5.1.1 (a) $\Phi(1.34) = 0.9099$

(b) $1 - \Phi(-0.22) = 0.5871$

(c) $\Phi(0.43) - \Phi(-2.19) = 0.6521$

(d) $\Phi(1.76) - \Phi(0.09) = 0.4249$

(e) $\Phi(0.38) - \Phi(-0.38) = 0.2960$

(f) Solving $\Phi(x) = 0.55$ gives $x = 0.1257$.

(g) Solving $1 - \Phi(x) = 0.72$ gives $x = -0.5828$.

(h) Solving $\Phi(x) - \Phi(-x) = (2 \times \Phi(x)) - 1 = 0.31$ gives $x = 0.3989$.

5.1.2 (a) $\Phi(-0.77) = 0.2206$

(b) $1 - \Phi(0.32) = 0.3745$

(c) $\Phi(-1.59) - \Phi(-3.09) = 0.0549$

(d) $\Phi(1.80) - \Phi(-0.82) = 0.7580$

(e) $1 - (\Phi(0.91) - \Phi(-0.91)) = 0.3628$

(f) Solving $\Phi(x) = 0.23$ gives $x = -0.7388$.

(g) Solving $1 - \Phi(x) = 0.51$ gives $x = -0.0251$.

(h) Solving $1 - (\Phi(x) - \Phi(-x)) = 2 - (2 \times \Phi(x)) = 0.42$ gives $x = 0.8064$.

5.1.3 (a) $P(X \le 10.34) = \Phi\left(\frac{10.34-10}{\sqrt{2}}\right) = 0.5950$

(b) $P(X \ge 11.98) = 1 - \Phi\left(\frac{11.98-10}{\sqrt{2}}\right) = 0.0807$

(c) $P(7.67 \le X \le 9.90) = \Phi\left(\frac{9.90-10}{\sqrt{2}}\right) - \Phi\left(\frac{7.67-10}{\sqrt{2}}\right) = 0.4221$

(d) $P(10.88 \le X \le 13.22) = \Phi\left(\frac{13.22-10}{\sqrt{2}}\right) - \Phi\left(\frac{10.88-10}{\sqrt{2}}\right) = 0.2555$

(e) $P(|X-10| \le 3) = P(7 \le X \le 13)$
$= \Phi\left(\frac{13-10}{\sqrt{2}}\right) - \Phi\left(\frac{7-10}{\sqrt{2}}\right) = 0.9662$

(f) Solving $P(N(10,2) \le x) = 0.81$ gives $x = 11.2415$.

(g) Solving $P(N(10,2) \ge x) = 0.04$ gives $x = 12.4758$.

(h) Solving $P(|N(10,2)-10| \ge x) = 0.63$ gives $x = 0.6812$.

5.1.4 (a) $P(X \le 0) = \Phi\left(\frac{0-(-7)}{\sqrt{14}}\right) = 0.9693$

(b) $P(X \ge -10) = 1 - \Phi\left(\frac{-10-(-7)}{\sqrt{14}}\right) = 0.7887$

(c) $P(-15 \le X \le -1) = \Phi\left(\frac{-1-(-7)}{\sqrt{14}}\right) - \Phi\left(\frac{-15-(-7)}{\sqrt{14}}\right) = 0.9293$

(d) $P(-5 \le X \le 2) = \Phi\left(\frac{2-(-7)}{\sqrt{14}}\right) - \Phi\left(\frac{-5-(-7)}{\sqrt{14}}\right) = 0.2884$

(e) $P(|X+7| \ge 8) = 1 - P(-15 \le X \le 1)$
$= 1 - \left(\Phi\left(\frac{1-(-7)}{\sqrt{14}}\right) - \Phi\left(\frac{-15-(-7)}{\sqrt{14}}\right)\right)$
$= 0.0326$

(f) Solving $P(N(-7,14) \le x) = 0.75$ gives $x = -4.4763$.

(g) Solving $P(N(-7,14) \ge x) = 0.27$ gives $x = -4.7071$.

(h) Solving $P(|N(-7,14)+7| \le x) = 0.44$ gives $x = 2.18064$.

5.1.5 Solving

$P(X \leq 5) = \Phi\left(\frac{5-\mu}{\sigma}\right) = 0.8$

and

$P(X \geq 0) = 1 - \Phi\left(\frac{0-\mu}{\sigma}\right) = 0.6$

gives $\mu = 1.1569$ and $\sigma = 4.5663$.

5.1.6 Solving

$P(X \leq 10) = \Phi\left(\frac{10-\mu}{\sigma}\right) = 0.55$

and

$P(X \leq 0) = \Phi\left(\frac{0-\mu}{\sigma}\right) = 0.4$

gives $\mu = 6.6845$ and $\sigma = 26.3845$.

5.1.7 $P(X \leq \mu + \sigma z_\alpha) = \Phi\left(\frac{\mu+\sigma z_\alpha - \mu}{\sigma}\right)$

$= \Phi(z_\alpha) = 1 - \alpha$

$P(\mu - \sigma z_{\alpha/2} \leq X \leq \mu + \sigma z_{\alpha/2}) = \Phi\left(\frac{\mu+\sigma z_{\alpha/2}-\mu}{\sigma}\right) - \Phi\left(\frac{\mu-\sigma z_{\alpha/2}-\mu}{\sigma}\right)$

$= \Phi(z_{\alpha/2}) - \Phi(-z_{\alpha/2})$

$= 1 - \alpha/2 - \alpha/2 = 1 - \alpha$

5.1.8 Solving $\Phi(x) = 0.75$ gives $x = 0.6745$.
Solving $\Phi(x) = 0.25$ gives $x = -0.6745$.
The interquartile range of a $N(0, 1)$ distribution is therefore
$0.6745 - (-0.6745) = 1.3490$.

The interquartile range of a $N(\mu, \sigma^2)$ distribution is $1.3490 \times \sigma$.

