Math 5710 Opportunity #4 ♬ Deadline: 1 p.m. on Thursday, August 6, 2020

1. Please print your name legibly. ____

2. Examine each of the following propositions, determine whether or not it is true, display your choice by circling either "T" or "F"; for each either prove that your decisions is correct or write at least one paragraph that explains why you decided that the proposition is true or why you decided that the proposition is false:

A. $X \in \{$ discrete random variables of $\Omega \} \Rightarrow X \in \{$ sets $\}$

T F

Sample proof:

Note the following lines from our *Glossary*:

012-A. Definition of a *relation from A to B*:
Given $A, B \in \{$ sets $\}$, (r is a *relation* from A to $B \Leftrightarrow r \subseteq A \times B$)

013-A. Definition of *function*:
Given $A, B \in \{$sets$\}$, "$f : A \rightarrow B$ " is read "f is a *function* from A to B "

Definition of *function*:
Given $A, B \in \{$sets$\}$, ($f : A \rightarrow B \Leftrightarrow$ (r is a relation from A to B) $\wedge$ ($\forall x \in A, \exists! y \in B \ni (x, y) \in f$))

038. Definition for *discrete random variable* :
$X \in \{$ discrete random variables of $\Omega \} \Leftrightarrow$
$(| \Omega |, | X | \in \{ \aleph_0, 0, 1, 2, 3, ... \} \wedge E = \{$ events of $\Omega \} \wedge X : E \rightarrow \mathbb{R})$

So X is a function and a function is a relation which is a set.

B. (A card is randomly drawn from standard 52-card poker deck) $\Rightarrow$ (the odds that a club is drawn is 0.25)

T F

Sample proof:

Since 25 percent of the cards are clubs, p(a club is drawn) = 0.25. And our definition of odds is as follows:

040. Definition for the odds of an event occurring:

Given E is an event of Ω , (the odds in favor of E occurring = $r : s$ where $r, s \in [\, 0, \infty\,) \Leftrightarrow \dfrac{r}{s} = \dfrac{p(E)}{1-p(E)}$)

Thus, the odds of the randomly drawn card is a club = 0.25 ÷ 0.75 = 1/3. And 1/3 $\neq$ 0.25.

C. $\{$ Bernoulli random variables $\} \cap \{$ binomial random variables $\} = \varnothing$

T F

Sample proof:

Compare the following two definitions:

042. Definition for a *Bernoulli random variable*:

$X \in \{$ Bernoulli random variables of $\Omega\ \} \Leftrightarrow$
$X \in \{$ discrete random variables of $\Omega\ \} \wedge$ the range of $X = \{ 0, 1 \}$

043-A. Definition for a binomial random variable: :

Given $n \in \mathbb{N} \wedge \Omega = \{ 1, 2, 3, ..., n \} \wedge X \in \{$ Bernoulli random variables of $\{ 0, 1\}\} \wedge$ (A string of n experiments are conducted with $X \ni (X(i)$ $= 0 \veebar (X(i) = 1$ depending on the results of the i^{th} experiment $\wedge$
$|\{ (i, X(i)) : X(i) = 1 \}| = k$),
($Y \in \{$ binomial random variables of $\Omega\ \} \Leftrightarrow$
$Y \in \{$ discrete random variables of $\Omega\ \} \ni Y(i) = \sum_{i}^{n} X(i) = k$

Thus, a binomial random variable in which $n = 1$ is also a Bernoulli random variable.

D. A number x is randomly selected from $[\,^{-}1,\ 1\,) \Rightarrow$
the probability that $x = {}^{-}1 >$ the probability that $x = 1$

T F

Sample proof:

Since $1 \notin [\,^{-}1,\ 1\,)$, the probability that $x = 1$ is 0. And since $|\,[\,^{-}1,\ 1\,)\,| = \mathcal{C}$, the probability of randomly selecting any single element of $[\,^{-}1,\ 1\,)$ is also 0.

E. A number x is randomly selected from $(\,0, 6\,) \Rightarrow$
the probability that $x \geq \pi$ is 0.5

T F

Sample proof quoted from Kameron Baird:

The probability of selecting a number within a range of a given interval cannot be determined without a sample space. Calculating the probability when dealing with intervals is calculating the area under the curve of the sample space with regards to that range. Without the sample space, our curve has no height and is therefore just a line from which we cannot determine the probability. However, if we assumed the set of all real numbers were included in the sample space on the interval from 0 to 6, and all entries had equal probability of being selected, it could be argued that the area from pi to 6 would be equal to less than 0.5 and the statement would still be considered false.

F. $X \in \{$ continuous random variables of $\Omega\ \} \Rightarrow |\ X\ | = \mathcal{C}$

T F

Sample proof quoted from Kameron Baird:

Statement: If X is an element of the set of all continuous random variables of the sample space, then the cardinality of X is equal to the continuum.

Definition of continuous random variable: X is an element of the set of all continuous random variables of the sample space if and only if the cardinality of the sample space and the cardinality of X are elements of the set of all continuum raised to the n such that n is an element of the set of all natural numbers and E is equal to the set off all events of the sample space and X is a function from E to the set of all real numbers.

