Review for Exam 2 KEY

In my other class we check "conditions" instead of "assumptions". So when you see the word "condition" you know it means "assumption".

I like saying "Condition" better because it reminds me that I shouldn't do the test if the conditions aren't met.

0.1 Gathering and Describing Data Review

1. Label each variable as numerical or categorical.

   If it is numerical, then break it down farther into continuous or discrete.

   (a) Your favorite brand of bread.
   Categorical

   (b) The percentage of whole wheat in your favorite bread.
   numerical ⟶ continuous
   (I would say you can have 99% or 99.3% or 99.37%.)

   (c) The brand of your cell phone.
   Categorical

   (d) The battery life of your cell phone.
   numerical ⟶ continuous
   (It doesn't say how you would measure it. If you want to measure in days, then you could have 0 days, 1 day, 2 days, etc. That would be discrete.
   But time can always be continuous because theoretically you can have 1 day and 3 hours and 2 minutes and 5 seconds and 3.12 milliseconds.)

   (e) Whether or not a person has a credit card.
   Categorical

   (f) The number of books you own.
   numerical ⟶ discrete
   (It has to be discrete because you have 0 or 1 or 2 or 3 or 4, etc. You don't have fractions of books.)

2. If we have a right skewed distribution, you would expect

   (a) the mean to be greater than the median.

   (b) the mean to be less than the median.

   (c) the mean to be equal to the median.
   mean greater than median– the right skew distribution has extreme high values which 'pull' the mean to the right.

3. The mean is a resistant measure of central tendency.

   (a) True

   (b) False
   False, the mean is not resistant.

4. The median is a resistant measure to outliers and extreme values.

   (a) True

   (b) False
   True, the median is much more resistant against outliers and extreme values than the mean.

5. The following distribution is

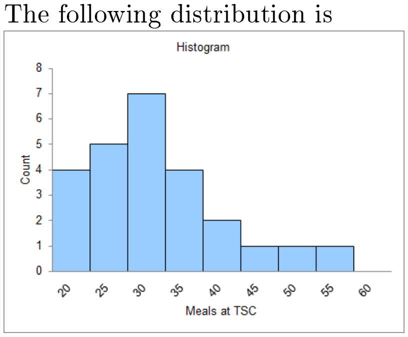
(a) left skew

(b) roughly symmetric

(c) right skew

It is left skew because the bulk of the distribution is on the right side, but there are a few values on the left. (It has a long tail on the left side.)

6. The following distribution is



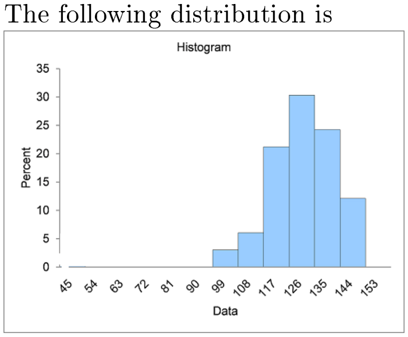
(a) left skew

(b) roughly symmetric

(c) right skew

It is right skew. (It has a long right tail.)

7. The following distribution is



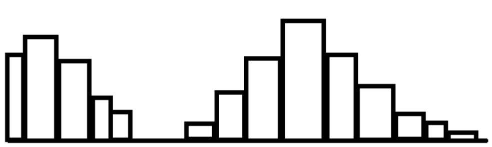
(a) unimodal

(b) bimodal

(c) three modes

It only has one major peak, so it is unimodal.

8. The following distribution is

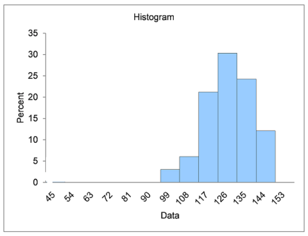


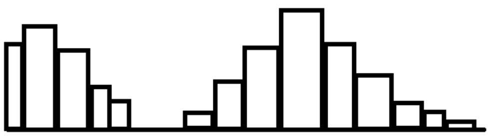
(a) unimodal

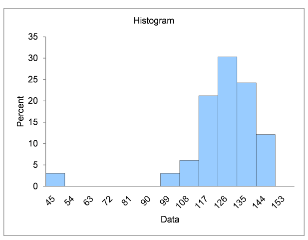
(b) bimodal

(c) three modes
It has two major peaks, so bimodal.

9. Which of the distributions has a potential outlier?

(a) 

(b) 

(c) 

(C). The last graph has a potential outlier on the left side of the graph around 45.

10. Label each variable as numerical or categorical.

If it is numerical, then break it down farther into continuous or discrete.

(a) The age of your puppy.
numerical$\longrightarrow$ continuous
(But time can always be continuous because theoretically you can have 1 day and 3 hours and 2 minutes and 5 seconds and 3.12 milliseconds.)

(b) The publisher of your textbooks.
Categorical

(c) The weight of your backpack.
numerical$\longrightarrow$ continuous
This is continuous because you can have 16 pounds or 16.1 pounds or 16.2321 pounds. Even though we often round to pounds, the true weight is continuous.

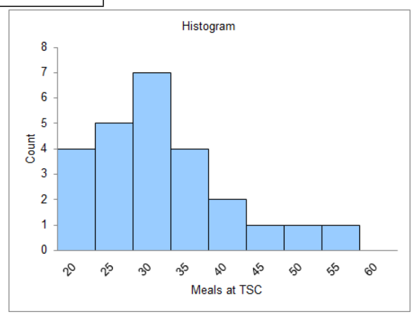
(d) Whether or not you will graduate this year.
Categorical

(e) The type of credit card a person owns.
Categorical

(f) The credit limit on a person's credit card. Assume credit card companies only do \$100 increments.
numerical$\longrightarrow$discrete
(Because it has to be \$100, \$200, ..., \$3000, ...\$6200, etc and it can't be values in between then it is discrete.)

11. Here is a data set for the number of times 25 students ate at the TSC during the year.
23, 23, 23, 24, 26, 26, 26, 28, 29, 30, 31, 32, 32, 32, 34, 34, 36, 36, 37, 38, 40, 44, 46, 50, 56
Create a histogram of the data. Don't forget to label your axes.

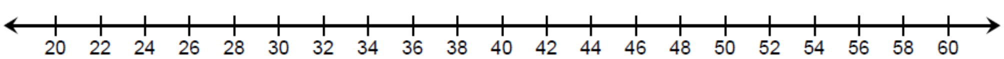
class		count
lower boundary	upper boundary	
20	24	
25	29	
30	34	
35	39	
40	44	
45	49	
50	54	
55	59	

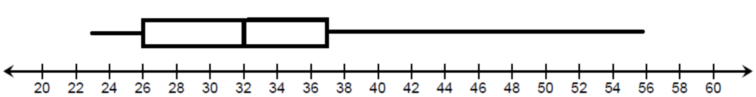


lower	**upper**	**count**
20	**24**	**4**
25	**29**	**5**
30	**34**	**7**
35	**39**	**4**
40	**44**	**2**
45	**49**	**1**
50	**54**	**1**
55	**59**	**1**

12. Use the summary information to create a boxplot for the meal data set.

Minimum	23
1st Quartile	26
Median	32
3rd Quartile	37
Maximum	56





13. The results of the test scores for a class are 81, 77, 99, 80, 95, 70, 43, 76, 95.

(a) What is the mean?

$$mean = \frac{81 + 77 + 99 + 80 + 95 + 70 + 43 + 76 + 95}{9} = \frac{716}{9} = 79.5555$$

(b) What is the median?
You first need to put the data in order.
43 70 76 77 80 81 95 95 99
The median is the 'middle' number, 80.

14. One student didn't take the test, the data set is now 0, 43, 70, 76, 77, 80, 81, 95, 95, 99.

(a) Find the mean.

$$mean = \frac{0 + 81 + 77 + 99 + 80 + 95 + 70 + 43 + 76 + 95}{10} = \frac{716}{10} = 71.6$$

(b) Find the median.
The data is already in order.
0 43 70 76 77 80 81 95 95 99
The median is the 'middle' number, but there are two middle numbers 77 and 80. We need to find the average of the two middle numbers.
The median is $\frac{77 + 80}{2} = 78.5$**.**

(c) Compare your results to the results in problem 13. What changed more by adding the zero, the mean or the median?
The mean changed more. This is why we say the mean is not a resistant measure of central tendency.

0.2 Normal Distribution

15. ACT scores are reported on a scale from 1 to 36. The distribution of ACT scores for more than 1 million students in a recent high school graduating class was roughly Normal with mean $\mu = 20.8$ and standard deviation $\sigma = 4.8$.

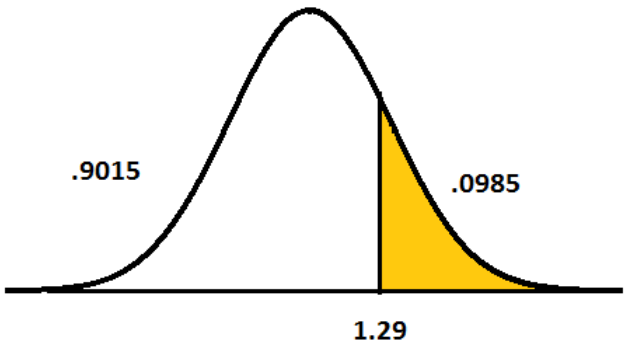
    (a) Allen scores 27 on the ACT. What is the percentage of ACT scores that are higher than his score?

    Allen is just one person, so we are looking for a probability for an individual value with a normal distribution.
    We are looking for $P(X > 27)$.

