9.2.1 The differences $z_i = x_i - y_i$ have a sample mean $\bar{z} = 4.257$ and a sample standard deviation $s = 37.65$.

Consider the hypotheses

$H_0 : \mu = \mu_A - \mu_B \leq 0$ versus $H_A : \mu = \mu_A - \mu_B > 0$

where the alternative hypothesis states that the new assembly method is quicker on average than the standard assembly method.

The test statistic is

$t = \frac{\sqrt{n}\,\bar{z}}{s} = \frac{\sqrt{35}\times 4.257}{37.65} = 0.669.$

The p-value is $P(t_{34} \geq 0.669) = 0.254$.

There is *not* sufficient evidence to conclude that the new assembly method is any quicker on average than the standard assembly method.

With $t_{0.05,34} = 1.691$ a one-sided 95% confidence level confidence interval

for $\mu = \mu_A - \mu_B$ is

$\left(4.257 - \frac{1.691\times 37.65}{\sqrt{35}}, \infty\right) = (-6.50, \infty).$

9.2.2 The differences $z_i = x_i - y_i$ have a sample mean $\bar{z} = -1.36$ and a sample standard deviation $s = 6.08$.

Consider the hypotheses

$H_0 : \mu = \mu_A - \mu_B = 0$ versus $H_A : \mu = \mu_A - \mu_B \neq 0.$

The test statistic is

$t = \frac{\sqrt{n}\,\bar{z}}{s} = \frac{\sqrt{14}\times(-1.36)}{6.08} = -0.837.$

The p-value is $2 \times P(t_{13} \leq -0.837) = 0.418$.

There is *not* sufficient evidence to conclude that the different stimulation conditions affect the adhesion of the red blood cells.

With $t_{0.025,13} = 2.160$ a two-sided 95% confidence level confidence interval

for $\mu = \mu_A - \mu_B$ is

$\left(-1.36 - \frac{2.160\times 6.08}{\sqrt{14}}, -1.36 + \frac{2.160\times 6.08}{\sqrt{14}}\right) = (-4.87, 2.15).$

9.2.3 The differences $z_i = x_i - y_i$ have a sample mean $\bar{z} = 0.570$ and a sample standard deviation $s = 0.813$.

Consider the hypotheses

$H_0 : \mu = \mu_A - \mu_B \leq 0$ versus $H_A : \mu = \mu_A - \mu_B > 0$

where the alternative hypothesis states that the new tires have a smaller average reduction in tread depth than the standard tires.

The test statistic is

$t = \frac{\sqrt{n}\,\bar{z}}{s} = \frac{\sqrt{20}\times 0.570}{0.813} = 3.14.$

The p-value is $P(t_{19} \geq 3.14) = 0.003$.

There is sufficient evidence to conclude that the new tires are better than the standard tires in terms of the average reduction in tread depth.

With $t_{0.05,19} = 1.729$ a one-sided 95% confidence level confidence interval

for $\mu = \mu_A - \mu_B$ is

$\left(0.570 - \frac{1.729\times 0.813}{\sqrt{20}}, \infty\right) = (0.256, \infty).$

9.2.4 The differences $z_i = x_i - y_i$ have a sample mean $\bar{z} = -7.70$ and a sample standard deviation $s = 14.64$.

Consider the hypotheses

$H_0 : \mu = \mu_A - \mu_B \geq 0$ versus $H_A : \mu = \mu_A - \mu_B < 0$

where the alternative hypothesis states that the new teaching method produces higher scores on average than the standard teaching method.

The test statistic is

$t = \frac{\sqrt{n}\,\bar{z}}{s} = \frac{\sqrt{40}\times(-7.70)}{14.64} = -3.33.$

The p-value is $P(t_{39} \leq -3.33) = 0.001$.

There is sufficient evidence to conclude that the new teaching method is better since it produces higher scores on average than the standard teaching method.

With $t_{0.05,39} = 1.685$ a one-sided 95% confidence level confidence interval

for $\mu = \mu_A - \mu_B$ is

$\left(-\infty, -7.70 + \frac{1.685\times 14.64}{\sqrt{40}}\right) = (-\infty, -3.80).$

9.2.5 The differences $z_i = x_i - y_i$ have a sample mean $\bar{z} = 2.20$ and a sample standard deviation $s = 147.8$.

Consider the hypotheses

$H_0 : \mu = \mu_A - \mu_B = 0$ versus $H_A : \mu = \mu_A - \mu_B \neq 0$.

The test statistic is

$t = \frac{\sqrt{n}\,\bar{z}}{s} = \frac{\sqrt{18} \times 2.20}{147.8} = 0.063.$

The p-value is $2 \times P(t_{17} \geq 0.063) = 0.95$.

There is *not* sufficient evidence to conclude that the two laboratories are any different in the datings that they provide.

With $t_{0.025,17} = 2.110$ a two-sided 95% confidence level confidence interval

for $\mu = \mu_A - \mu_B$ is

$\left(2.20 - \frac{2.110 \times 147.8}{\sqrt{18}}, 2.20 + \frac{2.110 \times 147.8}{\sqrt{18}}\right) = (-71.3, 75.7).$

9.2.6 The differences $z_i = x_i - y_i$ have a sample mean $\bar{z} = -1.42$ and a sample standard deviation $s = 12.74$.