By the definition of continuous random variables, we can see that the cardinality of X must be equal to the set of all continuum raised to the n. It may be reasoned that the set of all continuum raised to the n would be equal to the continuum. Therefore, the statement may be considered to be true.

G. $X \in \{$ random variables of $\Omega \} \Rightarrow X \in \{$ events of $\Omega \}$

T F

Sample proof:

Please keep the following definitions in mind:

029-E. Definition for *event*:

$A \in \{$ events of $\Omega \} \Leftrightarrow A \subseteq \Omega$

038. Definition for *discrete random variable* :

$X \in \{$ discrete random variables of $\Omega \} \Leftrightarrow$
$(|\, \Omega \,|, |\, X \,| \in \{ \aleph_o, 0, 1, 2, 3, \ldots \} \wedge E = \{$ events of $\Omega \} \wedge X : E \rightarrow \mathbb{R})$

045. Definition for *continuous random variable* :

$X \in \{$ continuous random variables of $\Omega \} \Leftrightarrow (|\, \Omega \,|, |\, X \,| \in \{ \mathcal{C}^n : n \in \mathbb{N} \} \wedge$
$E = \{$ events of $\Omega \} \wedge X : E \rightarrow \mathbb{R})$

Note that whether X is a continuous random variable or a discrete random variable, that X is a function (i.e., $X : E \rightarrow \mathbb{R}$). On the other hand, rather than functions, the elements of $\{$ events of $\Omega \}$ are sets of outcomes. And a set of outcomes is not necessarily a set of functions. For example, see Sybil's experiment described in Item #5-E of the agenda for Meeting #6.

H. Results from an interval measurement can be tenably interpreted as if they were nominal.

T F

Sample explanation:

An interval measurement generates numbers that can be modeled on a uniform scale so that the difference between a pair numbers is meaningful. Whereas a nominal measurement only generates results in whether two results are the same or different. Data from interval measurements are richer than data from nominal measurement. But some of the riches from an interval measurement can be discarded and only differences can be recorded. For example, an interval scale may result in two data points 45 and 51 and we can note that the two data points differ by 6. Because they differ by 6 also informs us that they are different. It’s tenable to report only that they are different.

J. Measurement usefulness is a necessary condition for measurement reliability.

T F

Sample explanation:

Examine the following lines from our *Glossary*:

033-E. Measurement usefulness:

i. Note: "$D_0 = D_t + D_E$" is read "data from a measurement equals the sum of what the data would be if the measurement were perfectly valid and the influence of measurement error."

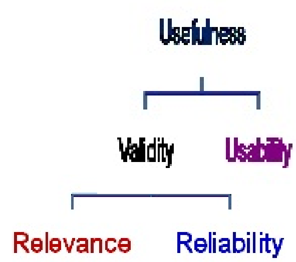
ii. Definition for *measurement relevance*: A measurement is *relevant* to the same degree that the data it generates are pertinent to the assessment that is influenced by those data.

iii. Definition for *measurement reliability*: A measurement is *reliable* to the same degree that it generates data that are internally consistent (i.e., the data reflects a non-contradictory pattern).

iv. Definition for *measurement validity*: A measurement is *valid* to the same degree that it both relevant and reliable.

v. Definition for *measurement usability*: A measurement is *usable* to the degree that it is practical to administer.

v. Definition for *measurement usefulness*: A measurement is *useful* to the same degree that it both valid and useable.



Note that for a measurement to be useful it must be usable. And a measurement could provide very consistent (i.e., reliable) results but it is too expensive and time-consuming to be usable. Thus, the results are reliable but since the measurement is not usable, it is not useful.

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3. Please solve the following problem; display the computations – including the probability distribution:

   A person is randomly selected from a population and tested for COVID-19 infection. A positive test result is labeled a “success” and coded as 1; a negative test result is labeled a “failure” and coded as 0. Again a person is randomly selected from that *same* population (Thus, the first person is still in the population; so the two events are independent). The trial is repeated twice more. The number of successes is recorded. As of June 2, 2020, one seemingly credible estimate is 20% of the people worldwide are infected; use that figure for this problem. Display the probability distribution for the random variable for this experiment.