    We need to standardize it.

    $$z = \frac{X - \mu}{\sigma} = \frac{27 - 20.8}{4.8} = 1.29$$

    So we are using the standard normal table and looking for the area to the right of $z = 1.29$.



We have to use the complement rule.

$$P(X > 27) = P(Z > 1.29) = .0985$$

****Here's a video for how to use the table for parts (a) and (b).**
https://youtu.be/rhFRdD3gIlY

So 9.82% of ACT scores are higher than Allen's score.

****

A student asked about the fact that if x can only be {1, 2, 3, ..., 36} wouldn't it be discrete. So if someone is higher than Allen they would have to be at least $X = 28$. So shouldn't we plug in 28 instead of 27?

Technically if x can only be {1, 2, 3, ..., 36}, then it is discrete and 28 is the next higher score. And if we were using a discrete distribution like a binomial distribution we would look for $P(X \geq 28)$. But since they said the distribution is approximately normal we are working with a continuous distribution so we just didn't worry about it because we only change to 28 for a discrete distribution. It is common practice to look at the test scores as a continuous normal distribution.

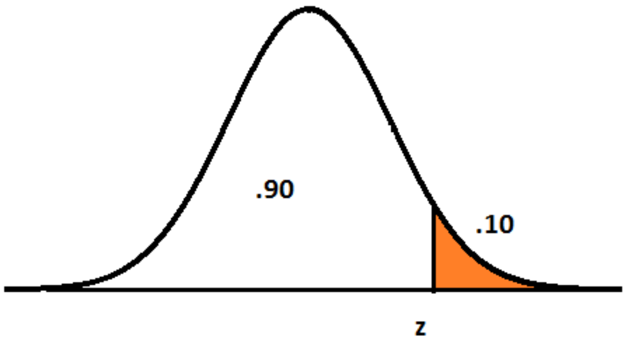
The best option would be if we only have the choices from 1 to 36, we should probably use a continuity correction. We didn't learn about it in this class, but it is made for situations when you use the normal distribution to approximate a discrete distribution like in this problem.

http://www.statisticshowto.com/what-is-the-continuity-correction-factor/ (Links to an external site.)

But on problem 30, scores go from 400 to 1600. So there are so many options that it wouldn't make a big difference if you use the continuity correction because there isn't much probability given to each single point because there are so many possible x values it may as well be continuous.

(b) John wants to be in the top 10%. What score does he need?
Here we know the probability and we need to find the SAT score.

First find the z score.
If $.10$ is the area to the right, then $1 - .10 = .90$ is the area to the left.



So $z \approx 1.28$. Then we solve the standardization equation for the ACT score.

$$z = \frac{ACT - 20.8}{4.8}$$

$$1.28 = \frac{ACT - 20.8}{4.8}$$

$$ACT = 1.28\,(4.8) + 20.8 = 26.944$$

We get

$$x = 26.95$$

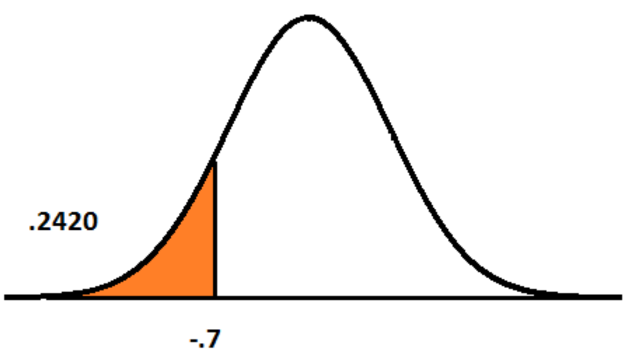
So he needs at to get at least 27 points.

16. SAT scores are reported on a scale from 400 to 1600. The distribution of SAT scores for 1.4 million students in the same graduating class was roughly Normal with mean $\mu = 1026$ and standard deviation $\sigma = 209$.

(a) Reports on a student's SAT usually give the percentile as well as the actual score. The percentile is just the cumulative proportion stated as a percent: the percent of all scores that were lower than this one. Jessica scores 880 on the SAT. What is her percentile?
Jessica is just one person, so we are looking for a probability for an individual value with a normal distribution.
We are looking for $P\,(X < 880)$.
We need to standardize it.

$$z = \frac{X - \mu}{\sigma} = \frac{880 - 1026}{209} = -.70$$

So we are using the standard normal table and looking for the area to the left of $z = -.7$.



$$P\,(X < 880) = P\,(Z < -.7) = .2420$$

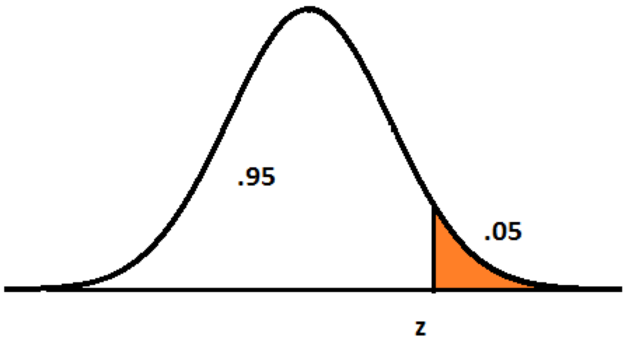
$$P\,(X < 880) = .2424$$

So she is in the $\approx 24^{th}$ percentile.

(b) Melissa is hoping to qualify for a scholarship. Those are only awarded to applicants who have a SAT score among the top 5% of all SAT scores. What is the lowest score that she can get that will be good enough to qualify for a scholarship?

Here we know the probability and we need to find the SAT score.

First find the z score.
If $.05$ is the area to the right, then $1 - .05 = .95$ is the area to the left.



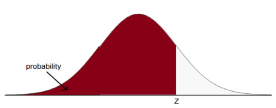
So $z \approx 1.645$. Then we solve the standardization equation for the SAT score.

$$z = \frac{SAT - 1026}{209}$$

$$1.645 = \frac{SAT - 1026}{209}$$

$$SAT = 1369.8$$

Standard Normal Table: Areas to the Left



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767

area is .95
z = 1.645
or 1.64
or 1.65

So she needs to get at least 1370.

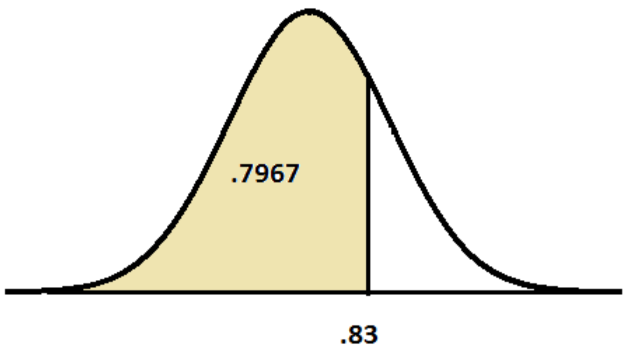
(c) We decide to randomly select one student. What is the probability that his score is less than 1200?

We are talking about an individual. So we will use the formula $z = \dfrac{x - \mu}{\sigma}$.
We are looking for $P(X < 1200)$.

We need to standardize it.

$$z = \frac{x - \mu}{\sigma} = \frac{1200 - 1026}{209} = .83$$

So we are using the standard normal table and looking for the area to the left of $z = .83$.



$$P\left(X < 1200\right) = P\left(Z < .83\right) = .7967$$

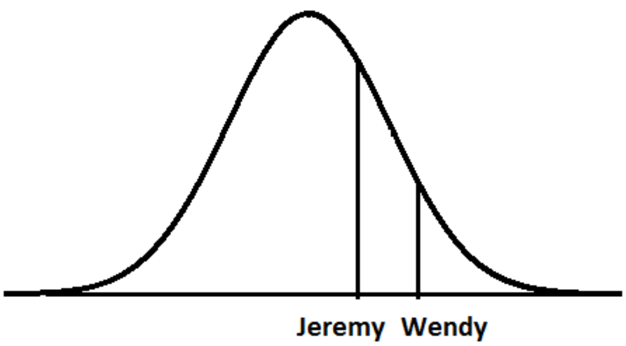
So 79.74% of the individual scores are less than 1200.

17. Compare a SAT with an ACT score: Wendy scores 1350 on the SAT. Jeremy scores 25 on the ACT. Assuming that both tests measure the same thing, who has the higher score – Wendy or Jeremy? Report the z-scores for both students.
Since their scores have different scales, to compare them, we should standardize both of their scores.

$$z_{Wendy} = \frac{1350 - 1026}{209} = 1.55$$

$$z_{jeremy} = \frac{25 - 20.8}{4.8} = .875$$

Since Wendy has the higher z score, she has the overall higher score.



18. Use the 68-95-99.7 rule (empirical rule) to describe each data set.

   (a) IQ test scores are supposed to be normally distributed with a mean of 100 and a standard deviation of 15.

   - **68% of the people should have IQ scores between 85 and 115.**
   - **95% of the people should have IQ scores between 70 and 130.**
   - **99.7% of people should have IQ scores between 55 and 145.**

   (b) Suppose teenage boys in Utah are normally distributed with a mean of 66.5 inches and a standard deviation of 3.5 inches.

   - **68% of the boys should be between 63 and 70 inches.**
   - **95% of the boys should be between 59.5 and 73.5 inches.**
   - **99.7% of the boys should be between 56 and 77 inches.**

   (c) Egg lengths are supposed to be normally distributed with a mean of 6 cm and a standard deviation of 1.4 cm.

