

8.1.1 With $t_{0.025,30} = 2.042$ the confidence interval is

$$\left(53.42 - \frac{2.042 \times 3.05}{\sqrt{31}}, 53.42 + \frac{2.042 \times 3.05}{\sqrt{31}} \right) = (52.30, 54.54).$$

8.1.2 With $t_{0.005,40} = 2.704$ the confidence interval is

$$\left(3.04 - \frac{2.704 \times 0.124}{\sqrt{41}}, 3.04 + \frac{2.704 \times 0.124}{\sqrt{41}} \right) = (2.99, 3.09).$$

The confidence interval does not contain the value 2.90, and so 2.90 is not a plausible value for the mean glass thickness.

8.1.3 At 90% confidence the critical point is $t_{0.05,19} = 1.729$ and the confidence interval is

$$\left(436.5 - \frac{1.729 \times 11.90}{\sqrt{20}}, 436.5 + \frac{1.729 \times 11.90}{\sqrt{20}} \right) = (431.9, 441.1).$$

At 95% confidence the critical point is $t_{0.025,19} = 2.093$ and the confidence interval is

$$\left(436.5 - \frac{2.093 \times 11.90}{\sqrt{20}}, 436.5 + \frac{2.093 \times 11.90}{\sqrt{20}} \right) = (430.9, 442.1).$$

At 99% confidence the critical point is $t_{0.005,19} = 2.861$ and the confidence interval is

$$\left(436.5 - \frac{2.861 \times 11.90}{\sqrt{20}}, 436.5 + \frac{2.861 \times 11.90}{\sqrt{20}} \right) = (428.9, 444.1).$$

Even the 99% confidence level confidence interval does not contain the value 450.0, and so 450.0 is not a plausible value for the average breaking strength.

8.1.4 With $t_{0.005,15} = 2.947$ the confidence interval is

$$\left(1.053 - \frac{2.947 \times 0.058}{\sqrt{16}}, 1.053 + \frac{2.947 \times 0.058}{\sqrt{16}} \right) = (1.010, 1.096).$$

The confidence interval contains the value 1.025, and so 1.025 is a plausible value for the average weight.

8.1.5 With $z_{0.025} = 1.960$ the confidence interval is

$$\left(0.0328 - \frac{1.960 \times 0.015}{\sqrt{28}}, 0.0328 + \frac{1.960 \times 0.015}{\sqrt{28}} \right) = (0.0272, 0.0384).$$

8.1.6 At 90% confidence the critical point is $z_{0.05} = 1.645$ and the confidence interval is

$$\left(19.50 - \frac{1.645 \times 1.0}{\sqrt{10}}, 19.50 + \frac{1.645 \times 1.0}{\sqrt{10}}\right) = (18.98, 20.02).$$

At 95% confidence the critical point is $z_{0.025} = 1.960$ and the confidence interval is

$$\left(19.50 - \frac{1.960 \times 1.0}{\sqrt{10}}, 19.50 + \frac{1.960 \times 1.0}{\sqrt{10}}\right) = (18.88, 20.12).$$

At 99% confidence the critical point is $z_{0.005} = 2.576$ and the confidence interval is

$$\left(19.50 - \frac{2.576 \times 1.0}{\sqrt{10}}, 19.50 + \frac{2.576 \times 1.0}{\sqrt{10}}\right) = (18.69, 20.31).$$

Even the 90% confidence level confidence interval contains the value 20.0, and so 20.0 is a plausible value for the average resilient modulus.

8.1.7 With $t_{0.025, n-1} \simeq 2.0$ a sufficient sample size can be estimated as

$$\begin{aligned} n &\geq 4 \times \left(\frac{t_{0.025, n-1} s}{L_0}\right)^2 \\ &= 4 \times \left(\frac{2.0 \times 10.0}{5}\right)^2 = 64. \end{aligned}$$

A sample size of about $n = 64$ should be sufficient.

8.1.8 With $t_{0.005, n-1} \simeq 3.0$ a sufficient sample size can be estimated as

$$\begin{aligned} n &\geq 4 \times \left(\frac{t_{0.005, n-1} s}{L_0}\right)^2 \\ &= 4 \times \left(\frac{3.0 \times 0.15}{0.2}\right)^2 = 20.25. \end{aligned}$$

A sample size slightly larger than 20 should be sufficient.

8.1.9 A total sample size of

$$\begin{aligned} n &\geq 4 \times \left(\frac{t_{0.025, n-1} s}{L_0}\right)^2 \\ &= 4 \times \left(\frac{2.042 \times 3.05}{2.0}\right)^2 = 38.8 \end{aligned}$$

is required.

Therefore, an additional sample of at least $39 - 31 = 8$ observations should be sufficient.

8.1.10 A total sample size of

$$\begin{aligned}n &\geq 4 \times \left(\frac{t_{0.005, n_1-1} s}{L_0} \right)^2 \\&= 4 \times \left(\frac{2.704 \times 0.124}{0.05} \right)^2 = 179.9\end{aligned}$$

is required.

Therefore, an additional sample of at least $180 - 41 = 139$ observations should be sufficient.

8.1.11 A total sample size of

$$\begin{aligned}n &\geq 4 \times \left(\frac{t_{0.005, n_1-1} s}{L_0} \right)^2 \\&= 4 \times \left(\frac{2.861 \times 11.90}{10.0} \right)^2 = 46.4\end{aligned}$$

is required.

