10.1.1 (a) With $z_{0.005} = 2.576$ the confidence interval is

$$\left(\frac{11}{32} - \frac{2.576}{32} \times \sqrt{\frac{11\times(32-11)}{32}}, \frac{11}{32} + \frac{2.576}{32} \times \sqrt{\frac{11\times(32-11)}{32}}\right)$$

$= (0.127, 0.560)$.

(b) With $z_{0.025} = 1.960$ the confidence interval is

$$\left(\frac{11}{32} - \frac{1.960}{32} \times \sqrt{\frac{11\times(32-11)}{32}}, \frac{11}{32} + \frac{1.960}{32} \times \sqrt{\frac{11\times(32-11)}{32}}\right)$$

$= (0.179, 0.508)$.

(c) With $z_{0.01} = 2.326$ the confidence interval is

$$\left(0, \frac{11}{32} + \frac{2.326}{32} \times \sqrt{\frac{11\times(32-11)}{32}}\right)$$

$= (0, 0.539)$.

(d) The exact p-value is $2 \times P(B(32, 0.5) \leq 11) = 0.110$.

The statistic for the normal approximation to the p-value is

$$z = \frac{x-np_0}{\sqrt{np_0(1-p_0)}} = \frac{11-(32\times0.5)}{\sqrt{32\times0.5\times(1-0.5)}} = -1.768$$

and the p-value is $2 \times \Phi(-1.768) = 0.077$.

10.1.2 (a) With $z_{0.005} = 2.576$ the confidence interval is

$$\left(\frac{21}{27} - \frac{2.576}{27} \times \sqrt{\frac{21\times(27-21)}{27}}, \frac{21}{27} + \frac{2.576}{27} \times \sqrt{\frac{21\times(27-21)}{27}}\right)$$

$= (0.572, 0.984)$.

(b) With $z_{0.025} = 1.960$ the confidence interval is

$$\left(\frac{21}{27} - \frac{1.960}{27} \times \sqrt{\frac{21\times(27-21)}{27}}, \frac{21}{27} + \frac{1.960}{27} \times \sqrt{\frac{21\times(27-21)}{27}}\right)$$

$= (0.621, 0.935)$.

(c) With $z_{0.05} = 1.645$ the confidence interval is

$$\left(\frac{21}{27} - \frac{1.645}{27} \times \sqrt{\frac{21\times(27-21)}{27}}, 1\right)$$

$= (0.646, 1)$.

(d) The exact p-value is $P(B(27, 0.6) \geq 21) = 0.042$.
The statistic for the normal approximation to the p-value is

$$z = \frac{x-np_0}{\sqrt{np_0(1-p_0)}} = \frac{21-(27\times0.6)}{\sqrt{27\times0.6\times(1-0.6)}} = 1.886$$

and the p-value is $1 - \Phi(1.886) = 0.030$.

10.1.3 (a) Let p be the probability that a value produced by the random number generator is a zero, and consider the hypotheses

$H_0 : p = 0.5$ versus $H_A : p \neq 0.5$

where the alternative hypothesis states that the random number generator is producing 0's and 1's with unequal probabilities.

The statistic for the normal approximation to the p-value is

$$z = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{25264 - (50000 \times 0.5)}{\sqrt{50000 \times 0.5 \times (1-0.5)}} = 2.361$$

and the p-value is $2 \times \Phi(-2.361) = 0.018$.

There is a fairly strong suggestion that the random number generator is producing 0's and 1's with unequal probabilities, although the evidence is not completely overwhelming.

(b) With $z_{0.005} = 2.576$ the confidence interval is

$$\left(\frac{25264}{50000} - \frac{2.576}{50000} \times \sqrt{\frac{25264 \times (50000-25264)}{50000}}, \frac{25264}{50000} + \frac{2.576}{50000} \times \sqrt{\frac{25264 \times (50000-25264)}{50000}} \right)$$

$= (0.4995, 0.5110)$.

(c) Using the worst case scenario

$\hat{p}(1-\hat{p}) = 0.25$

the total sample size required can be calculated as

$$n \geq \frac{4\, z_{\alpha/2}^2\, \hat{p}(1-\hat{p})}{L^2}$$

$$= \frac{4 \times 2.576^2 \times 0.25}{0.005^2} = 265431.04$$

so that an additional sample size of $265432 - 50000 \simeq 215500$ would be required.

10.1.4 With $z_{0.05} = 1.645$ the confidence interval is

$$\left(\frac{35}{44} - \frac{1.645}{44} \times \sqrt{\frac{35 \times (44-35)}{44}}, 1 \right)$$

$= (0.695, 1)$.

10.1.5 Let p be the probability that a six is scored on the die and consider the hypotheses

$H_0 : p \geq \frac{1}{6}$ versus $H_A : p < \frac{1}{6}$

where the alternative hypothesis states that the die has been weighted to reduce the chance of scoring a six.

In the first experiment the exact p-value is

$$P\left(B\left(50, \tfrac{1}{6}\right) \leq 2\right) = 0.0066$$

and in the second experiment the exact p-value is

$$P\left(B\left(100, \tfrac{1}{6}\right) \leq 4\right) = 0.0001$$

so that there is more support for foul play from the second experiment than from the first.

10.1.6 The exact p-value is

$$2 \times P\left(B\left(100, \tfrac{1}{6}\right) \geq 21\right) = 0.304$$

and the null hypothesis is accepted at size $\alpha = 0.05$.

10.1.7 Let p be the probability that a juror is selected from the county where the investigator lives, and consider the hypotheses

$H_0 : p = 0.14$ versus $H_A : p \neq 0.14$

where the alternative hypothesis implies that the jurors are not being randomly selected.

The statistic for the normal approximation to the p-value is

$$z = \frac{x-np_0}{\sqrt{np_0(1-p_0)}} = \frac{122-(1{,}386\times 0.14)}{\sqrt{1{,}386\times 0.14\times(1-0.14)}} = -5.577$$

and the p-value is $2 \times \Phi(-5.577) = 0.000$.

There is sufficient evidence to conclude that the jurors are not being randomly selected.

10.1.8 The statistic for the normal approximation to the p-value is

$$z = \frac{x-np_0}{\sqrt{np_0(1-p_0)}} = \frac{23-(324\times 0.1)}{\sqrt{324\times 0.1\times(1-0.1)}} = -1.741$$

and the p-value is $\Phi(-1.741) = 0.041$.

With $z_{0.01} = 2.326$ the confidence interval is

$$\left(0, \frac{23}{324} + \frac{2.326}{324} \times \sqrt{\frac{23\times(324-23)}{324}}\right)$$

$= (0, 0.104)$.

It has not been conclusively shown that the screening test is acceptable.

10.1.9 With $z_{0.025} = 1.960$ and $L = 0.02$

the required sample size for the worst case scenario with

$\hat{p}(1-\hat{p}) = 0.25$

can be calculated as

$$n \geq \frac{4\, z_{\alpha/2}^2\, \hat{p}(1-\hat{p})}{L^2} = \frac{4\times 1.960^2\times 0.25}{0.02^2} = 9604.$$

If it can be assumed that

$\hat{p}(1-\hat{p}) \leq 0.75 \times 0.25 = 0.1875$

then the required sample size can be calculated as

$$n \geq \frac{4\, z_{\alpha/2}^2\, \hat{p}(1-\hat{p})}{L^2} = \frac{4\times 1.960^2\times 0.1875}{0.02^2} = 7203.$$

10.1.10 With $z_{0.005} = 2.576$ and $L = 0.04$

the required sample size for the worst case scenario with

$\hat{p}(1-\hat{p}) = 0.25$

can be calculated as

$n \geq \frac{4\, z_{\alpha/2}^2\, \hat{p}(1-\hat{p})}{L^2} = \frac{4 \times 2.576^2 \times 0.25}{0.04^2} = 4148.$

If it can be assumed that

$\hat{p}(1-\hat{p}) \leq 0.4 \times 0.6 = 0.24$

then the required sample size can be calculated as

$n \geq \frac{4\, z_{\alpha/2}^2\, \hat{p}(1-\hat{p})}{L^2} = \frac{4 \times 2.576^2 \times 0.24}{0.04^2} = 3982.$

10.1.11 With $z_{0.005} = 2.576$ the confidence interval is

$\left(\frac{73}{120} - \frac{2.576}{120} \times \sqrt{\frac{73 \times (120-73)}{120}}, \frac{73}{120} + \frac{2.576}{120} \times \sqrt{\frac{73 \times (120-73)}{120}}\right)$

$= (0.494, 0.723).$

Using

$\hat{p}(1-\hat{p}) = \frac{73}{120} \times \left(1 - \frac{73}{120}\right) = 0.238$

the total sample size required can be calculated as

$n \geq \frac{4\, z_{\alpha/2}^2\, \hat{p}(1-\hat{p})}{L^2} = \frac{4 \times 2.576^2 \times 0.238}{0.1^2} = 631.7$

so that an additional sample size of $632 - 120 = 512$ would be required.

