Block Ciphers

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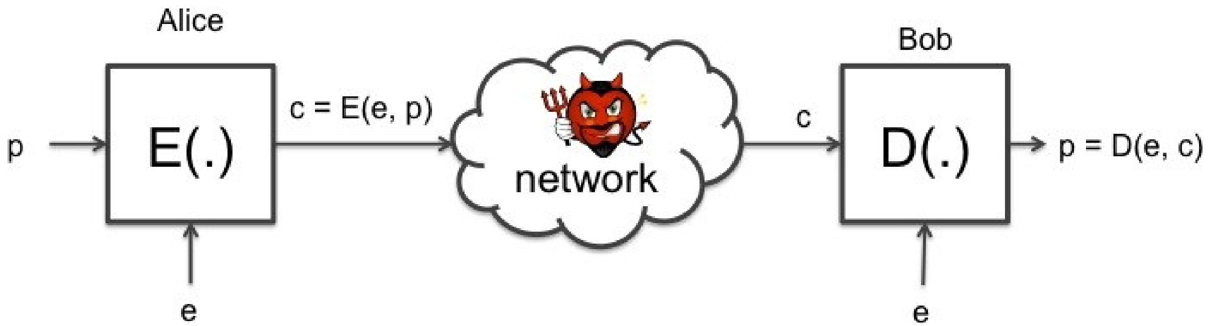
Version: 10/03/2025

Block Ciphers

GENERAL CONCEPTS

Block cipher

- Block ciphers break up the plaintext in blocks of fixed length *n* bits and encrypt one block at time



- $E_k: \{0,1\}^n \rightarrow \{0,1\}^n$ $D_k: \{0,1\}^n \rightarrow \{0,1\}^n$
- E is a keyed permutation: $E(k, p) = E_k(p) = Enc_k(p)$
- $E_K(\cdot)$ is a permutation

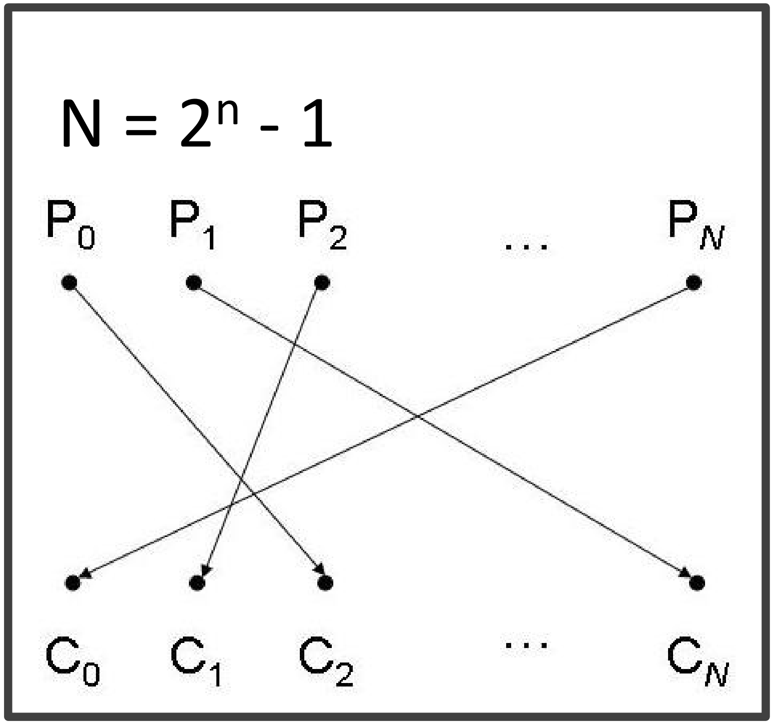
Permutation

- E_k is a permutation
  - E_K is efficiently computable
  - E_k is bijective
    - Surjective (or onto)
    - Injective (or one-to-one)
  - E_k^{-1} is efficiently computable

Examples

- Block ciphers
  - DES   n = 64 bits,   k = 56 bits
  - 3DES   n = 64 bits,   k = 168 bits
  - AES   n = 128 bits   k = 128, 192, 256 bits

Random permutations



A possible random permutation π

- Let Perm_n be the set of all permutations π: $\{0,1\}^n \rightarrow \{0,1\}^n$
- $|\text{Perm}_n| = 2^n!$
- A true random cipher
  – implements all the permutations in Perm_n
  – uniformly selects a permutation $\pi \in \text{Perm}_n$ at random

True Random Cipher

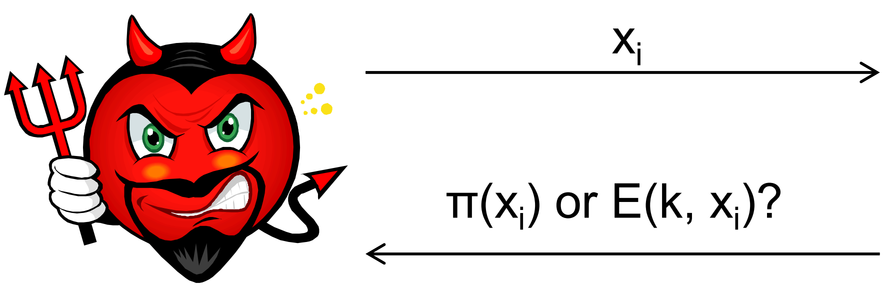
- A True random cipher is perfect
- A true random cipher implements all possible Random permutations ($2^n!$)
  - Need a uniform random key for each permutation (naming)
    - key size := $\log_2 (2^n!) \approx (n - 1.44)\, 2^n$
      - Exponential in the block size!
      - The block size cannot be small to avoid a dictionary attack
- A true random cipher cannot be implemented

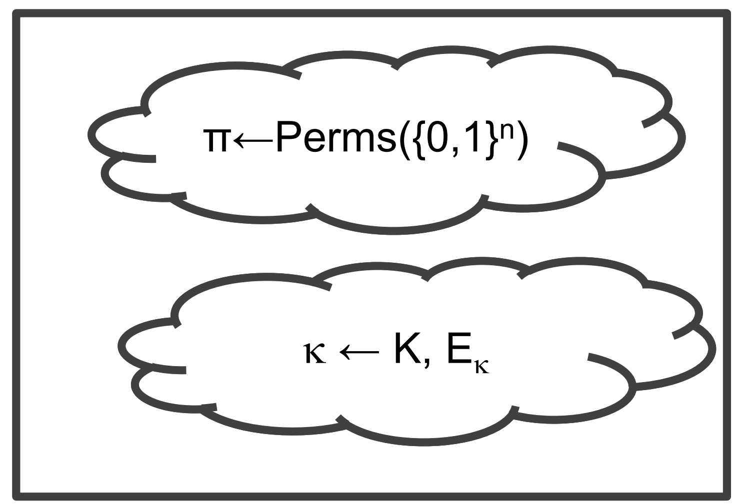
Pseudorandom permutations

- Consider a *family of permutations* parametrized by $\kappa \in \mathrm{K} = \{0, 1\}^k$, $E_\kappa: \{0,1\}^n \rightarrow \{0,1\}^n$
- A E_κ is a *pseudorandom permutation* (PRP) if it is indistinguishable from a uniform random permutation by a limited adversary
- $|\ \{E_\kappa\}\ | = 2^k << |Perm_n|$, with $|\kappa| = k$
- A block cipher is a practical instantiation of a PRP

Practical block cipher

- In practice, the encryption function corresponding to a randomly chosen key should appear as a randomly chosen permutation to a limited adversary





- Oracle access
  - adversary cannot look into the box

Exhaustive key search attack

- The attack
  - Given a pair (pt, ct), check whether ct == E_{ki}(pt), i = 0, 1, ..., $2^k - 1$
    - Known-plaintext attack
    - Time complexity: $O(2^k)$
- False positives
  - Do you expect that just one key k maps pt into ct?
  - How many keys (false positives) do we expect to map pt into ct?
  - How do you discriminate the good one?

