6/04/20

Agenda for Math 5710 ♬ Meeting #12 ☻☺ 7/08/20 (8:00 a.m. – 9:10 a.m.)

1. Hello:

Brigham City:	Adam Blakeslee	Ryan Johnson	Tyson Mortensen	
Logan:	David Allen	Natalie Anderson	Kameron Baird	Stephen Brezinski
	Zachary Ellis	Adam Flanders	Brock Francom	Xiang Gao
	Ryan Goodman	Janette Goodridge	Hadley Hamar	Phillip Leifer
	Brittney Miller	Jonathan Mousley	Erika Mueller	Shelby Simpson
	Steven Summers	Matthew White	Zhang Xiaomeng	

2. Note the syllabus' activity list for today:

12: W/7/08	1. Construct the concept of a permutation, focus on methods of counting, comprehend associated structures, and discover and prove a theorem with respect the permutations. 2. Take advantage of Quiz 12.

3. Briefly, raise and address issues and questions stimulated by the following homework assignment:

A. Study our notes from Meeting #11.

B. Comprehend Jim's sample response to Quiz 11.

C*. Three experiments are conducted:

Experiment 1: One card is randomly drawn from a well-shuffled poker deck consisting of 54 cards – including 2 jokers.

Experiment 2: A ball is randomly drawn from an urn that contains exactly 3 black balls, 3 green balls, 2 yellow balls, and 2 orange balls.

Experiment 3: Experiments 1 and 2 are combined.

What is the probability that Experiment 3 results in the event that both a joker is drawn and an orange ball is Not drawn?

Please display the computation that led to your solution; upload the resulting document on the indicated Assignment link on *Canvas*.

D. Comprehend the following entries from our Glossary:

034. A taste of Counting:

A. Multiplication principle theorem:

Theorem 05: Independent experiments #1 and #2 with respective finite sample spaces Ω_1 and Ω_2 are conducted $\Rightarrow |\ \Omega_1 \cap\ \Omega_2\ | = |\ \Omega_1\ | \cdot |\ \Omega_2\ |$

B. Definition of a *permutation* of a finite set:

$$\text{Given } A \in \{ \text{ finite sets } \}, (f \in \{ \text{ permutations of } A \} \Leftrightarrow f : A \underset{onto}{\overset{1:1}{\longrightarrow}} A)$$

E. From the Video Page of *Canvas*, view with comprehension the video "counting and the multiplication principle. "

F. Comprehend Jim's sample responses to the homework prompts that are posted on *Canvas*.

4. To help ourselves enter the spirit of counting principles techniques, peek into a time when Jim was a real teacher peek at the followings scenario (adapted from Cangelosi's *Teaching Mathematics in Secondary and Middle School: An Interactive Apprach, 3rd ed.* (Prentice Hall, New York, 2003):

The first few lessons of Mr. Polonia's unit on permutations and combinations include learning activities designed to help his algebra students achieve three objectives:

A. The student recognizes the advantages of being able compute permutations and combinations in real-life situations (appreciation).
B. The student discriminates between examples and nonexamples of each of the following two concepts: permutations and combinations (construct a concept).
C. The student explains why the following relationships hold:

$$ {}_nP_r = \frac{n!}{(n-r)!} \text{ and } {}_nC_r = \frac{n!}{(n-r)!\,r!} $$

(discover a relationship).

Mr. Polonia begins the first learning activity by telling the class: "Over the past two weeks, I've kept notes on comments I've overheard students make. Here, I'll show you five of them from students who gave me permission to share their comments with you." He reads each as he displays the following on the overhead projector:

α. "Did you notice that at the [school] dances, they never play two slow songs in a row? I think they're afraid of too much close dancing."
β. "One of us is bound to win the drawing; they pick five winners!"
γ. "Almost every time a teacher picks a group to do something, there are more non-blacks than blacks — like Johnson today, he picked me and two whites to supervise the drawing."
δ. "Ms. Simmons has never chosen one of my poems for the newspaper."
ε. "You ought to try the lunch room; there'll always be at least one thing you like."

Mr. Polonia continues: "Tomorrow, we will divide up into collaborative teams with each team assigned to analyze one of these statements for implications and causes. Let's take one now to show you what you'll be doing." He displays Statement γ, then initiates the following discussion:

Mr. Polonia: This statement hints at the possibility of racial bias influencing teachers' selection of student groups and committees. How might we examine the validity of that suggestion?
Theresa: We could keep a record of groups that teachers select over the next month or so and see how often blacks are in the minority.
Tracy: And if blacks are in the minority most of the time, then that would show bias.