10.1.12 Let p be the proportion of defective chips in the shipment.

With $z_{0.05} = 1.645$ a 95% upper confidence bound on p is

$\left(0, \frac{8}{200} + \frac{1.645}{200} \times \sqrt{\frac{8 \times (200-8)}{200}}\right)$

$= (0, 0.06279).$

A 95% upper confidence bound on the total number of defective chips in the shipment can therefore be calculated as

$0.06279 \times 100000 = 6279$ chips.

10.1.13 With $z_{0.025} = 1.960$ the confidence interval is

$$\left(\frac{12}{20} - \frac{1.960}{20} \times \sqrt{\frac{12\times(20-12)}{20}}, \frac{12}{20} + \frac{1.960}{20} \times \sqrt{\frac{12\times(20-12)}{20}}\right)$$

$= (0.385, 0.815)$.

10.1.14 Let p be the proportion of the applications that contained errors.

With $z_{0.05} = 1.645$ a 95% lower confidence bound on p is

$$\left(\frac{17}{85} - \frac{1.645}{85} \times \sqrt{\frac{17\times(85-17)}{85}}, 1\right)$$

$= (0.1286, 1)$.

A 95% lower confidence bound on the total number of applications which contained errors can therefore be calculated as

$0.1286 \times 7607 = 978.5$ or 979 applications.

10.1.15 With $z_{0.025} = 1.960$ and $L = 0.10$

the required sample size for the worst case scenario with

$\hat{p}(1 - \hat{p}) = 0.25$

can be calculated as

$$n \geq \frac{4\, z_{\alpha/2}^2\, \hat{p}(1-\hat{p})}{L^2} = \frac{4\times 1.960^2 \times 0.25}{0.10^2} = 384.2$$

or 385 householders.

If it can be assumed that

$\hat{p}(1 - \hat{p}) \leq 0.333 \times 0.667 = 0.222$

then the required sample size can be calculated as

$$n \geq \frac{4\, z_{\alpha/2}^2\, \hat{p}(1-\hat{p})}{L^2} = \frac{4\times 1.960^2 \times 0.222}{0.10^2} = 341.1$$

or 342 householders.

10.1.16 With $z_{0.005} = 2.576$ the confidence interval is

$$\left(\frac{22}{542} - \frac{2.576}{542} \times \sqrt{\frac{22\times(542-22)}{542}}, \frac{22}{542} + \frac{2.576}{542} \times \sqrt{\frac{22\times(542-22)}{542}}\right)$$

$= (0.019, 0.062)$.

10.1.17 The standard confidence interval is $(0.161, 0.557)$.

The alternative confidence interval is $(0.195, 0.564)$.

10.1.18 (a) Let p be the probability that the dielectric breakdown strength is below the threshold level, and consider the hypotheses

$H_0 : p \leq 0.05$ versus $H_A : p > 0.05$

where the alternative hypothesis states that the probability of an insulator of this type having a dielectric breakdown strength below the specified threshold level is larger than 5%.

The statistic for the normal approximation to the p-value is

$$z = \frac{x-np_0-0.5}{\sqrt{np_0(1-p_0)}} = \frac{13-(62\times 0.05)-0.5}{\sqrt{62\times 0.05\times(1-0.05)}} = 5.48$$

and the p-value is $1 - \Phi(5.48) = 0.000$.

There is sufficient evidence to conclude that the probability of an insulator of this type having a dielectric breakdown strength below the specified threshold level is larger than 5%.

(b) With $z_{0.05} = 1.645$ the confidence interval is

$$\left(\frac{13}{62} - \frac{1.645}{62}\sqrt{\frac{13(62-13)}{62}}, 1\right) = (0.125, 1).$$

10.1.19 $\hat{p} = \frac{31}{210} = 0.148$

With $z_{0.005} = 2.576$ the confidence interval is

$$p \in 0.148 \pm \frac{2.576}{210}\sqrt{\frac{31\times(210-31)}{210}}$$

$$= (0.085, 0.211).$$

10.1.20 Let p be the probability of preferring cushion type A.

Then

$\hat{p} = \frac{28}{38} = 0.737$

and the hypotheses of interest are

$H_0 : p \leq \frac{2}{3}$ versus $H_A : p > \frac{2}{3}$.

The test statistic is

$z = \frac{28-(38\times 2/3)-0.5}{\sqrt{38\times 2/3\times 1/3}} = 0.75$

and the p-value is $1 - \Phi(0.75) = 0.227$.

The data set does not provide sufficient evidence to establish that cushion type A is at least twice as popular as cushion type B.

10.1.21 If $793 = \frac{z^2_{\alpha/2}}{(2\times 0.035)^2}$

then $z^2_{\alpha/2} = 1.97$ so that $\alpha \simeq 0.05$.

Therefore, the margin of error was calculated with 95% confidence under the worst case scenario where the estimated probability could be close to 0.5.

10.1.22 A

10.2.1 (a) With $z_{0.005} = 2.576$ the confidence interval is

$$\frac{14}{37} - \frac{7}{26} \pm 2.576 \times \sqrt{\frac{14\times(37-14)}{37^3} + \frac{7\times(26-7)}{26^3}}$$

$= (-0.195, 0.413)$.

(b) With $z_{0.025} = 1.960$ the confidence interval is

$$\frac{14}{37} - \frac{7}{26} \pm 1.960 \times \sqrt{\frac{14\times(37-14)}{37^3} + \frac{7\times(26-7)}{26^3}}$$

$= (-0.122, 0.340)$.

(c) With $z_{0.01} = 2.326$ the confidence interval is

$$\left(\frac{14}{37} - \frac{7}{26} - 2.326 \times \sqrt{\frac{14\times(37-14)}{37^3} + \frac{7\times(26-7)}{26^3}}, 1\right)$$

$= (-0.165, 1)$.

(d) With the pooled probability estimate

$$\hat{p} = \frac{x+y}{n+m} = \frac{14+7}{37+26} = 0.333$$

the test statistic is

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{14}{37} - \frac{7}{26}}{\sqrt{0.333\times(1-0.333)\times\left(\frac{1}{37}+\frac{1}{26}\right)}} = 0.905$$

and the p-value is $2 \times \Phi(-0.905) = 0.365$.

10.2.2 (a) With $z_{0.005} = 2.576$ the confidence interval is

$$\frac{261}{302} - \frac{401}{454} \pm 2.576 \times \sqrt{\frac{261\times(302-261)}{302^3} + \frac{401\times(454-401)}{454^3}}$$

$= (-0.083, 0.045)$.

(b) With $z_{0.05} = 1.645$ the confidence interval is

$$\frac{261}{302} - \frac{401}{454} \pm 1.645 \times \sqrt{\frac{261\times(302-261)}{302^3} + \frac{401\times(454-401)}{454^3}}$$

$= (-0.060, 0.022)$.

(c) With $z_{0.05} = 1.645$ the confidence interval is

$$\left(-1, \frac{261}{302} - \frac{401}{454} + 1.645 \times \sqrt{\frac{261\times(302-261)}{302^3} + \frac{401\times(454-401)}{454^3}}\right)$$

$= (-1, 0.022)$.

(d) With the pooled probability estimate

$$\hat{p} = \frac{x+y}{n+m} = \frac{261+401}{302+454} = 0.876$$

the test statistic is

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{261}{302} - \frac{401}{454}}{\sqrt{0.876\times(1-0.876)\times\left(\frac{1}{302}+\frac{1}{454}\right)}} = -0.776$$

and the p-value is $2 \times \Phi(-0.776) = 0.438$.

10.2.3 (a) With $z_{0.005} = 2.576$ the confidence interval is

$$\frac{35}{44} - \frac{36}{52} \pm 2.576 \times \sqrt{\frac{35\times(44-35)}{44^3} + \frac{36\times(52-36)}{52^3}}$$

$= (-0.124, 0.331)$.

(b) With the pooled probability estimate

$$\hat{p} = \frac{x+y}{n+m} = \frac{35+36}{44+52} = 0.740$$

the test statistic is

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{35}{44} - \frac{36}{52}}{\sqrt{0.740\times(1-0.740)\times\left(\frac{1}{44}+\frac{1}{52}\right)}} = 1.147$$

and the p-value is $2 \times \Phi(-1.147) = 0.251$.

There is *not* sufficient evidence to conclude that one radar system is any better than the other radar system.

10.2.4 (a) With $z_{0.005} = 2.576$ the confidence interval is

$$\frac{4}{50} - \frac{10}{50} \pm 2.576 \times \sqrt{\frac{4\times(50-4)}{50^3} + \frac{10\times(50-10)}{50^3}}$$

$$= (-0.296, 0.056).$$

(b) With the pooled probability estimate

$$\hat{p} = \frac{x+y}{n+m} = \frac{4+10}{50+50} = 0.14$$

the test statistic is

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{4}{50}-\frac{10}{50}}{\sqrt{0.14\times(1-0.14)\times\left(\frac{1}{50}+\frac{1}{50}\right)}} = -1.729$$

and the p-value is $2 \times \Phi(-1.729) = 0.084$.

(c) In this case the confidence interval is

$$\frac{40}{500} - \frac{100}{500} \pm 2.576 \times \sqrt{\frac{40\times(500-40)}{500^3} + \frac{100\times(500-100)}{500^3}}$$

$$= (-0.176, -0.064).$$

With the pooled probability estimate

$$\hat{p} = \frac{x+y}{n+m} = \frac{40+100}{500+500} = 0.14$$

the test statistic is

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{40}{500}-\frac{100}{500}}{\sqrt{0.14\times(1-0.14)\times\left(\frac{1}{500}+\frac{1}{500}\right)}} = -5.468$$

and the p-value is $2 \times \Phi(-5.468) = 0.000$.

10.2.5 Let p_A be the probability of crystallization within 24 hours *without* seed crystals and let p_B be the probability of crystallization within 24 hours *with* seed crystals.

With $z_{0.05} = 1.645$ a 95% upper confidence bound for $p_A - p_B$ is

$$\left(-1, \frac{27}{60} - \frac{36}{60} + 1.645 \times \sqrt{\frac{27\times(60-27)}{60^3} + \frac{36\times(60-36)}{60^3}}\right)$$

$= (-1, -0.002)$.

Consider the hypotheses

$H_0 : p_A \geq p_B$ versus $H_A : p_A < p_B$

where the alternative hypothesis states that the presence of seed crystals increases the probability of crystallization within 24 hours.

With the pooled probability estimate

$$\hat{p} = \frac{x+y}{n+m} = \frac{27+36}{60+60} = 0.525$$

the test statistic is

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{27}{60} - \frac{36}{60}}{\sqrt{0.525\times(1-0.525)\times\left(\frac{1}{60}+\frac{1}{60}\right)}} = -1.645$$

and the p-value is $\Phi(-1.645) = 0.050$.

There is some evidence that the presence of seed crystals increases the probability of crystallization within 24 hours but it is not overwhelming.

10.2.6 Let p_A be the probability of an improved condition with the standard drug and let p_B be the probability of an improved condition with the new drug.

With $z_{0.05} = 1.645$ a 95% upper confidence bound for $p_A - p_B$ is

$$\left(-1, \frac{72}{100} - \frac{83}{100} + 1.645 \times \sqrt{\frac{72\times(100-72)}{100^3} + \frac{83\times(100-83)}{100^3}}\right)$$

$= (-1, -0.014)$.

Consider the hypotheses

$H_0 : p_A \geq p_B$ versus $H_A : p_A < p_B$

where the alternative hypothesis states that the new drug increases the probability of an improved condition.

With the pooled probability estimate

$$\hat{p} = \frac{x+y}{n+m} = \frac{72+83}{100+100} = 0.775$$

the test statistic is

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{72}{100} - \frac{83}{100}}{\sqrt{0.775\times(1-0.775)\times\left(\frac{1}{100}+\frac{1}{100}\right)}} = -1.863$$

and the p-value is $\Phi(-1.863) = 0.031$.

There is some evidence that the new drug increases the probability of an improved condition but it is not overwhelming.

10.2.7 Let p_A be the probability that a television set from production line A does not meet the quality standards and let p_B be the probability that a television set from production line B does not meet the quality standards.

With $z_{0.025} = 1.960$ a 95% two-sided confidence interval for $p_A - p_B$ is

$\frac{23}{1128} - \frac{24}{962} \pm 1.960 \times \sqrt{\frac{23\times(1128-23)}{1128^3} + \frac{24\times(962-24)}{962^3}}$

$= (-0.017, 0.008)$.

Consider the hypotheses

$H_0 : p_A = p_B$ versus $H_A : p_A \neq p_B$

where the alternative hypothesis states that there is a difference in the operating standards of the two production lines.

With the pooled probability estimate

$\hat{p} = \frac{x+y}{n+m} = \frac{23+24}{1128+962} = 0.022$

the test statistic is

$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{23}{1128} - \frac{24}{962}}{\sqrt{0.022\times(1-0.022)\times\left(\frac{1}{1128}+\frac{1}{962}\right)}} = -0.708$

and the p-value is $2 \times \Phi(-0.708) = 0.479$.

There is *not* sufficient evidence to conclude that there is a difference in the operating standards of the two production lines.

10.2.8 Let p_A be the probability of a successful outcome for the standard procedure and let p_B be the probability of a successful outcome for the new procedure.

With $z_{0.05} = 1.645$ a 95% upper confidence bound for $p_A - p_B$ is

$$\left(-1, \frac{73}{120} - \frac{101}{120} + 1.645 \times \sqrt{\frac{73\times(120-73)}{120^3} + \frac{101\times(120-101)}{120^3}}\right)$$

$= (-1, -0.142)$.

Consider the hypotheses

$H_0 : p_A \geq p_B$ versus $H_A : p_A < p_B$

where the alternative hypothesis states that the new procedure increases the probability of a successful outcome.

With the pooled probability estimate

$$\hat{p} = \frac{x+y}{n+m} = \frac{73+101}{120+120} = 0.725$$

the test statistic is

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{73}{120} - \frac{101}{120}}{\sqrt{0.725\times(1-0.725)\times\left(\frac{1}{120}+\frac{1}{120}\right)}} = -4.05$$

and the p-value is $\Phi(-4.05) \simeq 0.0000$.

There is sufficient evidence to conclude that the new procedure increases the probability of a successful outcome.

10.2.9 Let p_A be the probability that a computer chip from supplier A is defective and let p_B be the probability that a computer chip from supplier B is defective.

With $z_{0.025} = 1.960$ a 95% two-sided confidence interval for $p_A - p_B$ is

$$\frac{8}{200} - \frac{13}{250} \pm 1.960 \times \sqrt{\frac{8\times(200-8)}{200^3} + \frac{13\times(250-13)}{250^3}}$$

$= (-0.051, 0.027)$.

Consider the hypotheses

$H_0 : p_A = p_B$ versus $H_A : p_A \neq p_B$

where the alternative hypothesis states that there is a difference in the quality of the computer chips from the two suppliers.

With the pooled probability estimate

$$\hat{p} = \frac{x+y}{n+m} = \frac{8+13}{200+250} = 0.047$$

the test statistic is

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{8}{200} - \frac{13}{250}}{\sqrt{0.047\times(1-0.047)\times\left(\frac{1}{200}+\frac{1}{250}\right)}} = -0.600$$

and the p-value is $2 \times \Phi(-0.600) = 0.549$.

There is *not* sufficient evidence to conclude that there is a difference in the quality of the computer chips from the two suppliers.

10.2.10 Let p_A be the probability of an error in an application processed during the first two weeks and let p_B be the probability of an error in an application processed after the first two weeks.

With $z_{0.05} = 1.645$ a 95% lower confidence bound for $p_A - p_B$ is

$$\left(\frac{17}{85} - \frac{16}{132} - 1.645 \times \sqrt{\frac{17\times(85-17)}{85^3} + \frac{16\times(132-16)}{132^3}}, 1\right)$$

$= (-0.007, 1)$.

Consider the hypotheses

$H_0 : p_A \leq p_B$ versus $H_A : p_A > p_B$

where the alternative hypothesis states that the probability of an error in the processing of an application is larger during the first two weeks.

With the pooled probability estimate

$\hat{p} = \frac{x+y}{n+m} = \frac{17+16}{85+132} = 0.152$

the test statistic is

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{17}{85} - \frac{16}{132}}{\sqrt{0.152\times(1-0.152)\times\left(\frac{1}{85}+\frac{1}{132}\right)}} = 1.578$$

and the p-value is $1 - \Phi(1.578) = 0.057$.

There is some evidence that the probability of an error in the processing of an application is larger during the first two weeks but it is not overwhelming.