   - **68% of the eggs should be between 4.6 and 7.4 cm.**
   - **95% of the eggs should be between 3.2 and 8.8 cm.**
   - **99.7% of the eggs should be between 1.8 and 10.2 cm.**

19. The temperature at any random location in a kiln used for manufacturing bricks is normally distributed with a mean of 1000ºF and a standard deviation of 50ºF.

   (a) What is the z-score that relates to 1075ºF?
   Normal, $\mu = 1000$, $\sigma = 50$

$$\begin{aligned} z &= \frac{x-\mu}{\sigma} \\ &= \frac{1075-1000}{50} \\ &= 1.5 \end{aligned}$$

   z-score is 1.5

   (b) A z-score of -2.75 relates to a temperature of ______ºF?
   Now we know the z score and we solve the equation for the x value.

$$z = \frac{x-\mu}{\sigma}$$

$$-2.75 = \frac{x-1000}{50}$$

$$x = -2.75\,(50) + 1000$$

$$x = 862.5$$

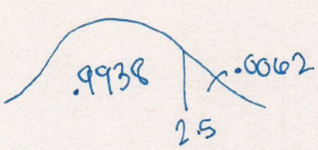
   The temperature is $862.5°$**F.**

   (c) If bricks are fired at a temperature above 1125ºF, they will crack and must be discarded. If the bricks are placed randomly throughout the kiln, what is the percentage of bricks that crack during the firing process?

$z = \frac{1125-1000}{50} = 2.5$

$P(X > 1125) = P(Z > 2.5) = 1 - P(Z \le 2.5)$

$= 1 - .9938 = .0062$

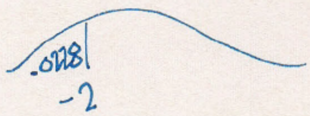


So .62% of bricks crack

   So about 0.62% of the bricks will crack.

(d) When glazed bricks are put in the oven, if the temperature is below 900ºF, they will discolor. If the bricks are placed randomly throughout the kiln, what percentage of glazed bricks will discolor?

$z = \frac{900-1000}{50} = -2$ $\qquad P(X<900) = P(Z<-2) = .0228$



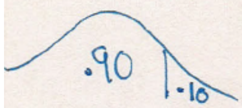
So $\underline{2.28\%}$ discolor.

So

$$P(X < 900) = .0227$$

So about 2.27% of bricks will discolor.

(e) When completely filled, bricks at the top 10% hottest locations will be exposed to temperatures of ________ºF (and higher).



need area to left to be .9

so $z \approx 1.28$

$1.28 = \frac{X-1000}{50}$ $\qquad$ so $X = \underline{1064}$

So $X = 1064.078$**.**
So the top 10% hottest locations are hotter than 1064 degrees.

20. All the books published by a certain publisher have a mean of 256 pages and a standard deviation of 23 pages. A random sample of books by the publisher have a mean of 243 pages and a standard deviation of 19 pages.

    (a) Summarize the given information with appropriate symbols.
    All the books are the population, so the population mean is $\mu = 256$ and the population standard deviation is $\sigma = 23$.
    The sample mean is $\overline{x} = 243$ and the sample standard deviation is $s = 19$.

    (b) Label each number as a parameter or statistic.
    Parameters are for populations and statistics are for samples.
    So $\mu = 256$ and $\sigma = 23$ are parameters.
    $\overline{x} = 243$ and $s = 19$ are statistics.

21. In a controversial election district, **73%** of registered voters are Democrats. A random survey of 500 voters had **68%** Democrats. Are the bold numbers parameters or statistics?

    (a) Both are statistics.
    (b) 73% is a parameter and 68% is a statistic.
    (c) 73% is a statistic and 68% is a parameter.
    (d) Both are parameters.
    (b) Parameters are for populations and statistics are for samples. So 73% is a parameter and 68% is a statistic.

22. SAT scores are reported on a scale from 400 to 1600. The distribution of SAT scores for 1.4 million students in the same graduating class was roughly Normal with mean $\mu = 1026$ and standard deviation $\sigma = 209$. Suppose we take a random sample of 13 seniors.

    (a) What is the mean of all the possible sample means?

    $$\mu_{\overline{x}} = \mu = 1026$$

    (b) What is the standard deviation of the sampling distribution of the sample means?

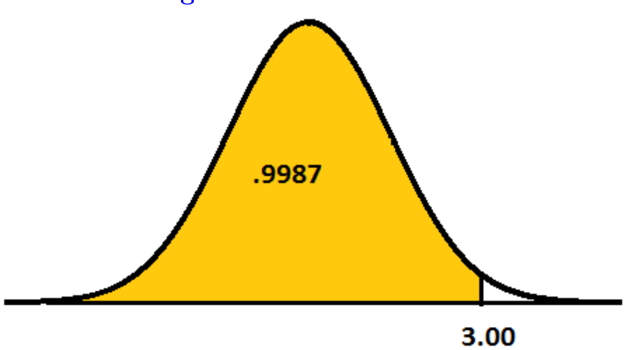
    $$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{209}{\sqrt{13}} = 57.97$$

    (c) What is the shape of the distribution of all the possible sample means?
    Normal
    Even though we have a small sample size, we can use the normal distribution for the sample means because the original population was normally distributed.

    (d) What is the probability that the sample mean is less than 1200?
    Notice that we are now talking about the sample mean.
    We are looking for $P\left(\overline{X} < 1200\right)$.
    We need to standardize it.

    $$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} = \frac{1200 - 1026}{209/\sqrt{13}} = 3.00$$

    So we are using the standard normal table and looking for the area to the left of $z = 3.00$.



$$P\left(\overline{X} < 1200\right) = P(Z < 3) = .9987$$

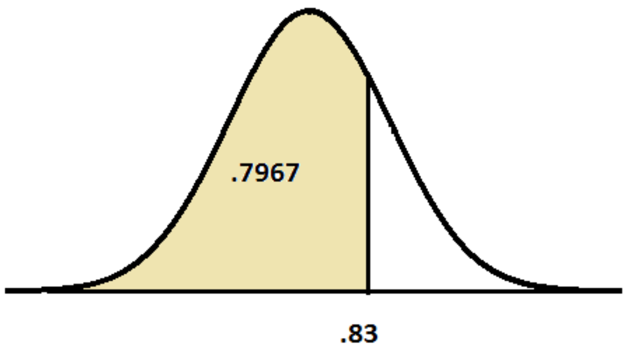
So 99.87% of the possible sample means are less than 1200.

(e) We decide to randomly select one student. What is the probability that his score is less than 1200?
We are looking for $P(X < 1200)$.
We are talking about an individual, NOT a sample.
This means it is a Normal Distribution Problem.

So we will use the formula $z = \dfrac{x-\mu}{\sigma}$.
We need to standardize it.

$$z = \frac{x-\mu}{\sigma} = \frac{1200-1026}{209} = .83$$

So we are using the standard normal table and looking for the area to the left of $z = .83$.



$$P(X < 1200) = P(Z < .83) = .7967$$

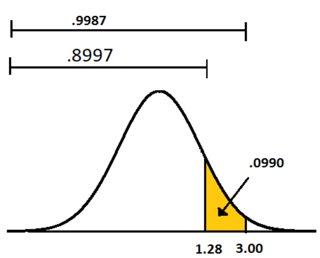
So 79.7% of the individual scores are less than 1200.
Or there is a 79.7% chance that the student we pick has a score less than 1200.

(f) What is the probability that the sample mean is between 1100 and 1200?
Notice that we are now talking about the sample mean.

Notice that we are now talking about the sample means. So we will use the formula $z = \dfrac{\overline{x}-\mu}{\sigma/\sqrt{n}}$.
We are looking for $P\left(1100 < \overline{X} < 1200\right)$. We will need to standardize both $\overline{X}$ values.

$$z_1 = \frac{\overline{x}_1-\mu}{\sigma/\sqrt{n}} = \frac{1100-1026}{209/\sqrt{13}} = 1.28$$

$$z_2 = \frac{\overline{x}_2-\mu}{\sigma/\sqrt{n}} = \frac{1200-1026}{209/\sqrt{13}} = 3.00$$



$$P\left(1100 < \overline{X} < 1200\right) = P(1.28 < Z < 3.00) = .9987 - .8997 = .099$$

So about 9.9% of all possible sample means are between 1100 and 1200.

***If you use a calculator you get a more accurate answer of 9.95% because there is no rounding.**

23. Packages of sugar bags for Sweeter Sugar Inc. are supposed to have an average weight of 16 ounces and a standard deviation of 0.3 ounces and are normally distributed. The company wants to find a sample of 15 bags and check the average weight.
The data for the population of individual values is $\mu = 16$ and $\sigma = .3$.

(a) Let X be the the weight of one bag of sugar. Why is X a random variable?
X is an individual observation. It is a random variable because the weight of the single bag changes based on which bag we choose.

(b) Why is $\overline{X}$ a random variable?
$\overline{X}$ is the sample mean of 15 bags. It is a random variable because the sample mean weight changes based on which sample we choose.

(c) What will the shape of the distribution of the population of all possible $\overline{X}$'s be?
If the original population is normally distributed or the sample size is at least 30, the sampling distribution will be normal.
In this case, $n = 15$ is too small to use the Central Limit Theorem. But the population is normally distributed, so the sampling distribution of $\overline{X}$ is normal as well.

