7.2.1 (a) $\text{bias}(\hat{\mu}_1) = 0$

The point estimate $\hat{\mu}_1$ is unbiased.

$\text{bias}(\hat{\mu}_2) = 0$

The point estimate $\hat{\mu}_2$ is unbiased.

$\text{bias}(\hat{\mu}_3) = 9 - \frac{\mu}{2}$

(b) $\text{Var}(\hat{\mu}_1) = 6.2500$

$\text{Var}(\hat{\mu}_2) = 9.0625$

$\text{Var}(\hat{\mu}_3) = 1.9444$

The point estimate $\hat{\mu}_3$ has the smallest variance.

(c) $\text{MSE}(\hat{\mu}_1) = 6.2500$

$\text{MSE}(\hat{\mu}_2) = 9.0625$

$\text{MSE}(\hat{\mu}_3) = 1.9444 + (9 - \frac{\mu}{2})^2$

This is equal to 26.9444 when $\mu = 8$.

7.2.2 (a) $\text{bias}(\hat{\mu}_1) = 0$

$\text{bias}(\hat{\mu}_2) = -0.217\mu$

$\text{bias}(\hat{\mu}_3) = 2 - \frac{\mu}{4}$

The point estimate $\hat{\mu}_1$ is unbiased.

(b) $\text{Var}(\hat{\mu}_1) = 4.444$

$\text{Var}(\hat{\mu}_2) = 2.682$

$\text{Var}(\hat{\mu}_3) = 2.889$

The point estimate $\hat{\mu}_2$ has the smallest variance.

(c) $\text{MSE}(\hat{\mu}_1) = 4.444$

$\text{MSE}(\hat{\mu}_2) = 2.682 + 0.0469\mu^2$

This is equal to 3.104 when $\mu = 3$.

$\text{MSE}(\hat{\mu}_3) = 2.889 + (2 - \frac{\mu}{4})^2$

This is equal to 4.452 when $\mu = 3$.

7.2.3 (a) $\text{Var}(\hat{\mu}_1) = 2.5$

(b) The value $p = 0.6$ produces the smallest variance which is $\text{Var}(\hat{\mu}) = 2.4$.

(c) The relative efficiency is $\frac{2.4}{2.5} = 0.96$.

7.2.4 (a) $\text{Var}(\hat{\mu}_1) = 2$

(b) The value $p = 0.875$ produces the smallest variance which is $\text{Var}(\hat{\mu}) = 0.875$.

(c) The relative efficiency is $\frac{0.875}{2} = 0.4375$.

7.2.5 (a) $a_1 + \ldots + a_n = 1$

(b) $a_1 = \ldots = a_n = \frac{1}{n}$

7.2.6 $\text{MSE}(\hat{\theta}_1) = 0.02\ \theta^2 + (0.13\ \theta)^2 = 0.0369\ \theta^2$

$\text{MSE}(\hat{\theta}_2) = 0.07\ \theta^2 + (0.05\ \theta)^2 = 0.0725\ \theta^2$

$\text{MSE}(\hat{\theta}_3) = 0.005\ \theta^2 + (0.24\ \theta)^2 = 0.0626\ \theta^2$

The point estimate $\hat{\theta}_1$ has the smallest mean square error.

7.2.7 $\text{bias}(\hat{\mu}) = \frac{\mu_0 - \mu}{2}$

$\text{Var}(\hat{\mu}) = \frac{\sigma^2}{4}$

$\text{MSE}(\hat{\mu}) = \frac{\sigma^2}{4} + \frac{(\mu_0 - \mu)^2}{4}$

$\text{MSE}(X) = \sigma^2$

7.2.8 (a) $\text{bias}(\hat{p}) = -\frac{p}{11}$

(b) $\text{Var}(\hat{p}) = \frac{10\,p\,(1-p)}{121}$

(c) $\text{MSE}(\hat{p}) = \frac{10\,p\,(1-p)}{121} + \left(\frac{p}{11}\right)^2 = \frac{10p-9p^2}{121}$

(d) $\text{bias}\left(\frac{X}{10}\right) = 0$

$\text{Var}\left(\frac{X}{10}\right) = \frac{p(1-p)}{10}$

$\text{MSE}\left(\frac{X}{10}\right) = \frac{p(1-p)}{10}$

7.2.9 $\text{Var}\left(\frac{X_1+X_2}{2}\right)$

$= \frac{\text{Var}(X_1)+\text{Var}(X_2)}{4}$

$= \frac{5.39^2+9.43^2}{4}$

$= 29.49$

The standard deviation is $\sqrt{29.49} = 5.43$.