Therefore, an additional sample of at least $47 - 20 = 27$ observations should be sufficient.

8.1.12 With $t_{0.05, 29} = 1.699$ the value of c is obtained as

$$c = \bar{x} + \frac{t_{\alpha, n-1} s}{\sqrt{n}} = 14.62 + \frac{1.699 \times 2.98}{\sqrt{30}} = 15.54.$$

The confidence interval does not contain the value 16.0, and so it is not plausible that $\mu \geq 16$.

8.1.13 With $t_{0.01, 60} = 2.390$ the value of c is obtained as

$$c = \bar{x} - \frac{t_{\alpha, n-1} s}{\sqrt{n}} = 0.768 - \frac{2.390 \times 0.0231}{\sqrt{61}} = 0.761.$$

The confidence interval contains the value 0.765, and so it is plausible that the average solution density is less than 0.765.

8.1.14 With $z_{0.05} = 1.645$ the value of c is obtained as

$$c = \bar{x} - \frac{z_{\alpha} \sigma}{\sqrt{n}} = 11.80 - \frac{1.645 \times 2.0}{\sqrt{19}} = 11.05.$$

8.1.15 With $z_{0.01} = 2.326$ the value of c is obtained as

$$c = \bar{x} + \frac{z_{\alpha} \sigma}{\sqrt{n}} = 415.7 + \frac{2.326 \times 10.0}{\sqrt{29}} = 420.0.$$

The confidence interval contains the value 418.0, and so it is plausible that the mean radiation level is greater than 418.0.

8.1.16 The interval (6.668, 7.054) is

$$(6.861 - 0.193, 6.861 + 0.193)$$

and

$$0.193 = \frac{1.753 \times 0.440}{\sqrt{16}}.$$

Since $1.753 = t_{0.05,15}$ it follows that the confidence level is $1 - (2 \times 0.05) = 0.90$.

8.1.17 Using the critical point $t_{0.005,9} = 3.250$ the confidence interval is

$$\left(2.752 - \frac{3.250 \times 0.280}{\sqrt{10}}, 2.752 + \frac{3.250 \times 0.280}{\sqrt{10}} \right) = (2.464, 3.040).$$

The value 3.1 is outside this confidence interval, and so 3.1 is not a plausible value for the average corrosion rate.

Note: The sample statistics for the following problems in this section and the related problems in this chapter depend upon whether any observations have been removed as outliers. To avoid confusion, the answers given here assume that **no** observations have been removed. Notice that removing observations as outliers reduces the sample standard deviation s as well as affecting the sample mean \bar{x} .

8.1.18 At 95% confidence the critical point is $t_{0.025,199} = 1.972$ and the confidence interval is

$$\left(69.35 - \frac{1.972 \times 17.59}{\sqrt{200}}, 69.35 + \frac{1.972 \times 17.59}{\sqrt{200}} \right) = (66.89, 71.80).$$

8.1.19 At 95% confidence the critical point is $t_{0.025,89} = 1.987$ and the confidence interval is

$$\left(12.211 - \frac{1.987 \times 2.629}{\sqrt{90}}, 12.211 + \frac{1.987 \times 2.629}{\sqrt{90}} \right) = (11.66, 12.76).$$

8.1.20 At 95% confidence the critical point is $t_{0.025,124} = 1.979$ and the confidence interval is

$$\left(1.11059 - \frac{1.979 \times 0.05298}{\sqrt{125}}, 1.11059 + \frac{1.979 \times 0.05298}{\sqrt{125}} \right) = (1.101, 1.120).$$

- 8.1.21 At 95% confidence the critical point is $t_{0.025,74} = 1.9926$ and the confidence interval is

$$\left(0.23181 - \frac{1.9926 \times 0.07016}{\sqrt{75}}, 0.23181 + \frac{1.9926 \times 0.07016}{\sqrt{75}}\right) = (0.2157, 0.2480).$$

- 8.1.22 At 95% confidence the critical point is $t_{0.025,79} = 1.9905$ and the confidence interval is

$$\left(9.2294 - \frac{1.9905 \times 0.0942}{\sqrt{80}}, 9.2294 + \frac{1.9905 \times 0.0942}{\sqrt{80}}\right) = (9.0419, 9.4169).$$

- 8.1.23 Since

$$2.773 = 2.843 - \frac{t_{\alpha,8} \times 0.150}{\sqrt{9}}$$

it follows that $t_{\alpha,8} = 1.40$ so that $\alpha = 0.10$.

Therefore, the confidence level of the confidence interval is 90%.

- 8.1.24 (a) The sample median is 34.

(b) $\sum_{i=1}^{15} x_i = 532$

$$\sum_{i=1}^{15} x_i^2 = 19336$$

$$\bar{x} = \frac{532}{15} = 35.47$$

$$s^2 = \frac{19336 - 532^2/15}{15-1} = 33.41$$

Using the critical point $z_{0.005} = 2.576$ the confidence interval is

$$35.47 \pm \frac{2.576 \times \sqrt{33.41}}{15} = (31.02, 39.91).$$

- 8.1.25 (a) Using the critical point $t_{0.025,13} = 2.160$ the confidence interval is

$$\mu \in 5437.2 \pm \frac{2.160 \times 376.9}{\sqrt{14}} = (5219.6, 5654.8).$$

- (b) With

$$4 \times \left(\frac{2.160 \times 376.9}{300}\right)^2 = 29.5$$

it can be estimated that an additional $30 - 14 = 16$ chemical solutions would need to be measured.