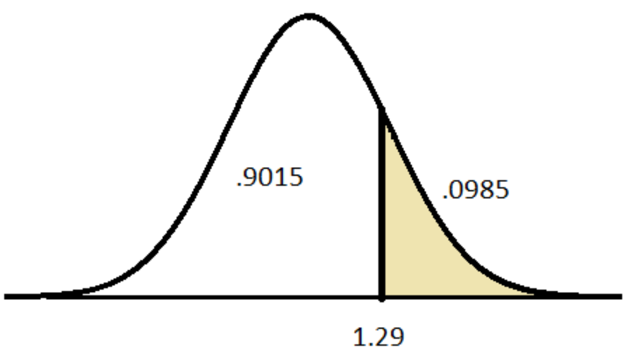
(d) What is $\mu_{\overline{X}}$?
$\mu_{\overline{X}} = \mu = 16$

(e) What is $\sigma_{\overline{X}}$?
$\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{.3}{\sqrt{15}} = .0775$

(f) What is the probability that 15 randomly selected packages will have a average weight in excess of 16.1 ounces?
Notice that we are talking about the sample means.

So we will use the formula $z = \dfrac{\overline{x} - \mu}{\sigma/\sqrt{n}}$.
We are looking for $P\left(\overline{X} > 16.1\right)$. We will need to standardize the $\overline{X}$ value.

$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} = \frac{16.1 - 16}{.3/\sqrt{15}} = 1.29$$



$$P\left(\overline{X} > 16.1\right) = P\left(Z > 1.29\right) = 1 - .9015 = .0985$$

(g) What is the probability that 1 randomly selected bag will have a weight between 15 and 17 ounces?
Notice that we are talking about the an individual value. So this is from the normal distribution chapter.

So we will use the formula $z = \dfrac{x - \mu}{\sigma}$. We can use this since the population is normally distributed.
We are looking for $P\left(15 < X < 17\right)$. We will need to standardize both the X values.

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{15 - 16}{.3} = -3.33$$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{17 - 16}{.3} = 3.33$$

$$P\left(15 < X < 17\right) = P\left(-3.33 < Z < 3.33\right) = .9996 - .0004 = .9992$$

24. The population lifetime of a certain battery has a mean of $\mu = 153$ hours and a standard deviation of $\sigma = 36$ hours. The distribution is right skewed. We decide to take a sample of 10 batteries and look at the sample mean.

   (a) What is the mean of the sampling distribution of $\overline{X}$?
   $\mu_{\overline{X}} = \mu = 153$

   (b) What is the standard deviation of the sampling distribution of $\overline{X}$?
   $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{36}{\sqrt{10}} = 11.38$

   (c) What is the shape of the sampling distribution of $\overline{X}$?
   The original population isn't normal and the sample size isn't big enough for the Central Limit Theorem to guarantee that the sampling distribution of $\overline{X}$ will be normal.
   So you don't know what the shape will be. It will be less skewed than the original distribution, but it won't be a normal distribution yet.

   *********NOTE******************
   So the interesting thing is that we have found that no matter the shape of the original population

   - **the average of the sample means is always equal to the population mean**
   - **and the SD of the sample mean is always equal to the population SD**

   Those two facts are always true no matter the shape.

   But if the original is normal or n>=30, then we can also say that all the samples means will be normal. Which is really cool too.

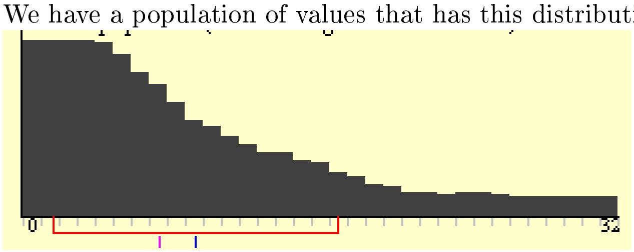
   (d) Find the probability that the sample mean is less than 140 hours.
   Since the sample size isn't big enough and the original population isn't normal, we can't use the normal distribution.
   That means that you can't do this problem!

25. Which distribution will have a smaller standard deviation:

   (a) a population of individual values

   (b) the sampling distribution of the sample mean for a sample of size $n = 2$?
   (b) the sample means always have less variation than the individual values

26. We have a population of values that has this distribution:



What will the shape of the sampling distribution of the sample mean for $n = 3$ look like?

   (a) normal

   (b) like the original population

   (c) still right skewed, but a little more normal than the original population is
   (c) even though the sample size isn't large enough to have a normal sampling distribution, it will already be a little more symmetric and a little more bell shaped than the original population is

27. Which sample size will give the smallest standard deviation for the sampling distribution of the sample mean?

   (a) $n = 100$.

(b) $n = 500$.

(c) $n = 1000$.

(d) It depends on the distribution, i.e., whether it is symmetric or skewed.

(c) $n = 1000$; Larger sample sizes always give smaller standard deviations for the sample mean.

28. The true percentage of red M&Ms is 15%. If six students took a sample of M&Ms and recorded the percentage of red M&Ms, which set of values are they most likely to obtain?

(a) 15%, 15%, 15%, 15%, 15%

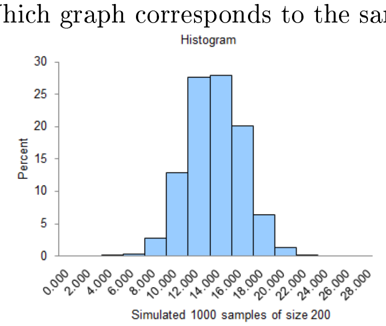
(b) 3%, 90%, 40%, 86%, 15%

(c) 14%, 13%, 15%, 14%, 16%

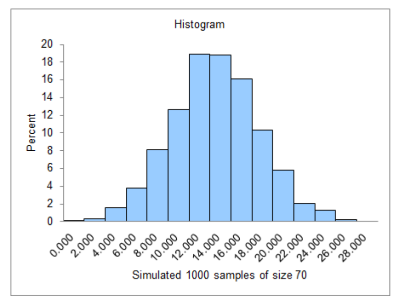
(d) Any of the above

(c) The sample proportion (or percentage) is a random variable. It varies based on which sample we choose, but the sample percentages should still be close to the true population percentage.

29. The graphs below are the sampling distributions for the percentage of red M&Ms for sample sizes of 70 and 200. Which graph corresponds to the sample size of 200?



A.



B.

You can tell that it is (A.) because bigger sample sizes always give smaller standard deviations (skinnier graphs)

30. We know that percentage of boy births in the US is 51.2%. At which hospital are we most likely to observe a week with 83% boy births?

(a) a small hospital with five births per week

(b) a medium hospital with fifty births per week

(c) a large hospital with a hundred births per week

(a) there is always more variation in small sample sizes

(bigger samples sizes have smaller standard deviation, this means that their proportions will be much closer to .512)

31. It is estimated that 75% of all young adults between the ages of 18-35 do not have a landline in their homes and only use a cell phone at home. We decide to take a random sample of 100 young adults and find the sample proportion $\hat{p}$.

$n = 100$ **and** $p = .75$

(a) What is the mean of the sampling distribution of $\hat{p}$?

$\mu_{\hat{p}} = p = .75$

(b) What is the standard deviation of the sampling distribution of $\hat{p}$?

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.75(1-.75)}{100}} = .043$$

(c) What is the shape of the sampling distribution of $\hat{p}$?

If $np \geq 5$ and $n(1-p) \geq 5$, then the distribution will be approximately normal.

$$100(.75) = 75 \ \surd$$

$$100(1-.75) = 25 \ \surd$$

32. We believe that the proportion of all teenagers who listen to streamed music is 30%. Let's take a sample of 1000 teenagers.

    (a) What is n? What is p? $n = 1000$ **and** $p = .3$

    (b) What is the mean of the sampling distribution of $\hat{p}$?
    $\mu_{\hat{p}} = p = .3$

    (c) What is the standard deviation of the sampling distribution of $\hat{p}$?
    $\sigma_{\hat{p}} = \sqrt{\frac{p\,(1-p)}{n}} = \sqrt{\frac{.3\,(.7)}{1000}} = .01449$

    (d) Can you use the normal distribution as an approximation for this problem?
    We can use the normal approximation if $np \geq 5$ **and** $n\,(1-p) \geq 5$**. Let's check if our conditions are met.**

    $$1000\,(.3) = 300 \checkmark$$
    $$1000\,(1-.3) = 700 \checkmark$$

    So we can use the normal distribution.

0.3 Confidence Intervals

Multiple Choice

33. Determine whether each of the following statements is true or false.
A) The margin of error for a 95% confidence interval for the mean μ increases as the sample size increases.
B) The margin of error for a confidence interval for the mean μ increases as the confidence level decreases.
C) The margin of error for a 95% confidence interval for the mean μ decreases as the population standard deviation decreases.
A) False, B) False, C) True

34. A small New England college requires the Math SAT for admission, and the population standard deviation is found to be 60. The formula for a 95% confidence interval yields the interval 634.12 to 645.88. Determine whether each of the following statements is true or false.
A) If we repeated this procedure many, many times, only 5% of all the possible confidence intervals would NOT include the mean Math SAT score of the population of all students at this college.
B) The probability that the population mean will fall between 634.12 and 645.88 is 0.95.
C) If we repeated this procedure many, many times, $\overline{x}$ would fall between 634.12 and 645.88 about 95% of the time.
A) True,
B) False, the population mean is a fixed number, there is no probability attached to it.
C) False, it is supposed to be the population mean μ, not $\overline{x}$.

