B*. Examine each of the following propositions, determine whether or not it is true, display your choice by circling either “T” or “F”; for each write at least two sentences that explains why you decided that the proposition is true or why you decided that the proposition is false (Please post the resulting document (as a PDF file) on the indicated *Assignment* link of *Canvas*.) :

i. $\{ \sqrt{3} \} \in \mathbb{R}$

T F

Although $\sqrt{3} \in \mathbb{R}$ because $\sqrt{3} \in \mathbb{R}$. However, $\{ \sqrt{3} \} \notin \mathbb{R}$ because $\{ \sqrt{3} \} \in \{ \text{sets} \}$ and $\mathbb{R}$ includes only numbers, not any sets.

ii. $\sqrt{3} \in \{ \text{sets} \}$

T F

$\sqrt{3} \in \mathbb{R} \wedge$ a number is not a set. Thus, the proposition is false.

iii. $\{ \sqrt{3} \} \in \{ \text{sets} \}$

T F

As argued in the explanation for why the proposition from #i above, any set is an element of the set of all sets. Thus, the proposition is true.

iv. $\sqrt{3} \in (-\infty, \infty)$

T F

Note that $(-\infty, \infty) = \mathbb{R}$ and we already made that argument in our response to Prompt #ii above, that $\sqrt{3} \in \mathbb{R}$. Thus, the proposition is true.

v. $\exists!\, x \in \mathbb{R} \ni x^2 < x$

T F

There does exist many real numbers with squares that are smaller than themselves. For example, $0.4 > 0.16 \ \wedge 0\,.01 > 0.00001$. So the reason that the proposition is false is not because there does not exist such an x but because such an x is not unique.

vi. $[\ 0, 36\ ] \subseteq \{\ n^2 : n \in \mathbb{Z}\ \}$

T F

Keeping in mind that $[\ 0, 36\ ] = \{\ x : 0 \le x \le 36\ \}$, $\pi \in [\ 0, 36\ ]$ but $\pi \notin \mathbb{Z}$ and every square of an integer is an integer. The proposition $\{\ 0, 36\ \} \subseteq \{\ n^2 : n \in \mathbb{Z}\ \}$ is true, but $\{\ 0, 36\ \} \neq [\ 0, 36\ ]$

vii. $\{\ \mathbb{Q}\ \} \subseteq \mathbb{Q}$

T F

$\{\ \mathbb{Q}\ \}$ is not a rational number; there are infinitely many rational numbers but there is only one set of all rational numbers. If we began listing rational numbers we include numbers (e.g., 0, $^-0.8$, and $.\overline{382}$) but we wouldn't include any sets in our listing of rational numbers.

viii. $\mathbb{Q} \subseteq \mathbb{Q}$

T F

Our definition of subset is as follows: Given $A, B \in \{\text{ sets }\}$, $(A \subseteq B \Leftrightarrow (\ x \in A \Rightarrow x \in B\ )$. Keep in mind that $x \in \mathbb{Q} \Rightarrow x \in \mathbb{Q}$. Thus, the proposion is true.

ix. $\varnothing \subseteq \mathbb{Q}$

T F

$\varnothing \subseteq B \ \forall \ B \in \{ \text{ sets } \}$. Two prove that, we need to call to mind two lines from our Glossary:

010A. Definition for *subset*:

Given $A, B \in \{ \text{ sets } \}$, $(A \subseteq B \Leftrightarrow (x \in A \Rightarrow x \in B))$

002H. The truth table for statements p and q:

p	q	$\overline{p}$	$p \vee q$	$p \wedge q$	$p \veebar q$	$p \Rightarrow q$	$q \Rightarrow p$	$\overline{q} \Rightarrow \overline{p}$	$p \Leftrightarrow q$
T	T	F	T	T	F	T	T	T	T
T	F	F	T	F	T	F	T	F	F
F	T	T	T	F	T	T	F	T	F
F	F	T	F	F	F	T	T	T	T

Now let A from our definition of subset be $\varnothing$. This gives us $(\varnothing \subseteq B \Leftrightarrow (x \in \varnothing \Rightarrow x \in B))$. But there are no elements in $\varnothing$, so $x \in \varnothing$ is false. Now visit the above truth table and focus on the entries that are colored red. Anytime the hypothesis of a conditional proposition is false, the conditional proposition is vacuously true. The truth value of the conclusion does not influence the truth value of the conditional proposition itself. Therefore, $\varnothing \subseteq \mathbb{Q}$ is true.

x. $\{ n^2 : n \in \mathbb{Z} \} \subset \omega$

T F

Note that $\{ n^2 : n \in \mathbb{Z} \} = \{ 0, 1, 4, 9, 16, ... \} \ \wedge \ \omega = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ... 16, ... \}$. Since each square of an integer is an integer, we have $\{ n^2 : n \in \mathbb{Z} \} \subseteq \omega$. And because ω includes integers that are not perfect squares of integers (e.g., 2), $\{ n^2 : n \in \mathbb{Z} \} \neq \omega$. Therefore, $\{ n^2 : n \in \mathbb{Z} \} \subset \omega$