7.3.1 $\text{Var}\left(\frac{X_1}{n_1}\right) = \frac{p(1-p)}{n_1}$

$\text{Var}\left(\frac{X_2}{n_2}\right) = \frac{p(1-p)}{n_2}$

The relative efficiency is the ratio of these two variances which is $\frac{n_1}{n_2}$.

7.3.2 (a) $P\left(\left|N\left(0, \frac{1}{10}\right)\right| \leq 0.3\right) = 0.6572$

(b) $P\left(\left|N\left(0, \frac{1}{30}\right)\right| \leq 0.3\right) = 0.8996$

7.3.3 (a) $P\left(\left|N\left(0, \frac{7}{15}\right)\right| \leq 0.4\right) = 0.4418$

(b) $P\left(\left|N\left(0, \frac{7}{50}\right)\right| \leq 0.4\right) = 0.7150$

7.3.4 (a) Solving

$$P\left(5 \times \frac{\chi^2_{30}}{30} \leq c\right) = P(\chi^2_{30} \leq 6c) = 0.90$$

gives $c = 6.709$.

(b) Solving

$$P\left(5 \times \frac{\chi^2_{30}}{30} \leq c\right) = P(\chi^2_{30} \leq 6c) = 0.95$$

gives $c = 7.296$.

7.3.5 (a) Solving

$$P\left(32 \times \frac{\chi^2_{20}}{20} \leq c\right) = P\left(\chi^2_{20} \leq \frac{5c}{8}\right) = 0.90$$

gives $c = 45.46$.

(b) Solving

$$P\left(32 \times \frac{\chi^2_{20}}{20} \leq c\right) = P\left(\chi^2_{20} \leq \frac{5c}{8}\right) = 0.95$$

gives $c = 50.26$.

7.3.6 (a) Solving

$$P(|t_{15}| \leq c) = 0.95$$

gives $c = t_{0.025,15} = 2.131$.

(b) Solving

$$P(|t_{15}| \leq c) = 0.99$$

gives $c = t_{0.005,15} = 2.947$.

7.3.7 (a) Solving

$P\left(\frac{|t_{20}|}{\sqrt{21}} \leq c\right) = 0.95$

gives $c = \frac{t_{0.025,20}}{\sqrt{21}} = 0.4552.$

(b) Solving

$P\left(\frac{|t_{20}|}{\sqrt{21}} \leq c\right) = 0.99$

gives $c = \frac{t_{0.005,20}}{\sqrt{21}} = 0.6209.$

7.3.8 $\hat{p} = \frac{234}{450} = 0.52$

s.e.$(\hat{p}) = \sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} = \sqrt{\frac{0.52\times0.48}{450}} = 0.0236$

7.3.9 $\hat{\mu} = \bar{x} = 974.3$

s.e.$(\hat{\mu}) = \frac{s}{\sqrt{n}} = \sqrt{\frac{452.1}{35}} = 3.594$

7.3.10 $\hat{p} = \frac{24}{120} = 0.2$

s.e.$(\hat{p}) = \sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} = \sqrt{\frac{0.2\times0.8}{120}} = 0.0365$

7.3.11 $\hat{p} = \frac{33}{150} = 0.22$

s.e.$(\hat{p}) = \sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} = \sqrt{\frac{0.22\times0.78}{150}} = 0.0338$

7.3.12 $\hat{p} = \frac{26}{80} = 0.325$

s.e.$(\hat{p}) = \sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} = \sqrt{\frac{0.325\times0.675}{80}} = 0.0524$

