6/03/20

Agenda for Math 5710 ♬ Meeting #6 ☻☺ 6/29/20 (8:00 a.m. – 9:10 a.m.)

1. Hello:

Brigham City:	Adam Blakeslee	Ryan Johnson	Tyson Mortensen	
Logan:	David Allen	Natalie Anderson	Kameron Baird	Stephen Brezinski
	Zachary Ellis	Adam Flanders	Brock Francom	Xiang Gao
	Ryan Goodman	Janette Goodridge	Hadley Hamar	Phillip Leifer
	Brittney Miller	Jonathan Mousley	Erika Mueller	Shelby Simpson
	Steven Summers	Matthew White	Zhang Xiaomeng	

2. Note the syllabus' activity list for today:

06: M/6/29	1. Construct the following concepts and comprehend associated communication structures: experiments, outcomes, randomness, sample spaces, and events. 2. Take advantage of Quiz 06.

3. Briefly, raise and address issues and questions stimulated by the following homework assignment:

A. Study our notes from Meeting #5 ; comprehend Jim's sample responses to the Quiz #5 prompts that are posted on *Canvas*.

B*. Examine each of the following propositions, determine whether or not it is true, display your choice by circling either "T" or "F," and prove that your choice is correct:

i. $(f \subseteq \{\,^{-}1, 0, 1\,\} \times \{\,^{-}1, 0, 1\,\} \ni f(x) = x^2\,) \Rightarrow$

$f: \{\,^{-}1, 0, 1\,\} \xrightarrow[onto]{1\text{-}1} \{\,^{-}1, 0, 1\,\}$

T F

ii. $\{\, n^2 : n \in \mathbb{N}\,\} \sim \mathbb{N}$

T F

iii. $\mathbb{Z} \sim \mathbb{N}$

T F

iv. $[\,0, 1\,] \sim [\,2, 3\,]$

T F

v. $[\,{}^{-}1,\ 0\,] \sim [\,0, 0.25\,]$

T F

vi. $V = \mathbb{Z} \Rightarrow \{\, {}^{-}n : n \in \omega \,\}^{c} = \mathbb{N}$

T F

C*. The following proposition is, of course, true:

$\mathbb{Z} \sim \mathbb{R} \ \veebar\ \mathbb{Z} \nsim \mathbb{R}$

Which one of the following propositions do you think is true? Circle either “$\mathbb{Z} \sim \mathbb{R}$” or “$\mathbb{Z} \nsim \mathbb{R}$” to indicate your choice: $\mathbb{Z} \sim \mathbb{R}$ $\mathbb{Z} \nsim \mathbb{R}$

Write a paragraph that explain the rationale for your choice.

D. Compare your responses to the homework prompts to those Jim posted in *Canvas* on the usual page.

E. Comprehend the entries from Lines #018–020 from our *Glossary* document.

4. Briskly pass our eyes over (without much reading) Item #029 of Glossary (which appears at the very beginning of the second section titled “Probability w/r Discrete Random Variables”:

029. Experiments, simulations, outcomes, random outcomes, sample spaces, events, and complements of events:

A. Experiment (undefined term)

B. A less-than rigorous definition for *simulation*:

A simulation is a procedure that facilitates addressing questions about real problems by running experiments that are algebraically equivalent to the real problem.

C. Outcome (undefined term)

D. Note and definition for *sample space*:

Note: “Ω” is often used as a symbol to denote a sample space.

Definition for sample space Ω:

$\Omega = \{ \text{ outcomes of an experiment } \}$

E. Definition for *event*:

$A \in \{ \text{ events of } \Omega \} \Leftrightarrow A \subseteq \Omega$

F. Definition and note for *complement of an event*:

Note: “A^c ” is read “the complement of event A”

Definition:

$A^c = \{ X : X \subseteq \Omega \ \wedge X \cap A = \varnothing \}$

5. Construct the concept of *experiment* and some related concepts (e.g., *outcome*):

A. The experiments we designed in response to the following homework prompt from Meeting #1:

E.* Design and describe an experiment that addresses a question about future events. The question should involve a prediction about some population – not about some unique individual member of that population. For example, rather than designing an experiment to help predict whether Jim becomes infected with COVID 19 before August 5, design an experiment to predict whether at least one member of our Math 5710 family will be infected with COVID 19 before August 5. Please post the resulting document (as a PDF file) on the indicated *Assignment* link of *Canvas*.

