MSc Course in *Cybersecurity*

Digital Signals

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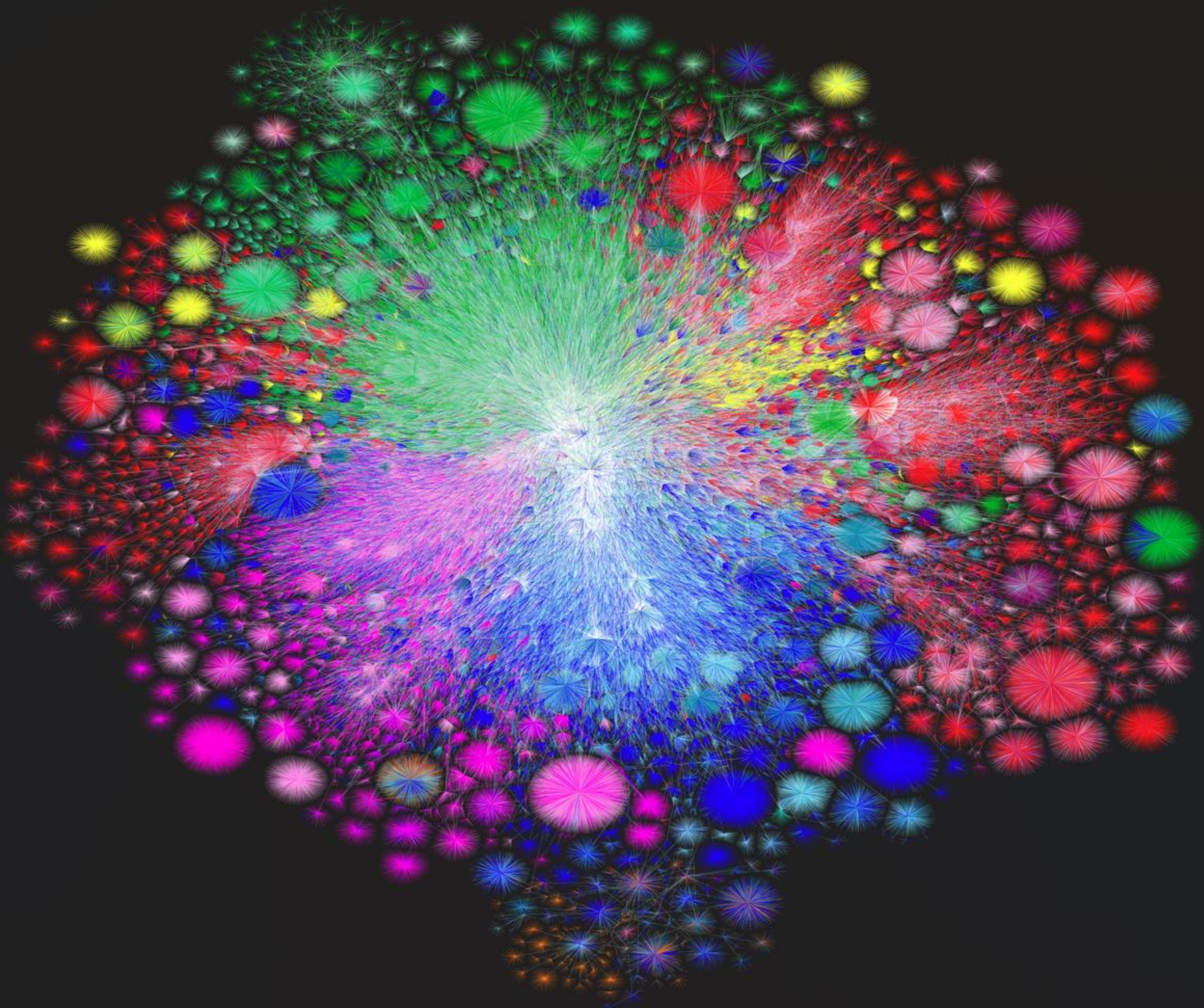
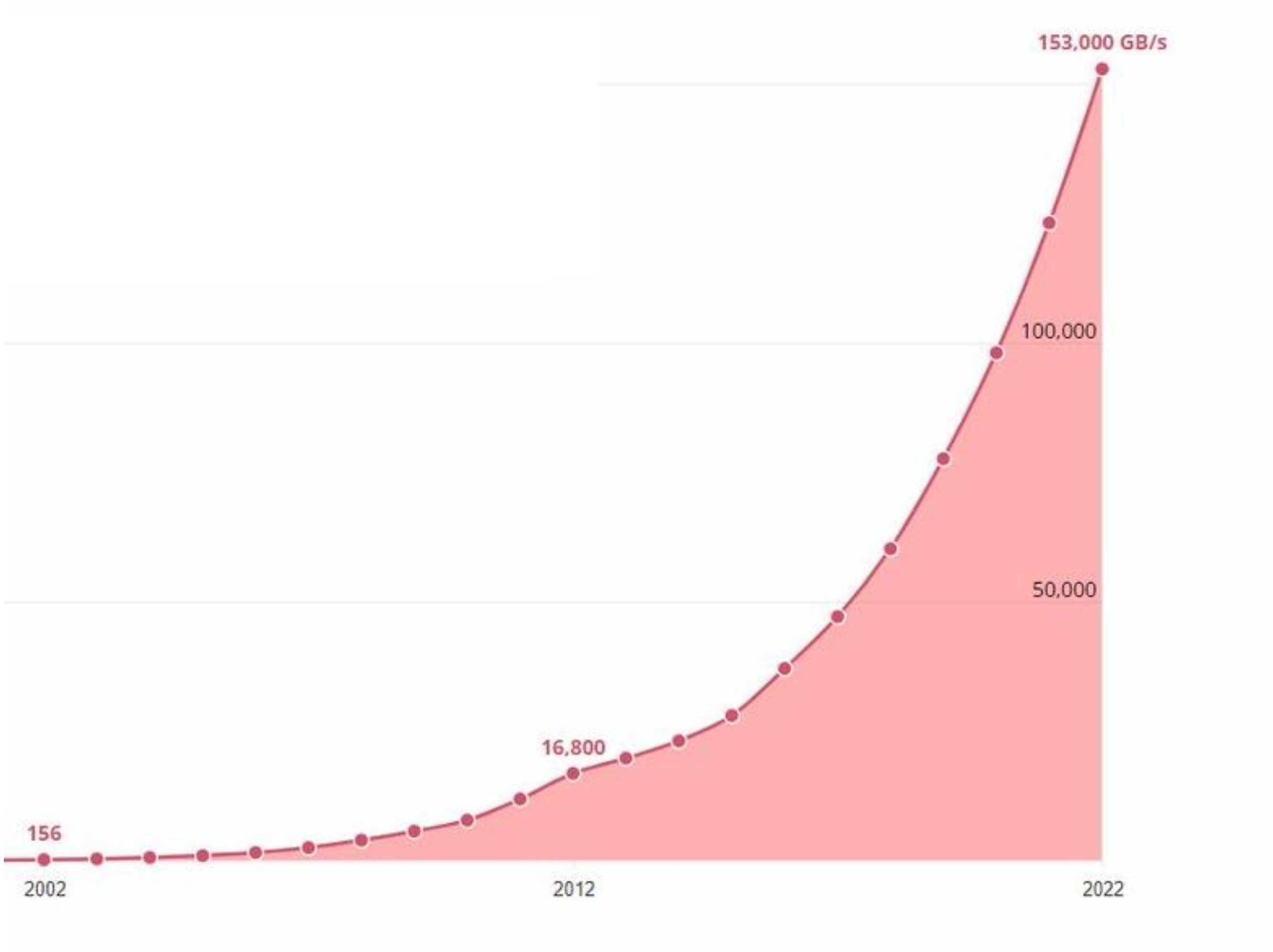
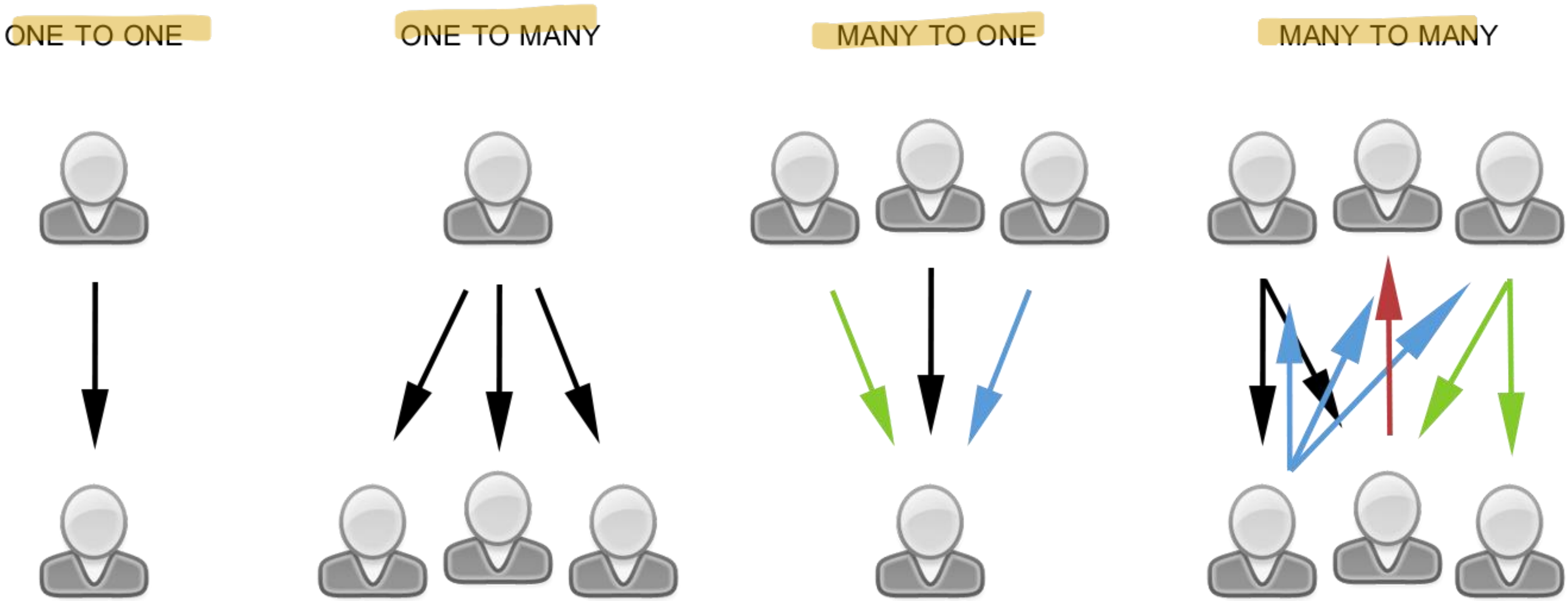