7.3.13 $\hat{\mu} = \bar{x} = 69.35$

s.e.$(\hat{\mu}) = \frac{s}{\sqrt{n}} = \frac{17.59}{\sqrt{200}} = 1.244$

7.3.14 $\hat{\mu} = \bar{x} = 3.291$

s.e.$(\hat{\mu}) = \frac{s}{\sqrt{n}} = \frac{3.794}{\sqrt{55}} = 0.512$

7.3.15 $\hat{\mu} = \bar{x} = 12.211$

$\text{s.e.}(\hat{\mu}) = \frac{s}{\sqrt{n}} = \frac{2.629}{\sqrt{90}} = 0.277$

7.3.16 $\hat{\mu} = \bar{x} = 1.1106$

$\text{s.e.}(\hat{\mu}) = \frac{s}{\sqrt{n}} = \frac{0.0530}{\sqrt{125}} = 0.00474$

7.3.17 $\hat{\mu} = \bar{x} = 0.23181$

$\text{s.e.}(\hat{\mu}) = \frac{s}{\sqrt{n}} = \frac{0.07016}{\sqrt{75}} = 0.00810$

7.3.18 $\hat{\mu} = \bar{x} = 9.2294$

$\text{s.e.}(\hat{\mu}) = \frac{s}{\sqrt{n}} = \frac{0.8423}{\sqrt{80}} = 0.0942$

7.3.19 If a sample of size $n = 100$ is used, then the probability is
$P(0.24 - 0.05 \leq \hat{p} \leq 0.24 + 0.05) = P(19 \leq B(100, 0.24) \leq 29)$.

Using a normal approximation this can be estimated as

$\Phi\left(\frac{29+0.5-100\times0.24}{\sqrt{100\times0.24\times0.76}}\right) - \Phi\left(\frac{19-0.5-100\times0.24}{\sqrt{100\times0.24\times0.76}}\right)$

$= \Phi(1.288) - \Phi(-1.288) = 0.8022$.

If a sample of size $n = 200$ is used, then the probability is
$P(38 \leq B(200, 0.24) \leq 58)$.

Using a normal approximation this can be estimated as

$\Phi\left(\frac{58+0.5-200\times0.24}{\sqrt{200\times0.24\times0.76}}\right) - \Phi\left(\frac{38-0.5-200\times0.24}{\sqrt{200\times0.24\times0.76}}\right)$

$= \Phi(1.738) - \Phi(-1.738) = 0.9178$.

7.3.20 $P(173 \leq \hat{\mu} \leq 175) = P(173 \leq \bar{X} \leq 175)$

where

$\bar{X} \sim N\left(174, \frac{2.8^2}{30}\right)$.

This is

$\Phi\left(\frac{175-174}{\sqrt{2.8^2/30}}\right) - \Phi\left(\frac{173-174}{\sqrt{2.8^2/30}}\right)$

$= \Phi(1.956) - \Phi(-1.956) = 0.9496.$

7.3.21 $P(0.62 \leq \hat{p} \leq 0.64)$

$= P(300 \times 0.62 \leq B(300, 0.63) \leq 300 \times 0.64)$

$\simeq P(185.5 \leq N(300 \times 0.63, 300 \times 0.63 \times 0.37) \leq 192.5)$

$= P\left(\frac{185.5-189}{\sqrt{69.93}} \leq N(0,1) \leq \frac{192.5-189}{\sqrt{69.93}}\right)$

$= \Phi(0.419) - \Phi(-0.419) = 0.324$

7.3.22 $P\left(109.9 \leq N\left(110.0, \frac{0.4^2}{22}\right) \leq 110.1\right)$

$= P\left(\frac{\sqrt{22}(109.9-110.0)}{0.4} \leq N(0,1) \leq \frac{\sqrt{22}(110.1-110.0)}{0.4}\right)$

$= \Phi(1.173) - \Phi(-1.173) = 0.759$

7.3.23 $\sqrt{\frac{0.126 \times 0.874}{360}} = 0.017$

7.3.24 $P\left(N\left(341, \frac{2^2}{20}\right) \leq 341.5\right)$

$= P\left(N(0.1) \leq \frac{\sqrt{20} \times (341.5-341)}{2}\right)$

$= \Phi(1.118) = 0.547$

7.3.25 $P\left(\mu - 2 \leq N\left(\mu, \frac{5.2^2}{18}\right) \leq \mu + 2\right)$

