HW: Normal Distribution Part 1

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5.1: 1,2   Due Jun 15

5.1

1a) $P(z \leq 1.34) = \boxed{.9099}$

b) $P(z \geq -.22) = \boxed{.5871}$

c) $P(-2.19 \leq z \leq .43) = .6664 - .0143 = \boxed{.6521}$

d) $P(.09 \leq z \leq 1.76) = .9608 - .5359 = \boxed{.4249}$

e) $P(-.38 \leq z \leq .38) = .6480 - .3520 = \boxed{.2960}$

f) $\Phi(x) = .55, \boxed{x = .1257}$

g) $1 - \Phi(x) = .72, \boxed{x = -.5828}$

h) $\Phi(x) - \Phi(-x) = .31, \boxed{x = .3989}$

2a) $P(z \leq -.77) = \boxed{.2206}$

b) $P(z \geq .32) = 1 - \Phi(.32) = \boxed{.3745}$

c) $P(-3.09 \leq z \leq -1.59) = \boxed{.0549}$

d) $P(-.82 \leq z \leq 1.80) = \boxed{.7580}$

e) $P(-.91 \geq z \geq .91) = \boxed{.3628}$

f) $\Phi(x) = .23, \boxed{x = -.7388}$

g) $1 - \Phi(x) = .51, \boxed{x = -.0251}$

h) $\Phi(x) - \Phi(-x) = .42, \boxed{x = .8064}$

Normal Distribution–Standard Normal Distribution

- There is a special normal distribution.
- We call it the standard normal distribution.
- The mean is $\mu = 0$.
- Then standard deviation is $\sigma = 1$.
- We call the random variable Z.

For this activity, you need the applet at https://www.geogebra.org/m/rDgGnMbw

If you have issues you can instead use https://wise1.cgu.edu/vis/p_z/

- The z value is for the standard normal distribution.
- You can drag the dividing line to change the Z value. This line divides the area under the normal curve into the area to the left and the area to the right.
- If you click the Both tails button, then you can see the area in the middle.
- You can click Right tail, you can see the area to the right.
- You can also use the left and right arrows on your keyboard to change Z.
- Remember that for a continuous distribution, $P(Z \geq 2)$ is the same as $P(Z > 2)$.
- **You won't be able to get the exact Z value you want by dragging the line with the mouse. But your answers should be close to mine.**

1. Play with the applet for a few minutes. Click the buttons and drag the dividing line until you feel comfortable working with it.

2. Use the applet to find the following probabilities. Draw a picture for each problem.

    (a) Find the area under the curve to the right of $z = 2$. In symbols this is $P(Z > 2)$.
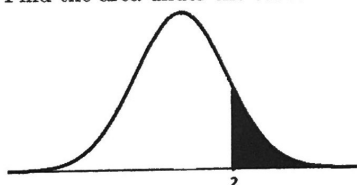
    $P(Z > 2) = .0226$

    (b) Find the area under the curve to the left of $z = 2$. In symbols this is $P(Z < 2)$.

    $P(Z < 2) = .977$

    (c) What is the relationship between $P(Z > 2)$ and $P(Z < 2)$?

    adding them together = 1

    (d) Find the area under the curve to the right of $z = -2$. In symbols this is $P(Z > -2)$.

    $P(Z > -2) = .972$

(e) $P(Z \geq -2)$

$$P(Z \geq -2) = .977$$

(f) $P(Z < 1)$ = $.841$

(g) Find the area under the curve between $z = -1$ and $z = 1$. In symbols this is $P(-1 < Z < 1)$.

$$P(-1 < Z < 1) = .683$$

(h) $P(-1.2 < Z < 1.2)$

$$P(-1.2 < Z < 1.2) = .770$$

3. The empirical rule says that

- approximately 68% of the data is within 1 standard deviation of the mean.
- approximately 95% of the data is within 2 standard deviations of the mean.
- approximately 99.7% of the data is within 3 standard deviations of the mean.

