3.1.1 (a) $P(X = 3) = \binom{10}{3} \times 0.12^3 \times 0.88^7 = 0.0847$

(b) $P(X = 6) = \binom{10}{6} \times 0.12^6 \times 0.88^4 = 0.0004$

(c) $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$
$= 0.2785 + 0.3798 + 0.2330$
$= 0.8913$

(d) $P(X \geq 7) = P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)$
$= 3.085 \times 10^{-5}$

(e) $E(X) = 10 \times 0.12 = 1.2$

(f) $\mathrm{Var}(X) = 10 \times 0.12 \times 0.88 = 1.056$

3.1.2 (a) $P(X = 4) = \binom{7}{4} \times 0.8^4 \times 0.2^3 = 0.1147$

(b) $P(X \neq 2) = 1 - P(X = 2)$

$= 1 - \binom{7}{2} \times 0.8^2 \times 0.2^5$

$= 0.9957$

(c) $P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 0.0334$

(d) $P(X \geq 6) = P(X = 6) + P(X = 7) = 0.5767$

(e) $E(X) = 7 \times 0.8 = 5.6$

(f) $\mathrm{Var}(X) = 7 \times 0.8 \times 0.2 = 1.12$

3.1.3 $X \sim B(6, 0.5)$

x_i	0	1	2	3	4	5	6
p_i	0.0156	0.0937	0.2344	0.3125	0.2344	0.0937	0.0156

$E(X) = 6 \times 0.5 = 3$

$\text{Var}(X) = 6 \times 0.5 \times 0.5 = 1.5$

$\sigma = \sqrt{1.5} = 1.22$

$X \sim B(6, 0.7)$

x_i	0	1	2	3	4	5	6
p_i	0.0007	0.0102	0.0595	0.1852	0.3241	0.3025	0.1176

$E(X) = 6 \times 0.7 = 4.2$

$\text{Var}(X) = 6 \times 0.7 \times 0.3 = 1.26$

$\sigma = \sqrt{1.5} = 1.12$

3.1.4 $X \sim B(9, 0.09)$

(a) $P(X = 2) = 0.1507$

(b) $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 0.1912$

$E(X) = 9 \times 0.09 = 0.81$

3.1.5 (a) $P\left(B\left(8, \frac{1}{2}\right) = 5\right) = 0.2187$

(b) $P\left(B\left(8, \frac{1}{6}\right) = 1\right) = 0.3721$

(c) $P\left(B\left(8, \frac{1}{6}\right) = 0\right) = 0.2326$

3.1.6 $P(B(10,0.2) \geq 7) = 0.0009$

$P(B(10,0.5) \geq 7) = 0.1719$

3.1.7 Let the random variable X be the number of employees taking sick leave.

Then $X \sim B(180, 0.35)$.

Therefore, the *proportion* of the workforce who need to take sick leave is

$Y = \frac{X}{180}$

so that

$E(Y) = \frac{E(X)}{180} = \frac{180 \times 0.35}{180} = 0.35$

and

$\text{Var}(Y) = \frac{\text{Var}(X)}{180^2} = \frac{180 \times 0.35 \times 0.65}{180^2} = 0.0013.$

In general, the variance is

$\text{Var}(Y) = \frac{\text{Var}(X)}{180^2} = \frac{180 \times p \times (1-p)}{180^2} = \frac{p \times (1-p)}{180}$

which is maximized when $p = 0.5$.