8.1.26 With $t_{0.025, n-1} \simeq 2$ the required sample size can be estimated to be about

$$\begin{aligned} n &= 4 \times \left(\frac{t \times \sigma}{L_0} \right)^2 \\ &= 4 \times \left(\frac{2 \times 0.2031}{0.1} \right)^2 \\ &= 66. \end{aligned}$$

Which just goes to show that you can get your kicks on Route 66.

8.1.27 B

8.2.1 (a) The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{18} \times (57.74 - 55.0)}{11.2} = 1.04.$$

The p -value is $2 \times P(t_{17} \geq 1.04) = 0.313$.

(b) The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{18} \times (57.74 - 65.0)}{11.2} = -2.75.$$

The p -value is $P(t_{17} \leq -2.75) = 0.0068$.

8.2.2 (a) The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{39} \times (5532 - 5680)}{287.8} = -3.21.$$

The p -value is $2 \times P(t_{38} \geq 3.21) = 0.003$.

(b) The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{39} \times (5,532 - 5,450)}{287.8} = 1.78.$$

The p -value is $P(t_{38} \geq 1.78) = 0.042$.

8.2.3 (a) The test statistic is

$$z = \frac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma} = \frac{\sqrt{13} \times (2.879 - 3.0)}{0.325} = -1.34.$$

The p -value is $2 \times \Phi(-1.34) = 0.180$.

(b) The test statistic is

$$z = \frac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma} = \frac{\sqrt{13} \times (2.879 - 3.1)}{0.325} = -2.45.$$

The p -value is $\Phi(-2.45) = 0.007$.

- 8.2.4 (a) The test statistic is

$$z = \frac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma} = \frac{\sqrt{44} \times (87.90 - 90.0)}{5.90} = -2.36.$$

The p -value is $2 \times \Phi(-2.36) = 0.018$.

- (b) The test statistic is

$$z = \frac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma} = \frac{\sqrt{44} \times (87.90 - 86.0)}{5.90} = 2.14.$$

The p -value is $1 - \Phi(2.14) = 0.016$.

- 8.2.5 (a) The critical point is $t_{0.05,40} = 1.684$
and the null hypothesis is accepted when $|t| \leq 1.684$.

- (b) The critical point is $t_{0.005,40} = 2.704$
and the null hypothesis is rejected when $|t| > 2.704$.

- (c) The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{41} \times (3.04 - 3.00)}{0.124} = 2.066.$$

The null hypothesis is rejected at size $\alpha = 0.10$
and accepted at size $\alpha = 0.01$.

- (d) The p -value is $2 \times P(t_{40} \geq 2.066) = 0.045$.

- 8.2.6 (a) The critical point is $t_{0.05,19} = 1.729$
and the null hypothesis is accepted when $|t| \leq 1.729$.

- (b) The critical point is $t_{0.005,19} = 2.861$
and the null hypothesis is rejected when $|t| > 2.861$.

- (c) The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{20} \times (436.5 - 430.0)}{11.90} = 2.443.$$

The null hypothesis is rejected at size $\alpha = 0.10$
and accepted at size $\alpha = 0.01$.

- (d) The p -value is $2 \times P(t_{19} \geq 2.443) = 0.025$.

8.2.7 (a) The critical point is $t_{0.05,15} = 1.753$
and the null hypothesis is accepted when $|t| \leq 1.753$.

(b) The critical point is $t_{0.005,15} = 2.947$
and the null hypothesis is rejected when $|t| > 2.947$.

(c) The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{16} \times (1.053 - 1.025)}{0.058} = 1.931.$$

The null hypothesis is rejected at size $\alpha = 0.10$
and accepted at size $\alpha = 0.01$.

(d) The p -value is $2 \times P(t_{15} \geq 1.931) = 0.073$.

8.2.8 (a) The critical point is $z_{0.05} = 1.645$
and the null hypothesis is accepted when $|z| \leq 1.645$.

(b) The critical point is $z_{0.005} = 2.576$
and the null hypothesis is rejected when $|z| > 2.576$.

(c) The test statistic is

$$z = \frac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma} = \frac{\sqrt{10} \times (19.50 - 20.0)}{1.0} = -1.581.$$

The null hypothesis is accepted at size $\alpha = 0.10$
and consequently also at size $\alpha = 0.01$.

(d) The p -value is $2 \times \Phi(-1.581) = 0.114$.

8.2.9 (a) The critical point is $t_{0.10,60} = 1.296$
and the null hypothesis is accepted when $t \leq 1.296$.

(b) The critical point is $t_{0.01,60} = 2.390$
and the null hypothesis is rejected when $t > 2.390$.

(c) The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{61} \times (0.0768 - 0.065)}{0.0231} = 3.990.$$

The null hypothesis is rejected at size $\alpha = 0.01$
and consequently also at size $\alpha = 0.10$.

(d) The p -value is $P(t_{60} \geq 3.990) = 0.0001$.

8.2.10 (a) The critical point is $z_{0.10} = 1.282$
and the null hypothesis is accepted when $z \geq -1.282$.

(b) The critical point is $z_{0.01} = 2.326$
and the null hypothesis is rejected when $z < -2.326$.

(c) The test statistic is

$$z = \frac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma} = \frac{\sqrt{29} \times (415.7 - 420.0)}{10.0} = -2.316.$$

The null hypothesis is rejected at size $\alpha = 0.10$
and accepted at size $\alpha = 0.01$.

(d) The p -value is $\Phi(-2.316) = 0.0103$.

8.2.11 Consider the hypotheses $H_0 : \mu = 44.350$ versus $H_A : \mu \neq 44.350$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{24} \times (44.364 - 44.350)}{0.019} = 3.61.$$

The p -value is $2 \times P(t_{23} \geq 3.61) = 0.0014$.

There is sufficient evidence to conclude that the machine is miscalibrated.

- 8.2.12 Consider the hypotheses $H_0 : \mu \leq 120$ versus $H_A : \mu > 120$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{36} \times (122.5 - 120.0)}{13.4} = 1.12.$$

The p -value is $P(t_{35} \geq 1.12) = 0.135$.

There is *not* sufficient evidence to conclude that the manufacturer's claim is incorrect.

- 8.2.13 Consider the hypotheses $H_0 : \mu \leq 12.50$ versus $H_A : \mu > 12.50$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{15} \times (14.82 - 12.50)}{2.91} = 3.09.$$

The p -value is $P(t_{14} \geq 3.09) = 0.004$.

There is sufficient evidence to conclude that the chemical plant is in violation of the working code.

- 8.2.14 Consider the hypotheses $H_0 : \mu \geq 0.25$ versus $H_A : \mu < 0.25$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{23} \times (0.228 - 0.250)}{0.0872} = -1.21.$$

The p -value is $P(t_{22} \leq -1.21) = 0.120$.

There is *not* sufficient evidence to conclude that the advertised claim is false.

- 8.2.15 Consider the hypotheses $H_0 : \mu \leq 2.5$ versus $H_A : \mu > 2.5$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{10} \times (2.752 - 2.5)}{0.280} = 2.846.$$

The p -value is $P(t_9 \geq 2.846) = 0.0096$.

There is sufficient evidence to conclude that the average corrosion rate of chilled cast iron of this type is larger than 2.5.

- 8.2.16 Consider the hypotheses $H_0 : \mu \leq 65$ versus $H_A : \mu > 65$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{200} \times (69.35 - 65.00)}{17.59} = 3.50.$$

The p -value is $P(t_{199} \geq 3.50) = 0.0003$.

There is sufficient evidence to conclude that the average service time is greater than 65 seconds and that the manager's claim is incorrect.

- 8.2.17 Consider the hypotheses $H_0 : \mu \geq 13$ versus $H_A : \mu < 13$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{90} \times (12.211 - 13.000)}{2.629} = -2.85.$$

The p -value is $P(t_{89} \leq -2.85) = 0.0027$.

There is sufficient evidence to conclude that the average number of calls taken per minute is less than 13 so that the manager's claim is false.

- 8.2.18 Consider the hypotheses $H_0 : \mu = 1.1$ versus $H_A : \mu \neq 1.1$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{125} \times (1.11059 - 1.10000)}{0.05298} = 2.23.$$

The p -value is $2 \times P(t_{124} \geq 2.23) = 0.028$.

There is some evidence that the manufacturing process needs adjusting but it is not overwhelming.

- 8.2.19 Consider the hypotheses $H_0 : \mu = 0.2$ versus $H_A : \mu \neq 0.2$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{75} \times (0.23181 - 0.22500)}{0.07016} = 0.841.$$

The p -value is $2 \times P(t_{74} \geq 0.841) = 0.40$.

There is not sufficient evidence to conclude that the spray painting machine is not performing properly.

- 8.2.20 Consider the hypotheses $H_0 : \mu \geq 9.5$ versus $H_A : \mu < 9.5$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{80} \times (9.2294 - 9.5000)}{0.8423} = -2.87.$$

The p -value is $P(t_{79} \leq -2.87) = 0.0026$.

There is sufficient evidence to conclude that the design criterion has not been met.

8.2.21 The hypotheses are $H_0 : \mu \leq 238.5$ versus $H_A : \mu > 238.5$
and the test statistic is

$$t = \frac{\sqrt{16}(239.13 - 238.50)}{2.80} = 0.90.$$

The p -value is $P(t_{15} > 0.90) = 0.191$.

There is not sufficient evidence to conclude that the average voltage of the batteries from the production line is at least 238.5.

8.2.22 (a) $0.002 \leq 2 \times P(t_{11} > 3.21) \leq 0.01$

(b) $0.05 \leq 2 \times P(t_{23} > 1.96) \leq 0.10$

(c) $2 \times P(t_{29} > 3.88) \leq 0.001$

- 8.2.23 The hypotheses are $H_0 : \mu = 82.50$ versus $H_A : \mu \neq 82.50$
and the test statistic is

$$t = \frac{\sqrt{25}(82.40 - 82.50)}{0.14} = -3.571.$$

The p -value is $2 \times P(t_{24} > 3.571) = 0.0015$.

There is sufficient evidence to conclude that the average length of the components is not 82.50.

