6/03/20

Agenda for Math 5710 ♬ Meeting #5 ☻☺ 6/26/20 (8:00 a.m. – 9:10 a.m.)

1. Hello:

Brigham City:	Adam Blakeslee	Ryan Johnson	Tyson Mortensen	
Logan:	David Allen	Natalie Anderson	Kameron Baird	Stephen Brezinski
	Zachary Ellis	Adam Flanders	Brock Francom	Xiang Gao
	Ryan Goodman	Janette Goodridge	Hadley Hamar	Phillip Leifer
	Brittney Miller	Jonathan Mousley	Erika Mueller	Shelby Simpson
	Steven Summers	Matthew White	Zhang Xiaomeng	

2. Note the syllabus' activity list for today:

05: F/6/26	1. Immerse ourselves into the modern algebra of bijections, equivalence of sets, and set cardinality 2. Take advantage of Quiz 05

3. Briefly, raise and address issues and questions stimulated by our homework from Meeting #4:

B*. Examine each one of the following propositions to determine whether or not its true; indicate your choice by circling either "T" or "F" then prove your determination is correct:

i. $f : \mathbb{R} \to \mathbb{R} \ni f(x) = x^2 - 1 \Rightarrow f : \mathbb{R} \xrightarrow{1:1} \mathbb{R}$

T F

ii. $f : \mathbb{R} \to \mathbb{R} \ni f(x) = x^2 - 1 \Rightarrow f : \mathbb{R} \xrightarrow{onto} \mathbb{R}$

T F

iii. $g : \mathbb{R} \to \mathbb{R} \ni g(x) = \sqrt[3]{x} \Rightarrow g : \mathbb{R} \xrightarrow{1:1} \mathbb{R}$

T F

iv. $g : \mathbb{R} \to \mathbb{R} \ni g(x) = \sqrt[3]{x} \Rightarrow g : \mathbb{R} \xrightarrow{onto} \mathbb{R}$

T F

v. $h : \mathbb{R} \to \mathbb{R} \ni h(x) = \sqrt{x} \Rightarrow h : \mathbb{R} \xrightarrow{1:1} \mathbb{R}$

T F

vi. $(\, s \subseteq (\, \mathbb{Q} \times \mathbb{Q}\,) \times \mathbb{Q} \ni s = \{\, (\, (x, y),\ x{\cdot}y\,) : x, y \in \mathbb{Q}\,\}) \Rightarrow s : \mathbb{Q} \times \mathbb{Q} \xrightarrow{1:1} \mathbb{Q}$

T F

C. Compare your responses to the six homework prompts from Item #7B to the sample responses and accompanying explanations posted on *Canvas*.

D. Comprehend the entries from Lines #013–015 from our *Glossary* document.

4. Call to mind the type of functions known as "*one-to-one and onto function*s" or "*bijection*s."

A. Given $A, B \in$ {sets}, discuss the idea of a one-to-one and onto function from A to B.

B. Note: "$f : A \xrightarrow[onto]{1:1} B$" or "$f : A \xrightarrow[1:1]{onto} B$" is read "*f* is a one-to-one and onto function from A to B." It's also read, "*f* is a *bijection* from A to B."

Also note the following definition of a *bijection*:

Given $A, B \in$ {sets}, $(\, f : A \xrightarrow[onto]{1:1} B \Leftrightarrow f : A \xrightarrow{onto} B \wedge f : A \xrightarrow{1:1} B\,)$

C. Examine each of the following propositions, determine whether or not it is true, display your choice by circling either "T" or "F":

i. $\exists f \ni f : \{\, 1, 2, 7\} \xrightarrow[onto]{1:1} \{\, 0, 7\,\}$

T F

ii. $g \subseteq \mathbb{Z} \times \mathbb{N} \ni g = \{ (z, g(z)) : z \in \mathbb{Z} \wedge g(z) = {}^{-}2z \; \forall\, z < 0 \;\wedge g(z) = 2z + 1 \; \forall\, z \geq 0 \,) \Rightarrow\; g : \mathbb{Z} \xrightarrow[onto]{1:1} \mathbb{N}$

T F

5. Bring to mind the interrelation between sets commonly referred to as a "one-to-one correspondence."

A. The idea of *one-to-one correspondence* between two sets.

i. Examples

ii. Non-examples

B. Formulate a definition for *one-to-one correspondence* between sets:

Given A, $B \in$ {sets}, there is a *one-to-one correspondence* between A and $B \Leftrightarrow$