3.1.8 The random variable Y can be considered to be the number of successes out of $n_1 + n_2$ trials.

3.1.9 $X \sim B(18, 0.6)$

(a) $P(X = 8) + P(X = 9) + P(X = 10)$

$$= \binom{18}{8} \times 0.6^8 \times 0.4^{10} + \binom{18}{9} \times 0.6^9 \times 0.4^9 + \binom{18}{10} \times 0.6^{10} \times 0.4^8$$

$= 0.0771 + 0.1284 + 0.1734 = 0.3789$

(b) $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$$= \binom{18}{0} \times 0.6^0 \times 0.4^{18} + \binom{18}{1} \times 0.6^1 \times 0.4^{17} + \binom{18}{2} \times 0.6^2 \times 0.4^{16}$$

$$+ \binom{18}{3} \times 0.6^3 \times 0.4^{15} + \binom{18}{4} \times 0.6^4 \times 0.4^{14}$$

$= 0.0013$

3.1.10 A

3.1.11 $P(B(10, 0.65) \geq 5) = 0.905$

3.1.12 0.9532

3.2.1 (a) $P(X = 4) = (1 - 0.7)^3 \times 0.7 = 0.0189$

(b) $P(X = 1) = (1 - 0.7)^0 \times 0.7 = 0.7$

(c) $P(X \leq 5) = 1 - (1 - 0.7)^5 = 0.9976$

(d) $P(X \geq 8) = 1 - P(X \leq 7) = (1 - 0.7)^7 = 0.0002$

3.2.2 (a) $P(X = 5) = \binom{4}{2} \times (1 - 0.6)^2 \times 0.6^3 = 0.2074$

(b) $P(X = 8) = \binom{7}{2} \times (1 - 0.6)^5 \times 0.6^3 = 0.0464$

(c) $P(X \leq 7) = P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7)$
$= 0.9037$

(d) $P(X \geq 7) = 1 - P(X = 3) - P(X = 4) - P(X = 5) - P(X = 6)$
$= 0.1792$

3.2.4 Notice that a negative binomial distribution with parameters p and r can be thought of as the number of trials up to and including the r^{th} success in a sequence of independent Bernoulli trials with a constant success probability p, which can be considered to be the number of trials up to and including the first success, plus the number of trials after the first success and up to and including the second success, plus the number of trials after the second success and up to and including the third success, and so on. Each of these r components has a geometric distribution with parameter p.

3.2.5 (a) Consider a geometric distribution with parameter $p = 0.09$.
$(1 - 0.09)^3 \times 0.09 = 0.0678$

(b) Consider a negative binomial distribution with parameters $p = 0.09$ and $r = 3$.
$\binom{9}{2} \times (1 - 0.09)^7 \times 0.09^3 = 0.0136$

(c) $\frac{1}{0.09} = 11.11$

(d) $\frac{3}{0.09} = 33.33$

3.2.6 (a) $\frac{1}{0.37} = 2.703$

(b) $\frac{3}{0.37} = 8.108$

(c) The required probability is
$P(X \leq 10) = 0.7794$
where the random variable X has a negative binomial distribution with parameters $p = 0.37$ and $r = 3$.

Alternatively, the required probability is
$P(Y \geq 3) = 0.7794$
where the random variable Y has a binomial distribution with parameters
$n = 10$ and $p = 0.37$.

(d) $P(X = 10) = \binom{9}{2} \times (1 - 0.37)^7 \times 0.37^3 = 0.0718$

3.2.7 (a) Consider a geometric distribution with parameter $p = 0.25$.
$(1 - 0.25)^2 \times 0.25 = 0.1406$

(b) Consider a negative binomial distribution with parameters $p = 0.25$ and $r = 4$.
$\binom{9}{3} \times (1 - 0.25)^6 \times 0.25^4 = 0.0584$

The expected number of cards drawn before the fourth heart is obtained is the expectation of a negative binomial distribution with parameters $p = 0.25$ and $r = 4$, which is $\frac{4}{0.25} = 16$.

If the first two cards are spades then the probability that the first heart card is obtained on the fifth drawing is the same as the probability in part (a).

3.2.8 (a) $\frac{1}{0.77} = 1.299$

(b) Consider a geometric distribution with parameter $p = 0.23$.
$(1 - 0.23)^4 \times 0.23 = 0.0809$

(c) Consider a negative binomial distribution with parameters $p = 0.77$ and $r = 3$.
$\binom{5}{2} \times (1 - 0.77)^3 \times 0.77^3 = 0.0555$

(d) $P(B(8, 0.77) \geq 3) = 0.9973$

3.2.9 (a) Consider a geometric distribution with parameter $p = 0.6$.
$P(X = 5) = (1 - 0.6)^4 \times 0.6 = 0.01536$

(b) Consider a negative binomial distribution with parameters $p = 0.6$ and $r = 4$.
$P(X = 8) = \binom{7}{3} \times 0.6^4 \times 0.4^4 = 0.116$

3.2.10 $E(X) = \frac{r}{p} = \frac{3}{1/6} = 18$

3.2.11 $P(X = 10) = \binom{9}{4} \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^5 = 0.123$

3.2.12 C

3.2.13 $\frac{5}{0.65} = 7.69$

3.2.14 $\frac{10}{1-0.93} = 142.9$

3.3.1 (a) $P(X=4) = \frac{\binom{6}{4} \times \binom{5}{3}}{\binom{11}{7}} = \frac{5}{11}$

(b) $P(X=5) = \frac{\binom{6}{5} \times \binom{5}{2}}{\binom{11}{7}} = \frac{2}{11}$

(c) $P(X \leq 3) = P(X=2) + P(X=3) = \frac{23}{66}$

3.3.2

x_i	0	1	2	3	4	5
p_i	$\frac{3}{429}$	$\frac{40}{429}$	$\frac{140}{429}$	$\frac{168}{429}$	$\frac{70}{429}$	$\frac{8}{429}$

3.3.3 (a) $\frac{\binom{10}{3} \times \binom{7}{2}}{\binom{17}{5}} = \frac{90}{221}$

(b) $\frac{\binom{10}{1} \times \binom{7}{4}}{\binom{17}{5}} = \frac{25}{442}$

(c) $P(\text{no red balls}) + P(\text{one red ball}) + P(\text{two red balls}) = \frac{139}{442}$

3.3.4 $$\frac{\binom{16}{5}\times\binom{18}{7}}{\binom{34}{12}} = 0.2535$$

$$P\left(B\left(12, \tfrac{18}{34}\right) = 7\right) = 0.2131$$

3.3.5 $$\frac{\binom{12}{3}\times\binom{40}{2}}{\binom{52}{5}} = \tfrac{55}{833}$$

The number of picture cards X in a hand of 13 cards has a hypergeometric distribution with $N = 52$, $n = 13$, and $r = 12$. The expected value is

$E(X) = \frac{13\times 12}{52} = 3$

and the variance is

$\mathrm{Var}(X) = \left(\frac{52-13}{52-1}\right) \times 13 \times \frac{12}{52} \times \left(1 - \frac{12}{52}\right) = \frac{30}{17}$.

3.3.6 $$\frac{\binom{4}{1}\times\binom{5}{2}\times\binom{6}{2}}{\binom{15}{5}} = \tfrac{200}{1001}$$

3.3.7 (a) $$\frac{\binom{7}{3}\times\binom{4}{0}}{\binom{11}{3}} = \tfrac{7}{33}$$

(b) $$\frac{\binom{7}{1}\times\binom{4}{2}}{\binom{11}{3}} = \tfrac{14}{55}$$

3.3.8 $P(5 \leq X \leq 7) = P(X = 5) + P(X = 6) + P(X = 7)$

$$= \frac{\binom{9}{4} \times \binom{6}{1}}{\binom{15}{5}} + \frac{\binom{9}{3} \times \binom{6}{2}}{\binom{15}{5}} + \frac{\binom{9}{2} \times \binom{6}{3}}{\binom{15}{5}}$$

$= 0.911$

3.3.9 (a) $\frac{\binom{8}{2} \times \binom{8}{2}}{\binom{16}{4}} = \frac{28}{65} = 0.431$