$= P\left(\frac{-\sqrt{18} \times 2}{5.2} \leq N(0.1) \leq \frac{\sqrt{18} \times 2}{5.2}\right)$

$= \Phi(1.632) - \Phi(-1.632) = 0.103$

7.3.26 The largest standard error is obtained when $\hat{p} = 0.5$ and is equal to

$\sqrt{\frac{0.5\times0.5}{1400}} = 0.0134.$

7.3.27 $P(X \geq 60) = e^{-0.02\times60} = 0.301$

Let Y be the number of components that last longer than one hour.

$P\left(0.301 - 0.05 \leq \frac{Y}{110} \leq 0.301 + 0.05\right)$

$= P(27.6 \leq Y \leq 38.6)$

$= P(28 \leq B(110, 0.301) \leq 38)$

$\simeq P\left(27.5 \leq N(110 \times 0.301, 110 \times 0.301 \times 0.699) \leq 38.5\right)$

$= P\left(\frac{27.5-33.11}{\sqrt{23.14}} \leq N(0,1) \leq \frac{38.5-33.11}{\sqrt{23.14}}\right)$

$= \Phi(1.120) - \Phi(-1.166)$

$= 0.869 - 0.122 = 0.747$

7.3.28 (a) $P(\mu - 0.5 \leq \bar{X} \leq \mu + 0.5)$

$= P\left(\mu - 0.5 \leq N\left(\mu, \frac{0.82^2}{5}\right) \leq \mu + 0.5\right)$

$= \Phi\left(\frac{0.5\sqrt{5}}{0.82}\right) - \Phi\left(\frac{-0.5\sqrt{5}}{0.82}\right) = 0.827$

(b) $P(\mu - 0.5 \leq \bar{X} \leq \mu + 0.5)$

$= P\left(\mu - 0.5 \leq N\left(\mu, \frac{0.82^2}{10}\right) \leq \mu + 0.5\right)$

$= \Phi\left(\frac{0.5\sqrt{10}}{0.82}\right) - \Phi\left(\frac{-0.5\sqrt{10}}{0.82}\right) = 0.946$

(c) In order for

$P\left(\mu - 0.5 \leq N\left(\mu, \frac{0.82^2}{n}\right) \leq \mu + 0.5\right)$

$= \Phi\left(\frac{0.5\sqrt{n}}{0.82}\right) - \Phi\left(\frac{-0.5\sqrt{n}}{0.82}\right) \geq 0.99$

it is necessary that

$\frac{0.5\sqrt{n}}{0.82} \geq z_{0.005} = 2.576$

which is satisfied for a sample size n of at least 18.

7.3.29 (a) $p = \frac{592}{3288} = 0.18$

$P(p - 0.1 \leq \hat{p} \leq p + 0.1)$

$= P\left(0.08 \leq \frac{X}{20} \leq 0.28\right)$

$= P(1.6 \leq X \leq 5.6)$

where $X \sim B(20, 0.18)$.

This probability is

$P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$

$= \binom{20}{2} \times 0.18^2 \times 0.82^{18} + \binom{20}{3} \times 0.18^3 \times 0.82^{17}$

$+ \binom{20}{4} \times 0.18^4 \times 0.82^{16} + \binom{20}{5} \times 0.18^5 \times 0.82^{15}$

$= 0.7626.$

(b) The probability that a sampled meter is operating outside the acceptable tolerance limits is now

$p^* = \frac{184}{2012} = 0.09.$

$P(p - 0.1 \leq \hat{p} \leq p + 0.1)$

$= P\left(0.08 \leq \frac{Y}{20} \leq 0.28\right)$

$= P(1.6 \leq Y \leq 5.6)$

where $Y \sim B(20, 0.09)$.

