E*. Please solve the following problems; display the computations, and upload the resulting pdf document on the appropriate Canvas assignment link:

i. A person is randomly selected from a population and tested for COVID-19 infection. A positive test result is labeled a "success" and coded as 1; a negative test result is labeled a "failure" and coded as 0. Again a person is randomly selected from that *same* population (Thus, the first person is still in the population; so the two events are independent). The trial is repeated another 3 times. The number of successes is recorded. As of May 26, 2020, one seemingly credible estimate is 30% of the people worldwide are infected; use that figure for this problem. Display the probability distribution for the random variable for this experiment.

Sample computation:

From Theorem 11 we have: $(X \in \{ \text{Bernoulli random variables of } \{ 0, 1 \}\} \wedge m : X \to [0, 1] \wedge (\Omega = \{ 0, 1, 2, ..., n \} \wedge Y \in \{ \text{binomial random variables of } \Omega \}) \wedge p : Y \to [0, 1]) \Rightarrow$ $p(k) = \binom{n}{k} m(1)^k (1 - m(1))^{n-k}$ where $n = 5 \wedge m(1) = 0.30$. Our probability distribution is as follows:

$$p(0) = \binom{5}{0} m(1)^0 (1 - m(1))^5 = (1)(1)(.70)^5 \approx 0.1681$$

$$p(1) = \binom{5}{1} m(1)^1 (1 - m(1))^4 = (5)(.30)(.70)^4 \approx 0.3602$$

$$p(2) = \binom{5}{2} m(1)^2 (1 - m(1))^3 = (10)(.30)^2(.70)^3 \approx 0.3087$$

$$p(3) = \binom{5}{3} m(1)^3 (1 - m(1))^2 = (10)(.30)^3(.70)^2 \approx 0.1323$$

$$p(4) = \binom{5}{4} m(1)^4 (1 - m(1))^1 = (5)(.30)^4(.70)^1 \approx 0.0284$$

$$p(5) = \binom{5}{5} m(1)^5 (1 - m(1))^0 = (1)(.30)^5(.70)^0 \approx 0.0024$$

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ii. Three fair dice are randomly rolled and the sum of the dots on the three upper-facing surfaces is recorded. An even sum is considered a success and coded 1; an odd sum is considered a failure and coded 0. The number of success is recorded. The trial is repeated 5 more times. The number of successes is recorded. Display the probability distribution for the random variable for this experiment.

Sample computation:

From Theorem 11 we have: ($X \in$ { Bernoulli random variables of { 0, 1 }} $\wedge$ $m : X \to [\,0, 1]$ $\wedge$ (Ω = { 0, 1, 2, ..., n } $\wedge$ $Y \in$ { binomial random variables of Ω }) $\wedge$ $p : Y \to [\,0, 1]$) $\Rightarrow$ $p(\,k\,) = \binom{n}{k} m(1)^k\,(\,1 - m(1))^{n-k}$ where $n = 6 \wedge m(1) = 0.50$. Our probability distribution is as follows:

$$p(\,0\,) = \binom{6}{0} m(1)^0\,(\,1 - m(1))^6 = (1)(1)(.50)^6 \approx 0.0156$$

$$p(\,1\,) = \binom{6}{1} m(1)^1\,(\,1 - m(1))^5 = (6)(.50)(.50)^5 \approx 0.0938$$

$$p(\,2\,) = \binom{6}{2} m(1)^2\,(\,1 - m(1))^4 = (15)(.50)^2(.50)^4 \approx 0.2344$$

$$p(\,3\,) = \binom{6}{3} m(1)^3\,(\,1 - m(1))^3 = (20)(.50)^3(.50)^3 \approx 0.3125$$

$$p(\,4\,) = \binom{6}{4} m(1)^4\,(\,1 - m(1))^2 = (15)(.50)^4(.50)^2 \approx 0.2340$$

$$p(\,5\,) = \binom{6}{5} m(1)^5\,(\,1 - m(1))^1 = (6)(.50)^5(.50)^1 \approx 0.0936$$

$$p(\,6\,) = \binom{6}{6} m(1)^6\,(\,1 - m(1))^0 = (1)(.50)^6(.50)^0 \approx 0.0156$$