(b) $P\left(B\left(4, \frac{1}{2}\right) = 2\right) = \binom{4}{2} \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^2 = \frac{3}{8} = 0.375$

3.3.10 $\frac{\binom{19}{4} \times \binom{6}{1}}{\binom{25}{5}} + \frac{\binom{19}{5} \times \binom{6}{0}}{\binom{25}{5}} = 0.4377 + 0.2189 = 0.6566$

3.3.11 $P(2) = \frac{\binom{7}{2} \times \binom{3}{2}}{\binom{10}{4}} = \frac{3}{10}$

$P(3) = \frac{\binom{7}{1} \times \binom{3}{3}}{\binom{10}{4}} = \frac{1}{30}$

$P(2) + P(3) = \frac{3}{10} + \frac{1}{30} = \frac{1}{3}$

3.4.1 (a) $P(X = 1) = \frac{e^{-3.2} \times 3.2^1}{1!} = 0.1304$

(b) $P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 0.6025$

(c) $P(X \geq 6) = 1 - P(X \leq 5) = 0.1054$

(d) $P(X = 0 | X \leq 3) = \frac{P(X=0)}{P(X \leq 3)} = \frac{0.0408}{0.6025} = 0.0677$

3.4.2 (a) $P(X=0) = \frac{e^{-2.1} \times 2.1^0}{0!} = 0.1225$

(b) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = 0.6496$

(c) $P(X \geq 5) = 1 - P(X \leq 4) = 0.0621$

(d) $P(X=1 | X \leq 2) = \frac{P(X=1)}{P(X \leq 2)} = \frac{0.2572}{0.6496} = 0.3959$

3.4.4 $P(X=0) = \frac{e^{-2.4} \times 2.4^0}{0!} = 0.0907$

$P(X \geq 4) = 1 - P(X \leq 3) = 0.2213$

3.4.5 It is best to use a Poisson distribution with $\lambda = \frac{25}{100} = 0.25$.

$P(X=0) = \frac{e^{-0.25} \times 0.25^0}{0!} = 0.7788$

$P(X \leq 1) = P(X=0) + P(X=1) = 0.9735$

3.4.6 It is best to use a Poisson distribution with $\lambda = 4$.

(a) $P(X=0) = \frac{e^{-4} \times 4^0}{0!} = 0.0183$

(b) $P(X \geq 6) = 1 - P(X \leq 5) = 0.2149$

3.4.7 A $B(500, 0.005)$ distribution can be approximated by a
Poisson distribution with $\lambda = 500 \times 0.005 = 2.5$.
Therefore,
$P(B(500, 0.005) \leq 3)$
$\simeq \frac{e^{-2.5} \times 2.5^0}{0!} + \frac{e^{-2.5} \times 2.5^1}{1!} + \frac{e^{-2.5} \times 2.5^2}{2!} + \frac{e^{-2.5} \times 2.5^3}{3!}$
$= 0.7576$

3.4.8 $X \sim P(9.2)$

(a) $P(X=6)+P(X=7)+P(X=8)+P(X=9)+P(X=10)$

$= \frac{e^{-9.2}\times 9.2^6}{6!} + \frac{e^{-9.2}\times 9.2^7}{7!} + \frac{e^{-9.2}\times 9.2^8}{8!} + \frac{e^{-9.2}\times 9.2^9}{9!} + \frac{e^{-9.2}\times 9.2^{10}}{10!}$

$= 0.0851 + 0.1118 + 0.1286 + 0.1315 + 0.1210$

$= 0.5780$

(b) $P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= \frac{e^{-9.2}\times 9.2^0}{0!} + \frac{e^{-9.2}\times 9.2^1}{1!} + \frac{e^{-9.2}\times 9.2^2}{2!} + \frac{e^{-9.2}\times 9.2^3}{3!} + \frac{e^{-9.2}\times 9.2^4}{4!}$