- 8.2.24 The hypotheses are $H_0 : \mu \leq 70$ versus $H_A : \mu > 70$
and the test statistic is

$$t = \frac{\sqrt{25}(71.97 - 70)}{7.44} = 1.324.$$

The p -value is $P(t_{24} > 1.324) = 0.099$.

There is some evidence to conclude that the components have an average weight larger than 70, but the evidence is not overwhelming.

- 8.2.25 The hypotheses are $H_0 : \mu = 7.000$ versus $H_A : \mu \neq 7.000$
and the test statistic is

$$t = \frac{\sqrt{28}(7.442 - 7.000)}{0.672} = 3.480.$$

The p -value is $2 \times P(t_{27} > 3.480) = 0.002$.

There is sufficient evidence to conclude that the average breaking strength is not 7.000.

- 8.2.26 The hypotheses are $H_0 : \mu \leq 50$ versus $H_A : \mu > 50$
and the test statistic is

$$t = \frac{\sqrt{25}(53.43 - 50)}{3.93} = 4.364.$$

The p -value is $P(t_{24} > 4.364) = 0.0001$.

There is sufficient evidence to conclude that average failure time of this kind of component is at least 50 hours.

- 8.2.27 The hypotheses are $H_0 : \mu \geq 25$ versus $H_A : \mu < 25$.

8.2.28 (a) The t -statistic is

$$t = \frac{\sqrt{20}(12.49-10)}{1.32} = 8.44$$

and the p -value is $2 \times P(t_{19} > 8.44)$ which is less than 1%.

(b) The t -statistic is

$$t = \frac{\sqrt{43}(3.03-3.2)}{0.11} = -10.13$$

and the p -value is $P(t_{42} > -10.13)$ which is greater than 10%.

(c) The t -statistic is

$$t = \frac{\sqrt{16}(73.43-85)}{16.44} = -2.815$$

and the p -value is $P(t_{15} < -2.815)$ which is less than 1%.

8.2.29 (a) The sample mean is $\bar{x} = 11.975$

and the sample standard deviation is $s = 2.084$

so that the t -statistic is

$$t = \frac{\sqrt{8}(11.975-11)}{2.084} = 1.32.$$

The p -value is $P(t_7 > 1.32)$ which is greater than 10%.

Consequently, the experiment does not provide sufficient evidence to conclude that the average time to toxicity of salmon fillets under these storage conditions is more than 11 days.

(b) With $t_{0.005,7} = 3.499$ the confidence interval is

$$11.975 \pm \frac{3.499 \times 2.084}{\sqrt{8}} = (9.40, 14.55).$$

8.2.30 A

8.2.31 D

8.2.32 B

8.2.33 A

8.2.34 D

8.2.35 B

- 8.6.1 (a) Consider the hypotheses $H_0 : \mu \leq 65$ versus $H_A : \mu > 65$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{15} \times (67.42 - 65.00)}{4.947} = 1.89.$$

The p -value is $P(t_{14} \geq 1.89) = 0.040$.

There is some evidence that the average distance at which the target is detected is at least 65 miles although the evidence is not overwhelming.

- (b) With $t_{0.01,14} = 2.624$ the confidence interval is

$$\left(67.42 - \frac{2.624 \times 4.947}{\sqrt{15}}, \infty \right) = (64.07, \infty).$$

- 8.6.2 (a) Consider the hypotheses $H_0 : \mu \geq 10$ versus $H_A : \mu < 10$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{40} \times (9.39 - 10.00)}{1.041} = -3.71.$$

The p -value is $P(t_{39} \leq -3.71) = 0.0003$.

The company can safely conclude that the telephone surveys will last on average less than ten minutes each.

- (b) With $t_{0.01,39} = 2.426$ the confidence interval is

$$\left(-\infty, 9.39 + \frac{2.426 \times 1.041}{\sqrt{40}} \right) = (-\infty, 9.79).$$

- 8.6.3 (a) Consider the hypotheses $H_0 : \mu = 75.0$ versus $H_A : \mu \neq 75.0$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{30} \times (74.63 - 75.00)}{2.095} = -0.9673.$$

The p -value is $2 \times P(t_{29} \geq 0.9673) = 0.341$.

There is not sufficient evidence to conclude that the paper does not have an average weight of 75.0 g/m².

- (b) With $t_{0.005, 29} = 2.756$ the confidence interval is

$$\left(74.63 - \frac{2.756 \times 2.095}{\sqrt{30}}, 74.63 + \frac{2.756 \times 2.095}{\sqrt{30}} \right) = (73.58, 75.68).$$

- (c) A total sample size of

$$n \geq 4 \times \left(\frac{t_{0.005, n_1-1} s}{L_0} \right)^2 = 4 \times \left(\frac{2.756 \times 2.095}{1.5} \right)^2 = 59.3$$

is required.

Therefore, an additional sample of at least $60 - 30 = 30$ observations should be sufficient.

- 8.6.4 (a) Consider the hypotheses $H_0 : \mu \geq 0.50$ versus $H_A : \mu < 0.50$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{14} \times (0.497 - 0.500)}{0.0764} = -0.147.$$

The p -value is $P(t_{13} \leq -0.147) = 0.443$.

There is not sufficient evidence to establish that the average deformity value of diseased arteries is less than 0.50.

- (b) With $t_{0.005, 13} = 3.012$ the confidence interval is

$$\left(0.497 - \frac{3.012 \times 0.0764}{\sqrt{14}}, 0.497 + \frac{3.012 \times 0.0764}{\sqrt{14}} \right) = (0.435, 0.559).$$

- (c) A total sample size of

$$n \geq 4 \times \left(\frac{t_{0.005, n_1-1} s}{L_0} \right)^2 = 4 \times \left(\frac{3.012 \times 0.0764}{0.10} \right)^2 = 21.2$$

is required.

Therefore, an additional sample of at least $22 - 14 = 8$ observations should be sufficient.

- 8.6.5 At a 90% confidence level the critical point is $t_{0.05,59} = 1.671$ and the confidence interval is

$$\left(69.618 - \frac{1.671 \times 1.523}{\sqrt{60}}, 69.618 + \frac{1.671 \times 1.523}{\sqrt{60}}\right) = (69.29, 69.95).$$

At a 95% confidence level the critical point is $t_{0.025,59} = 2.001$ and the confidence interval is

$$\left(69.618 - \frac{2.001 \times 1.523}{\sqrt{60}}, 69.618 + \frac{2.001 \times 1.523}{\sqrt{60}}\right) = (69.23, 70.01).$$

At a 99% confidence level the critical point is $t_{0.005,59} = 2.662$ and the confidence interval is

$$\left(69.618 - \frac{2.662 \times 1.523}{\sqrt{60}}, 69.618 + \frac{2.662 \times 1.523}{\sqrt{60}}\right) = (69.10, 70.14).$$

There is not strong evidence that 70 inches is not a plausible value for the mean height because it is included in the 95% confidence level confidence interval.

- 8.6.6 At a 90% confidence level the critical point is $t_{0.05,39} = 1.685$ and the confidence interval is

$$\left(32.042 - \frac{1.685 \times 5.817}{\sqrt{40}}, 32.042 + \frac{1.685 \times 5.817}{\sqrt{40}}\right) = (30.49, 33.59).$$

At a 95% confidence level the critical point is $t_{0.025,39} = 2.023$ and the confidence interval is

$$\left(32.042 - \frac{2.023 \times 5.817}{\sqrt{40}}, 32.042 + \frac{2.023 \times 5.817}{\sqrt{40}}\right) = (30.18, 33.90).$$

At a 99% confidence level the critical point is $t_{0.005,39} = 2.708$ and the confidence interval is

$$\left(32.042 - \frac{2.708 \times 5.817}{\sqrt{40}}, 32.042 + \frac{2.708 \times 5.817}{\sqrt{40}}\right) = (29.55, 34.53).$$

Since 35 and larger values are not contained within the 99% confidence level confidence interval they are not plausible values for the mean shoot height, and so these new results contradict the results of the previous study.

8.6.7 The interval $(472.56, 486.28)$ is
 $(479.42 - 6.86, 479.42 + 6.86)$

and

$$6.86 = \frac{2.787 \times 12.55}{\sqrt{26}}.$$

Since $2.787 = t_{0.005, 25}$ it follows that the confidence level is
 $1 - (2 \times 0.005) = 0.99$.

8.6.8 (a) Consider the hypotheses $H_0 : \mu \geq 0.36$ versus $H_A : \mu < 0.36$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{18} \times (0.337 - 0.36)}{0.025} = -3.903.$$

The p -value is $P(t_{17} \leq -3.903) = 0.0006$.

There is sufficient evidence to conclude that the average weight gain for composites of this kind is smaller than 0.36%.

(b) Using the critical point $t_{0.01, 17} = 2.567$ the confidence interval is

$$\left(-\infty, 0.337 + \frac{2.567 \times 0.025}{\sqrt{18}}\right) = (-\infty, 0.352).$$

8.6.9 Using the critical point $t_{0.01, 43} = 2.416$ the confidence interval is

$$\left(-\infty, 25.318 + \frac{2.416 \times 0.226}{\sqrt{44}}\right) = (-\infty, 25.400).$$

Consider the hypotheses $H_0 : \mu \geq 25.5$ versus $H_A : \mu < 25.5$.

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{44} \times (25.318 - 25.5)}{0.226} = -5.342.$$

The p -value is $P(t_{43} \leq -5.342) = 0.000$.

There is sufficient evidence to conclude that the average soil compressibility is no larger than 25.5.

8.6.11 At a 95% confidence level the critical points are

$$\chi_{0.025,17}^2 = 30.19 \text{ and } \chi_{0.975,17}^2 = 7.564$$

so that the confidence interval is

$$\left(\frac{(18-1) \times 6.48^2}{30.19}, \frac{(18-1) \times 6.48^2}{7.564} \right) = (23.6, 94.4).$$

At a 99% confidence level the critical points are

$$\chi_{0.005,17}^2 = 35.72 \text{ and } \chi_{0.995,17}^2 = 5.697$$

so that the confidence interval is

$$\left(\frac{(18-1) \times 6.48^2}{35.72}, \frac{(18-1) \times 6.48^2}{5.697} \right) = (20.0, 125.3).$$

8.6.12 At a 99% confidence level the critical points are

$$\chi_{0.005,40}^2 = 66.77 \text{ and } \chi_{0.995,40}^2 = 20.71$$

so that the confidence interval is

$$\left(\sqrt{\frac{(41-1) \times 0.124^2}{66.77}}, \sqrt{\frac{(41-1) \times 0.124^2}{20.71}} \right) = (0.095, 0.170).$$

8.6.13 At a 95% confidence level the critical points are

$$\chi_{0.025,19}^2 = 32.85 \text{ and } \chi_{0.975,19}^2 = 8.907$$

so that the confidence interval is

$$\left(\frac{(20-1) \times 11.90^2}{32.85}, \frac{(20-1) \times 11.90^2}{8.907} \right) = (81.9, 302.1).$$

8.6.14 At a 90% confidence level the critical points are

$$\chi_{0.05,15}^2 = 25.00 \text{ and } \chi_{0.95,15}^2 = 7.261$$

so that the confidence interval is

$$\left(\sqrt{\frac{(16-1) \times 0.058^2}{25.00}}, \sqrt{\frac{(16-1) \times 0.058^2}{7.261}} \right) = (0.045, 0.083).$$

At a 95% confidence level the critical points are

$$\chi_{0.025,15}^2 = 27.49 \text{ and } \chi_{0.975,15}^2 = 6.262$$

so that the confidence interval is

$$\left(\sqrt{\frac{(16-1) \times 0.058^2}{27.49}}, \sqrt{\frac{(16-1) \times 0.058^2}{6.262}} \right) = (0.043, 0.090).$$

At a 99% confidence level the critical points are

$$\chi_{0.005,15}^2 = 32.80 \text{ and } \chi_{0.995,15}^2 = 4.601$$

so that the confidence interval is

$$\left(\sqrt{\frac{(16-1) \times 0.058^2}{32.80}}, \sqrt{\frac{(16-1) \times 0.058^2}{4.601}} \right) = (0.039, 0.105).$$

8.6.15 (a) The p -value is $2 \times P(t_7 > 1.31)$ which is more than 0.20.

(b) The p -value is $2 \times P(t_{29} > 2.82)$ which is between 0.002 and 0.01.

(c) The p -value is $2 \times P(t_{24} > 1.92)$ which is between 0.05 and 0.10.

8.6.16 The hypotheses are $H_0 : \mu \geq 81$ versus $H_A : \mu < 81$ and the test statistic is

$$t = \frac{\sqrt{16} \times (76.99 - 81.00)}{5.37} = -2.987$$

so that the p -value is $P(t_{15} \leq -2.987) = 0.005$.

There is sufficient evidence to conclude that the average clay compressibility at the location is less than 81.

8.6.17 The hypotheses are $H_0 : \mu \leq 260.0$ versus $H_A : \mu > 260.0$ and the test statistic is

$$t = \frac{\sqrt{14} \times (266.5 - 260.0)}{18.6} = 1.308$$

so that the p -value is $P(t_{13} \geq 1.308) = 0.107$.

There is not sufficient evidence to conclude that the average strength of fibers of this type is at least 260.0.

8.6.18 (a) $n = 18$

(b) $\frac{50+52}{2} = 51$

(c) $\bar{x} = 54.61$

(d) $s = 19.16$

(e) $s^2 = 367.07$

(f) $\frac{s}{\sqrt{n}} = 4.52$

(g) With $t_{0.01,17} = 2.567$ the confidence interval is

$$\mu \in \left(54.61 - \frac{2.567 \times 19.16}{\sqrt{18}}, \infty \right) = (43.02, \infty).$$

(h) The test statistic is

$$t = \frac{\sqrt{18}(54.61 - 50)}{19.16} = 1.021$$

and the p -value is $2 \times P(t_{17} \geq 1.021)$.

The critical points in Table III imply that the p -value is larger than 0.20.

8.6.19 (a) True

(b) False

(c) True

(d) True

(e) True

(f) True

(g) True

8.6.20 The hypotheses are $H_0 : \mu = 200.0$ versus $H_A : \mu \neq 200.0$ and the test statistic is

$$t = \frac{\sqrt{22} \times (193.7 - 200)}{11.2} = -2.639$$

so that the p -value is $2 \times P(t_{21} \geq 2.639) = 0.015$.

There is some evidence that the average resistance of wires of this type is not 200.0 but the evidence is not overwhelming.

8.6.21 (a) Since

$$L = 74.5 - 72.3 = 2.2 = 2 \times \frac{t_{0.005,9}s}{\sqrt{10}} = 2 \times \frac{3.250 \times s}{\sqrt{10}}$$

it follows that $s = 1.070$.

(b) Since

$$4 \times \frac{3.250^2 \times 1.070^2}{1^2} = 48.4$$

it can be estimated that a further sample of size $49 - 10 = 39$ will be sufficient.

8.6.22 (a) The hypotheses are $H_0 : \mu = 600$ versus $H_A : \mu \neq 600$ and the test statistic is

$$t = \frac{\sqrt{10}(614.5 - 600)}{42.9} = 1.069$$

The p -value is $2 \times P(t_9 \geq 1.069) = 0.313$.

There is not sufficient evidence to establish that the population average is not 600.

(b) With $t_{0.01,9} = 3.250$ the confidence interval is

$$\mu \in 614.5 \pm \frac{3.250 \times 42.9}{\sqrt{10}} = (570.4, 658.6)$$

(c) With

$$4 \times \left(\frac{3.250 \times 42.9}{30} \right)^2 = 86.4$$

it can be estimated that about $87 - 10 = 77$ more items would need to be sampled.

