Review for Exam 1

0.1 Probability Review

Statistical Name	Words	Symbols	Find Probability	Key Words
Intersection	A and B	$A \cap B$	Multiply	and
Union	A or B or both	$A \cup B$	Add	or
Conditional	A given B	$A \mid B$	Divide	if/then/already know

- The sample space is all the possible outcomes.
- All probabilities are between 0 and 1.
- The total probability of all the possible outcomes is 1.
- The complement of event A is everything in the sample space that is not in event A.
- The union of A and B is everything in A *or* B or both.
- The intersection of A and B is the outcomes that are in both A *and* B.
- The conditional probability $P(A \mid B)$ means what is the probability of A if we already know B.
- **Complement Rule:**
$$P(A^c) = 1 - P(A)$$
- **Union:**
$$P(\text{A or B}) = P(A) + P(B) - P(\text{A and B})$$
- **Intersection:**
$$P(\text{A and B}) = P(A \mid B) \cdot P(B)$$
- **Conditional Probability:**
$$P(\text{A}|\text{B}) = \frac{P(\text{A and B})}{P(B)}$$
- **Bayes' Theorem:** (Use to "reverse" the conditional probability.)
$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}$$
- Two events are **disjoint (mutually exclusive)** if they have no outcomes in common, i.e. they can't happen at the same time.
  - If A and B are disjoint, $P(\text{A and B}) = 0$.
  - If A and B are disjoint, $P(\text{A or B}) = P(A) + P(B)$.
- Two events are **independent** if they don't affect each other.
  - A and B are independent if $P(A \mid B) = P(A)$.
  - If A and B are independent, $P(\text{A and B}) = P(A) \cdot P(B)$.

0.1.1 Counting Possibilities

Drawing objects with replacement

If we want to draw k objects from a total pool of n objects, with replacement, then there are

$$n^k$$

possible ways to draw the objects.

Permutations

We use a permutation when:

- we want to select k objects from n objects
- we are selecting the objects without replacement
- the order of the objects matters

The number of possible permutations of k objects from n objects is

$$P_k^n = \frac{n!}{(n-k)!}$$

Combinations

We use a combination when:

- we want to select k objects from n objects
- we are selecting the objects without replacement
- the order of the objects doesn't matter

The number of possible combinations of k objects from n objects is

$$C_k^n = \binom{n}{k} = \frac{n!}{(n-k)!k!}$$

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1. Jim and Mary are married.
   The probability that Jim watches TV after work is .6.
   The probability that Mary watches TV is .5.
   The probability that Jim watches TV given that Mary does is .8.
   (a) What is the probability that both Jim *and* Mary watch TV after work tonight?
   (b) What is the probability that Mary watches TV tonight if Jim turns it on and sits down to watch?
   (c) What is the probability that either Jim *or* Mary watch TV tonight?
2. You decide to roll 5 six sided fair dice. What is the probability that you get at least one 3 out of the five dice?

3. A contingency table for injuries in the United States is shown below. Frequencies are in millions.

	Work	Home	Other	**Total**
Male	8	9.8	17.8	**35.6**
Female	1.3	11.6	12.9	**25.8**
Total	**9.3**	**21.4**	**30.7**	**61.4**

(a) What is the probability an injury occurs at home?

(b) Find $P\,(\text{Female})$.

(c) What is the probability that an injury occurs at home, if you know the person was a female?

(d) Are the events "injury at home" and "injury to a female" independent? Justify your answer by comparing two probabilities.

(e) Are you more likely to be injured at work if you are a male or female? What two probabilities should you compare?

(f) Find $P\,(\text{Male and Work})$.

(g) Are the events "Male" and "Work" disjoint? Give a numerical justification.

(h) Find $P\,(\text{Male or Work})$.

(i) Find $P\,(\text{Female}|\text{Home})$.

(j) Based on your gender, where are you more likely to be injured? (Hint: You will need to compute 3 probabilities.)

4. For space shuttles, NASA standards state that each critical component of the mission must have 99.99% reliability, or that the probability of failure is only .0001. The space mission on which the Challenger exploded had 748 critical components.

   (a) What is the probability that none of the critical components would fail?

   (b) What is the probability that at least one critical component would fail?

5. In the game Epic Duels, a person gets two actions per turn. For each action they can attack, draw a card, or heal. Suppose attacking, drawing a card, and healing are equally likely.

   (a) What is the sample space?

   (b) What is the probability of each possible outcome?

6. Is it more likely to get 4 heads out of 5 coin tosses or 400 heads out of 500 coin tosses?

7. Here is the information for a class on campus. A student can't be a freshman and a sophomore at the same time, etc.

Class	Frequency
Freshman	21
Sophomore	14
Junior	9
Senior	6

(a) What is the probability that a randomly selected student is a Sophomore?

(b) Are the events "Sophomore" and "Junior" disjoint?

(c) What is the probability that a randomly selected student is a Sophomore or a Junior?

(d) What is the probability that a randomly selected student is not a Senior?

(e) We hold a drawing for prizes at the end of the semester. We give out 3 prizes to randomly selected students and students can be chosen more than once.

i. Are the three student selections going to be independent?

ii. What is the probability that we choose a Freshman, then a Sophomore, then a Junior?

iii. What is the probability that we choose three Seniors?

iv. What is the probability that we choose at least one Senior?

8. Here is the information for a class on campus. We need to randomly select two students to interview. (We don't want to interview the same student twice.)

Class	Frequency
Freshman	21
Sophomore	14
Junior	9
Senior	6

(a) Are the two student selections going to be independent?

(b) What is the probability that we choose a Freshman and then a Senior?

(c) What is the probability that we choose a Sophomore and then another Sophomore?

(d) What is the probability that we choose a Freshman and a Senior in any order?

(e) What is the probability that both students are at the same class level?

(f) What is the probability that both students are not at the same class level?

(g) We decide we need to interview more students. We decide to interview a total of 6 randomly chosen students. What is the probability that all the students chosen are freshmen?

9. The National Safety council reported that 14.1% of drivers responsible for fatal crashes are between 16-20 years old; of those drivers, 12.7% had an elevated blood alcohol level.

   (a) Which of the following probability symbols represents the 12.7%?

      i. $P(16-20)$
      ii. $P(\text{blood alcohol})$
      iii. $P(16-20 \mid \text{blood alcohol})$
      iv. $P(\text{blood alcohol} \mid 16-20)$
      v. $P(\text{16-20 and blood alcohol})$

   (b) Find $P(\text{blood alcohol and 16-20})$.

   (c) We know that the probability that a driver responsible for a fatal crash is between 21 and 24 year old is .114. We also know that the probability that a driver responsible for a fatal crash is between 21-24 years old *and* has an elevated blood alcohol level is .0317.
   A crash occurred. We discover that the driver was between 21-24 year old. What is the probability that the driver had an elevated blood alcohol level?

   (d) Is it more likely that a driver responsible for a fatal crash has an elevated blood alcohol level if they are between 16-20 or if they are between 21-24 years old? (What two probabilities do you need to compare?)

10. According to the Medical College of Wisconsin, 9% of men are color blind. If we randomly select four men, find the probability that:

   (a) all of the men are color blind.

   (b) at least one man is color blind.

   (c) the first three men are not color blind and the last man is color blind.

   (d) exactly one of the men is color blind.

11. Cards numbered 1,2,3,...,10 are placed in a box. The box is shaken and a person selects two cards with replacement.

    (a) Are the card draws independent?

    (b) What is the probability that the first card is a 6?

    (c) If the first card is a 6, what is the probability of that the second card is a 9?

    (d) What is the probability of the first card being a 6 and the second card being a 9?

    (e) What is the probability that both cards selected have the numbers 6, 7, 8, 9, or 10?

12. Cards numbered 1,2,3,...,10 are placed in a box. The box is shaken and a person selects two cards without replacement.

    (a) Are the card draws independent?

    (b) What is the probability that the first card is a 6?

    (c) If the first card is a 6, what is the probability of that the second card is a 9?

    (d) What is the probability of the first card being a 6 and the second card being a 9?

    (e) What is the probability that both cards selected have the numbers 6, 7, 8, 9, or 10?

13. The Gallup Organization conducted a study of U.S. adults on their attitudes toward the death penalty. Here are the results.

	In Favor	Not in Favor	Not Sure	**Total**
Northeast	68	29	3	**100**
Midwest	69	29	2	**100**
South	72	23	5	**100**
West	80	19	1	**100**
Total	**289**	**100**	**11**	**400**

(a) Suppose we randomly select one U.S. adult. Find each probability.

i. $P\,(\text{In Favor})$

ii. The person doesn't say "not sure".

iii. The person is from the Northeast.

iv. $P\,(\text{Northeast or In Favor})$

v. $P\,(\text{In Favor}|\text{Northeast})$

vi. $P\,(\text{Northeast}|\text{In Favor})$

vii. $P(\text{In Favor}|\text{South})$

viii. $P(\text{South}|\text{In Favor})$

ix. $P(\text{In Favor and West})$

(b) Is a person more likely to be in favor of the death penalty if they are in the Northeast or the South? Justify your answer.

(c) Are the events "Northeast" and "In Favor" independent? Justify your answer.

(d) Are the events "In Favor" and "West" disjoint? Justify your answer.

14. Suppose we toss a biased coin 3 times. This biased coin has 0.7 probability of being a head on any coin toss.

    (a) What is the sample space? (Hint: draw a tree diagram.)

    (b) What is the probability of getting a tail on one coin toss?

    (c) Should the results of the three coin tosses be independent?

    (d) Fill in the probability table for the possible outcomes:

Outcome	(Show your work)	Probability
HHH		
HHT		
HTH		
HTT		
THH		
THT		
TTH		
TTT		

    (e) Find the probabilities for the following events:

        i. we get exactly 2 heads.

        ii. we get at least 2 heads.

        iii. we get at least one tail.

        iv. we get a tail on the second toss.

        v. we get the same result on all three coin tosses.

15. A construction company employs several welders who do thousands of welds each year. From previous records, the company knows that 5% of all welds done by their welders do not meet industry safety requirements; that is, the welds are considered defective (D). As such, each weld that is done on a construction project is examined by the company's inspector. The inspector's performance has also been monitored, and over several years the following characteristics have been observed:

    - Any weld that is in fact defective (D) will be correctly classified as defective (CD) by the inspector 96% of the time; in the remaining cases, the defective weld will be classified as being good (CG);
    - Any weld that is good (G) will be incorrectly classified as being defective (CD) 2% of the time; in the remaining cases, the good weld will be classified as being good (CG).

    (a) Draw a tree diagram that represents all the possible outcomes associated with the experiment of selecting a weld at random for inspection and the classification of the weld by the inspector.

    (b) If a randomly selected weld is classified by the inspector as being good (CG), what is the probability that the weld is defective (D)?

16. The probability that Shylor stops for a soda on his way to work is .23. If he stops for a soda there is a .76 chance he is late. Overall he shows up late 34% of the time. Today he is late, what is the chance that he stopped for a soda?

17. Among people infected with a certain virus, 32% have strain A, 59% have strain B, and 9% have strain C.

    21% of people with strain A exhibit symptoms, 16% with strain B exhibit symptoms, 63% of people with strain C exhibit symptoms.

    (a) Draw a tree diagram representing all the possible outcomes of which strain a person has and if they exhibit symptoms.

    (b) If they have strain B, what is the probability that they do NOT show symptoms?

    (c) Find the probability that they have strain B *and* they do NOT show symptoms.

    (d) Find the probability that they have strain C and they show symptoms.

    (e) Find the overall probability that an infected person shows symptoms.

    (f) If someone shows symptoms, find the probability that they have strain C.

    (g) Now in parts 17d and 17f we found $P\,(C\,\text{and}\,S)$ and then used that to find $P\,(C \mid S)$. Sometimes we want to combine those two steps into 1.
    Using Bayes' Theorem, find $P\,(C \mid S)$ in one step.

0.1.2 Practice Problems for Sections 1.6 and 1.7

I don't have problems written up, so you need to do these problems from the book.

Section 1.6: 1.10.33 (use tree diagram), 1.10.36a (use tree diagrams)

Section 1.7: 1.7.7, 1.7.5, 1.10.22

0.2 Random Variables, Expected Value and Variance, and Binomial Distribution

0.2.1 Population Versus Sample

	size	mean	standard deviation
Population **parameters**		μ	σ
Sample **statistics**	n	$\overline{x}$	s

0.2.2 Random Variable

Random Variable: A variable that assigns numerical values to outcomes of a study or experiment.

0.2.3 Discrete Random Variables

Discrete Random Variable: the variable X can only be certain values (usually means you are counting)

- number of kids at park
- number of broken tvs
- score on exam (when you only get graded in half point increments)

Probability Mass Function (pmf): The set of probability values p_i assigned to the x_i values for the random variable X, for **discrete** random variables.

The probabilities p_i must satisfy two requirements:

- $0 \leq p_i \leq 1$
- $p_1 + p_2 + \cdots + p_k = 1$.

We write the probability that the random variable X takes the value x_i is p_i as

$$P\left(X = x_i\right) = p_i$$

X refers to the random variable

x_i is a specific value of x

Cumulative distribution function (cdf): a function that describes the random variable X

$$F\left(x\right) = P\left(X \leq x\right)$$

Cumulative distribution functions are always defined for all real numbers.

PMF to CDF: If we have a discrete random variable, then we can find the cdf by adding up the probabilities $P\left(X = k\right)$ for k values that are less than or equal to x.

$$F\left(x\right) = \sum_{k: k \leq x} P\left(X = k\right)$$

CDF to PMF: If you want to go back to a pmf

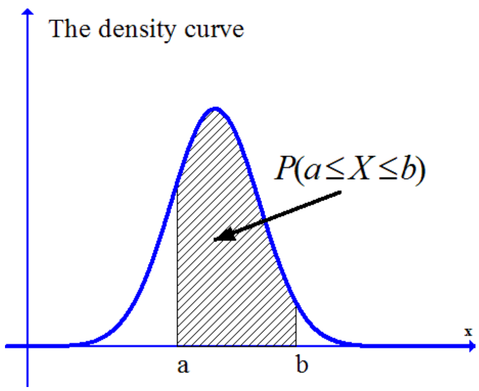
$$P\left(X = k\right) = F\left(k\right) - F\left(\text{last } x \text{ value}\right)$$

0.2.4 Continuous Random Variables

Continuous random variable: the variable X can be *any* values in an interval

- temperature of cup of hot chocolate
- weight of snakes
- age of kindergarteners

The **probability distribution** of X is described by a density curve.



The shaded area is the probability that X will be between a and b.

- We think of probabilities of events as areas under a density curve.
- Any density curve has a total area of 1 corresponding to a total probability 1.
- We can't assign probability to each individual outcome because there are infinitely many possible values.
- So the probability of any individual outcome is 0. i.e. $P(X = 8) = 0$.
- Instead we assign probabilities to intervals of numbers. e.g. $P(X \geq 8)$, $P(3 < X < 8)$, etc.
- Because $P(X = 8) = 0$, we know that $P(X \geq 8) = P(X > 8)$.

0.2.4.1 pdf

The probability density function $f(x)$ describes the distribution of a *continuous* random variable.

A pdf must meet the two rules to be a valid distribution.

- $f(x) \geq 0$
- $\int_{-\infty}^{\infty} f(x)\, dx = 1$

0.2.4.2 cdf

The cumulative distribution function (cdf) is

$$F(x) = P(X \leq x)$$

0.2.4.3 Converting between cdf, pdf, and probabilities

- pdf to cdf
$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(y)\,dy$$
- cdf to pdf
$$f(x) = \frac{d}{dx} F(x)$$
- pdf to probabilities
$$P(a \leq X \leq b) = \int_{a}^{b} f(x)\,dx$$
- cdf to probabilities
$$P(a \leq X \leq b) = P(X \leq b) - P(X \leq a) = F(b) - F(a)$$

0.2.5 Quantiles

p*th* quantile: the value of x for which $F(x) = p$.

0.2.6 Mean, Variance, Standard Deviation

The mean (expected value) of a random variable X is an average of the possible values of X, but we need to take into account the fact that not all outcomes have to be equally likely.

The variance measure the spread of the distribution about the mean value. Larger values of the variance indicate the distribution is more spread out.

- Mean $E(X) = \mu$
- Variance $Var(X) = \sigma^2$
- Standard Deviation σ

0.2.6.1 Mean and Standard Deviation of a Discrete Random Variable

Suppose that X is a discrete random variable whose distribution is

Value of X	x_1	x_2	x_3	$\cdots$	x_k
Probability	p_1	p_2	p_3	$\cdots$	p_k

Expected Value:

To find the mean of X (expected value of X), multiply each possible value by its probability, then add all the products.

$$\begin{aligned} \mu_X = E(X) &= x_1 p_1 + x_2 p_2 + \cdots + x_k p_k \\ &= \sum x_i p_i \end{aligned}$$

Variance:

$$\begin{aligned} \sigma_X^2 = Var(X) &= (x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + \cdots + (x_k - \mu)^2 p_k \\ &= \sum (x_i - \mu)^2 p_i \end{aligned}$$

The **standard deviation** of X is the square root of the variance.

0.2.6.2 Mean and Standard Deviation of a Continuous Random Variable

$$\mu_X = E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

$$\sigma^2 = Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

You can also use the formula

$$Var(X) = E\left(X^2\right) - [E(X)]^2$$

0.2.7 Linear Combinations of Random Variables

Rules for Means

Rule 1: If X is a random variable and a and b are constants, then

$$E(aX+b) = a \cdot E(X) + b$$

Rule 2: If X and Y are random variables, then

$$E(X+Y) = E(X) + E(Y)$$

Rules for Variances and Standard Deviations

Rule 1: If X is a random variable and a and b are constants, then

$$Var(aX+b) = a^2 \cdot Var(X)$$

Rule 2: If X and Y are **independent** random variables, then

$$Var(X+Y) = Var(X) + Var(Y)$$

To find the standard deviation, take the square root of the variance.

0.2.8 Functions of Random Variables

*Linear Combinations of Random Variables are special circumstances of this rule.

Expected Value of a Function of a Random Variable:

If you have a function $h(X)$ of a random variable, you can find the expected value by

$$E(h(X)) = \sum_i h(x_i) \cdot p_i$$

$$E(h(X)) = \int_{-\infty}^{\infty} h(x)\, f(x)\, dx$$

0.2.9 Binomial Distribution

The binomial random variable counts the number of "successes" .

The requirements to be a binomial random variable:

- There is a fixed number of observations n.
- The n observations are all independent.
- Each observation is classified as either 'success' or 'failure'.
- The probability of a success, p, is the same for each observation.

If we want to find the probability of k successes from a binomial distribution with n trials and p probability of success

$$P\left(X=k\right)=\binom{n}{k}p^{k}\left(1-p\right)^{n-k}$$

0.2.9.1 Mean and Standard Deviation for a Binomial Random Variable

$$\mu_X = np$$

$$\sigma_X = \sqrt{np\left(1-p\right)}$$

18. Remember Problem 14.
Suppose we toss a biased coin 3 times. This biased coin has 0.7 probability of being a head on any coin toss.
We found the sample space (all the possible outcomes) and the probabilities.

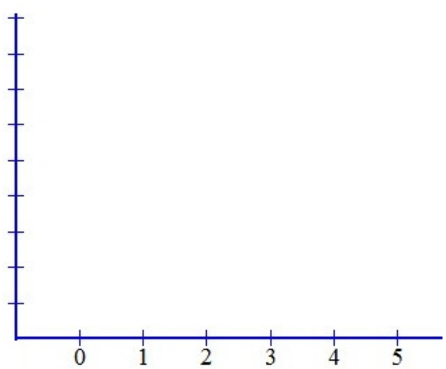
Outcome	(Show your work)	Probability
HHH	$P(HHH) = P(H) \cdot P(H) \cdot P(H) = (0.7)(0.7)(0.7) = 0.343$	0.343
HHT	(.7) (.7) (.3)	0.147
HTH	(.7)(.3)(.7)	0.147
HTT	(.7)(.3)(.3)	0.063
THH	(.3)(.7)(.7)	0.147
THT	(.3)(.7)(.3)	0.063
TTH	(.3)(.3)(.7)	0.063
TTT	(.3)(.3)(.3)	0.027

Let X be the number of heads in 3 tosses.