   We can find the exact percentages for a normal distribution using the applet.
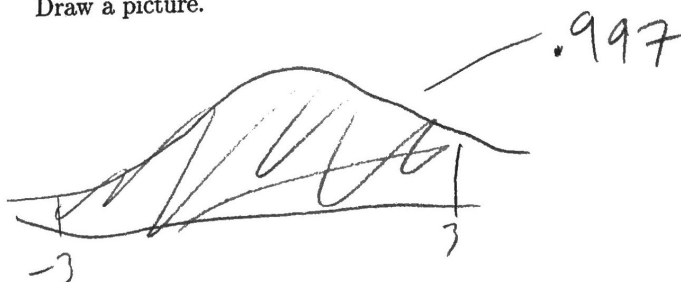
(a) To find the percentage of data within 1 standard deviation of the mean, we need to find $P(-1 < Z < 1)$. Draw a picture.

$.682$
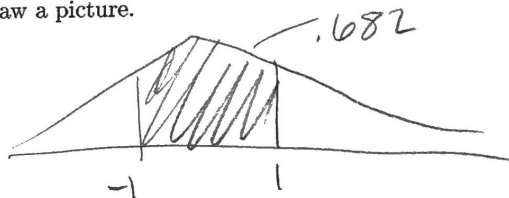
(b) What is the exact percentage of data within 2 standard deviations of the mean? Find $P(-2 < Z < 2)$. Draw a picture.
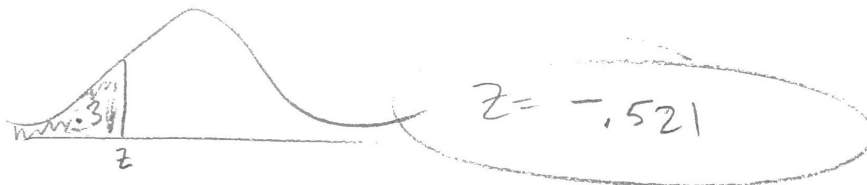
$.954$

(c) What is the exact percentage of data within 3 standard deviations of the mean? Find $P(-3 < Z < 3)$. Draw a picture.
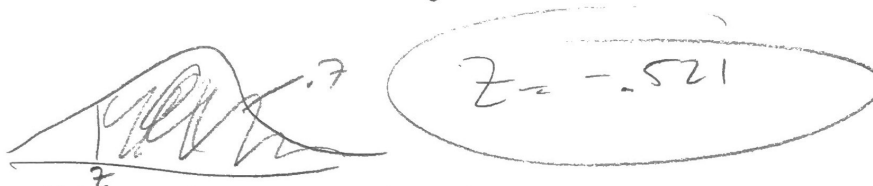
$.997$

4. Sometimes we want to know the z value that divides the curve into certain probabilities. Draw a picture for each problem.

(a) What z value has 0.3 area to the left?
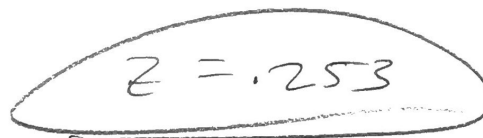
$z = -.521$

(b) What z value has 0.7 area to the right?

.7

$z = -.521$

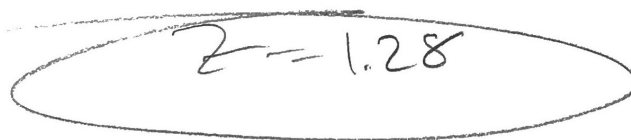(c) What z value has 0.3 area to the right?

.3

$z = -.521$

(d) Find the z value such that $P(Z > z) = .4$.
(This is really fancy notation to say find the z value that has .4 probability to the right. Some books say find the k value such that $P(Z > k) = .4$.)

$z = .253$

(e) What z value has 0.1 area to the right?

$z = 1.28$

(f) What z value do we need so that the area in the middle between z and $-z$ is 0.5?

.5

$z = .675$