B. Since Jim built this agenda and recorded Meeting #6's lecture prior to our posting these experiments on Canvas, let’s read the following excerpt from about an experiment presented by Grinstead and Snell in their manuscript *Introduction to Probability* 2006):

> *During the Second World War, physicists at the Los Alamos Scientific Laboratory needed to know, for purposes of shielding, how far neutrons travel through various materials. This question was beyond the reach of theoretical calculations. Daniel McCracken, writing in the Scientific American, states:*
>
> > *The physicists had most of the necessary data: they knew the average distance a neutron of a given speed would travel in a given substance before it collided with an atomic nucleus, what the probabilities were that the neutron would bounce off*

instead of being absorbed by the nucleus, how much energy the neutron was likely to lose after a given a collision and so on.

John von Neumann and Stanislas Ulam suggested that the problem be solved by modeling the experiment by chance devices on a computer. Their work being secret, it was necessary to give it a code name. Von Neumann chose the name "Monte Carlo." Since that time, this method of simulation has been called the Monte Carlo Method.

C. Discuss the role of *simulation* in the development, application, and pedagogy of probability.

D. Consider simulated problems to help us conceptualize probability theory that is applicable to problems that are important in a variety of fields (e.g., physics, health science, psychology, meteorology, population biology, ..., demography) and why we need to address simulations that are comprehensible to a group of people with diverse specialties. Although, gambling and games of chance were the initial motivation for the development of probability theory and practices, simulations such as the following in some people's mind (e.g., Jim's) pale in comparison to seemingly more important problems (e.g., developing models to predict spreads of populations of corona-viruses). Note and read Item #9 of this agenda.

E. Comprehend the following problem:

For the purpose of formulating the rules of a game of chance in which a pair of fair dice (a red die and a yellow die) are rolled one time, Sybil wants to determine the likelihood of each of the possible *events* determined by the sum of the number of dots that appear on the top face of the yellow die and on the top face of the red die.

Sybil thinks, "Each die has six faces – a face with one dot, a face with two dots, a face with three dots, a face with four dots, a face with five dots, and a face with six dots. So there are 36 possible *outcomes* since 36 is the cardinality of the following set:

{ (r, y) : r = the number of dots on the red die's top face $\wedge$
y = the number of dots on the red die's top face } =
{ (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) }

And since I'm interested in the *random probability* of each of the possible *events*, the *events* of interest are the sum of numbers associated with the two top faces. So each event is associated with one element in the following set: { 2, 3, 4, 5, ..., 12 }.

I'll count the number outcomes that are associated with each of the 12 events:

Let X_i = the event that the sum is i. Thus,
$| X_2 | = | \{ (1, 1) \} | = 1$
$| X_3 | = | \{ (1, 2), (2, 1) \} | = 2$
$| X_4 | = | \{ (1, 3), (2, 2), (3, 1) \} | = 3$
$| X_5 | = | \{ (1, 4), (2, 3), (3, 2), (4, 1) \} | = 4$
$| X_6 | = | \{ (1, 5), (2, 4), (3, 3), (4, 2), (5, 1) \} | = 5$
$| X_7 | = | \{ (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) \} | = 6$
$| X_8 | = | \{ (2, 6), (3, 5), (4, 4), (5, 3), (6, 2) \} | = 5$
$| X_9 | = | \{ (3, 6), (4, 5), (5, 4), (6, 3) \} | = 4$
$| X_{10} | = | \{ (4, 6), (5, 5), (6, 4) \} | = 3$
$| X_{11} | = | \{ (5, 6), (6, 5) \} | = 2$
$| X_{12} | = | \{ (6, 6) \} | = 1$

So the most likely event is X_7 and the two events that are less likely than any of the others are X_2 and X_{11}. And my chart above shows the scale for each events. Then I'll use the chart to compute the exact theoretical probabilities by reporting the probabilities as the ratio of each of those events and 36 (i.e., $p(X_i) = X_i \div 36$."