Consider the hypotheses

$H_0 : \mu = \mu_A - \mu_B \geq 0$ versus $H_A : \mu = \mu_A - \mu_B < 0$

where the alternative hypothesis states that the new golf balls travel further on average than the standard golf balls.

The test statistic is

$t = \frac{\sqrt{n}\,\bar{z}}{s} = \frac{\sqrt{24} \times (-1.42)}{12.74} = -0.546.$

The p-value is $P(t_{23} \leq -0.546) = 0.30$.

There is *not* sufficient evidence to conclude that the new golf balls travel further on average than the standard golf balls.

With $t_{0.05,23} = 1.714$ a one-sided 95% confidence level confidence interval

for $\mu = \mu_A - \mu_B$ is

$\left(-\infty, -1.42 + \frac{1.714 \times 12.74}{\sqrt{24}}\right) = (-\infty, 3.04).$

9.2.7 The differences $z_i = x_i - y_i$ have a sample mean $\bar{z} = -2.800$ and a sample standard deviation $s = 6.215$.

The hypotheses are

$H_0 : \mu = \mu_A - \mu_B = 0$ versus $H_A : \mu = \mu_A - \mu_B \neq 0$

and the test statistic is

$t = \frac{\sqrt{10}\times(-2.800)}{6.215} = -1.425.$

The p-value is $2 \times P(t_9 \geq 1.425) = 0.188$.

There is not sufficient evidence to conclude that procedures A and B give different readings on average.

The reviewer's comments are plausible.

9.2.8 The differences $z_i = x_i - y_i$ have a sample mean $\bar{z} = 1.375$ and a sample standard deviation $s = 1.785$.

Consider the hypotheses

$H_0 : \mu = \mu_S - \mu_N \leq 0$ versus $H_A : \mu = \mu_S - \mu_N > 0$

where the alternative hypothesis states that the new antibiotic is quicker than the standard antibiotic.

The test statistic is

$t = \frac{\sqrt{n}\,\bar{z}}{s} = \frac{\sqrt{8}\times 1.375}{1.785} = 2.18.$

The p-value is $P(t_7 \geq 2.18) = 0.033$.

Consequently, there is some evidence that the new antibiotic is quicker than the standard antibiotic, but the evidence is not overwhelming.

9.2.9 The differences $z_i = x_i - y_i$ have a sample mean $\bar{z} = 0.85$ and a sample standard deviation $s = 4.283$.

Consider the hypotheses

$H_0 : \mu = \mu_A - \mu_B = 0$ versus $H_A : \mu = \mu_A - \mu_B \neq 0$

where the alternative hypothesis states that the addition of the surfactant has an effect on the amount of uranium-oxide removed from the water.

The test statistic is

$t = \frac{\sqrt{n}\,\bar{z}}{s} = \frac{\sqrt{6}\times 0.85}{4.283} = 0.486.$

The p-value is $2 \times P(t_5 \geq 0.486) = 0.65$.

Consequently, there is *not* sufficient evidence to conclude that the addition of the surfactant has an effect on the amount of uranium-oxide removed from the water.

9.2.10 B

9.2.11 D

9.3.2 (a) The pooled variance is

$$s_p^2 = \frac{(n-1)s_x^2+(m-1)s_y^2}{n+m-2} = \frac{((14-1)\times 4.30^2)+((14-1)\times 5.23^2)}{14+14-2} = 22.92.$$

With $t_{0.005,26} = 2.779$ a 99% two-sided confidence interval for $\mu_A - \mu_B$ is

$$32.45 - 41.45 \pm 2.779 \times \sqrt{22.92} \times \sqrt{\frac{1}{14} + \frac{1}{14}}$$

$$= (-14.03, -3.97).$$

(b) Since

$$\frac{\left(\frac{4.30^2}{14}+\frac{5.23^2}{14}\right)^2}{\frac{4.30^4}{14^2\times(14-1)}+\frac{5.23^4}{14^2\times(14-1)}} = 25.06$$

the degrees of freedom are $\nu = 25$.

Using a critical point $t_{0.005,25} = 2.787$ a 99% two-sided confidence interval for $\mu_A - \mu_B$ is

$$32.45 - 41.45 \pm 2.787 \times \sqrt{\frac{4.30^2}{14} + \frac{5.23^2}{14}}$$

$$= (-14.04, -3.96).$$

(c) The test statistic is

$$t = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{s_x^2}{n}+\frac{s_y^2}{m}}} = \frac{32.45-41.45}{\sqrt{\frac{4.30^2}{14}+\frac{5.23^2}{14}}} = 4.97.$$

The null hypothesis is rejected since $|t| = 4.97$ is larger than the critical point $t_{0.005,26} = 2.779$.
The p-value is $2 \times P(t_{26} \geq 4.97) = 0.000$.

9.3.3 (a) The pooled variance is

$$s_p^2 = \frac{(n-1)s_x^2+(m-1)s_y^2}{n+m-2} = \frac{((8-1)\times 44.76^2)+((17-1)\times 38.94^2)}{8+17-2} = 1664.6.$$

With $t_{0.005,23} = 2.807$ a 99% two-sided confidence interval for $\mu_A - \mu_B$ is

$$675.1 - 702.4 \pm 2.807 \times \sqrt{1664.6} \times \sqrt{\tfrac{1}{8} + \tfrac{1}{17}}$$

$$= (-76.4, 21.8).$$

(b) Since

$$\frac{\left(\frac{44.76^2}{8}+\frac{38.94^2}{17}\right)^2}{\frac{44.76^4}{8^2\times(8-1)}+\frac{38.94^4}{17^2\times(17-1)}} = 12.2$$

the degrees of freedom are $\nu = 12$.

Using a critical point $t_{0.005,12} = 3.055$
a 99% two-sided confidence interval for $\mu_A - \mu_B$ is

$$675.1 - 702.4 \pm 3.055 \times \sqrt{\tfrac{44.76^2}{8} + \tfrac{38.94^2}{17}}$$

$$= (-83.6, 29.0).$$

(c) The test statistic is

$$t = \frac{\bar{x}-\bar{y}}{s_p\sqrt{\frac{1}{n}+\frac{1}{m}}} = \frac{675.1-702.4}{\sqrt{1664.6}\times\sqrt{\frac{1}{8}+\frac{1}{17}}} = -1.56.$$

The null hypothesis is accepted since $|t| = 1.56$ is smaller than the critical point $t_{0.005,23} = 2.807$.
The p-value is $2 \times P(t_{23} \geq 1.56) = 0.132$.

9.3.4 (a) Since

$$\frac{\left(\frac{1.07^2}{10}+\frac{0.62^2}{9}\right)^2}{\frac{1.07^4}{10^2\times(10-1)}+\frac{0.62^4}{9^2\times(9-1)}} = 14.7$$

the degrees of freedom are $\nu = 14$.

Using a critical point $t_{0.01,14} = 2.624$
a 99% one-sided confidence interval for $\mu_A - \mu_B$ is

$$\left(7.76 - 6.88 - 2.624 \times \sqrt{\frac{1.07^2}{10} + \frac{0.62^2}{9}}, \infty\right)$$

$= (-0.16, \infty)$.

(b) The value of c increases with a confidence level of 95%.

(c) The test statistic is

$$t = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{s_x^2}{n}+\frac{s_y^2}{m}}} = \frac{7.76-6.88}{\sqrt{\frac{1.07^2}{10}+\frac{0.62^2}{9}}} = 2.22.$$

The null hypothesis is accepted since
$t = 2.22 \leq t_{0.01,14} = 2.624$.
The p-value is $P(t_{14} \geq 2.22) = 0.022$.

9.3.5 (a) The pooled variance is

$$s_p^2 = \frac{(n-1)s_x^2+(m-1)s_y^2}{n+m-2} = \frac{((13-1)\times 0.00128^2)+((15-1)\times 0.00096^2)}{13+15-2}$$

$$= 1.25 \times 10^{-6}.$$

With $t_{0.05,26} = 1.706$ a 95% one-sided confidence interval for $\mu_A - \mu_B$ is

$$\left(-\infty, 0.0548 - 0.0569 + 1.706 \times \sqrt{1.25 \times 10^{-6}} \times \sqrt{\frac{1}{13} + \frac{1}{15}}\right)$$

$$= (-\infty, -0.0014).$$

(b) The test statistic is

$$t = \frac{\bar{x}-\bar{y}}{s_p\sqrt{\frac{1}{n}+\frac{1}{m}}} = \frac{0.0548-0.0569}{\sqrt{1.25\times 10^{-6}}\times\sqrt{\frac{1}{13}+\frac{1}{15}}} = -4.95.$$

The null hypothesis is rejected at size $\alpha = 0.01$ since $t = -4.95 < -t_{0.01,26} = -2.479$.
The null hypothesis is consequently also rejected at size $\alpha = 0.05$.
The p-value is $P(t_{26} \leq -4.95) = 0.000$.

9.3.6 (a) The pooled variance is

$$s_p^2 = \frac{(n-1)s_x^2+(m-1)s_y^2}{n+m-2} = \frac{((41-1)\times 0.124^2)+((41-1)\times 0.137^2)}{41+41-2} = 0.01707.$$

The test statistic is

$$t = \frac{\bar{x}-\bar{y}}{s_p\sqrt{\frac{1}{n}+\frac{1}{m}}} = \frac{3.04-3.12}{\sqrt{0.01707}\times\sqrt{\frac{1}{41}+\frac{1}{41}}} = -2.77.$$

The null hypothesis is rejected at size $\alpha = 0.01$ since $|t| = 2.77$ is larger than $t_{0.005,80} = 2.639$.
The p-value is $2 \times P(t_{80} \leq -2.77) = 0.007$.

(b) With $t_{0.005,80} = 2.639$ a 99% two-sided confidence interval for $\mu_A - \mu_B$ is

$$3.04 - 3.12 \pm 2.639 \times \sqrt{0.01707} \times \sqrt{\frac{1}{41} + \frac{1}{41}}$$

$$= (-0.156, -0.004).$$

(c) There is sufficient evidence to conclude that the average thicknesses of sheets produced by the two processes are different.

9.3.7 (a) Since

$$\frac{\left(\frac{11.90^2}{20}+\frac{4.61^2}{25}\right)^2}{\frac{11.90^4}{20^2\times(20-1)}+\frac{4.61^4}{25^2\times(25-1)}} = 23.6$$

the degrees of freedom are $\nu = 23$.

Consider the hypotheses
$H_0 : \mu = \mu_A - \mu_B \geq 0$ versus $H_A : \mu = \mu_A - \mu_B < 0$
where the alternative hypothesis states that the synthetic fiber bundles have an average breaking strength larger than the wool fiber bundles.

The test statistic is

$$t = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{s_x^2}{n}+\frac{s_y^2}{m}}} = \frac{436.5-452.8}{\sqrt{\frac{11.90^2}{20}+\frac{4.61^2}{25}}} = -5.788.$$

The null hypothesis is rejected at size $\alpha = 0.01$ since
$t = -5.788 < -t_{0.01,23} = -2.500$.
The p-value is $P(t_{23} \leq -5.788) = 0.000$.

(b) With a critical point $t_{0.01,23} = 2.500$
a 99% one-sided confidence interval for $\mu_A - \mu_B$ is

$$\left(-\infty, 436.5 - 452.8 + 2.500 \times \sqrt{\frac{11.90^2}{20} + \frac{4.61^2}{25}}\right)$$

$= (-\infty, -9.3)$.

(c) There is sufficient evidence to conclude that the synthetic fiber bundles have an average breaking strength larger than the wool fiber bundles.

9.3.8 Since

$$\frac{\left(\frac{0.058^2}{16}+\frac{0.062^2}{16}\right)^2}{\frac{0.058^4}{16^2\times(16-1)}+\frac{0.062^4}{16^2\times(16-1)}} = 29.9$$

the appropriate degrees of freedom for a general analysis without assuming equal population variances are $\nu = 29$.

Consider the hypotheses

$H_0 : \mu = \mu_A - \mu_B \geq 0$ versus $H_A : \mu = \mu_A - \mu_B < 0$

where the alternative hypothesis states that the brand B sugar packets weigh slightly more on average than brand A sugar packets.

The test statistic is

$$t = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{s_x^2}{n}+\frac{s_y^2}{m}}} = \frac{1.053-1.071}{\sqrt{\frac{0.058^2}{16}+\frac{0.062^2}{16}}} = -0.848$$

and the p-value is $P(t_{29} \leq -0.848) = 0.202$.

There is *not* sufficient evidence to conclude that the brand B sugar packets weigh slightly more on average than brand A sugar packets.

9.3.9 (a) The test statistic is

$$z = \frac{\bar{x}-\bar{y}-\delta}{\sqrt{\frac{\sigma_A^2}{n}+\frac{\sigma_B^2}{m}}} = \frac{100.85-89.32-3}{\sqrt{\frac{25^2}{47}+\frac{20^2}{62}}} = 1.92$$

and the p-value is $2 \times \Phi(-1.92) = 0.055$.

(b) With a critical point $z_{0.05} = 1.645$ a 90% two-sided confidence interval for $\mu_A - \mu_B$ is

$$100.85 - 89.32 \pm 1.645 \times \sqrt{\frac{25^2}{47} + \frac{20^2}{62}}$$

$$= (4.22, 18.84).$$

9.3.10 (a) The test statistic is

$$z = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{\sigma_A^2}{n}+\frac{\sigma_B^2}{m}}} = \frac{5.782-6.443}{\sqrt{\frac{2.0^2}{38}+\frac{2.0^2}{40}}} = -1.459$$

and the p-value is $\Phi(-1.459) = 0.072$.

(b) With a critical point $z_{0.01} = 2.326$ a 99% one-sided confidence interval for $\mu_A - \mu_B$ is

$$\left(-\infty, 5.782 - 6.443 + 2.326 \times \sqrt{\frac{2.0^2}{38} + \frac{2.0^2}{40}}\right)$$

$= (-\infty, 0.393)$.

9.3.11 (a) The test statistic is

$$z = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{\sigma_A^2}{n}+\frac{\sigma_B^2}{m}}} = \frac{19.50-18.64}{\sqrt{\frac{1.0^2}{10}+\frac{1.0^2}{12}}} = 2.009$$

and the p-value is $2 \times \Phi(-2.009) = 0.045$.

(b) With a critical point $z_{0.05} = 1.645$ a 90% two-sided confidence interval for $\mu_A - \mu_B$ is

$$19.50 - 18.64 \pm 1.645 \times \sqrt{\frac{1.0^2}{10} + \frac{1.0^2}{12}}$$

$= (0.16, 1.56)$.

With a critical point $z_{0.025} = 1.960$ a 95% two-sided confidence interval for $\mu_A - \mu_B$ is

$$19.50 - 18.64 \pm 1.960 \times \sqrt{\frac{1.0^2}{10} + \frac{1.0^2}{12}}$$

$= (0.02, 1.70)$.

With a critical point $z_{0.005} = 2.576$ a 99% two-sided confidence interval

for $\mu_A - \mu_B$ is

$$19.50 - 18.64 \pm 2.576 \times \sqrt{\frac{1.0^2}{10} + \frac{1.0^2}{12}}$$

$= (-0.24, 1.96)$.

9.3.12 Using 2.6 as an upper bound for $t_{0.005,\nu}$ equal sample sizes of

$$n = m \geq \frac{4\,t^2_{\alpha/2,\nu}\,(\sigma_A^2+\sigma_B^2)}{L_0^2} = \frac{4\times 2.6^2\times(10.0^2+15.0^2)}{10.0^2} = 87.88$$

should be sufficient.

Equal sample sizes of at least 88 can be recommended.

9.3.13 Using 2.0 as an upper bound for $t_{0.025,\nu}$ equal sample sizes of

$$n = m \geq \frac{4\,t^2_{\alpha/2,\nu}\,(\sigma_A^2+\sigma_B^2)}{L_0^2} = \frac{4\times 2.0^2\times(1.2^2+1.2^2)}{1.0^2} = 46.08$$

should be sufficient.

Equal sample sizes of at least 47 can be recommended.

9.3.14 Using $t_{0.005,26} = 2.779$ equal total sample sizes of

$$n = m \geq \frac{4\,t^2_{\alpha/2,\nu}\,(s_x^2+s_y^2)}{L_0^2} = \frac{4\times 2.779^2\times(4.30^2+5.23^2)}{5.0^2} = 56.6$$

should be sufficient.

Additional sample sizes of at least $57 - 14 = 43$ from each population can be recommended.

9.3.15 Using $t_{0.005,80} = 2.639$ equal total sample sizes of

$$n = m \geq \frac{4\,t^2_{\alpha/2,\nu}\,(s_x^2+s_y^2)}{L_0^2} = \frac{4\times 2.639^2\times(0.124^2+0.137^2)}{0.1^2} = 95.1$$

should be sufficient.

Additional sample sizes of at least $96 - 41 = 55$ from each population can be recommended.

9.3.16 (a) The appropriate degrees of freedom are

$$\frac{\left(\frac{0.315^2}{12}+\frac{0.297^2}{13}\right)^2}{\frac{0.315^4}{12^2\times(12-1)}+\frac{0.297^4}{13^2\times(13-1)}} = 22.5$$

which should be rounded down to $\nu = 22$.

Consider the two-sided hypotheses

$H_0 : \mu_A = \mu_B$ versus $H_A : \mu_A \neq \mu_B$

for which the test statistic is

$$t = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{s_x^2}{n}+\frac{s_y^2}{m}}} = \frac{2.462-2.296}{\sqrt{\frac{0.315^2}{12}+\frac{0.297^2}{13}}} = 1.35$$

and the p-value is $2 \times P(t_{22} \geq 1.35) = 0.190$.

There is not sufficient evidence to conclude that the amount of chromium content has an effect on the average corrosion rate of chilled cast iron.

(b) With a critical point $t_{0.005,22} = 2.819$ a 99% two-sided confidence interval for the difference of the average corrosion rates of chilled cast iron at the two levels of chromium content is

$$2.462 - 2.296 \pm 2.819 \times \sqrt{\frac{0.315^2}{12} + \frac{0.297^2}{13}}$$

$= (-0.180, 0.512)$.

9.3.17 There is sufficient evidence to conclude that the paving slabs from company A weigh more on average than the paving slabs from company B.

There is also more variability in the weights of the paving slabs from company A.

9.3.18 There is a fairly strong suggestion that the paint thicknesses from production line A are larger than those from production line B, although the evidence is not completely overwhelming (the p-value is 0.011).

9.3.19 There is sufficient evidence to conclude that the damped feature is effective in reducing the heel-strike force.

9.3.20 The high level of hydrogen peroxide seems to produce more variability in the whiteness measurements than the low level.

There is not sufficient evidence to conclude that the high level of hydrogen peroxide produces a larger average whiteness measurement than the low level of hydrogen peroxide.

9.3.21 There is not sufficient evidence to conclude that the average service times are any different at these two times of day.

9.3.22 The hypotheses are

$H_0 : \mu_N \leq \mu_S$ versus $H_A : \mu_N > \mu_S$

and

$$\frac{\left(\frac{6.30^2}{14}+\frac{7.15^2}{20}\right)^2}{\frac{6.30^4}{14^2\times(14-1)}+\frac{7.15^4}{20^2\times(20-1)}} = 30.2$$

so that the degrees of freedom are $\nu = 30$.

The test statistic is

$$t = \frac{56.43-62.11}{\sqrt{\frac{6.30^2}{14}+\frac{7.15^2}{20}}} = -2.446$$

and the p-value is $P(t_{30} < -2.446) = 0.0103$.

Since the p-value is almost equal to 0.01, there is sufficient evidence to conclude that the new procedure has a larger breaking strength on average than the standard procedure.

9.3.23 $\bar{x}_A = 142.4$

$s_A = 9.24$

$n_A = 10$

$\bar{x}_B = 131.6$

$s_B = 7.97$

$n_B = 10$

The hypotheses are

$H_0 : \mu_A \leq \mu_B$ versus $H_A : \mu_A > \mu_B$

and

$$\frac{\left(\frac{9.24^2}{10}+\frac{7.97^2}{10}\right)^2}{\frac{9.24^4}{10^2\times(10-1)}+\frac{7.97^4}{10^2\times(10-1)}} = 17.6$$

so that the degrees of freedom are $\nu = 17$.

The test statistic is

$$t = \frac{142.4-131.6}{\sqrt{\frac{9.24^2}{10}+\frac{7.97^2}{10}}} = 2.799$$

and the p-value is $P(t_{17} > 2.799) = 0.006$.

There is sufficient evidence to conclude that on average medicine A provides a higher response than medicine B.

9.3.24 (a) $\bar{x}_M = 132.52$
$s_M = 1.31$
$n_M = 8$
$\bar{x}_A = 133.87$
$s_A = 1.72$
$n_A = 10$

The hypotheses are
$H_0 : \mu_M = \mu_A$ versus $H_A : \mu_M \neq \mu_A$
and

$$\frac{\left(\frac{1.31^2}{8}+\frac{1.72^2}{10}\right)^2}{\frac{1.31^4}{8^2\times(8-1)}+\frac{1.72^4}{10^2\times(10-1)}} = 15.98$$

so that the degrees of freedom are $\nu = 15$.

The test statistic is

$$t = \frac{132.52-133.87}{\sqrt{\frac{1.31^2}{8}+\frac{1.72^2}{10}}} = -1.89$$

and the p-value is $2 \times P(t_{15} > 1.89)$ which is between 5% and 10%.
There is some evidence to suggest that there is a difference between the running times in the morning and afternoon, but the evidence is not overwhelming.

(b) With $t_{0.005,15} = 2.947$ the confidence interval is

$$\mu_M - \mu_A \in 132.52 - 133.87 \pm 2.947 \times \sqrt{\frac{1.31^2}{8} + \frac{1.72^2}{10}} = (-3.46, 0.76).$$

9.3.25 $\bar{x}_A = 152.3$

$s_A = 1.83$

$n_A = 10$

$s_B = 1.94$

$n_B = 8$

The hypotheses are

$H_0 : \mu_A \leq \mu_B$ versus $H_A : \mu_A > \mu_B$

and

$$\frac{\left(\frac{1.83^2}{10}+\frac{1.94^2}{8}\right)^2}{\frac{1.83^4}{10^2\times(10-1)}+\frac{1.94^4}{8^2\times(8-1)}} = 14.7$$

so that the degrees of freedom are $\nu = 14$.

Since the p-value is $P(t_{14} > t) < 0.01$, it follows that

$$t = \frac{\bar{x}_A-\bar{x}_B}{\sqrt{\frac{s_A^2}{n_A}+\frac{s_B^2}{n_B}}} = \frac{152.3-\bar{x}_B}{0.8974} > t_{0.01,14} = 2.624$$

so that $\bar{x}_B < 149.9$.

9.3.26 C

9.7.1 The differences $z_i = x_i - y_i$ have a sample mean $\bar{z} = 2.85$ and a sample standard deviation $s = 5.30$.

Consider the hypotheses

$H_0 : \mu = \mu_A - \mu_B \leq 0$ versus $H_A : \mu = \mu_A - \mu_B > 0$

where the alternative hypothesis states that the color displays are more effective than the black and white displays.

The test statistic is

$t = \frac{\sqrt{n}\,\bar{z}}{s} = \frac{\sqrt{22}\times 2.85}{5.30} = 2.52$

and the p-value is $P(t_{21} \geq 2.52) = 0.010$.

There is sufficient evidence to conclude that the color displays are more effective than the black and white displays.

With $t_{0.05,21} = 1.721$ a one-sided 95% confidence level confidence interval

for $\mu = \mu_A - \mu_B$ is

$\left(2.85 - \frac{1.721\times 5.30}{\sqrt{22}}, \infty\right)$

$= (0.91, \infty)$.

9.7.2 The differences $z_i = x_i - y_i$ have a sample mean $\bar{z} = 7.50$ and a sample standard deviation $s = 6.84$.

Consider the hypotheses

$H_0 : \mu = \mu_A - \mu_B = 0$ versus $H_A : \mu = \mu_A - \mu_B \neq 0$.

The test statistic is

$t = \frac{\sqrt{n}\,\bar{z}}{s} = \frac{\sqrt{14}\times 7.50}{6.84} = 4.10$

and the p-value is $2 \times P(t_{13} \geq 4.10) = 0.001$.

There is sufficient evidence to conclude that the water absorption properties of the fabric are different for the two different roller pressures.

With $t_{0.025,13} = 2.160$ a two-sided 95% confidence level confidence interval

for $\mu = \mu_A - \mu_B$ is

$\left(7.50 - \frac{2.160\times 6.84}{\sqrt{14}}, 7.50 + \frac{2.160\times 6.84}{\sqrt{14}}\right)$

$= (3.55, 11.45)$.

9.7.3 (a) Since

$$\frac{\left(\frac{5.20^2}{35}+\frac{3.06^2}{35}\right)^2}{\frac{5.20^4}{35^2\times(35-1)}+\frac{3.06^4}{35^2\times(35-1)}} = 55.03$$

the degrees of freedom are $\nu = 55$.

The test statistic is

$$t = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{s_x^2}{n}+\frac{s_y^2}{m}}} = \frac{22.73-12.66}{\sqrt{\frac{5.20^2}{35}+\frac{3.06^2}{35}}} = 9.87$$

and the p-value is $2 \times P(t_{55} \geq 9.87) = 0.000$.
It is not plausible that the average crystal size does not depend upon the pre-expansion temperature.

(b) With a critical point $t_{0.005,55} = 2.668$ a 99% two-sided confidence interval for $\mu_A - \mu_B$ is

$$22.73 - 12.66 \pm 2.668 \times \sqrt{\frac{5.20^2}{35} + \frac{3.06^2}{35}}$$

$$= (7.35, 12.79).$$

(c) Using $t_{0.005,55} = 2.668$ equal total sample sizes of

$$n = m \geq \frac{4\, t_{\alpha/2,\nu}^2\, (s_x^2+s_y^2)}{L_0^2} = \frac{4\times 2.668^2\times(5.20^2+3.06^2)}{4.0^2} = 64.8$$

should be sufficient.
Additional sample sizes of at least $65 - 35 = 30$ from each population can be recommended.

9.7.4 Since

$$\frac{\left(\frac{20.39^2}{48}+\frac{15.62^2}{10}\right)^2}{\frac{20.39^4}{48^2\times(48-1)}+\frac{15.62^4}{10^2\times(10-1)}}=16.1$$

the appropriate degrees of freedom for a general analysis without assuming equal population variances are $\nu = 16$.

Consider the hypotheses

$H_0 : \mu = \mu_A - \mu_B \leq 0$ versus $H_A : \mu = \mu_A - \mu_B > 0$

where the alternative hypothesis states that the new driving route is quicker on average than the standard driving route.

The test statistic is

$$t = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{s_x^2}{n}+\frac{s_y^2}{m}}} = \frac{432.7-403.5}{\sqrt{\frac{20.39^2}{48}+\frac{15.62^2}{10}}} = 5.08$$

and the p-value is $P(t_{16} \geq 5.08) = 0.000$.

There is sufficient evidence to conclude that the new driving route is quicker on average than the standard driving route.

9.7.5 There is sufficient evidence to conclude that the additional sunlight results in larger heights on average.

9.7.6 There is *not* sufficient evidence to conclude that the reorganization has produced any improvement in the average waiting time.

However, the variability in the waiting times has been reduced following the reorganization.

9.7.7 This is a paired data set.

There is not any evidence of a difference in the average ocular motor measurements after reading a book and after reading a computer screen.

9.7.8 The variabilities in the viscosities appear to be about the same for the two engines, but there is sufficient evidence to conclude that the average viscosity is higher after having been used in engine 2 than after having been used in engine 1.

9.7.10 With $F_{0.05,17,20} = 2.1667$ and $F_{0.05,20,17} = 2.2304$

the confidence interval is

$$\left(\frac{6.48^2}{9.62^2 \times 2.1667}, \frac{6.48^2 \times 2.2304}{9.62^2}\right) = (0.21, 1.01).$$

9.7.11 With $F_{0.05,40,40} = 1.6928$

the confidence interval is

$$\left(\frac{0.124^2}{0.137^2 \times 1.6928}, \frac{0.124^2 \times 1.6928}{0.137^2}\right) = (0.484, 1.387).$$

9.7.12 With $F_{0.05,19,24} = 2.0399$ and $F_{0.05,24,19} = 2.1141$

the 90% confidence interval is

$$\left(\frac{11.90^2}{4.61^2 \times 2.0399}, \frac{11.90^2 \times 2.1141}{4.61^2}\right) = (3.27, 14.09).$$

With $F_{0.025,19,24} = 2.3452$ and $F_{0.025,24,19} = 2.4523$

the 95% confidence interval is

$$\left(\frac{11.90^2}{4.61^2 \times 2.3452}, \frac{11.90^2 \times 2.4523}{4.61^2}\right) = (2.84, 16.34).$$

With $F_{0.005,19,24} = 3.0920$ and $F_{0.005,24,19} = 3.3062$

the 99% confidence interval is

$$\left(\frac{11.90^2}{4.61^2 \times 3.0920}, \frac{11.90^2 \times 3.3062}{4.61^2}\right) = (2.16, 22.03).$$

9.7.13 $\bar{x}_A = 327433$

$s_A = 9832$

$n_A = 14$

$\bar{x}_B = 335537$

$s_B = 10463$

$n_B = 12$

The hypotheses are

$H_0 : \mu_A = \mu_B$ versus $H_A : \mu_A \neq \mu_B$

and since

$$\frac{\left(\frac{9832^2}{14}+\frac{10463^2}{12}\right)^2}{\frac{9832^4}{14^2\times(14-1)}+\frac{10463^4}{12^2\times(12-1)}} = 22.8$$

the degrees of freedom are $\nu = 22$.

