6/02/20

Agenda for Math 5710 ♬ Meeting #22 ☻☺ 7/22/20 (8:00 a.m. – 9:10 a.m.)

1. Hello:

Brigham City:	Adam Blakeslee	Ryan Johnson	Tyson Mortensen	
Logan:	David Allen	Natalie Anderson	Kameron Baird	Stephen Brezinski
	Zachary Ellis	Adam Flanders	Brock Francom	Xiang Gao
	Ryan Goodman	Janette Goodridge	Hadley Hamar	Phillip Leifer
	Brittney Miller	Jonathan Mousley	Erika Mueller	Shelby Simpson
	Steven Summers	Matthew White	Zhang Xiaomeng	

2. Note the syllabus' activity list for today:

22 W/7/22	1. Acquaint ourselves and comprehend the role of the following types of discrete random variables and their related probability distributions: geometric, negative binomial, hypergeometric, and Poisson 2. Take advantage of Quiz 22.

3. Briefly raise issues and questions prompted by the following homework assignment:

A. Study our notes from Meeting #21.

B. Comprehend Jim's sample response to Quiz 21.

C. Comprehend Entries #042 & #43A–B of our *Glossary*.

D. From the Video Page of *Canvas*, view with comprehension the videos named "intro bernoulli distributions" and "visualizing binomial distributions."

E*. Please solve the following problems; display the computations, and upload the resulting pdf document on the appropriate Canvas assignment link:

i. A person is randomly selected from a population and tested for COVID-19 infection. A positive test result is labeled a "success" and coded as 1; a negative test result is labeled a "failure" and coded as 0. Again a person is randomly selected from that *same* population (Thus, the first person is still the population; so the two events are independent). The trial is repeated another 3 times. The number of successes is recorded. As of May 26, 2020, one seemingly credible estimate is 30% of the people worldwide are infected; use that figure for this problem. Display the probability distribution for the random variable for this experiment.

ii. Three fair dice are randomly rolled and the sum of the dots on the three upper-facing surfaces is recorded. An even sum is considered a success and coded 1; an odd sum is considered a failure and coded 0. The number of success is recorded. The trial is repeated 5 more times. The number of successes is recorded. Display the probability distribution for the random variable for this experiment.

E. Comprehend Jim's sample responses to the homework prompts that are posted on *Canvas*.

4. Acquaint ourselves and comprehend the role of the following types of discrete random variables and their related probability distributions: geometric, negative binomial, hypergeometric, and Poisson.

A. Comprehend why we're not digging in as deep with these four types of random variables as we have with previous ones; begin with the following note from our Glossary:

> 044A. Note on these Notes #044B–E: Geometric, negative binomial, hypergeometric, and Poisson random variables and their respective probability distributions, like a myriad of other random variables and probability functions are worthy of our attention. These particular four (random variable, probability distributions) ordered pairs are applicable to addressing important real-life problems and for our purposes of Math 5710 they are vital in our quest toward the *Central Limit Theorem*. However, We won't attend to them as we'd like because of course time constraints but we will become aware of their existence for future reference and to help us develop a better feel why Glossary Entry #49 is what it is.

B. Comprehend features of *geometric* random values beginning with the following note from our Glossary:

> 044B. Note on *geometric* experiments and related discrete probability distributions: John R. Rice on p. 40 of *Mathematical Statistics and Data Analysis* (3^{rd}. Ed.), begins the following statement by comparing geometric experiments to binomial experiments: " The **geometric distribution** is also constructed from independent Bernoulli trials but from an infinite sequence. On each trial, a success occurs with probability p, and X is the total number of trials up to and including the first success. So that $X = k$, there must be $k - 1$ failures followed by a success. From the independence of the trials, this occurs with probability
>
> $$P(k) = P(X = k) = (1 - p)^{k-1}p, \qquad k = 1, 2, 3, \ldots$$
>
> Note that these probabilities sum to 1: $\sum_{k=1}^{\infty}(1-p)^{k-1}p = p\sum_{j=0}^{\infty}(1-p)^{j} = 1$"

Further note that Rice's use of the notations "P" and "p", although a popular convention, differs from our usage. We will clarify during one of our class meetings.

