Discover Binomial Distribution

Example 1. Let's consider a student taking a multiple choice quiz. Each question has 3 choices. Assume the student is blindly guessing. We will let X be the number of right answers.

1. Let's have the student take a quiz with just one question.

   (a) Fill out the table of individual probabilities and the probability distribution.

sample space outcomes	value of X	probability of individual outcome
Right	1	1/3
Wrong	0	2/3

2. Now what if the student takes a quiz with three questions.

   (a) Fill out the table of probabilities.

sample space outcomes	probability of individual outcome	probability of individual outcome (rewrite using exponents)	value of X
RRR	$\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} = \frac{1}{27}$	$\left(\frac{1}{3}\right)^3\cdot\left(\frac{2}{3}\right)^0$	3
RRW	$\frac{1}{3}\ \frac{1}{3}\cdot\frac{2}{3} = \frac{2}{27}$	$\left(\frac{1}{3}\right)^2\cdot\left(\frac{2}{3}\right)^1$	2
RWR	$\frac{2}{27}$	$\left(\frac{1}{3}\right)^2\cdot\left(\frac{2}{3}\right)^1$	2
RWW	$\left(\frac{1}{3}\right)\cdot\left(\frac{2}{3}\right)\cdot\left(\frac{2}{3}\right) = \frac{4}{27}$	$\left(\frac{1}{3}\right)^1\cdot\left(\frac{2}{3}\right)^2$	1
WRR	$\left(\frac{2}{3}\right)\cdot\left(\frac{1}{3}\right)\cdot\left(\frac{1}{3}\right) = \frac{2}{27}$	$\left(\frac{1}{3}\right)^2\cdot\left(\frac{2}{3}\right)^1$	2
WRW	$\left(\frac{2}{3}\right)\cdot\left(\frac{1}{3}\right)\cdot\left(\frac{2}{3}\right) = \frac{4}{27}$	$\left(\frac{1}{3}\right)^1\cdot\left(\frac{2}{3}\right)^2$	1
WWR	$\left(\frac{2}{3}\right)\cdot\left(\frac{2}{3}\right)\cdot\left(\frac{1}{3}\right) = \frac{4}{27}$	$\left(\frac{1}{3}\right)^1\cdot\left(\frac{2}{3}\right)^2$	1
WWW	$\left(\frac{2}{3}\right)\cdot\left(\frac{2}{3}\right)\cdot\left(\frac{2}{3}\right) = \frac{8}{27}$	$\left(\frac{1}{3}\right)^0\cdot\left(\frac{2}{3}\right)^3$	0

   (b) Fill out the probability distribution (pmf) of X.

value of X	Probability of X
0	8/27
1	(4/27)·3 = 12/27
2	(2/27)·3 = 6/27
3	1/27

   (c) Rewrite the probability distribution using exponents and multiplication.

value of X	Probability of X
0	$1\cdot(1/3)^0\cdot(2/3)^3$
1	$3\cdot(1/3)^1\cdot(2/3)^2$
2	$3\cdot(1/3)^2\cdot(2/3)^1$
3	$1\cdot(1/3)^3\cdot(2/3)^0$

   (d) What do the exponents always add up to? 3, number of trials

   (e) Compare the exponent on the $\frac{1}{3}$ to the x value. What do you notice?

   It's the same as the value of X

   (f) For the value of $X = 2$, why do you need the exponent of 2 on the $\frac{1}{3}$? X=2 you need 2 correct, $\frac{1}{3}\cdot\frac{1}{3}$

   (g) Why do we have to multiply the probabilities by 3 for $X = 2$?

   There's 3 ways to get 2 correct answers.

3.18 (adef) 4,5,6,9 + PDF
Due [illegible]

3. Now the student has to take a quiz with 10 questions. (Each question still has 3 choices.)

(a) How many possible outcomes are there?

$2^{10} = 1024$ outcomes

(b) Do you want to fill out a table for all the possible outcomes and probabilities?

no!