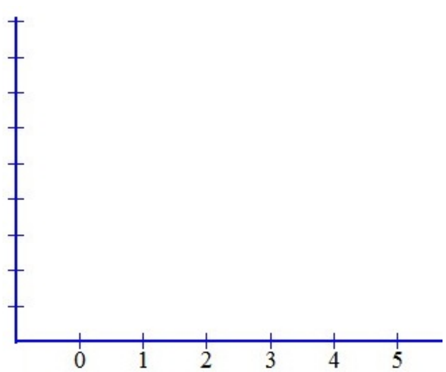
(a) Find the probability distribution (pmf) of X.

(b) Check that this is a valid probability distribution.

(c) Graph the probability distribution (pmf) of X. (Don't forget to label the axes.)



(d) Find and graph the cdf of X.



19. A basketball player believes that the probability distribution for the number of free throws he makes out of two attempts is:

Value of X	0	1	2
Probability	.36	.28	.36

   (a) Calculate the mean (expected value) of the number of hits.

   (b) Calculate the standard deviation σ_X of the number of hits.

20. Which variables are discrete?

   (a) The number of four patients taking a new antibiotic who experience gastrointestinal distress as a side effect.

   (b) The number of the next three customers entering the store who will make a purchase.

   (c) The weight of strawberry preserves dispensed by an automatic filling machine into a 16-ounce jar.

   (d) The score on a test that is graded in half point increments.

   (e) The number of fires in a large city in the next two months.

   (f) The time that a customer in a store must wait to receive a credit card authorization.

21. Which of the following might be reasonably modeled by the binomial distribution?

   (a) The number of customers that enter a store in a one–hour period, assuming customers enter independently.

   (b) The number of questions you get correct on a 100–question multiple choice exam in which each question has four possible answers and exactly one of these answer is correct. Assume you have not studied at all for the test and have to guess each answer.

   (c) A couple will have children until they have three girls or five children. X is the number of children in the family.

   (d) A really nervous basketball player is making two free throw attempts. The probability of him making the first shot is .4, but if he misses the first shot, the probability of him making the second shot is only .1. X is the number of shots he makes.

22. The weight of medium-size tomatoes selected at random from a large bin at the local supermarket is a random variable X with mean $\mu_X = 10$oz. and standard deviation $\sigma_X = 1.2$oz.

 (a) Let the random variable W be the weight of the tomatoes in pounds (1 pound = 16 oz). What is the expected value and standard deviation σ_W of the random variable W (in pounds)?

 (b) Suppose we pick four tomatoes from the bin at random and put them in a bag. Define the random variable Y as the weight of the content of the bag containing the four tomatoes. What is the mean μ_Y of the random variable Y (in oz.)? What is the standard deviation?

 (c) Suppose we pick two tomatoes at random from the bin. Let the random variable V be the difference in the weights (in oz.) of the two tomatoes selected (i.e., the weight of the first tomato minus the weight of the second tomato). What is the expected value μ_V of the random variable V (in oz.)? What is the standard deviation?

23. A student takes a multiple choice test and guesses on 3 questions. Each question has **4 possibilities.**

 (a) Is $X =$ the number of correct answers a binomial distribution? If so, what are n and p?

 (b) Find the probability distribution of X.

X	probability

 (c) Is this a discrete or continuous distribution?

 (d) Find μ_X.

 (e) Find σ_X.

24. During the last student elections at a certain college, 45% of the students voted for the democratic student party. A simple random sample of students from this college is to be selected.

    (a) If 12 students are to be selected, what is the distribution of the number of students in the sample who voted for the democratic student party?

    (b) What is the expected number of students in the sample that voted for the democratic student party?

    (c) What is the standard deviation of the random variable, σ_X?

    (d) What is the variance of the random variable, σ_X^2?

    (e) If 12 students are to be selected, what is the probability that more than 7 students in the sample voted for the democratic student party?

25. An insurance company will insure a \$75,000 Hummer for its full value against theft at a premium of \$1500 per year. Suppose that the probability that the Hummer will be stolen is 0.0075.

    (a) Calculate the insurance company's expected net profit for one year.

    (b) Interpret your results.

26. Universal blood donors: People with type O–negative blood are universal donors. That is, any patient can receive a transfusion of O–negative blood. About 6% of a particular population have O–negative blood. If 12 people appear at random to give blood:

    (a) What is the probability that at least 1 of them is a universal donor?