8.6.23 (a) The hypotheses are $H_0 : \mu \geq 750$ versus $H_A : \mu < 750$ and the t -statistic is

$$t = \frac{\sqrt{12}(732.9 - 750)}{12.5} = -4.74$$

so that the p -value is $P(t_{11} < -4.74) = 0.0003$.

There is sufficient evidence to conclude that the flexibility of this kind of metal alloy is smaller than 750.

(b) With $t_{0.01,11} = 2.718$ the confidence interval is

$$\left(-\infty, 732.9 + \frac{2.718 \times 12.5}{\sqrt{12}}\right) = (-\infty, 742.7).$$

8.6.24 (a) $\bar{x} = \frac{\sum_{i=1}^9 x_i}{9} = \frac{4047.4}{9} = 449.71$

(b) The ordered data are: 402.9 418.4 423.6 442.3 453.2 459 477.7 483 487.3
Therefore, the sample median is 453.2.

(c) $s^2 = \frac{(\sum_{i=1}^9 x_i^2) - (\sum_{i=1}^9 x_i)^2/9}{8} = 913.9111$
 $s = 30.23$

(d) $\left(\bar{x} - \frac{t_{0.005,8} \times 30.23}{\sqrt{9}}, \bar{x} + \frac{t_{0.005,8} \times 30.23}{\sqrt{9}} \right)$
 $= \left(449.71 - \frac{3.355 \times 30.23}{3}, 449.71 + \frac{3.355 \times 30.23}{3} \right)$
 $= (415.9, 483.52)$

(e) $\left(-\infty, \bar{x} + \frac{t_{0.05,8} \times 30.23}{\sqrt{9}} \right)$
 $= \left(-\infty, 449.71 + \frac{1.86 \times 30.23}{3} \right)$
 $= (-\infty, 468.45)$

(f) $n \geq 4 \times \left(\frac{t_{0.005,8} \times 30.23}{L_0} \right)^2$
 $= 4 \times \left(\frac{3.355 \times 30.23}{50} \right)^2$
 $= 16.459 \simeq 17$

Therefore, $17 - 9 = 8$ additional samples should be sufficient.

(g) The hypotheses are $H_0 : \mu = 440$ versus $H_A : \mu \neq 440$.

The t -statistic is

$$t = \frac{\sqrt{9} \times (449.71 - 440)}{30.23} = 0.9636$$

so that the p -value is $2 \times P(t_8 \geq 0.9636) > 0.2$.

This large p -value indicates that H_0 should be accepted.

(h) The hypotheses are $H_0 : \mu \geq 480$ versus $H_A : \mu < 480$.

The t -statistic is

$$t = \frac{\sqrt{9} \times (449.71 - 480)}{30.23} = -3.006$$

so that the p -value is $P(t_8 \leq -3.006) < 0.01$.

This small p -value indicates that H_0 should be rejected.

8.6.25 (a) The sample mean is $\bar{x} = 3.669$

and the sample standard deviation is $s = 0.2531$.

The hypotheses are $H_0 : \mu \leq 3.50$ versus $H_A : \mu > 3.50$ and the t -statistic is

$$t = \frac{\sqrt{8}(3.669 - 3.50)}{0.2531} = 1.89$$

so that the p -value is $P(t_7 \geq 1.89) = 0.51$.

There is some evidence to establish that the average density of these kind of compounds is larger than 3.50, but the evidence is not overwhelming.

(b) With $t_{0.01,7} = 2.998$ the confidence interval is

$$\mu \in \left(3.669 - \frac{0.2531 \times 2.998}{\sqrt{8}}, \infty \right) = (3.40, \infty).$$

8.6.26 (a) The hypotheses are $H_0 : \mu = 385$ versus $H_A : \mu \neq 385$ and the t -statistic is

$$t = \frac{\sqrt{33}(382.97 - 385.00)}{3.81} = -3.06$$

so that the p -value is $2 \times P(t_{32} \geq 3.06) = 0.004$.

There is sufficient evidence to establish that the population mean is not 385.

(b) With $t_{0.005,32} = 2.738$ the confidence interval is

$$\mu \in \left(382.97 - \frac{3.81 \times 2.738}{\sqrt{33}}, 382.97 + \frac{3.81 \times 2.738}{\sqrt{33}} \right) = (381.1, 384.8).$$

8.6.27 (a) The t -statistic is

$$t = \frac{\sqrt{24} \times (2.39 - 2.5)}{0.21} = -2.566$$

so that the p -value is $2 \times P(t_{23} \geq |-2.566|)$.

The critical points in Table III imply that the p -value is between 0.01 and 0.02.

(b) The t -statistic is

$$t = \frac{\sqrt{30} \times (0.538 - 0.54)}{0.026} = -0.421$$

so that the p -value is $P(t_{29} \leq -0.421)$.

The critical points in Table III imply that the p -value is larger than 0.1.

(c) The t -statistic is

$$t = \frac{\sqrt{10} \times (143.6 - 135)}{4.8} = 5.67$$

so that the p -value is $P(t_9 \geq 5.67)$.

The critical points in Table III imply that the p -value is smaller than 0.0005.

8.6.36 B

8.6.37 C

8.6.38 A

8.6.39 E

8.6.40 A

8.6.41 B

8.6.46 A

8.6.47 C

8.6.48 B

8.6.49 D