https://www.opte.org/

Global Internet Traffic



Communications…



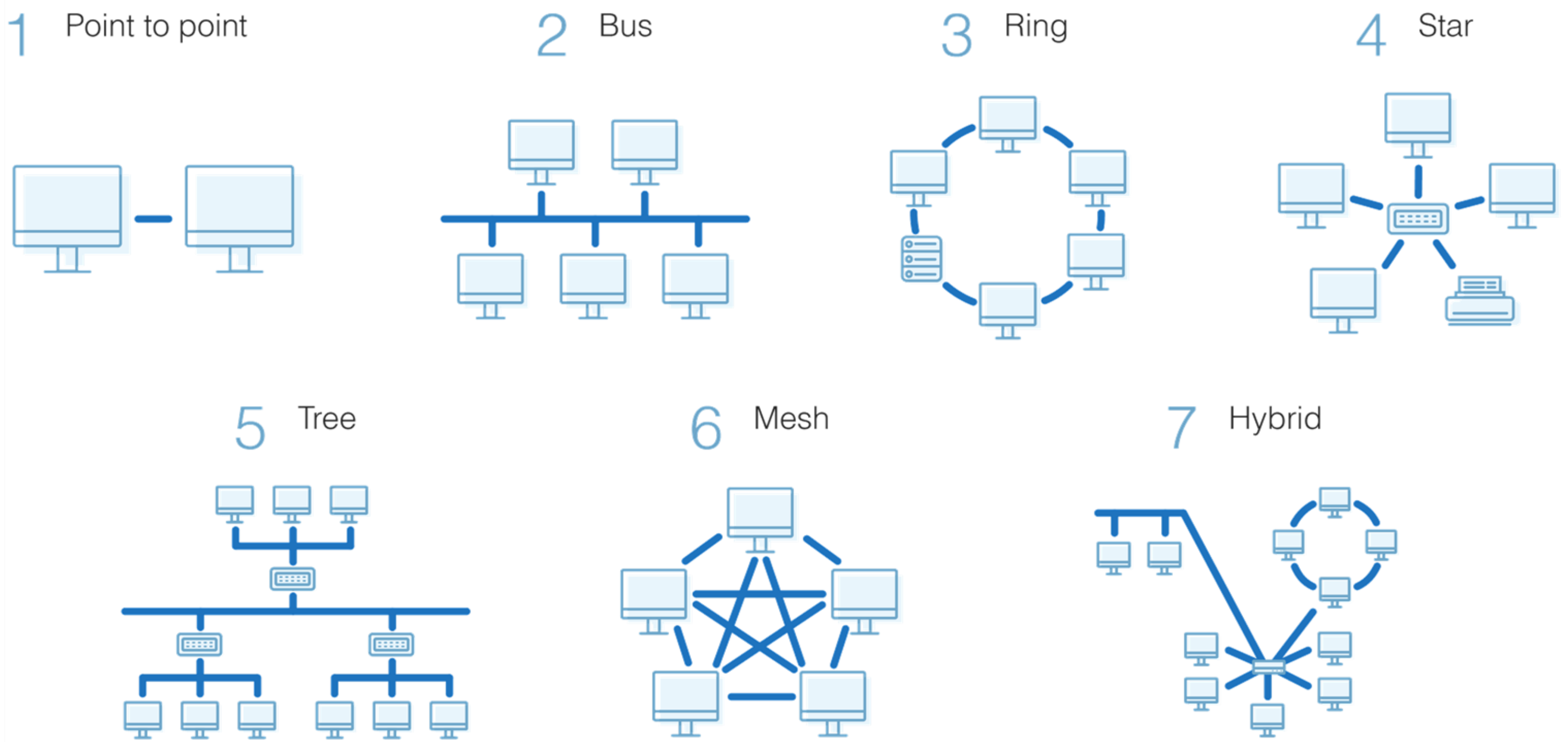
Communication is data. We wish to send data. We have different kinds of communications, one to one, one to many (broadcasting), many to one (cellular tower is an example: many smartphones communicate with the tower*), many to many (like on the internet, WAN).

NO ASSUMPTION ON THE PHYSICAL MEAN OF COMMUNICATION.

* THAT ONE is called Base transceiver station, technical name for the tower.

transmits and receive at the same time.

Communication Networks Architecture



(2) How to implement those communication systems? Architecture describes the characteristics of the network to achieve the way a communication.

Two points are connected by a link.

(13 Kbits/s is the bitrate for standard calls). Network is a connection of point to point links.

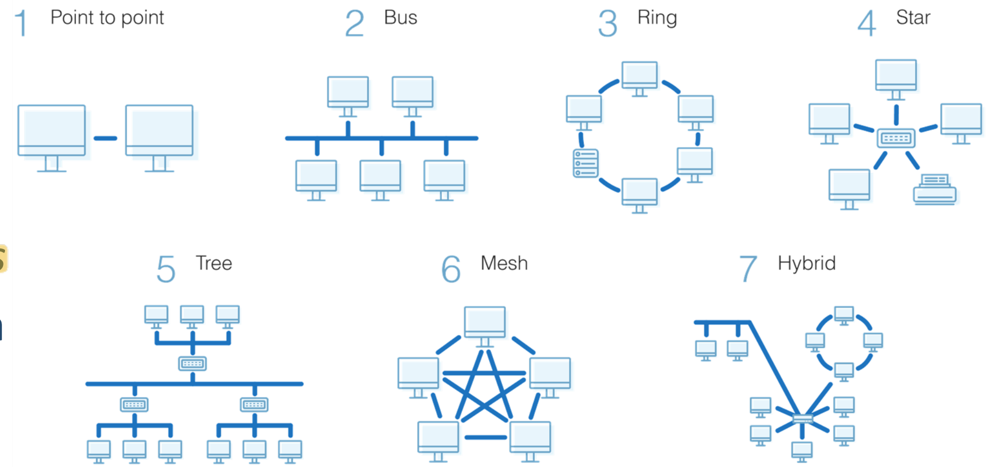
Ring, Star, Bus can do one to one but also broadcast.

Ethernet is Bus network.

Mesh is ideal but full mesh is very expensive.

Reality is hybrid network, different architectures interconnected.

Point-to-Point Multimedia Digital Communications



- Any network topology is made of a collection of point-to-point digital *links*
- The links are implemented exploiting a *physical medium*
- Different physical media require different technologies
  - Wireless
  - Copper
  - Fibers
- Whatever the medium, any link transfers *digital information* carrying *multimedia messages* (audio, images, video, text, pure data)
- How can everything be reduced to the same *digital format* ?

③ Wireless also means Fixed wireless access, it's a wireless comm. between fixed points (home and a station). Has nothing to do with mobile network.

- Fibers were built to carry up to terabits per second, as the backbone transfer technology. Now we have FTTH, fiber to the home, up to 1 Gigabit/s.

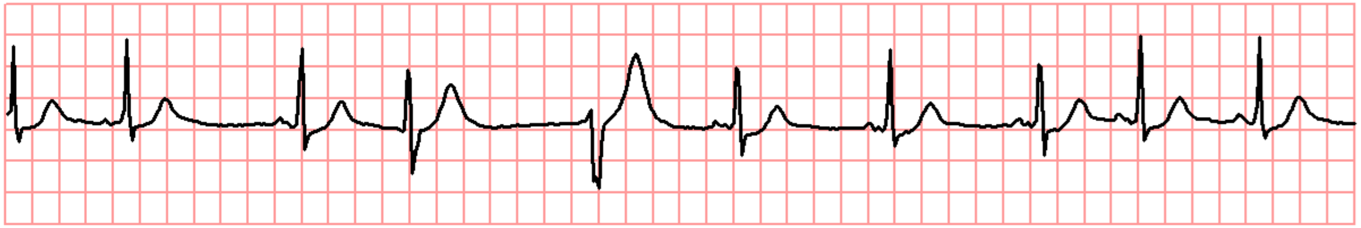
In general, we send multimedia messages over the internet.

