11.1.1 (a) $P(X \geq 4.2) = 0.0177$

(b) $P(X \geq 2.3) = 0.0530$

(c) $P(X \geq 31.7) \leq 0.0001$

(d) $P(X \geq 9.3) = 0.0019$

(e) $P(X \geq 0.9) = 0.5010$

11.1.2

Source	df	SS	MS	F	p-value
Treatments	5	557.0	111.4	5.547	0.0017
Error	23	461.9	20.08		
Total	28	1018.9			

11.1.3

Source	df	SS	MS	F	p-value
Treatments	7	126.95	18.136	5.01	0.0016
Error	22	79.64	3.62		
Total	29	206.59			

11.1.4

Source	df	SS	MS	F	p-value
Treatments	6	7.66	1.28	0.78	0.59
Error	77	125.51	1.63		
Total	83	133.18			

11.1.5

Source	df	SS	MS	F	p-value
Treatments	3	162.19	54.06	6.69	0.001
Error	40	323.34	8.08		
Total	43	485.53			

11.1.6

Source	df	SS	MS	F	p-value
Treatments	2	46.8	23.4	2.7	0.08
Error	52	451.2	8.7		
Total	54	498.0			

11.1.7

Source	df	SS	MS	F	p-value
Treatments	3	0.0079	0.0026	1.65	0.189
Error	52	0.0829	0.0016		
Total	55	0.0908			

11.1.8 (a) $\mu_1 - \mu_2 \in \left(48.05 - 44.74 - \frac{\sqrt{4.96}\times 3.49}{\sqrt{11}}, 48.05 - 44.74 + \frac{\sqrt{4.96}\times 3.49}{\sqrt{11}}\right)$

$= (0.97, 5.65)$

$\mu_1 - \mu_3 \in \left(48.05 - 49.11 - \frac{\sqrt{4.96}\times 3.49}{\sqrt{11}}, 48.05 - 49.11 + \frac{\sqrt{4.96}\times 3.49}{\sqrt{11}}\right)$

$= (-3.40, 1.28)$

$\mu_2 - \mu_3 \in \left(44.74 - 49.11 - \frac{\sqrt{4.96}\times 3.49}{\sqrt{11}}, 44.74 - 49.11 + \frac{\sqrt{4.96}\times 3.49}{\sqrt{11}}\right)$

$= (-6.71, -2.03)$

(c) The total sample size required from each factor level can be estimated as

$n \geq \frac{4\, s^2\, q^2_{\alpha,k,\nu}}{L^2} = \frac{4\times 4.96\times 3.49^2}{2.0^2} = 60.4$

so that an additional sample size of $61 - 11 = 50$ observations from each factor level can be recommended.

11.1.9 (a) $\mu_1 - \mu_2 \in \left(136.3 - 152.1 - \frac{\sqrt{15.95}\times 4.30}{\sqrt{6}}, 136.3 - 152.1 + \frac{\sqrt{15.95}\times 4.30}{\sqrt{6}}\right)$

$= (-22.8, -8.8)$

$\mu_1 - \mu_3 \in (3.6, 17.6)$

$\mu_1 - \mu_4 \in (-0.9, 13.1)$

$\mu_1 - \mu_5 \in (-13.0, 1.0)$

$\mu_1 - \mu_6 \in (1.3, 15.3)$

$\mu_2 - \mu_3 \in (19.4, 33.4)$

$\mu_2 - \mu_4 \in (14.9, 28.9)$

$\mu_2 - \mu_5 \in (2.8, 16.8)$

$\mu_2 - \mu_6 \in (17.1, 31.1)$

$\mu_3 - \mu_4 \in (-11.5, 2.5)$

$\mu_3 - \mu_5 \in (-23.6, -9.6)$

$\mu_3 - \mu_6 \in (-9.3, 4.7)$

$\mu_4 - \mu_5 \in (-19.1, -5.1)$

$\mu_4 - \mu_6 \in (-4.8, 9.2)$

$\mu_5 - \mu_6 \in (7.3, 21.3)$

(c) The total sample size required from each factor level can be estimated as

$n \geq \frac{4\, s^2\, q^2_{\alpha,k,\nu}}{L^2} = \frac{4\times 15.95\times 4.30^2}{10.0^2} = 11.8$

so that an additional sample size of $12 - 6 = 6$ observations from each factor level can be recommended.