This probability is

$P(Y = 2) + P(Y = 3) + P(Y = 4) + P(Y = 5)$

$= \binom{20}{2} \times 0.09^2 \times 0.91^{18} + \binom{20}{3} \times 0.09^3 \times 0.91^{17}$

$+ \binom{20}{4} \times 0.09^4 \times 0.91^{16} + \binom{20}{5} \times 0.09^5 \times 0.91^{15}$

$= 0.5416.$

7.3.30 D

7.3.31 D

7.3.32 D

7.3.33 B

7.3.34 D

7.3.35 A

7.4.1 $\hat{\lambda} = \bar{x} = 5.63$

s.e.$(\hat{\lambda}) = \sqrt{\frac{\hat{\lambda}}{n}} = \sqrt{\frac{5.63}{23}} = 0.495$

7.4.2 Using the method of moments the point estimates $\hat{a}$ and $\hat{b}$ are the solutions to the equations

$\frac{a}{a+b} = 0.782$

and

$\frac{ab}{(a+b)^2(a+b+1)} = 0.0083$

which are $\hat{a} = 15.28$ and $\hat{b} = 4.26$.

7.4.3 Using the method of moments

$E(X) = \frac{1}{\lambda} = \bar{x}$

which gives $\hat{\lambda} = \frac{1}{\bar{x}}$.

The likelihood is

$L(x_1, \ldots, x_n, \lambda) = \lambda^n \, e^{-\lambda(x_1 + \ldots + x_n)}$

which is maximized at $\hat{\lambda} = \frac{1}{\bar{x}}$.

7.4.4 $\hat{p}_i = \frac{x_i}{n}$ for $1 \leq i \leq n$

7.4.5 Using the method of moments

$E(X) = \frac{5}{\lambda} = \bar{x}$

which gives $\hat{\lambda} = \frac{5}{\bar{x}}$.

The likelihood is

$L(x_1, \ldots, x_n, \lambda) = \left(\frac{1}{24}\right)^n \times \lambda^{5n} \times x_1^4 \times \ldots \times x_n^4 \times e^{-\lambda(x_1 + \ldots + x_n)}$

which is maximimized at $\hat{\lambda} = \frac{5}{\bar{x}}$.

7.7.1 $\text{bias}(\hat{\mu}_1) = 5 - \frac{\mu}{2}$

$\text{bias}(\hat{\mu}_2) = 0$

$\text{Var}(\hat{\mu}_1) = \frac{1}{8}$

$\text{Var}(\hat{\mu}_2) = \frac{1}{2}$

$\text{MSE}(\hat{\mu}_1) = \frac{1}{8} + (5 - \frac{\mu}{2})^2$

$\text{MSE}(\hat{\mu}_2) = \frac{1}{2}$

7.7.2 (a) $\text{bias}(\hat{p}) = -\frac{p}{7}$

(b) $\text{Var}(\hat{p}) = \frac{3p(1-p)}{49}$

(c) $\text{MSE}(\hat{p}) = \frac{3p(1-p)}{49} + (\frac{p}{7})^2 = \frac{3p-2p^2}{49}$

(d) $\text{MSE}\left(\frac{X}{12}\right) = \frac{p(1-p)}{12}$

7.7.3 (a) $F(t) = P(T \leq t) = P(X_1 \leq t) \times \ldots \times P(X_n \leq t)$

$= \frac{t}{\theta} \times \ldots \times \frac{t}{\theta} = (\frac{t}{\theta})^n$

for $0 \leq t \leq \theta$

(b) $f(t) = \frac{dF(t)}{dt} = n\ \frac{t^{n-1}}{\theta^n}$

for $0 \leq t \leq \theta$

(c) Notice that

$E(T) = \int_0^\theta\ t\ f(t)\ dt = \frac{n}{n+1}\theta$

so that $E(\hat{\theta}) = \theta$.

(d) Notice that

$E(T^2) = \int_0^\theta\ t^2\ f(t)\ dt = \frac{n}{n+2}\theta^2$

so that

$\text{Var}(T) = \frac{n}{n+2}\theta^2 - \left(\frac{n}{n+1}\theta\right)^2$

$= \frac{n\theta^2}{(n+2)(n+1)^2}$.