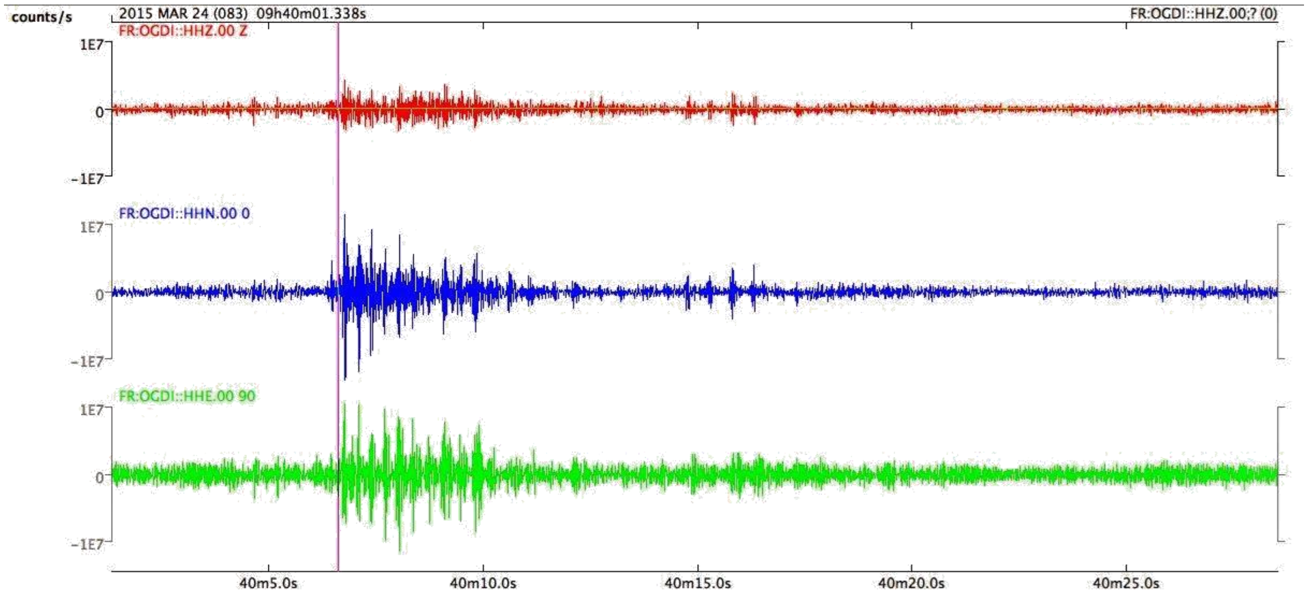
Everything that runs on the internet is DIGITAL. The large part of things we send on the net is something PHYSICAL, IT HAS TO BE REDUCED TO DIGITAL FORM.

- Video signal is a collection of images one after the other. Usually we have 25 fps, so 25 pics per second.

Signals…

A signal is *any variable physical quantity* that carries a certain amount of *information*, that makes the signal itself «interesting»





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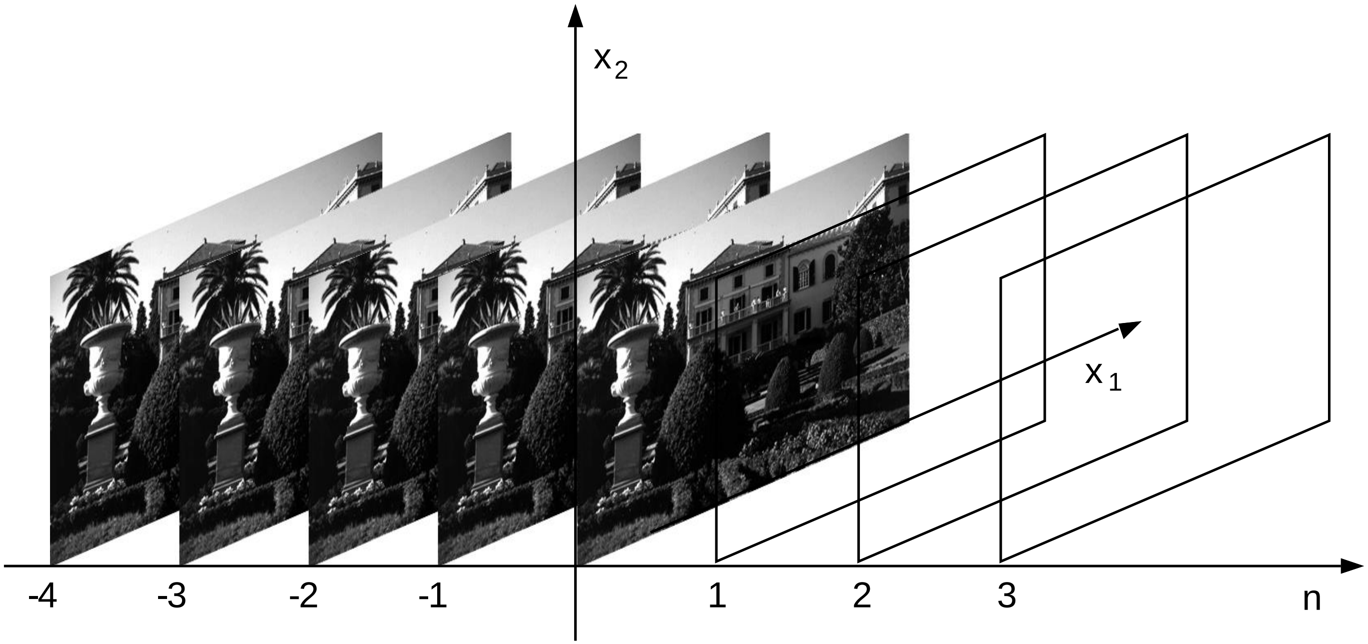
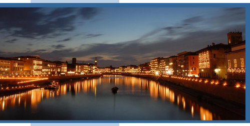
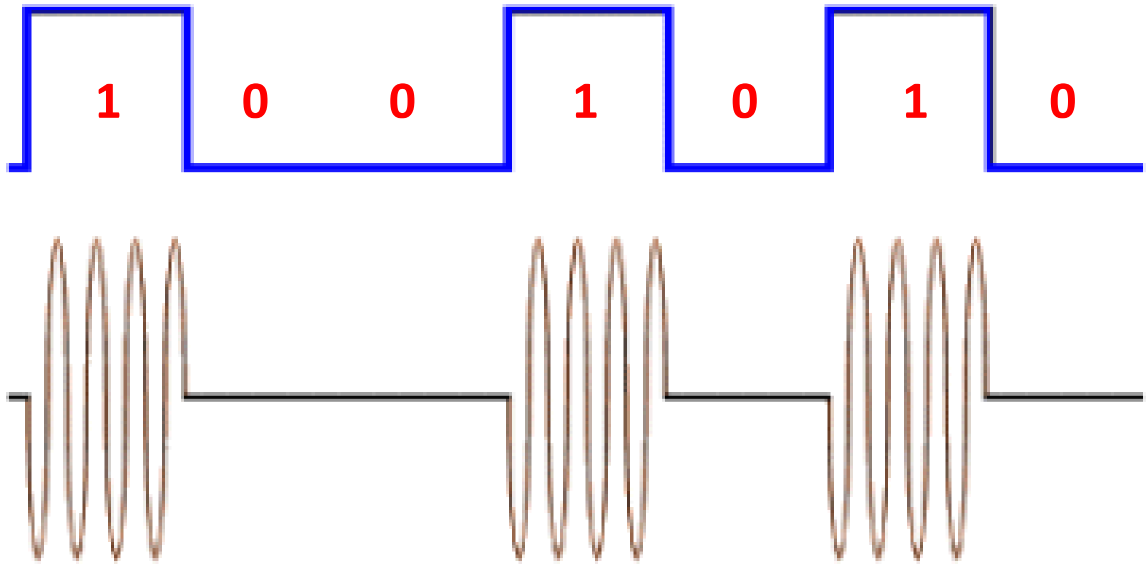
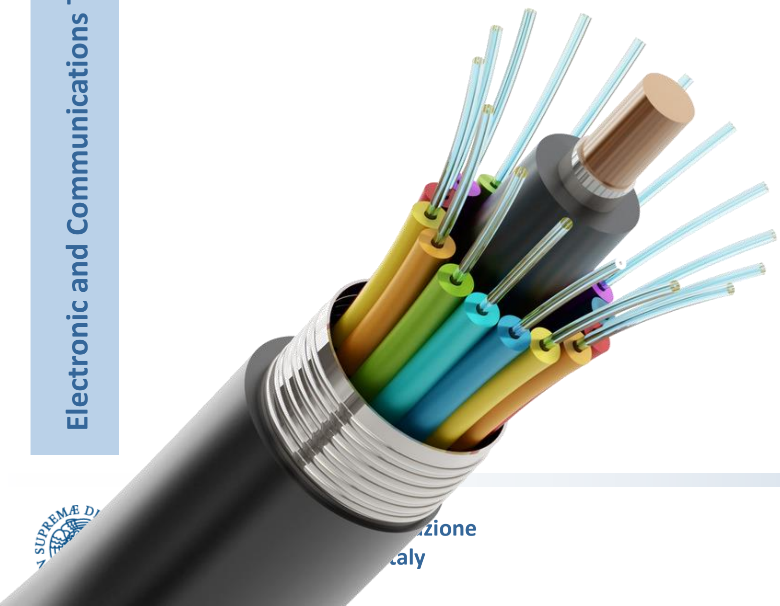