$= 0.0001 + 0.0009 + 0.0043 + 0.0131 + 0.0302$

$= 0.0486$

3.4.9 B

3.4.10 B

3.5.1 (a) $\frac{11!}{4!\times 5!\times 2!} \times 0.23^4 \times 0.48^5 \times 0.29^2 = 0.0416$

(b) $P(B(7, 0.23) < 3) = 0.7967$

3.5.2 (a) $\frac{15!}{3!\times 3!\times 9!} \times \left(\frac{1}{6}\right)^3 \times \left(\frac{1}{6}\right)^3 \times \left(\frac{2}{3}\right)^9 = 0.0558$

(b) $\frac{15!}{3!\times 3!\times 4!\times 5!} \times \left(\frac{1}{6}\right)^3 \times \left(\frac{1}{6}\right)^3 \times \left(\frac{1}{6}\right)^4 \times \left(\frac{1}{2}\right)^5 = 0.0065$

(c) $\frac{15!}{2!\times 13!} \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^{13} = 0.2726$

The expected number of sixes is $\frac{15}{6} = 2.5$.

3.5.3 (a) $\frac{8!}{2!\times 5!\times 1!} \times 0.09^2 \times 0.79^5 \times 0.12^1 = 0.0502$

(b) $\frac{8!}{1!\times 5!\times 2!} \times 0.09^1 \times 0.79^5 \times 0.12^2 = 0.0670$

(c) $P(B(8, 0.09) \geq 2) = 0.1577$

The expected number of misses is $8 \times 0.12 = 0.96$.

3.5.4 The expected number of dead seedlings is $22 \times 0.08 = 1.76$
the expected number of slow growth seedlings is $22 \times 0.19 = 4.18$
the expected number of medium growth seedlings is $22 \times 0.42 = 9.24$
and the expected number of strong growth seedlings is $22 \times 0.31 = 6.82$.

(a) $\frac{22!}{3!\times4!\times6!\times9!} \times 0.08^3 \times 0.19^4 \times 0.42^6 \times 0.31^9 = 0.0029$

(b) $\frac{22!}{5!\times5!\times5!\times7!} \times 0.08^5 \times 0.19^5 \times 0.42^5 \times 0.31^7 = 0.00038$

(c) $P(B(22, 0.08) \leq 2) = 0.7442$

3.5.5 The probability that an order is received over the internet and it is large is $0.6 \times 0.3 = 0.18$.

The probability that an order is received over the internet and it is small is $0.6 \times 0.7 = 0.42$.

The probability that an order is not received over the internet and it is large is $0.4 \times 0.4 = 0.16$.

The probability that an order is not received over the internet and it is small is $0.4 \times 0.6 = 0.24$.

The answer is $\frac{8!}{2!\times2!\times2!\times2!} \times 0.18^2 \times 0.42^2 \times 0.16^2 \times 0.24^2 = 0.0212$.

3.5.6 $\frac{10!}{2!\times 2!\times 2!\times 2!\times 2!} \times 0.2^2 \times 0.2^2 \times 0.2^2 \times 0.2^2 \times 0.2^2 = 0.0116$

3.8.1 (a) $P(B(18, 0.085) \geq 3) = 1 - P(B(18, 0.085) \leq 2) = 0.1931$

(b) $P(B(18, 0.085) \leq 1) = 0.5401$

(c) $18 \times 0.085 = 1.53$

3.8.2 $P(B(13, 0.4) \geq 3) = 1 - P(B(13, 0.4) \leq 2) = 0.9421$
The expected number of cells is $13 + (13 \times 0.4) = 18.2$.

