6/06/20

Agenda for Math 5710 ♬ Meeting #14 ☻☺ 7/10/20 (8:00 a.m. – 9:10 a.m.)

1. Hello:

Brigham City:	Adam Blakeslee	Ryan Johnson	Tyson Mortensen	
Logan:	David Allen	Natalie Anderson	Kameron Baird	Stephen Brezinski
	Zachary Ellis	Adam Flanders	Brock Francom	Xiang Gao
	Ryan Goodman	Janette Goodridge	Hadley Hamar	Phillip Leifer
	Brittney Miller	Jonathan Mousley	Erika Mueller	Shelby Simpson
	Steven Summers	Matthew White	Zhang Xiaomeng	

2. Note the syllabus' activity list for today:

14: F/7/10	1. Rediscover and prove the binomial theorem and relate it its role in our counting techniques as well as its role in our march toward the *Central Limit Theorem* (i.e., Theorem 15 (Glossary entry # 050)). 2. Take advantage of Quiz 14.

3. Briefly, raise and address issues and questions stimulated by the following homework assignment:

A. Study our notes from Meeting #13.

B. Comprehend Jim's sample response to Quiz 13.

C*. Solve Lanfen's problem and display your computation (As usual upload the resulting pdf document on the appropriate *Canvas* Assignment link):

In a lottery, players pick six different numbers from { 1, 2, 3, ..., 49 }; the order in which a player picks them is irrelevant. The lottery manager randomly selects (without replacement) six of the numbers from { 1, 2, 3, ..., 49 }; the six selected numbers are referred to as "winning numbers." A player wins the grand prize if they/he/she picked all of the winning numbers. A player wins the second prize exactly if five of her/his/their picks match five of the winning numbers. A player wins the third prize if exactly four of his/their/her picks match four of the winning numbers. Lanfen wants to know the probability the pick of a player wins the first prize, the probability that it wins the second prize, and that the probability that it wins the third prize.

D. From the Video Page of *Canvas*, view with comprehension "combinations" and then do the same for "probability using combinations."

E. Comprehend Jim's sample responses to the homework prompts that are posted on *Canvas*.

4. Muse a bit about why we're about to review the binomial theorem:

   A. The binomial coefficient's role in counting and, thus, computing probabilities.

   B. Binomial experiments, binomial distributions, our Quiz 10, our experiences transitioning from Riemann sums to integrals, and our transition between discrete distributions (e.g., binomial distributions) and continuous distributions (e.g., normal distributions).

   C. Display some appreciation for an ancient convenience; read the following treatise adapted from *Wikipedia*:

   Special cases of the binomial theorem were known since at least the 4th century BC when Greek mathematician Euclid mentioned the special case of the binomial theorem for exponent 2. There is evidence that the binomial theorem for cubes was known by the 6th century AD in India. Binomial coefficients, as combinatorial quantities expressing the number of ways of selecting k objects out of n without replacement, were of interest to ancient Indian mathematicians. The earliest known reference to this combinatorial problem is the Chanda śāstra by the Indian lyricist Pingala (c. 200 BC), which contains a method for its solution. The commentator Halayudha from the 10th century AD explains this method using what is now known as Pascal's triangle. By the 6th century AD, the Indian mathematicians probably knew how to express this as a quotient

   $$\frac{n!}{(n-k)!k!}$$

   and a clear statement of this rule can be found in the 12th century text *Lilavati* by Bhaskara.

   The first formulation of the binomial theorem and the table of binomial coefficients, to our knowledge, can be found in a work by Al-Karaji, quoted by Al-Samaw'al in his *al-Bahir*. Al-Karaji described the triangular pattern of the binomial coefficients and also provided a mathematical proof of both the binomial theorem and Pascal's triangle, using an early form of mathematical induction. The Persian poet and mathematician Omar Khayyam was probably familiar with the formula to higher orders, although many of his mathematical works are lost. The binomial expansions of small degrees were known in the 13th century mathematical works of Yang Hui and also Chu Shih-Chieh.Yang Hui attributes the method to a much earlier 11th century text of Jia Xian, although those writings are now also lost. In 1544, Michael Stifel introduced the term "binomial coefficient" and showed how to use them to express $(1 + a)^n$ in terms of $(1 + a)^{n-1}$, via Pascal's triangle. Blaise Pascal studied the eponymous triangle comprehensively in the treatise *Traité du triangle arithmétique* (1665). However, the pattern of numbers was already known to the European mathematicians of the late Renaissance, including Stifel, Niccolò Fontana Tartaglia, and Simon Stevin. Isaac Newton is generally credited with the generalized binomial theorem, valid for any rational exponent.

6. After a nod to the long and storied history of the binomial theorem, consider how we might go about proving it:

   Soon-to-be Theorem 8:

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$$

7. Take advantage of Quiz 14.

8. Complete the following assignment prior to Meeting #14:

   A. Study our notes from Meeting #14.

   B. Comprehend Jim's sample response to Quiz 14.

   C*. Express (in simplified form) the coefficient of x^2y^3 in the expansion of $(2x + 3y)^5$. Please display the computations in a pdf document uploaded to the appropriate Canvas assignment link.

   D*. Express (in simplified form) the following; display the computations in a pdf document uploaded to the appropriate Canvas assignment link:

$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\binom{n}{3}+\ldots+\binom{n}{n}$$

   E. From the Video Page of *Canvas*, view with comprehension "binomial theorem."

   F. Comprehend Jim's sample responses to the homework prompts that are posted on *Canvas*.

9. And from *XKCD* :