6/04/20

Agenda for Math 5710 ♬ Meeting #11 ☻☺ 7/07/20 (8:00 a.m. – 9:10 a.m.)

1. Hello:

Brigham City:	Adam Blakeslee	Ryan Johnson	Tyson Mortensen	
Logan:	David Allen	Natalie Anderson	Kameron Baird	Stephen Brezinski
	Zachary Ellis	Adam Flanders	Brock Francom	Xiang Gao
	Ryan Goodman	Janette Goodridge	Hadley Hamar	Phillip Leifer
	Brittney Miller	Jonathan Mousley	Erika Mueller	Shelby Simpson
	Steven Summers	Matthew White	Zhang Xiaomeng	

2. Note the syllabus' activity list for today:

11: T/7/07	1. Focus on methods of counting, comprehend associated structures, and discover and prove theorems with respect the following: multiplication principle, permutations, and combinations 2. Take advantage of Quiz 11.

3. Briefly, raise and address issues and questions stimulated by the following homework assignment:

A. Study our notes from Meeting #10 ; comprehend Jim's sample responses to the Quiz #10 prompts that are posted on *Canvas*.

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B. Comprehend the entry from Line #03Eii–v from our *Glossary* document.

C*. For each of the following multiple-choice prompts, select the one best response that either answers the question or completes the statement so that it is true; circle the lower-case letter in front of your choice (Upload the resulting document on Canvas.):

i. Which one of the following is a *necessary* condition for measurement relevance?
  a) Scorer consistency
  b) Reliability
  c) Measurement usefulness
  d) Learning-level relevance
  e) Validity

ii. Which one of the following is a *sufficient* condition for measurement relevance?
  a) Scorer consistency
  b) Reliability
  c) Measurement usefulness
  d) Learning-level relevance
  e) Content relevance

iii. Which one of the following is a *necessary* condition for reliability?

a) Internal consistency
b) Learning-level relevance
c) Usability
d) Pertinence to the intended content

iv. Which one of the following is a *sufficient* condition for measurement reliability?
a) Validity
b) Relevance
c) Usability
d) Internal consistency

v. Which one of the following is a *sufficient* condition for a measurement to be useful?
a) Usability, internal consistency, scorer consistency, and relevance
b) Relevance, reliability, validity, and scorer consistency
c) Usability, content relevance, and learning-level relevance

vi. Which one of the following variables depends on the stated purpose of the measurement?
a) Reliability
b) Usability
c) Usefulness
d) Scorer consistency

vii. Which one of the following variables depends on the time it takes to administer a test?
a) Content relevance
b) Scorer consistency
c) Usefulness

viii. By designing a measurement in a way to enhance its relevance (e.g., by using a measurement blueprint) and reliability (e.g., by carefully wording directions for prompts so that they are less likely to be misinterpreted), an experimenter is attempting to accomplish which one of the following regarding $D_o = D_t + D_E$:
a) Solve for D_E
b) Increase D_t
c) Increase D_o
d) Decrease D_E
e) Decrease $| D_E |$

ix. By conducting a validation study of a measurement, a teacher is attempting to accomplish which one of the following regarding $D_o = D_t + D_E$

a) Solve for D_E
b) Increase D_t
c) Increase D_o
d) Decrease D_E
e) Decrease $| D_E |$

D. Comprehend Jim's sample responses to the homework prompts that are posted on *Canvas*.

4. Let's think about the art of counting:

A. Note: Typically, counting involves easier algorithms for simulated experiments than experiments that directly address real-life problems (e.g., an experiment involving DNA molecules that have four types of nucleotides that can be millions units long).

B. Begin our journey into the art of counting with simulated experiment as presented by John A. Rice in *Mathematical Statistics and Data Analysis (3rd ed.*, 2007, p. 7):

Simpson's Paradox

A black urn contains 5 red and 6 green balls, and a white urn contains 3 red and 4 green balls. You are allowed to choose an urn and then choose a ball at random from the urn. If you choose a red ball you get a prize. Which urn should you choose to draw from? If you draw from the black urn, the probability of choosing a red ball is 5/11 = .455 (the number of ways you can draw a red ball divided by the total number of outcomes). If you choose to draw from the white urn, the probability of choosing a red ball is 3/7 = .429, so you choose to draw from the black urn.

Now consider another game in which a second black urn has 6 red and 3 green balls, and a second white urn has 9 red and 3 green balls. If you draw from the black urn, the probability of a red ball is 6/9 = .667, whereas if you choose to draw from the white urn, the probability is 9/14 = .643. So, again you choose to draw from the black urn.

In the final game, the contents of the second black urn are added to the first black urn and the contents of the second white urn are added to the first white urn. Again, you can choose which urn to draw from. Which should you choose? Intuition says choose the black urn, but let's calculate the probabilities. The black urn now contains 11 red and 9 green balls, so the probability , so the probability of drawing a red ball is 11/20 =.55. The white urn now contains 12 red and 9 green balls, so the probability of drawing a red ball from it is 12/21 =.571. So you should choose the white urn. This counterintuitive result is an example Simpson's paradox.

Note: Examples of *Simpson's paradox* in real-life problems are plentiful.

C. Visit two of our old friends:

i. Retrieve the following example of two independent experiments:

Experiment #1:

Staggerlee wants to conduct a coin-flipping experiment for the purpose of determining the probabilities of randomly obtaining various events when a fair coin is flipped exactly three times in succession. He plans to use the resulting probability distributions to hedge his bets in a variety of games of chance.

Note: Here's the homework prompt that accompanied this experiment: Please design the experiment for him so that it yields probability values for the each of the following events: X_j is the event in which exactly j tails turn up for $j \in \{ 0, 1, 2, 3 \}$. Describe the experiment – identifying the sample space and discrete probability distribution.

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Note: Following is one of our homework samples; the sample spaced has been renamed " Ω_1" to distinguish this sample space from the one the Sybil's experiment.

Let $\Omega_1 = \{$ TTT, TTH, THT, HTT, HHH, HHT, HTH, THH $\}$

Let X_j = the event that there exactly j tails. Thus,
$| X_0 | = | \{ \text{HHH} \} | = 1$
$| X_1 | = | \{ \text{HHT, HTH, THH} \} | = 3$
$| X_2 | = | \{ \text{TTH, THT, HTT} \} | = 3$
$| X_3 | = | \{ \text{TTT} \} | = 1$

Let p be our random probability function. Since $|\Omega| = 8$, we have the following probability values:

$$p(X_0) = \frac{1}{8} \wedge p(X_1) = \frac{3}{8} \wedge p(X_2) = \frac{3}{8} \wedge p(X_3) = \frac{1}{8}$$

Experiment #2:

For the purpose of formulating the rules of a game of chance in which a pair of fair dice (a red die and a yellow die) are rolled one time, Sybil wants to determine the likelihood of each of the possible *events* determined by the sum of the number of dots that appear on the top face of the yellow die and on the top face of the red die.

Sybil thinks, "Each die has six faces – a face with one dot, a face with two dots, a face with three dots, a face with four dots, a face with five dots, and a face with six dots. So there are 36 possible *outcomes* since 36 is the cardinality of the following set:

$\{ (r, y) : r =$ the number of dots on the red die's top face $\wedge$
$y =$ the number of dots on the red die's top face $\} =$
$\{ (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),$
$(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),$
$(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \}$

And since I'm interested in the *random probability* of each of the possible *events*, the *events* of interest are the sums of numbers associated with the two top faces. So each event is associated with one element in the following set: $\{ 2, 3, 4, 5, ..., 12 \}$. I'll count the number outcomes that are associated with each of the 12 events:

Let X_i = the event that the sum is i. Thus,
$| X_2 | = | \{ (1, 1) \} | = 1$
$| X_3 | = | \{ (1, 2), (2, 1) \} | = 2$
$| X_4 | = | \{ (1, 3), (2, 2), (3, 1) \} | = 3$
$| X_5 | = | \{ (1, 4), (2, 3), (3, 2), (4, 1) \} | = 4$
$| X_6 | = | \{ (1, 5), (2, 4), (3, 3), (4, 2), (5, 1) \} | = 5$
$| X_7 | = | \{ (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) \} | = 6$
$| X_8 | = | \{ (2, 6), (3, 5), (4, 4), (5, 3), (6, 2) \} | = 5$
$| X_9 | = | \{ (3, 6), (4, 5), (5, 4), (6, 3) \} | = 4$
$| X_{10} | = | \{ (4, 6), (5, 5), (6, 4) \} | = 3$
$| X_{11} | = | \{ (5, 6), (6, 5) \} | = 2$
$| X_{12} | = | \{ (6, 6) \} | = 1$

So the most likely event is X_7 and the two events that are less likely than any of the others are X_2 and X_{11}. And my chart above shows the scale for each events. Then I'll use the chart to compute the exact theoretical probabilities by reporting the probabilities as the ratio of each of those events and 36 (i.e., $p(X_i) = X_i \div 36$."

D. Combine Experiments #1 and #2 into Experiment 3 and go to work on it:

i <u>Experiment #3 (combined from Experiments #1 & #2)</u>:

Torialba wants to build upon Staggerlee's and Sybil's work to address the following question: What is the random probability of the event of both the red and green dice to come up with same number of dots on the top surfaces AND flipping exactly two tails?

ii. Let's go to work on Torialba's problem:

Begin by identifying $| \Omega_3 |$:

And then let's compute $p(S \cap T)$ where S is the event that both the red and green dice to come up with same number and T is the event that exactly two tails are flipped:

5. Take advantage of Quiz 11.

6. Complete the following homework assignment prior to Meeting #12:

   A. Study our notes from Meeting #11.

   B. Comprehend Jim's sample response to Quiz 11.

   C*. Three experiments are conducted:

   Experiment 1: One card is randomly drawn from a well-shuffled poker deck consisting of 54 cards – including 2 jokers.

   Experiment 2: A ball is randomly drawn from an urn that contains exactly 3 black balls, 3 green balls, 2 yellow balls, and 2 orange balls.

   Experiment 3: Experiments 1 and 2 are combined.

   What is the probability that Experiment 3 results in the event that both a joker is drawn and an orange ball is Not drawn?

   Please display the computation that led to your solution (Please post the resulting PDF using the appropriate Canvas Assignment link.) .

   D. Comprehend the following entries from our Glossary:

   034. A taste of Counting:

   A. Multiplication principle theorem:

   Theorem 05: Independent experiments #1 and #2 with respective finite sample spaces Ω_1 and Ω_2 are conducted $\Rightarrow |\ \Omega_1 \cap \ \Omega_2\ | = |\ \Omega_1\ |\cdot|\ \Omega_2\ |$

   B. Definition of a *permutation* of a finite set:

   Given $A \in$ { finite sets }, ($f \in$ { permutations of A } $\Leftrightarrow f : A \underset{onto}{\overset{1:1}{\longrightarrow}} A$)

   C. Note: Given $n, r \in \omega \ni r \leq n$, "${}_nP_r$" is read "the number of all possible permutations of n elements taken r at a time.

   E. From the Video Page of *Canvas*, view with comprehension the video "counting and the multiplication principle. "

   F. Comprehend Jim's sample responses to the homework prompts that are posted on *Canvas*.

7. Discourage one another from lying – including lying to children.