HW: Functions of Random Variables

Brock Francom
A02052161

2.6: 1, 3, 4, 5, 6, 8ab, 12, 13, 15, 19

2.6

1a) $E(3x+7) = 3E(x) + 7 = \boxed{13}$

$Var(3x+7) = 3^2 Var(x) = \boxed{36}$

b) $E(5x-9) = 5E(x) - 9 = \boxed{1}$

$Var(5x-9) = 5^2 Var(x) = \boxed{100}$

c) $E(2x+6y) = 2E(x) + 6E(y) = \boxed{-14}$

$Var(2x+6y) = 2^2 Var(x) + 6^2 Var(y) = \boxed{88}$

d) $E(4x-3y) = 4(E(x)) + -3E(y) = \boxed{17}$

$Var(4x-3y) = 4^2 Var(x) + (-3)^2 Var(y) = \boxed{82}$

e) $E(5x-9z+8) = 5E(x) - 9E(z) + 8 = \boxed{-54}$

$Var(5x-9z+8) = 5^2 Var(x) + (-9)^2 E(z) = \boxed{669}$

f) $E(-3y-z-5) = -3E(y) - E(z) - 5 = \boxed{-4}$

$Var(-3y-z-5) = (-3)^2 E(y) + (-1)^2 E(z) = \boxed{25}$

g) $E(x+2y+3z) = E(x) + 2E(y) + 3E(z) = \boxed{20}$

$Var(x+2y+3z) = Var(x) + 2^2 Var(y) + 3^2 Var(z) = \boxed{75}$

h) $E(6x+2y-z+16) = 6E(x) + 2E(y) - E(z) + 16 = \boxed{14}$

$Var(6x+2y-z+16) = 6^2 Var(x) + 2^2 Var(y) + (-1)^2 Var(z) = \boxed{159}$

3) $E(Y) = 3E(x_1) = 3\mu$, $Var(Y) = 3^2 Var(x_1) = 9\sigma^2$

$E(\bar{x}) = E(x_1) + E(x_2) + E(x_3) = 3\mu$, $Var(\bar{x}) = Var(x_1) + Var(x_2) + Var(x_3) = 3\sigma^2$

4) $E(A_1 + A_2 + B) = 37 + 37 + 24 = \boxed{98}$

$Var(A_1 + A_2 + B) = .7^2 + .7^2 + .3^2 = \boxed{1.07}$

5) $E(x) = 10\left(\frac{1}{8}\right) + (-1)\left(\frac{7}{8}\right) = \frac{3}{8}$

$E(x^2) = 10^2\left(\frac{1}{8}\right) + (-1)^2\left(\frac{7}{8}\right) = \frac{107}{8}$

$Var(x) = \frac{107}{8} - \left(\frac{3}{8}\right)^2 = \frac{847}{64}$

total winnings $= x_1 + \ldots + x_{50} = Y$

$E(Y) = E(x_1) + \ldots + E(x_{50}) = \frac{3}{8} \cdot 50 = \boxed{18.75 \text{ dollars}}$

$Var(Y) = Var(x_1) + \ldots + Var(x_{50}) = 50\left(\frac{847}{64}\right) = 661.72$

$\sigma_Y = \sqrt{661.72} = \boxed{25.72 \text{ dollars}}$

6a) E(Average weight) = 1.12 kg

Var(average weight) = $\frac{.03^2}{25}$

$\sigma = \sqrt{\frac{.03^2}{25}} = .006$

b) $.005^2 = \frac{.03^2}{n}$ ⟶ $n(.005)^2 = .03^2$ → $\frac{.03^2}{.005^2} = n$ $n = 36$

8a) $E(Y) = E(X^3) = \int_0^1 x^3 \cdot 2x\,dx = 0.4$

b) $E(Y) = E(\sqrt{X}) = \int_0^1 \sqrt{x}\, 2x\,dx = 0.8$

12) $Y = X_1 + X_2 + X_3 + \ldots + X_5$

$E(Y) = E(X_1) + E(X_2) + \ldots = 5 \cdot E(X) = 52.09$

$Var(Y) = 5 \cdot Var(X) = .379$

$\sigma_Y = \sqrt{.379} = 0.616$

13a) $E(X) = \frac{59 + 67 + 72}{3} = 66$

$Var(X) = \left(\frac{1}{3}\right)^2 \cdot Var(x_1) + \left(\frac{1}{3}\right) \cdot Var(x_2) + \left(\frac{1}{3}\right) Var(x_3) = \frac{95}{3}$

$\sigma_X = \sqrt{\frac{95}{3}} = 5.63$

b) $E(x) = .4(59) + .4(67) + .2(72) = 64.8$

$Var(x) = .4^2(10)^2 + .4^2(13)^2 + .2^2(4)^2 = 43.68$

$\sigma_X = \sqrt{43.68} = 6.61$

15a) $\mu = 65.90$, $\sigma = \frac{\sigma}{\sqrt{5}} = .143$

b) $\mu = 8\mu = 527.2$ $\sigma = \sqrt{8}\sigma = 0.905$

19) $A_C = \frac{5(110-32)}{9} = 43.33$ $\quad 5(F-32) = \frac{9C}{5}$

$Var(C) = Var\left(\frac{5(F-32)}{9}\right) = Var\left(\frac{5^2 Var(F)}{9^2}\right) = \frac{5^2 \cdot 2.2^2}{9^2} = 1.49$

0.2 Rules for Average and Standard Deviation

Example 4. The factory manager has figured out that average number of equipment breakdowns per day is 0.25, and the standard deviation is 0.536 breakdowns.

1. He estimates that each breakdown costs \$500 in repairs and production costs.

   (a) What is the average cost for breakdowns each day?

$E(C) = .25(500) = 125$

   (b) What is the standard deviation of the costs?

$Var(C) = 500^2(var(B)) = 71,825$

$\sigma_C = 268.0018$

Example 5. An insurance company currently has an average profit of \$325 per policy and the standard deviation is \$40. The shareholders decide that they want more profit. They decide to charge \$50 more for each policy.

1. What is the new average profit per policy?

$E(N) = E(P+50) = \$375$

2. What is the new standard deviation?

$Var(N) = Var(P+50) = Var(P) = 40^2 = 1600$

$\sigma_N = \sqrt{1600} = \$40$