35. A sprinkler system is being installed in a large office complex. Based on a series of test runs, a 99% confidence interval for μ, the average activation time of the sprinkler system (in seconds), is found to be (22, 28). Determine whether each of the following statements is true or false.
A) The 99% confidence level implies that $P(22 < \mu < 28) = 0.99$.
B) The 99% confidence level implies that 99% of the sample means ($\overline{x}$) obtained from repeated sampling would fall between 22 and 28.
A) False, B) False

36. Central Middle School has calculated a 95% confidence interval for the mean height μ of eleven- year-old boys at their school and found it to be 56 ± 2 inches.

    (a) Determine whether each of the following statements is true or false.
    A) There is a 95% probability that μ is between 54 and 58.
    B) There is a 95% probability that the true mean is 56.
    C) If we took many additional random samples of the same size and from each computed a 95% confidence interval for μ, approximately 95% of these intervals would contain μ.
    D) If we took many additional random samples of the same size and from each computed a 95% confidence interval for μ, approximately 95% of the time μ would fall between 54 and 58.
    A) False, B) False, C) true, D)false
    Remember, the population mean is a fixed number, we shouldn't talk about probability.

    The population mean μ isn't changing. We don't actually know what it is, but that's just because we don't have all the data. If we had all the data we would know exactly what it is. In other words μ isn't random. There isn't any chance involved and so we don't talk about probability.

    Now the sample mean $\overline{X}$ does change based on which sample we choose. So there is probability attached to $\overline{X}$.

    So for the confidence interval, μ is fixed and unchanging. But each possible sample mean has a different confidence interval. And 95% of those possible confidence intervals actually contain μ.

    That's why we say we are 95% confident that μ is in the confidence interval.

    (b) Which of the following could be the 90% confidence interval based on the same data?
    A) 56 ± 1

B) 56 ± 2
C) 56 ± 3
D) Without knowing the sample size, any of the above answers could be the 90% confidence interval.
(A) 56± 1
Since getting more confident requires a wider interval, being less confident requires a smaller interval. Since it is the same data set, the sample size is the same.

37. An agricultural researcher plants 25 plots with a new variety of yellow corn. Assume that the yield per acre for the new variety of yellow corn follows a Normal distribution with an unknown mean of μ and a standard deviation of $\sigma = 10$ bushels per acre.

(a) If the average yield for these 25 plots is $\overline{x} = 150$ bushels per acre, what is a 90% confidence interval for μ?
A) 150 ± 0.784
B) 150 ± 2.00
C) 150 ± 3.29
D) 150 ± 3.92
(C) We are given $n = 25$, $\sigma = 10$, **Normal,** $\overline{x} = 150$
Find the critical value for a 90% confidence interval.
$z^* = 1.645$

The confidence interval is

$$\overline{x} \pm z^* \left(\frac{\sigma}{\sqrt{n}}\right)$$

$$150 \pm 1.645 \left(\frac{10}{\sqrt{25}}\right)$$

$$150 \pm 3.29$$

Standard Normal Table: Areas to the Left

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379

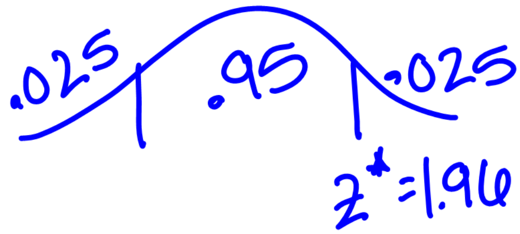
(b) Which of the following would produce a confidence interval with a smaller margin of error than the 90% confidence interval?
A) Plant only five plots rather than 25, because five are easier to manage and control.
B) Plant 10 plots rather than 25, because a smaller sample size will result in a smaller margin of error.
C) Plant 100 plots rather than 25, because a larger sample size will result in a smaller margin of error.
D) Compute a 99% confidence interval rather than a 90% confidence interval, because a higher confidence level will result in a smaller margin of error.
(C)

38. To assess the accuracy of a laboratory scale, a standard weight that is supposed to weigh exactly 1 gram is repeatedly weighed a total of n times and the mean $\overline{x}$ is computed. Suppose the scale readings are Normally distributed with an unknown mean of μ and a standard deviation of $\sigma = 0.01$ g. How large should n be so that a 95% confidence interval for μ has a margin of error no larger than $\pm$ 0.0001?
A) $n = 100$
B)$n = 196$
C) $n = 10000$
D)$n = 38416$
(D) This is a problem when we want to find the sample size for a margin of error.
We are told: normal, $\sigma = .01$, want margin of error $m = .0001$, 95% confidence

Notice on page 18 we have a formula for sample size for desired margin of error.

$$n = \left(\frac{z^* \cdot \sigma}{m}\right)^2$$

All you need to do is find z^* just like you do for a 95% confidence interval. So $z_{.025} = 1.96$.



Now plug it all in

$$\begin{aligned} n &= \left(\frac{z^* \cdot \sigma}{m}\right)^2 \\ &= \left(\frac{1.96 \cdot .01}{.0001}\right)^2 \\ &= 38,416 \end{aligned}$$

So to get a margin of error less than .0001, we need a sample size of at least 38,416.

39. The heights of a simple random sample of 400 male high school sophomores in a Midwestern state are measured. The sample mean is $\overline{x} = 66.2$ inches. Suppose that the heights of male high school sophomores follow a Normal distribution with a standard deviation of $\sigma = 4.1$ inches.

    (a) What is a 95% confidence interval for μ?
    A) (58.16, 74.24)
    B) (59.46, 72.94)
    C) (65.80, 66.60)
    D) (65.86, 66.54)
    (C)

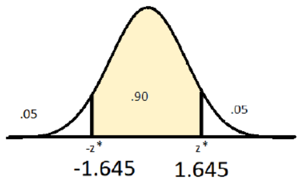
    (b) Suppose the heights of a simple random sample of 100 male sophomores were measured rather than 400. Which of the following statements is true?
    A) The margin of error for the 95% confidence interval would increase.
    B) The margin of error for the 95% confidence interval would decrease.
    C) The margin of error for the 95% confidence interval would stay the same because the level of confidence has not changed.
    (A) Larger sample sizes give us more precise results, meaning smaller margin of error. So smaller sample sizes give us bigger margin of errors.

40. Suppose we wish to calculate a 90% confidence interval for the average amount spent on books by freshmen in their first year at a major university. The interval is to have a margin of error of \$2. Assume that the amount spent on books by freshmen has a Normal distribution with a standard deviation of $\sigma = \$30$. How many observations are required to achieve this margin of error?
    A) 25
    B) 608
    C) 609
    D) 865
    (C) We want 90% confidence, a margin of error of $m = 2$, and we know $\sigma = 30$.

    To find the needed sample size, use the formula

    $$n = \left(\frac{z^* \sigma}{m}\right)^2$$

    and our critical value is $z^* = 1.645$.



So we have

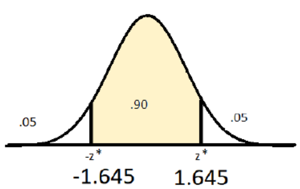
$$n = \left(\frac{1.645 \cdot 30}{2}\right)^2 = 608.85$$

Now if you are finding the necessary sample size you should always round up the number of people to make sure you have enough. Because if you need 608.85 then 608 is not quite enough. But if you round up you will have enough and a little extra.

So we need $n = 609$.

41. Battery packs in radio-controlled racing cars need to be able to last pretty long. The distribution of the lifetimes of battery packs made by Lectric Co. is slightly left-skewed. Assume that the standard deviation of the lifetime distribution is $\sigma = 2.5$ hours. A simple random sample of 75 battery packs results in a mean of $\overline{x} = 29.6$ hours.

(a) What is a 90% confidence interval for μ, the true average lifetime of the battery packs made by Lectric Co.?
A) (29.13, 30.07)
B) (29.03, 30.17)
C) (28.86, 30.34)
D) The confidence interval cannot be calculated because the population distribution is not Normal.
(A)
(It doesn't matter that the distribution isn't normal because the sample size is bigger than 30.)
We use a Z confidence interval because we know the POPULATION standard deviation $\sigma = 2.5$.
Our critical value is $z^* = 1.645$.



$$\overline{x} \pm z^* \left(\frac{\sigma}{\sqrt{n}}\right)$$

$$29.6 \pm 1.645 \left(\frac{2.5}{\sqrt{75}}\right)$$

$$29.125 \ to \ 30.074$$

(b) Which statement is true:
A) If a 95% confidence interval had been calculated, the margin of error would have been larger.
B) If many more samples of 75 battery packs were taken, 90% of the resulting confidence intervals would have a sample mean between 29.13 and 30.07.
C) If the sample size had been 150 and not 75, the margin of error would have been larger.
(A) is CORRECT. If you go from 90% to 95% you are getting more confident. The way you get more confident is to get a bigger margin of error.

B. The interpretation should be: If you take many samples, 90% of the resulting confidence intervals will contain the POPULATION MEAN.
The interval 29.13 to 30.07 is based on our current sample mean. The other sample means will be based on the POPULATION MEAN, not our current *sample* mean. Most of the possible sample means will be within a 1 or 2 standard deviations of our POPULATION mean. But

we don't know what μ actually is to calculate an interval that would contain most of our sample means.

C. If your sample size gets bigger, then the margin of error gets smaller.

42. A nationally distributed college newspaper conducts a survey among students nationwide every year. This year, responses from a simple random sample of 204 college students to the question "About how many CDs do you own?" resulted a 95% confidence interval for the mean number of CDs owned by all college students. The interval is (71.8, 73.8).