The test statistic is

$$t = \frac{327433-335537}{\sqrt{\frac{9832^2}{14}+\frac{10463^2}{12}}} = -2.024$$

and the p-value is $2 \times P(t_{22} > 2.024)$ which is between 5% and 10%.

There is some evidence to suggest that there is a difference between the strengths of the two canvas types, but the evidence is not overwhelming.

9.7.14 Let x_i be the strength of the cement sample using procedure 1 and let y_i be the strength of the cement sample using procedure 2.

With $z_i = x_i - y_i$ it can be found that

$$\bar{z} = \frac{\sum_{i=1}^{9} z_i}{9} = -0.0222$$

and

$$s_z = \sqrt{\frac{\sum_{i=1}^{9} (z_i - \bar{z})^2}{8}} = 0.5911.$$

For the hypotheses

$H_0 : \mu_x = \mu_y$ versus $H_A : \mu_x \neq \mu_y$

the test statistic is

$$t = \frac{\sqrt{9}(-0.0222-0)}{0.5911} = -0.113$$

and the p-value is $2 \times P(t_8 \geq 0.113) = 0.91$.

Therefore, there is no evidence of any difference between the two procedures.

9.7.15 (a) False

(b) True

(c) True

(d) False

(e) False

(f) True

(g) True

(h) True

(i) True

9.7.16 Let x_i be the data obtained using therapy 1 and let y_i be the data obtained using therapy 2.

With $z_i = x_i - y_i$ it can be found that

$$\bar{z} = \frac{\sum_{i=1}^{8} z_i}{8} = 1.000$$

and

$$s_z = \sqrt{\frac{\sum_{i=1}^{8}(z_i - \bar{z})^2}{7}} = 5.757.$$

For the hypotheses

$H_0 : \mu_x = \mu_y$ versus $H_A : \mu_x \neq \mu_y$

the test statistic is

$$t = \frac{\sqrt{8}(1.000-0)}{5.757} = 0.491$$

and the p-value is $2 \times P(t_7 \geq 0.491) = 0.638$.

Therefore, there is not sufficient evidence to conclude that there is a difference between the two experimental drug therapies.

9.7.17 (a) The hypotheses are

$H_0 : \mu_A \geq \mu_B$ versus $H_A : \mu_A < \mu_B$

and the appropriate degrees of freedom are

$$\frac{\left(\frac{24.1^2}{20} + \frac{26.4^2}{24}\right)^2}{\frac{24.1^4}{20^2 \times (20-1)} + \frac{26.4^4}{24^2 \times (24-1)}} = 41.6$$

which should be rounded down to $\nu = 41$.

The test statistic is

$$t = \frac{2376.3 - 2402.0}{\sqrt{\frac{24.1^2}{20} + \frac{26.4^2}{24}}} = -3.37$$

and the p-value is $P(t_{41} \leq -3.37) = 0.0008$.

There is sufficient evidence to conclude that the items from manufacturer B provide larger measurements on average than the items from manufacturer A.

(b) With $t_{0.05,41} = 1.683$ the confidence interval is

$$\mu_B - \mu_A \in \left(-\infty, 2402.0 - 2376.3 + 1.683\sqrt{\frac{24.1^2}{20} + \frac{26.4^2}{24}}\right) = (-\infty, 38.5).$$

9.7.20 Let x_i be the mean error measurement for patient i using joystick design 1 and let y_i be the mean error measurement for patient i using joystick design 2.

With $z_i = x_i - y_i$ it can be found that

$\bar{z} = \frac{\sum_{i=1}^{9} z_i}{9} = 0.02067$

and

$s_z = \sqrt{\frac{\sum_{i=1}^{9}(z_i - \bar{z})^2}{8}} = 0.03201.$

For the hypotheses

$H_0 : \mu_x = \mu_y$ versus $H_A : \mu_x \neq \mu_y$

the test statistic is

$t = \frac{\sqrt{9}(0.02067-0)}{0.03201} = 1.937$

and the p-value is $2 \times P(t_8 \geq 1.937)$ which is between 5% and 10%.

Therefore, there is some evidence that the the two joystick designs result in different error rate measurements, but the evidence is not overwhelming.

With $t_{0.005,8} = 3.355$ a 99% confidence interval for the difference between the mean error measurements obtained from the two designs is

$0.02067 \pm \frac{3.355 \times 0.03201}{\sqrt{9}} = (-0.015, 0.056).$

9.7.23 A

9.7.24 E

9.7.30 D

9.7.31 B

9.7.32 B

9.7.33 C

9.7.34 D