Sybil smiles.

6. Relate the entries from Lines 29A–F of our Glossary to Sybil's experiment:

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Note: "Ω" is often used as a symbol to denote a sample space.

Definition for sample space Ω:

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E. Definition for *event*:

$$A \in \{\text{ events of } \Omega\} \Leftrightarrow A \subseteq \Omega$$

F. Definition and note for *complement of an event*:

Note: "A^c " is read "the complement of event A"

Definition:

$$A^c = \{X : X \subseteq \Omega \wedge X \cap A = \varnothing\}$$

7. Take advantage of Quiz 06.

8. Complete the following assignment prior to Meeting #7:

   A. Study our notes from Meeting #6 ; comprehend Jim's sample responses to the Quiz #6 prompts that are posted on *Canvas*.

   B. Comprehend the entries from Lines #029A–F from our *Glossary* document.

   C. From the Video Page of *Canvas*, view with comprehension the video "khan intro to probability - basic. "

9. Please read the following excerpt from "A Short History of Probability" from *Calculus, Volume II by Tom M. Apostol* (2nd edition, John Wiley & Sons, 1969):

A gambler's dispute in 1654 led to the creation of a mathematical theory of probability by two famous French mathematicians, Blaise Pascal and Pierre de Fermat. Antoine Gombaud, Chevalier de Méré, a French nobleman with an interest in gaming and gambling questions, called Pascal's attention to an apparent contradiction concerning a popular dice game. The game consisted in throwing a pair of dice 24 times; the problem was to decide whether or not to bet even money on the occurrence of at least one "double six" during the 24 throws. A seemingly well-established gambling rule led de Méré to believe that betting on a double six in 24 throws would be profitable, but his own calculations indicated just the opposite.

This problem and others posed by de Méré led to an exchange of letters between Pascal and Fermat in which the fundamental principles of probability theory were formulated for the first time. Although a few special problems on games of chance had been solved by some Italian mathematicians in the 15th and 16th centuries, no general theory was developed before this famous correspondence.

The Dutch scientist Christian Huygens, a teacher of Leibniz, learned of this correspondence and shortly thereafter (in 1657) published the first book on probability; entitled De Ratiociniis in Ludo Aleae, it was a treatise on problems associated with gambling. Because of the *inherent appeal of games of chance, probability theory soon became popular, and the subject developed rapidly during the 18th century. The major contributors during this period were Jakob Bernoulli (1654-1705) and Abraham de Moivre (1667-1754).*

In 1812 Pierre de Laplace (1749-1827) introduced a host of new ideas and mathematical techniques in his book, Théorie Analytique des Probabilités. Before Laplace, probability theory was solely concerned with developing a mathematical analysis of games of chance. Laplace applied probabilistic ideas to many scientific and practical problems. The theory of errors, actuarial mathematics, and statistical mechanics are examples of some of the important applications of probability theory developed in the l9th century.

Like so many other branches of mathematics, the development of probability theory has been stimulated by the variety of its applications. Conversely, each advance in the theory has enlarged the scope of its influence. Mathematical statistics is one important branch of applied probability; other applications occur in such widely different fields as genetics, psychology, economics, and engineering. Many workers have contributed to the theory since Laplace's time; among the most important are Chebyshev, Markov, von Mises, and Kolmogorov.

One of the difficulties in developing a mathematical theory of probability has been to arrive at a definition of probability that is precise enough for use in mathematics, yet comprehensive enough to be applicable to a wide range of phenomena. The search for a widely acceptable definition took nearly three centuries and was marked by much controversy. The matter was finally resolved in the 20th century by treating probability theory on an axiomatic basis. In 1933 a monograph by a Russian mathematician A. Kolmogorov outlined an axiomatic approach that forms the basis for the modern theory. (Kolmogorov's monograph is available in English translation as Foundations of Probability Theory, Chelsea, New York, 1950.) Since then the ideas have been refined somewhat and probability theory is now part of a more general discipline known as measure theory.