5.1.9 (a) $P(N(3.00, 0.12^2) \geq 3.2) = 0.0478$

(b) $P(N(3.00, 0.12^2) \leq 2.7) = 0.0062$

(c) Solving
$P(3.00 - c \leq N(3.00, 0.12^2) \leq 3.00 + c) = 0.99$
gives
$c = 0.12 \times z_{0.005} = 0.12 \times 2.5758 = 0.3091.$

5.1.10 (a) $P(N(1.03, 0.014^2) \leq 1) = 0.0161$

(b) $P(N(1.05, 0.016^2) \leq 1) = 0.0009$
There is a decrease in the proportion of underweight packets.

(c) The expected excess weight is $\mu - 1$ which is 0.03 and 0.05.

5.1.11 (a) Solving $P(N(4.3, 0.12^2) \leq x) = 0.75$ gives $x = 4.3809$.
Solving $P(N(4.3, 0.12^2) \leq x) = 0.25$ gives $x = 4.2191$.

(b) Solving
$P(4.3 - c \leq N(4.3, 0.12^2) \leq 4.3 + c) = 0.80$
gives
$c = 0.12 \times z_{0.10} = 0.12 \times 1.2816 = 0.1538.$

5.1.12 (a) $P(N(0.0046, 9.6 \times 10^{-8}) \leq 0.005) = 0.9017$

(b) $P(0.004 \leq N(0.0046, 9.6 \times 10^{-8}) \leq 0.005) = 0.8753$

(c) Solving $P(N(0.0046, 9.6 \times 10^{-8}) \leq x) = 0.10$ gives $x = 0.0042$.

(d) Solving $P(N(0.0046, 9.6 \times 10^{-8}) \leq x) = 0.99$ gives $x = 0.0053$.

5.1.13 (a) $P(N(23.8, 1.28) \leq 23.0) = 0.2398$

(b) $P(N(23.8, 1.28) \geq 24.0) = 0.4298$

(c) $P(24.2 \leq N(23.8, 1.28) \leq 24.5) = 0.0937$

(d) Solving $P(N(23.8, 1.28) \leq x) = 0.75$ gives $x = 24.56$.

(e) Solving $P(N(23.8, 1.28) \leq x) = 0.95$ gives $x = 25.66$.

5.1.14 Solving

$P(N(\mu, 0.05^2) \leq 10) = 0.01$

gives

$\mu = 10 + (0.05 \times z_{0.01}) = 10 + (0.05 \times 2.3263) = 10.1163$.

5.1.15 (a) $P(2599 \leq X \leq 2601) = \Phi\left(\frac{2601-2600}{0.6}\right) - \Phi\left(\frac{2599-2600}{0.6}\right)$

$= 0.9522 - 0.0478 = 0.9044$

The probability of being outside the range is $1 - 0.9044 = 0.0956$.

(b) It is required that

$P(2599 \leq X \leq 2601) = \Phi\left(\frac{2601-2600}{\sigma}\right) - \left(\frac{2599-2600}{\sigma}\right)$

$= 1 - 0.001 = 0.999$.

Consequently,

$\Phi\left(\frac{1}{\sigma}\right) - \Phi\left(\frac{-1}{\sigma}\right)$

$= 2\Phi\left(\frac{1}{\sigma}\right) - 1 = 0.999$

so that

$\Phi\left(\frac{1}{\sigma}\right) = 0.9995$

which gives

$\frac{1}{\sigma} = \Phi^{-1}(0.9995) = 3.2905$

with

$\sigma = 0.304$.

5.1.16 $P(N(1320,15^2) \leq 1300) = P\left(N(0,1) \leq \frac{1300-1320}{15}\right)$

$= \Phi(-1.333) = 0.0912$

$P(N(1320,15^2) \leq 1330) = P\left(N(0,1) \leq \frac{1330-1320}{15}\right)$

$= \Phi(0.667) = 0.7475$

Using the multinomial distribution the required probability is

$\frac{10!}{3!\times 4!\times 3!} \times 0.0912^3 \times (0.7475 - 0.0912)^4 \times (1 - 0.7475)^3 = 0.0095.$

5.1.17 $0.95 = P(N(\mu,4.2^2) \leq 100) = P\left(N(0,1) \leq \frac{100-\mu}{4.2}\right)$

Therefore,

$\frac{100-\mu}{4.2} = z_{0.05} = 1.645$

so that $\mu = 93.09$.

5.1.18 0.894

5.2.1 (a) $P(N(3.2 + (-2.1), 6.5 + 3.5) \geq 0) = 0.6360$

(b) $P(N(3.2 + (-2.1) - (2 \times 12.0), 6.5 + 3.5 + (2^2 \times 7.5)) \leq -20) = 0.6767$

(c) $P(N((3 \times 3.2) + (5 \times (-2.1)), (3^2 \times 6.5) + (5^2 \times 3.5)) \geq 1) = 0.4375$

(d) The mean is $(4 \times 3.2) - (4 \times (-2.1)) + (2 \times 12.0) = 45.2$.
The variance is $(4^2 \times 6.5) + (4^2 \times 3.5) + (2^2 \times 7.5) = 190$.
$P(N(45.2, 190) \leq 25) = 0.0714$

(e) $P(\mid N(3.2 + (6 \times (-2.1)) + 12.0, 6.5 + (6^2 \times 3.5) + 7.5) \mid \geq 2) = 0.8689$

(f) $P(\mid N((2 \times 3.2) - (-2.1) - 6, (2^2 \times 6.5) + 3.5) \mid \leq 1) = 0.1315$

5.2.2 (a) $P(N(-1.9-3.3, 2.2+1.7) \geq -3) = 0.1326$

(b) The mean is $(2 \times (-1.9)) + (3 \times 3.3) + (4 \times 0.8) = 9.3$.
The variance is $(2^2 \times 2.2) + (3^2 \times 1.7) + (4^2 \times 0.2) = 27.3$.
$P(N(9.3, 27.3) \leq 10) = 0.5533$

(c) $P(N((3 \times 3.3) - 0.8, (3^2 \times 1.7) + 0.2) \leq 8) = 0.3900$

(d) The mean is $(2 \times (-1.9)) - (2 \times 3.3) + (3 \times 0.8) = -8.0$.
The variance is $(2^2 \times 2.2) + (2^2 \times 1.7) + (3^2 \times 0.2) = 17.4$.
$P(N(-8.0, 17.4) \leq -6) = 0.6842$

(e) $P(\mid N(-1.9+3.3-0.8, 2.2+1.7+0.2) \mid \geq 1.5) = 0.4781$

(f) $P(\mid N((4 \times (-1.9)) - 3.3 + 10, (4^2 \times 2.2) + 1.7) \mid \leq 0.5) = 0.0648$

5.2.3 (a) $\Phi(0.5) - \Phi(-0.5) = 0.3830$

(b) $P\left(\left|N\left(0, \frac{1}{8}\right)\right| \leq 0.5\right) = 0.8428$

(c) It is required that

$0.5\sqrt{n} \geq z_{0.005} = 2.5758$

which is satisfied for $n \geq 27$.

5.2.4 (a) $N(4.3+4.3, 0.12^2+0.12^2) = N(8.6, 0.0288)$

(b) $N\left(4.3, \frac{0.12^2}{12}\right) = N(4.3, 0.0012)$

(c) It is required that

$z_{0.0015} \times \frac{0.12}{\sqrt{n}} = 2.9677 \times \frac{0.12}{\sqrt{n}} \leq 0.05$

which is satisfied for $n \geq 51$.

5.2.5 $P(144 \leq N(37+37+24+24+24, 0.49+0.49+0.09+0.09+0.09) \leq 147) = 0.7777$

5.2.6 (a) $\text{Var}(Y) = (p^2 \times \sigma_1^2) + ((1-p)^2 \times \sigma_2^2)$

The minimum variance is

$\frac{1}{\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2}} = \frac{\sigma_1^2\,\sigma_2^2}{\sigma_1^2+\sigma_2^2}$.

(b) In this case

$\text{Var}(Y) = \sum_{i=1}^n \; p_i^2\, \sigma_i^2$.

The variance is minimized with

$p_i = \frac{\frac{1}{\sigma_i^2}}{\frac{1}{\sigma_1^2}+\ldots+\frac{1}{\sigma_n^2}}$

and the minimum variance is

$\frac{1}{\frac{1}{\sigma_1^2}+\ldots+\frac{1}{\sigma_n^2}}$.

5.2.7 (a) $1.05y + 1.05(1000-y) = \$1050$

(b) $0.0002y^2 + 0.0003(1000-y)^2$

(c) The variance is minimized with $y = 600$ and the minimum variance is 120.

$P(N(1050, 120) \geq 1060) = 0.1807$

5.2.8 (a) $P(N(3.00 + 3.00 + 3.00, 0.12^2 + 0.12^2 + 0.12^2) \geq 9.50) = 0.0081$

(b) $P\left(N\left(3.00, \frac{0.12^2}{7}\right) \leq 3.10\right) = 0.9863$

5.2.9 (a) $N(22 \times 1.03, 22 \times 0.014^2) = N(22.66, 4.312 \times 10^{-3})$

(b) Solving $P(N(22.66, 4.312 \times 10^{-3}) \leq x) = 0.75$ gives $x = 22.704$.
Solving $P(N(22.66, 4.312 \times 10^{-3}) \leq x) = 0.25$ gives $x = 22.616$.

5.2.10 (a) Let the random variables X_i be the widths of the components.
Then

$P(X_1 + X_2 + X_3 + X_4 \leq 10402.5) = P(N(4 \times 2600, 4 \times 0.6^2) \leq 10402.5)$

$= \Phi\left(\frac{10402.5-10400}{1.2}\right) = \Phi(2.083) = 0.9814.$

(b) Let the random variable Y be the width of the slot.
Then

$P(X_1 + X_2 + X_3 + X_4 - Y \leq 0)$

$= P(N((4 \times 2600) - 10402.5, (4 \times 0.6^2) + 0.4^2) \leq 0)$

$= \Phi\left(\frac{2.5}{1.2649}\right) = \Phi(1.976) = 0.9759.$

5.2.11 (a) $P\left(4.2 \leq N\left(4.5, \frac{0.88}{15}\right) \leq 4.9\right)$

$= P\left(\frac{\sqrt{15}(4.2-4.5)}{\sqrt{0.88}} \leq N(0,1) \leq \frac{\sqrt{15}(4.9-4.5)}{\sqrt{0.88}}\right)$

$= \Phi(1.651) - \Phi(-1.239)$

$= 0.951 - 0.108 = 0.843$

(b) $0.99 = P\left(4.5 - c \leq N\left(4.5, \frac{0.88}{15}\right) \leq 4.5 + c\right)$

$= P\left(\frac{-c\sqrt{15}}{\sqrt{0.88}} \leq N(0,1) \leq \frac{c\sqrt{15}}{\sqrt{0.88}}\right)$

Therefore,

$\frac{c\sqrt{15}}{\sqrt{0.88}} = z_{0.005} = 2.576$

so that $c = 0.624$.

5.2.12 (a) $P(X_1 + X_2 + X_3 + X_4 + X_5 \geq 45)$

$= P(N(8+8+8+8+8, 2^2+2^2+2^2+2^2+2^2) \geq 45)$

$= P\left(N(0,1) \geq \frac{45-40}{\sqrt{20}}\right)$

$= 1 - \Phi(1.118) = 0.132$

(b) $P(N(28, 5^2) \geq N(8+8+8, 2^2+2^2+2^2))$

$= P(N(28-24, 25+12) \geq 0)$

$= P\left(N(0,1) \geq \frac{-4}{\sqrt{37}}\right)$

$= 1 - \Phi(-0.658) = 0.745$

5.2.13 The height of a stack of 4 components of type A has a normal distribution with mean $4 \times 190 = 760$ and a standard deviation $\sqrt{4} \times 10 = 20$.

The height of a stack of 5 components of type B has a normal distribution with mean $5 \times 150 = 750$ and a standard deviation $\sqrt{5} \times 8 = 17.89$.

$P(N(760, 20^2) > N(750, 17.89^2))$

$= P(N(760-750, 20^2+17.78^2) > 0)$

$= P\left(N(0,1) > \frac{-10}{\sqrt{720}}\right)$

$= 1 - \Phi(-0.373) = 0.645$

5.2.14 Let the random variables X_i be the times taken by worker 1 to perform a task and let the random variables Y_i be the times taken by worker 2 to perform a task.

$P(X_1 + X_2 + X_3 + X_4 - Y_1 - Y_2 - Y_3 \leq 0)$

$= P(N(13+13+13+13-17-17-17, 0.5^2+0.5^2+0.5^2+0.5^2+0.6^2+0.6^2+0.6^2) \leq 0)$

$= P(N(1, 2.08) \leq 0)$

$= P\left(N(0,1) \leq \frac{-1}{\sqrt{2.08}}\right)$

$= \Phi(-0.693) = 0.244$

5.2.15 It is required that

$P\left(N\left(110, \frac{4}{n}\right) \leq 111\right)$

$= P\left(N(0,1) \leq \frac{\sqrt{n}(111-110)}{2}\right) \geq 0.99.$

Therefore,

$\frac{\sqrt{n}(111-110)}{2} \geq z_{0.01} = 2.326$

which is satisfied for $n \geq 22$.

5.2.16 If X has mean of 7.2 m and a standard deviation of 0.11 m,
then $\frac{X}{2}$ has a mean of $\frac{7.2}{2} = 3.6$ m and a standard deviation of $\frac{0.11}{2} = 0.055$ m.

5.2.17 (a) $E(X) = 20\mu = 20 \times 63400 = 1268000$
The standard deviation is $\sqrt{20}\ \sigma = \sqrt{20} \times 2500 = 11180$.

(b) $E(X) = \mu = 63400$
The standard deviation is $\frac{\sigma}{\sqrt{30}} = \frac{2500}{\sqrt{30}} = 456.4$.

5.2.18 (a) $P(X < 800) = \Phi\left(\frac{800-T}{47}\right) = 0.1$

so that

$\frac{800-T}{47} = -z_{0.1} = -1.282.$

This gives $T = 860.3$.

(b) The average Y is distributed as a $N\left(850, \frac{47^2}{10}\right)$ random variable.

Therefore,

$P(Y < 875) = \Phi\left(\frac{875-850}{47/\sqrt{10}}\right) = 0.954.$

5.2.19 $P\left(N\left(30000 + 45000, 4000^2 + 3000^2\right) \geq 85000\right)$
$= P\left(N\left(0,1\right) \geq 2\right) = 0.023$

5.3.1 (a) The exact probability is 0.3823.

The normal approximation is

$1 - \Phi\left(\frac{8-0.5-(10\times0.7)}{\sqrt{10\times0.7\times0.3}}\right) = 0.3650.$

(b) The exact probability is 0.9147.

The normal approximation is

$\Phi\left(\frac{7+0.5-(15\times0.3)}{\sqrt{15\times0.3\times0.7}}\right) - \Phi\left(\frac{1+0.5-(15\times0.3)}{\sqrt{15\times0.3\times0.7}}\right) = 0.9090.$

(c) The exact probability is 0.7334.

The normal approximation is

$\Phi\left(\frac{4+0.5-(9\times0.4)}{\sqrt{9\times0.4\times0.6}}\right) = 0.7299.$

(d) The exact probability is 0.6527.

The normal approximation is

$\Phi\left(\frac{11+0.5-(14\times0.6)}{\sqrt{14\times0.6\times0.4}}\right) - \Phi\left(\frac{7+0.5-(14\times0.6)}{\sqrt{14\times0.6\times0.4}}\right) = 0.6429.$

5.3.2 (a) The exact probability is 0.0106.

The normal approximation is

$1 - \Phi\left(\frac{7-0.5-(10\times 0.3)}{\sqrt{10\times 0.3\times 0.7}}\right) = 0.0079.$

(b) The exact probability is 0.6160.

The normal approximation is

$\Phi\left(\frac{12+0.5-(21\times 0.5)}{\sqrt{21\times 0.5\times 0.5}}\right) - \Phi\left(\frac{8+0.5-(21\times 0.5)}{\sqrt{21\times 0.5\times 0.5}}\right) = 0.6172.$

(c) The exact probability is 0.9667.

The normal approximation is

$\Phi\left(\frac{3+0.5-(7\times 0.2)}{\sqrt{7\times 0.2\times 0.8}}\right) = 0.9764.$

(d) The exact probability is 0.3410.

The normal approximation is

$\Phi\left(\frac{11+0.5-(12\times 0.65)}{\sqrt{12\times 0.65\times 0.35}}\right) - \Phi\left(\frac{8+0.5-(12\times 0.65)}{\sqrt{12\times 0.65\times 0.35}}\right) = 0.3233.$

5.3.3 The required probability is

$\Phi\left(0.02\sqrt{n} + \frac{1}{\sqrt{n}}\right) - \Phi\left(-0.02\sqrt{n} - \frac{1}{\sqrt{n}}\right)$

which is equal to

0.2358 for $n = 100$

0.2764 for $n = 200$

0.3772 for $n = 500$

0.4934 for $n = 1000$

and 0.6408 for $n = 2000$.

5.3.4 (a) $\Phi\left(\frac{180+0.5-(1{,}000\times 1/6)}{\sqrt{1{,}000\times 1/6\times 5/6}}\right) - \Phi\left(\frac{149+0.5-(1{,}000\times 1/6)}{\sqrt{1{,}000\times 1/6\times 5/6}}\right) = 0.8072$

(b) It is required that

$1 - \Phi\left(\frac{50-0.5-n/6}{\sqrt{n\times 1/6\times 5/6}}\right) \geq 0.99$

which is satisfied for $n \geq 402$.

5.3.5 (a) A normal distribution can be used with

$\mu = 500 \times 2.4 = 1200$

and

$\sigma^2 = 500 \times 2.4 = 1200.$

(b) $P(N(1200, 1200) \geq 1250) = 0.0745$

5.3.6 The normal approximation is

$$1 - \Phi\left(\frac{135-0.5-(15{,}000\times 1/125)}{\sqrt{15{,}000\times 1/125\times 124/125}}\right) = 0.0919.$$

5.3.7 The normal approximation is

$$\Phi\left(\frac{200+0.5-(250{,}000\times 0.0007)}{\sqrt{250{,}000\times 0.0007\times 0.9993}}\right) = 0.9731.$$

5.3.8 (a) The normal approximation is

$$1 - \Phi\left(\frac{30-0.5-(60\times 0.25)}{\sqrt{60\times 0.25\times 0.75}}\right) \simeq 0.$$

(b) It is required that $P(B(n, 0.25) \leq 0.35n) \geq 0.99$.

Using the normal approximation this can be written

$$\Phi\left(\frac{0.35n+0.5-0.25n}{\sqrt{n\times 0.25\times 0.75}}\right) \geq 0.99$$

which is satisfied for $n \geq 92$.

5.3.9 The yearly income can be approximated by a normal distribution with

$$\mu = 365 \times \frac{5}{0.9} = 2027.8$$

and

$$\sigma^2 = 365 \times \frac{5}{0.9^2} = 2253.1.$$

$$P(N(2027.8, 2253.1) \geq 2000) = 0.7210$$

5.3.10 The normal approximation is

$P(N(1500 \times 0.6, 1500 \times 0.6 \times 0.4) \geq 925 - 0.5)$

$= 1 - \Phi(1.291) = 0.0983.$

5.3.11 The expectation of the strength of a chemical solution is

$E(X) = \frac{18}{18+11} = 0.6207$

and the variance is

$\text{Var}(X) = \frac{18\times11}{(18+11)^2(18+11+1)} = 0.007848.$

Using the central limit theorem the required probability can be estimated as

$P\left(0.60 \leq N\left(0.6207, \frac{0.007848}{20}\right) \leq 0.65\right)$

$= \Phi(1.479) - \Phi(-1.045) = 0.7824.$

5.3.12 $P(B(1550, 0.135) \geq 241)$

$\simeq P(N(1550 \times 0.135, 1550 \times 0.135 \times 0.865) \geq 240.5)$

$= P\left(N(0,1) \geq \frac{240.5-209.25}{\sqrt{181.00}}\right)$

$= 1 - \Phi(2.323) = 0.010$

5.3.13 $P(60 \leq X \leq 100) = (1 - e^{-100/84}) - (1 - e^{-60/84}) = 0.1855$

$P(B(350, 0.1855) \geq 55)$

$\simeq P(N(350 \times 0.1855, 350 \times 0.1855 \times 0.8145) \geq 54.5)$

$= P\left(N(0,1) \geq \frac{54.5-64.925}{7.272}\right)$

$= 1 - \Phi(-1.434) = 0.92$

5.3.14 $P(X \geq 20) = e^{-(0.056\times 20)^{2.5}} = 0.265$

$P(B(500, 0.265) \geq 125)$

$\simeq P(N(500 \times 0.265, 500 \times 0.265 \times 0.735) \geq 124.5)$

$= P\left(N(0,1) \geq \frac{124.5-132.57}{9.871}\right)$

$= 1 - \Phi(-0.818) = 0.79$

5.3.15 (a) $P(X \geq 891.2) = \frac{892-891.2}{892-890} = 0.4$

Using the negative binomial distribution the required probability is

$\binom{5}{2} \times 0.4^3 \times 0.6^3 = 0.138.$

(b) $P(X \geq 890.7) = \frac{892-890.7}{892-890} = 0.65$

$P(B(200, 0.65) \geq 100)$

$\simeq P(N(200 \times 0.65, 200 \times 0.65 \times 0.35) \geq 99.5)$

$= P\left(N(0,1) \geq \frac{99.5-130}{\sqrt{45.5}}\right)$

$= 1 - \Phi(-4.52) \simeq 1$

5.3.16 $P(\text{spoil within 10 days}) = 1 - e^{10/8} = 0.713$

The number of packets X with spoiled food has a binomial distribution with $n = 100$ and $p = 0.713$,
so that the expectation is $100 \times 0.713 = 71.3$
and the standard deviation is $\sqrt{100 \times 0.713 \times 0.287} = 4.52$.

$P(X \geq 75) \simeq P(N(71.3, 4.52^2) \geq 74.5)$

$= 1 - \Phi\left(\frac{74.5-71.3}{4.52}\right) = 1 - \Phi(0.71) = 0.24$

5.4.1 (a) $E(X) = e^{1.2+(1.5^2/2)} = 10.23$

(b) $\text{Var}(X) = e^{(2\times1.2)+1.5^2} \times (e^{1.5^2} - 1) = 887.69$

(c) Since $z_{0.25} = 0.6745$ the upper quartile is
$e^{1.2+(1.5\times0.6745)} = 9.13$.

(d) The lower quartile is
$e^{1.2+(1.5\times(-0.6745))} = 1.21$.

(e) The interquartile range is $9.13 - 1.21 = 7.92$.

(f) $P(5 \leq X \leq 8) = \Phi\left(\frac{\ln(8)-1.2}{1.5}\right) - \Phi\left(\frac{\ln(5)-1.2}{1.5}\right) = 0.1136$.

5.4.2 (a) $E(X) = e^{-0.3+(1.1^2/2)} = 1.357$

(b) $\text{Var}(X) = e^{(2\times(-0.3))+1.1^2} \times (e^{1.1^2} - 1) = 4.331$

(c) Since $z_{0.25} = 0.6745$ the upper quartile is

$e^{-0.3+(1.1\times 0.6745)} = 1.556.$

(d) The lower quartile is

$e^{-0.3+(1.1\times(-0.6745))} = 0.353.$

(e) The interquartile range is $1.556 - 0.353 = 1.203$.

(f) $P(0.1 \leq X \leq 7) = \Phi\left(\frac{\ln(7)-(-0.3)}{1.1}\right) - \Phi\left(\frac{\ln(0.1)-(-0.3)}{1.1}\right) = 0.9451.$

5.4.4 (a) $E(X) = e^{2.3+(0.2^2/2)} = 10.18$

(b) The median is $e^{2.3} = 9.974$.

(c) Since $z_{0.25} = 0.6745$ the upper quartile is

$e^{2.3+(0.2\times 0.6745)} = 11.41.$

(d) $P(X \geq 15) = 1 - \Phi\left(\frac{\ln(15)-2.3}{0.2}\right) = 0.0207$

(e) $P(X \leq 6) = \Phi\left(\frac{\ln(6)-2.3}{0.2}\right) = 0.0055$

5.4.5 (a) $\chi^2_{0.10,9} = 14.68$

(b) $\chi^2_{0.05,20} = 31.41$

(c) $\chi^2_{0.01,26} = 45.64$

(d) $\chi^2_{0.90,50} = 37.69$

(e) $\chi^2_{0.95,6} = 1.635$

5.4.6 (a) $\chi^2_{0.12,8} = 12.77$

(b) $\chi^2_{0.54,19} = 17.74$

(c) $\chi^2_{0.023,32} = 49.86$

(d) $P(X \leq 13.3) = 0.6524$

(e) $P(9.6 \leq X \leq 15.3) = 0.4256$

5.4.7 (a) $t_{0.10,7} = 1.415$

(b) $t_{0.05,19} = 1.729$

(c) $t_{0.01,12} = 2.681$

(d) $t_{0.025,30} = 2.042$

(e) $t_{0.005,4} = 4.604$

5.4.8 (a) $t_{0.27,14} = 0.6282$

(b) $t_{0.09,22} = 1.385$

(c) $t_{0.016,7} = 2.670$

(d) $P(X \leq 1.78) = 0.9556$

(e) $P(-0.65 \leq X \leq 2.98) = 0.7353$

(f) $P(\mid X \mid \geq 3.02) = 0.0062$

5.4.9 (a) $F_{0.10,9,10} = 2.347$

(b) $F_{0.05,6,20} = 2.599$

(c) $F_{0.01,15,30} = 2.700$

(d) $F_{0.05,4,8} = 3.838$

(e) $F_{0.01,20,13} = 3.665$

5.4.10 (a) $F_{0.04,7,37} = 2.393$

(b) $F_{0.87,17,43} = 0.6040$

(c) $F_{0.035,3,8} = 4.732$

(d) $P(X \geq 2.35) = 0.0625$

(e) $P(0.21 \leq X \leq 2.92) = 0.9286$

5.4.11 This follows from the definitions

$t_\nu \sim \frac{N(0,1)}{\sqrt{\chi^2_\nu/\nu}}$

and

$F_{1,\nu} \sim \frac{\chi^2_1}{\chi^2_\nu/\nu}$.

5.4.12 (a) $x = t_{0.05,23} = 1.714$

(b) $y = -t_{0.025,60} = -2.000$

(c) $\chi^2_{0.90,29} = 19.768$ and $\chi^2_{0.05,29} = 42.557$
so
$P(19.768 \leq \chi^2_{29} \leq 42.557) = 0.95 - 0.10 = 0.85$

5.4.13 $P(F_{5,20} \geq 4.00) = 0.011$

5.4.14 $P(t_{35} \geq 2.50) = 0.009$

5.4.15 (a) $P(F_{10,50} \geq 2.5) = 0.016$

(b) $P(\chi^2_{17} \leq 12) = 0.200$

(c) $P(t_{24} \geq 3) = 0.003$

(d) $P(t_{14} \geq -2) = 0.967$

5.4.16 (a) $P(t_{21} \leq 2.3) = 0.984$

(b) $P(\chi^2_6 \geq 13.0) = 0.043$

(c) $P(t_{10} \leq -1.9) = 0.043$

(d) $P(t_7 \geq -2.7) = 0.985$

5.4.17 (a) $P(t_{16} \leq 1.9) = 0.962$

(b) $P(\chi^2_{25} \geq 42.1) = 0.018$

(c) $P(F_{9,14} \leq 1.8) = 0.844$

(d) $P(-1.4 \leq t_{29} \leq 3.4) = 0.913$

5.4.18 A

5.7.1 (a) $P(N(500, 50^2) \geq 625) = 0.0062$

(b) Solving $P(N(500, 50^2) \leq x) = 0.99$ gives $x = 616.3$.

(c) $P(N(500, 50^2) \geq 700) \simeq 0$
There is a strong suggestion that an eruption is imminent.

5.7.2 (a) $P(N(12500, 200000) \geq 13000) = 0.1318$

(b) $P(N(12500, 200000) \leq 11400) = 0.0070$

(c) $P(12200 \leq N(12500, 200000) \leq 14000) = 0.7484$

(d) Solving $P(N(12500, 200000) \leq x) = 0.95$ gives $x = 13200$.

5.7.3 (a) $P(N(70, 5.4^2) \geq 80) = 0.0320$

(b) $P(N(70, 5.4^2) \leq 55) = 0.0027$

(c) $P(65 \leq N(70, 5.4^2) \leq 78) = 0.7536$

(d) $c = \sigma \times z_{0.025} = 5.4 \times 1.9600 = 10.584$

5.7.4 (a) $P(X_1 - X_2 \geq 0) = P(N(0, 2 \times 5.4^2) \geq 0) = 0.5$

(b) $P(X_1 - X_2 \geq 10) = P(N(0, 2 \times 5.4^2) \geq 10) = 0.0952$

(c) $P\left(\frac{X_1+X_2}{2} - X_3 \geq 10\right) = P(N(0, 1.5 \times 5.4^2) \geq 10) = 0.0653$

5.7.5 $P(\mid X_1 - X_2 \mid \leq 3)$
$= P(\mid N(0, 2 \times 2^2) \mid \leq 3)$
$= P(-3 \leq N(0, 8) \leq 3) = 0.7112$

5.7.6 $E(X) = \frac{1.43+1.60}{2} = 1.515$

$\text{Var}(X) = \frac{(1.60-1.43)^2}{12} = 0.002408$

Therefore, the required probability can be estimated as

$P(180 \leq N(120 \times 1.515, 120 \times 0.002408) \leq 182) = 0.6447.$

5.7.7 $E(X) = \frac{1}{0.31} = 3.2258$

$\text{Var}(X) = \frac{1}{0.31^2} = 10.406$

Therefore, the required probability can be estimated as

$P\left(3.10 \leq N\left(3.2258, \frac{10.406}{2000}\right) \leq 3.25\right) = 0.5908.$

5.7.8 The required probability is $P(B(350000, 0.06) \geq 20,800)$.

The normal approximation is

$1 - \Phi\left(\frac{20800-0.5-(350000\times0.06)}{\sqrt{350000\times0.06\times0.94}}\right) = 0.9232.$

5.7.9 (a) The median is $e^{5.5} = 244.7$.

