Math 5710 Opportunity #3 ♬ W/07/15/20 – H/07/16/20

1. Please print your name legibly. ______Jim Cangelosi ________________________________

2. Examine each of the following propositions, determine whether or not it is true, display your choice by circling either "T" or "F"; for each either prove that your decisions is correct or write at least one paragraph that explains why you decided that the proposition is true or why you decided that the proposition is false:

A. $\varnothing \in [\,0,\ \infty\,)$

T F

Sample proof:

$[\,0,\ \infty\,) = \{\, x \in \mathbb{R} : 0 \le x \,\}$. Thus, only real number are elements of $[\,0,\ \infty\,)$ but $\varnothing$ is not a real number; it is a set. Of course, we proved that $\varnothing \subseteq \mathbb{R}$ but $\varnothing \notin \mathbb{R}$.

B. $(\, V \text{ is the universe} \wedge A \in \{\text{ sets }\} \wedge A^c = A\,) \Rightarrow V = \varnothing$

T F

Sample proof:

From our definition of complement of a set (011F) we have $A^c = \mathrm{V} - A$. And $A^c = \mathrm{V} - A \Rightarrow A \cap A^c = \varnothing$. $\therefore\ A^c = A \Rightarrow \nexists\, x \in A \ni x \notin A^c$. So $\nexists\, x \ni x \in V$.

C. $s, t \in \{\text{ sequences }\} \Rightarrow |\, s \,| = |\, t \,|$

T F

Sample proof:

From our definition of sequence (019A), given $A \in \{\text{sets}\}$, (u is a sequence $\Leftrightarrow u : \mathbb{N} \rightarrow A$). Since $s, t \in \{\text{ sequences }\}$, we have $|\, s \,| = \aleph_0 \wedge |\, t \,| = \aleph_0$.

D. (Ω is a sample space $\wedge$ A is an event of Ω $\wedge$ p is a probability distribution on Ω }) $\Rightarrow$ (x is an outcome of Ω $\Rightarrow$ x is a set)

T F

Sample proof:

An outcome of outcome is an element of Ω. To prove that (x is an outcome of Ω $\Rightarrow$ x is a set) is false, we need a counterexample such as the following one:

In Sybil's experiment, described by Meeting #6's Agenda Item # 5E, Ω = { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) }.

Notice that none of the elements of the sample space is a set; each one is a vector.

E. (Ω is a sample space $\wedge$ A is an event of Ω $\wedge$ p is a probability distribution on Ω }) $\Rightarrow A$ is a set

T F

Sample proof:

From Glossary Item #029E, our definition of event is as follows:

$A \in$ { events of Ω } $\Leftrightarrow A \subseteq \Omega$

Since an event is a subset, an even is a set.

F. (Ω is a sample space $\wedge$ A is an event of Ω $\wedge$ p is a probability distribution on Ω }) $\Rightarrow p$ is a set

T F

Sample proof:

Glossary Item #030, states our definition of probability distribution on Ω (i.e., probability measure):

Definition for *probability measure*:

Given Ω is a sample space $\wedge E = \{$ events of $\Omega \}$,
$(p \in \{$ probability measures on $\Omega \} \Leftrightarrow$
$p : \{ E \rightarrow [\, 0, 1 \,] \ni (p(\Omega) = 1 \wedge (A_1 \subseteq E \wedge A_2 \subseteq E \ \wedge A_1 \cap A_2 = \varnothing) \Rightarrow$
$p(A_1 \cup A_2)) = p(A_1) + p(A_2))$

Thus, p is a function and every function is a set. Let's polish off this proof by reminding ourselves of our definitions for relation and function:

012A. Definition of a *relation from A to B*:

Given $A, B \in \{$ sets $\}$, (r is a *relation* from A to $B \Leftrightarrow r \subseteq A \times B$)

013A. Definition of *function*:

Given $A, B \in \{$sets$\}$, $(f : A \rightarrow B \Leftrightarrow ($ r is a relation from A to B $) \wedge$
$(\forall x \in A, \exists! y \in B \ni (x, y) \in f))$

G. (Ω is a sample space $\wedge A$ is an event of Ω $\wedge$ p is a probability distribution on Ω })
$\Rightarrow p(A)$ is a set

T F

Sample proof:

Once again, let's remind ourselves of our definition of probability distribution (i.e, measure):

Given Ω is a sample space $\wedge E = \{$ events of $\Omega \}$,
$(p \in \{$ probability measures on $\Omega \} \Leftrightarrow$
$p : \{ E \rightarrow [\, 0, 1 \,] \ni (p(\Omega) = 1 \wedge (A_1 \subseteq E \wedge A_2 \subseteq E \ \wedge A_1 \cap A_2 = \varnothing) \Rightarrow$
$p(A_1 \cup A_2)) = p(A_1) + p(A_2))$

Note that $p(A)$ is the probability of Event A occurring. Also note that the codomain of the function p is $[\, 0, 1 \,]$. $\therefore$ $p(A) \in [\, 0, 1 \,]$. As an unspecified element of the range of p, $p(A)$ is a real-numbered variable, not a set.

H. (Ω is a sample space $\wedge$ A is an event of Ω $\wedge$ p is a probability distribution on Ω }) $\Rightarrow$ A is the domain of p

T F

Sample proof:

Once again, revisit our definition of probability distribution. Note that the domain of $p = E$ $= \{$ events of Ω $\}$. Since A is an event of Ω, rather than the set of all events, $A \neq E$.

I. Results from an interval measurement can tenably interpreted as if they were ratio.

T F

Sample explanation:

An interval measurement does not provide data with an absolute zero as does a ratio measurement. So data f rom an interval measurement should not be interpreted as if the data were scaled to the existence of an absolute zero. One can scale down a measurement by using less than what's available but one cannot tenably scale it up by trying to use more than what is available. Thus a ratio measurement could be interpreted as though it were only interval but not vice-versa.

J. Measurement relevance is a sufficient condition for measurement reliability.

T F

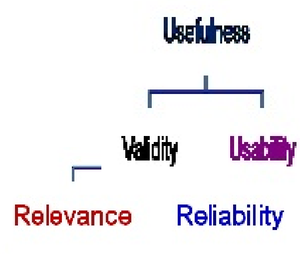
Sample explanation:

Check out Glossary entries 033Ei and 033Eii:

i. Definition for *measurement relevance*: A measurement is *relevant* to the same degree that the data it generates are pertinent to the assessment that is influenced by those data.

ii. Definition for *measurement reliability*: A measurement is *reliable* to the same degree that it generates data that are internally consistent (i.e., the data reflects a non-contradictory pattern).

Measurement relevance and measurement reliability are not interdependent. Relevance is dependent on the association between the qualitative variable to be assessed and the quantitative variable to be measured. Theoretically, they are independent variables. This diagram from the Glossary might help:



K. $(A, B \in \{$ non-empty subsets of $\Omega \wedge A$ and B are mutually-exclusive relative to one another $) \Rightarrow A$ and B are independent of one another.

T F

Sample proof:

As we work through the logical deductions of this proof, keep the following definitions in mind:

037. Independent, dependent, and mutually-exclusive events:

A. Definition for *independent events*:

Given $A \subseteq \Omega \wedge B \subseteq \Omega$, (A and B are independent of one another $\Leftrightarrow$ $p(A \cap B) = p(A)\cdot p(B)$

B. Definition for *dependent events*:

Given $A \subseteq \Omega \wedge B \subseteq \Omega$, (A and B are dependent of one another $\Leftrightarrow$ $p(A \cap B) \neq p(A)\cdot p(B)$)

D. Definition of *mutually exclusive events*:

Given $A \subseteq \Omega \wedge B \subseteq \Omega$, (A and B are mutually-exclusive relative to one another $\Leftrightarrow p(A \cap B) = 0$)

Since $A, B \in \{$ non-empty subsets of Ω , ($p(A) > 0 \wedge p(B) > 0$), thus $p(A)\cdot p(B) \neq 0$.
However, A and B are mutually-exclusive relative to one another) $\Rightarrow p(A \cap B) = 0$.
$\therefore\ p(A \cap B) \neq p(A)\cdot p(B) \Rightarrow A$ and B are dependent of one another.

L. $(D \in \{ \text{ finite sets } \} \wedge t \in \{ \text{ permutations of } D \}) \Rightarrow t \in \{ \text{ finite sets } \}$

T F

Sample proof:

Our definition for permutation is as follows:

34B. Definition of a *permutation* of a finite set:

$$\text{Given } A \in \{ \text{ finite sets } \}, (f \in \{ \text{ permutations of } A \} \Leftrightarrow f : A \underset{onto}{\overset{1:1}{\rightarrow}} A)$$

So a permutation is a function and as we argued in our response to Prompt #2F from this Opportunity #2, $t \in \{ \text{ finite sets } \}$. And to make sure we're convinced that t is finite, note that by the following theorem, $| t | \in \omega$:

034D. Theorem 06: $_nP_r = \dfrac{n!}{(n-r)!}$

M. $(n, r \in \omega \ni r \leq n) \Rightarrow \binom{n}{r} \in \{ \text{ finite sets } \}$

T F

Sample proof:

$\binom{n}{r}$ is not a set because it a natural number as indicated by the following theorem:

34G. Theorem 07: $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

---

3. Fawn teaches an ESOL (English for speakers of other languages) to 30 students. Five of the students only write in Portuguese, 10 only write in Korean, and 15 only write in Spanish. She finds a document belonging to one of the students but she doesn't know whom. The document is not related to the class, so she doesn't think she should read it but she readily sees that it is not written in Korean. Address the following question and display your computation: What is the probability that it is written in Portuguese?

Sample computation:

Let A be the event that it is written in Portuguese and B the event it is NOT written in Korean. Before calculating $p(A|B)$, note that $A \cap B = A$ because a document that is not Korean and Portuguese is written in Portuguese. So we have

$$p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{p(A)}{p(B)} = \frac{5/30}{20/30} = 0.25$$

---

4. Three experiments are conducted:

Experiment 1: One card is randomly drawn from a well-shuffled poker deck consisting of 52 cards – no jokers).

Experiment 2: A ball is randomly drawn from an urn that contains exactly 4 black balls, 3 green balls, 3 yellow balls, and 2 orange balls.

Experiment 3: Experiments 1 and 2 are combined.

What is the probability that Experiment 3 results in the event that an ace is drawn and a black ball is drawn?

Please display the computation that led to your solution.

Sample computation:

$|\,\Omega_1\,| = 52 \;\wedge\; p_1(\text{ an ace drawn }) = 4/52 = 1/13 \approx 0.080$

$|\,\Omega_2\,| = 12 \;\wedge\; p_2(\text{ a back ball is drawn }) = 4/12 = 1/3 \approx 0.333$

$|\,\Omega_3\,| = 624 \;\wedge\; p_3(\text{ an ace is drawn and a ball is drawn }) \approx 0.0370 \times 0.8 = 0.027$

The solution is approximately equal to 0.027

---

5. When you completed our homework assignment for Meeting #1, you described an experiment in response to the following prompt:

E.* Design and describe an experiment that addresses a question about future events. The question should involve a prediction about some population – not about some unique individual member of that population. For example, rather than designing an experiment to help predict whether Jim becomes infected with COVID 19 before August 5, design an experiment to predict whether at least one member of our Math 5710 family will be infected with COVID 19 before August 5. Please post the resulting document (as a PDF file) on the indicated *Assignment* link of *Canvas*.

Although you've already posted your description on the Canvas Assignment link, please either paste it herein or attach it to this document; and then respond to the following prompt:

Write a paragraph that explains how your experiences in Math 5710 up to this point in time have influenced how you would approach or modify your Homework #1-related experiment.

Note: Jim did not offer a sample paragraph because responses are unique to each one of us.

6. Smile you finished taking advantage of Opportunity #3.