11.1.10 The p-value remains unchanged.

11.1.11 (a) $\bar{x}_{1.} = 5.633$
$\bar{x}_{2.} = 5.567$
$\bar{x}_{3.} = 4.778$

(b) $\bar{x}_{..} = 5.326$

(c) $SSTR = 4.076$

(d) $\sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij}^2 = 791.30$

(e) $SST = 25.432$

(f) $SSE = 21.356$

(g)

Source	df	SS	MS	F	p-value
Treatments	2	4.076	2.038	2.29	0.123
Error	24	21.356	0.890		
Total	26	25.432			

(h) $\mu_1 - \mu_2 \in \left(5.633 - 5.567 - \frac{\sqrt{0.890} \times 3.53}{\sqrt{9}}, 5.633 - 5.567 + \frac{\sqrt{0.890} \times 3.53}{\sqrt{9}}\right)$

$= (-1.04, 1.18)$

$\mu_1 - \mu_3 \in \left(5.633 - 4.778 - \frac{\sqrt{0.890} \times 3.53}{\sqrt{9}}, 5.633 - 4.778 + \frac{\sqrt{0.890} \times 3.53}{\sqrt{9}}\right)$

$= (-0.25, 1.97)$

$\mu_2 - \mu_3 \in \left(5.567 - 4.778 - \frac{\sqrt{0.890} \times 3.53}{\sqrt{9}}, 5.567 - 4.778 + \frac{\sqrt{0.890} \times 3.53}{\sqrt{9}}\right)$

$= (-0.32, 1.90)$

(j) The total sample size required from each factor level can be estimated as

$n \geq \frac{4\, s^2\, q_{\alpha,k,\nu}^2}{L^2} = \frac{4 \times 0.890 \times 3.53^2}{1.0^2} = 44.4$

so that an additional sample size of $45 - 9 = 36$ observations from each factor level can be recommended.

11.1.12 (a) $\bar{x}_{1.} = 10.560$

$\bar{x}_{2.} = 15.150$

$\bar{x}_{3.} = 17.700$

$\bar{x}_{4.} = 11.567$

(b) $\bar{x}_{..} = 14.127$

(c) $SSTR = 364.75$

(d) $\sum_{i=1}^{k} \sum_{j=1}^{n_i} x_{ij}^2 = 9346.74$

(e) $SST = 565.23$

(f) $SSE = 200.47$

(g)

Source	df	SS	MS	F	p-value
Treatments	3	364.75	121.58	24.26	0.000
Error	40	200.47	5.01		
Total	43	565.23			

(h) $\mu_1 - \mu_2 \in (-7.16, -2.02)$

$\mu_1 - \mu_3 \in (-9.66, -4.62)$

$\mu_1 - \mu_4 \in (-3.76, 1.75)$

$\mu_2 - \mu_3 \in (-4.95, -0.15)$

$\mu_2 - \mu_4 \in (0.94, 6.23)$

$\mu_3 - \mu_4 \in (3.53, 8.74)$

Note: In the remainder of this section the confidence intervals for the pairwise differences of the factor level means are provided with an overall confidence level of 95%.

11.1.13

Source	df	SS	MS	F	p-value
Treatments	2	0.0085	0.0042	0.24	0.787
Error	87	1.5299	0.0176		
Total	89	1.5384			

$\mu_1 - \mu_2 \in (-0.08, 0.08)$

$\mu_1 - \mu_3 \in (-0.06, 0.10)$

$\mu_2 - \mu_3 \in (-0.06, 0.10)$

There is *not* sufficient evidence to conclude that there is a difference between the three production lines.

11.1.14

Source	df	SS	MS	F	p-value
Treatments	2	278.0	139.0	85.4	0.000
Error	50	81.3	1.63		
Total	52	359.3			

$\mu_1 - \mu_2 \in (3.06, 5.16)$

$\mu_1 - \mu_3 \in (4.11, 6.11)$

$\mu_2 - \mu_3 \in (-0.08, 2.08)$

There is sufficient evidence to conclude that Monday is slower than the other two days.

11.1.15

Source	df	SS	MS	F	p-value
Treatments	2	0.0278	0.0139	1.26	0.299
Error	30	0.3318	0.0111		
Total	32	0.3596			

$\mu_1 - \mu_2 \in (-0.15, 0.07)$

$\mu_1 - \mu_3 \in (-0.08, 0.14)$

$\mu_2 - \mu_3 \in (-0.04, 0.18)$

There is *not* sufficient evidence to conclude that the radiation readings are affected by the background radiation levels.

11.1.16

Source	df	SS	MS	F	p-value
Treatments	2	121.24	60.62	52.84	0.000
Error	30	34.42	1.15		
Total	32	155.66			

$\mu_1 - \mu_2 \in (-5.12, -2.85)$

$\mu_1 - \mu_3 \in (-0.74, 1.47)$

$\mu_2 - \mu_3 \in (3.19, 5.50)$

There is sufficient evidence to conclude that layout 2 is slower than the other two layouts.

11.1.17

Source	df	SS	MS	F	p-value
Treatments	2	0.4836	0.2418	7.13	0.001
Error	93	3.1536	0.0339		
Total	95	3.6372			

$\mu_1 - \mu_2 \in (-0.01, 0.22)$

$\mu_1 - \mu_3 \in (0.07, 0.29)$

$\mu_2 - \mu_3 \in (-0.03, 0.18)$

There is sufficient evidence to conclude that the average particle diameter is larger at the low amount of stabilizer than at the high amount of stabilizer.