 (a) Answer each of the following questions with yes, no, or can't tell.
 A) Does the sample mean lie in the 95% confidence interval?
 B) Does the population mean lie in the 95% confidence interval?
 C) If we were to use a 92% confidence level, would the confidence interval from the same data produce an interval wider than the 95% confidence interval?
 D) With a smaller sample size, all other things being the same, would the 95% confidence interval be wider?
 (A) Yes: the confidence interval is always centered at the sample mean so of course the interval contains the sample mean.
 (B) Can't tell: We don't know. We know that 95% of all the possible intervals contain the population mean, but we don't know if this interval contains the population mean.
 (C) No: smaller confidence levels give smaller confidence intervals
 (D) Yes: increasing the sample size makes the interval smaller, so decreasing the sample size makes the interval wider.

 (b) Which of the following interpretations of the interval is correct?
 A) 95% of all students own between 71.8 and 73.8 CDs
 B) the probability that the population mean is between 71.8 and 73.8 CDs is 95%
 C) we are 95% confident that the sample mean is between 71.8 and 73.8
 D) we are 95% confident that the population mean number of CDs is between 71.8 and 73.8
 (D)

Show Your Work

43. A total of 114 male athletes from Canadian sports centers were surveyed. The average calorie intake was 3077 kcal/day with a sample standard deviation of 987.

 (a) Summarize the given information with appropriate symbols.
 The sample mean is $\overline{x} = 3077$ and the sample standard deviation is $s = 987$.
 The sample size is $n = 114$.

 (b) Which confidence interval should you use?
 The T Confidence Interval for Mean

$$\overline{x} \pm t^* \frac{s}{\sqrt{n}}$$

 (c) Check conditions.
 You have to check if the data is normal or the sample size is at least 30.
 Since the sample size is greater than 30, we can use it. $\surd$

 (d) Find the **99%** confidence interval for the population mean calorie intake μ.
 $n = 114$**, The degrees of freedom are 113, but that isn't on the table. Since we always round down to be conservative, use 100 degrees of freedom. The critical value is $t^* = 2.626$.**

$$\overline{x} \pm t^* \frac{s}{\sqrt{n}}$$

$$3077 \pm 2.626 \left(\frac{987}{\sqrt{114}} \right)$$

$$(2834, 3319)$$

(e) What is the margin of error?
The margin of error is always the part you add or subtract in a confidence interval.

$$t^* \frac{s}{\sqrt{n}}$$

$$2.626\left(\frac{987}{\sqrt{114}}\right)$$

$$242.75$$

(f) Interpret your result.
We are 99% confident that mean intake for all Canadian male athletes is between 2831 and 3322.

44. A nationally distributed college newspaper conducts a survey among students nationwide every year. This year, responses from a simple random sample of 204 college students to the question "About how many CDs do you own?" resulted in a sample mean of $\overline{x} = 72.8$. Based on data from previous years, the editors of the newspaper will assume that $\sigma = 7.2$. They want to find a confidence interval.

(a) Which formula should you use?

$$\overline{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

(b) Check conditions.
You have to check if the data is normal or the sample size is at least 30.
The sample size $n = 204$ is bigger than 30. $\surd$

(c) Find the 97% confidence interval for the population mean number of CDs μ.
The critical value is $z^* = 2.17$.

1 - .97 = .03
both tails

.03/2 = .015
one tail

look up area .015 left → z = -2.17 symmetry z* = 2.17
.97 center → z = 2.17
.985 left → z = 2.17
.015 right → z = 2.17

$$\overline{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

$$72.8 \pm 2.17\left(\frac{7.2}{\sqrt{204}}\right)$$
$$72.8 \pm 1.094$$
$$(71.706, 73.894)$$

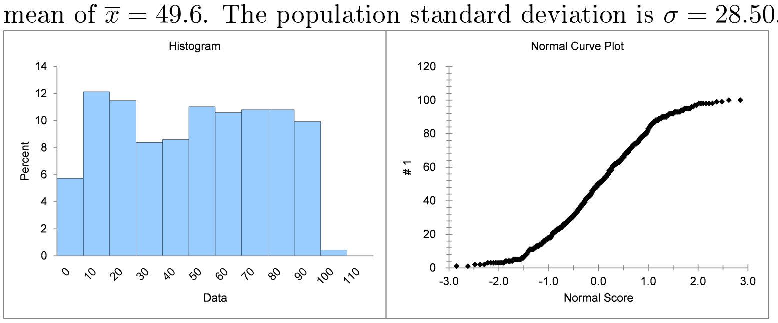
(d) What is the margin of error?

$$m = z^* \frac{\sigma}{\sqrt{n}}$$
$$m = 2.17\left(\frac{7.2}{\sqrt{204}}\right)$$
$$m = 1.094$$

(e) Interpret your confidence interval.
We are 97% confident that the population mean number of CD's is between 71.706 and 73.894.

(f) What does the 97% confidence level mean?
Of all the possible samples we could have chosen, 97% of the sample means would result in confidence intervals that contain the population mean μ.

45. You want to find a 90% confidence interval for the population mean μ. The sample size is $n = 21$ with a sample mean of $\overline{x} = 49.6$. The population standard deviation is $\sigma = 28.50$.



(a) Which formula should you use?

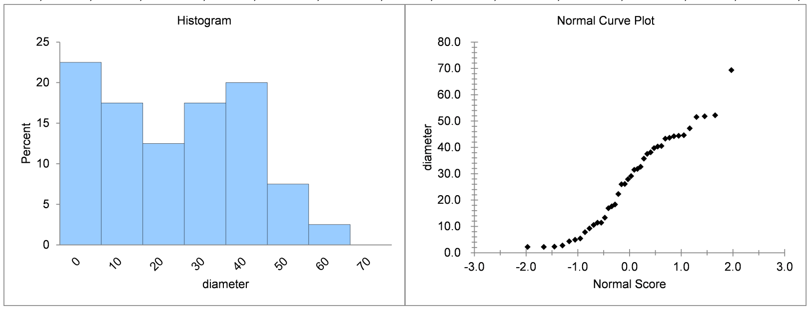
$$\overline{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

(b) Check conditions.
You have to check if the data is normal or the sample size is at least 30.
The sample size $n = 21$ is small so we have to check the data to see if it looks normal. The histogram doesn't look normal and the normal quantile plot is definitely not a straight line. So our data isn't normal. ✗
We can't find the confidence interval. Stop here.

(c) Find the 90% confidence interval for the population mean μ.
Trick question. Since the conditions aren't met, you can't find the confidence interval.

(d) ***NOTE!!! The plot above is called a NORMAL CURVE PLOT or a NORMAL QUANTILE PLOT. A lot of students have thought that if the problem on the exam says **normal** quantile plot that the data is normal. That is not true. It is just the name of the plot. You have to look at the plot to see if it is close to a straight line.

46. We have a data set of the diameter of trees in Georgia. Here is a sample of 40 trees.
Data Set: 10.5, 13.3, 26, 18.3, 52.2, 9.2, 26.1, 17.6, 40.5, 31.8, 47.2, 11.4, 2.7, 69.3, 44.4, 16.9, 35.7, 5.4, 44.2, 2.2, 4.3, 7.8, 38.1, 2.2, 11.4, 51.5, 4.9, 39.7, 32.6, 51.8, 43.6, 2.3, 44.6, 31.5, 40.3, 22.3, 43.3, 37.5, 29.1, 27.9



The sample mean is 27.69 and the standard deviation is 17.706.

(a) Summarize the given information with appropriate symbols.
$n = 40$, $\overline{x} = 27.69$ **and** $s = 17.706$

(b) Which confidence interval formula should you use?
T confidence Interval for Mean

$$\overline{x} \pm t^* \frac{s}{\sqrt{n}}$$

(c) Check conditions.
You have to check if the data is normal or the sample size is at least 30.
Since the sample size is bigger than 30, we don't care what the data looks like. (This is a good thing because the data is definitely right skewed and not even roughly bell shaped. The normal quantile plot doesn't look like a straight line.) So even though the data doesn't look normal, the sample size is large so we can use the T confidence interval.

(d) Find the 95% confidence interval for the population mean tree diameter μ.
$n = 40$, **The degrees of freedom are 39, but that isn't on the table. Since we always round down to be conservative, use 30 degrees of freedom so the critical value is** $t^* = 2.042$.

$$\overline{x} \pm t^* \frac{s}{\sqrt{n}}$$

$$27.69 \pm 2.042 \left(\frac{17.706}{\sqrt{40}} \right)$$

$$(21.97, 33.41)$$

****If you use a calculator you get** $t^* = 2.022$ **which changes your answer a bit to** $(22.03, 33.35)$

(e) Interpret your result.
We are 95% confident that mean diameter for all the trees is between 22.02 to 33.35.

0.4 Review for Hypothesis Tests

47. The cost of a one-carat VS2 clarity, H color diamond from Diamond Source USA is \$5600.
A Midwestern jeweler makes calls to contacts in the diamond district of New York City to see whether the mean price of diamonds there differs from \$5600. He calls 35 jewelers and the results are a sample mean of \$5835. He believes the population standard deviation is \$520.
Conduct a test to see if the mean diamond price in New York City is different from \$5600.

$n = 35$, $\overline{x} = 5835$, $\sigma = 520$

Test: one sample Z test for mean

condition: sample size at least 30 $\surd$

Level of significance: I choose $\alpha = .05$

Hypotheses: $H_0 : \mu = 5600$ versus $H_A : \mu \neq 5600$

Test Statistic:

$$z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{5835 - 5600}{520/\sqrt{35}} = 2.67$$

P-value: Use the Z table.
Look for the area in both tails.

