4.1.1 (a) $E(X) = \frac{-3+8}{2} = 2.5$

(b) $\sigma = \frac{8-(-3)}{\sqrt{12}} = 3.175$

(c) The upper quartile is 5.25.

(d) $P(0 \leq X \leq 4) = \int_0^4 \frac{1}{11}\, dx = \frac{4}{11}$

4.1.2 (a) $E(X) = \frac{1.43+1.60}{2} = 1.515$

(b) $\sigma = \frac{1.60-1.43}{\sqrt{12}} = 0.0491$

(c) $F(x) = \frac{x-1.43}{1.60-1.43} = \frac{x-1.43}{0.17}$

for $1.43 \leq x \leq 1.60$

(d) $F(1.48) = \frac{1.48-1.43}{0.17} = \frac{0.05}{0.17} = 0.2941$

(e) $F(1.5) = \frac{1.5-1.43}{0.17} = \frac{0.07}{0.17} = 0.412$

The number of batteries with a voltage less than 1.5 Volts has a binomial distribution with parameters $n = 50$ and $p = 0.412$ so that the expected value is

$E(X) = n \times p = 50 \times 0.412 = 20.6$

and the variance is

$\text{Var}(X) = n \times p \times (1-p) = 50 \times 0.412 \times 0.588 = 12.11.$

4.1.3 (a) These four intervals have probabilities 0.30, 0.20, 0.25, and 0.25 respectively, and the expectations and variances are calculated from the binomial distribution.

The expectations are:
$20 \times 0.30 = 6$
$20 \times 0.20 = 4$
$20 \times 0.25 = 5$
$20 \times 0.25 = 5$

The variances are:
$20 \times 0.30 \times 0.70 = 4.2$
$20 \times 0.20 \times 0.80 = 3.2$
$20 \times 0.25 \times 0.75 = 3.75$
$20 \times 0.25 \times 0.75 = 3.75$

(b) Using the multinomial distribution the probability is

$\frac{20!}{5!\times5!\times5!\times5!} \times 0.30^5 \times 0.20^5 \times 0.25^5 \times 0.25^5 = 0.0087.$

4.1.4 (a) $E(X) = \frac{0.0+2.5}{2} = 1.25$

$\text{Var}(X) = \frac{(2.5-0.0)^2}{12} = 0.5208$

(b) The probability that a piece of scrap wood is longer than 1 meter is

$\frac{1.5}{2.5} = 0.6.$

The required probability is

$P(B(25, 0.6) \geq 20) = 0.0294.$

4.1.5 (a) The probability is $\frac{4.184-4.182}{4.185-4.182} = \frac{2}{3}$.

(b) $P(\text{difference} \leq 0.0005 \mid \text{fits in hole}) = \frac{P(4.1835 \leq \text{diameter} \leq 4.1840)}{P(\text{diameter} \leq 4.1840)}$

$= \frac{4.1840-4.1835}{4.1840-4.1820} = \frac{1}{4}$

4.1.6 (a) $P(X \leq 85) = \frac{85-60}{100-60} = \frac{5}{8}$

$$P\left(B\left(6, \tfrac{5}{8}\right) = 3\right) = \binom{6}{3} \times \left(\tfrac{5}{8}\right)^3 \times \left(1 - \tfrac{5}{8}\right)^3 = 0.257$$

(b) $P(X \leq 80) = \frac{80-60}{100-60} = \frac{1}{2}$

$P(80 \leq X \leq 90) = \frac{90-60}{100-60} - \frac{80-60}{100-60} = \frac{1}{4}$

$P(X \geq 90) = 1 - \frac{90-60}{100-60} = \frac{1}{4}$

Using the multinomial distribution the required probability is

$\frac{6!}{2! \times 2! \times 2!} \times \left(\frac{1}{4}\right)^2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{4}\right)^2 = 0.088.$

(c) The number of employees that need to be tested before 3 are found with a score larger than 90 has a negative binomial distribution with $r = 3$ and $p = \frac{1}{4}$, which has an expectation of $\frac{r}{p} = 12$.

4.2.2 (a) $E(X) = \frac{1}{0.1} = 10$

(b) $P(X \geq 10) = 1 - F(10) = 1 - (1 - e^{-0.1 \times 10}) = e^{-1} = 0.3679$

(c) $P(X \leq 5) = F(5) = 1 - e^{-0.1 \times 5} = 0.3935$

(d) The *additional* waiting time also has an exponential distribution with parameter $\lambda = 0.1$.

The probability that the total waiting time is longer than 15 minutes is the probability that the *additional* waiting time is longer than 10 minutes, which is 0.3679 from part (b).

(e) $E(X) = \frac{0+20}{2} = 10$ as in the previous case.

However, in this case the *additional* waiting time has a $U(0, 15)$ distribution.

4.2.3 (a) $E(X) = \frac{1}{0.2} = 5$

(b) $\sigma = \frac{1}{0.2} = 5$

(c) The median is $\frac{0.693}{0.2} = 3.47$.

(d) $P(X \geq 7) = 1 - F(7) = 1 - (1 - e^{-0.2\times 7}) = e^{-1.4} = 0.2466$

(e) The memoryless property of the exponential distribution implies that the required probability is
$P(X \geq 2) = 1 - F(2) = 1 - (1 - e^{-0.2\times 2}) = e^{-0.4} = 0.6703$.

4.2.4 (a) $P(X \leq 5) = F(5) = 1 - e^{-0.31\times 5} = 0.7878$

(b) Consider a binomial distribution with parameters $n = 12$ and $p = 0.7878$.
The expected value is
$E(X) = n \times p = 12 \times 0.7878 = 9.45$
and the variance is
$\text{Var}(X) = n \times p \times (1 - p) = 12 \times 0.7878 \times 0.2122 = 2.01$.

(c) $P(B(12, 0.7878) \leq 9) = 0.4845$

4.2.5 $F(x) = \int_{-\infty}^{x} \frac{1}{2}\lambda e^{-\lambda(\theta - y)}\, dy = \frac{1}{2}e^{-\lambda(\theta - x)}$

for $-\infty \leq x \leq \theta$, and

$F(x) = \frac{1}{2} + \int_{\theta}^{x} \frac{1}{2}\lambda e^{-\lambda(y-\theta)}\, dy = 1 - \frac{1}{2}e^{-\lambda(x-\theta)}$

for $\theta \leq x \leq \infty$.

(a) $P(X \leq 0) = F(0) = \frac{1}{2}e^{-3(2-0)} = 0.0012$

(b) $P(X \geq 1) = 1 - F(1) = 1 - \frac{1}{2}e^{-3(2-1)} = 0.9751$

4.2.6 (a) $E(X) = \frac{1}{2} = 0.5$

(b) $P(X \geq 1) = 1 - F(1) = 1 - (1 - e^{-2\times 1}) = e^{-2} = 0.1353$

(c) A Poisson distribution with parameter $2 \times 3 = 6$.

(d) $P(X \leq 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$= \frac{e^{-6}\times 6^0}{0!} + \frac{e^{-6}\times 6^1}{1!} + \frac{e^{-6}\times 6^2}{2!} + \frac{e^{-6}\times 6^3}{3!} + \frac{e^{-6}\times 6^4}{4!} = 0.2851$

4.2.7 (a) $\lambda = 1.8$

(b) $E(X) = \frac{1}{1.8} = 0.5556$

(c) $P(X \geq 1) = 1 - F(1) = 1 - (1 - e^{-1.8\times 1}) = e^{-1.8} = 0.1653$

(d) A Poisson distribution with parameter $1.8 \times 4 = 7.2$.

(e) $P(X \geq 4) = 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3)$

$= 1 - \frac{e^{-7.2}\times 7.2^0}{0!} - \frac{e^{-7.2}\times 7.2^1}{1!} - \frac{e^{-7.2}\times 7.2^2}{2!} - \frac{e^{-7.2}\times 7.2^3}{3!} = 0.9281$

4.2.8 (a) Solving
$F(5) = 1 - e^{-\lambda\times 5} = 0.90$
gives $\lambda = 0.4605$.

(b) $F(3) = 1 - e^{-0.4605\times 3} = 0.75$

4.2.9 (a) $P(X \geq 1.5) = e^{-0.8\times 1.5} = 0.301$

(b) The number of arrivals Y has a Poisson distribution with parameter
$0.8 \times 2 = 1.6$
so that the required probability is

$P(Y \geq 3) = 1 - P(Y = 0) - P(Y = 1) - P(Y = 2)$

$= 1 - \left(e^{-1.6} \times \frac{1.6^0}{0!}\right) - \left(e^{-1.6} \times \frac{1.6^1}{1!}\right) - \left(e^{-1.6} \times \frac{1.6^2}{2!}\right) = 0.217$

4.2.10 $P(X \leq 1) = 1 - e^{-0.3\times 1} = 0.259$
$P(X \leq 3) = 1 - e^{-0.3\times 3} = 0.593$

Using the multinomial distribution the required probability is

$\frac{10!}{2!\times 4!\times 4!} \times 0.259^2 \times (0.593 - 0.259)^4 \times (1 - 0.593)^4 = 0.072.$

4.2.11 (a) $P(X \leq 6) = 1 - e^{-0.2\times 6} = 0.699$

(b) The number of arrivals Y has a Poisson distribution with parameter
$0.2 \times 10 = 2$
so that the required probability is
$P(Y = 3) = e^{-2} \times \frac{2^3}{3!} = 0.180$

4.2.12 $P(X \geq 150) = e^{-0.0065\times 150} = 0.377$
The number of components Y in the box with lifetimes longer than 150 days has a $B(10, 0.377)$ distribution.

$P(Y \geq 8) = P(Y = 8) + P(Y = 9) + P(Y = 10)$

$= \binom{10}{8} \times 0.377^8 \times 0.623^2 + \binom{10}{9} \times 0.377^9 \times 0.623^1 + \binom{10}{10} \times 0.377^{10} \times 0.623^0$

$= 0.00713 + 0.00096 + 0.00006 = 0.00815$

4.2.13 The number of signals X in a 100 meter stretch has a Poisson distribution with mean $0.022 \times 100 = 2.2$.

$P(X \leq 1) = P(X = 0) + P(X = 1)$

$= \left(e^{-2.2} \times \frac{2.2^0}{0!}\right) + \left(e^{-2.2} \times \frac{2.2^1}{1!}\right)$

$= 0.111 + 0.244 = 0.355$

4.2.14 Since

$F(263) = \frac{50}{90} = 1 - e^{-263\lambda}$

it follows that $\lambda = 0.00308$.

Therefore,
$F(x) = \frac{80}{90} = 1 - e^{-0.00308x}$
gives $x = 732.4$.

4.3.1 $\Gamma(5.5) = 4.5 \times 3.5 \times 2.5 \times 1.5 \times 0.5 \times \sqrt{\pi} = 52.34$

4.3.3 (a) $f(3) = 0.2055$
$F(3) = 0.3823$
$F^{-1}(0.5) = 3.5919$

(b) $f(3) = 0.0227$
$F(3) = 0.9931$
$F^{-1}(0.5) = 1.3527$

(c) $f(3) = 0.2592$
$F(3) = 0.6046$
$F^{-1}(0.5) = 2.6229$

In this case

$f(3) = \frac{1.4^4 \times 3^{4-1} \times e^{-1.4\times 3}}{3!} = 0.2592.$

4.3.4 (a) $E(X) = \frac{5}{0.9} = 5.556$

(b) $\sigma = \frac{\sqrt{5}}{0.9} = 2.485$

(c) From the computer the lower quartile is
$F^{-1}(0.25) = 3.743$
and the upper quartile is
$F^{-1}(0.75) = 6.972.$

(d) From the computer $P(X \geq 6) = 0.3733.$

4.3.5 (a) A gamma distribution with parameters $k = 4$ and $\lambda = 2$.

(b) $E(X) = \frac{4}{2} = 2$

(c) $\sigma = \frac{\sqrt{4}}{2} = 1$

(d) The probability can be calculated as
$P(X \geq 3) = 0.1512$
where the random variable X has a gamma distribution with parameters
$k = 4$ and $\lambda = 2$.
The probability can also be calculated as
$P(Y \leq 3) = 0.1512$
where the random variable Y has a Poisson distribution with parameter
$2 \times 3 = 6$
which counts the number of imperfections in a 3 meter length of fiber.

4.3.6 (a) A gamma distribution with parameters $k = 3$ and $\lambda = 1.8$.

(b) $E(X) = \frac{3}{1.8} = 1.667$

(c) $\text{Var}(X) = \frac{3}{1.8^2} = 0.9259$

(d) The probability can be calculated as
$P(X \geq 3) = 0.0948$
where the random variable X has a gamma distribution with parameters
$k = 3$ and $\lambda = 1.8$.

The probability can also be calculated as
$P(Y \leq 2) = 0.0948$
where the random variable Y has a Poisson distribution with parameter
$1.8 \times 3 = 5.4$
which counts the number of arrivals in a 3 hour period.

4.3.7 (a) The expectation is $E(X) = \frac{44}{0.7} = 62.86$

the variance is $\text{Var}(X) = \frac{44}{0.7^2} = 89.80$

and the standard deviation is $\sqrt{89.80} = 9.48$.

(b) $F(60) = 0.3991$

4.4.2 (a) $\frac{(-\ln(1-0.5))^{1/4.9}}{0.22} = 4.218$

(b) $\frac{(-\ln(1-0.75))^{1/4.9}}{0.22} = 4.859$

$\frac{(-\ln(1-0.25))^{1/4.9}}{0.22} = 3.525$

(c) $F(x) = 1 - e^{-(0.22x)^{4.9}}$

$P(2 \leq X \leq 7) = F(7) - F(2) = 0.9820$

4.4.3 (a) $\frac{(-\ln(1-0.5))^{1/2.3}}{1.7} = 0.5016$

(b) $\frac{(-\ln(1-0.75))^{1/2.3}}{1.7} = 0.6780$

$\frac{(-\ln(1-0.25))^{1/2.3}}{1.7} = 0.3422$

(c) $F(x) = 1 - e^{-(1.7x)^{2.3}}$

$P(0.5 \leq X \leq 1.5) = F(1.5) - F(0.5) = 0.5023$

4.4.4 (a) $\frac{(-\ln(1-0.5))^{1/3}}{0.5} = 1.77$

(b) $\frac{(-\ln(1-0.01))^{1/3}}{0.5} = 0.43$

(c) $E(X) = \frac{1}{0.5}\,\Gamma\left(1 + \frac{1}{3}\right) = 1.79$

$\text{Var}(X) = \frac{1}{0.5^2}\left\{\Gamma\left(1 + \frac{2}{3}\right) - \Gamma\left(1 + \frac{1}{3}\right)^2\right\} = 0.42$

(d) $P(X \leq 3) = F(3) = 1 - e^{-(0.5\times 3)^3} = 0.9658$

The probability that at least one circuit is working after three hours is $1 - 0.9688^4 = 0.13$.

4.4.5 (a) $\frac{(-\ln(1-0.5))^{1/0.4}}{0.5} = 0.8000$

(b) $\frac{(-\ln(1-0.75))^{1/0.4}}{0.5} = 4.5255$

$\frac{(-\ln(1-0.25))^{1/0.4}}{0.5} = 0.0888$

(c) $\frac{(-\ln(1-0.95))^{1/0.4}}{0.5} = 31.066$

$\frac{(-\ln(1-0.99))^{1/0.4}}{0.5} = 91.022$

(d) $F(x) = 1 - e^{-(0.5x)^{0.4}}$

$P(3 \leq X \leq 5) = F(5) - F(3) = 0.0722$

4.4.6 (a) $\frac{(-\ln(1-0.5))^{1/1.5}}{0.03} = 26.11$

$\frac{(-\ln(1-0.75))^{1/1.5}}{0.03} = 41.44$

$\frac{(-\ln(1-0.99))^{1/1.5}}{0.03} = 92.27$

(b) $F(x) = 1 - e^{-(0.03x)^{1.5}}$

$P(X \geq 30) = 1 - F(30) = 0.4258$

The number of components still operating after 30 minutes has a binomial distribution with parameters $n = 500$ and $p = 0.4258$.
The expected value is
$E(X) = n \times p = 500 \times 0.4258 = 212.9$
and the variance is
$\text{Var}(X) = n \times p \times (1 - p) = 500 \times 0.4258 \times 0.5742 = 122.2.$

4.4.7 The probability that a culture has developed within four days is
$F(4) = 1 - e^{-(0.3\times 4)^{0.6}} = 0.672.$

Using the negative binomial distribution, the probability that exactly ten cultures are opened is

$$\binom{9}{4} \times (1 - 0.672)^5 \times 0.672^5 = 0.0656.$$

4.4.8 A Weibull distribution can be used with

$F(7) = 1 - e^{-(7\lambda)^a} = \frac{9}{82}$

and

$F(14) = 1 - e^{-(14\lambda)^a} = \frac{24}{82}$.

This gives $a = 1.577$ and $\lambda = 0.0364$ so that the median time is the solution to

$1 - e^{-(0.0364x)^{1.577}} = 0.5$

which is 21.7 days.

4.5.1 (a) Since

$\int_0^1 A\, x^3(1-x)^2\, dx = 1$

it follows that $A = 60$.

(b) $E(X) = \int_0^1 60\, x^4(1-x)^2\, dx = \frac{4}{7}$

$E(X^2) = \int_0^1 60\, x^5(1-x)^2\, dx = \frac{5}{14}$

Therefore,

$\mathrm{Var}(X) = \frac{5}{14} - \left(\frac{4}{7}\right)^2 = \frac{3}{98}$.

(c) This is a beta distribution with $a = 4$ and $b = 3$.

$E(X) = \frac{4}{4+3} = \frac{4}{7}$

$\mathrm{Var}(X) = \frac{4\times 3}{(4+3)^2\times(4+3+1)} = \frac{3}{98}$

4.5.2 (a) This is a beta distribution with $a = 10$ and $b = 4$.

(b) $A = \frac{\Gamma(10+4)}{\Gamma(10)\times\Gamma(4)} = \frac{13!}{9!\times 3!} = 2860$

(c) $E(X) = \frac{10}{10+4} = \frac{5}{7}$

(d) $\mathrm{Var}(X) = \frac{10\times 4}{(10+4)^2\times(10+4+1)} = \frac{2}{147}$

$\sigma = \sqrt{\frac{2}{147}} = 0.1166$

(e) $F(x) = \int_0^x\ 2860\ y^9\ (1-y)^3\ dy$

$= 2860\left(\frac{x^{10}}{10} - \frac{3x^{11}}{11} + \frac{x^{12}}{4} - \frac{x^{13}}{13}\right)$

for $0 \leq x \leq 1$

4.5.3 (a) $f(0.5) = 1.9418$
$F(0.5) = 0.6753$
$F^{-1}(0.75) = 0.5406$

(b) $f(0.5) = 0.7398$
$F(0.5) = 0.7823$
$F^{-1}(0.75) = 0.4579$

(c) $f(0.5) = 0.6563$
$F(0.5) = 0.9375$
$F^{-1}(0.75) = 0.3407$

In this case

$f(0.5) = \frac{\Gamma(2+6)}{\Gamma(2)\times\Gamma(6)} \times 0.5^{2-1} \times (1-0.5)^{6-1} = 0.65625.$

4.5.4 (a) $3 \leq y \leq 7$

(b) $E(X) = \frac{2.1}{2.1+2.1} = \frac{1}{2}$

Therefore, $E(Y) = 3 + (4 \times E(X)) = 5$.

$\text{Var}(X) = \frac{2.1\times2.1}{(2.1+2.1)^2\times(2.1+2.1+1)} = 0.0481$

Therefore, $\text{Var}(Y) = 4^2 \times \text{Var}(X) = 0.1923$.

(c) The random variable X has a symmetric beta distribution so $P(Y \leq 5) = P(X \leq 0.5) = 0.5$.

4.5.5 (a) $E(X) = \frac{7.2}{7.2+2.3} = 0.7579$

$\text{Var}(X) = \frac{7.2\times2.3}{(7.2+2.3)^2\times(7.2+2.3+1)} = 0.0175$

(b) From the computer $P(X \geq 0.9) = 0.1368$.