11.1.18

Source	df	SS	MS	F	p-value
Treatments	2	135.15	67.58	19.44	0.000
Error	87	302.50	3.48		
Total	89	437.66			

$\mu_1 - \mu_2 \in (-1.25, 1.04)$

$\mu_1 - \mu_3 \in (1.40, 3.69)$

$\mu_2 - \mu_3 \in (1.50, 3.80)$

There is sufficient evidence to conclude that method 3 is quicker than the other two methods.

11.1.19 $\bar{x}_{..} = \frac{(8\times 42.91)+(11\times 44.03)+(10\times 43.72)}{8+11+10} = \frac{1264.81}{29} = 43.61$

$SSTr = (8 \times 42.91^2) + (11 \times 44.03^2) + (10 \times 43.72^2) - \frac{1264.81^2}{29} = 5.981$

$SSE = (7 \times 5.33^2) + (10 \times 4.01^2) + (9 \times 5.10^2) = 593.753$

Source	df	SS	MS	F	p-value
Treatments	2	5.981	2.990	0.131	0.878
Error	26	593.753	22.837		
Total	28	599.734			

There is not sufficient evidence to conclude that there is a difference between the catalysts in terms of the strength of the compound.

11.1.20 (a) $\bar{x}_{1.} = 33.6$

$\bar{x}_{2.} = 40.0$

$\bar{x}_{3.} = 20.4$

$\bar{x}_{4.} = 31.0$

$\bar{x}_{5.} = 26.5$

Source	df	SS	MS	F	p-value
Treatments	4	1102.7	275.7	18.51	0.000
Error	20	297.9	14.9		
Total	24	1400.6			

(b) $q_{0.05,5,20} = 4.23$

$s = \sqrt{MSE} = \sqrt{14.9} = 3.86$

The pairwise comparisons which contain zero are:
treatment 1 and treatment 2
treatment 1 and treatment 4
treatment 3 and treatment 5
treatment 4 and treatment 5

The treatment with the largest average quality score is either treatment 1 or treatment 2.

The treatment with the smallest average quality score is either treatment 3 or treatment 5.

11.1.21 $q_{0.05,5,43} = 4.04$

With a 95% confidence level the pairwise confidence intervals that contain zero are:

$\mu_1 - \mu_2$

$\mu_2 - \mu_5$

$\mu_3 - \mu_4$

It can be inferred that the largest mean is either μ_3 or μ_4
and that the smallest mean is either μ_2 or μ_5.

11.1.22 (a) $\bar{x}_{..} = \frac{(8\times10.50)+(8\times9.22)+(9\times6.32)+(6\times11.39)}{31}$

$= 9.1284$

$SSTr = (8 \times 10.50^2) + (8 \times 9.22^2) + (9 \times 6.32^2) + (6 \times 11.39^2) - (31 \times 9.1284^2)$

$= 116.79$

$SSE = (7 \times 1.02^2) + (7 \times 0.86^2) + (8 \times 1.13^2) + (5 \times 0.98^2)$

$= 27.48$

Source	df	SS	MS	F	p-value
Alloy	3	116.79	38.93	38.3	0.000
Error	27	27.48	1.02		
Total	30	144.27			

There is sufficient evidence to conclude that the average strengths of the four metal alloys are not all the same.

(b) $q_{0.05,4,27} = 3.88$

$\mu_1 - \mu_2 \in 10.50 - 9.22 \pm \frac{\sqrt{1.02}\times3.88}{\sqrt{2}}\sqrt{\frac{1}{8}+\frac{1}{8}} = (-0.68, 3.24)$

$\mu_1 - \mu_3 \in 10.50 - 6.32 \pm \frac{\sqrt{1.02}\times3.88}{\sqrt{2}}\sqrt{\frac{1}{8}+\frac{1}{9}} = (2.28, 6.08)$

$\mu_1 - \mu_4 \in 10.50 - 11.39 \pm \frac{\sqrt{1.02}\times3.88}{\sqrt{2}}\sqrt{\frac{1}{8}+\frac{1}{6}} = (-3.00, 1.22)$

$\mu_2 - \mu_3 \in 9.22 - 6.32 \pm \frac{\sqrt{1.02}\times3.88}{\sqrt{2}}\sqrt{\frac{1}{8}+\frac{1}{9}} = (1.00, 4.80)$

$\mu_2 - \mu_4 \in 9.22 - 11.39 \pm \frac{\sqrt{1.02}\times3.88}{\sqrt{2}}\sqrt{\frac{1}{8}+\frac{1}{6}} = (-4.28, -0.06)$

$\mu_3 - \mu_4 \in 6.32 - 11.39 \pm \frac{\sqrt{1.02}\times3.88}{\sqrt{2}}\sqrt{\frac{1}{9}+\frac{1}{6}} = (-7.13, -3.01)$

The strongest metal alloy is either type A or type D.

The weakest metal alloy is type C.

