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A02052161

HW: Expected Value and Variance

2.3: 1, 3, 4, 5, 7, 8, 10, 11, 17, 19
2.4: 1, 2, 3, 5, 6, 7, 13, 15

2.3

1) $E(x) = (0 \cdot .08) + (1 \cdot .11) + (2 \cdot .27) + (3 \cdot .33) + (4 \cdot .21) = \boxed{2.48}$

3) w/ replacement $E(x) = (0 \cdot .5625) + (1 \cdot .3750) + (2 \cdot .0625) = \boxed{.5}$
w/o replacement $E(x) = (0 \cdot .5588) + (1 \cdot .3824) + (2 \cdot .0588) = \boxed{.5}$
No

4) $E(x) = (1 \cdot \frac{2}{5}) + (2 \cdot \frac{3}{10}) + (3 \cdot \frac{1}{5}) + (4 \cdot \frac{1}{10}) = \boxed{2}$

5) $E(x) = 2(\frac{1}{13}) + 3(\frac{1}{13}) + 4(\frac{1}{13}) + 5(\frac{1}{13}) + 6(\frac{1}{13}) + 7(\frac{1}{13}) + 8(\frac{1}{13}) +$
$9(\frac{1}{13}) + 10(\frac{1}{13}) + 15(\frac{4}{13}) = \$8.77$
if you pay \$9, you will lose .23 cents on average per game.

7) $E(x) = (\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}) \cdot 499 + (\frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6})(-1) = \$1.31$
yes, you need to play a lot though.

8) negative, because they (the people) want to get rich

10a) $E(X) = \int_4^6 x \frac{1}{x \ln(1.5)} dx = \boxed{4.94}$
b) $F(x) = .5$, $x = \frac{1}{\ln(1.5)} \times (\ln(.5) - \ln(4)) = \boxed{4.90}$

11a) $E(x) = \int_0^4 x \left(\frac{x}{8}\right) dx = \boxed{2.67}$
b) $F(x) = .5$, $x = \frac{.5}{8} = \boxed{2.83}$

17a) $E(x) = \int_{10}^{11} \frac{4x^2(130 - x^2)}{819} dx = \boxed{10.418234}$
b) $F(x) = .5$, $x = \frac{4}{819}(65(.5)^2 - \frac{(.5)^4}{4} - 4000) = \boxed{10.385}$

19) $E(x) = (0 \cdot .38) + (1 \cdot .44) + (2 \cdot .15) + (3 \cdot .03) = \boxed{0.83}$

1, 2, 3, 5, 6, 7, 13, 15

2.4

1a) $E(x) = \left(-2 \cdot \frac{1}{3}\right) + \left(1 \cdot \frac{1}{6}\right) + \left(4 \cdot \frac{1}{3}\right) + \left(6 \cdot \frac{1}{6}\right) = \boxed{\frac{11}{6}}$

b) $\theta^2 = \left(-2 - \frac{11}{6}\right)^2 \cdot \frac{1}{3} + \left(1 - \frac{11}{6}\right)^2 \cdot \frac{1}{6} + \left(4 - \frac{11}{6}\right)^2 \cdot \frac{1}{3} + \left(6 - \frac{11}{6}\right)^2 \cdot \frac{1}{6} = \boxed{\frac{341}{36}}$

c) $E(x^2) = \frac{1}{3} \cdot -2^2 + \frac{1}{6} \cdot 1^2 + \frac{1}{3} \cdot 4^2 + \frac{1}{6} \cdot 6^2 = \frac{77}{6}$

$\theta^2 = \frac{77}{6} - \left(\frac{11}{6}\right)^2 = \boxed{\frac{341}{36}}$

2) $\theta^2 = ?$, $E(x) = (0 \cdot .08) + (1 \cdot .11) + (2 \cdot .27) + (3 \cdot .33) + (4 \cdot .21) = 2.48$

$E(x^2) = (0^2 \cdot .08) + (1^2 \cdot .11) + (2^2 \cdot .27) + (3^2 \cdot .33) + (4^2 \cdot .21) = 7.52$

$\theta^2 = E(x^2) - (E(x))^2 = 7.52 - 6.1504 = \boxed{1.37}$

$\theta = \sqrt{1.37} = \boxed{1.17}$

3) $E(x^2) = \left(1^2 \cdot \frac{2}{5}\right) + \left(2^2 \cdot \frac{3}{10}\right) + \left(3^2 \cdot \frac{1}{5}\right) + \left(4^2 \cdot \frac{1}{10}\right) = 5$

$E(x) = \left(1 \cdot \frac{2}{5}\right) + \left(2 \cdot \frac{3}{10}\right) + \left(3 \cdot \frac{1}{5}\right) + \left(4 \cdot \frac{1}{10}\right) = 2$

$\theta^2 = 5 - 2^2 = \boxed{1}$

$\boxed{\theta = 1}$

5a) $\mu = 4.94$, $\theta^2 = \int_4^6 (x - 4.94)^2 \left(\frac{1}{x \ln(1.5)}\right) dx = \boxed{0.332478}$

b) $\theta = \sqrt{.3325} = \boxed{0.5766}$

c) $F(x) = .25$, $\boxed{x = 4.43}$

$F(x) = .75$, $\boxed{x = 5.42}$

d) interquartile range $= 5.42 - 4.43 = \boxed{0.99}$

6a) $\mu = 2.67$, $\theta^2 = \int_0^4 (x - 2.67)^2 \left(\frac{x^2}{16}\right) dx = \boxed{.889}$

b) $\theta = \sqrt{.889} = \boxed{0.94}$

c) $F(x) = .25$, $\boxed{x = 2}$

$F(x) = .75$, $\boxed{x = 3.46}$

d) interquartile $= 3.46 - 2 = \boxed{1.46}$

7a) $E(x) = \int_{.125}^{.5} x \left(5.5054\left(.5 - (x - .25)^2\right)\right) dx = 0.3095$

$\theta^2 = \int_{.125}^{.5} (x - .3095)^2 \left(5.5054\left(.5 - (x - .25)^2\right)\right) dx = \boxed{0.0115}$

b) $\theta = \sqrt{.0115} = \boxed{0.1072}$

c) $F(x) = .25$, $\boxed{x = 0.217}$, $F(x) = .75$, $\boxed{x = 0.401}$

d) interquartile $= .401 - .217 = \boxed{0.184}$

13a) $\mu = 10.4182$, $\sigma^2 = \int_{10}^{11} (x-10.4182)^2 \frac{4x(130-x^2)}{819} = 0.0758$

$\sigma = \sqrt{.0758} = \boxed{0.275}$

b) $F(x) = .8$, $\boxed{x = 10.69}$

$F(x) = .1$, $\boxed{x = 10.07}$

15) $E(x) = (-1)(.25) + (1)(.4) + (4)(.35) = \boxed{1.55}$

$\sigma^2 = (-1-1.55)^2(.25) + (1-1.55)^2(.4) + (4-1.55)^2(.35) = 3.8475$

$\sigma = \sqrt{3.8475} = \boxed{1.96}$