10.2.11 With the pooled probability estimate

$\hat{p} = \frac{x+y}{n+m} = \frac{159+138}{185+185} = 0.803$

the test statistic is

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{159}{185} - \frac{138}{185}}{\sqrt{0.803\times(1-0.803)\times\left(\frac{1}{185}+\frac{1}{185}\right)}} = 2.745$$

and the two-sided p-value is $2 \times \Phi(-2.745) = 0.006$.

The two-sided null hypothesis $H_0 : p_A = p_B$ is rejected and there is sufficient evidence to conclude that machine A is better than machine B.

10.2.12 Let p_A be the probability of a link being followed with the original design and let p_B be the probability of a link being followed with the modified design.

With $z_{0.05} = 1.645$ a 95% upper confidence bound for $p_A - p_B$ is

$$\left(-1, \frac{22}{542} - \frac{64}{601} + 1.645 \times \sqrt{\frac{22\times(542-22)}{542^3} + \frac{64\times(601-64)}{601^3}}\right)$$

$= (-1, -0.041)$.

Consider the hypotheses

$H_0 : p_A \geq p_B$ versus $H_A : p_A < p_B$

where the alternative hypothesis states that the probability of a link being followed is larger after the modifications.

With the pooled probability estimate

$$\hat{p} = \frac{x+y}{n+m} = \frac{22+64}{542+601} = 0.0752$$

the test statistic is

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{22}{542} - \frac{64}{601}}{\sqrt{0.0752\times(1-0.0752)\times\left(\frac{1}{542}+\frac{1}{601}\right)}} = -4.22$$

and the p-value is $\Phi(-4.22) \simeq 0.000$.

There is sufficient evidence to conclude that the probability of a link being followed has been increased by the modifications.

10.2.13 (a) Consider the hypotheses

$H_0 : p_{180} \geq p_{250}$ versus $H_A : p_{180} < p_{250}$

where the alternative hypothesis states that the probability of an insulator of this type having a dielectric breakdown strength below the specified threshold level is larger at 250 degrees Centigrade than it is at 180 degrees Centigrade.

With the pooled probability estimate

$\frac{x+y}{n+m} = \frac{13+20}{62+70} = 0.25$

the test statistic is

$z = \frac{\hat{p}_{180} - \hat{p}_{250}}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{13}{62} - \frac{20}{70}}{\sqrt{0.25\times(1-0.25)\left(\frac{1}{62}+\frac{1}{70}\right)}} = -1.007$

and the p-value is $\Phi(-1.007) = 0.1570$.

There is not sufficient evidence to conclude that the probability of an insulator of this type having a dielectric breakdown strength below the specified threshold level is larger at 250 degrees Centigrade than it is at 180 degrees Centigrade.

(b) With $z_{0.005} = 2.576$ the confidence interval is

$\frac{13}{62} - \frac{20}{70} \pm 2.576 \times \sqrt{\frac{13\times(62-13)}{62^3} + \frac{20\times(70-20)}{70^3}}$

$= (-0.269, 0.117)$.

10.2.14 $\hat{p}_A = \frac{72}{125} = 0.576$

$\hat{p}_B = \frac{60}{125} = 0.480$

The pooled estimate is

$\hat{p} = \frac{72+60}{125+125} = 0.528$

and the hypotheses are

$H_0 : p_A = p_B$ versus $H_A : p_A \neq p_B$.

The test statistic is

$z = \frac{0.576-0.480}{\sqrt{0.528\times0.472\times\left(\frac{1}{125}+\frac{1}{125}\right)}} = 1.520$

and the p-value is $2 \times \Phi(-1.520) = 0.128$.

There is not sufficient evidence to conclude that there is a difference between the two treatments.

10.2.15 $\hat{p}_1 = \frac{76}{243} = 0.313$

$\hat{p}_2 = \frac{122}{320} = 0.381$

With $z_{0.005} = 2.576$ the confidence interval is

$p_1 - p_2 \in 0.313 - 0.381 \pm 2.576 \times \sqrt{\frac{76\times(243-76)}{243^3} + \frac{122\times(320-122)}{320^3}}$

$= (-0.172, 0.036)$

The confidence interval contains zero so there is not sufficient evidence to conclude that the failure rates due to operator misuse are different for the two products.

10.2.16 C

10.3.1 (a) The expected cell frequencies are $e_i = \frac{500}{6} = 83.33$.

(b) The Pearson chi-square statistic is

$X^2 = \frac{(80-83.33)^2}{83.33} + \frac{(71-83.33)^2}{83.33} + \frac{(90-83.33)^2}{83.33} + \frac{(87-83.33)^2}{83.33}$

$+ \frac{(78-83.33)^2}{83.33} + \frac{(94-83.33)^2}{83.33} = 4.36.$

(c) The likelihood ratio chi-square statistic is

$G^2 = 2 \times \left(80 \ln\left(\frac{80}{83.33}\right) + 71 \ln\left(\frac{71}{83.33}\right) + 90 \ln\left(\frac{90}{83.33}\right)\right.$

$\left. + 87 \ln\left(\frac{87}{83.33}\right) + 78 \ln\left(\frac{78}{83.33}\right) + 94 \ln\left(\frac{94}{83.33}\right)\right) = 4.44.$

(d) The p-values are $P(\chi^2_5 \geq 4.36) = 0.499$ and $P(\chi^2_5 \geq 4.44) = 0.488$.

A size $\alpha = 0.01$ test of the null hypothesis that the die is fair is accepted.

(e) With $z_{0.05} = 1.645$ the confidence interval is

$\left(\frac{94}{500} - \frac{1.645}{500} \times \sqrt{\frac{94\times(500-94)}{500}}, \frac{94}{500} + \frac{1.645}{500} \times \sqrt{\frac{94\times(500-94)}{500}}\right)$

$= (0.159, 0.217).$

10.3.2 The expected cell frequencies are

1	2	3	4	5	6	7	8	9	$\geq$10
50.00	41.67	34.72	28.94	24.11	20.09	16.74	13.95	11.62	58.16

The Pearson chi-square statistic is $X^2 = 10.33$.

The p-value is $P(\chi_9^2 \geq 10.33) = 0.324$.

The geometric distribution with $p = \frac{1}{6}$ is plausible.

10.3.3 (a) The expected cell frequencies are:

$e_1 = 221 \times \frac{4}{7} = 126.29$

$e_2 = 221 \times \frac{2}{7} = 63.14$

$e_3 = 221 \times \frac{1}{7} = 31.57$

The Pearson chi-square statistic is

$X^2 = \frac{(113-126.29)^2}{126.29} + \frac{(82-63.14)^2}{63.14} + \frac{(26-31.57)^2}{31.57} = 8.01.$

The p-value is $P(\chi_2^2 \geq 8.01) = 0.018$.

There is a fairly strong suggestion that the supposition is not plausible although the evidence is not completely overwhelming.

(b) With $z_{0.005} = 2.576$ the confidence interval is

$\left(\frac{113}{221} - \frac{2.576}{221} \times \sqrt{\frac{113\times(221-113)}{221}}, \frac{113}{221} + \frac{2.576}{221} \times \sqrt{\frac{113\times(221-113)}{221}}\right)$

$= (0.425, 0.598).$

10.3.4 The expected cell frequencies are:

$e_1 = 964 \times 0.14 = 134.96$

$e_2 = 964 \times 0.22 = 212.08$

$e_3 = 964 \times 0.35 = 337.40$

$e_4 = 964 \times 0.16 = 154.24$

$e_5 = 964 \times 0.13 = 125.32$

The Pearson chi-square statistic is $X^2 = 14.6$.

The p-value is $P(\chi_4^2 \geq 14.6) = 0.006$.

There is sufficient evidence to conclude that the

jurors have not been selected randomly.

10.3.5 (a) The expected cell frequencies are:

$e_1 = 126 \times 0.5 = 63.0$

$e_2 = 126 \times 0.4 = 50.4$

$e_3 = 126 \times 0.1 = 12.6$

The likelihood ratio chi-square statistic is

$$G^2 = 2 \times \left(56 \ln\left(\frac{56}{63.0}\right) + 51 \ln\left(\frac{51}{50.4}\right) + 19 \ln\left(\frac{19}{12.6}\right)\right) = 3.62.$$

The p-value is $P(\chi_2^2 \geq 3.62) = 0.164$.

These probability values are plausible.

(b) With $z_{0.025} = 1.960$ the confidence interval is

$$\left(\frac{56}{126} - \frac{1.960}{126} \times \sqrt{\frac{56\times(126-56)}{126}}, \frac{56}{126} + \frac{1.960}{126} \times \sqrt{\frac{56\times(126-56)}{126}}\right)$$

$= (0.358, 0.531)$.

10.3.6 If the three soft drink formulations are equally likely
then the expected cell frequencies are

$e_i = 600 \times \frac{1}{3} = 200.$

The Pearson chi-square statistic is

$X^2 = \frac{(225-200)^2}{200} + \frac{(223-200)^2}{200} + \frac{(152-200)^2}{200} = 17.29.$

The p-value is $P(\chi_2^2 \geq 17.29) = 0.0002.$

It is not plausible that the three soft drink formulations are equally likely.

10.3.7 The first two cells should be pooled so that there are 13 cells altogether.

The Pearson chi-square statistic is $X^2 = 92.9$
and the p-value is $P(\chi_{12}^2 \geq 92.9) = 0.0000.$

It is not reasonable to model the number of arrivals with a
Poisson distribution with mean $\lambda = 7$.

10.3.8 A Poisson distribution with mean $\lambda = \bar{x} = 4.49$ can be considered.
The first two cells should be pooled and the last two cells should be pooled
so that there are 9 cells altogether.

The Pearson chi-square statistic is $X^2 = 8.3$
and the p-value is $P(\chi_7^2 \geq 8.3) = 0.307.$

It is reasonable to model the number of radioactive particles emitted
with a Poisson distribution.

10.3.9 According to the genetic theory the probabilities are $\frac{9}{16}$, $\frac{3}{16}$, $\frac{3}{16}$ and $\frac{1}{16}$, so that the expected cell frequencies are:

$e_1 = \frac{9\times727}{16} = 408.9375$

$e_2 = \frac{3\times727}{16} = 136.3125$

$e_3 = \frac{3\times727}{16} = 136.3125$

$e_4 = \frac{1\times727}{16} = 45.4375$

The Pearson chi-square statistic is

$X^2 = \frac{(412-408.9375)^2}{408.9375} + \frac{(121-136.3125)^2}{136.3125}$

$+\frac{(148-136.3125)^2}{136.3125} + \frac{(46-45.4375)^2}{45.4375} = 2.75$

and the likelihood ratio chi-square statistic is

$G^2 = 2 \times \left(412 \ln\left(\frac{412}{408.9375}\right) + 121 \ln\left(\frac{121}{136.3125}\right)\right.$

$\left.+148 \ln\left(\frac{148}{136.3125}\right) + 46 \ln\left(\frac{46}{45.4375}\right)\right) = 2.79.$

The p-values are $P(\chi_3^2 \geq 2.75) = 0.432$ and $P(\chi_3^2 \geq 2.79) = 0.425$ so that the data set is consistent with the proposed genetic theory.

10.3.10 $e_1 = e_2 = e_3 = 205 \times \frac{1}{3} = 68.33$

The Pearson chi-square statistic is

$X^2 = \frac{(83-68.33)^2}{68.33} + \frac{(75-68.33)^2}{68.33} + \frac{(47-68.33)^2}{68.33} = 10.46$

so that the p-value is $P(X_2^2 \geq 10.46) = 0.005$.

There is sufficient evidence to conclude that the three products do not have equal probabilities of being chosen.

10.3.11 (a) $\hat{p}_3 = \frac{489}{630} = 0.776$

The hypotheses are
$H_0 : p_3 = 0.80$ versus $H_A : p_3 \neq 0.80$
and the test statistic is

$z = \frac{489-(630\times 0.8)}{\sqrt{630\times 0.8\times 0.2}} = -1.494.$

The p-value is $2 \times \Phi(-1.494) = 0.135$.

There is not sufficient evidence to conclude that the probability that a solution has normal acidity is not 0.80.

(b) $e_1 = 630 \times 0.04 = 25.2$
$e_2 = 630 \times 0.06 = 37.8$
$e_3 = 630 \times 0.80 = 504.0$
$e_4 = 630 \times 0.06 = 37.8$
$e_5 = 630 \times 0.04 = 25.2$

The Pearson chi-square statistic is

$X^2 = \frac{(34-25.2)^2}{25.2} + \frac{(41-37.8)^2}{37.8} + \frac{(489-504.0)^2}{504.0} + \frac{(52-37.8)^2}{37.8} + \frac{(14-25.2)^2}{25.2} = 14.1$

so that the p-value is $P(X_4^2 \geq 14.1) = 0.007$.

The data are not consistent with the claimed probabilities.

10.3.12 $P(X \leq 24) = 1 - e^{-(0.065\times 24)^{0.45}} = 0.705$

$P(X \leq 48) = 1 - e^{-(0.065\times 48)^{0.45}} = 0.812$

$P(X \leq 72) = 1 - e^{-(0.065\times 72)^{0.45}} = 0.865$

The observed cell frequencies are $x_1 = 12$, $x_2 = 53$, $x_3 = 39$, and $x_4 = 21$.

The expected cell frequencies are:

$e_1 = 125 \times 0.705 = 88.125$

$e_2 = 125 \times (0.812 - 0.705) = 13.375$

$e_3 = 125 \times (0.865 - 0.812) = 6.625$

$e_4 = 125 \times (1 - 0.865) = 16.875$

The Pearson chi-square statistic is

$X^2 = \frac{(12-88.125)^2}{88.125} + \frac{(53-13.375)^2}{13.375} + \frac{(39-6.625)^2}{6.625} + \frac{(21-16.875)^2}{16.875} = 342$

so that the p-value is $P(\chi_3^2 \geq 342) \simeq 0$.

It is not plausible that for these batteries under these storage conditions the time in hours until the charge drops below the threshold level has a Weibull distribution with parameters $\lambda = 0.065$ and $a = 0.45$.

10.3.13 The total sample size is $n = 76$.

Under the specified Poisson distribution the expected cell frequencies are:

$e_1 = 76 \times e^{-2.5} \times \frac{2.5^0}{0!} = 6.238$

$e_2 = 76 \times e^{-2.5} \times \frac{2.5^1}{1!} = 15.596$

$e_3 = 76 \times e^{-2.5} \times \frac{2.5^2}{2!} = 19.495$

$e_4 = 76 \times e^{-2.5} \times \frac{2.5^3}{3!} = 16.246$

$e_5 = 76 \times e^{-2.5} \times \frac{2.5^4}{4!} = 10.154$

$e_6 = 76 - e_1 - e_2 - e_3 - e_4 - e_5 = 8.270$

The Pearson chi-square statistic is

$X^2 = \frac{(3-6.238)^2}{6.238} + \frac{(12-15.596)^2}{15.596} + \frac{(23-19.495)^2}{19.495}$

$+ \frac{(18-16.246)^2}{16.246} + \frac{(13-10.154)^2}{10.154} + \frac{(7-8.270)^2}{8.270} = 4.32$

so that the p-value is $P(\chi_5^2 \geq 4.32) = 0.50$.

It is plausible that the number of shark attacks per year follows a Poisson distribution with mean 2.5.

10.3.14 A

10.3.15 D

10.4.1 (a) The expected cell frequencies are

	Acceptable	Defective
Supplier A	186.25	13.75
Supplier B	186.25	13.75
Supplier C	186.25	13.75
Supplier D	186.25	13.75

(b) The Pearson chi-square statistic is $X^2 = 7.087$.

(c) The likelihood ratio chi-square statistic is $G^2 = 6.889$.

(d) The p-values are $P(\chi_3^2 \geq 7.087) = 0.069$ and $P(\chi_3^2 \geq 6.889) = 0.076$ where the degrees of freedom of the chi-square random variable are calculated as $(4-1) \times (2-1) = 3$.

(e) The null hypothesis that the defective rates are identical for the four suppliers is accepted at size $\alpha = 0.05$.

(f) With $z_{0.025} = 1.960$ the confidence interval is

$\frac{10}{200} \pm \frac{1.960}{200} \times \sqrt{\frac{10\times(200-10)}{200}}$

$= (0.020, 0.080)$.

(g) With $z_{0.025} = 1.960$ the confidence interval is

$\frac{15}{200} - \frac{21}{200} \pm 1.960 \times \sqrt{\frac{15\times(200-15)}{200^3} + \frac{21\times(200-21)}{200^3}}$

$= (-0.086, 0.026)$.

10.4.2 The expected cell frequencies are

	Dead	Slow growth	Medium growth	Strong growth
No fertilizer	57.89	93.84	172.09	163.18
Fertilizer I	61.22	99.23	181.98	172.56
Fertilizer II	62.89	101.93	186.93	177.25

The Pearson chi-square statistic is $X^2 = 13.66$.

The p-value is $P(\chi_6^2 \geq 13.66) = 0.034$ where the degrees of freedom of the chi-square random variable are $(3-1) \times (4-1) = 6$.

There is a fairly strong suggestion that the seedlings growth pattern is different for the different growing conditions, although the evidence is not overwhelming.

10.4.3 The expected cell frequencies are

	Formulation I	Formulation II	Formulation III
10-25	75.00	74.33	50.67
26-50	75.00	74.33	50.67
≥ 51	75.00	74.33	50.67

The Pearson chi-square statistic is $X^2 = 6.11$.

The p-value is $P(\chi_4^2 \geq 6.11) = 0.191$ where the degrees of freedom of the chi-square random variable are calculated as $(3-1) \times (3-1) = 4$.

There is *not* sufficient evidence to conclude that the preferences for the different formulations change with age.

10.4.4 (a) The expected cell frequencies are

	Pass	Fail
Line 1	166.2	13.8
Line 2	166.2	13.8
Line 3	166.2	13.8
Line 4	166.2	13.8
Line 5	166.2	13.8

The Pearson chi-square statistic is $X^2 = 13.72$.

The p-value is $P(\chi_4^2 \geq 13.72) = 0.008$ where the degrees of freedom of the chi-square random variable are calculated as $(5-1) \times (2-1) = 4$.

There is sufficient evidence to conclude that the pass rates are different for the five production lines.

(b) With $z_{0.025} = 1.960$ the confidence interval is

$\frac{11}{180} - \frac{15}{180} \pm 1.960 \times \sqrt{\frac{11 \times (180-11)}{180^3} + \frac{15 \times (180-15)}{180^3}}$

$= (-0.076, 0.031)$.

10.4.5 The expected cell frequencies are

	Completely satisfied	Somewhat satisfied	Not satisfied
Technician 1	71.50	22.36	4.14
Technician 2	83.90	26.24	4.86
Technician 3	45.96	14.37	2.66
Technician 4	57.64	18.03	3.34

The Pearson chi-square statistic is $X^2 = 32.11$.

The p-value is $P(\chi_6^2 \geq 32.11) = 0.000$ where the degrees of freedom of the chi-square random variable are calculated as $(4-1) \times (3-1) = 6$.

There is sufficient evidence to conclude that some technicians are better than others in satisfying their customers.

Note: In this analysis 4 of the cells have expected values less than 5 and it may be preferable to pool together the categories "somewhat satisfied" and "not satisfied". In this case the Pearson chi-square statistic is $X^2 = 31.07$ and comparison with a chi-square distribution with 3 degrees of freedom again gives a p-value of 0.000. The conclusion remains the same.

10.4.7 (a) The expected cell frequencies are

	Less than one week	More than one week
Standard drug	88.63	64.37
New drug	79.37	57.63

The Pearson chi-square statistic is $X^2 = 15.71$.

The p-value is $P(\chi_1^2 \geq 15.71) = 0.0000$ where the degrees of freedom of the chi-square random variable are calculated as $(2-1) \times (2-1) = 1$.

There is sufficient evidence to conclude that $p_s \neq p_n$.

(b) With $z_{0.005} = 2.576$ the confidence interval is

$\frac{72}{153} - \frac{96}{137} \pm 2.576 \times \sqrt{\frac{72\times(153-72)}{153^3} + \frac{96\times(137-96)}{137^3}}$
$= (-0.375, -0.085)$.

10.4.8 The Pearson chi-square statistic is

$X^2 = \frac{1986\times(1078\times 111-253\times 544)^2}{1331\times 655\times 1622\times 364} = 1.247$

which gives a p-value of $P(\chi_1^2 \geq 1.247) = 0.264$ where the degrees of freedom of the chi-square random variable are calculated as $(2-1) \times (2-1) = 1$.

It is plausible that the completeness of the structure and the etch depth are independent factors.

10.4.9 The expected cell frequencies are

Type	Warranty purchased	Warranty not purchased
A	34.84	54.16
B	58.71	91.29
C	43.45	67.55

The Pearson chi-square statistic is $X^2 = 2.347$.

The p-value is $P(\chi_2^2 \geq 2.347) = 0.309$.

The null hypothesis of independence is plausible and there is not sufficient evidence to conclude that the probability of a customer purchasing the extended warranty is different for the three product types.

10.4.10 The expected cell frequencies are

Type	Minor cracking	Medium cracking	Severe cracking
A	35.77	13.09	8.14
B	30.75	11.25	7.00
C	56.48	20.66	12.86

The Pearson chi-square statistic is $X^2 = 5.024$.

The p-value is $P(\chi_4^2 \geq 5.024) = 0.285$.

The null hypothesis of independence is plausible and there is not sufficient evidence to conclude that the three types of asphalt are different with respect to cracking.

10.4.11 A

10.7.1 With $z_{0.025} = 1.960$ the confidence interval is

$$\left(\frac{27}{60} - \frac{1.960}{60} \times \sqrt{\frac{27\times(60-27)}{60}}, \frac{27}{60} + \frac{1.960}{60} \times \sqrt{\frac{27\times(60-27)}{60}}\right)$$

$= (0.324, 0.576)$.

10.7.2 Let p be the probability that a bag of flour is underweight and consider the hypotheses

$H_0 : p \leq \frac{1}{40} = 0.025$ versus $H_A : p > \frac{1}{40} = 0.025$

where the alternative hypothesis states that the consumer watchdog organization can take legal action.

The statistic for the normal approximation to the p-value is

$$z = \frac{x-np_0}{\sqrt{np_0(1-p_0)}} = \frac{18-(500\times 0.025)}{\sqrt{500\times 0.025\times(1-0.025)}} = 1.575$$

and the p-value is $1 - \Phi(1.575) = 0.058$.

There is a fairly strong suggestion that the proportion of underweight bags is more than 1 in 40 although the evidence is not overwhelming.

10.7.3 Let p be the proportion of customers who request the credit card.

With $z_{0.005} = 2.576$ a 99% two-sided confidence interval for p is

$$\left(\frac{384}{5000} - \frac{2.576}{5000} \times \sqrt{\frac{384\times(5000-384)}{5000}}, \frac{384}{5000} + \frac{2.576}{5000} \times \sqrt{\frac{384\times(5000-384)}{5000}}\right)$$

$= (0.0671, 0.0865)$.

The number of customers out of 1,000,000 who request the credit card can be estimated as being between 67,100 and 86,500.

10.7.4 Let p_A be the probability that an operation performed in the morning is a total success and let p_B be the probability that an operation performed in the afternoon is a total success.

With $z_{0.05} = 1.645$ a 95% lower confidence bound for $p_A - p_B$ is

$$\left(\frac{443}{564} - \frac{388}{545} - 1.645 \times \sqrt{\frac{443\times(564-443)}{564^3} + \frac{388\times(545-388)}{545^3}}, 1\right)$$

$= (0.031, 1)$.

Consider the hypotheses

$H_0 : p_A \leq p_B$ versus $H_A : p_A > p_B$

where the alternative hypothesis states that the probability that an operation is a total success is smaller in the afternoon than in the morning.

With the pooled probability estimate

$$\hat{p} = \frac{x+y}{n+m} = \frac{443+388}{564+545} = 0.749$$

the test statistic is

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{443}{564} - \frac{388}{545}}{\sqrt{0.749\times(1-0.749)\times\left(\frac{1}{564}+\frac{1}{545}\right)}} = 2.822$$

and the p-value is $1 - \Phi(2.822) = 0.002$.

There is sufficient evidence to conclude that the probability that an operation is a total success is smaller in the afternoon than in the morning.

10.7.5 Let p_A be the probability that a householder with an income above \$60,000 supports the tax increase and let p_B be the probability that a householder with an income below \$60,000 supports the tax increase.

With $z_{0.025} = 1.960$ a 95% two-sided confidence interval for $p_A - p_B$ is

$$\frac{32}{106} - \frac{106}{221} \pm 1.960 \times \sqrt{\frac{32\times(106-32)}{106^3} + \frac{106\times(221-106)}{221^3}}$$

$= (-0.287, -0.068)$.

Consider the hypotheses

$H_0 : p_A = p_B$ versus $H_A : p_A \neq p_B$

where the alternative hypothesis states that the support for the tax increase does depend upon the householder's income.

With the pooled probability estimate

$$\hat{p} = \frac{x+y}{n+m} = \frac{32+106}{106+221} = 0.422$$

the test statistic is

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n}+\frac{1}{m}\right)}} = \frac{\frac{32}{106} - \frac{106}{221}}{\sqrt{0.422\times(1-0.422)\times\left(\frac{1}{106}+\frac{1}{221}\right)}} = -3.05$$

and the p-value is $2 \times \Phi(-3.05) = 0.002$.

There is sufficient evidence to conclude that the support for the tax increase does depend upon the householder's income.

10.7.6 The expected cell frequencies are:

$e_1 = 619 \times 0.1 = 61.9$

$e_2 = 619 \times 0.8 = 495.2$

$e_3 = 619 \times 0.1 = 61.9$

The Pearson chi-square statistic is

$$X^2 = \frac{(61-61.9)^2}{61.9} + \frac{(486-495.2)^2}{495.2} + \frac{(72-61.9)^2}{61.9} = 1.83$$

so that the p-value is $P(\chi_2^2 \geq 3.62) = 0.400$.

These probability values are plausible.

10.7.7 A Poisson distribution with mean $\lambda = \bar{x} = 2.95$ can be considered.

The last two cells can be pooled so that there are 8 cells altogether.

The Pearson chi-square statistic is $X^2 = 13.1$ and the p-value is
$P(\chi_6^2 \geq 13.1) = 0.041$.

There is some evidence that a Poisson distribution is not appropriate although the evidence is not overwhelming.

10.7.8 If the random numbers have a uniform distribution then the expected cell frequencies are $e_i = 1000$.

The Pearson chi-square statistic is $X^2 = 9.07$ and the
p-value is $P(\chi_9^2 \geq 9.07) = 0.431$.

There is no evidence that the random number generator is not operating correctly.

10.7.10 The expected cell frequencies are

	A	B	C
This year	112.58	78.18	30.23
Last year	211.42	146.82	56.77

The Pearson chi-square statistic is $X^2 = 1.20$.

The p-value is $P(\chi_2^2 \geq 1.20) = 0.549$ where the degrees of freedom of the chi-square random variable are calculated as $(2-1) \times (3-1) = 2$.

There is *not* sufficient evidence to conclude that there has been a change in preferences for the three types of tire between the two years.

10.7.11 The expected cell frequencies are

	Completely healed	Partially healed	No change
Treatment 1	19.56	17.81	6.63
Treatment 2	22.22	20.24	7.54
Treatment 3	14.22	12.95	4.83

The Pearson chi-square statistic is $X^2 = 5.66$.

The p-value is $P(\chi_4^2 \geq 5.66) = 0.226$ where the degrees of freedom of the chi-square random variable are calculated as $(3-1) \times (3-1) = 4$.

There is *not* sufficient evidence to conclude that the three medications are not equally effective.

10.7.12 The expected cell frequencies are

	Computers	Library
Engineering	72.09	70.91
Arts & Sciences	49.91	49.09

The Pearson chi-square statistic is $X^2 = 4.28$.

The p-value is $P(\chi_1^2 \geq 4.28) = 0.039$ where the degrees of freedom of the chi-square random variable are calculated as $(2-1) \times (2-1) = 1$.

There is a fairly strong suggestion that the opinions differ between the two colleges but the evidence is not overwhelming.

10.7.13 (a) Let p be the probability that a part has a length outside the specified tolerance range, and consider the hypotheses

$H_0 : p \leq 0.10$ versus $H_A : p > 0.10$

where the alternative hypothesis states that the probability that a part has a length outside the specified tolerance range is larger than 10%.

The statistic for the normal approximation to the p-value is

$$z = \frac{x - np_0 - 0.5}{\sqrt{np_0(1-p_0)}} = \frac{445 - (3877 \times 0.10) - 0.5}{\sqrt{3877 \times 0.10 \times (1 - 0.10)}} = 3.041$$

and the p-value is $1 - \Phi(3.041) = 0.0012$.

There is sufficient evidence to conclude that the probability that a part has a length outside the specified tolerance range is larger than 10%.

(b) With $z_{0.01} = 2.326$ the confidence interval is

$$\left(\frac{445}{3877} - \frac{2.326}{3877}\sqrt{\frac{445(3877-445)}{3877}}, 1\right)$$

$= (0.103, 1)$.

(c) The Pearson chi-square statistic is

$$X^2 = \frac{3877 \times (161 \times 420 - 3271 \times 25)^2}{186 \times 3691 \times 3432 \times 445} = 0.741$$

which gives a p-value of $P(\chi_1^2 \geq 0.741) = 0.389$ where the degrees of freedom of the chi-square random variable are calculated as $(2-1) \times (2-1) = 1$.

It is plausible that the acceptability of the length and the acceptability of the width of the parts are independent of each other.

10.7.14 (a) The expected cell frequencies are:

$800 \times 0.80 = 640$

$800 \times 0.15 = 120$

$800 \times 0.05 = 40$

The Pearson chi-square statistic is

$X^2 = \frac{(619-640)^2}{640} + \frac{(124-120)^2}{120} + \frac{(57-40)^2}{40} = 8.047$

and the likelihood ratio chi-square statistic is

$G^2 = 2 \times \left(619 \ln\left(\frac{619}{640}\right) + 124 \ln\left(\frac{124}{120}\right) + 57 \ln\left(\frac{57}{40}\right)\right) = 7.204.$

The p-values are $P(\chi_2^2 \geq 8.047) = 0.018$ and $P(\chi_2^2 \geq 7.204) = 0.027$.

There is some evidence that the claims made by the research report are incorrect, although the evidence is not overwhelming.

(b) With $z_{0.01} = 2.326$ the confidence interval is

$\left(0, \frac{57}{800} + \frac{2.326}{800}\sqrt{\frac{57(800-57)}{800}}\right)$

$= (0, 0.092).$

10.7.15 The expected cell frequencies are

	Weak	Satisfactory	Strong
Preparation method 1	13.25	42.65	15.10
Preparation method 2	23.51	75.69	26.80
Preparation method 3	13.25	42.65	15.10

The Pearson chi-square statistic is $X^2 = 16.797$ and the p-value is

$P(\chi_4^2 \geq 16.797) = 0.002$

where the degrees of freedom of the chi-square random variable are calculated as $(3-1) \times (3-1) = 4$.

There is sufficient evidence to conclude that the three preparation methods are not equivalent in terms of the quality of chemical solutions which they produce.

10.7.16 (a) The expected cell frequencies are

	No damage	Slight damage	Medium damage	Severe damage
Type I	87.33	31.33	52.00	49.33
Type II	87.33	31.33	52.00	49.33
Type III	87.33	31.33	52.00	49.33

The Pearson chi-square statistic is $X^2 = 50.08$ so that the p-value is

$P(\chi_6^2 \geq 50.08) = 0.000$

where the degrees of freedom of the chi-square random variable are calculated as $(4-1) \times (3-1) = 6$.

Consequently, there is sufficient evidence to conclude that the three types of metal alloy are not all the same in terms of the damage that they suffer.

(b) $\hat{p}_{Se1} = \frac{42}{220} = 0.1911$

$\hat{p}_{Se3} = \frac{32}{220} = 0.1455$

The pooled estimate is

$\hat{p} = \frac{42+32}{220+220} = 0.1682.$

The test statistic is

$z = \frac{0.1911-0.1455}{\sqrt{0.1682\times0.8318\times\left(\frac{1}{220}+\frac{1}{220}\right)}} = 1.27$

and the p-value is $2 \times \Phi(-1.27) = 0.20$.

There is not sufficient evidence to conclude that the probability of suffering severe damage is different for alloys of type I and type III.

(c) $\hat{p}_{N2} = \frac{52}{220} = 0.236$

With $z_{0.005} = 2.576$ the confidence interval is

$0.236 \pm \frac{2.576}{220}\sqrt{\frac{52\times(220-52)}{220}}$

$= (0.163, 0.310).$

10.7.17 (a) The expected cell frequencies are:

$e_1 = 655 \times 0.25 = 163.75$

$e_2 = 655 \times 0.10 = 65.50$

$e_3 = 655 \times 0.40 = 262.00$

$e_4 = 655 \times 0.25 = 163.75$

The Pearson chi-square statistic is

$X^2 = \frac{(119-163.75)^2}{163.75} + \frac{(54-65.50)^2}{65.50} + \frac{(367-262.00)^2}{262.00} + \frac{(115-163.75)^2}{163.75} = 70.8$

so that the p-value is $P(\chi^2_3 \geq 70.8) = 0.000$.

The data is not consistent with the claimed probabilities.

(b) With $z_{0.005} = 2.576$ the confidence interval is

$p_C \in \frac{367}{655} \pm \frac{2.576}{655}\sqrt{\frac{367\times(655-367)}{655}}$

$= (0.510, 0.610)$.