Sample computation:

From Theorem 11 we have: $(X \in \{ \text{Bernoulli random variables of } \{ 0, 1 \}\} \wedge m : X \rightarrow [0, 1] \wedge (\Omega = \{ 0, 1, 2, ..., n \} \wedge Y \in \{ \text{binomial random variables of } \Omega \}) \wedge p : Y \rightarrow [0, 1]) \Rightarrow$
$p(k) = \binom{n}{k} m(1)^k (1 - m(1))^{n-k}$ where $n = 4 \wedge m(1) = 0.20$. Our probability distribution is as follows:

$$p(0) = \binom{4}{0} m(1)^0 (1 - m(1))^4 = (1)(1)(.80)^4 = 0.4096$$

$$p(1) = \binom{4}{1} m(1)^1 (1 - m(1))^3 = (4)(.20)(.80)^3 = 0.4096$$

$$p(2) = \binom{4}{2} m(1)^2 (1 - m(1))^2 = (6)(.20)^2(.80)^2 = 0.1536$$

$$p(3) = \binom{4}{3} m(1)^3 (1 - m(1))^1 = (4)(.20)^3(.80)^1 = 0.0256$$

$$p(4) = \binom{4}{4} m(1)^4 (1 - m(1))^0 = (1)(.20)^4(.80)^0 = 0.0016$$

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4. Comprehend the following case:

An educational H&PE researcher conducted a study to assess the relationship between the psychomotor agility of second-grade children and their computational fluency. She administered a psychomotor agility test as well as a computational fluency test to a single random sample of 86 second-grade students. The resulting string of bivariate data X is of the following form:

$$X = (\,(v_1, d_1), (v_2, d_2), (v_3, d_3), \ldots, (v_{86}, d_{86})\,)$$

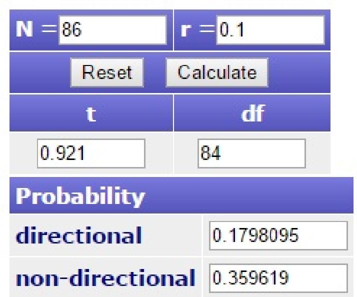
The resulting sample statistics are as follows:

$$n = 86 \wedge r = .10$$

She tested the following null hypothesis via a *t*-test for correlations:

$$H_o : \rho_X = 0$$

The calculation from http://vassarstats.net/textbook/ch4apx.html provided the following results:



Because the t value was such that $p > 0.05$, the researcher failed to reject H_o.

Examine each of the following propositions to determine its truth value; indicate your choice by circling either “T” or “F” and then write a paragraph defending your choice:

i. The results of the *t*-test indicated that the correlation coefficient is not statistically significant.

T F

ii. The results of the *t*-test indicated that there is no relationship between second grade students’ psychomotor agility and their computational fluency at least among those children represented by the study sample.

T F

iii. The results of the t-test indicated that $|r|$ is so close to 0, that H_o should be accepted.

T F

iv. Based on the results of the study, a Type II error is possible but it is impossible to have a Type I error.

T F

Sample responses:

i. The results of the t-test indicated that the correlation coefficient is statistically insignificant.

T F

The proposition is true because $p > 0.05$ and it seems that the research set $p < 0.05$ as the standard for rejecting the null hypothesis. The p-values of 0.18 (directional) and 0.36 (non-directional) leads to a failure to reject the null hypothesis (i.e., 0.1 is statistically insignificant for this particular study).

ii. The results of the t-test indicated that there is no relationship between second grade students' psychomotor agility and their computational fluency at least among those children represented by the study sample.

T F

Since the null-hypothesis was not rejected, there is not strong enough evidence to suggest that there is a relationship between second grade students' psychomotor agility and their computational fluency at least among those children represented by the study sample. But because of the high probability of a Type II error, one should not conclude that there is no relationship. The results are inconclusive.

iii. The results of the t-test indicated that $|r|$ is so close to 0, that H_o should be accepted.

T F

The results of the t-test indicated that $|r|$ is so close to 0, that H_o should not be rejected. But as suggested by response to the previous prompt, H_o should not be accepted either. Again, the possibility of Type II error looms and the evidence is inconclusive as to whether or not H_o is true.

iv. Based on the results of the study, a Type II error is possible but it is impossible to have a Type I error.

T F

Type I error occurs when a true null-hypothesis is rejected. Since it was not rejected in this case, there can be no Type I error. But a Type II error is possible since a Type II error occurs when one fails to reject a false null hypothesis and in this case the null hypothesis was not rejected. So there might have been a Type II error but we don't know whether or not there was a Type II error.

5. Consider the graph of $f \ni f : [\,^-2, 2\,] \to \mathbb{R} \ni f(x) = x^2$. H is a region that is bounded under the curve of f and is determined by the following 4 points with coordinates (⁻2, 0), (⁻2, 4), (2, 0), and (2, 4). An experiment is conducted in which a point is randomly selected from H. What is the probability that the selected point lies in the subset of H that is determined by 4 points with coordinates (1, 0), (1, 1), (2, 0), and (2, 4)?

Display an illustration that is useful in helping people comprehend the question. And display the computations that will help people understand how you computed the problem in question.

Sample computation:

The area of H $= \int_{-2}^{2} x^2 dx = \left.\frac{x^3}{3}\right]_{-2}^{2} = \frac{8}{3} - \frac{-8}{3} = \frac{16}{3}$.

Let J = the subset in question. The area of $J = \int_{1}^{2} x^2 dx = \left.\frac{x^3}{3}\right]_{1}^{2} = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$.

And the probability of the randomly selected point being in $J = \dfrac{\frac{7}{3}}{\frac{16}{3}} = \frac{7}{16}$