4.5.6 (a) $E(X) = \frac{8.2}{8.2+11.7} = 0.4121$

(b) $\text{Var}(X) = \frac{8.2\times11.7}{(8.2+11.7)^2\times(8.2+11.7+1)} = 0.0116$

$\sigma = \sqrt{0.0116} = 0.1077$

(c) From the computer $F^{-1}(0.5) = 0.4091$.

4.8.1 $F(0) = P(\text{winnings} = 0) = \frac{1}{4}$

$F(x) = P(\text{winnings} \leq x) = \frac{1}{4} + \frac{x}{720} \quad \text{for} \quad 0 \leq x \leq 360$

$F(x) = P(\text{winnings} \leq x) = \frac{\sqrt{x+72540}}{360} \quad \text{for} \quad 360 \leq x \leq 57060$

$F(x) = 1 \quad \text{for} \quad 57060 \leq x$

4.8.2 (a) Solving

$\frac{0.693}{\lambda} = 1.5$

gives $\lambda = 0.462$.

(b) $P(X \geq 2) = 1 - F(2) = 1 - (1 - e^{-0.462\times2}) = e^{-0.924} = 0.397$

$P(X \leq 1) = F(1) = 1 - e^{-0.462\times1} = 0.370$

4.8.3 (a) $E(X) = \frac{1}{0.7} = 1.4286$

(b) $P(X \geq 3) = 1 - F(3) = 1 - (1 - e^{-0.7\times 3}) = e^{-2.1} = 0.1225$

(c) $\frac{0.693}{0.7} = 0.9902$

(d) A Poisson distribution with parameter $0.7 \times 10 = 7$.

(e) $P(X \geq 5) = 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3) - P(X = 4)$
$= 0.8270$

(f) A gamma distribution with parameters $k = 10$ and $\lambda = 0.7$.

$E(X) = \frac{10}{0.7} = 14.286$

$\text{Var}(X) = \frac{10}{0.7^2} = 20.408$

4.8.4 (a) $E(X) = \frac{1}{5.2} = 0.1923$

(b) $P\left(X \leq \frac{1}{6}\right) = F\left(\frac{1}{6}\right) = 1 - e^{-5.2\times 1/6} = 0.5796$

(c) A gamma distribution with parameters $k = 10$ and $\lambda = 5.2$.

(d) $E(X) = \frac{10}{5.2} = 1.923$

(e) The probability is
$P(X > 5) = 0.4191$
where the random variable X has a Poisson distribution with parameter 5.2.

4.8.5 (a) The total area under the triangle is equal to 1 so the height at the midpoint is $\frac{2}{b-a}$.

(b) $P\left(X \leq \frac{a}{4} + \frac{3b}{4}\right) = P\left(X \leq a + \frac{3(b-a)}{4}\right) = \frac{7}{8}$

(c) $\mathrm{Var}(X) = \frac{(b-a)^2}{24}$

(d) $F(x) = \frac{2(x-a)^2}{(b-a)^2}$

for $a \leq x \leq \frac{a+b}{2}$

and

$F(x) = 1 - \frac{2(b-x)^2}{(b-a)^2}$

for $\frac{a+b}{2} \leq x \leq b$

4.8.6 (a) $\frac{(-\ln(1-0.5))^{1/4}}{0.2} = 4.56$

$\frac{(-\ln(1-0.75))^{1/4}}{0.2} = 5.43$

$\frac{(-\ln(1-0.95))^{1/4}}{0.2} = 6.58$

(b) $E(X) = \frac{1}{0.2}\,\Gamma\left(1 + \frac{1}{4}\right) = 4.53$

$\mathrm{Var}(X) = \frac{1}{0.2^2}\left\{\Gamma\left(1 + \frac{2}{4}\right) - \Gamma\left(1 + \frac{1}{4}\right)^2\right\} = 1.620$

(c) $F(x) = 1 - e^{-(0.2x)^4}$

$P(5 \leq X \leq 6) = F(6) - F(5) = 0.242$

4.8.7 (a) $E(X) = \frac{2.7}{2.7+2.9} = 0.4821$

(b) $\mathrm{Var}(X) = \frac{2.7 \times 2.9}{(2.7+2.9)^2 \times (2.7+2.9+1)} = 0.0378$

$\sigma = \sqrt{0.0378} = 0.1945$

(c) From the computer $P(X \geq 0.5) = 0.4637$.

4.8.8 Let the random variable Y be the starting time of the class in minutes after 10 o'clock, so that $Y \sim U(0,5)$.

If $x \leq 0$, the expected penalty is
$A_1(|x| + E(Y)) = A_1(|x| + 2.5)$.

If $x \geq 5$, the expected penalty is
$A_2(x - E(Y)) = A_2(x - 2.5)$.

If $0 \leq x \leq 5$, the penalty is
$A_1(Y - x)$ for $Y \geq x$ and $A_2(x - Y)$ for $Y \leq x$.

The expected penalty is therefore

$\int_x^5 A_1(y-x)f(y)\ dy + \int_0^x A_2(x-y)f(y)\ dy$

$= \int_x^5 A_1(y-x)\frac{1}{5}\ dy + \int_0^x A_2(x-y)\frac{1}{5}\ dy$

$= \frac{A_1(5-x)^2}{10} + \frac{A_2 x^2}{10}$.

The expected penalty is minimized by taking

$x = \frac{5A_1}{A_1+A_2}$.

4.8.9 (a) Solving simultaneously

$F(35) = 1 - e^{-(\lambda \times 35)^a} = 0.25$

and

$F(65) = 1 - e^{-(\lambda \times 65)^a} = 0.75$

gives $\lambda = 0.0175$ and $a = 2.54$.

(b) Solving

$F(x) = 1 - e^{-(0.0175 \times x)^{2.54}} = 0.90$

gives x as about 79 days.

4.8.10 For this beta distribution $F(0.5) = 0.0925$ and $F(0.8) = 0.9851$
so that the probability of a solution being too weak is 0.0925
the probability of a solution being satisfactory is $0.9851 - 0.0925 = 0.8926$
and the probability of a solution being too strong is $1 - 0.9851 = 0.0149$.

Using the multinomial distribution, the required answer is

$\frac{10!}{1!\times 8!\times 1!} \times 0.0925 \times 0.8926^8 \times 0.0149 = 0.050$.

4.8.11 (a) The number of visits within a two hour interval has a Poisson distribution with parameter $2 \times 4 = 8$.

$P(X = 10) = e^{-8} \times \frac{8^{10}}{10!} = 0.099$

(b) A gamma distribution with $k = 10$ and $\lambda = 4$.

4.8.12 (a) $\frac{1}{\lambda} = \frac{1}{0.48} = 2.08$ cm

(b) $\frac{10}{\lambda} = \frac{10}{0.48} = 20.83$ cm

(c) $P(X \leq 0.5) = 1 - e^{-0.48\times 0.5} = 0.213$

(d) $P(8 \leq X \leq 12) = \sum_{i=8}^{12} e^{-0.48\times 20} \frac{(0.48\times 20)^i}{i!} = 0.569$

4.8.13 (a) False

(b) True

(c) True

(d) True

4.8.14 Using the multinomial distribution the probability is

$\frac{5!}{2!\times 2!\times 1!} \times \left(\frac{2}{5}\right)^2 \times \left(\frac{2}{5}\right)^2 \times \left(\frac{1}{5}\right)^2 = \frac{96}{625} = 0.154.$

4.8.15 (a) The number of events in the interval has a Poisson distribution with parameter $8 \times 0.5 = 4$.

$P(X = 4) = e^{-4} \times \frac{4^4}{4!} = 0.195$

(b) The probability is obtained from an exponential distribution with $\lambda = 8$ and is

$1 - e^{-8\times 0.2} = 0.798.$

4.8.16 $P(X \leq 8) = 1 - e^{-(0.09\times 8)^{2.3}} = 0.375$

$P(8 \leq X \leq 12) = 1 - e^{-(0.09\times 12)^{2.3}} - 0.375 = 0.322$

$P(X \geq 12) = 1 - 0.375 - 0.322 = 0.303$

Using the multinomial distribution the required probability is

$\frac{10!}{3!\times 4!\times 3!} \times 0.375^3 \times 0.322^4 \times 0.303^3 = 0.066.$