11.1.23 $\bar{x}_{1.} = 40.80$

$\bar{x}_{2.} = 32.80$

$\bar{x}_{3.} = 25.60$

$\bar{x}_{4.} = 50.60$

$\bar{x}_{5.} = 41.80$

$\bar{x}_{6.} = 31.80$

Source	df	SS	MS	F	p-value
Physician	5	1983.8	396.8	15.32	0.000
Error	24	621.6	25.9		
Total	29	2605.4			

The p-value of 0.000 implies that there is sufficient evidence to conclude that the times taken by the physicians for the investigatory surgical procedures are different.

Since

$\frac{s \times q_{0.05,6,24}}{\sqrt{5}} = \frac{\sqrt{25.9} \times 4.37}{\sqrt{5}} = 9.95$

it follows that two physicians cannot be concluded to be different if their sample averages have a difference of less than 9.95.

The slowest physician is either physician 1, physician 4, or physician 5.

The quickest physician is either physician 2, physician 3, or physician 6.

11.1.24 $\bar{x}_{1.} = 29.00$

$\bar{x}_{2.} = 28.75$

$\bar{x}_{3.} = 28.75$

$\bar{x}_{4.} = 37.00$

$\bar{x}_{5.} = 44.00$

$\bar{x}_{6.} = 28.00$

Source	df	SS	MS	F	p-value
Treatments	5	852.33	170.47	19.99	0.000
Error	18	153.50	8.53		
Total	23	1005.83			

The small p-value in the analysis of variance table implies that there is sufficient evidence to conclude that the E. Coli pollution levels are not the same at all six locations.

Since

$$\frac{s \times q_{0.05,6,18}}{\sqrt{n}} = \frac{\sqrt{8.53} \times 4.49}{\sqrt{4}} = 6.56$$

the pairwise comparisons reveal that the pollution levels at both locations 4 and 5 are larger than the pollution levels at the other four locations.

The highest E. Coli pollution level is at location 5, and the smallest E. Coli pollution level is at either location 1, 2, 3, or 6. The pollution has returned to a background level (location 6) by location 3.

11.1.25 (a) $\bar{x}_{1.} = 46.83$

$\bar{x}_{2.} = 47.66$

$\bar{x}_{3.} = 48.14$

$\bar{x}_{4.} = 48.82$

$\bar{x}_{..} = 47.82$

$SSTr = \sum_{i=1}^{4} n_i(\bar{x}_{i.} - \bar{x}_{..})^2 = 13.77$

Since the p-value is 0.01, the F-statistic in the analysis of variance table must be $F_{0.01,3,24} = 4.72$ so that the complete analysis of variance table is

Source	df	SS	MS	F	p-value
Treatments	3	13.77	4.59	4.72	0.01
Error	24	23.34	0.97		
Total	27	37.11			

(b) With $s = \sqrt{MSE} = 0.986$ and $q_{0.05,4,24} = 3.90$ the pairwise confidence intervals for the treatment means are:

$\mu_1 - \mu_2 \in (-2.11, 0.44)$

$\mu_1 - \mu_3 \in (-2.63, 0.01)$

$\mu_1 - \mu_4 \in (-3.36, -0.62)$

$\mu_2 - \mu_3 \in (-1.75, 0.79)$

$\mu_2 - \mu_4 \in (-2.48, 0.18)$

$\mu_3 - \mu_4 \in (-2.04, 0.70)$

There is sufficient evidence to establish that μ_4 is larger than μ_1.

11.1.26 A

11.1.27 B

11.1.28 B

11.2.1

Source	df	SS	MS	F	p-value
Treatments	3	10.15	3.38	3.02	0.047
Blocks	9	24.53	2.73	2.43	0.036
Error	27	30.24	1.12		
Total	39	64.92			

11.2.2

Source	df	SS	MS	F	p-value
Treatments	7	26.39	3.77	3.56	0.0036
Blocks	7	44.16	6.31	5.95	0.0000
Error	49	51.92	1.06		
Total	63	122.47			

11.2.3

Source	df	SS	MS	F	p-value
Treatments	3	58.72	19.57	0.63	0.602
Blocks	9	2839.97	315.55	10.17	0.0000
Error	27	837.96	31.04		
Total	39	3736.64			

11.2.4

Source	df	SS	MS	F	p-value
Treatments	4	240.03	60.01	18.59	0.0000
Blocks	14	1527.12	109.08	33.80	0.0000
Error	56	180.74	3.228		
Total	74	1947.89			

11.2.5 (a)

Source	df	SS	MS	F	p-value
Treatments	2	8.17	4.085	8.96	0.0031
Blocks	7	50.19	7.17	15.72	0.0000
Error	14	6.39	0.456		
Total	23	64.75			

(b) $\mu_1 - \mu_2 \in \left(5.93 - 4.62 - \frac{\sqrt{0.456}\times 3.70}{\sqrt{8}}, 5.93 - 4.62 + \frac{\sqrt{0.456}\times 3.70}{\sqrt{8}}\right)$

