6/07/20

Agenda for Math 5710 ♬ Meeting #21 ☻☺ 7/21/20 (8:00 a.m. – 9:10 a.m.)

1. Hello:

Brigham City:	Adam Blakeslee	Ryan Johnson	Tyson Mortensen	
Logan:	David Allen	Natalie Anderson	Kameron Baird	Stephen Brezinski
	Zachary Ellis	Adam Flanders	Brock Francom	Xiang Gao
	Ryan Goodman	Janette Goodridge	Hadley Hamar	Phillip Leifer
	Brittney Miller	Jonathan Mousley	Erika Mueller	Shelby Simpson
	Steven Summers	Matthew White	Zhang Xiaomeng	

2. Note the syllabus' activity list for today:

21 T/7/21	1. Construct the following concepts, comprehend associated communication structures, discover associated relationships, and employ associated algorithms: Bernoulli trials, binomial probabilities, 2. Comprehend and design a proof for Theorem 11. 3. Take advantage of Quiz 21.

3. Briefly raise issues and questions prompted by the following homework assignment:

   A. Study our notes from Meeting #20.

   B. Comprehend Entry #041A–C of our *Glossary*.

   C. From the Video Page of *Canvas*, view with comprehension the video named "expected value and variance of discrete random variables (better)."

   D. Re-analyze our old friend Sybil's problem and ponder the following questions:

      i. What is the random variable X?

      ii. How does μ_X and σ_X^2 compare to $E(X)$ and $V(X)$?

   And for the sake of convenience, here is a copy of Sybil's problem:

For the purpose of formulating the rules of a game of chance in which a pair of fair dice (a red die and a yellow die) are rolled one time, Sybil wants to determine the likelihood of each of the possible *events* determined by the sum of the number of dots that appear on the top face of the yellow die and on the top face of the red die.

Sybil thinks, "Each die has six faces – a face with one dot, a face with two dots, a face with three dots, a face with four dots, a face with five dots, and a face with six dots. So there are 36 possible *outcomes* since 36 is the cardinality of the following set:

> { (r, y) : r = the number of dots on the red die's top face $\wedge$
> y = the number of dots on the red die's top face } =
> { (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5),
> (6, 6) }

And since I'm interested in the *random probability* of each of the possible *events*, the *events* of interest are the sum of numbers associated with the two top faces. So each event is associated with one element in the following set: { 2, 3, 4, 5, ..., 12 }. I'll count the number outcomes are associated with each of the 12 events:

> Let X_i = the event that the sum is i. Thus,
> $|X_2|$ = |{ (1, 1) }| = 1
> $|X_3|$ = |{ (1, 2), (2, 1) }| = 2
> $|X_4|$ = |{ (1, 3), (2, 2), (3, 1) }| = 3
> $|X_5|$ = |{ (1, 4), (2, 3), (3, 2), (4, 1) }| = 4
> $|X_6|$ = |{ (1, 5), (2, 4), (3, 3), (4, 2), (5, 1) }| = 5
> $|X_7|$ = |{ (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) }| = 6
> $|X_8|$ = |{ (2, 6), (3, 5), (4, 4), (5, 3), (6, 2) }| = 5
> $|X_9|$ = |{ (3, 6), (4, 5), (5, 4), (6, 3) }| = 4
> $|X_{10}|$=|{ (4, 6), (5, 5), (6, 4) }| = 3
> $|X_{11}|$=|{ (5, 6), (6, 5) }| = 2
> $|X_{12}|$=|{ (6, 6) }| = 1

So the most likely event is X_7 and the two events that are less likely than any of the others are X_2 and X_{11}. And my chart above shows the scale for each events. Then I'll use the chart to compute the exact theoretical probabilities by reporting the probabilities as the ratio of each of those events and 36 (i.e., $p(X_i) = X_i \div 36$."

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4. Consider the common attributes of the following experiments and then compare those attributes to our definition of a *Bernoulli random variable*:

   A. A person is randomly selected from a population and tested for COVID-19 infection. A positive test result is labeled a “success” and coded as 1; a negative test result is labeled a “failure” and coded as 0.

   B. Three fair dice are randomly rolled and the sum of the dots on the three upper-facing surfaces is recorded. An even sum is considered a success and coded 1; an odd sum is considered a failure and coded 0

   C. 042. Definition for a *Bernoulli random variable*:

      $X \in \{$ Bernoulli random variables of $\Omega \} \Leftrightarrow$
      $X \in \{$ discrete random variables of $\Omega \} \wedge X : \Omega \rightarrow \{ 0, 1 \}$

5. Consider the common attributes of the following experiments and then compare those attributes to our definition of a *binomial random variable*:

   A. A person is randomly selected from a population and tested for COVID-19 infection. A positive test result is labeled a “success” and coded as 1; a negative test result is labeled a “failure” and coded as 0. Again a person is randomly selected from that *same* population (Thus, the first person is still the population; so the two events are independent). The trial is repeated another 98 times. The number of success is recorded.

   B. Three fair dice are randomly rolled and the sum of the dots on the three upper-facing surfaces is recorded. An even sum is considered a success and coded 1; an odd sum is considered a failure and coded 0. The number of success is recorded. The trial is repeated 49 more times. The number of successes is recorded.

   C. 43A. Definition for a Binomial random variable: :

      Given $n \in \mathbb{N} \wedge \Omega = \{ 1, 2, 3, ..., n \} \wedge$
      $X \in \{$ Bernoulli random variables of $\{ 0, 1\}\} \wedge$
      (A string of n experiments are conducted with $X \ni (X(i) = 0 \veebar (X(i) = 1$
      depending on the results of the i^{th} experiment $\wedge |\{ (i, X(i)) : X(i) = 1 \}| = k$),
      ($Y \in \{$ binomial random variables of $\Omega \} \Leftrightarrow$

      $$Y : \Omega \rightarrow \omega \ni Y(i) = \sum_{i}^{n} X(i) = k$$

6. Comprehend the following proposition and design a proof for elevating it to a theorem:

43B. Theorem 11:

$$(X \in \{ \text{Bernoulli random variables of } \{ 0, 1 \}\} \wedge m : X \rightarrow [0, 1] \wedge$$
$$(\Omega = \{ 0, 1, 2, ..., n \} \wedge Y \in \{ \text{binomial random variables of } \Omega \}) \wedge p : Y \rightarrow [0, 1]) \Rightarrow$$
$$p(k) = \binom{n}{k} m(1)^k (1 - m(1))^{n-k}$$

7. Take advantage of Quiz 21.

8. Complete the following assignment prior to Meeting #22:

A. Study our notes from Meeting #21.

B. Comprehend Jim's sample response to Quiz 21.

C. Comprehend Entries #042 & #43A–B of our *Glossary*.

D. From the Video Page of *Canvas*, view with comprehension the videos named "intro bernoulli distributions" and "visualizing binomial distributions."

E*. Please solve the following problems; display the computations, and upload the resulting pdf document on the appropriate Canvas assignment link:

i. A person is randomly selected from a population and tested for COVID-19 infection. A positive test result is labeled a "success" and coded as 1; a negative test result is labeled a "failure" and coded as 0. Again a person is randomly selected from that *same* population (Thus, the first person is still the population; so the two events are independent). The trial is repeated another 3 times. The number of successes is recorded. As of May 26, 2020, one seemingly credible estimate is 30% of the people worldwide are infected; use that figure for this problem. Display the probability distribution for the random variable for this experiment.

ii. Three fair dice are randomly rolled and the sum of the dots on the three upper-facing surfaces is recorded. An even sum is considered a success and coded 1; an odd sum is considered a failure and coded 0. The number of success is recorded. The trial is repeated 5 more times. The number of successes is recorded. Display the probability distribution for the random variable for this experiment.

E. Comprehend Jim's sample responses to the homework prompts that are posted on *Canvas*.

9. Consider the source (i.e., Hernando de Soto (1497–1542) of the following quote:

Blood and cruelty are the foundations of all good things.