3.8.3 (a) $\frac{8!}{2!\times 3!\times 3!} \times 0.40^2 \times 0.25^3 \times 0.35^3 = 0.0600$

(b) $\frac{8!}{3!\times 1!\times 4!} \times 0.40^3 \times 0.25^1 \times 0.35^4 = 0.0672$

(c) $P(B(8, 0.35) \leq 2) = 0.4278$

3.8.4 (a) $P(X = 0) = \frac{e^{-2/3}\times(2/3)^0}{0!} = 0.5134$

(b) $P(X = 1) = \frac{e^{-2/3}\times(2/3)^1}{1!} = 0.3423$

(c) $P(X \geq 3) = 1 - P(X \leq 2) = 0.0302$

3.8.5 $P(X = 2) = \frac{e^{-3.3}\times(3.3)^2}{2!} = 0.2008$

$P(X \geq 6) = 1 - P(X \leq 5) = 0.1171$

3.8.6 (a) Consider a negative binomial distribution with parameters $p = 0.55$ and $r = 4$.

(b) $P(X = 7) = \binom{6}{3} \times (1 - 0.55)^3 \times 0.55^4 = 0.1668$

(c) $P(X = 6) = \binom{5}{3} \times (1 - 0.55)^2 \times 0.55^4 = 0.1853$

(d) The probability that team A wins the series in game 5 is

$\binom{4}{3} \times (1 - 0.55)^1 \times 0.55^4 = 0.1647.$

The probability that team B wins the series in game 5 is

$\binom{4}{3} \times (1 - 0.45)^1 \times 0.45^4 = 0.0902.$

The probability that the series is over after game five is $0.1647 + 0.0902 = 0.2549$.

(e) The probability that team A wins the series in game 4 is $0.55^4 = 0.0915$.
The probability that team A wins the series is
$0.0915 + 0.1647 + 0.1853 + 0.1668 = 0.6083.$

3.8.7 (a) Consider a negative binomial distribution with parameters $p = 0.58$ and $r = 3$.

$P(X = 9) = \binom{8}{2} \times (1 - 0.58)^6 \times 0.58^3 = 0.0300$

(b) Consider a negative binomial distribution with parameters $p = 0.42$ and $r = 4$.

$P(X \leq 7) = P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7) = 0.3294$

3.8.8 $P(\text{two red balls} \mid \text{head}) = \frac{\binom{6}{2} \times \binom{5}{1}}{\binom{11}{3}} = \frac{5}{11}$

$P(\text{two red balls} \mid \text{tail}) = \frac{\binom{5}{2} \times \binom{6}{1}}{\binom{11}{3}} = \frac{4}{11}$

Therefore,

$P(\text{two red balls}) = (P(\text{head}) \times P(\text{two red balls} \mid \text{head}))$

$+ (P(\text{tail}) \times P(\text{two red balls} \mid \text{tail}))$

$= \left(0.5 \times \frac{5}{11}\right) + \left(0.5 \times \frac{4}{11}\right) = \frac{9}{22}$

and

$P(\text{head} \mid \text{two red balls}) = \frac{P(\text{head and two red balls})}{P(\text{two red balls})}$

$= \frac{P(\text{head}) \times P(\text{two red balls}|\text{head})}{P(\text{two red balls})} = \frac{5}{9}.$

3.8.9 Using the hypergeometric distribution, the answer is

$P(X = 0) + P(X = 1) = \frac{\binom{36}{5} \times \binom{4}{0}}{\binom{40}{5}} + \frac{\binom{36}{4} \times \binom{4}{1}}{\binom{40}{5}} = 0.9310.$

For a collection of 4,000,000 items of which 400,000 are defective

a $B(5, 0.1)$ distribution can be used.

