow elements (no pressure drop/change across a node). This equation is not explicitely formulation but will be used to eliminate variables in the solution process.

1.2.3 Closing/complementory equations

The system itself needs additional constraints to be solved, usually a reference pressure defined at *exactly one node*, for example the node connected to an inlet of a pump. In the solution process it might then possible to eliminate this pressure from the connected flow element's equations. However, a more pragmatic approach from point of view of implementing such a network solver is to define a pressure "boundary condition" as discussed below.

1.3 Temperature/Pressure Controlled Flow Network Elements

Some flow network elements may be controlled, for example pumps and valves. The control equation may use pressures or temperatures of the flow network itself, thus creating a tightly coupled problem. With respect to engineering requirements, often these direct feedback formulations can be softened by introducing typical time scales of reaction. For example, if a signal for pump activation is received (e.g. temperature trigger point passed), it may be possible to introduce a delay of some minutes until the pump has reached the required speed. The time delay must be selected such that the network itself is solved sufficiently correctly.

Suppose we name a control variable $r\left(p,T\right)$ and denote by the parameters p and T some dependence of flow network pressures/temperatures. Then the system equation for a pump might read:

$$0 = f\left(\dot{m}, p_{in}, p_{out}, r\left(p,T\right)\right)$$

and the resulting network will be fully coupled. If, however, we introduce a time variation on the control variable, we obtain a differential equation:

$$\frac{dr}{dt} = \tau\left(r_{set}\left(p,T\right) - r\right)$$

with τ being some time constant parameter. $r_{set}\left(p,T\right)$ will be the target control signal obtained from the control algorithm equations that use the current network's state. $r\left(t\right)$ will be a transient quantity that more or less slowly approaches this set point value. With respect to the steady-state solution of the flow network, the control variable r is now again a constant, which simplifies solution of the network significantly.

2 Solution procedure

We will derive the general solution procedure for all kinds of thermo-hydraulic networks by starting with a simple example.

2.1 Example 1: Pump with Pipes

The first example consists of three nodes and three flow elements (Fig. 2). This is an adiabatic model without energy balances.

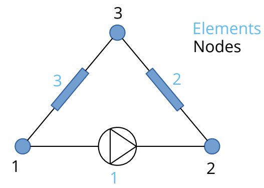

Figure 2: Example 1: Simple flow network with a pump and two pipes

For each of the flow elements we have the system equation. When formulating the equations, we already substitude p_{in} and p_{out} with the pressure at the connected node.

$$0 = f_1 \left(\dot{m}_1, p_1, p_2\right)$$
$$0 = f_2 \left(\dot{m}_2, p_2, p_3\right)$$
$$0 = f_3 \left(\dot{m}_3, p_3, p_1\right)$$

In addition, we have the nodal equations:

$$0 = n_1 \left(m\right)$$

2.2 Generic Solution and Equation System Setup

TODO : Andreas

3 Flow Element Equations

The sections below detail the flow equations for the various elements in the network.

3.1 Pipes

3.1.1 General Hydraulic Pipe Flow Equations

3.1.2 Adiabatic pipes

3.1.3 Pipes with heat exchange with the surroundings

3.1.4 Spatially discretized pipes

... still a general flow element, but now exporting/formulating time state variables to be integrated outside the actual network solution algorithm, pretty much like the control variables.

3.2 Pumps

3.3 Heat Exchangers

3.4 Soil Heat Collectors

3.5 Valves