Homework Confidence Intervals

Confidence Interval for Population Mean μ (σ is known)

$$\overline{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

Assumption: normal population or $n \geq 30$

Confidence Interval for Population Mean μ (σ is unknown)

$$\overline{x} \pm t^* \frac{s}{\sqrt{n}}$$

Degrees of freedom: $n - 1$

Assumption: normal population or $n \geq 30$

Homework Z Confidence Interval

1. A study measured the tartrate resistant acid phosphatase (TRAP) in blood of young women. There were 31 subjects in the sample and the mean was 13.2 U/l. Assume the population standard deviation is 6.5 U/l.

   (a) Check the conditions. $n=31 \geq 30$ ✓ normal

   (b) What is the margin of error? $z^* = 1.96$, $m = 1.96\left(\frac{6.5}{\sqrt{31}}\right) = 2.3$

   (c) Find the 95% confidence interval. $13.2 \pm 2.3 \rightarrow (10.9, 15.5)$

   (d) Interpret the interval. we are 95% sure the average TRAP level in all young women is between 10.9 and 15.5

   (e) If you wanted to do the study again, but you want a margin of error less than .2 U/l, what sample size do you need? $.2 = 1.96\left(\frac{6.5}{\sqrt{n}}\right)$, $n = 4057.69$, 4058

2. The same study measured osteocalcin (OC). There were still 31 young women with a mean of 33.4 ng/ml. Assume the population standard deviation is 19.6 ng/ml.

   (a) Check the conditions. $n=31 \geq 30$ ✓ normal

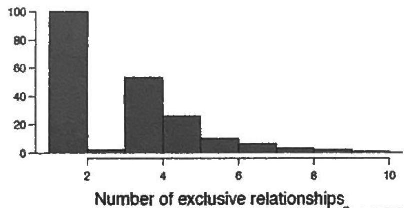
   (b) What is the margin of error? $z^* = 2.576$, $m = 2.576\left(\frac{33.4}{\sqrt{31}}\right) = 9.1$

   (c) Find the 99% confidence interval. 33.4 ± 9.1 $(24.3, 42.5)$

   (d) Interpret the interval. we are 99% sure the average OC in all yw is between 24.3 and 42.5

   (e) If you wanted to do the study again, but you want a margin of error less than 1.5ng/ml, what sample size do you need? $1.5 = 2.576\left(\frac{33.4}{\sqrt{n}}\right)$, $n = 1132.97$, 1133

3. A survey conducted on a random sample of 203 undergraduates asked about the number of exclusive relationships these students have been in. The histogram below shows the distribution of the data from this sample. The sample average is 3.2 with a population standard deviation of 1.97.
   Estimate the average number of exclusive relationships students have been in using a 90% confidence interval.



   $n = 203 \geq 30$ ✓ normal

   (a) Check the conditions. $z^* = 1.645$, $3.2 \pm 1.645\left(\frac{1.97}{\sqrt{203}}\right)$ $(2.973, 3.427)$

   (b) Find the 90% confidence interval.

   (c) Interpret the interval in context. we are 90% confident that the average # of relations for all college kids is between 2.973 and 3.427.

   (d) What does the 90% confidence level mean? If we took lots of samples 90% would actually contain the pop. mean μ.

4. **OPTIONAL:** The Student monitor surveyed 1200 undergraduates from 100 colleges each year. This year they found the average amount of time spent per week on the internet was 19 hours. Assume the population standard deviation is 5.5 hours.

   (a) Check the conditions. normal

   (b) What is the margin of error?

   (c) Find the 99% confidence interval.

   (d) Interpret the interval.

   (e) They want to do the study again, but they've decided they don't actually need as much accuracy because this isn't really that important (it's not like it's a medical study that will affect a person's health). They want to save money next year and only sample enough students to get a margin of error of 1.1 or less.

5. The 2010 General Social Survey asked the question: "After an average work day, about how many hours do you have to relax or pursue activities that you enjoy?" to a random sample of 1,155 Americans.
A 95% confidence interval for the mean number of hours spent relaxing or pursuing activities they enjoy was (1.38, 1.92). They are 95% confident that All americans spend an average of 1.38 to 1.92 hours per day relaxing.
   (a) Interpret this interval in context of the data.
   (b) Suppose another set of researchers reported a confidence interval with a larger margin of error based on the same sample of 1,155 Americans. If it was the same sample, how did they get a larger margin of error? They had a bigger confidence level. 99% or something.
   (c) If they took a new sample with 2500 Americans, how would the margin of error change?
   It would decrease

6. **OPTIONAL:** The National Survey of Family Growth conducted by the Centers for Disease Control gathers information on family life, marriage and divorce, pregnancy, infertility, use of contraception, and men's and women's health. One of the variables collected on this survey is the age at first marriage. The histogram below shows the distribution of ages at first marriage of 5,534 randomly sampled women between 2006 and 2010. The average age at first marriage among these women is 23.44. The population standard deviation is 4.72.
Estimate the average age at first marriage of women using a 95% confidence interval, and interpret this interval in context. Discuss any relevant assumptions.

   (a) Check the conditions.
   (b) Find the 95% confidence interval.
   (c) Interpret the interval in context.
   (d) What does the 95% confidence level mean?

7. The 2010 General Social Survey asked the question: "For how many days during the past 30 days was your mental health, which includes stress, depression, and problems with emotions, not good?" Based on responses from 1,151 US residents, the survey reported a 95% confidence interval of 3.40 to 4.24 days in 2010. They are 95% confident that all US residents, they experience mental health 3.4 to 4.24 days each month
   (a) Interpret this interval in context of the data.
   (b) What does "95% confident" mean? That if they did this a lot, they would have the pop μ in their interval 95% of the time.
   (c) Suppose the researchers think a 99% confidence level would be more appropriate for this interval. Will this new interval be smaller or larger than the 95% confidence interval? Larger
   (d) If a new survey were to be done with 500 Americans, would the width of the confidence interval be larger, smaller, or about the same. larger

8. A hospital administrator hoping to improve wait times decides to estimate the average emergency room waiting time at her hospital. She collects a simple random sample of 64 patients and determines the time (in minutes) between when they checked in to the ER until they were first seen by a doctor. They do a Z 95% confidence interval based on this sample giving (128 minutes, 147 minutes).
Determine whether the following statements are true or false, and explain your reasoning.

   (a) This confidence interval is not valid since we do not know if the population distribution of the ER wait times is nearly Normal. False, 68 > 30 ✓ normal

   (b) We are 95% confident that the average waiting time of these 64 emergency room patients is between 128 and 147 minutes. False, you are 100% sure of that.

   (c) We are 95% confident that the average waiting time of all patients at this hospital's emergency room is between 128 and 147 minutes. True.

   (d) 95% of random samples have a sample mean between 128 and 147 minutes. False.

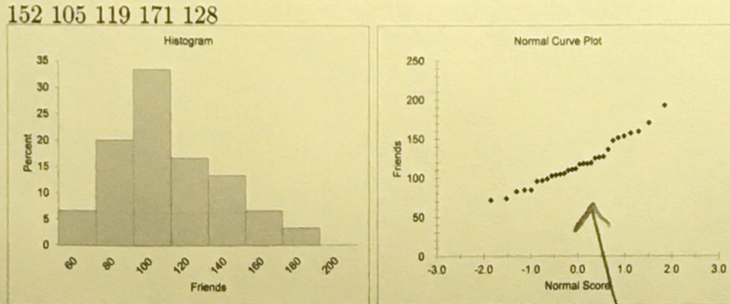
   (e) A 99% confidence interval would be narrower than the 95% confidence interval since we need to be more sure of our estimate. False. It's bigger

9. **OPTIONAL:** We want to find a 95% confidence interval for the mean dust exposure for drill and blast workers. Assume the population standard deviation is $\sigma = 7.8$. How big of a sample do you need if you want a margin of error less than 1.5?

10. **OPTIONAL:** We want to find a 99% confidence interval for the population mean self efficacy of eating low fat foods. Assume the population standard deviation is $\sigma = 1.12$. How big of a sample do you need if you want a margin of error less than .5?

T confidence interval for the Mean

11. We have the data set for the number of friends for $n = 30$ facebook users.
Data Set: 99 148 158 126 118 112 103 111 154 85 120 127 137 74 85 104 106 72 119 160 83 110 97 193 96 152 105 119 171 128



The summary data is $\overline{x} = 119.07$ and $s = 29.57$.

(a) Does the sample data look normally distributed?
yes straight.

(b) Can we use the T confidence interval for this problem? (Is the population normally distributed or $n \geq 30$?)
$n = 30$ ✓

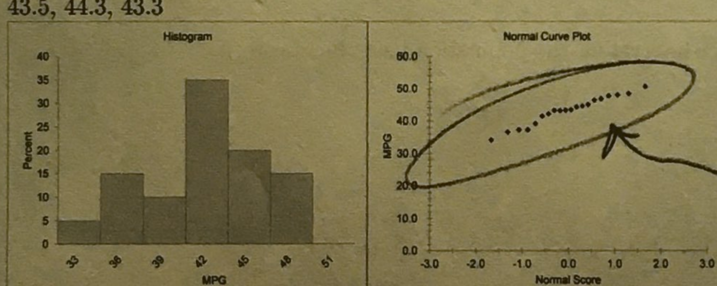
(c) What is the 95% confidence interval for the average number of friends for all Face book users?
$df = 29$, $t^* = 2.045$

$$119.07 \pm 2.045\left(\frac{29.57}{\sqrt{30}}\right)$$

$(108.03, 130.11)$

(d) Interpret your results.
we are 95% confidence that the mean number of Facebook friends is between 108.03 and 130.11

12. We have a data set of 20 gas mileages.
Data Set: 41.5, 50.7, 36.6, 37.3, 34.2, 45.0, 48.0, 43.2, 47.7, 42.2, 43.2, 44.6, 48.4, 46.4, 46.8, 39.2, 37.3, 43.5, 44.3, 43.3



The summary data is $\overline{x} = 43.17$ and $s = 4.415$.

(a) Can we use the T confidence interval for this problem?
It looks semi normal, yes.

(b) Find the 95% confidence interval for the population mean mileage μ.
$df = 19$ $t^* = 2.093$

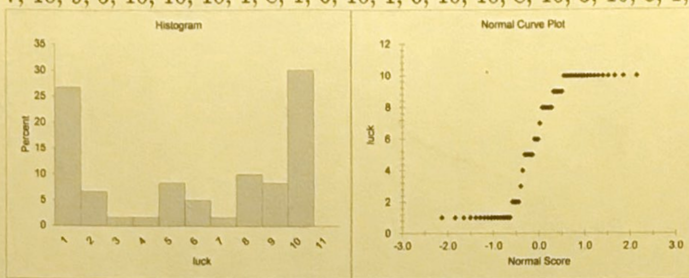
$$43.17 \pm 2.093\left(\frac{4.415}{\sqrt{20}}\right)$$

$(41.1038, 45.2363)$

(c) Interpret your result.
we are 95% confident that the population mean milage is between 41.1038 and 45.2363 miles

13. Children in a study were asked to solve some puzzles and were then given feedback on their performance. Then they were asked to rate how luck played a role in determining their scores. The scores were on a scale from 1 to 10.

Data Set:1, 10, 1, 10, 1, 1, 10, 5, 1, 1, 8, 1, 10, 2, 1, 9, 5, 2, 1, 8, 10, 5, 9, 10, 10, 9, 6, 10, 1, 5, 1, 9, 2, 1, 7, 10, 9, 5, 10, 10, 10, 1, 8, 1, 6, 10, 1, 6, 10, 10, 8, 10, 3, 10, 8, 1, 8, 10, 4, 2



The summary data is $n = 60$, $\overline{x} = 5.9$ and $s = 3.77$.

(a) Why do you think the normal quantile plot looks like it has steps instead of being continuous?

It is discrete data.

(b) Can we use the T confidence interval for this problem?

Sample size = 60 so yes.

(c) Could you have used the T confidence interval for this problem if the sample size was $n = 15$?

nope.

(d) Find the 95% confidence interval for the population mean luck score μ.

$n = 60$ $df = 59$, but 50. $t^* = 2.009$

$5.9 \pm 2.009\left(\frac{3.77}{\sqrt{60}}\right)$

$(4.92, 6.88)$

(e) Interpret your result.

we are 95% confident that the mean luck score is between 4.92–6.88

14. In a study of children with ADHD, parents were asked to rate their child on "Has difficulty organizing work" rated on a scale from 0 to 4.

The summary data is $n = 282$, $\overline{x} = 2.22$ and the sample standard deviation is $s = 1.03$.

(a) Check assumptions.

$n = 282 > 30$ ✓

(b) Could you have used the T confidence interval for this problem if the sample size was $n = 15$?

nope

(c) Find the 99% confidence interval for the population mean rating μ.

df = 281 use 200 $t^* = 2.601$

$$2.22 \pm 2.601\left(\frac{1.03}{\sqrt{282}}\right)$$

$(2.06, 2.38)$

(d) Interpret your result.

we are 99% confident that the mean rating for all ADHD kids is between 2.06 - 2.38.

(e) Find the 90% confidence interval for μ.

$t^* = 1.653$

$$2.22 \pm 1.653\left(\frac{1.03}{\sqrt{282}}\right)$$

$(2.12, 2.32)$

(f) Find the 95% confidence interval for μ.

$t^* = 1.971$

$$2.22 \pm 1.971\left(\frac{1.03}{\sqrt{282}}\right)$$

$(2.099, 2.341)$

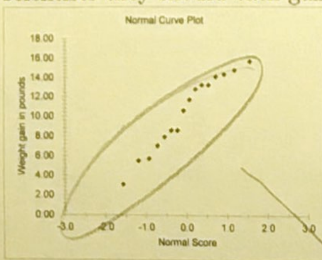
(g) Fill out the table.

Confidence Level	Margin of Error
90%	.10
95%	.12
99%	.16

(h) What happens to the margin of error as the confidence level increases?

It gets bigger

15. In a study 16 adults aged 25-36 years were fed an extra 1000 calories for 8 weeks. According to nutritional scientists they should each gain 16 pounds.



From the sample of 16 adults the mean was 10.41 and the standard deviation was 3.84.

(a) Check assumptions.

n=16 semi straight, but I'm not confident with it. So no.

(b) Find the 90% confidence interval for the population mean weight gain μ.

no, not normal

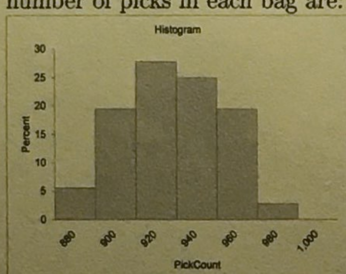
(c) What is the margin of error?

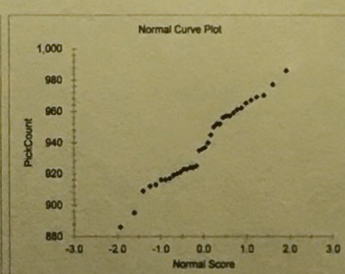
not normal

(d) Interpret your result.

(e) Does your result agree with the proposed weight gain of 16 pounds per person?

16. A guitar supply company advertises that their bags contain between 900 and 1000 picks. A simple random sample of thirty-six bags of pics were collected as part of a Six Sigma Quality Improvement effort. The number of picks in each bag are:





The sample mean is 938.22 picks and the sample standard deviation is 24.3.

(a) Summarize the given information in appropriate symbols.

$\bar{x} = 938.22$, $s = 24.3$ $n=36$

(b) Check assumptions.

$36 \geq 30$ ✓

(c) Find the 98% confidence interval for the population mean number of picks μ.

$df=35$, use 30 $t^* = 2.457$

$938.22 \pm 2.457\left(\frac{24.3}{\sqrt{36}}\right)$ (928.27, 948.17)

(d) Interpret your result.

we are 98% confident that the mean # of picks is between 928 and 948.

(e) Does the confidence interval make you think that the company is on target for having between 900 and 1000 picks per bag?

yes. because they are in the range

17. A brand group is considering a new bottle design for a popular soft drink. They found a random sample of $n = 60$ consumer ratings of this new bottle design. The sample mean is 30.35 and the sample standard deviation is 3.1073.
Let μ denote the mean rating of the new bottle design that would be given by all consumers.

(a) Summarize the given information with appropriate symbols.

$n=60 \quad \bar{x}=30.35 \quad s=3.1073$

(b) Which formula should we use?

t, sample st. dev.

(c) Check assumptions.

$60 > 30$ ✓

(d) Find the critical value for a 90% confidence interval.

df = 59, use 50

Crit value, $t^* = 1.676$

(e) Find the 90% confidence interval for the population mean rating μ.

$30.35 \pm 1.676\left(\frac{3.1037}{\sqrt{60}}\right)$

$(29.68, 31.02)$

(f) Find the margin of error for a 90% confidence interval.

$m = 1.676\left(\frac{3.1037}{\sqrt{60}}\right)$

$m = .67$

(g) Interpret your confidence interval.

we are 90% confident that the population mean rating is between 29.68 and 31.02.

Find Sample Size for T Confidence Interval

18. Book Problem 8.1.10

$n=41$ 99% t^* lenth $\leq .05$

$$n=\left(\frac{t^*\cdot s}{m}\right)^2$$

$$\left(\frac{2.704(.124)}{.05}\right)^2 = 179.9 \approx 180$$

so $180-41 = 139$ more

Find One Sided T Confidence Interval

19. Book Problem 8.1.13

$n=61$ $\bar{x}=.768$ $s=.0231$

(C,∞) 99% $t^*=2.39$

$$C = .768 - 2.39\left(\frac{.0231}{\sqrt{61}}\right) \approx .761$$

Plausible.