Eva: I don't think so.
Tracy: Why not?
Eva: We African Americans are a minority in the school, so you'd expect most of the groups would have more non-blacks.
Milton: I think it's because the teachers always want to have one black in a group, so they have to spread us out in all the groups.
Mr. Polonia: How often would groups have black students in the majority if the teachers never considered color when they picked groups?
Don: That's impossible. A few teachers are out-and-out prejudiced, but the others bend over backwards to show they're not.
Mr. Polonia: Maybe so, but if we figured what the numbers would be if the choices were never biased, then we'd have something to compare with the actual choices. Theresa suggests we keep a record.
Tracy: Well, if there were no bias, then the percentage of groups with a majority of African-American students should equal the percentage of African-American students in school.
Mr. Polonia: Okay, in this class we have 9 black students and 15 non-black students. That's—
Eva: 9 out of 24 is 37.5%.
Tracy: So, 37.5% of the groups in this class ought to have an African-American majority and the other—
Eva: 62.5%.
Tracy: The other 62.5% should have an African-American minority.
Estelle: I don't think it's that simple because ...

After a few more minutes Mr. Polonia calls a halt to the discussion and directs the students as follows: "I would like for us to continue to work on this problem, but to move us toward developing a model; let's limit the situation for now to selecting groups of 3 people each from this class. Remember 9 of us are black and 15 of us are non-black. The question is, if there is no bias in the selection of a group of 3, what are the chances that either 2 or 3 of the 3 will be black? What's the first thing we need to do to figure that chance?

Eva: Make a sample space.
Mr. Polonia: The sample space for this problem might be quite long. So, for homework let's divide up the task by having each of you list all possible groups of 3 from the class of which you yourself are a member. Tomorrow, we'll eliminate the duplications, combine the rest, and voilà! We'll have our sample space.

After further clarification of the assignment, the students begin the task, returning the next day to learn that there are many more possible groups of 3 than they had expected — 2,024 in all. In class they complete the arduous task of counting the number of groups for each relevant category and to the surprise of most, discover the following:

> 4% of the groups are all black, 27% contain 2 blacks and 1 non-black, 47% contain 2 non-blacks and 1 black, and 22% contain all non-blacks.

Thus, they conclude that under the no-bias supposition, 31% of the time a group of 3 would have a black majority. After further discussion regarding the implication of their findings (i.e., how much above or below the 31% figure should be tolerated before the figures are indicative of bias), Mr. Polonia directs their attention to the process by which they obtained the 31% figure. All agree that the process was quite tedious and that they should search for easier ways.

From work with other examples, Mr. Polonia leads the students over the next few days to construct the concepts of permutations and combinations, and to discover relationships on which they base algorithms they invent for computing them.

Because Mr. Polonia was concerned with the appreciation objective as well as his cognitive objectives, he carefully chose initial examples that would get students' attention. Once he had them working on a problem, the mathematical content to be taught (i.e., permutation and combination formulas) came as a welcome tool for making their work easier and more efficient.

5. With an example or two, wrap our minds around and deep inside the concept of a permutation.

6. Compare out conception of permutation to the following definition; resolve the question of their compatibility:

   34-B. Definition of a *permutation* of a finite set:

   $$\text{Given } A \in \{\text{ finite sets }\}, (f \in \{\text{ permutations of } A\} \Leftrightarrow f : A \underset{onto}{\overset{1:1}{\longrightarrow}} A)$$

7. Comprehend and prove the following theorem:

   34-D. Theorem 06: ${}_nP_r = \dfrac{n!}{(n-r)!}$

8. Take advantage of Quiz 12.

9. Complete the following assignment prior to Meeting #12:

   A. Study our notes from Meeting #12.

   B. Comprehend Jim's sample response to Quiz 12.

   C*. Solve Lunar's problem and display your computation (As usual upload the resulting pdf document on the appropriate *Canvas* Assignment link):

   > The Osceola jail contains a row of 10 cells. Each of 10 inmates is randomly assigned to exactly one of those side-by-side cells. None of the 10 prisoners are exhibiting symptoms of the COVID-19 disease. However, unbeknownst to the correction officers who manage the jail, 5 of the 10 prisoners are carriers of the COVID-19 virus. Lunar would like to determine the probability of the event that no uninfected person occupies a cell next to an uninfected person.

   E. From the Video Page of *Canvas*, view with comprehension the video "permutations."

   F. Comprehend Jim's sample responses to the homework prompts that are posted on *Canvas*.

10. Enjoy: