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Homework Introduction to Hypothesis Testing

1. Write the null and alternative hypotheses in words and then symbols for each of the following situations.

   (a) New York is known as "the city that never sleeps". A random sample of 25 New Yorkers were asked how much sleep they get per night. Do these data provide convincing evidence that New Yorkers on average sleep less than 8 hours a night?

   $H_0: \mu = 8$ hours a night

   $H_A: \mu < 8$ hours a night.

   (b) Since 2008, chain restaurants in California have been required to display calorie counts of each menu item. Prior to menus displaying calorie counts, the average calorie intake of diners at a restaurant was 1100 calories. After calorie counts started to be displayed on menus, a nutritionist collected data on the number of calories consumed at this restaurant from a random sample of diners. Do these data provide convincing evidence of a *difference* in the average calorie intake of a diners at this restaurant?

   $H_0: \mu = 1100$

   $H_A: \mu \neq 1100$

   (c) Employers at a firm are worried about the effect of March Madness, a basketball championship held each spring in the US, on employee productivity. They estimate that on a regular business day employees spend on average 15 minutes of company time checking personal email, making personal phone calls, etc. (So normally the average time NOT working is 15 minutes.) They also collect data on how much company time employees spend on such non- business activities during March Madness. They want to determine if these data provide convincing evidence that employee productivity decreases during March Madness.

   $H_0: \mu = 15$

   $H_A: \mu > 15$

   (d) Based on the performance of those who took the GRE exam between July 1, 2004 and June 30, 2007, the average Verbal Reasoning score was calculated to be 462. In 2011 the average verbal score was slightly higher. Do these data provide convincing evidence that the average GRE Verbal Reasoning score has changed since 2004?

   $H_0: \mu = 462$

   $H_A: \mu \neq 462$

2. The _______ the p-value, the stronger the evidence against the null hypothesis provided by the data.

   (a) smaller

   (b) bigger

3. If the p-value is smaller than α, we say our results are ______ .

   (a) practically significant

   (b) statistically significant

   (c) absolutely significant

4. The square footage of several thousands apartments in a new development is advertised to be 1250 square feet. A tenant group thinks the apartments are smaller than advertised. They measure a sample of apartments. Let μ represent the true average area of all the apartments. What are the appropriate null and alternative hypotheses?

   $H_0: \mu = 1250$

   $H_A: \mu < 1250$

5. Let μ represent the population mean height of all adult American males between 18 and 21. What are the appropriate null and alternative hypotheses to answer "Is the mean height for all adult American males between 18 and 21 now over 6 feet?".

   $H_0: \mu = 6$

   $H_A: \mu > 6$

6. What does the p-value tell us?

   (a) the probability the null hypotheses is true
   (b) the probability the null hypotheses is false
   (c) the probability the alternative hypothesis is true
   (d) the probability the alternative hypothesis is false
   (e) the probability of getting our sample data, or something more extreme, IF the null hypothesis is true

7. If the p-value is smaller than α, you should

   (a) reject the null hypothesis
   (b) fail to reject the null hypothesis
   (c) reject the alternative hypothesis
   (d) fail to reject the alternative hypothesis

8. A study suggests that the average college student spends 10 hours per week communicating with others online. You believe that this is an underestimate and decide to collect your own sample for a hypothesis test. You randomly sample 60 students from your dorm and find that on average they spent 13.5 hours a week communicating with others online. A friend of yours, who offers to help you with the hypothesis test, comes up with the following set of hypotheses. Indicate any errors you see.

$$H_0 : \cancel{\bar{x} \leq 10\ hours} \longrightarrow \mu = 10$$
$$\cancel{H_A : \bar{x} > 13.5\ hours} \quad \mu > 10$$

9. Which of the following statements about a p-value is true?

   (a) The p-value is the probability that the null hypothesis is rejected even if the null hypothesis is actually true.
   (b) The p-value tells us the strength of the evidence against the null hypothesis. The smaller the p-value, the stronger the evidence against the null hypothesis.

10. What is the level of significance?

α tells you how much evidence you need to reject null hypothesis

11. What is a type I error?

Reject a true null hypothesis

12. What is a type II error?

fail to reject a false Ho

13. Above, we wanted to see if American males are over 6 feet. What would be a type I error in this situation? What would be a type II error?

mean is 6 feet, reject that

mean is not 6 feet, no evidence that height it is true.

14. A report indicated that automobiles manufactured in North America are less fuel efficient than automobiles manufactured in Asia. It is known that automobiles from Asia have a mean fuel efficiency of 22 miles per gallon. To determine if there is evidence to support this claim, a random sample of automobiles manufactured in North American is to be selected and their fuel efficiency determined. The appropriate hypotheses to be tested are:

(a) $H_0 : \mu = 22$ versus $H_a : \mu > 22$
(b) $H_0 : \mu = 22$ versus $H_a : \mu \neq 22$
(c) $H_0 : \overline{X} = 22$ versus $H_a : \overline{X} \neq 22$
(d) $H_0 : \mu = 22$ versus $H_a : \mu < 22$
(e) $H_0 : \overline{X} = 22$ versus $H_a : \overline{X} < 22$

15. The results of a 2006 - 2010 survey showed that the average age of women at first marriage is 23.44. Suppose a social scientist believes that this value has increased in 2012, but she would also be interested if she found a decrease. Below is how she set up her hypotheses. Indicate any errors you see.

$H_0 : \cancel{\overline{x} =} 23.44$ *years old* $\mu = 23.44$

$H_A : \overline{x} > 23.44$ *years old* $\mu \neq 23.44$

Homework Z Test for Mean

Instructions: For each problem, do the following for every hypothesis test. Then do any additional parts.

- **Check the conditions.**
- **Pick α.**
- **State the null hypothesis H_0 and the alternative hypothesis H_a.**
- **Compute the value of the test statistic.**
- **Calculate the P-value.**
- **Reject H_0 if the P-value is less than α.**
- **Interpret your statistical results in real world terms.**
- ****Interpret the p-value.**

****Remember if you are using a calculator you might get slightly different answers than using a table.**

16) normal ✓

• $\alpha = .1$

• $H_0: \mu = 330$

$H_A: \mu \neq 330$

• test stat:

$z = \frac{326-330}{40/\sqrt{10}} = -.32$

• p-val: $.3745 + .3745 = .7490$

• fail to reject H_0

• we didn't find enough evidence that the population mean candy sales is $\neq 330$.

16. A candy company wants to see how many sales they need to prepare for each store. Assume that the population standard deviation is 40 boxes and the candy sales are normally distributed. They take a sample of 10 stores and find a sample mean of 326 boxes per store. Assess whether this year's sales of valentine chocolate will be different than a mean of 330 boxes per store at a significance level of $\alpha = .10$. Make sure you follow all the steps.

17) $64 \geq 30$ ✓ normal

• $\alpha = .01$

• $H_0: \mu = 127$

$H_A: \mu > 127$

• test stat:

$z = \frac{157.5-127}{39/\sqrt{64}} = 6.26$

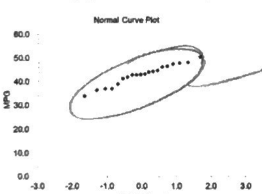
• pval: $1 - \Phi(6.26) = 0$

• reject H_0

• we found extremely strong evidence that the mean wait time for all patients is greater than 127 mins.

17. A hospital administrator randomly selected 64 patients and measured the time (in minutes) between when they checked in to the ER and the time they were first seen by a doctor. The average time is 157.5 minutes. She thinks the population standard deviation is 39 minutes. She is getting grief from her supervisor on the basis that the wait times in the ER has increased greatly from last year's average of 127 minutes. However, she claims that the increase is probably just due to chance. Conduct a hypothesis test to see if the wait times have increased or if the sample mean of 157.5 being higher than 127 is just due to chance and which sample she chose. Use $\alpha = .01$.

18. A random sample of gas mileages was taken for a certain model of vehicle. We want to know if the population mean gas mileage is different from the advertised 44 mpg. We believe that the population standard deviation is 4.5 mpg. The sample size is $n = 20$ and the sample mean is $\overline{x} = 43.2$.



• normal,

• $\alpha = .05$

• $H_0: \mu = 44$

$H_A: \mu \neq 44$

• test stat:

$z = \frac{43.2-44}{4.5/\sqrt{20}} = -.795$

• p-val: $.2133 + .2133 = .4266$

• Fail to reject null hypothesis.

(a) Conduct a hypothesis test with $\alpha = .05$.

we didn't find evidence that the mean pop. gas milage is different from 44.

(b) Calculate a 95% confidence interval for the population mean gas mileage.

$43.2 \pm 1.96\left(\frac{4.5}{\sqrt{20}}\right) \Rightarrow (41.2, 45.2)$

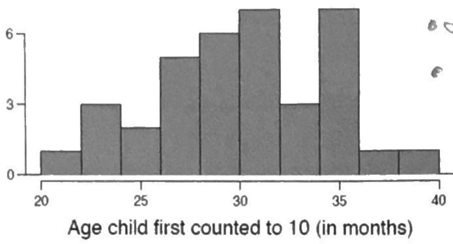
(c) Interpret the confidence interval.

we are 95% confident that the average mpg is between 41.2 − 45.2 mpg

(d) Forget you did the hypothesis test. Based on your confidence interval, do you think 44mpg is a plausible value for the population mean?

yes, it's in range.

19. Researchers investigating characteristics of gifted children collected data from schools in a large city on a random sample of thirty-six children who were identified as gifted children soon after they reached the age of four. The following histogram shows the distribution of the ages (in months) at which these children first counted to 10 successfully. The sample size was 36 and the sample mean is 30.69. They believe the *population* standard deviation is 4.3.



• 36 ≥ 30 ✓

• $\alpha = .1$

• $H_0: \mu = 32$

$H_A: \mu < 32$

• test stat:

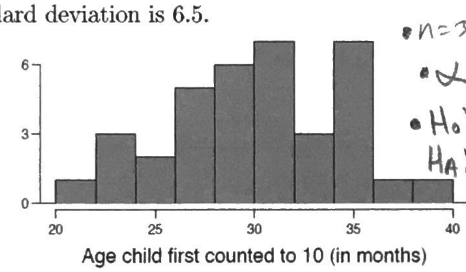
$$Z = \frac{30.69 - 32}{4.3/\sqrt{36}} = -1.83$$

• pval: .0336

we found evidence that the average age to count to 10 for all gifted kids is less than 32 months.

Suppose you read online that children first count to 10 successfully when they are 32 months old, on average. Perform a hypothesis test to evaluate if these data provide convincing evidence that the average age at which gifted children first count to 10 successfully is less than the general average of 32 months. Use a significance level of 0.10.

20. In the gifted child study, along with variables on the children, the researchers also collected data on the mother's and father's IQ of the 36 randomly sampled gifted children. The histogram below shows the distribution of mother's IQ. The sample mean of the mothers' IQ is 118.2. They believe the population standard deviation is 6.5.



• n = 36 ≥ 30 ✓

• $\alpha = .05$

• $H_0: \mu = 100$

$H_A: \mu > 100$

• test stat:

$$Z = \frac{118.2 - 100}{6.5/\sqrt{36}} = 16.8$$

• pvalue = 0

reject null.

we found evidence that the mean IQ for mothers is higher than 100.

Perform a hypothesis test to evaluate if these data provide convincing evidence that the average IQ of mothers of gifted children is greater than the IQ of the general population which is 100.

(Interesting thought, perhaps the gifted children are gifted because their mother are smarter than average. However, this study isn't set up to prove such a hypothesis. All we are showing here is whether or not the gifted children's mothers have a higher IQ than normal.)

21. The nutrition label on a bag of potato chips says that a 28 gram serving of potato chips has 130 calories and contains ten grams of fat, with three grams of saturated fat.
A random sample of 35 bags yielded a sample mean of 134 calories. From past experience, we believe the *population* standard deviation is 17 calories. Is there evidence that the nutrition label does not provide an accurate measure of calories in the bags of potato chips?

22. A patient named Diana was diagnosed with Fibromyalgia, a long-term syndrome of body pain, and was prescribed anti-depressants. If she wanted to test if the anti-depressants helped, her what would be the hypotheses?

(a) Write the hypotheses in words. Ho: Anti depressants don't do anything

HA: Anti depressants work

(b) What is a Type 1 Error in this context?

Anti depressants don't work, but we find they do

(c) What is a Type 2 Error in this context?

Anti depressants don't work but we find they do.

Acceptance and Rejection Regions

There are two ways to do hypothesis tests.

- p-value approach: find the p-value and if it is smaller than α reject the null hypothesis
- acceptance/rejection regions
  - figure out what values of Z you would accept/reject the null hypothesis
  - calculate the test statistic z
  - compare it to the acceptance/rejection regions
  - reject or fail to reject the null hypothesis

Both methods are valid approaches. The acceptance/rejection region approach is the old approach when we had to use tables.

Now that we have calculators we much prefer the p-value approach because it gives us much more information.

In practice, the p-value approach would always be better.

But there are quite a few classes that still ask you to do the acceptance/rejection region approach because that's the way they learned.

Let's practice the acceptance/rejection region approach on a few problems you already did.

https://youtu.be/yInBFv-juJQ

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23. A hospital administrator randomly selected 64 patients and measured the time (in minutes) between when they checked in to the ER and the time they were first seen by a doctor. The average time is 157.5 minutes. She thinks the population standard deviation is 39 minutes. She is getting grief from her supervisor on the basis that the wait times in the ER has increased greatly from last year's average of 127 minutes. However, she claims that the increase is probably just due to chance. Conduct a hypothesis test to see if the wait times have increased or if the sample mean of 157.5 being higher than 127 is just due to chance and which sample she chose. Use $\alpha = .01$.
    (a) You would check the assumptions, write the hypotheses, and pick α just like any problem.
    (b) For what z values will you reject the null hypothesis? (Hint: what z values give you .01 are in the right?)
    (c) For what z values will you fail to reject the null hypothesis?
    (d) Remember she got a sample mean of $\overline{x} = 157.5$. Calculate the test statistic.
    (e) Should you reject or fail to reject the null hypothesis?
    (f) Now interpret the results as normal.

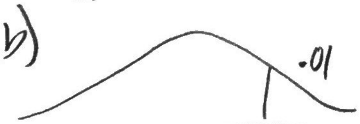
a) $64 \geq 30$ ✓

$\alpha = .01$

$H_0: \mu = 127$

$H_A: \mu > 127$

b)



$z = 2.326$

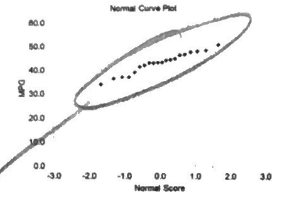
c) fail to reject if $z < 2.326$

d) $z = \dfrac{157.5 - 127}{39/\sqrt{64}} = 6.26$

e) fail to reject

f) we found evidence that pop mean wait time for all patients is greater than 127 mins

24. A random sample of gas mileages was taken for a certain model of vehicle. We want to know if the population mean gas mileage is different from the advertised 44 mpg. We believe that the population standard deviation is 4.5 mpg. The sample size is $n = 20$ and the sample mean is $\overline{x} = 43.2$. Use $\alpha = .05$.



(a) You would check the assumptions, write the hypotheses, and pick α just like any problem.

(b) For what z values will you reject the null hypothesis? (Hint: what z values give you a total of .05 area in both tails?)

(c) For what z values will you fail to reject the null hypothesis?

(d) Remember she got a sample mean of $\overline{x} = 157.5$. Calculate the test statistic.

(e) Should you reject or fail to reject the null hypothesis?

(f) Now interpret the results as normal.

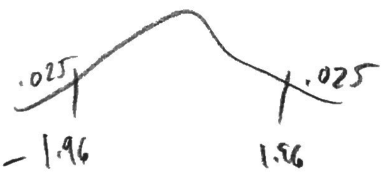
a) normalish ✓

$\alpha = .05$

$H_0: \mu = 44$

$H_A: \mu \neq 44$

b) $\frac{.05}{2} = .025$



c) if $-1.96 \leq z \leq 1.96$

d) $z = \frac{43.2 - 44}{4.5/\sqrt{20}} = -.795$

e) Fail to reject

f) we didn't find evidence that the population mean gas milage is different from 44 mpg.