Consequently,

$\text{Var}(\hat{\theta}) = \frac{(n+1)^2}{n^2}\text{Var}(T) = \frac{\theta^2}{n(n+2)}$

and

$\text{s.e.}(\hat{\theta}) = \frac{\hat{\theta}}{\sqrt{n(n+2)}}$.

(e) $\hat{\theta} = \frac{11}{10} \times 7.3 = 8.03$

$\text{s.e.}(\hat{\theta}) = \frac{8.03}{\sqrt{10 \times 12}} = 0.733$

7.7.4 Recall that $f(x_i, \theta) = \frac{1}{\theta}$ for $0 \leq x_i \leq \theta$

(and $f(x_i, \theta) = 0$ elsewhere)

so that the likelihood is $\frac{1}{\theta^n}$

as long as $x_i \leq \theta$ for $1 \leq i \leq n$

and is equal to zero otherwise.

$\text{bias}(\hat{\theta}) = -\frac{\theta}{n+1}$

7.7.5 Using the method of moments

$E(X) = \frac{1}{p} = \bar{x}$

which gives $\hat{p} = \frac{1}{\bar{x}}$.

The likelihood is

$L(x_1, \ldots, x_n, \lambda) = p^n (1-p)^{x_1+\ldots+x_n-n}$

which is maximimized at $\hat{p} = \frac{1}{\bar{x}}$.

7.7.6 $\hat{p} = \frac{35}{100} = 0.35$

s.e.$(\hat{p}) = \sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} = \sqrt{\frac{0.35 \times 0.65}{100}} = 0.0477$

7.7.7 $\hat{\mu} = \bar{x} = 17.79$

s.e.$(\bar{x}) = \frac{s}{\sqrt{n}} = \frac{6.16}{\sqrt{24}} = 1.26$

7.7.8 $\hat{\mu} = \bar{x} = 1.633$

s.e.$(\bar{x}) = \frac{s}{\sqrt{n}} = \frac{0.999}{\sqrt{30}} = 0.182$

7.7.9 $\hat{\mu} = \bar{x} = 69.618$

s.e.$(\bar{x}) = \frac{s}{\sqrt{n}} = \frac{1.523}{\sqrt{60}} = 0.197$

7.7.10 $\hat{\mu} = \bar{x} = 32.042$

s.e.$(\bar{x}) = \frac{s}{\sqrt{n}} = \frac{5.817}{\sqrt{40}} = 0.920$

7.7.11 $\text{Var}(s_1^2) = \text{Var}\left(\frac{\sigma^2 \chi^2_{n_1-1}}{n_1-1}\right)$

$= \left(\frac{\sigma^2}{n_1-1}\right)^2 \text{Var}(\chi^2_{n_1-1})$

$= \left(\frac{\sigma^2}{n_1-1}\right)^2 2(n_1-1) = \frac{2\sigma^4}{n_1-1}$

Similarly, $\text{Var}(s_2^2) = \frac{2\sigma^4}{n_2-1}$.

The ratio of these two variances is $\frac{n_1-1}{n_2-1}$.

7.7.12 The true proportion of "very satisfied" customers is

$p = \frac{11842}{24839} = 0.4768.$

The probability that the manager's estimate of the proportion of "very satisfied" customers is within 0.10 of $p = 0.4768$ is

$P(0.4768 - 0.10 \leq \hat{p} \leq 0.4768 + 0.10)$

$= P(0.3768 \times 80 \leq X \leq 0.5768 \times 80)$

$= P(30.144 \leq X \leq 46.144) = P(31 \leq X \leq 46)$

where $X \sim B(80, 0.4768)$.

This probability is 0.9264.

7.7.13 When a sample of size $n = 15$ is used

$P(62.8 - 0.5 \leq \hat{\mu} \leq 62.8 + 0.5)$

$= P(62.3 \leq \bar{X} \leq 63.3)$

where $\bar{X} \sim N(62.8, 3.9^2/15)$.