    (b) What is the probability that 3 of them are universal donors?

    (c) What is the probability that there are 3 or fewer universal donors?

27. Portfolio analysis. Here are the means and standard deviations for the annual returns from three Fidelity mutual funds for the 10 years ending in February 2004. Assume that the stocks returns are independent of each other.

$$W = \text{annual return on 500 Index Fund}$$
$$X = \text{annual return on Investment Grade Bond Fund}$$
$$Y = \text{annual return on Diversified International Fund}$$
$$\mu_W = 10.12\%,\ \sigma_W = 17.46\%$$
$$\mu_X = 6.46\%,\ \sigma_X = 4.18\%$$
$$\mu_Y = 11.10\%,\ \sigma_Y = 15.62\%$$

    Assume that a portfolio contains 70% Investment Grade Bond Fund (X) and 30% Diversified International Fund (Y) stocks. Calculate the mean and standard deviation of the returns for this portfolio.

28. A variable is called random if

    (a) Individual outcomes are uncertain but happen in a predictable manner through time.
    (b) One has no idea what will happen.
    (c) Outcomes happen in a 50–50 split.
    (d) I flip a coin and get 5 heads on my first 5 flips, I am very likely to get 5 tails on my next 5 flips.

29. Mandy usually gets paid \$5 from customers on her newspaper route. One day a customer offers Mandy a deal. He will place three \$1 bills, one \$5 bill, and one \$10 bill in a paper bag. Mandy can then draw and keep one bill from the bag instead of her usual \$5. She could do this every week from now on.

    (a) **Gut feeling:** Do you think Mandy should take the deal? Will she make more or less money in the long run?

    (b) Find the expected value of Mandy's earnings if she takes the deal.

Mandy's Possible Earnings	Probability

    (c) Interpret your result.

    (d) Will Mandy make more or less money in the long run if she takes the deal?

30. Remember the sentences given by a second judge have a distribution given by

$$f(y) = A\sqrt{y}, \qquad 0 \leq y \leq 4$$

(a) What must A be in order for this to be a pdf?

(b) Find the cdf.

(c) Use the cdf to find the probability that a prison term is no more than 2.5 years.

(d) Use the pdf to find the probability that a prison term is at least 2.5 years.

(e) Use the cdf to find the probability that a prison term is more than 2.5 years.

31. A cdf is

$$F(y) = \frac{y^{3/2}}{8}, \qquad 0 \le y \le 4$$

(a) Use the cdf to find $P(-2 \le Y < 3.6)$.

(b) Find the probability density function.

(c) Use the pdf to find the probability that Y is between 1.2 and 3.

32. Remember the sentences given by a second judge have a distribution given by

$$f(y) = \frac{3}{16}\sqrt{y}, \qquad 0 \le y \le 4$$

Find the standard deviation of the length of the sentences.

33. A pdf is

$$f(x) = \frac{3}{1000}x^2$$

for $0 \le x \le 10$ and $f(x) = 0$ otherwise.

(a) Find the cdf.

(b) What is the median?

(c) Find the .37 quantile.

34. We have a random variable X with pdf

$$f(x) = 3x^2 \qquad \text{for } 0 \le x \le 1$$

and $Y = X^3$. Find $E(4Y+2)$.

35. We have a random variable X with pdf

$$f_X(x) = 2x \qquad 0 \le x \le 1$$

and $Y = \sqrt{X}$.

(a) Find the cdf of X.

(b) Find the cdf of Y.

(c) Find the pdf of Y.

(d) Find $E(Y)$.

(e) Find $P(Y > .3)$.

0.3 Book Problems

Here are more practice problems from the book.

Section 1.10: 1, 4, 5, 6, 7, 10, 14, 17, 18, 22, 23, 25, 26, 28, 30, 32, 33, 34, 35, 36, 37

Problem 1.10.32 has a typo in the book solutions. It should be $1 - (.0005 \times .001) = .9999995$

Section 2.9: 2, 4, 5, 7, 11, 12, *14, 15, 16, 17, 18, 20, 21, 22, 23, 26, 28

Section 3.8: 1, 2, 6, 7a, 10, 13ab

Skip sections 3.4 (Poisson), 4.1 (uniform), 4.2 (exponential),

*Do the book problems last