$= (0.43, 2.19)$

$\mu_1 - \mu_3 \in \left(5.93 - 4.78 - \frac{\sqrt{0.456}\times 3.70}{\sqrt{8}}, 5.93 - 4.78 + \frac{\sqrt{0.456}\times 3.70}{\sqrt{8}}\right)$

$= (0.27, 2.03)$

$\mu_2 - \mu_3 \in \left(4.62 - 4.78 - \frac{\sqrt{0.456}\times 3.70}{\sqrt{8}}, 4.62 - 4.78 + \frac{\sqrt{0.456}\times 3.70}{\sqrt{8}}\right)$

$= (-1.04, 0.72)$

11.2.6 The numbers in the "Blocks" row change (except for the degrees of freedom) and the total sum of squares changes.

11.2.7 (a) $\bar{x}_{1.} = 6.0617$

$\bar{x}_{2.} = 7.1967$

$\bar{x}_{3.} = 5.7767$

(b) $\bar{x}_{.1} = 7.4667$

$\bar{x}_{.2} = 5.2667$

$\bar{x}_{.3} = 5.1133$

$\bar{x}_{.4} = 7.3300$

$\bar{x}_{.5} = 6.2267$

$\bar{x}_{.6} = 6.6667$

(c) $\bar{x}_{..} = 6.345$

(d) $SSTr = 6.7717$

(e) $SSBl = 15.0769$

(f) $\sum_{i=1}^{3} \sum_{j=1}^{6} x_{ij}^2 = 752.1929$

(g) $SST = 27.5304$

(h) $SSE = 5.6818$

(i)

Source	df	SS	MS	F	p-value
Treatments	2	6.7717	3.3859	5.96	0.020
Blocks	5	15.0769	3.0154	5.31	0.012
Error	10	5.6818	0.5682		
Total	17	27.5304			

(j) $\mu_1 - \mu_2 \in \left(6.06 - 7.20 - \frac{\sqrt{0.5682}\times 3.88}{\sqrt{6}}, 6.06 - 7.20 + \frac{\sqrt{0.5682}\times 3.88}{\sqrt{6}}\right)$

$= (-2.33, 0.05)$

$\mu_1 - \mu_3 \in \left(6.06 - 5.78 - \frac{\sqrt{0.5682}\times 3.88}{\sqrt{6}}, 6.06 - 5.78 + \frac{\sqrt{0.5682}\times 3.88}{\sqrt{6}}\right)$

$= (-0.91, 1.47)$

$\mu_2 - \mu_3 \in \left(7.20 - 5.78 - \frac{\sqrt{0.5682}\times 3.88}{\sqrt{6}}, 7.20 - 5.78 + \frac{\sqrt{0.5682}\times 3.88}{\sqrt{6}}\right)$

$= (0.22, 2.61)$

(l) The total sample size required from each factor level (number of blocks) can be estimated as

$n \geq \frac{4\, s^2\, q^2_{\alpha,k,\nu}}{L^2} = \frac{4\times 0.5682\times 3.88^2}{2.0^2} = 8.6$

so that an additional $9 - 6 = 3$ blocks can be recommended.

11.2.8

Source	df	SS	MS	F	p-value
Treatments	3	67.980	22.660	5.90	0.004
Blocks	7	187.023	26.718	6.96	0.000
Error	21	80.660	3.841		
Total	31	335.662			

$\mu_1 - \mu_2 \in (-2.01, 3.46)$

$\mu_1 - \mu_3 \in (-5.86, -0.39)$

$\mu_1 - \mu_4 \in (-3.95, 1.52)$

$\mu_2 - \mu_3 \in (-6.59, -1.11)$

$\mu_2 - \mu_4 \in (-4.68, 0.79)$

$\mu_3 - \mu_4 \in (-0.83, 4.64)$

The total sample size required from each factor level (number of blocks) can be estimated as

$n \geq \frac{4\, s^2\, q^2_{\alpha,k,\nu}}{L^2} = \frac{4 \times 3.841 \times 3.95^2}{4.0^2} = 14.98$

so that an additional $15 - 8 = 7$ blocks can be recommended.

Note: In the remainder of this section the confidence intervals for the pairwise differences of the factor level means are provided with an overall confidence level of 95%.

11.2.9

Source	df	SS	MS	F	p-value
Treatments	2	17.607	8.803	2.56	0.119
Blocks	6	96.598	16.100	4.68	0.011
Error	12	41.273	3.439		
Total	20	155.478			

$\mu_1 - \mu_2 \in (-1.11, 4.17)$

$\mu_1 - \mu_3 \in (-0.46, 4.83)$

$\mu_2 - \mu_3 \in (-1.99, 3.30)$

There is *not* sufficient evidence to conclude that the calciners are operating at different efficiencies.

11.2.10

Source	df	SS	MS	F	p-value
Treatments	2	133.02	66.51	19.12	0.000
Blocks	7	1346.76	192.39	55.30	0.000
Error	14	48.70	3.48		
Total	23	1528.49			

$\mu_1 - \mu_2 \in (-8.09, -3.21)$

$\mu_1 - \mu_3 \in (-4.26, 0.62)$

$\mu_2 - \mu_3 \in (1.39, 6.27)$

There is sufficient evidence to conclude that radar system 2 is better than the other two radar systems.