$P(X = 0) + P(X = 1) = \binom{5}{0} \times 0.1^0 \times 0.9^5 + \binom{5}{1} \times 0.1^1 \times 0.9^4 = 0.9185$

3.8.10 (a) $P\left(B\left(22,\frac{1}{6}\right)=3\right)=\binom{22}{3}\times\left(\frac{1}{6}\right)^3\times\left(\frac{5}{6}\right)^{19}=0.223$

(b) Using a negative binomial distribution with $p=\frac{1}{6}$ and $r=3$ the required probability is

$$P(X=10)=\binom{9}{2}\times\left(\frac{1}{6}\right)^3\times\left(\frac{5}{6}\right)^7=0.047$$

(c) $P(B(11,0.5)\le 3)$

$$=\binom{11}{0}\times 0.5^0\times 0.5^{11}+\binom{11}{1}\times 0.5^1\times 0.5^{10}+\binom{11}{2}\times 0.5^2\times 0.5^9+\binom{11}{3}\times 0.5^3\times 0.5^8$$

$=0.113$

3.8.11 $$\frac{\binom{11}{3}\times\binom{8}{3}}{\binom{19}{6}}=0.3406$$

3.8.12 (a) True

(b) True

(c) True

(d) True

3.8.13 (a) $P\left(B\left(10,\frac{1}{6}\right)=3\right)=\binom{10}{3}\times\left(\frac{1}{6}\right)^3\times\left(\frac{5}{6}\right)^7=0.155$

(b) Using a negative binomial distribution with $p=\frac{1}{6}$ and $r=4$ the required probability is

$$P(X=20)=\binom{19}{3}\times\left(\frac{1}{6}\right)^4\times\left(\frac{5}{6}\right)^{16}=0.040$$

(c) Using the multinomial distribution the required probability is

$$\frac{9!}{5!\times 2!\times 2!}\times\left(\frac{2}{3}\right)^5\times\left(\frac{1}{6}\right)^2\times\left(\frac{1}{6}\right)^2=0.077$$

3.8.14 (a) $P(\text{top quality}) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= \left(e^{-8.3} \times \frac{8.3^0}{0!}\right) + \left(e^{-8.3} \times \frac{8.3^1}{1!}\right) + \left(e^{-8.3} \times \frac{8.3^2}{2!}\right) + \left(e^{-8.3} \times \frac{8.3^3}{3!}\right) + \left(e^{-8.3} \times \frac{8.3^4}{4!}\right)$

$= 0.0837$

$P(\text{good quality}) = P(X=5) + P(X=6) + P(X=7) + P(X=8)$

$= \left(e^{-8.3} \times \frac{8.3^5}{5!}\right) + \left(e^{-8.3} \times \frac{8.3^6}{6!}\right) + \left(e^{-8.3} \times \frac{8.3^7}{7!}\right) + \left(e^{-8.3} \times \frac{8.3^8}{8!}\right)$

$= 0.4671$

$P(\text{normal quality}) = P(X=9) + P(X=10) + P(X=11) + P(X=12)$

$= \left(e^{-8.3} \times \frac{8.3^9}{9!}\right) + \left(e^{-8.3} \times \frac{8.3^{10}}{10!}\right) + \left(e^{-8.3} \times \frac{8.3^{11}}{11!}\right) + \left(e^{-8.3} \times \frac{8.3^{12}}{12!}\right)$

$= 0.3699$

$P(\text{bad quality}) = 1 - 0.0837 - 0.4671 - 0.3699 = 0.0793$

Using the multinomial distribution the required probability is

$\frac{7!}{2! \times 2! \times 2! \times 1!} \times 0.0837^2 \times 0.4671^2 \times 0.3699^2 \times 0.0793 = 0.0104$

(b) The expectation is $10 \times 0.3699 = 3.699$.

The standard deviation is $\sqrt{10 \times 0.3699 \times (1 - 0.3699)} = 1.527$.

(c) The probability of being either top quality or good quality is $0.0837 + 0.4671 = 0.5508$.

$P(B(8, 0.5508) \leq 3) = \binom{8}{0} \times 0.5508^0 \times 0.4492^8 + \binom{8}{1} \times 0.5508^1 \times 0.4492^7$

$+ \binom{8}{2} \times 0.5508^2 \times 0.4492^6 + \binom{8}{3} \times 0.5508^3 \times 0.4492^5$

$= 0.2589$

3.8.15 D

3.8.16 A