Exhaustive key search

- False positives
  - Do you expect that just one key k maps pt into ct?
  - How many keys (false positives) do we expect to map pt into ct?
  - How do you discriminate the good one?

False positives

- Problem: Given (ct, pt) s.t. ct = E_{k*}(pt) for a given k*, determine the number of keys that map pt into ct
- Solution.
  - Given a certain key k, P(k) = Pr[E_k(pt) == ct] = $1/2^n$
  - The *expected* number of keys that map pt into ct is $2^k \times 1/2^n = 2^{k-n}$

False positives

- Example 1 – DES with n = 64 and k = 56
  - On average 2^{-8} keys map pt into ct
  - One pair (pt, ct) is sufficient for an exhaustive key search
- Example 2 – Skipjack with n = 64 and k = 80
  - On average 2^{16} keys map pt into ct
  - Two or more plaintext-ciphertext pairs are necessary for an exhaustive key search

False positives

- Consider now t pairs (pt_i, ct_i), i = 1, 2,…, t
  - Given k, $\Pr[E_k(pt_i) = ct_i$, for all i = 1, 2,…, t] = $(1/2^n)^t = 1/2^{tn}$
  - Expected number of keys that map pt_i into ct_i, for all i = 1, 2, …, t, is $2^k/2^{tn} = 2^{k-tn}$
- Example 3 – Skypjack with k = 80, n = 64, t = 2
  - The expected number of keys is = $2^{80-2\times64} = 2^{-48}$
  - Two pairs are sufficient for an exhaustive key search

False positives

- THEOREM
  - Given a block cipher with a key lenght of *k* bits and a block size of *n* bits, as well as *t* plaintext-ciphertext pairs, (pt_1, ct_1),…, (pt_t, ct_t), the expected number of false keys which encrypt all plaintexts to the corresponding ciphertexts is $2^{k - tn}$
- FACT
  - Two input-output pairs are generally enough for exhaustive key search

Block ciphers

EXERCISES

Exercise 1 - Exhaustive key search

- Exhaustive key search is a known-plaintext attack
- However, the adversary can mount a cyphertext-only attack if (s)he has some knowledge on PT

Exercise 1 – exhaustive key search

- Assume DES is used to encrypt 64-bit blocks of 8 ASCII chars, with one bit per char serving as parity bit
- How many CT blocks the adversary needs to remove false positives with a probability smaller than ε?
- Answer: $2^{-8t} < \varepsilon$, with t number of ct-blocks
  - With DES, t = 10 is sufficient for the most practical uses

Exercise 2 - dictionary attack

- Consider a block cipher with k and n.
- The adversary has collected D pairs (pt_i, ct_i), i = 1,…, D, with $D << 2^n$ (the dictionary)
- Now the adversary reads C newly produced cyphertexts ct^*_j, j = 1,…, C.
- Determine the value of C s.t. the Pr[Exists j, j = 1, 2,… C, s.t. c^*_j is in the dictionary] = P
- Answer: $C = 2^n/D$

Exercise 3 - Rekeying

- An adversary can successfully perform an exhaustive key search in a month.
- Our security policy requires that keys are changed every hour.
- What is the probability P that, in a month, the adversary is able to find any key before it is changed?
  - For simplicity assume that every month is composed of 30 days.
- What if we refresh key every minute?
- Answer: P = 0.63.

Symmetric Encryption

MULTIPLE ENCRYPTION AND KEY WHITENING

Increasing the Security of Block Ciphers

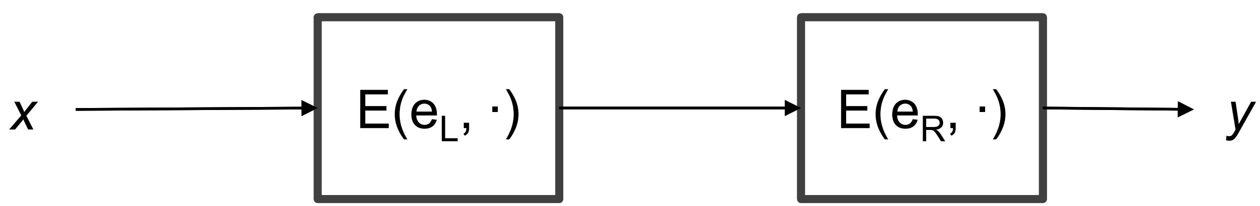
- DES is a secure cipher, no efficient cryptanalys is known
- DES does not define a group
- DES key has become too short
- Can we improve the security of DES?
- Yes, by means of two techniques
  - Multiple encryption
  - Key whitening

DES does not define a group

- If DES were a group then $\forall k_1, k_2 \in \mathcal{K}, \exists k_3 \in \mathcal{K}$ s.t. $\forall x \in \mathcal{M}, E_{k_2}\left(E_{k_1}(x)\right) = E_{k_3}(x)$
- So, double (multiple) encryption would be useless
- Furthermore, DES would be vulnerable to Meet-in-the-Middle attack that runs in 2^{28}

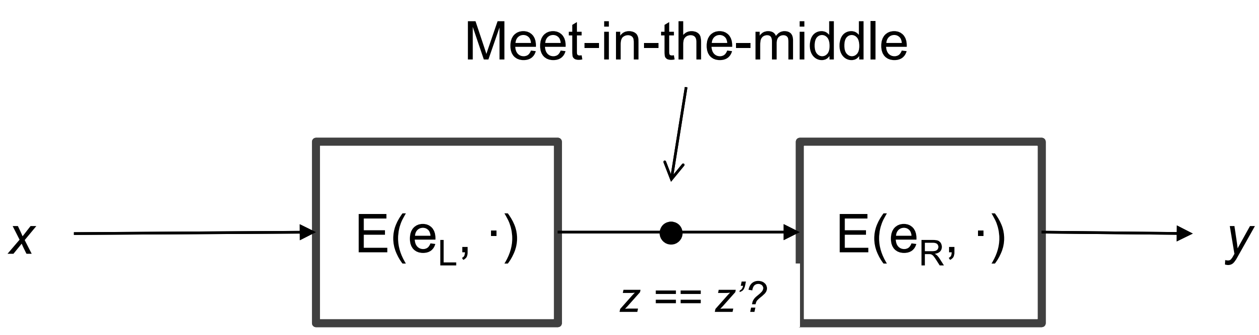
Two-times Encryption (2E)

- $y = 2E((e_L, e_R), m) = E(e_R, E(e_L, x))$
  - key size is 2k bits
  - Brute force attack requires 2^{2k} steps
  - 2E is two times slower than E
- Is it really more secure than single encryption?
- Meet-in-the-middle attack



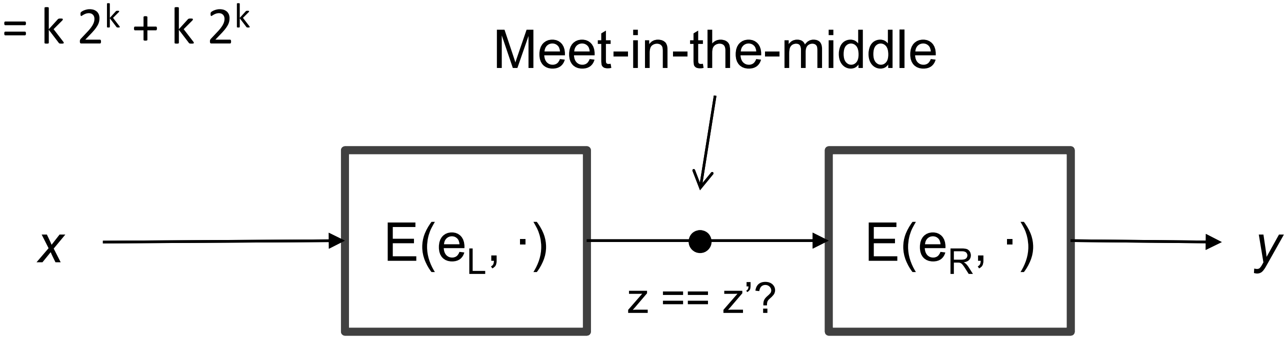
Meet-in-the-middle attack

- Attack Sketch
  1. Build a table T containing $z = E(e_L, x)$ for all possible keys e_L. Keep T sorted according to z.
  2. Check whether $z' = D(e_R, y)$ is contained in the table T, for all possible key e_R.
     1. If z' in contained in T then (e_L, e_R) maps x into y with e_L s.t. $T[e_L] = z'$.



Meet-in-the-middle attack

- Attack complexity
  - Data complexity: negligible.
  - Storage complexity: $O(2^k)$.
    - Storage necessary for table T.
  - Time complexity: $O(k2^k)$.
    - Time complexity for step 1 + Time complexity for step 2 = Time for building and sorting the table + Time for searching in a sorted table = $k\ 2^k + k\ 2^k$



Two-times DES

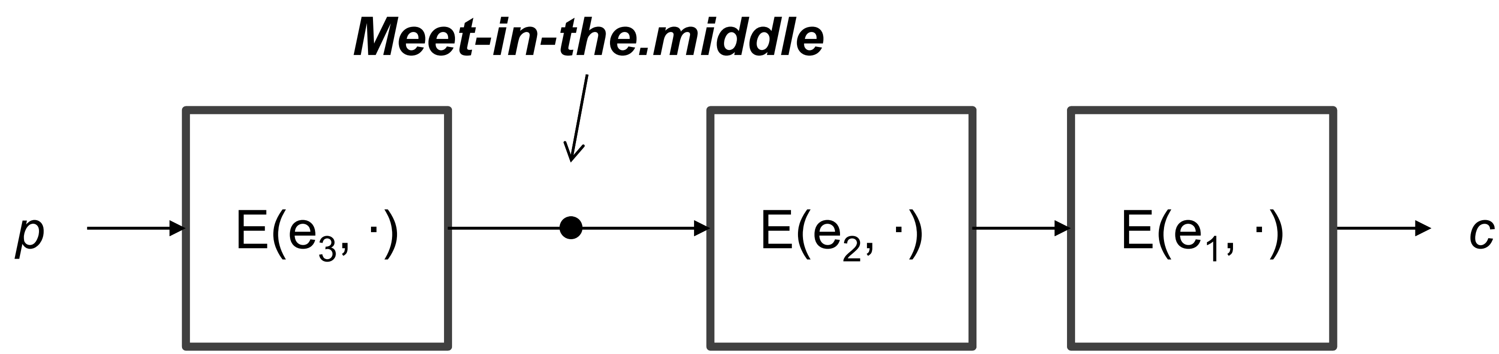
- 2DES
  - Time complexity: 2^{56} (doable nowadays!)
  - Space complexity: 2^{56} (lot of space!)
  - 2DES brings no advantage

Triple DES (3DES)

- EDE scheme
  - Standard ANSI X9.17 and ISO 8732
  - $Y = 3E((e_1, e_2, e_3), x) = E(e_1, D(e_2, E(e_3, x)))$
    - If $e_1 = e_2 = e_3$, 3DES becomes DES
      - backward compatibility
  - Key size = 168-bits
  - 3 times slower than DES
  - Simple attack $\approx 2^{118}$

3DES – meet-in-the-middle attack

- Time = 2^{112} (undoable!)
- Space = 2^{56} (lot of space!)



False positives for multiple encryption

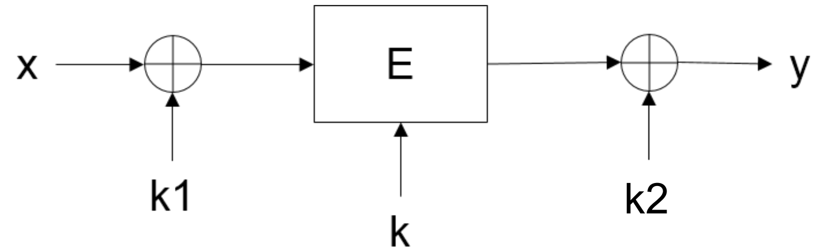
- THEOREM
  - Given there are r subsequent encyptions with a block cipher with a key lenght of k bits and a block size of n bits, as well as t plaintext-ciphertext pairs, $(pt_1, ct_2),\ldots, (pt_t, ct_t)$, the expected number of false keys which encrypt all plaintext to the corresponsig ciphertext is 2^{rk-tn}

Limitations of 3DES

- 3DES resists brute force but
  - It is not efficient regarding software implementation
  - It has a short block size (n = 64)
    - A drawback if you want to make a hash function from 3DES, for example
  - Key lengths of at least 256-bit are necessary to resist quantum computing attack

Key whitening

- Considerations
  - KW is not a “cure” for weak ciphers
- Applications
  - DESX: a variant of DES
  - AES: uses KW internally
- Performance
  - Negliglible overhead w.r.t. E (Just two XOR’s!)



Definition 5.3.1 Key whitening for block ciphers
Encryption: $y = e_{k,k_1,k_2}(x) = e_k(x \oplus k_1) \oplus k_2$.
Decryption: $x = e^{-1}_{k,k_1,k_2}(x) = e_k^{-1}(y \oplus k_2) \oplus k_1$

Key whitening

- Attacks
  - Brute-force attack
    - Time complexity: 2^{k+2n} encryption ops
  - Meet-in-the-middle:
    - Time complexity 2^{k+n}
    - Storage complexity: 2^n data sets
  - The most efficient attack
    - If the adversary can collect 2^m pt-ct pairs, then time complexity becomes 2^{k+n-m}
      - The adversary cannot control m (rekeying)
    - Example: DES (m = 32)
      - Time complexity 2^{88} encryptions (nowadays, out of reach)
      - Storage complexity 2^{32} pairs = 64 GBytes of data (!!!)

Symmetric Encryption

ENCRYPTION MODES

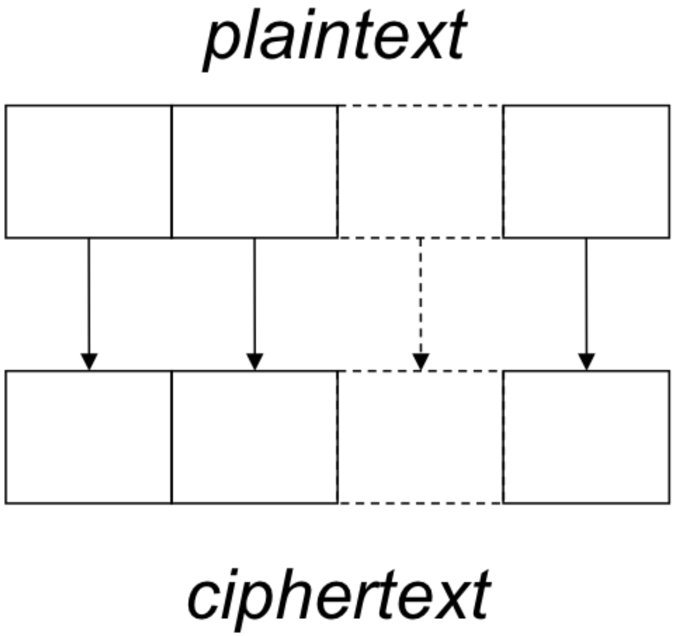
Encryption Modes

- A block cipher encrypts PT in fixed-size *n*-bit blocks
- When the PT len exceeds n bits, there are several modes to use the block cipher
  - Electronic Codebook (ECB)
  - Cipher-block Chaining (CBC)

Other encryption modes

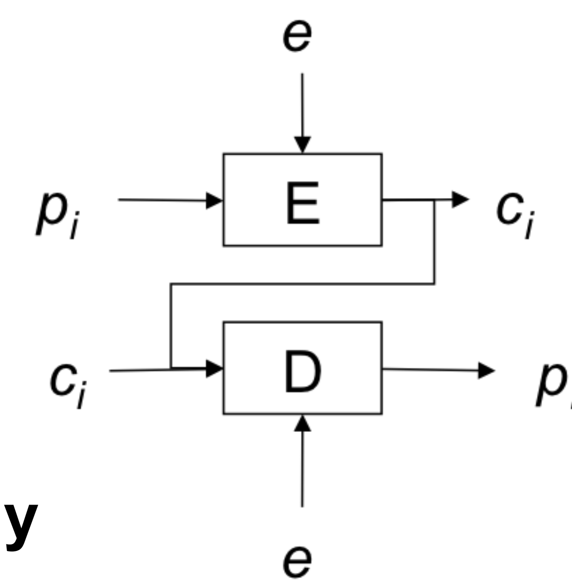
- Other encryption modes
  - To build a stream cipher out of a block cipher
    - Cipher Feedback mode (CFB)
    - Output Feedback mode (OFB)
    - Counter mode (CTR)
  - Authenticated encryption
    - Galois Counter mode (GCM, CCM, …)
  - and many others (e.g., CTS, …)
- Block ciphers are very versatile components

Electronic codebook



$$\forall 1 \le i \le t, c_i \leftarrow E(e, p_i)$$

$$\forall 1 \le i \le t, p_i \leftarrow D(e, c_i)$$



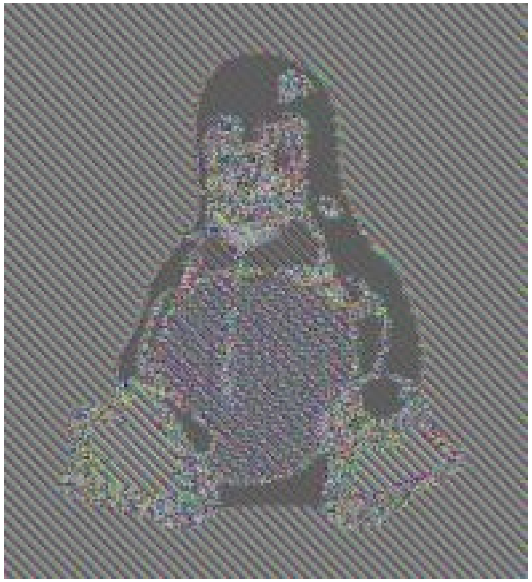
PT blocks are encrypted separately

ECB - properties

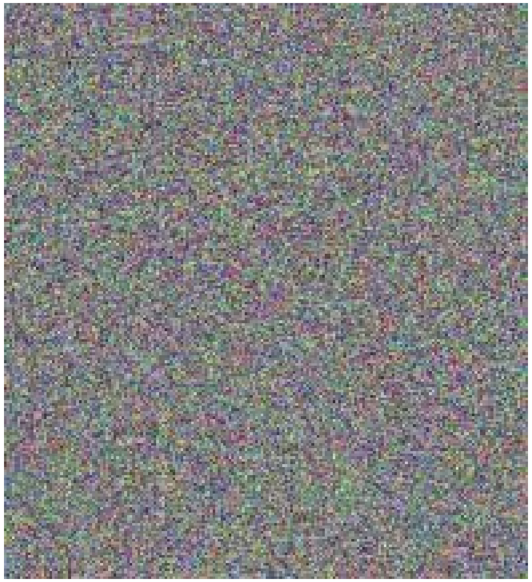
- PROS
  - No error propagation
    - One or more bits in a single CT block affects decryption of that block only
  - Enc & Dec can be parallelized
- CONS (it is insecure)
  - Blocks are encrypted separately
    - Identical PT results in identical CT
      - ECB doesn't hide data pattern
      - ECB allows traffic analysis
    - ECB allows block re-ordering and substitution

ECB doesn’t hide data patterns

Plaintext



ECB encrypted



Non-ECB encrypted

ECB – block attack

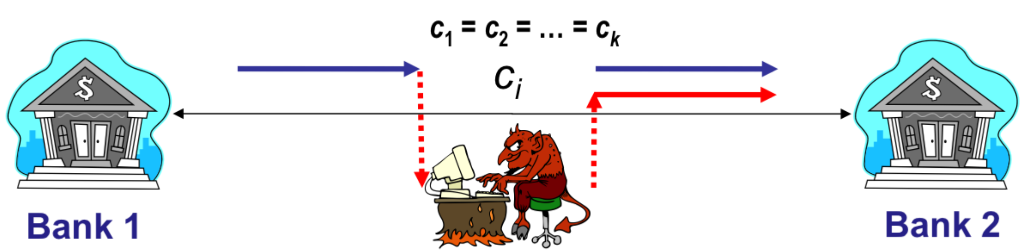
- Bank transaction that transfers a customer C's amount of money D from bank B1 to bank B2
  - Bank B1 debits D to C
  - Bank B1 sends the "credit D to C" message to bank B2
  - Upon receiving the message, Bank B2 credits D to C
- Credit message format
  - Src bank: M (12 byte)
  - Rcv banck: R (12 byte)
  - Customer: C (48 byte)
  - Bank account number: N (16 byte)
  - Amount of money: D (8 byte)
- Cipher: n = 64 bit; ECB mode

ECB – block attack

- Mr. Lou Cipher is a client of the banks and wants to make a fraud
- Attack aim
  - To replay Bank B1’s message "credit 100$ to Lou Cipher" many times
- Attack strategy
  - Lou Cipher activates multiple transfers of 100$ so that multiple messages "credit 100$ to Lou Cipher" are sent from B1 to B2
  - The adversary identifies at least one of these messages
  - The adversary replies the message several times

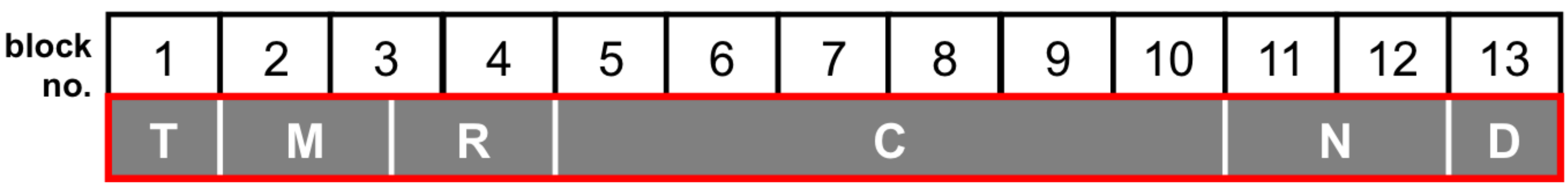
ECB – block attack

- The fraud
  1. Mr. Lou Cipher performs k equal transfers
     - credit 100$ to Lou Cipher → c1
     - credit 100$ to Lou Cipher → c2
     - …
     - credit 100$ to Lou Cipher → c_k
  2. Then, he searches for “his own” CTs, namely k equal CTs!
  3. Finally he replies one of these cryptograms (many times)



ECB – block attack

- The message lacks any notion of time so it can be easily replied
- An 8-byte timestamp field T (block #1) is added to the message to prevent replay attacks
- A replied message can now be discarded



ECB – block attack

- However, Mr Lou Cipher can still perform the attack
  1. Identify “his own” CTs by inspecting blocks #2-#13
  2. Select any his-own-CT
  3. Substitute block #1 of his-own-CT with block #1 of any intercepted “fresh” block
  4. Replay the resulting CT

ECB is disallowed

Table 2. Approval status of the block cipher modes of operation for AES encryption and decryption

Publication	Mode	Status
SP 800-38A	ECB	Disallowed for data encryption Legacy use for decryption
	CBC	Acceptable
	CFB	Acceptable
	CTR	Acceptable
	OFB	Acceptable

NIST Special Publication 800
NIST SP 800-131Ar3 ipd

Transitioning the Use of Cryptographic Algorithms and Key Lengths

Initial Public Draft

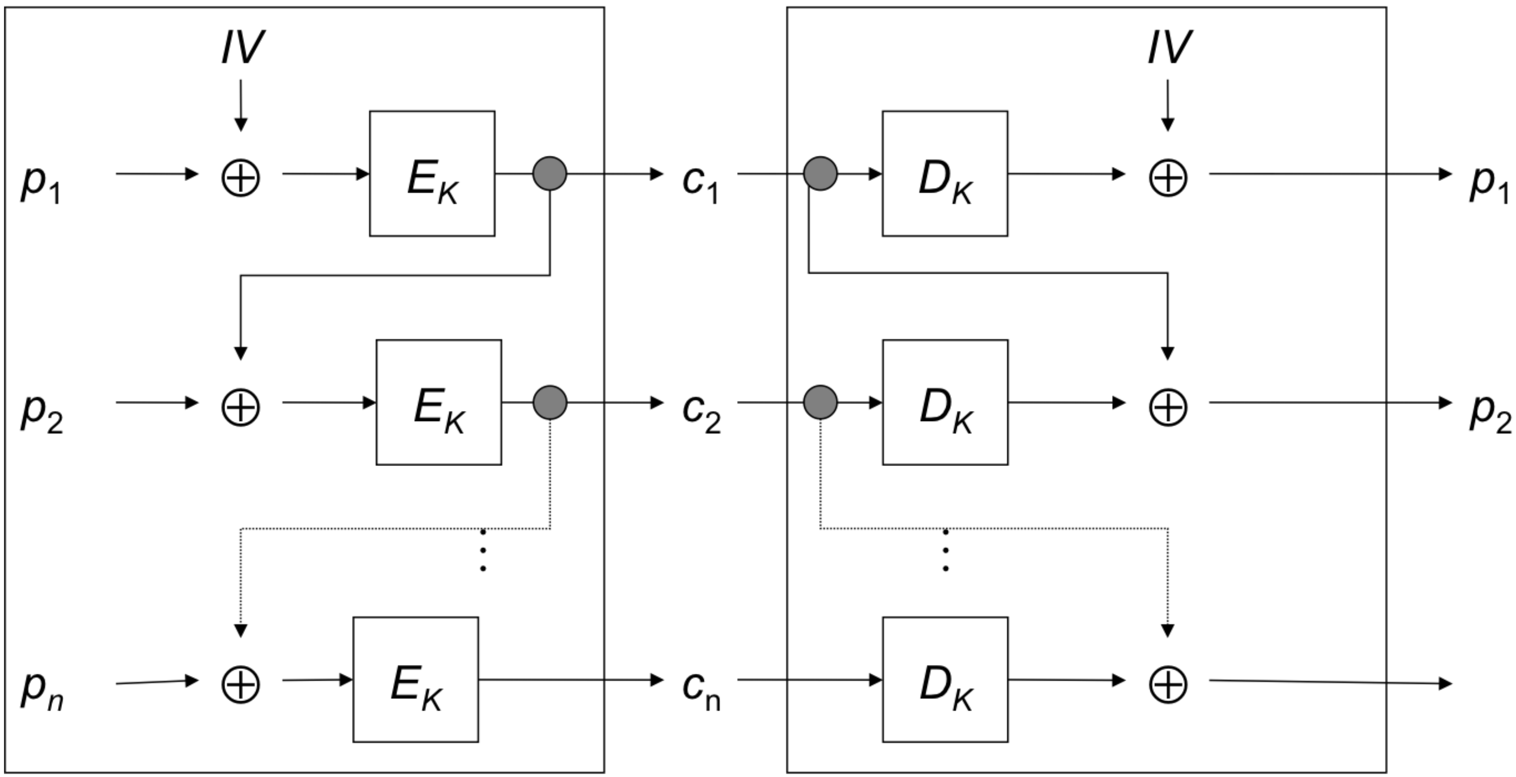
Elaine Barker
Allen Roginsky

This publication is available free of charge from:
https://doi.org/10.6028/NIST.SP.800-131Ar3.ipd

Cipher block chaining (CBC)

Encryption: $c_0 \leftarrow IV. \forall 1 \leq i \leq t, c_i \leftarrow E_k(p_i \oplus c_{i-1})$

Decryption: $c_0 \leftarrow IV. \forall 1 \leq i \leq t, p_i \leftarrow c_{i-1} \oplus D_k(c_i)$



CBC – properties (→)

- CBC mode is CPA-secure.
- CBC-Enc is *randomized* by using IV (nonce).
  - Identical ciphertext results from the same PT under the same key and IV.
- Chaining dependencies: c_i depends on p_i and the preceding CT block c_{i-1}
- CT-block reordering affects decryption
- CBC suffers from Error propagation
  - Bit errors in c_i affect p_i and p_{i+1} (*error propagation*)

CBC – properties

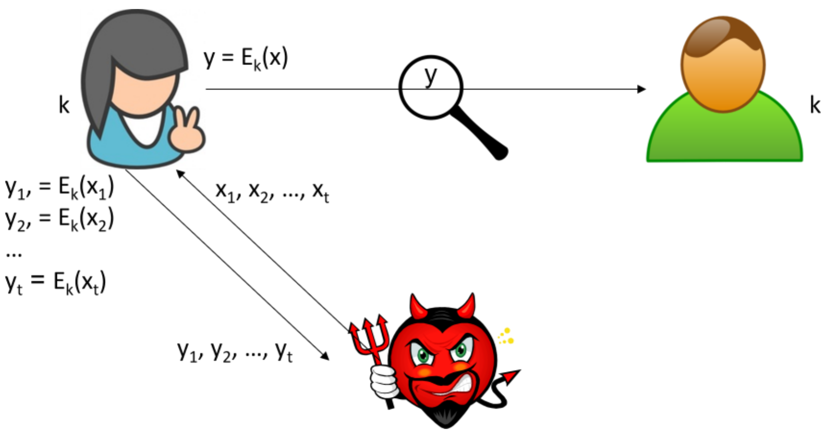
- IV can be sent in the clear but its integrity must be guaranteed
- Cyphertext expansion is just one block (IV)
- Only CBC-dec can be parallelized

CBC – block attack

- If Bank A chooses a random IV for each wire transfer the attack will not work.
- However, if Lou Cipher substitutes blocks #5–10 and #13, bank B would decrypt *account number* and *deposit amount* to random numbers ➔
  - This is highly undesirable!
  - Encryption itself is not sufficient, we need additional mechanisms (MDC, MAC, digsig) to protect integrity

Chosen-Plaintext Attack (Informal)

- CPA Attack
  - Attacker *makes* the sender to encrypt $x_1, \ldots, x_t$
    - The attacker may influence or control encryption
  - The sender encrypts and transmits $y_1 = E_k(x_1), \ldots, y_t = E_k(x_t)$
  - Later on, the sender encrypts x and transmits $y = E_k(x)$
- CPA-security guarantees that the adversary cannot learn anything about x
- The encryption scheme must be randomized



CBC is acceptable

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Check for updates

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NIST SP 800-131Ar3 ipd

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Block Ciphers

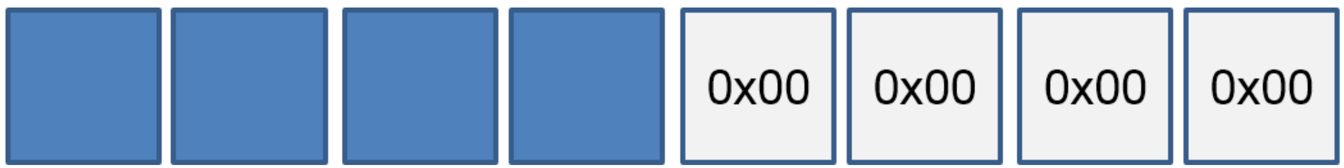
PADDING

Padding

- Padding is necessary when PT len is not an integer multiple of the block

A naïve (wrong) solution

Pad the message with zeroes to the right, without ambiguous boundaries



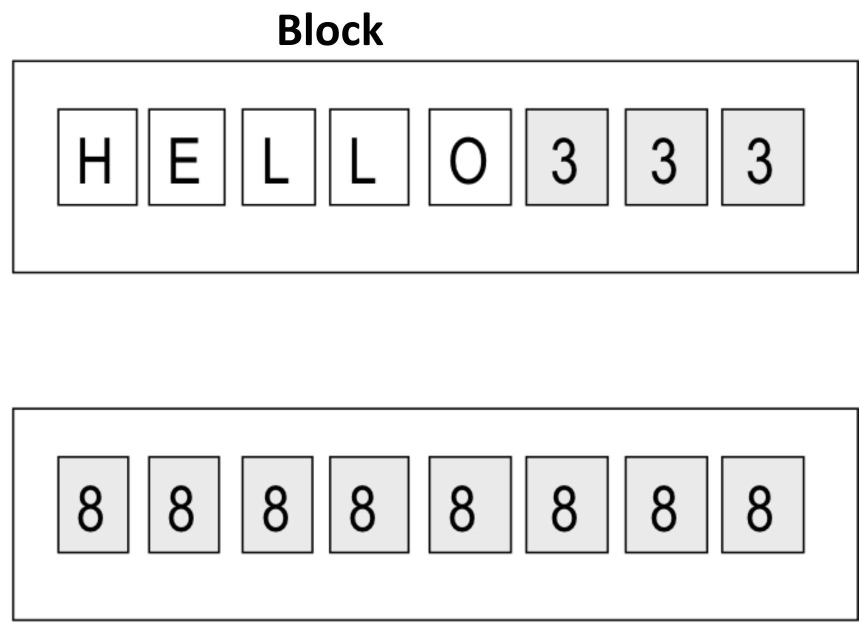
Problem: What if the message was a NULL-terminated string?

At the receiving side: Was it a NULL-terminated string or a 7-bytes pt?

The PKCS #5 padding scheme

If PT len is NOT a block multiple
- We need *b* padding bytes
- Fill each padding byte by *b*

Example: b = 3 then append 0x030303



If PT len is a block multiple
Padding = block
Fill each padding block by 8

Padding causes *ciphertext expansion*

PKCS #5: encryption & decryption →

- Let *L* be the block length (in bytes) of the cipher
- Let *b* be the # of bytes that need to be appended to the plaintext to get its length a multiple of L, $1 \leq b \leq L$
- Before encryption
  - Append *b* (encoded in 1 byte), *b* times
  - Example: if b = 3, append 0x030303

PKCS #5: encryption & decryption

- After decryption, say the final byte has value *b*
  - If b == 0 or b > L, return “error”
  - If the trailing *b* bytes are not all equal to *b*, return “error”
  - Strip off the trailing b bytes and output the left as the message

PKCS #5 vs PKCS #7

- Difference between PKCS#5 and PKCS#7
- PKCS#5: padding is defined for 8-byte block sizes (RFC 2898)
- PKCS#7: padding is defined for block of any size ranging from 1 to 255 bytes (RFC 2315)

Block Ciphers | Padding

PADDING ORACLE ATTACK

Padding Oracle Attack (CCA)

Attack against CBC encryption mode

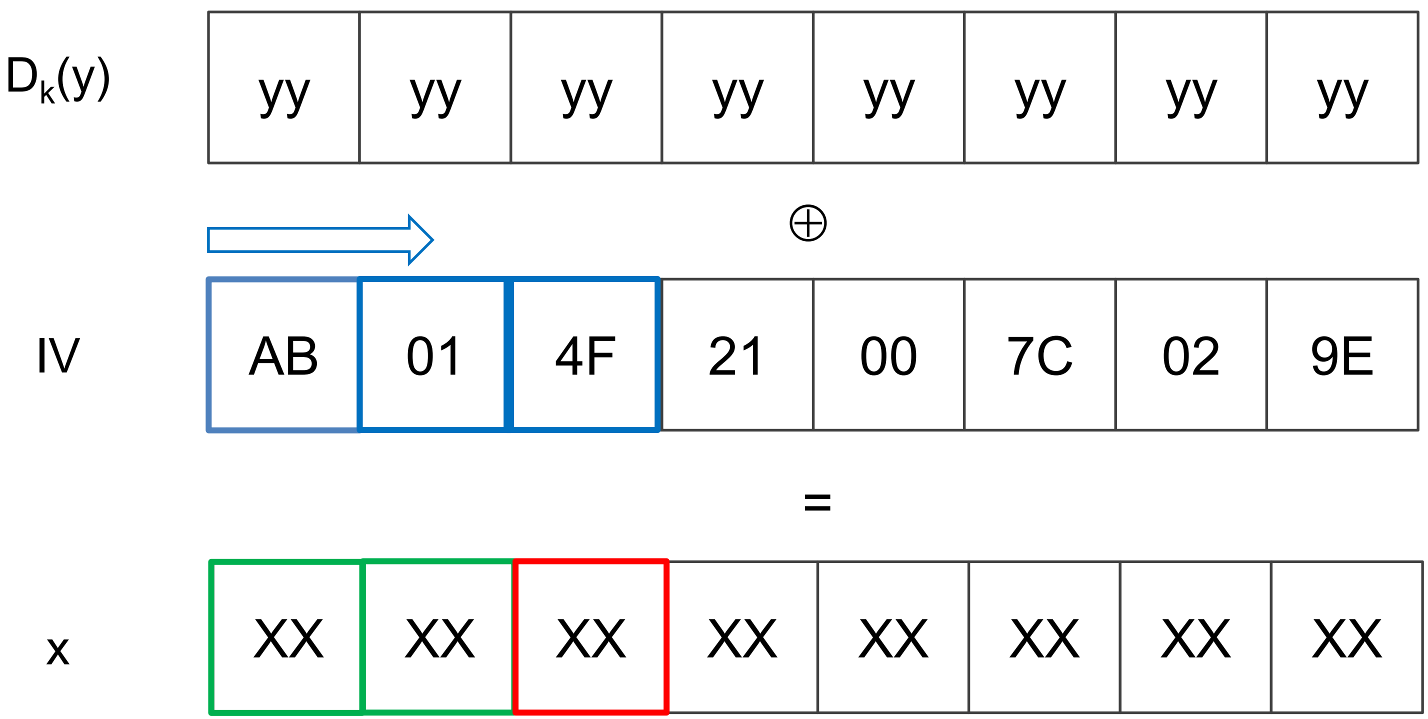
- The attacker
  - intercepts y and wants to obtain x (*ciphertext-only attack*)
  - modifies y into y' and submits to the receiver
- The receiver (the padding oracle) it might be an oracle if you receive an explicit error, but also looking at the timing for response
  - Receiver decrypts y' and returns "error", if x' is not properly formatted (padding)
- On padding oracles
  - Frequently present in web applications
  - Error, receiver timing, receiver behaviour,…

Receiver can behave like an oracle more generally

Main idea of the attack

- For simplicity, let CT be a two-block ciphertext (IV, y), with $y = Enc_k(x \oplus IV)$ equation to explain how we obtain CT.
- At the receiving site: $x = D_k(y) \oplus IV$
- Assume message x is well formatted in terms of padding by means of PKCS #5
- Main intuition of the attack
  - If the attacker changes the i-th byte of IV, this causes a predictable change (only) to the i-th byte of x

The attack – step 1 – determine padding lenght



Goal: obtaining x. At the receiving side, receiver computes $D_k(y)$ [UNKNOWN TO ADVERSARY] and then compute $x = D_k(y) \oplus IV$.

- STEP 1 FOR ADV: Determine length of padding

We start by modifying first byte of IV. Receiver decrypts y and obtains a different value for x. 2 possibilities: after decryption receiver checks wether x' is well formatted. Either yes [MODIFICATION HAS NOT CHANGED A PADDING BYTE], so the padding is at most 7 bytes, or no [THE PADDING IS 8 BYTES].

Repeat until you get an error and determine length of bytes.

CLASS of CHOSEN CYPHERTEXT ATTACK

The attack – step 2a – determine pt

$D_k(y)$	yy	yy	yy	yy	yy	yy	yy	yy

$\oplus$

IV	AB	01	4F	21	00	7C	02	9E	←9F = 9E⊕06⊕07

MODIFY THEM IN A WAY THAT GIVES US 07 (WE KNOW 06)

=

x	XX	XX	06	06	06	06	06	06	←each blue byte of x' becomes 0x07

(A)

$yy \oplus 9F = yy \oplus 9E \oplus 06 \oplus 07 = 06 \oplus 06 \oplus 07 = 07$

$9F = 9E \oplus \underbrace{06 \oplus 07}_{P}$

P = Actual byte ⊕ target byte

• STEP 2: Determine the real plaintext

Change IV in such a way that receiver obtains X' after decryption in a way that has increased padding.

If we know padding is 06, perturbation is [IV actual byte] ⊕ 07 ⊕ 06

We modify then the 2nd byte until we don't get an error.

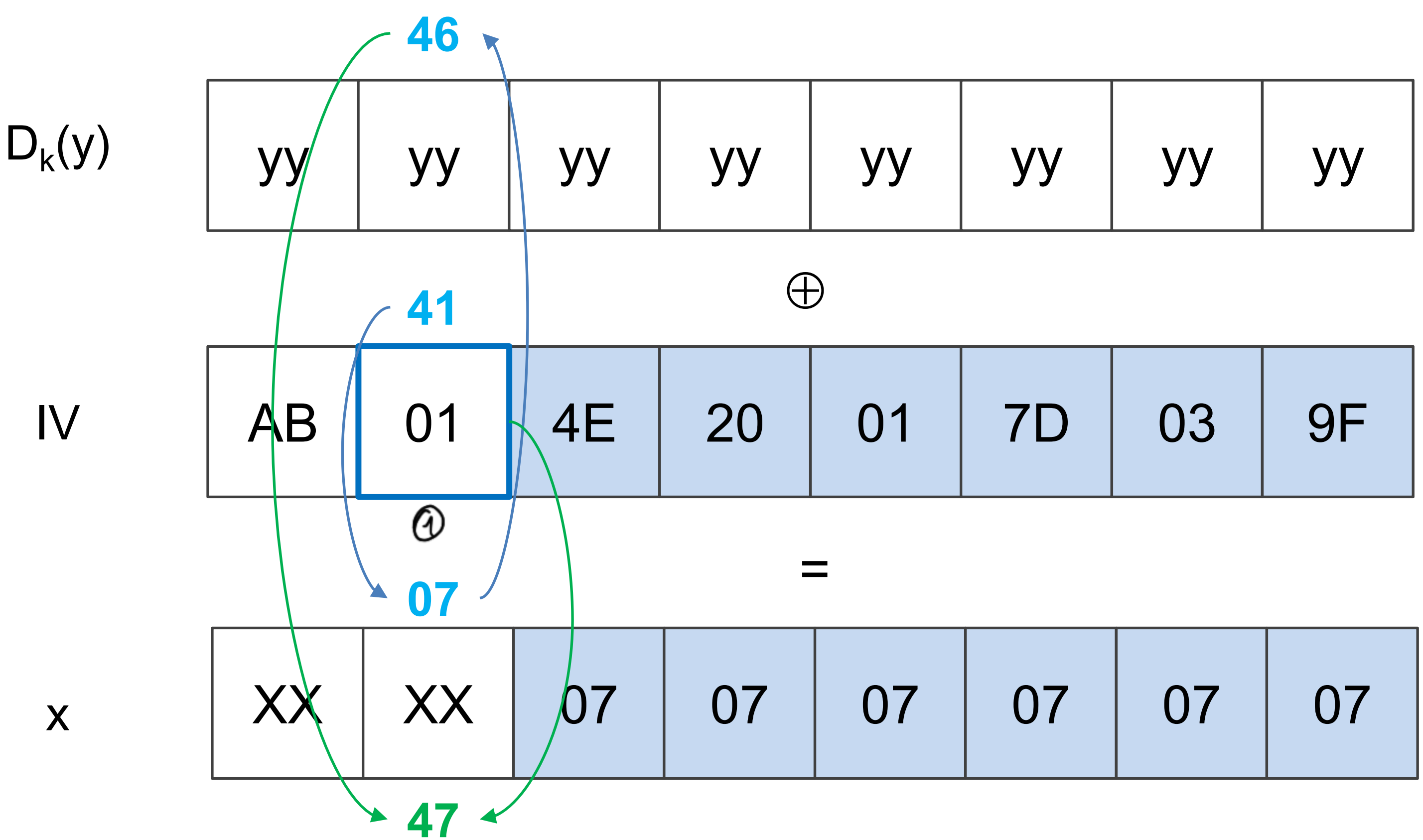
If we get an error, the 2nd byte doesn't contain 07.

If it does contain 07, we save the IV' byte we inserted to get the value (1). So, we now know that $yy \oplus 1 = 07$ (A)

• yy is the decrypted 2nd byte of CT. So using the actual IV, we can compute the actual 2nd byte of PT.

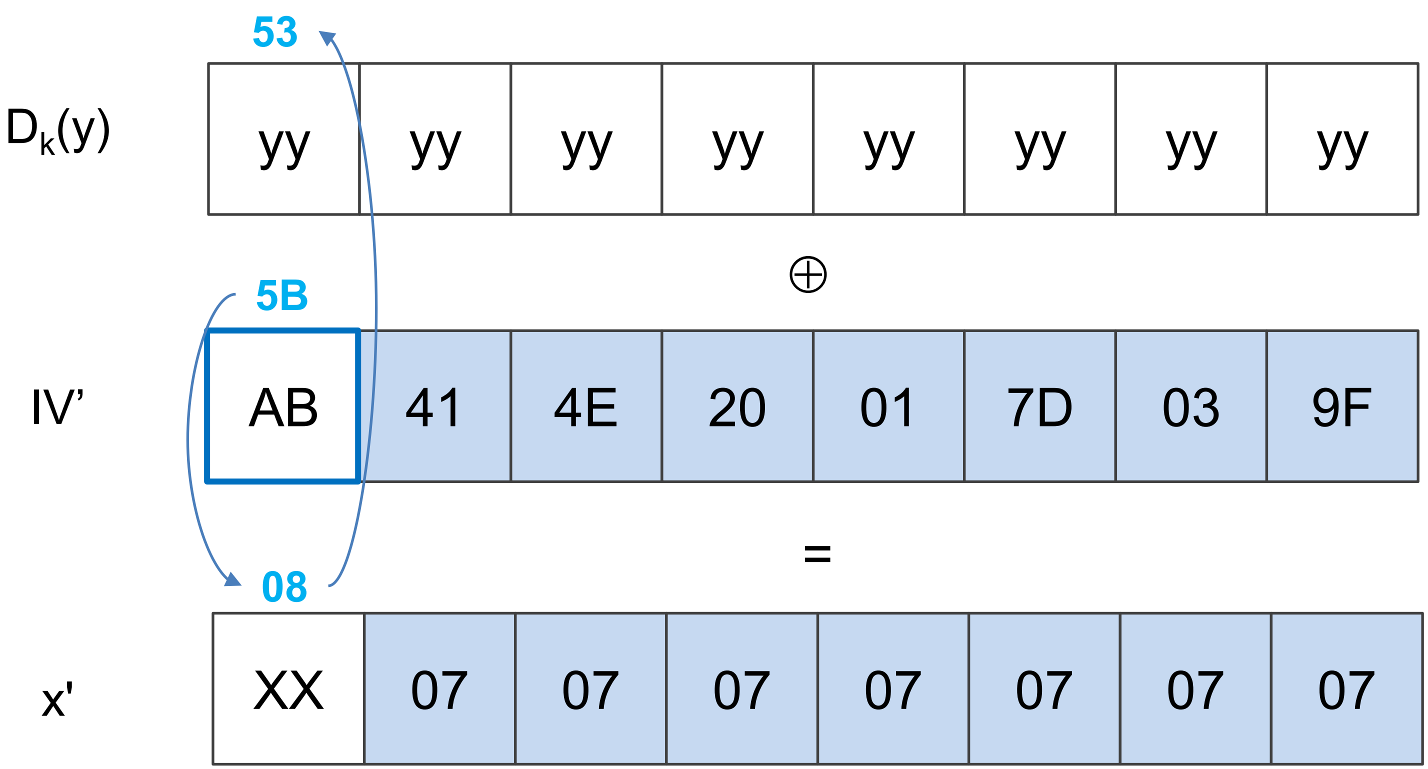
• Then we repeat same approach for 1st byte. We type 08 instead of 07.

The attack – step 2b – determine pt



$0x41 \oplus yy = 0x07$ $yy = 0x46$ $xx = yy \oplus 01$

The attack – step 2c – determine pt



Attack complexity

- At most L tries to learn the # of padding bytes
- At most $2^8 = 256$ tries to learn each plaintext byte

• Complexity: $L + (8 - L) \cdot 256$

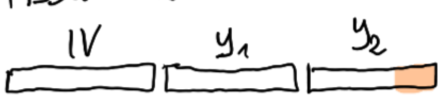
• L = size of padding

• Much faster than bruteforce

[THIS IS FOR LAST BLOCK]

FOR PREVIOUS BLOCKS YOU HAVE COMPLEXITY OF $8 \cdot 256$ PER BLOCK

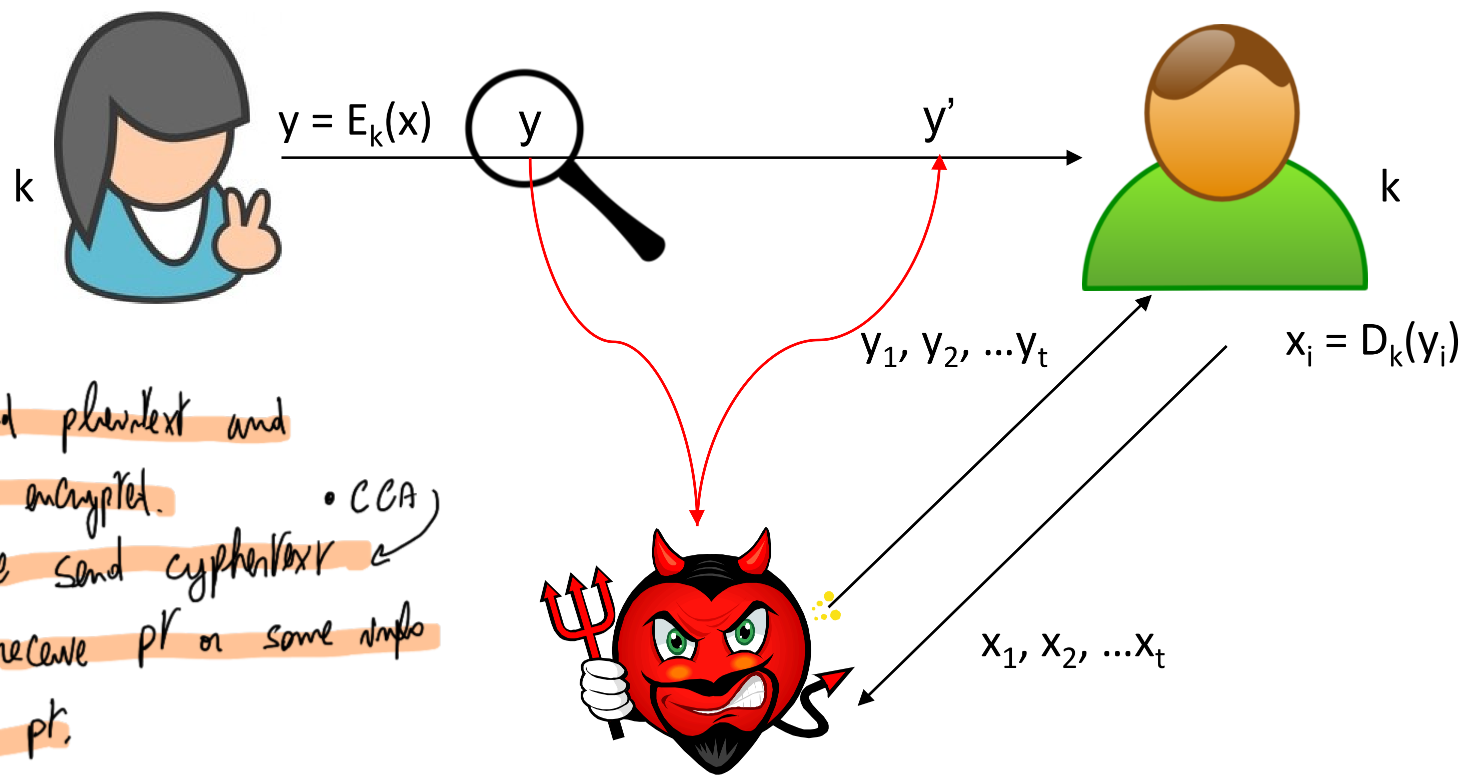
Assume a 3 block CT:



• Here you use y_1 as initialization vector

• To decrypt y_1 you cheat receiver to think you have 1 byte of padding

CCA model



- CPA
  We send plaintext and have it encrypted.
- CCA
  Here we send cyphertext and receive pt or some info about pt.

To avoid this you could introduce a MAC

Chosen-ciphertext attack

- Now the attacker becomes active
- The CCA
  - The attacker intercepts $y = E_k(x)$ and modifies it into y'
  - The receiver decrypts y' and returns (the attacker) either x' or some information about x'
  - The adversary can derive either x or some information about x
- CCA and malleability
  - CCA-security implies non-malleability

CCA-security

- Chosen-ciphertext attacks represent a significant, real-world threat
- Modern encryption schemes are designed to be CCA-secure