11.2.11

Source	df	SS	MS	F	p-value
Treatments	3	3231.2	1,077.1	4.66	0.011
Blocks	8	29256.1	3,657.0	15.83	0.000
Error	24	5545.1	231.0		
Total	35	38032.3			

$\mu_1 - \mu_2 \in (-8.20, 31.32)$

$\mu_1 - \mu_3 \in (-16.53, 22.99)$

$\mu_1 - \mu_4 \in (-34.42, 5.10)$

$\mu_2 - \mu_3 \in (-28.09, 11.43)$

$\mu_2 - \mu_4 \in (-45.98, -6.46)$

$\mu_3 - \mu_4 \in (-37.65, 1.87)$

There is sufficient evidence to conclude that driver 4 is better than driver 2.

11.2.12

Source	df	SS	MS	F	p-value
Treatments	2	7.47	3.73	0.34	0.718
Blocks	9	313.50	34.83	3.15	0.018
Error	18	199.20	11.07		
Total	29	520.17			

$\mu_1 - \mu_2 \in (-3.00, 4.60)$

$\mu_1 - \mu_3 \in (-2.60, 5.00)$

$\mu_2 - \mu_3 \in (-3.40, 4.20)$

There is *not* sufficient evidence to conclude that there is any difference between the assembly methods.

11.2.13

Source	df	SS	MS	F	p-value
Treatments	4	8.462×10^8	2.116×10^8	66.55	0.000
Blocks	11	19.889×10^8	1.808×10^8	56.88	0.000
Error	44	1.399×10^8	3.179×10^6		
Total	59	29.750×10^8			

$\mu_1 - \mu_2 \in (4372, 8510)$

$\mu_1 - \mu_3 \in (4781, 8919)$

$\mu_1 - \mu_4 \in (5438, 9577)$

$\mu_1 - \mu_5 \in (-3378, 760)$

$\mu_2 - \mu_3 \in (-1660, 2478)$

$\mu_2 - \mu_4 \in (-1002, 3136)$

$\mu_2 - \mu_5 \in (-9819, -5681)$

$\mu_3 - \mu_4 \in (-1411, 2727)$

$\mu_3 - \mu_5 \in (-10228, -6090)$

$\mu_4 - \mu_5 \in (-10886, -6748)$

There is sufficient evidence to conclude that either agent 1 or agent 5 is the best agent.

The worst agent is either agent 2, 3 or 4.

11.2.14

Source	df	SS	MS	F	p-value
Treatments	3	10.637	3.546	2.01	0.123
Blocks	19	169.526	8.922	5.05	0.000
Error	57	100.641	1.766		
Total	79	280.805			

$\mu_1 - \mu_2 \in (-1.01, 1.21)$

$\mu_1 - \mu_3 \in (-1.89, 0.34)$

$\mu_1 - \mu_4 \in (-1.02, 1.20)$

$\mu_2 - \mu_3 \in (-1.98, 0.24)$

$\mu_2 - \mu_4 \in (-1.12, 1.11)$

$\mu_3 - \mu_4 \in (-0.24, 1.98)$

There is *not* sufficient evidence to conclude that there is any difference between the four formulations.

11.2.15 (a)

Source	df	SS	MS	F	p-value
Treatments	3	0.151	0.0503	5.36	0.008
Blocks	6	0.324	0.054	5.75	0.002
Error	18	0.169	0.00939		
Total	27	0.644			

(b) With $q_{0.05,4,18} = 4.00$ and

$\frac{\sqrt{MSE} \times q_{0.05,4,18}}{\sqrt{b}} = \frac{\sqrt{0.00939} \times 4.00}{\sqrt{7}} = 0.146$

the pairwise confidence intervals are:

$\mu_2 - \mu_1 \in 0.630 - 0.810 \pm 0.146 = (-0.326, -0.034)$

$\mu_2 - \mu_3 \in 0.630 - 0.797 \pm 0.146 = (-0.313, -0.021)$

$\mu_2 - \mu_4 \in 0.630 - 0.789 \pm 0.146 = (-0.305, -0.013)$

None of these confidence intervals contains zero so there is sufficient evidence to conclude that treatment 2 has a smaller mean value than each of the other treatments.