C. Comprehend features of *negative binomial* random values beginning with the following note from our Glossary:

> 044C. Note on *negative binomial* experiments and related discrete probability distributions: John R. Rice on p. 41 of *Mathematical Statistics and Data Analysis* (3rd. Ed.) states that, "The **negative binomial distribution** arises as a generalization of the geometric distribution. Suppose that a sequence of independent trials, each with probability of success *p*, is performed until there are *r* successes in all; let *X* denote the total number of trials. To find $P(X = k)$, we can argue in the following way: Any such sequence has probability $p^r(1-p)^{k-r}$, from the independence assumption. The last trial is a success, and the remaining $r – 1$ successes can be assigned to the remaining $k – 1$ trials in $\binom{k-1}{r-1}$ ways. Thus, $P(X = k) = \binom{k-1}{r-1} p^r(1-p)^{k-r}$."

D. Comprehend features of *hypergeometric* random values beginning with the following note from our Glossary:

> 044D. Note on *hypergeometric* experiments and related discrete probability distributions: A hypergeometric experiment is quite similar to a binomial experiment but with one crucial exception: In a binomial experiment, the selected events are independent from one another; whereas in a hypergeometric experiment the selected events are dependent one another because events are selected one at a time without replacement. Consider the following example of hypergeometric experiment:
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> > Five cards are randomly selected from a standard 52-card poker deck and this is done *without replacement*. The goal of the experiment is to determine the probability that exactly two of the selected cards are red.
>
> Now consider a similar experiment that is binomial rather than hypergeometric:
>
> > Five cards are randomly selected one at a time from a standard 52-card poker deck; after the first card is selected, its color is recorded and returned to the deck, and the deck is reshuffled. The same algorithm is repeated until five cards have been drawn. Thus, this is done *with replacement*. The goal of the experiment is to determine the probability that exactly two of the selected cards are red.
>
> For a hypergeometric random variable *X*, the formula for the discrete probability function *p* can be expressed as follows where *N* is the number of elements in the population (e.g., 52), k = the number of successful events in the population (e.g., the number of possible events in which exactly two cards are red), *n* = the number of element in each event (e.g., 5) , and *x* is the number of successes in the random sample (e.g., 2):

$$p(X = x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

E. Comprehend features of *Poisson* random values beginning with the following note from our Glossary:

044E. Binomial random variables are applicable to a myriad of real-world problems. Often we need to calculate $p(k)$ with parameter specified by Glossary line 043A in the following formula:

$$p(k) = \binom{n}{k} m(1)^k (1 - m(1))^{n-k}$$

However, in many cases this calculation is inordinately time-consuming (even for a computer) due the magnitude of n! Poisson random variables and their discrete probability distributions provide approximations that can be within ε of the binomial probabilities (Yes, we are referring to ε from Glossary Lines 023A and 026A). The formula for a discrete probability distribution for a Poisson random variable X is as follows with λ a constant positive real number whose value is selected based on parameters of the experiment including the number of trials:

$$p(X - k) = \frac{\lambda^k}{k!} e^{-k} \text{ where } k \in \omega$$

5. Take advantage of Quiz 22.

6. Complete the following assignment prior to Meeting #23:

   A. Study our notes from Meeting #22.

   B. Comprehend Jim's sample response to Quiz 22.

   C. Comprehend Entry #044A–E of our *Glossary*.

   D. From the Video Page of *Canvas*, view with comprehension the videos named "geometric random variables," "negative binomial distribution," "hypergeometric distributions," and "Poisson process 1 probability and statistics Kahn Academy."

   E*. Please solve the following problems; display the computations, and upload the resulting pdf document on the appropriate Canvas assignment link:

   i. A person is randomly selected from a population and tested for COVID-19 infection. A positive test result is labeled a "success" and coded as 1; a negative test result is labeled a "failure" and coded as 0. If the first person selected is infected, then the experiment is completed. If the first person is not infected than the experiment continues with the same population. This process is repeated until an infected person is selected. As of May 26, 2020, one seemingly credible estimate is 30% of the people worldwide are infected; use that figure for this problem. Compute the probability that exactly 4 trials are executed before an infected person is identified.

ii. Five cards are randomly selected from a standard 52-card poker deck and this is done *without replacement*. Determine the probability that exactly two of the selected cards are red.

E. Comprehend Jim's sample responses to the homework prompts that are posted on *Canvas*.

7. And from Dilbert: