Homework Two Sample Tests for Mean

Instructions: For each problem, do the following for every hypothesis test. Then do any additional parts as asked.

- Check the conditions.
- Specify the level of significance α.
- State the null hypothesis H_0 and the alternative hypothesis H_a.
- Calculate the test statistic.
- Calculate degrees of freedom.
- Find the p-value.
- Reject H_0 if the P-value is less than α.
- Interpret your statistical results in real world terms.
- **Interpret the p-value.

Paired Samples (Matched Pairs)

1. Air quality measurements were collected in a random sample of 25 country capitals in 2013, and then again in the same cities in 2014. We would like to use the sample average and sample standard deviation to compare average air quality between the two years.

   (a) Should we use a one-sided or a two-sided test? Explain your reasoning. 2 sided, they want to see a difference.

   (b) Should we use a paired or non-paired test? Explain your reasoning. Paired, record data in same places.

   (c) Should we use a t-test or a z-test? Explain your reasoning.
   ttest, you know sample σ

2. In each of the following scenarios, determine if the data are paired.

   (a) Compare pre-test (beginning of semester) and post-test (end of semester) scores of students. Paired

   (b) Assess gender-related salary gap by comparing salaries of randomly sampled men and women. not paired

   (c) Compare artery thicknesses at the beginning of a study and after 2 years of taking Vitamin E for the same group of patients. Paired

   (d) Assess effectiveness of a diet regimen by comparing the before and after weights of subjects. paired

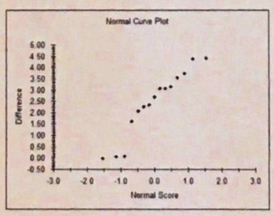
   (e) We would like to know if Intel's stock and Southwest Airlines' stock have similar rates of return. To find out, we take a random sample of 50 days, and record Intel's and Southwest's stock on those same days. paired

   (f) We randomly sample 50 items from Target stores and note the price for each. Then we visit Walmart and collect the price for each of those same 50 items. Paired

   (g) A school board would like to determine whether there is a difference in average SAT scores for students at one high school versus another high school in the district. To check, they take a simple random sample of 100 students from each high school. not paired

3. Some people think mental patients are influenced by the moon. We conducted a study of dementia patients in nursing homes. For each patient, we counted the average number of disruptive behaviors on days with a full moon and also the average number of disruptive behaviors on days without a full moon. Conduct a hypothesis test to determine if there are more disruptive behaviors on days with a full moon.

Patient	Moon Days	Other Days	Difference
1	3.33	0.27	3.06
2	3.67	0.59	3.08
3	2.67	0.32	2.35
4	3.33	0.19	3.14
5	3.33	1.26	2.07
6	3.67	0.11	3.56
7	4.67	0.3	4.37
8	2.67	0.4	2.27
9	6	1.59	4.41
10	4.33	0.6	3.73
11	3.33	0.65	2.68
12	0.67	0.69	-0.02
13	1.33	1.26	0.07
14	0.33	0.23	0.1
15	2	0.38	1.62



	Difference
count	15
mean	2.4327
sample variance	2.1325
sample standard deviation	1.4603

   (a) Why is this a paired sample? patients are compared to themselves.

   (b) Conduct a hypothesis test with $\alpha = .05$.

   (c) Find a 95% confidence interval for the mean difference in aggressive behaviors per day. Interpret the interval.

b) Fairly normal

$\alpha = .05$

$H_0: \mu_d = 0$

$H_a: \mu_d > 0$

One sample T test

$$t = \frac{2.4327}{1.4603/\sqrt{15}} = 6.4519$$

df = 14

Pval ≈ 0

Small, reject null

we have strong evidence that the mean difference is greater than 0.

c) $t^* = 2.145$

$$2.4327 \pm 2.145\left(\frac{1.4603}{\sqrt{15}}\right)$$

$(2.4327 \pm .8088)$

we are 95% confident that the population mean difference is between 1.6239 and 3.2415.

4. We want to know if the September 11 terrorist attack had an effect on U.S. airline demand. We found a sample of 12 airline routes whose passenger miles were tracked for one year before the attack and for one year afterward.
We subtracted the post attack mileages from the pre attack mileages for each airline route. The mean of the sample of paired differences was 29.7 million miles and the standard deviation of the sample was 2.975 million miles.
Test to see if the attack had a negative impact on how much passengers fly. Assume normality.

normal ✓ $\mu_d = 29.7$, $S_d = 2.975$

$\alpha = .05$

One sample T test

$H_0 = \mu_{diff} = 0$

$H_A = \mu_{diff} > 0$

test stat:

$$t = \frac{29.7}{2.975/\sqrt{12}} = 34.58$$

$df = 11$

$pval \approx 0$

small pval, reject null

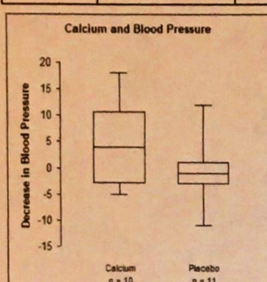
we found strong evidence that $\mu_{diff} > 0$.

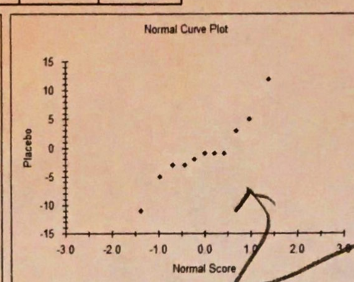
5. **OPTIONAL:** Because of skyrocketing health-care costs, many hospital administrators are working to contain costs. They want to know if they can treat OSAS (obstructive sleep apnea syndrome) at home effectively. They take a sample of 9 patients and count the number of obstructions before treatment and after treatment. If the treatment is effective there should be less obstructions after treatment. They find the paired differences (post treatment-pre treatment) and get a sample mean of -86 and standard deviation of 101.83.
Conduct a hypothesis test with $\alpha = .05$ to determine if the home treatment was effective. (Assume normality)

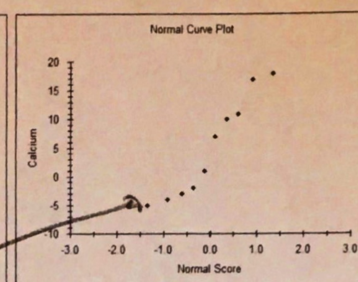
Independent Two Sample Tests

6. A study was conducted to see if taking calcium reduces blood pressure. Two independent groups were chosen. One group was given calcium and one group was given a placebo. The decrease in their blood pressure was recorded. Conduct a hypothesis test to see if taking calcium reduced blood pressure. (A positive number represents a decrease and a negative number represents an increase in blood pressure).

Group	Treatment	n	$\overline{x}$	s
1	Calcium	10	5	8.743
2	Placebo	11	-.273	5.901







$n=10$
$n=11$ Small not normal

Can't do it!

7. The "misery is not miserly" phenomenon refers to a person's spending judgment going haywire when sad. In a recent study, 31 young adults were randomly assigned to watch a sad movie or a neutral movie. After the video, the researchers kept track of how much the subjects were willing to spend on an insulated water bottle.
Assume both populations are normally distributed.

	Neutral Group	Sad Group
count	14	17
mean	0.5714	2.1176
sample variance	0.5330	1.5478

(a) Conduct a hypothesis test to determine if the sad group is willing to spend more money. Use $\alpha = .05$.

(b) Interpret the p-value.

(c) I used my calculator to find a 95% confidence interval for the difference in the population means. It is -2.28 to -.81. How do you think we could interpret it?

a) T test, unequal variances

both are normal ✓

$\alpha = .05$

$H_0: \mu_N = \mu_S$

$H_A: \mu_N < \mu_S$

test stat:

$$t = \frac{.5714 - 2.1176}{\sqrt{\frac{.5330}{14} + \frac{1.5478}{17}}} = -4.303$$

df = 26.47
= 26

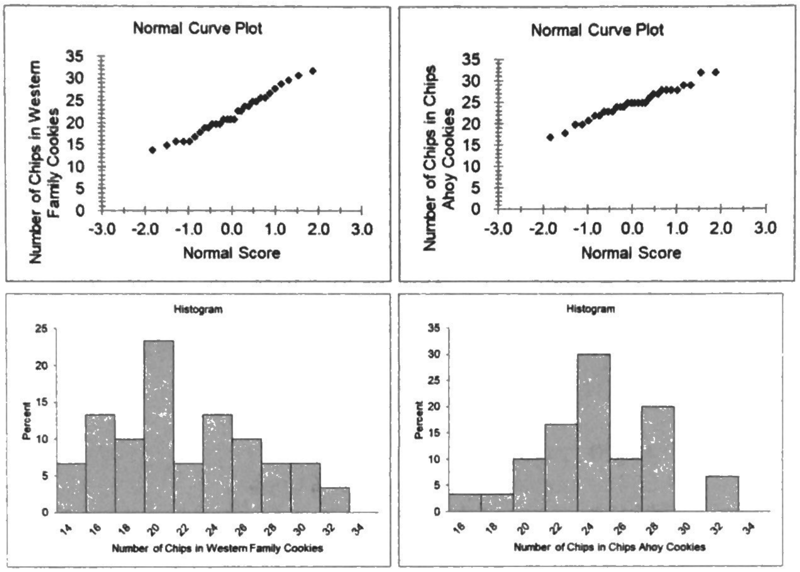
Pval = less than .0005
small, reject null.

We have strong evidence that the population mean of neutral group is less than sad group.

b) If the population means are equal we have less than a .05% chance of getting sample mean difference that big.

c) we are 95% confident that the difference between population means is between -2.28 and -.81

8. In previous assignments we have discussed the number of chocolate chips in cookies. My previous class took a sample of chips ahoy cookies and found the average. They also took a sample of Western Family cookies and found the average. Shawn insists that Chips Ahoy is worth the extra money and has more chocolate chips. Conduct a hypothesis test with $\alpha = .05$ to determine if he is right.

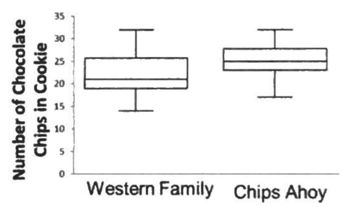


Hypothesis Test: Independent Groups (t-test, unequal variance)

Western Family	Chips Ahoy	
22.23	24.83	mean
4.95	3.65	std. dev.
30	30	n

53 df
-2.600 difference (Western Family - Chips Ahoy)
1.122 standard error of difference
0 hypothesized difference

-2.32 t
.0122 p-value (one-tailed, lower)



T test,
n=30 normal ✓
$\alpha = .05$
$H_0: \mu_{WF} = \mu_{CA}$
$H_A: \mu_{WF} < \mu_{CA}$
test stat:
$t = -2.32$
$df = 53$
pval = .0122
small, reject null
we found strong evidence that on average of all cookies, Chips ahoy would have more chips per cookie than western family.

9. Participants were asked to rate labels on a scale of 1-7 with 7 being the best. They were divided into two groups. One group was primed and the other group was not primed before they were shown the labels. Assume Normality and use the computer output.

Hypothesis Test: Independent Groups (t-test, unequal variance)

Primed	Not Primed	
4	2.95	mean
0.92736185	0.943398113	std. dev.
22	20	n

39	df
1.05000	difference (primed - not primed)
0.28912	standard error of difference
0	hypothesized difference
3.63	t
.0004	p-value (one-tailed, upper)
0.46520	confidence interval 95.% lower
1.63480	confidence interval 95.% upper
0.58480	margin of error

(a) Conduct a hypothesis test to determine if the primed group gives a higher rating. Use $\alpha = .05$.

(b) Interpret the p-value.

(c) Find and interpret a 95% confidence interval for the difference in the population means.

a) T test

Assume normality ✓

$\alpha = .05$

$H_0: \mu_p = \mu_{NP}$

$H_A: \mu_p > \mu_{NP}$

test stat: $t = 3.63$, $df = 39$

Pval = .0004 small, reject null.

we have strong evidence that the population mean of the primed group is greater than the population mean of the not primed group.

b) If the population means are equal, the probability of getting a difference as big or bigger is .0004.

c) (.465 to 1.64) we are 95% confident that the primed population mean is between greater than the not primed.

10. Researchers want to compare the long-term psychological effects of dieters on a high-carbohydrate, low-fat (LF) diet with those on a high-fat, low-carbohydrate (LC) diet. Overweight and obese participants were randomly assigned to one of these diets. After a year, their mood was assessed. (A lower score is associated with a positive mood.) Conduct a hypothesis test with $\alpha = .01$ to determine if there is a difference in the mean moods for the different diets.

Group	n	$\bar{x}$	s
High Carb, Low Fat (LF)	33	19.3	25.8
High Fat, Low Carb (LC)	32	47.3	28.3

T test, equal variance

$n \geq 30$ for both ✓

$\alpha = .01$

$H_0: \mu_{LC} = \mu_{LF}$

$H_A: \mu_{LC} \neq \mu_{LF}$

$$s_p^2 = \frac{(33-1)(25.8)^2 + (32-1)(28.3)^2}{33+32-2} = 732.2$$

$$t = \frac{19.3 - 47.3}{\sqrt{732.2\left(\frac{1}{33}+\frac{1}{32}\right)}} = -4.17, \quad df = 63 \approx 60$$

Pval = less than .001

small, reject null

we found evidence that the population means for the 2 groups are different.

11. A study compared the total cholesterol between sedentary females and sedentary males.

Group	n	$\overline{x}$	s
Female	71	173.7	34.79
Male	37	171.81	33.24

(a) Conduct a hypothesis test to determine if there is a difference in the population means for the males and females. Use $\alpha = .05$.

(b) The 95% confidence interval for the difference in the population means is -12 to 15.9. Interpret it.

(c) Compare the information given by the confidence interval and the hypothesis test.

a) T test, equal variances

n > 71 ✓ normal
n > 37 ✓

$\alpha = .05$

$H_0: \mu_F = \mu_M$

$H_A: \mu_F \neq \mu_M$

test stat

$$S_p^2 = \frac{(71-1)(34.79)^2 + (37-1)(33.24)^2}{71+37-2} = 1174.5$$

$$t = \frac{173.7 - 171.81}{\sqrt{1174.5\left(\frac{1}{71} + \frac{1}{37}\right)}} = .27$$

$df = 106$
≈ 100

Pval = bigger than .50
big fail to reject.

we didn't find evidence that the population means of the two groups are different

b) we are 95% confident that the difference between the population means is between -12 and 15.9.

c) they match, and tell us we can't tell a difference between the groups.

12. A multimedia company produced a intervention to try to improve dietary behavior among low-income women. They want to see if their intervention is successful. They use the intervention on 165 women and also have a control group of 212 women. All the women take a test two months later. The intervention group has a sample mean of 5.08 and the control group has a sample mean of 4.33. Somehow they know that the population standard deviation of the intervention group is 1.15 and the population standard deviation of the control group is 1.16.

Conduct a hypothesis test to determine if the intervention group has higher test scores. Use $\alpha = .10$.

Z test

212 > 30 normal ✓
165 > 30

$\alpha = .1$

$H_0: \mu_I = \mu_C$

$H_A: \mu_I > \mu_C$

test stat

$$Z = \frac{5.08 - 4.33}{\sqrt{\frac{1.15^2}{165} + \frac{1.16^2}{212}}} = 6.25$$

Pval = 0

small pval
reject null.

$\boxed{\text{we have very strong evidence that } \mu_I > \mu_C.}$

13. **OPTIONAL:** The same study compared the LDL cholesterol levels. (LDL is the "bad cholesterol") The researchers specifically want to know if the LDL levels are higher in sedentary males than sedentary females.

Group	n	$\overline{x}$	s
Female	71	96.38	29.78
Male	37	109.44	31.05

(a) Conduct a hypothesis test to determine if the male group has higher LDL levels. Use $\alpha = .05$.

(b) Interpret the 95% confidence interval which is -25.74 to -.38.