(c) Instead we should come up with a formula to find the probabilities.

$$\binom{n}{x} \cdot (p(\text{right}))^x \, (1-p(\text{right}))^{n-x}$$

(d) How do we find the number of ways to get x right answers out of 10 questions? (Hint: think of combinations.)

nCr

(e) Write a formula for the pdf of X.

$$P(X=x) = \binom{10}{x} \cdot \left(\frac{1}{3}\right)^x \cdot \left(\frac{2}{3}\right)^{10-x}$$

4. Fill out the probability distribution for the number of correct guesses if 10 students take the taste test.

X	(number of ways to get x correct guesses)	probability of X
0	$\binom{10}{0} = 1$	.0173
1	$\binom{10}{1} = 10$	.0867
2	$\binom{10}{2} = 45$	.1951
3	$\binom{10}{3} = 120$	.2601
4	$\binom{10}{4} = 210$	.2276
5	$\binom{10}{5}$ =252	$252 \cdot \left(\frac{1}{3}\right)^5 \cdot \left(\frac{2}{3}\right)^5 = .1355$
6	$\binom{10}{6}$ =210	$210 \cdot \left(\frac{1}{3}\right)^6 \cdot \left(\frac{2}{3}\right)^4 = .0569$
7	$\binom{10}{7}$ =120	$120 \cdot \left(\frac{1}{3}\right)^7 \cdot \left(\frac{2}{3}\right)^3 = .0163$
8	$\binom{10}{8}$ =45	$45 \cdot \left(\frac{1}{3}\right)^8 \cdot \left(\frac{2}{3}\right)^2 = .0031$
9	$\binom{10}{9}$ =10	$10 \cdot \left(\frac{1}{3}\right)^9 \cdot \left(\frac{2}{3}\right)^1 = .00034$
10	$\binom{10}{10}$ =1	$1 \cdot \left(\frac{1}{3}\right)^{10} \cdot \left(\frac{2}{3}\right)^0 = .00002$

HW2: Binomial Distribution

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3.1: 1adef, 4, 5, 6, 9 + PDF

Due Jun 1

3.1

a) $P(X=3) = \binom{10}{3} .12^3 (.88)^7 = \boxed{0.0847}$

d) $P(X \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$= \boxed{3.085 \times 10^{-5}}$

e) $E(X) = n \times p = 10 \cdot .12 = \boxed{1.2}$

f) $Var(X) = np(1-p) = 10 \cdot .12(.88) = \boxed{1.056}$

4a) indep, n=9, success = hit bullseye $p = 0.09$.

$P(X=2) = \binom{9}{2}(.09)^2(1-.91)^7 = \boxed{0.1507}$

b) $P(X \geq 2) = 1 - (P(X=0) + P(X=1)) = \boxed{0.1912}$

$E(X) = 9(.09) = \boxed{0.81}$

5a) Indep, n=8, success = even #'s, p=.5

$P(X=5) = \binom{8}{5}(.5)^5(1-.5)^3 = \boxed{0.2187}$

b) $p = \frac{1}{6}$, success = 1 "6"

$P(X=1) = \binom{8}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^7 = \boxed{0.3721}$

c) Success = no 4's, $p = \frac{1}{6}$

$P(X=0) = \binom{8}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^8 = \boxed{0.2326}$

6) indep, n=10, success = correct, $p = \frac{1}{5}$

$P(X \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10) = \boxed{0.0009}$

$p = .5$

$P(X \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10) = \boxed{0.1719}$

9a) $P = .6$, indep, success = placed over internet, n = 18

$P(8 \leq X \leq 10) = P(X=8) + P(X=9) + P(X=10) =$

$= \binom{18}{8}(.6)^8(1-.6)^{10} + \binom{18}{9}(.6)^9(1-.6)^9 + \binom{18}{10}(.6)^{10}(1-.6)^8 = \boxed{0.3789}$

b) $P(X \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= \boxed{0.0013}$