11.2.16 $\bar{x}_{..} = \frac{107.68+109.86+111.63}{3} = \frac{329.17}{3} = 109.72$

$SSTR = 4 \times (107.68^2 + 109.86^2 + 111.63^2) - 12 \times \left(\frac{329.17}{3}\right)^2 = 31.317$

$MSE = \hat{\sigma}^2 = 1.445^2 = 2.088$

Source	df	SS	MS	F	p-value
Treatments	2	31.317	15.659	7.50	0.023
Blocks	3	159.720	53.240	25.50	0.001
Error	6	12.528	2.088		
Total	11	203.565			

11.2.17 The new analysis of variance table is

Source	df	SS	MS	F	p-value
Treatments	same	a^2 SSTr	a^2 MSTr	same	same
Blocks	same	a^2 SSBl	a^2 MSBl	same	same
Error	same	a^2 SSE	a^2 MSE		
Total	same	a^2 SST			

11.2.18 $\bar{x}_{..} = \frac{\bar{x}_{1.}+\bar{x}_{2.}+\bar{x}_{3.}+\bar{x}_{4.}}{4} = \frac{3107.3}{4} = 776.825$

$SSTr = 7 \times (763.9^2 + 843.9^2 + 711.3^2 + 788.2^2) - 4 \times 7 \times 776.825^2 = 63623.2$

Source	df	SS	MS	F	p-value
Treatments	3	63623.2	21207.7	54.13	0.000
Blocks	6	13492.3	2248.7	5.74	0.002
Error	18	7052.8	391.8		
Total	27	84168.3			

There is sufficient evidence to conclude that the treatments are not all the same.

Since

$\frac{s \times q_{0.05,4,18}}{\sqrt{b}} = \frac{\sqrt{391.8} \times 4.00}{\sqrt{7}} = 29.9$

it follows that treatments are only known to be different if their sample averages are more than 29.9 apart.

It is known that treatment 2 has the largest mean, and that treatment 3 has the smallest mean.

Treatments 1 and 4 are indistinguishable.

11.2.19 $\bar{x}_{1.} = 23.18$

$\bar{x}_{2.} = 23.58$

$\bar{x}_{3.} = 23.54$

$\bar{x}_{4.} = 22.48$

Source	df	SS	MS	F	p-value
Locations	3	3.893	1.298	0.49	0.695
Time	4	472.647	118.162	44.69	0.000
Error	12	31.729	2.644		
Total	19	508.270			

The p-value of 0.695 implies that there is not sufficient evidence to conclude that the pollution levels are different at the four locations.

The confidence intervals for all of the pairwise comparisons contain zero, so the graphical representation has one line joining all four sample means.

11.5.1

Source	df	SS	MS	F	p-value
Treatments	3	1.9234	0.6411	22.72	0.000
Error	16	0.4515	0.0282		
Total	19	2.3749			

$\mu_1 - \mu_2 \in (-0.35, 0.26)$

$\mu_1 - \mu_3 \in (0.38, 0.99)$

$\mu_1 - \mu_4 \in (-0.36, 0.25)$

$\mu_2 - \mu_3 \in (0.42, 1.03)$

$\mu_2 - \mu_4 \in (-0.31, 0.30)$

$\mu_3 - \mu_4 \in (-1.04, -0.43)$

There is sufficient evidence to conclude that type 3 has a lower average Young's modulus.

11.5.2

Source	df	SS	MS	F	p-value
Treatments	3	5.77	1.92	0.49	0.690
Error	156	613.56	3.93		
Total	159	619.33			

$\mu_1 - \mu_2 \in (-1.27, 1.03)$

$\mu_1 - \mu_3 \in (-0.82, 1.61)$

$\mu_1 - \mu_4 \in (-1.16, 1.17)$

$\mu_2 - \mu_3 \in (-0.64, 1.67)$

$\mu_2 - \mu_4 \in (-0.97, 1.22)$

$\mu_3 - \mu_4 \in (-1.55, 0.77)$

There is *not* sufficient evidence to conclude that any of the cars is getting better gas mileage than the others.

11.5.3

Source	df	SS	MS	F	p-value
Treatments	4	2,716.8	679.2	3.57	0.024
Blocks	5	4,648.2	929.6	4.89	0.004
Error	20	3,806.0	190.3		
Total	29	11,171.0			

There is not conclusive evidence that the different temperature levels have an effect on the cement strength.

11.5.4

Source	df	SS	MS	F	p-value
Treatments	4	10,381.4	2,595.3	25.70	0.000
Blocks	9	6,732.7	748.1	7.41	0.000
Error	36	3,635.8	101.0		
Total	49	20,749.9			

There is sufficient evidence to conclude that either fertilizer type 4 or type 5 provides the highest yield.

11.5.5

Source	df	SS	MS	F	p-value
Treatments	3	115.17	38.39	4.77	0.007
Blocks	11	4,972.67	452.06	56.12	0.000
Error	33	265.83	8.06		
Total	47	5,353.67			

There is sufficient evidence to conclude that clinic 3 is different from clinics 2 and 4.

11.5.6

Source	df	SS	MS	F	p-value
Treatments	2	142.89	71.44	16.74	0.000
Error	24	102.42	4.27		
Total	26	245.31			

$\mu_h - \mu_a \in (-5.13, -0.27)$

$\mu_h - \mu_b \in (0.50, 5.36)$

$\mu_a - \mu_b \in (3.20, 8.06)$

There is sufficient evidence to conclude that each of the three positions produce different average insertion gains.