Since $z_{0.25} = 0.6745$ the upper quartile is
$e^{5.5+(2.0\times 0.6745)} = 942.9$.

The lower quartile is
$e^{5.5-(2.0\times 0.6745)} = 63.50$.

(b) $P(X \geq 75000) = 1 - \Phi\left(\frac{\ln(75000)-5.5}{2.0}\right) = 0.0021$

(c) $P(X \leq 1000) = \Phi\left(\frac{\ln(1000)-5.5}{2.0}\right) = 0.7592$

5.7.10 Using the central limit theorem the required probability can be estimated as
$P(N(100 \times 9.2, 100 \times 9.2) \leq 1000) = \Phi(2.638) = 0.9958$.

5.7.11 If the variables are measured in minutes after 2pm, the probability of making the connection is
$P(X_1 + 30 - X_2 \leq 0)$
where $X_1 \sim N(47, 11^2)$ and $X_2 \sim N(95, 3^2)$.

This probability is
$P(N(47 + 30 - 95, 11^2 + 3^2) \leq 0) = \Phi(1.579) = 0.9428$.

5.7.12 The normal approximation is
$P(N(80 \times 0.25, 80 \times 0.25 \times 0.75) \geq 25 - 0.5)$
$= 1 - \Phi(1.162) = 0.1226$.

If physician D leaves the clinic, then the normal approximation is
$P(N(80 \times 0.3333, 80 \times 0.3333 \times 0.6667) \geq 25 - 0.5)$
$= 1 - \Phi(-0.514) = 0.6963$.

5.7.13 (a) $P(B(235, 0.9) \geq 221)$
$\simeq P(N(235 \times 0.9, 235 \times 0.9 \times 0.1) \geq 221 - 0.5)$
$= 1 - \Phi(1.957) = 0.025$

(b) If n passengers are booked on the flight, it is required that
$P(B(n, 0.9) \geq 221)$
$\simeq P(N(n \times 0.9, n \times 0.9 \times 0.1) \geq 221 - 0.5) \leq 0.25.$

This is satisfied at $n = 241$ but not at $n = 242$.
Therefore, the airline can book up to 241 passengers on the flight.

5.7.14 (a) $P(0.6 \leq N(0, 1) \leq 2.2)$
$= \Phi(2.2) - \Phi(0.6)$
$= 0.9861 - 0.7257 = 0.2604$

(b) $P(3.5 \leq N(4.1, 0.25^2) \leq 4.5)$

$= P\left(\frac{3.5-4.1}{0.25} \leq N(0, 1) \leq \frac{4.5-4.1}{0.25}\right)$

$= \Phi(1.6) - \Phi(-2.4)$

$= 0.9452 - 0.0082 = 0.9370$

(c) Since $\chi^2_{0.95,28} = 16.928$ and $\chi^2_{0.90,28} = 18.939$ the required probability is $0.95 - 0.90 = 0.05$.

(d) Since $t_{0.05,22} = 1.717$ and $t_{0.005,22} = 2.819$ the required probability is $(1 - 0.005) - 0.05 = 0.945$.

5.7.15 $P(X \geq 25) = 1 - \Phi\left(\frac{\ln(25)-3.1}{0.1}\right)$

$= 1 - \Phi(1.189) = 0.117$

$P(B(200, 0.117) \geq 30)$

$\simeq P(N(200 \times 0.117, 200 \times 0.117 \times 0.883) \geq 29.5)$

$= P\left(N(0, 1) \geq \frac{29.5-23.4}{\sqrt{20.66}}\right)$

$= 1 - \Phi(1.342) = 0.090$

5.7.16 (a) True

(b) True

(c) True

(d) True

(e) True

5.7.17 $P(B(400, 0.2) \geq 90)$

$\simeq P(N(400 \times 0.2, 400 \times 0.2 \times 0.8) \geq 89.5)$

$= P\left(N(0,1) \geq \frac{89.5-80}{\sqrt{64}}\right)$

$= 1 - \Phi(1.1875) = 0.118$

5.7.18 (a) The probability that an expression is larger than 0.800 is

$P(N(0.768, 0.083^2) \geq 0.80) = P\left(N(0,1) \geq \frac{0.80-0.768}{0.083}\right)$

$= 1 - \Phi(0.386) = 0.350$

If Y measures the number of samples out of six that have an expression larger than 0.80, then Y has a binomial distribution with $n = 6$ and $p = 0.350$.

$P(Y \geq 3) = 1 - P(Y < 3)$

$= 1 - \left(\binom{6}{0} \times (0.35)^0 \times (0.65)^6 + \binom{6}{1} \times (0.35)^1 \times (0.65)^5\right.$

$\left. + \binom{6}{2} \times (0.35)^2 \times (0.65)^4\right)$

$= 0.353$

(b) Let Y_1 be the number of samples that have an expression smaller than 0.70, let Y_2 be the number of samples that have an expression between 0.70 and 0.75, let Y_3 be the number of samples that have an expression between 0.75 and 0.78, and let Y_4 be the number of samples that have an expression larger than 0.78.

$P(X_i \leq 0.7) = \Phi(-0.819) = 0.206$
$P(0.7 \leq X_i \leq 0.75) = \Phi(-0.217) - \Phi(-0.819) = 0.414 - 0.206 = 0.208$
$P(0.75 \leq X_i \leq 0.78) = \Phi(0.145) - \Phi(-0.217) = 0.557 - 0.414 = 0.143$
$P(X_i \geq 0.78) = 1 - \Phi(0.145) = 1 - 0.557 = 0.443$

$P(Y_1 = 2, Y_2 = 2, Y_3 = 0, Y_4 = 2)$

$= \frac{6!}{2! \times 2! \times 2! \times 0!} \times 0.206^2 \times 0.208^2 \times 0.143^0 \times 0.443^2$

$= 0.032$

(c) A negative binomial distribution can be used with $r = 3$ and
$p = P(X \leq 0.76) = \Phi(-0.096) = 0.462.$