The area in one tail is .0038. But the p-value is area in both tails. The p-value is .0076.

Conclusion: small p-value; reject $H_0 : \mu = 5600$

Interpret: He has very strong evidence that the population mean diamond price in New York is different from \$5600.

48. A hamburger restaurant wants to form a partnership with a potato company. The potatoes come in supposedly 20 pound bags. Of course the hamburger restaurant doesn't want to buy underweight bags of potatoes. Before they sign a contract, they weigh a sample of 40 bags and find an average weight of 19.7 pounds with a standard deviation of 0.1 pounds. Conduct a test to see if the population mean of the bag weights will be less than 20 pounds. **Interpret the p-value.**

$n = 40$, $\overline{x} = 19.7$, $s = .1$

Test: One sample T test for mean

condition: sample size is at least 30. $\surd$

Level of significance: I choose $\alpha = .05$

Hypotheses: $H_0 : \mu = 20$ versus $H_A : \mu < 20$

Test Statistic:

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{19.7 - 20}{.1/\sqrt{40}} = -18.9736$$

$$df = n - 1 = 39$$

we will have to round down to $df = 30$ to use the T table

P-value: use t table; area in left tail; p-value is less than .0005
(we can use the computer to find the exact p-value is 1.35092×10^{-21})

**Video for p-value: https://youtu.be/5Bz0GnAAxOo

Conclusion: small p-value; reject $H_0 : \mu = 20$

Interpret: we have extremely strong evidence that the population mean is less than 20 pounds.

Interpret P-value: If the population mean really is 20 pounds, then the probability of getting a sample mean of $\overline{x} = 19.7$ or less, is less than .0005.

49. Nike, thinks it has found a way to make running shoes that last longer than the current average of 750 miles. A sample of 35 runners are selected to wear the new design and the shoes last an average of 815 miles. Assume that the population standard deviation is 25 miles. Conduct a test to see if the average distance the new design will last is greater than 750 miles. **Interpret the p-value.**

$n = 35$, $\overline{x} = 815$, $\sigma = 25$

Test: One Sample Z Test for Mean

condition: sample sizes is at least 30 $\surd$

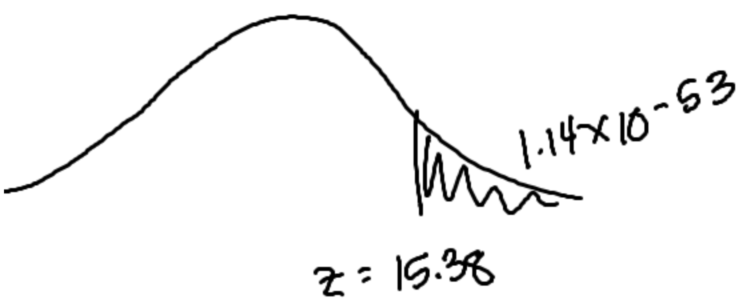
Level of significance: I choose $\alpha = .05$

Hypotheses: $H_0 : \mu = 750$ versus $H_A : \mu > 750$

Test Statistic:

$$z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{815 - 750}{25/\sqrt{35}} = 15.38$$

P-value: use z table; area in right tail; p-value is essentially 0
(computer says p-value is 1.14×10^{-53})



Conclusion: small p-value, reject null hypothesis

Interpret: we have extremely strong evidence that the new design will have a population mean longer than 750 miles.

Interpret p-value: If the population mean really is 750, then the probability of observing a sample mean of $\overline{x} = 815$ or greater is basically 0.

50. The Employment and Training Administration reported the US mean unemployment insurance benefit of \$238 per week. A researcher in Virginia wonders if they have a different benefit than the national average.
He takes a sample of 85 individuals in Virginia and finds the sample mean weekly unemployment benefit was \$231 with a sample standard deviation of \$80.
Conduct a hypothesis test to see if Virginia has a different mean benefit than the national average.

$n = 85$, $\overline{x} = 231$, $s = 80$

Test: one sample T test for mean

condition: $n \geq 30$ $\surd$

Level of significance: I chose $\alpha = .05$

Hypotheses: $H_0 : \mu = 238$ versus $H_A : \mu \neq 238$

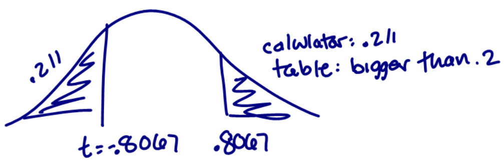
Test Statistic:

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{231 - 238}{80/\sqrt{85}} = -.8067$$

$$df = n - 1 = 84$$

we will round down to $df = 80$ to use the T table

P-value: p-value is area in both tails.



(*Remember that you look along the $df = 80$ line for your t-value. The T table only does area to the right, so use symmetry. Look for positive t=.8067. It would give you a p-value bigger than .2. So the area to the right of .8067 is the same as the area to the left of -.8067.)

degrees of freedom	α							
v	.20	.10	.05	.025	.01	.005	.001	.0005
1	1.376	3.078	6.314	12.71	31.82	63.66	318.3	636.6
40	0.851	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.849	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.848	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.846	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.845	1.290	1.660	1.984	2.364	2.626	3.174	3.390
200	0.843	1.286	1.653	1.972	2.345	2.601	3.131	3.340
∞	0.842	1.282	1.645	1.960	2.326	2.576	3.090	3.291

https://youtu.be/0VhUp0RO9Z4

The area in the right tail is bigger than .20.
The total p-value is bigger than .40.

$$p - value\ is\ bigger\ than\ .40$$

Using a calculator:

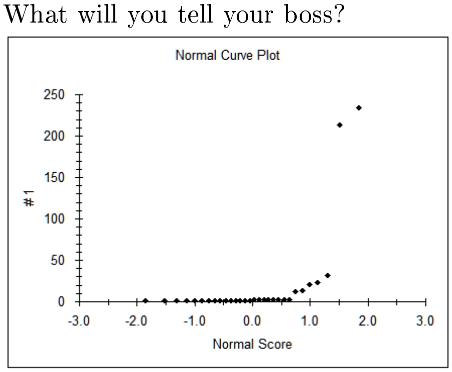
$$p - value = .422$$

Conclusion: big p-value; fail to reject $H_0 : \mu = 238$

Interpret: we didn't find any evidence that the population mean benefit in Virginia is different from the population mean benefit nationwide.

51. Your boss wants you to run a hypothesis test to see if $\mu < 87$ for a data set with sample size 20. You found the normal curve plot below.

(a) What will you tell your boss?



We have a small sample size and the data is definitely not normal. We can't use the T test here. So we can't do this problem.
(You would have to use non-parametric techniques.)

(a) ***NOTE!!!** The plot above is called a NORMAL CURVE PLOT or a NORMAL QUANTILE PLOT. A lot of students have thought that if the problem on the exam says **normal** quantile plot that the data is normal. That is not true. It is just the name of the plot. You have to look at the plot to see if it is close to a straight line.

More Review for Hypothesis Tests

52. A statistics class also took a sample of 30 Chips Ahoy Cookies. They claim to have 1000 chocolate chips per bag which means an average of 33 chocolate chips per cookie. We want to know if there are less than an average of 33 chocolate chips per cookie.

    (a) What are the null and alternative hypotheses in words?
    Null: the mean number of chocolate chips in all the cookies is 33
    Alternative: the mean number of chocolate chips in all the cookies is less than 33

    (b) The sample mean is $\overline{x} = 24.83$ and the sample standard deviation is $s = 3.65$. The p-value is 2.7×10^{-13}. Interpret the p-value.
    If the mean number of chocolate chips is really $\mu = 33$, then the probability of getting a sample mean of $\overline{x} = 24.83$ or less, is only .00000000000027.

53. A hamburger restaurant wants to form a partnership with a potato company.
    The potatoes come in supposedly 20 pound bags. Of course the hamburger restaurant doesn't want to buy underweight bags of potatoes.
    Before they sign a contract, they weigh a sample of 40 bags and find an average weight of 19.7 pounds with a standard deviation of 0.1 pounds.
    They conduct a test to see if the population mean of the bag weights will be less than 20 pounds.
    Their p-value is 1.35×10^{-21}.

    (a) The small p-value means we have __________________ significance.
        i. practical
        ii. statistical
        statistical

    (b) Which interpretation of the p-value is correct?
        i. The probability that the mean weight is 20 pounds is 1.35×10^{-21}.
        ii. The probability that the mean weight is less than 20 pounds is 1.35×10^{-21}.
        iii. If the population mean really is 20 pounds, then the probability of getting a sample mean of $\overline{x} = 19.7$ or less, is 1.35×10^{-21}.
        If the population mean really is 20 pounds, then the probability of getting a sample mean of $\overline{x} = 19.7$ or less, is 1.35×10^{-21}

54. What does the level of significance $\alpha = .05$ tell us?

    - **How much evidence we need against the null hypothesis before we can reject it.**
    - **We need strong evidence against the null hypothesis before we can reject it.**
    - **We will only reject the null hypothesis if our sample data is so extreme, it would only happen 5% of the time when the null hypothesis is true.**
    - **The probability of a Type I error is 5%.**

55. What is the p-value?

    (a) The probability that the null hypothesis is true.
    (b) The probability that the alternative hypothesis is true.
    (c) The probability that the null hypothesis is false.
    (d) The probability of observing our sample data, or something more extreme, if the null hypothesis is true.
    The probability of observing our sample data, or something more extreme, if the null hypothesis is true.

