

Complete the following assignment prior to Meeting #14:

- C*. Express (in simplified form) the coefficient of x^2y^3 in the expansion of $(2x + 3y)^5$. Please display the computations in a pdf document uploaded to the appropriate Canvas assignment link.
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Sample computation:

Let $u = 2x$ and $v = 3y$, then $(2x + 3y)^5 = (u + v)^5$. The coefficient of u^2v^3 in the expansion of $(u + v)^5$ is $\binom{5}{3}$ and $u^2v^3 = (2^2 \cdot 3^3) \cdot x^2y^3$. Therefore the coefficient of x^2y^3 in the expansion of $(2x + 3y)^5$ is $\binom{5}{3} \cdot (2^2 \cdot 3^3) = 1080$.

- D*. Express (in simplified form) the following; display the computations in a pdf document uploaded to the appropriate Canvas assignment link:

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots + \binom{n}{n}$$

Sample computation:

We may recall the following theorem we proved a previous life:

$$A \in \{ \text{finite sets} \} \Rightarrow |\{ \text{subsets of } A \}| = 2^{|A|}.$$

Since $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots + \binom{n}{n}$ is the total number of subsets of a set whose cardinality is n , we have $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots + \binom{n}{n} = 2^n$.

But for those of us who didn't enjoy a life previous to Math 571, here is an alternative computation:

$$\text{Note that } \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \dots + \binom{n}{n} = \sum_{i=0}^n \binom{n}{i} 1^{n-i} 1^i = (1+1)^n = 2^n$$