11.5.7

Source	df	SS	MS	F	p-value
Treatments	3	1175.3	391.8	8.11	0.000
Error	33	1595.1	48.3		
Total	36	2770.4			

$\mu_1 - \mu_2 \in (3.45, 21.32)$

$\mu_1 - \mu_3 \in (3.29, 20.13)$

$\mu_1 - \mu_4 \in (-6.94, 10.36)$

$\mu_2 - \mu_3 \in (-9.61, 8.26)$

$\mu_2 - \mu_4 \in (-19.82, -1.53)$

$\mu_3 - \mu_4 \in (-18.65, -1.35)$

The drags for designs 1 and 4 are larger than the drags for designs 2 and 3.

11.5.8

Source	df	SS	MS	F	p-value
Treatments	3	0.150814	0.050271	5.39	0.008
Blocks	6	0.325043	0.054174	5.80	0.002
Error	18	0.167986	0.009333		
Total	27	0.643843			

There is sufficient evidence to conclude that the shrinkage from preparation method 2 is smaller than from the other preparation methods.

11.5.9 (a) True

(b) False

(c) True

(d) True

(e) True

(f) True

(g) False

(h) False

11.5.10 (a) $\bar{x}_{1.} = 16.667$

$\bar{x}_{2.} = 19.225$

$\bar{x}_{3.} = 14.329$

Source	df	SS	MS	F	p-value
Alloys	2	89.83	44.91	13.84	0.000
Error	18	58.40	3.24		
Total	20	148.23			

There is sufficient evidence to establish that the alloys are not all the same with respect to their hardness measurements.

(b) With $q_{0.05,3,18} = 3.61$ the pairwise confidence intervals are:

$$\mu_1 - \mu_2 \in 16.667 - 19.225 \pm \tfrac{3.61 \times \sqrt{3.24}}{\sqrt{2}} \sqrt{\tfrac{1}{6} + \tfrac{1}{8}} = (-5.042, -0.075)$$

$$\mu_1 - \mu_3 \in 16.667 - 14.329 \pm \tfrac{3.61 \times \sqrt{3.24}}{\sqrt{2}} \sqrt{\tfrac{1}{6} + \tfrac{1}{7}} = (-0.220, 4.896)$$

$$\mu_2 - \mu_3 \in 19.225 - 14.329 \pm \tfrac{3.61 \times \sqrt{3.24}}{\sqrt{2}} \sqrt{\tfrac{1}{8} + \tfrac{1}{7}} = (2.517, 7.276)$$

These confidence intervals show that alloy 2 has larger hardness measurements than both alloys 1 and 3, which are indistinguishable.

Alloy 2 has the largest mean.

Either alloy 1 or alloy 3 has the smallest mean.

11.5.11 $\bar{x}_{..} = \frac{\bar{x}_{1.}+\bar{x}_{2.}+\bar{x}_{3.}+\bar{x}_{4.}}{4} = \frac{50.1}{4} = 12.525$

$SSTr = 9 \times (11.43^2 + 12.03^2 + 14.88^2 + 11.76^2) - 4 \times 9 \times 12.525^2 = 68.18$

Source	df	SS	MS	F	p-value
Treatments	3	68.18	22.73	38.63	0.000
Blocks	8	53.28	6.66	11.32	0.000
Error	24	14.12	0.588		
Total	35	135.58			

There is sufficient evidence to conclude that the treatments are not all the same.

Since

$\frac{s \times q_{0.05,4,24}}{\sqrt{b}} = \frac{\sqrt{0.588} \times 3.90}{\sqrt{9}} = 0.997$

it follows that two treatments are only known to be different if their sample averages are more than 0.997 apart.

Therefore, treatment 3 is known to have a larger mean than treatments 1, 2, and 4, which are indistinguishable.

11.5.12 $\bar{x}_{1.} = 310.83$

$\bar{x}_{2.} = 310.17$

$\bar{x}_{3.} = 315.33$

$\bar{x}_{4.} = 340.33$

$\bar{x}_{5.} = 300.00$

Source	df	SS	MS	F	p-value
Rivers	4	5442.3	1360.6	20.71	0.000
Error	25	1642.3	65.7		
Total	29	7084.7			

There is sufficient evidence to conclude that the average radon levels in the five rivers are different.

Since

$\frac{s \times q_{0.05,5,25}}{\sqrt{n}} = \frac{\sqrt{65.7} \times 4.165}{\sqrt{6}} = 13.7$

it follows that rivers are only known to be different if their sample averages are more than 13.7 apart.

River 4 can be determined to be the river with the highest radon level.

11.5.13 $\mu_1 - \mu_2 \in (3.23, 11.57)$

$\mu_1 - \mu_3 \in (4.32, 11.68)$

$\mu_1 - \mu_4 \in (-5.85, 1.65)$

$\mu_2 - \mu_3 \in (-3.44, 4.64)$

$\mu_2 - \mu_4 \in (-13.60, -5.40)$

$\mu_3 - \mu_4 \in (-13.70, -6.50)$

11.5.15 A

11.5.16 D