56. In the cookie problem, $H_0 : \mu = 33$ versus $H_A : \mu < 33$, we got a p-value of essentially 0 (software tells us it was 1.95×10^{-32}). What is an appropriate conclusion?

(a) We have proved that the population mean number of chocolate chips is 33.

(b) We have proved that the population mean number of chocolate chips is less than 33.

(c) We found extremely strong evidence that the population mean number of chocolate chips is 33.

(d) We found extremely strong evidence that the population mean number of chocolate chips is less than 33.
(D) Remember that hypothesis tests deal with probabilities, not certainties. So we prefer to say we find or don't find evidence instead of saying we have proven something. Proving something should only occur when we know absolutely that we are right, like with a mathematical proof.

57. A KFC manager conducts a hypothesis test with $H_0 : \mu = 1$ versus $H_A : \mu > 1$. He decided ahead of time to use a significance level of $\alpha = .01$. His p-value is .0367.

(a) Did he reject or fail to reject the null hypothesis?
The p-value of .0367 was bigger than $\alpha = .01$ so he failed to reject the null hypothesis. So he didn't find evidence that the mean waiting time is greater than 1 minute.

(b) Is it ethical for him to change his significance level after he sees the results?
No! You have to pick the significance level and hypotheses before you see the results of the data.

(c) If he had chosen a significance level of .05 instead of $\alpha = .01$ ahead of time, would he reject or fail to reject the null hypothesis?
Then the p-value of .0367 would be smaller than $\alpha = .05$ so he would have rejected the null hypothesis. Then he would say he found evidence that the mean waiting time is greater than 1 minute.

58. Suppose unbeknownst to our manager, managers at several different KFCs did the same experiment. The different p-values that they found are shown below. Which manager found the strongest evidence that the $\mu > 1$.

(a) .023

(b) .564

(c) .001

(d) .00034
The smaller the p-value, the stronger the evidence against the null hypothesis and for the alternative hypothesis.

59. Statistically significant means that it is very unlikely that we would get such an extreme sample statistic if the null hypothesis were true.
We say that our results are statistically significant if we get a ___________ p-value.

(a) big

(b) small
Small: statistically significant means that it is very unlikely that we would get such an extreme result if the null hypothesis were true. This happens for small p-values.

60. We conducted a hypothesis test to determine if a dice was unfair.

(a) What would be the null hypothesis (in words)?
The dice is fair.

(b) What would be the alternative hypothesis (in words)?
The dice is not fair.

(c) What would be a Type I error?
A Type I Error in this situation is deciding that a fair dice is unfair.

(d) What would be a Type II error?
A Type II Error in this situation is deciding that a unfair dice is fair.

61. The ____ the P-value, the stronger the evidence against the null hypothesis provided by the data.
A) larger
B) smaller
(B) Small p-values give evidence against the null hypothesis.

62. When performing significance tests at level α, if the P-value obtained is _____ α, the null hypothesis will not be rejected.
A) larger than
B) smaller than
C) the same as
D) None of the above.
(A) We reject for small p-values and fail to reject for big p-values.

63. When performing significance tests at level α, if the P-value obtained is _____ α, the null hypothesis will be rejected.
A) larger than
B) smaller than
C) the same as
D) None of the above.
(B) We reject for small p-values and fail to reject for big p-values.

64. If the P-value is as small as or smaller than a specified value of α, then the data are _______ significant.
A) practically
B) statistically
C) absolutely
(B) statistically

65. A test of significance for a null hypothesis has been conducted and the P-value determined. Which of the following statements about a P-value is (are) TRUE?
A) The P-value is the probability that the null hypothesis is false.
B) The P-value is the probability that the alternative hypothesis is true.
C) The P-value is the probability that the null hypothesis is rejected even if that hypothesis is actually true.
D) The P-value tells us the strength of the evidence against the null hypothesis.
E)The larger the P-value the stronger the evidence against the null hypothesis.

Keep in mind that we set up our hypothesis tests to control for the probability of making a wrong decision, but we never actually know if the null hypothesis is true or false, so we don't know anything about the probability of the null hypothesis being true or false.
(A) This is False! We don't know anything about the probability that the null hypothesis is true or false.
(B) This is False! We don't know anything about the probability that the null hypothesis is true or false.
(C) This is False! This is the definition of the probability of the Type I error α, or the significance level.
(D) This is TRUE!
(E) This is False! Smaller P-values give stronger evidence against the null hypothesis.

66. If a statistical significance is found when performing a test of significance, then practical significance will also be found.
A) True
B) False
(B) False: statistical significance doesn't mean that our results will have any practical significance. If you have a large enough sample, very small differences can become statistically significant.

67. If you reject the null hypothesis when in fact the null hypothesis is true it is called __________.
A) a Type I error
B) a Type II error
Type I error

68. If you accept the null hypothesis when in fact the alternative hypothesis is true is called __________.
A) a Type I error.

B) a Type II error.
Type II error

69. A small company consists of 25 employees. As a service to the employees, the company arranges for each of the employees to have a complete physical exam for free. Among other things, the weight of each employee is measured. The mean weight is found to be 165 pounds. The standard deviation of the weight measurements is 20 pounds. It is believed that a mean weight of 160 pounds would be expected for this group. To see if there is evidence that the mean weight of the population of all employees of the company is significantly larger than 160, the hypotheses $H_0 : \mu = 160$ versus $H_a : \mu > 160$ are tested. You obtain a P-value of about 0.106. Determine which statement is true for a significance level of $\alpha = .05$.

    (a) You have proved H_0 is true.
    (b) You have proved H_a is true.
    (c) You have proved H_0 is false.
    (d) You failed to find evidence against H_0.
    (D) Remember that we are dealing with probabilities and we don't like to say we prove anything. Instead we find evidence for or against the null hypothesis.

    In this case the p-value is bigger than $\alpha = .05$, so we fail to reject the null hypothesis. So we didn't find any evidence against H_0.

    "We didn't find any evidence that the mean weight is greater than 160 pounds."

70. Ten years ago, at a small high school in Alabama, the mean Math SAT score of all high school students who took the exam was 490 with a standard deviation of 80. This year, the Math SAT scores of a random sample of 25 students who took the exam are obtained. The mean score of these 25 students is $\overline{x} = 525$. We assume the population standard deviation continues to be $\sigma = 80$. To determine if there is evidence that the scores in the district have improved, the hypotheses $H_0 : \mu = 490$ versus $H_a : \mu > 490$ are tested. The P-value is found to be 0.014. Use the significance level of $\alpha = .05$. What conclusion can we draw?

    (a) We have proved that the mean score is 490.
    (b) We have proved that the mean score is greater than 490.
    (c) We found evidence that the mean score is greater than 490.
    (d) We didn't find any evidence that the mean score is greater than 490.
    (C) Remember that we are dealing with probabilities and we don't like to say we prove anything. Instead we find evidence for or against the hypotheses.

    In this case the p-value is smaller than $\alpha = .05$, so we reject the null hypothesis. So we found evidence against H_0.

    "We found evidence that the mean score is greater than 490."

71. A study was conducted to determine the effect of the addition of a particular supplement to a well-known low-density lipoprotein (LDL) cholesterol lowering drug. 5000 randomly selected patients were studied on this combination of drugs over a one-year period. An increased reduction of 5 mg/dL of LDL cholesterol was achieved with the combination of drugs relative to the well-known drug alone. The reduction resulted in a P-value of 0.001.
    A) This P-value result shows that the combination drug should be prescribed widely.
    B) Clearly, this study has demonstrated the effectiveness of this new treatment.
    C) The result should be viewed with caution because one year is not a long time to study such an important matter as cholesterol.
    D) A sample of this size can detect very small differences and hence care should be taken to determine if a reduction of this amount has any practical significance with respect to LDL cholesterol reduction.
    (D) No matter how small a difference is, if you make the sample size large enough, the difference will be statistically significant. (Remember, statistically significant means that such a large difference would rarely occur by chance.) So even though we are very sure that the new drugs lower the cholesterol, we don't know if lowering it by 5mg/dL over a year will really affect our health. We would need to consult a medical expert for this.

72. What is power?

    (a) The probability of rejecting a true null hypothesis.
    (b) The probability of rejecting a false null hypothesis.
    (c) The probability of accepting a true null hypothesis.
    (d) The probability of accepting a false null hypothesis.
    (B) Power is correctly rejecting a false null hypothesis.

73. Higher power is a good thing. How can we increase the power and lower the probability of making an error?
    Increase the sample size.

74. Read this: Does taking ginkgo tablets twice a day provide significant improvement in mental performance? To investigate this issue, a researcher conducted a study with 150 adult subjects who took ginkgo tablets twice a day for a period of six months.

    At the end of the study, 200 different variables related to the mental performance of the subjects were measured on each subject and the means compared to known means for these variables in the population of all adults.

    (So 200 hypothesis tests were conducted.)

    Nine of these variables were significantly better (in the sense of statistical significance) at the 5% level for the group taking the ginkgo tablets as compared to the population as a whole.

    Do you think this means that we can say taking ginkgo tablets helps your mental performance? Probability not.

    Remember that if we use $\alpha = .05$, we have a 5% probability of deciding the null hypothesis is false even if it is true. Or we will make a type I error 5% of the time.
    So if we conduct 200 tests, we would expect to get $200\,(.05) = 10$ significant results even if the ginkgo tablets don't do anything.
    So the fact that we got 9 significant results means it was probably those expected type I errors.