10.7.18 $P(N(120, 4^2) \leq 115) = P\left(N(0,1) \leq \frac{115-120}{4}\right) = \Phi(-1.25) = 0.1056$

$P(N(120, 4^2) \leq 120) = P\left(N(0,1) \leq \frac{120-120}{4}\right) = \Phi(0) = 0.5000$

$P(N(120, 4^2) \leq 125) = P\left(N(0,1) \leq \frac{125-120}{4}\right) = \Phi(1.25) = 0.8944$

The observed cell frequencies are $x_1 = 17$, $x_2 = 32$, $x_3 = 21$, and $x_4 = 14$.

The expected cell frequencies are:

$e_1 = 84 \times 0.1056 = 8.87$

$e_2 = 84 \times (0.5000 - 0.1056) = 33.13$

$e_3 = 84 \times (0.8944 - 0.5000) = 33.13$

$e_4 = 84 \times (1 - 0.8944) = 8.87$

The Pearson chi-square statistic is

$X^2 = \frac{(17-8.87)^2}{8.87} + \frac{(32-33.13)^2}{33.13} + \frac{(21-33.13)^2}{33.13} + \frac{(14-8.87)^2}{8.87} = 14.88$

so that the p-value is $P(\chi^2_3 \geq 14.88) = 0.002$.

There is sufficient evidence to conclude that the breaking strength of concrete of this type is not normally distributed with a mean of 120 and a standard deviation of 4.

10.7.19 (a) $\hat{p}_M = \frac{28}{64} = 0.438$

$\hat{p}_F = \frac{31}{85} = 0.365$

The hypotheses are

$H_0 : p_M = p_F$ versus $H_A : p_M \neq p_F$

and the pooled estimate is

$\hat{p} = \frac{28+31}{64+85} = 0.396.$

The test statistic is

$z = \frac{0.438-0.365}{\sqrt{0.396\times0.604\times\left(\frac{1}{64}+\frac{1}{85}\right)}} = 0.90$

and the p-value is $2 \times \Phi(-0.90) = 0.37$.

There is not sufficient evidence to conclude that the support for the proposal is different for men and women.

(b) With $z_{0.005} = 2.576$ the confidence interval is

$p_M - p_F \in 0.438 - 0.365 \pm 2.576\sqrt{\frac{28\times36}{64^3} + \frac{31\times54}{85^3}}$

$= (-0.14, 0.28).$

10.7.20 (a) $\hat{p}_A = \frac{56}{94} = 0.596$

$\hat{p}_B = \frac{64}{153} = 0.418$

The hypotheses are

$H_0 : p_A \leq 0.5$ versus $H_A : p_A > 0.5$

and the test statistic is

$z = \frac{56-(94\times0.5)-0.5}{\sqrt{94\times0.5\times0.5}} = 1.753$

so that the p-value is $1 - \Phi(1.753) = 0.040$.

There is some evidence that the chance of success for patients with Condition A is better than 50%, but the evidence is not overwhelming.

(b) With $z_{0.005} = 2.576$ the confidence interval is

$p_A - p_B \in 0.596 - 0.418 \pm 2.576\sqrt{\frac{56\times38}{94^3} + \frac{64\times89}{153^3}}$

$= (0.012, 0.344)$.

(c) The Pearson chi-square statistic is

$X^2 = \frac{n(x_{11}x_{22}-x_{12}x_{21})^2}{x_{1.}x_{.1}x_{2.}x_{.2}} = \frac{247\times(56\times89-38\times64)^2}{94\times120\times153\times127} = 7.34$

and the p-value is $P(\chi_1^2 \geq 7.34) = 0.007$.

There is sufficient evidence to conclude that the success probabilities are different for patients with Condition A and with Condition B.

10.7.21 (a) True

(b) True

(c) False

(d) False

(e) True

(f) True

(g) True

(h) True

(i) True

(j) False

10.7.22 (a) $\hat{p} = \frac{485}{635} = 0.764$

With $z_{0.025} = 1.960$ the confidence interval is

$0.764 \pm \frac{1.960}{635} \times \sqrt{\frac{485 \times (635-485)}{635}}$

$= 0.764 \pm 0.033$

$= (0.731, 0.797)$.

(b) The hypotheses are
$H_0 : p \leq 0.75$ versus $H_A : p > 0.75$
and the test statistic is

$z = \frac{485-(635 \times 0.75)-0.5}{\sqrt{635 \times 0.75 \times 0.25}} = 0.756$

so that the p-value is $1 - \Phi(0.756) = 0.225$.

There is not sufficient evidence to establish that at least 75% of the customers are satisfied.

10.7.23 (a) The expected cell frequencies are

	Hospital 1	Hospital 2	Hospital 3	Hospital 4	Hospital 5
Admitted	38.07	49.60	21.71	74.87	50.75
Returned home	324.93	423.40	185.29	639.13	433.25

The Pearson chi-square statistic is $X^2 = 26.844$
so that the p-value is $P(\chi_4^2 \geq 26.844) \simeq 0$.

Consequently, there is sufficient evidence to support the claim that the hospital admission rates differ between the five hospitals.

(b) $\hat{p}_3 = \frac{42}{207} = 0.203$

$\hat{p}_4 = \frac{57}{714} = 0.080$

With $z_{0.25} = 1.960$ the confidence interval is

$p_3 - p_4 \in 0.203 - 0.080 \pm 1.960\sqrt{\frac{0.203\times0.797}{207} + \frac{0.080\times0.920}{714}}$

$= 0.123 \pm 0.058$

$= (0.065, 0.181)$.

(c) $\hat{p}_1 = \frac{39}{363} = 0.107$

The hypotheses are
$H_0 : p_1 \leq 0.1$ versus $H_A : p_1 > 0.1$
and the test statistic is

$z = \frac{39-(363\times0.1)-0.5}{\sqrt{363\times0.1\times0.9}} = 0.385$

so that the p-value is $1 - \Phi(0.385) = 0.35$.

There is not sufficient evidence to conclude that the admission rate for hospital 1 is larger than 10%.

10.7.24 (a) The expected cell frequencies are

	Minimal scour depth	Substantial scour depth	Severe scour depth
Pier design 1	7.86	13.82	7.32
Pier design 2	8.13	14.30	7.57
Pier design 3	13.01	22.88	12.11

The Pearson chi-square statistic is

$X^2 = \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{(x_{ij}-e_{ij})^2}{e_{ij}} = 17.41$

so that the p-value is $P(\chi_4^2 \geq 17.41) = 0.002$.

Consequently, there is sufficient evidence to conclude that the pier design has an effect on the amount of scouring.

The likelihood ratio chi-square statistic is

$G^2 = 2 \sum_{i=1}^{3} \sum_{j=1}^{3} x_{ij} \ln\left(\frac{x_{ij}}{e_{ij}}\right) = 20.47$

which provides a similar conclusion.

(b) The expected cell frequencies are

$e_1 = e_2 = e_3 = \frac{29}{3}$

and the Pearson chi-square statistic is

$X^2 = \frac{(12-\frac{29}{3})^2}{\frac{29}{3}} + \frac{(15-\frac{29}{3})^2}{\frac{29}{3}} + \frac{(2-\frac{29}{3})^2}{\frac{29}{3}} = 9.59$

so that the p-value is $P(\chi_2^2 \geq 9.59) = 0.008$.

Consequently, the hypothesis of homogeneity is not plausible and the data set provides sufficient evidence to conclude that for pier design 1 the three levels of scouring are not equally likely.

(c) Let p_{3m} be the probability of minimal scour depth with pier design 3, so that the hypotheses of interest are

$H_0 : p_{3m} = 0.25$ versus $H_A : p_{3m} \neq 0.25$.

Since

$\hat{p}_{3m} = \frac{x}{n} = \frac{15}{48} = 0.3125 > 0.25$

the exact p-value is $2 \times P(B(48, 0.25) \geq 15)$.

With a test statistic

$z = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{15-48(0.25)}{\sqrt{48(0.25)(1-0.25)}} = \frac{3}{3} = 1$

the normal approximation to the p-value is

$2\Phi(-|z|) = 2\Phi(-1) = 0.3174.$

Consequently, the null hypothesis is not rejected and it is plausible that the probability of minimal scour for pier design 3 is 25%.

(d) $\hat{p}_{1s} = \frac{2}{29} = 0.0690$

$\hat{p}_{2s} = \frac{8}{30} = 0.2667$

With $z_{0.005} = 2.576$ the confidence interval is

$p_{1s} - p_{2s} \in 0.0690 - 0.2667 \pm 2.576\sqrt{\frac{0.0690(1-0.0690)}{29} + \frac{0.2667(1-0.2667)}{30}}$

$= -0.1977 \pm 0.2407$

$= (-0.4384, 0.0430).$

10.7.25 A

10.7.26 A

10.7.27 B

10.7.28 C

10.7.29 A

10.7.30 B