This probability is equal to

$$\Phi\left(\frac{63.3-62.8}{\sqrt{3.9^2/15}}\right) - \Phi\left(\frac{62.3-62.8}{\sqrt{3.9^2/15}}\right)$$

$= \Phi(0.4965) - \Phi(-0.4965) = 0.3804.$

When a sample of size $n = 40$ is used

$P(62.8 - 0.5 \leq \hat{\mu} \leq 62.8 + 0.5)$

$= P(62.3 \leq \bar{X} \leq 63.3)$

where $\bar{X} \sim N(62.8, 3.9^2/40)$.

This probability is equal to

$$\Phi\left(\frac{63.3-62.8}{\sqrt{3.9^2/40}}\right) - \Phi\left(\frac{62.3-62.8}{\sqrt{3.9^2/40}}\right)$$

$= \Phi(0.8108) - \Phi(-0.8108) = 0.5826.$

7.7.14 $\hat{\mu} = \bar{x} = 25.318$

$\text{s.e.}(\bar{x}) = \frac{s}{\sqrt{n}} = \frac{0.226}{\sqrt{44}} = 0.0341$

The upper quartile of the distribution of soil compressibilities can be estimated by the upper sample quartile 25.50.

7.7.15 Probability theory

7.7.16 Probability theory

7.7.17 $\hat{p} = \frac{39}{220} = 0.177$

$\text{s.e.}(\hat{p}) = \sqrt{\frac{0.177 \times 0.823}{220}} = 0.026$

7.7.18 Let X be the number of cases where the treatment was effective.

$P\left(0.68 - 0.05 \leq \frac{X}{140} \leq 0.68 + 0.05\right)$

$= P(88.2 \leq X \leq 102.2)$

$= P(89 \leq B(140, 0.68) \leq 102)$

$\simeq P(88.5 \leq N(140 \times 0.68, 140 \times 0.68 \times 0.32) \leq 102.5)$

$= P\left(\frac{88.5-95.2}{5.519} \leq N(0,1) \leq \frac{102.5-95.2}{5.519}\right)$

$= \Phi(1.268) - \Phi(-1.268) = 0.80$

7.7.19 (a) $\hat{\mu} = \bar{x} = 70.58$

(b) $\frac{s}{\sqrt{n}} = \frac{12.81}{\sqrt{12}} = 3.70$

(c) $\frac{67+70}{2} = 68.5$

7.7.20 Statistical inference

7.7.21 Statistical inference

7.7.22 (a) True

(b) True

(c) True

(d) True

7.7.23 $P\left(722 \leq \bar{X} \leq 724\right)$

$= P\left(722 \leq N\left(723, \frac{3^2}{11}\right) \leq 724\right)$

$= P\left(\frac{-1\times\sqrt{11}}{3} \leq N(0,1) \leq \frac{1\times\sqrt{11}}{3}\right)$

$= \Phi(1.106) - \Phi(-1.106) = 0.73$

7.7.24 (a) $P\left(\mu - 20.0 \leq \bar{X} \leq \mu + 20.0\right)$

$= P\left(\mu - 20.0 \leq N\left(\mu, \frac{40.0^2}{10}\right) \leq \mu + 20.0\right)$

$= P\left(\frac{-20.0\times\sqrt{10}}{40.0} \leq N(0,1) \leq \frac{20.0\times\sqrt{10}}{40.0}\right)$

$= \Phi(1.58) - \Phi(-1.58) = 0.89$

(b) $P\left(\mu - 20.0 \leq \bar{X} \leq \mu + 20.0\right)$

$= P\left(\mu - 20.0 \leq N\left(\mu, \frac{40.0^2}{20}\right) \leq \mu + 20.0\right)$

$= P\left(\frac{-20.0\times\sqrt{20}}{40.0} \leq N(0,1) \leq \frac{20.0\times\sqrt{20}}{40.0}\right)$

$= \Phi(2.24) - \Phi(-2.24) = 0.97$

7.7.25 $\hat{p}_A = \frac{852}{1962} = 0.434$

$\text{s.e.}(\hat{p}_A) = \sqrt{\frac{0.434\times(1-0.434)}{1962}} = 0.011$

7.7.34 A

7.7.35 A

7.7.36 D

7.7.37 A

7.7.38 A