The required probability is

$$P(Y = 6) = \binom{5}{3} \times (1 - 0.462)^3 \times 0.462^3 = 0.154.$$

(d) A geometric distribution can be used with
$p = P(X \leq 0.68) = \Phi(-1.060) = 0.145.$

The required probability is
$P(Y = 5) = (1 - 0.145)^4 \times 0.145 = 0.077.$

(e) Using the hypergeometric distribution the required probability is

$$\frac{\binom{5}{3} \times \binom{5}{3}}{\binom{10}{6}} = 0.476.$$

5.7.19 (a) $P(X \leq 8000) = \Phi\left(\frac{8000-8200}{350}\right)$

$= \Phi(-0.571) = 0.284$

$P(8000 \leq X \leq 8300) = \Phi\left(\frac{8300-8200}{350}\right) - \Phi\left(\frac{8000-8200}{350}\right)$

$= \Phi(0.286) - \Phi(-0.571) = 0.330$

$P(X \geq 8300) = 1 - \Phi\left(\frac{8300-8200}{350}\right)$

$= 1 - \Phi(0.286) = 0.386$

Using the multinomial distribution the required probability is

$\frac{3!}{1!\times 1!\times 1!} \times 0.284^1 \times 0.330^1 \times 0.386^1 = 0.217.$

(b) $P(X \leq 7900) = \Phi\left(\frac{7900-8200}{350}\right) = \Phi(-0.857) = 0.195$

Using the negative binomial distribution the required probability is

$\binom{5}{1} \times (1 - 0.195)^4 \times 0.195^2 = 0.080.$

(c) $P(X \geq 8500) = 1 - \Phi\left(\frac{8500-8200}{350}\right) = 1 - \Phi(0.857) = 0.195$

Using the binomial distribution the required probability is

$\binom{7}{3} \times (0.195)^3 \times (1 - 0.195)^4 = 0.109.$

5.7.20 $0.90 = P(X_A \leq X_B)$

$= P(N(220, 11^2) \leq N(t + 185, 9^2))$

$= P(N(220 - t - 185, 11^2 + 9^2) \leq 0)$

$= P\left(N(0,1) \leq \frac{t-35}{\sqrt{202}}\right)$

Therefore,

$\frac{t-35}{\sqrt{202}} = z_{0.10} = 1.282$

so that $t = 53.22$.

Consequently, operator B started working at 9:53 am.

5.7.21 (a) $P(X \leq 30) = 1 - e^{-(0.03\times 30)^{0.8}} = 0.601$

Using the binomial distribution the required probability is

$\binom{5}{2} \times 0.601^2 \times (1 - 0.601)^3 = 0.23.$

(b) $P(B(500, 0.399) \leq 210)$

$\simeq P(N(500 \times 0.399, 500 \times 0.399 \times 0.601) \leq 210.5)$

$= P\left(N(0,1) \leq \frac{210.5-199.5}{10.95}\right)$

$= \Phi(1.005) = 0.843$

5.7.22 $P(N(3 \times 45.3, 3 \times 0.02^2) \leq 135.975)$

$= P\left(N(0,1) \leq \frac{135.975-135.9}{\sqrt{3}\times 0.02}\right)$

$= \Phi(2.165) = 0.985$

5.7.23 $P(X_A - X_{B1} - X_{B2} \geq 0)$

$= P(N(67.2, 1.9^2) - N(33.2, 1.1^2) - N(33.2, 1.1^2) \geq 0)$

$= P(N(67.2 - 33.2 - 33.2, 1.9^2 + 1.1^2 + 1.1^2) \geq 0)$

$= P(N(0.8, 6.03) \geq 0)$

$= P\left(N(0,1) \geq \frac{-0.8}{\sqrt{6.03}}\right)$

$= 1 - \Phi(-0.326) = 0.628$

5.7.24 $P(X \geq 25) = e^{-25/32} = 0.458$

$P(B(240, 0.458) \geq 120)$

$\simeq P\left(N(240 \times 0.458, 240 \times 0.458 \times 0.542\right) \geq 119.5)$

$= P\left(N(0,1) \geq \frac{119.5 - 109.9}{\sqrt{59.57}}\right)$

$= 1 - \Phi(1.24) = 0.108$

5.7.25 (a) $P(N(55980, 10^2) \geq N(55985, 9^2))$

$= P(N(55980 - 55985, 10^2 + 9^2) \geq 0)$

$= P(N(-5, 181) \geq 0)$

$= P\left(N(0,1) \geq \frac{5}{\sqrt{181}}\right)$

$= 1 - \Phi(0.372) = 0.355$

(b) $P(N(55980, 10^2) \leq N(56000, 10^2))$

$= P(N(55980 - 56000, 10^2 + 10^2) \leq 0)$

$= P(N(-20, 200) \leq 0)$

$= P\left(N(0,1) \leq \frac{20}{\sqrt{200}}\right)$

$= \Phi(1.414) = 0.921$

(c) $P(N(56000, 10^2) \leq 55995) \times P(N(56005, 8^2) \leq 55995)$

$= P\left(N(0,1) \leq \frac{55995-56000}{10}\right) \times P\left(N(0,1) \leq \frac{55995-56005}{8}\right)$

$= \Phi(-0.5) \times \Phi(-1.25)$

$= 0.3085 \times 0.1056 = 0.033$

5.7.26 (a) $t_{0.10,40} = 1.303$ and $t_{0.025,40} = 2.021$ so that
$P(-1.303 \leq t_{40} \leq 2.021) = 0.975 - 0.10 = 0.875$

(b) $P(t_{17} \geq 2.7) = 0.008$

5.7.27 (a) $P(F_{16,20} \leq 2) = 0.928$

(b) $P(\chi^2_{28} \geq 47) = 0.014$

(c) $P(t_{29} \geq 1.5) = 0.072$

(d) $P(t_7 \leq -1.3) = 0.117$

(e) $P(t_{10} \geq -2) = 0.963$

5.7.28 (a) $P(\chi^2_{40} > 65.0) = 0.007$

(b) $P(t_{20} < -1.2) = 0.122$

(c) $P(t_{26} < 3.0) = 0.997$

(d) $P(F_{8,14} > 4.8) = 0.0053.$