Math 5710 Thursday, 6/25/20 Quiz 04

1. What is your name?

2. Write two paragraphs that explain why the following proposition is true:

$$k \subseteq \omega \times \mathbb{N} \ni k = \{ (n, k(n)) : n \in \omega \ \wedge \ k(n) = n + 1 \} \ \Rightarrow (k : \omega \xrightarrow{1:1} \mathbb{N} \ \wedge \ k : \omega \xrightarrow{onto} \mathbb{N})$$

Sample explanation:

It helps me to express k in roster form:

$$k = \{ (0, 1), (1, 2), (2, 3), (3, 4), \ ... \}$$

This pattern suggests that each element of the domain $\{ 0, 1, 2, 3, 4, ...)$ is paired to exactly one element of the codomain (i.e., $\{ 1, 2, 3, 4, 5, ... \}$. So we see that k is a relation from ω to $\mathbb{N}$.

k is an injection because each element of the range is paired with only one element of the domain. Here is a proof of that fact:

Suppose $\exists \ n_1, n_2 \in \omega \ni k(n_1) = k(n_2)$, then we have $n_1 + 1 = n_2 + 1 \Rightarrow n_1 = n_2$

k is a surjection because the codomain is the range. Here is a proof of that fact:

Suppose $y \in \mathbb{N}$ (i.e., an arbitrary element in the codomain), we need to demonstrate that $\exists \, n \in \omega$ (i.e., an element in the domain) $\ni k (n) = y$. Consider $n = y - 1$. Note that since $y \in \mathbb{N}$, we know that $y - 1 \in \omega$. Also $k (y - 1) = (y - 1) + 1 = y$. That bit of deduction makes me smile.

Although revealing the scratch work behind my choice of $y - 1$ for n in the part of the proof above w/r k being a surjection is an unnecessary part of the proof itself, I want to show it to you anyway:

$$\boldsymbol{k (n) = y \ \Rightarrow y = n + 1 \ \Rightarrow n = y - 1}$$

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3. Smile.