6. Focus on *set cardinality*.

A. Note: Given $A \in$ {sets}, "$|A|$" is read "the *cardinality* of A."

B. Note the following definition for *sets having the same cardinality*:

Given A, $B \in$ {sets}, $|A| = |B| \Leftrightarrow$
There is a *one-to-one correspondence* between A and B

C. Note: Given $A, B \in$ {sets}, "$A \sim B$" is read "A is *equivalent* to B"

D. Note the following definition for *equivalent* sets:

$$\text{Given } A, B \in \{\text{sets}\},\ A \sim B \Leftrightarrow |A| = |B|$$

7. Take a tiny sip of transfinite arithmetic as we walk through the following entries from out Glossary:

018. The eerie world of transfinite arithmetic:

A. Definition of *infinite set* that Georg Cantor (1845–1918) formulated:

$$S \in \{\text{infinite sets}\} \Leftrightarrow S \in \{\text{sets}\} \ni \exists\, T \in \{\text{sets}\} \ni (\, T \subset S \ \wedge\ T \sim S\,)$$

B. Definition of *finite set*:

$$S \in \{\text{finite sets}\} \Leftrightarrow (\, S \in \{\text{sets}\} \wedge\ S \notin \{\text{infinite sets}\})$$

C. Axiom w/r *cardinality of finite sets*:

$$S \in \{\text{finite sets}\} \Rightarrow |S| \in\ \omega$$

D. Note: "$\aleph_o$" is read "aleph-naught." And $\aleph_o$ is defined as follows:

$$\text{Given } A \in \{\text{ sets }\},\ (\ |A| =\ \aleph_o \Leftrightarrow A \sim \mathbb{N}\ )$$

E. Definition of a *countably infinite set* (i.e., *countable set*):

$$|A| =\ \aleph_o \Leftrightarrow A \in \{\text{ countably infinite sets }\}\ )$$

F. Note: Some infinite sets (e.g., $\mathbb{I}$ and $\mathbb{R}$) are not countably infinite. If an infinite set is not countably infinite it is an element of { uncountable sets }. Definition of an *uncountably infinite set*:

Given $A \in$ { infinite sets }, (A is uncountably infinite $\Leftrightarrow$ A is not countably infinite).

Note: "A is uncountably infinite" is expressed "$|\ A\ | = \mathcal{C}$" and "$\mathcal{C}$" is referred to as "*the continuum*."

8. Take advantage of Quiz 05.

9. Complete the following assignment prior to Meeting #6:

   A. Study our notes from Meeting #5 ; comprehend Jim's sample responses to the Quiz #5 prompts that are posted on *Canvas*.

   B*. Examine each of the following propositions, determine whether or not it is true, display your choice by circling either "T" or "F," and prove that your choice is correct (Please post the resulting PDF file using the indicated Assignment link on Canvas):

      i. $(f \subseteq \{\,^{-}1, 0, 1\,\} \times \{\,^{-}1, 0, 1\,\} \ni f(x) = x^2\,) \Rightarrow$

      $f: \{\,^{-}1, 0, 1\,\} \xrightarrow[onto]{1\text{-}1} \{\,^{-}1, 0, 1\,\}$

      T F

      ii. $\{\, n^2 : n \in \mathbb{N}\,\} \sim \mathbb{N}$

      T F

iii. $\mathbb{Z} \sim \mathbb{N}$

T F

iv. $[\,0, 1\,] \sim [\,2, 3\,]$

T F

v. $[\,{}^{-}1,\ 0\,] \sim [\,0, 0.25\,]$

T F

vi. $V = \mathbb{Z} \Rightarrow \{ {}^{-}n : n \in \omega \}^{c} = \mathbb{N}$

T F

C*. The following proposition is, of course, true:

$$\mathbb{Z} \sim \mathbb{R} \ \veebar \ \mathbb{Z} \nsim \mathbb{R}$$

Which one of the following propositions do you think is true? Circle either "$\mathbb{Z} \sim \mathbb{R}$" or "$\mathbb{Z} \nsim \mathbb{R}$" to indicate your choice: $\mathbb{Z} \sim \mathbb{R}$ $\quad$ $\mathbb{Z} \nsim \mathbb{R}$

Write a paragraph that explain the rationale for your choice.

D. Compare your responses to the homework prompts to those Jim posted in *Canvas* on the usual page.

E. Comprehend the entries from Lines #018–020 from our *Glossary* document.

10. Check out the following;