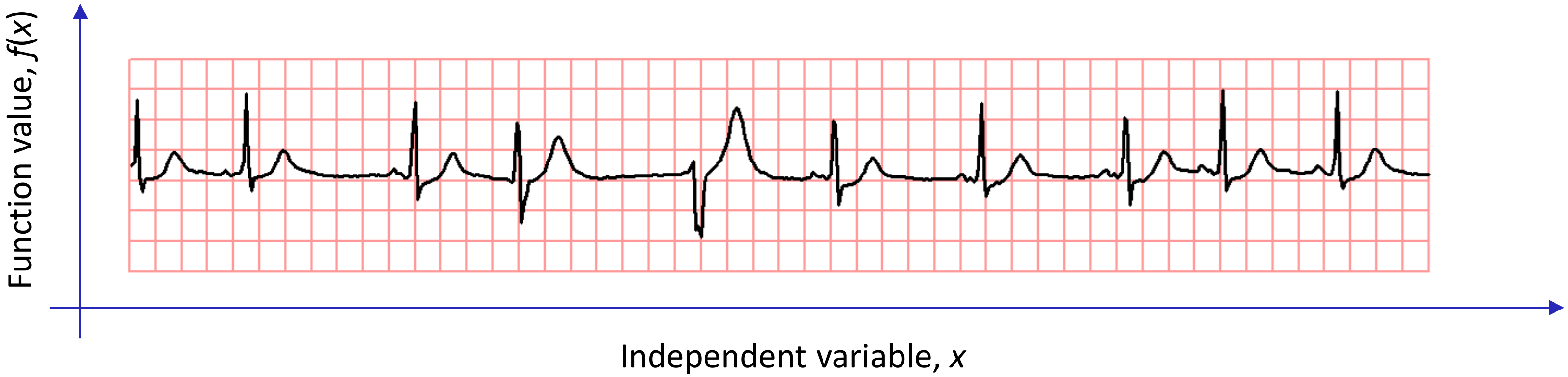
Signals…



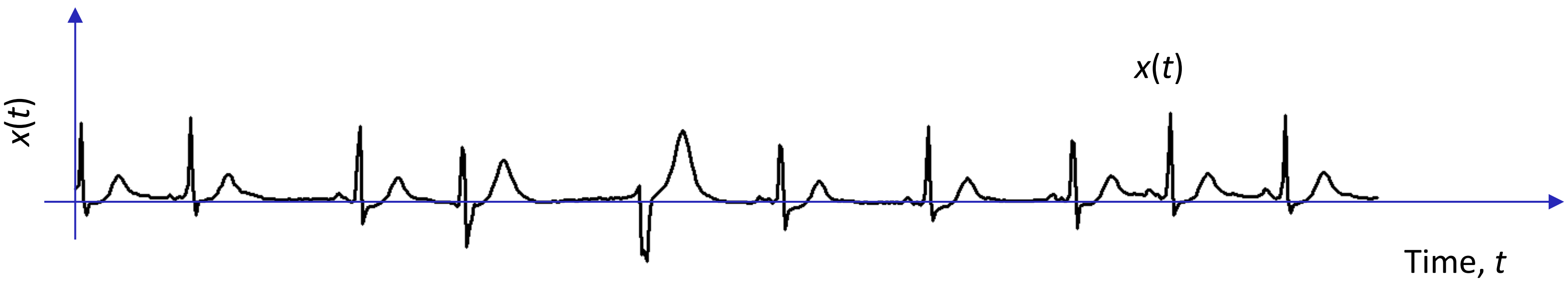
194 THz Frequency – 1.55 μm wavelength



Mathematical representation of a signal



- The “signal” is the value of a mathematical function of a certain independent variable
- Here (ECG), the independent variable is *time*
- We will reserve the letter *f* for the *frequency*, and we will call the signal *x, y, z*



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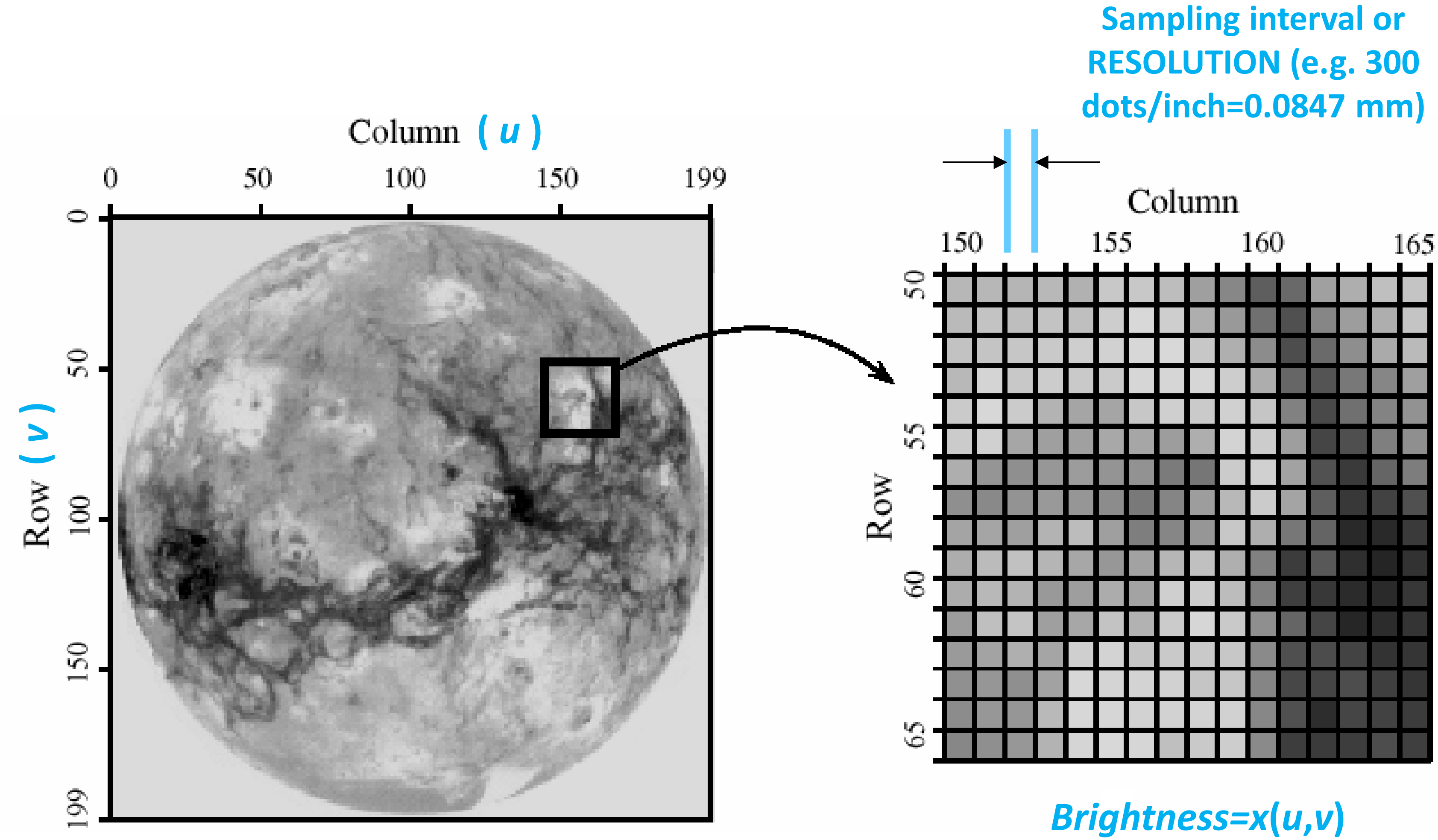
• Optical fiber is implemented through pulses of light that carry information.
1 = switch light on, 2 = switch light off. This is in fact called ON-OFF keying (keying means encoding).

How to convert physical signal to a digital form?

↳ Simple example: digitalization of a signal. We can identify a point in the image with 2 coordinates (u, v, that start in the upper left of the image).

In a ECG we can represent the heartbeat with a function of time, x(t). The value of the function is x(t), the name of the function. So that signal can be represented with a voltage collected on the sensor.

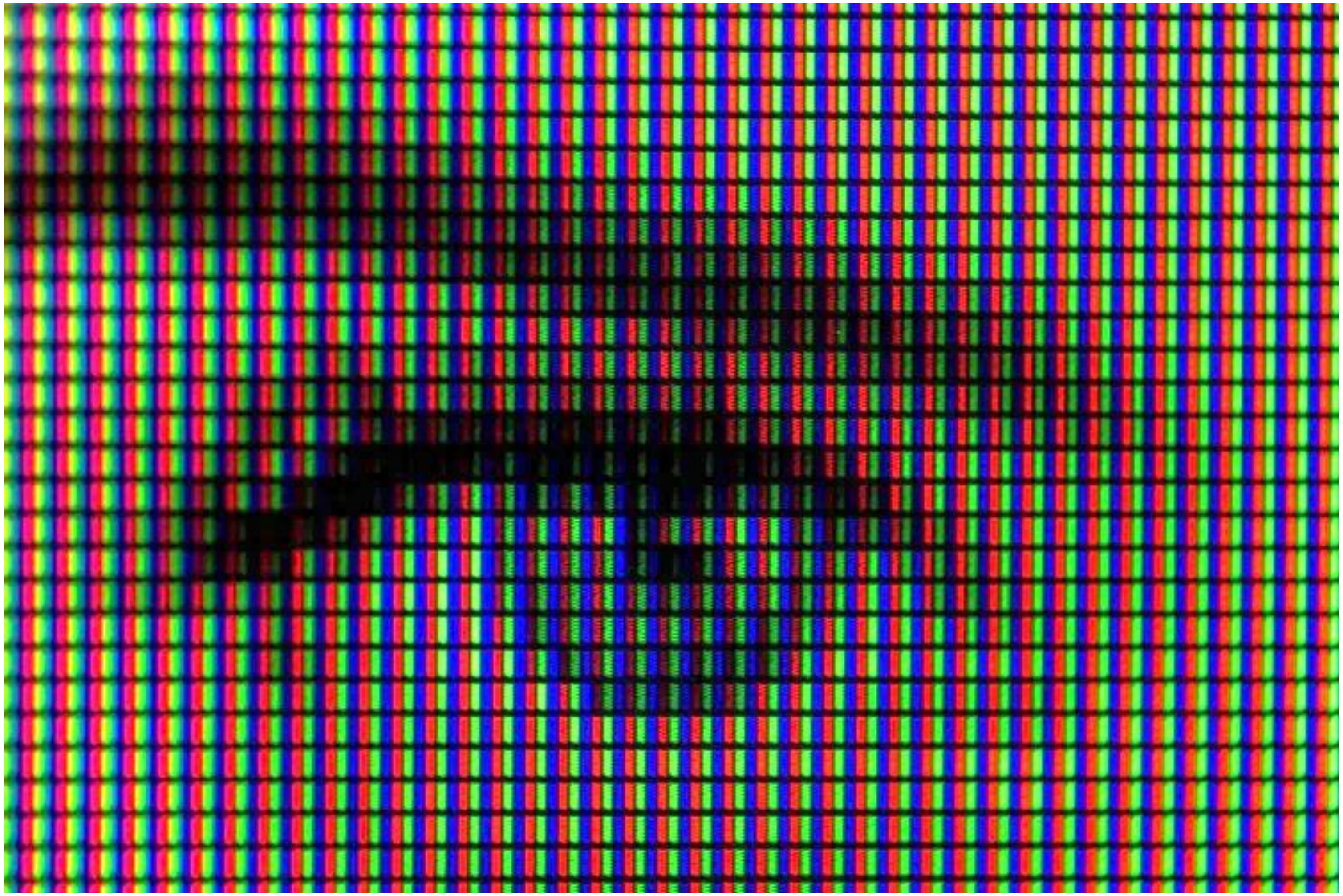
B/W Satellite Image for Weather Forecast



Creating PIXELS: «Sampling» the values of the image along the two u-v SPACE dimensions

Color Image…

… = 3 x monochromatic RGB images



In a picture, I have two coordinates, u and v and an infinite amount of points.
The value of the signal is the pic's brightness.

NOTE: theoretically in a continuous function (heartbeat) we have infinitely many points with infinitely many values. Same with the image. We would need infinite memory.

So we do SAMPLING: I can't store all values of the signal, so I SAMPLE the signal (select the value of specific points) and save it.

SAMPLING creates "PIXELS", so we set the resolution of the image = create a grid of pixels that will assume values. PIXEL = picture element. So now we have a finite number of values.

HD standard is 1920 x 1080 pixels. 4K is 3840×2160 (or UHD).

MP's the resolution with which you sample pictures.

What about coloured pictures? BW pics are a collection of values that represent brightness. Those values represent shades of grey.

- Colored images are a collection of 3 different images in 3 different sets of colours, red, green and blue levels. This is represented in phone specs as COLOR DEPTH.

B/W Original image: digital, but you cannot perceive it

Sampling: (too) Low resolution

Quantization: Different numbers of bits/sample

4 bits/pixel

2 bits/pixel

ANOTHER PROBLEM: We only store sampled values of a signal. But the sampled value is a real number, again, unrepresentable. What do we do? I have a range of values (0-1 for brightness for example) and we approximate the assumed value on finite possibilities. This is called QUANTIZATION. Not every possible value is allowed, but only approximations. To store those levels, we attach to each of it a digital code, could be the number of the level. Now we really have the digital image.

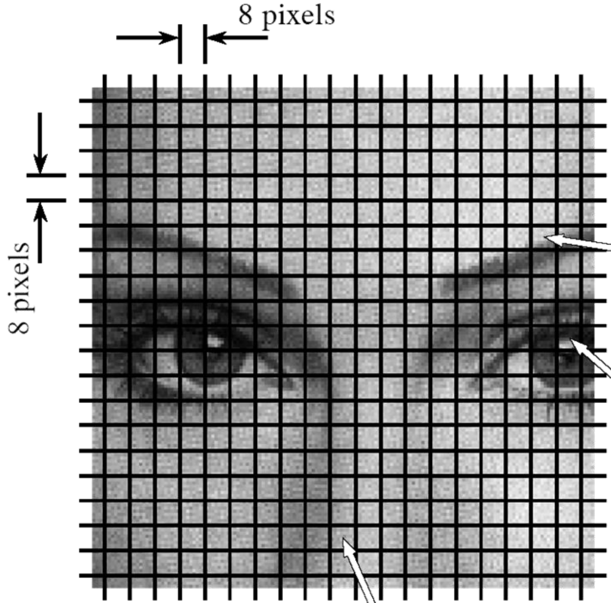
So the depth of the image is how many bits i attach to each pixel of the image.

Usually we have 256 levels associated to each sampled value, so $3 \cdot 8 = 24$ bits for RGB.

Even though nowadays we have HDR, which means 10 bits/color/pixel, so 30 bits in total.

Price to pay: the size will be bigger.

Image segments (JPEG)

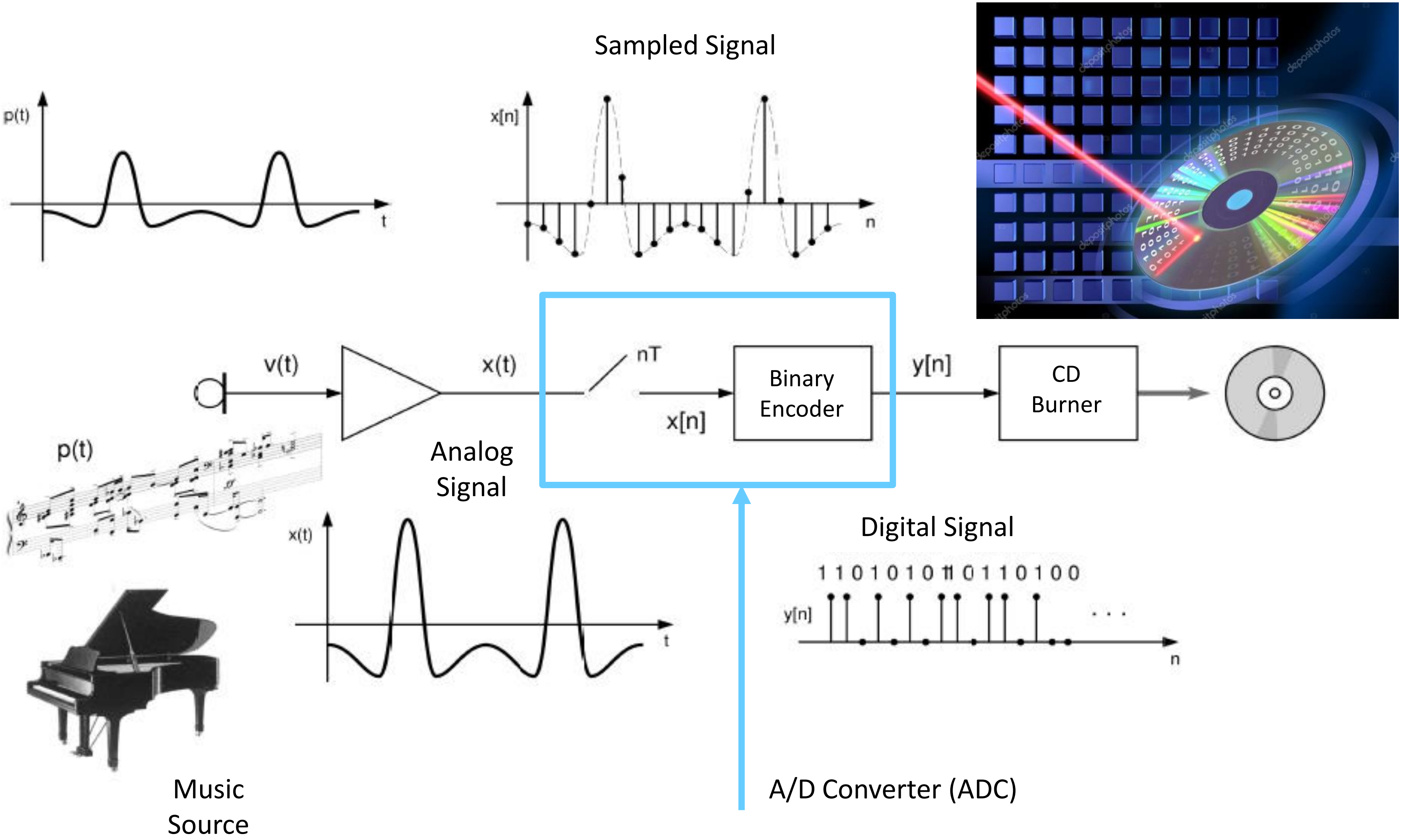


231	224	224	217	217	203	189	196
210	217	203	189	203	224	217	224
196	217	210	224	203	203	196	189
210	203	196	203	182	203	182	189
203	224	203	217	196	175	154	140
182	189	168	161	154	126	119	112
175	154	126	105	140	105	119	84
154	98	105	98	105	63	112	84

42	28	35	28	42	49	35	42
49	49	35	28	35	35	35	42
42	21	21	28	42	35	42	28
21	35	35	42	42	28	28	14
56	70	77	84	91	28	28	21
70	126	133	147	161	91	35	14
126	203	189	182	175	175	35	21
49	189	245	210	182	84	21	35

154	154	175	182	189	168	217	175
154	147	168	154	168	168	196	175
175	154	203	175	189	182	196	182
175	168	168	168	140	175	168	203
133	168	154	196	175	189	203	154
168	161	161	168	154	154	189	189
147	161	175	182	189	175	217	175
175	175	203	175	189	175	175	182

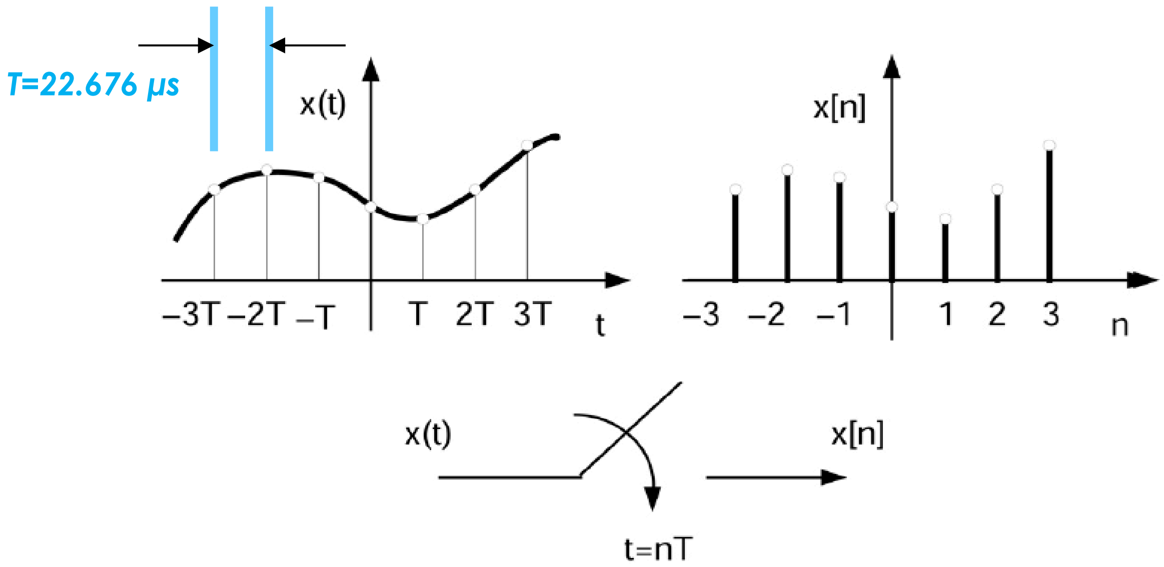
Digital Music Recording



NOTE: This is true for music recording too. 1: SAMPLE the time signal (signal is variation of acoustic pressure), the mic turns into voltage, it is amplified and sampled. We change (on the switch) the signal into a sampled signal. Then we quantize the signal and represent it with bits. We will have bits per sample.
With the specific example of CD recording we have 44100 samples per second (sampling rate) (44.1 KHz), agreed by Philips and Sony. Since we want to avoid misrepresentations we use 16 bits/second. The ear is more sensitive than eyes.

So we have, for each second, $44100 \times 16 = 705.600$ bits/s, 705 Kbits/s. Since we are recording a stereo sound, we have twice as much. $R_b \simeq 1.4$ Mbit/s.
So for every second we need to store 176 KBytes/s. This is the .wav standard.
We can also compress it if it is too much. .wav is the raw format.

Sampling an analog signal



Sampling an analog signal $x(t)$ means collecting one after the other the sequence $x[n]$ of values of that signal at the time instants nT, i.e, taken at the rate $f_c=1/T$ that is called the *sampling frequency* (Hz or samples/s)

Audio recording: f_c=44.1 kHz, T=22.676 μs

- f_c = SAMPLING RATE, n. of samples per second. T = time resolution.

CONCLUSION: why video traffic is saturating the internet. High quality music requires 1.4 Mbits/s to be transferred. (Digital voice is 13 Kbits/s).

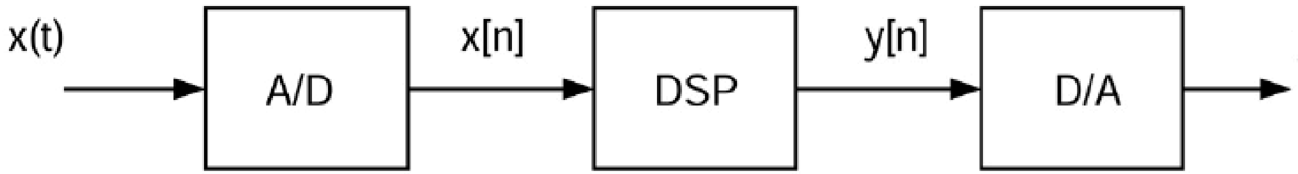
What about HDTV? It's 25 frames (images) per second, being 1920×1080 with 3·8 bits/pixel
→ pixels/image.

Total bitrate of HDTV: $25 \cdot 1920 \cdot 1080 \cdot 3 \cdot 8$ bits/s = 1.244 Gbits/s. Raw format video.

We need compression! MP4 does that. There might be ORDERS of MAGNITUDE of difference between digitalization of signals. With video compression we go down to 20 Mbits/s.

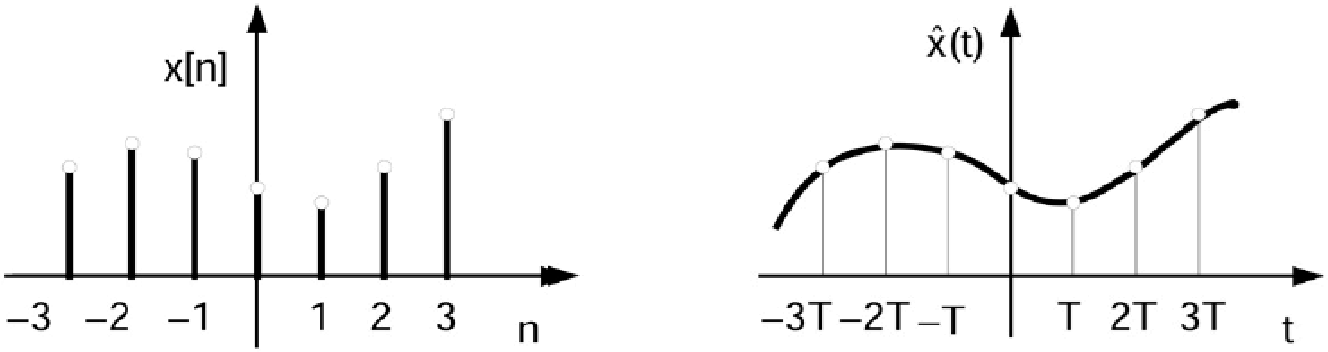
ADC & DAC

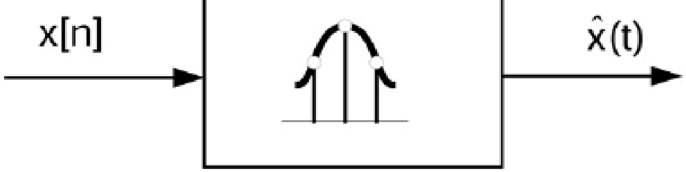
Signal processing is performed on the digital values extracted from the signal itself after A / D conversion, and results in the execution of an appropriate program by a digital processor. This structure is extremely flexible, in the sense that different processing functions can be realized simply by changing the processing program (software) without having to modify the physical structure (hardware) of the system.



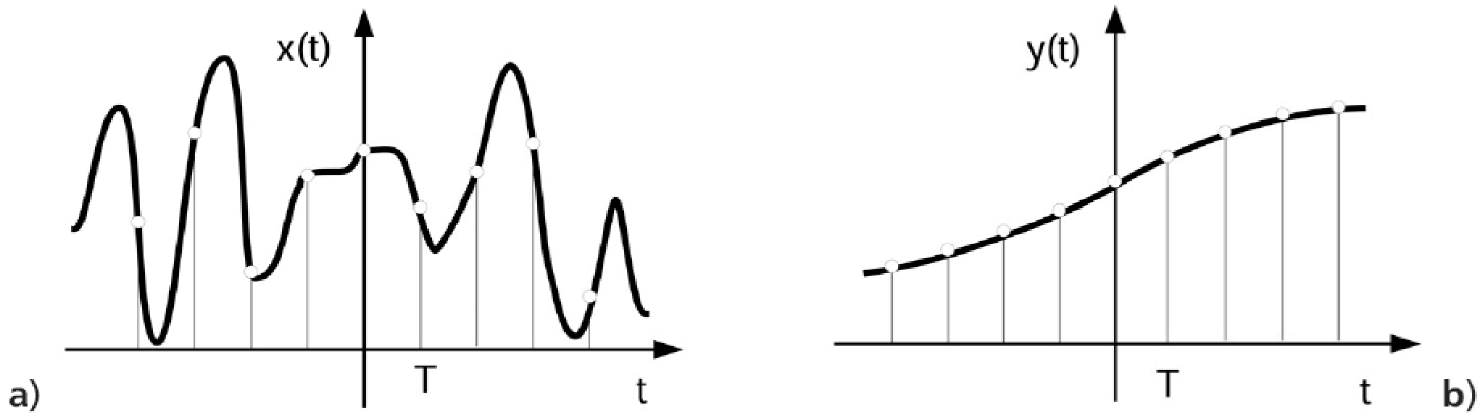
Sampling (ADC) & Interpolation (DAC)

(Time continuous) interpolation of a time-discrete signal (sequence)





«Fast» and «Slow» Signals



The sampling period is adequate for y(t), but it is clearly too large for x(t). In this sense, the sampling frequency must be commensurate with the "speed" (i.e., with the rate of change) of the signal – how can we quantify this feature?

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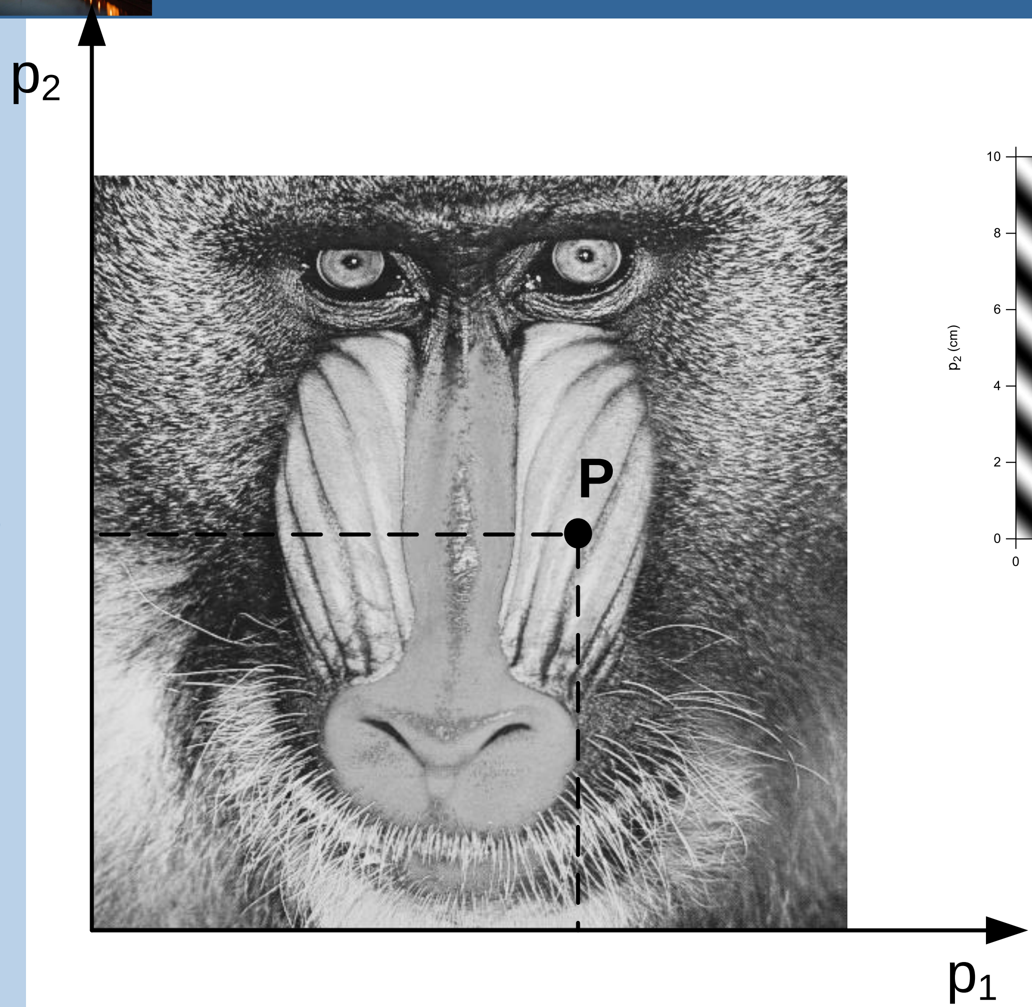
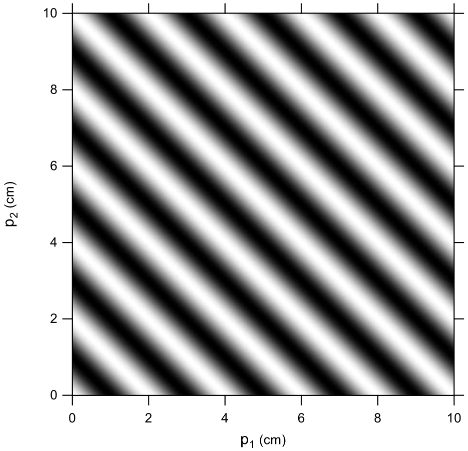
NOTE: there must be a criterion for sampling. "Fast" signals need more frequent samples because they change very quickly. The time resolution might fail to capture the signal characteristics. The same goes for pictures. Fast changing images need careful sampling.

«Fast» and «Slow» Signals





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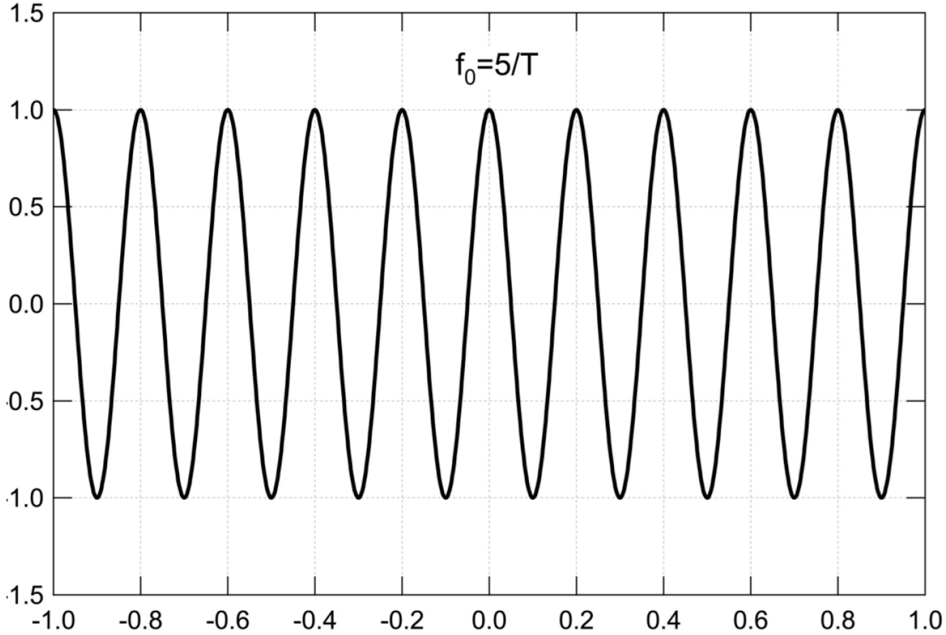
ONE (Simple) Sinusoidal Signal

$$x(t) = \cos\left(2\pi t \cdot \frac{5}{T}\right)$$



Repeats itselfs periodically

PERIODIC signal: the repetition period is

$T_0 = 5/T$

The oscillation frequency f_0 is the inverse of the period:

$f_0 = 1/T_0$

The amplitude A is the peak value (maximum value) of the signal, in the example A=1

Whistle and flute are good approximation of sine wave

In general,

$$x(t) = A\cos(2\pi f_0 t + \vartheta)$$

ONE (Simple) Sinusoidal Signal

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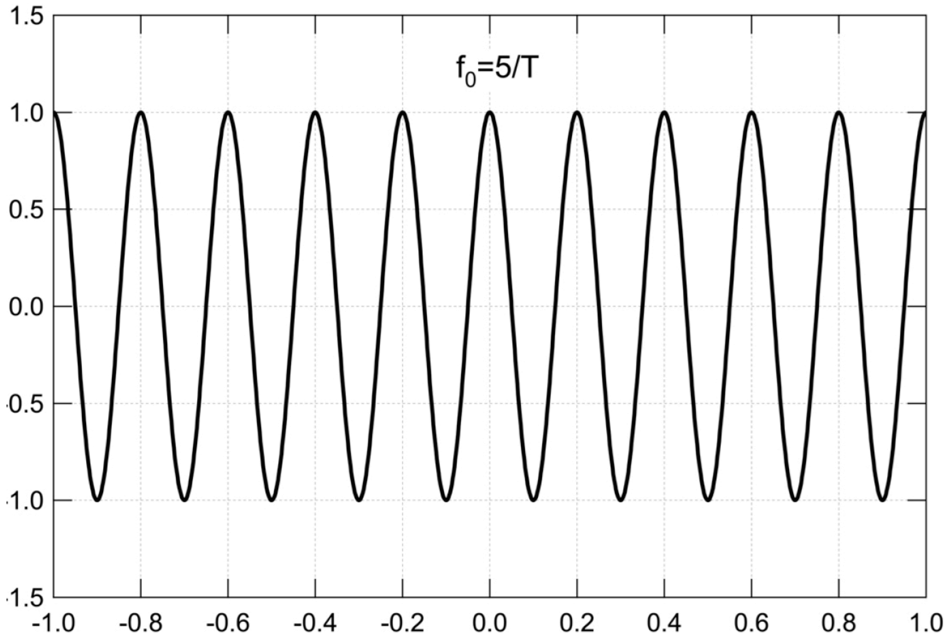
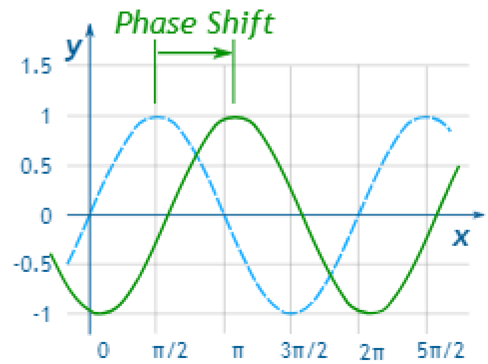


The phase-shift q indicates the initial point or initial value of the waveform at t=0:

$$x(0) = A\cos(\vartheta)$$



In general,

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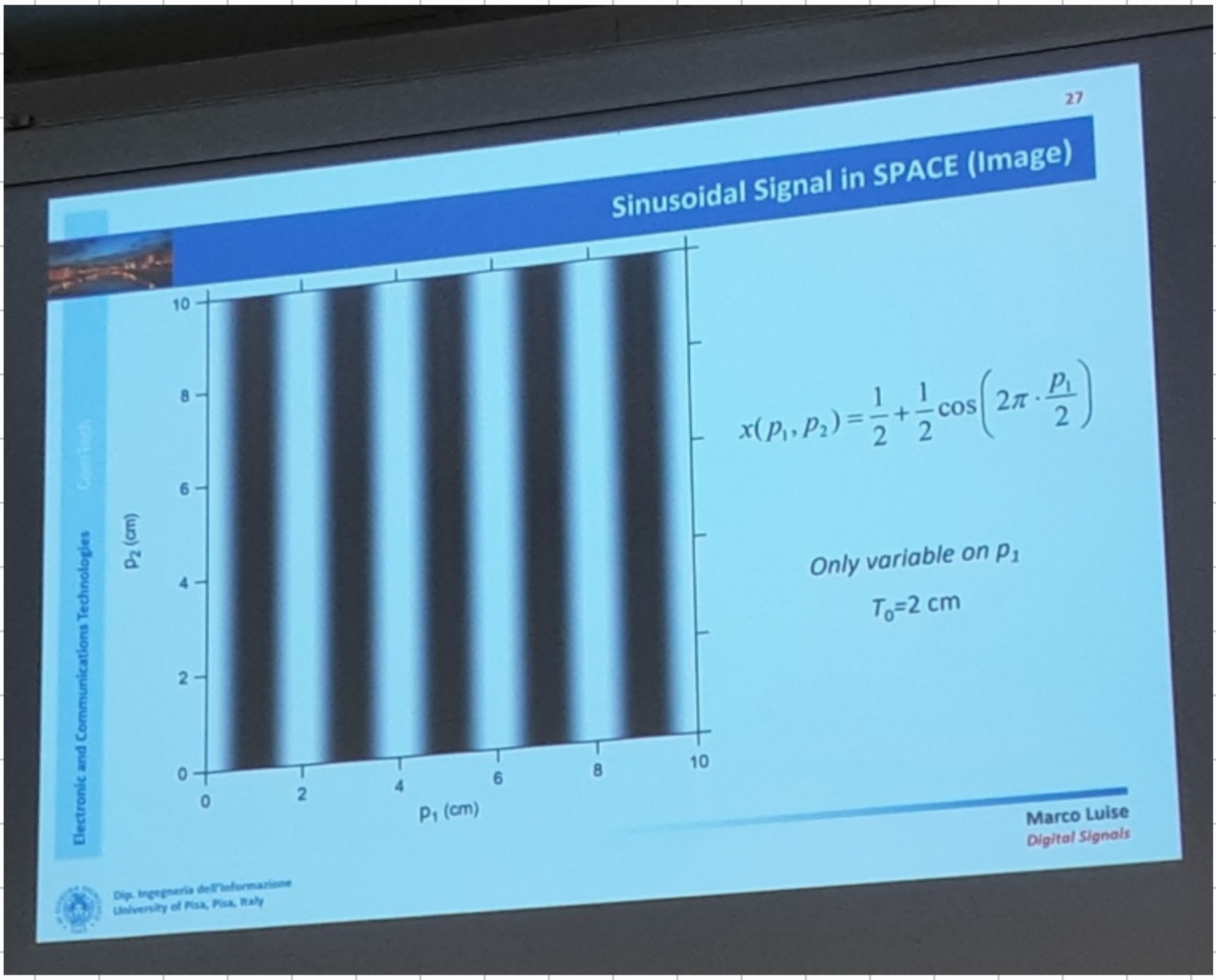
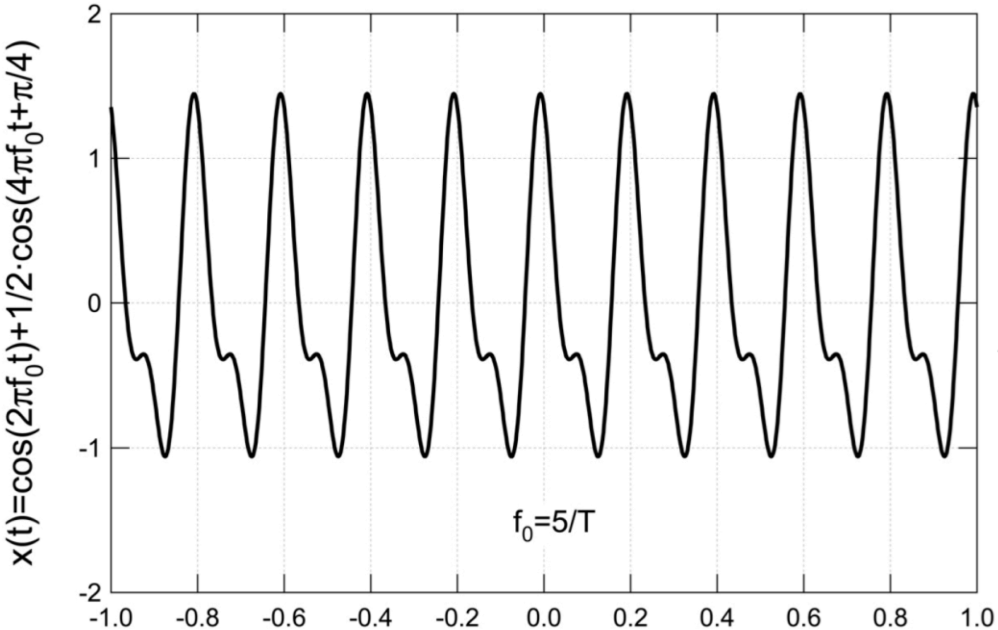


Signals from now on will be time functions.

TWO (Simple) Sinusoidal Signals

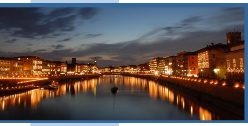
$$x(t) = \cos(2\pi 5t / T) + 0.5\cos(2\pi 10t / T + \pi / 4)$$



local maxima and minima emerge.

$$x(t) = A_1\cos(2\pi f_0 t) + A_2\cos(2\pi 2 f_0 t + \pi / 4)$$

Fourier analysis: you can deconstruct every periodic signal into a sum of infinite simpler signals whose frequency is a multiple of the original frequency.



VERY MANY (Simple) Sinusoidal Signals…

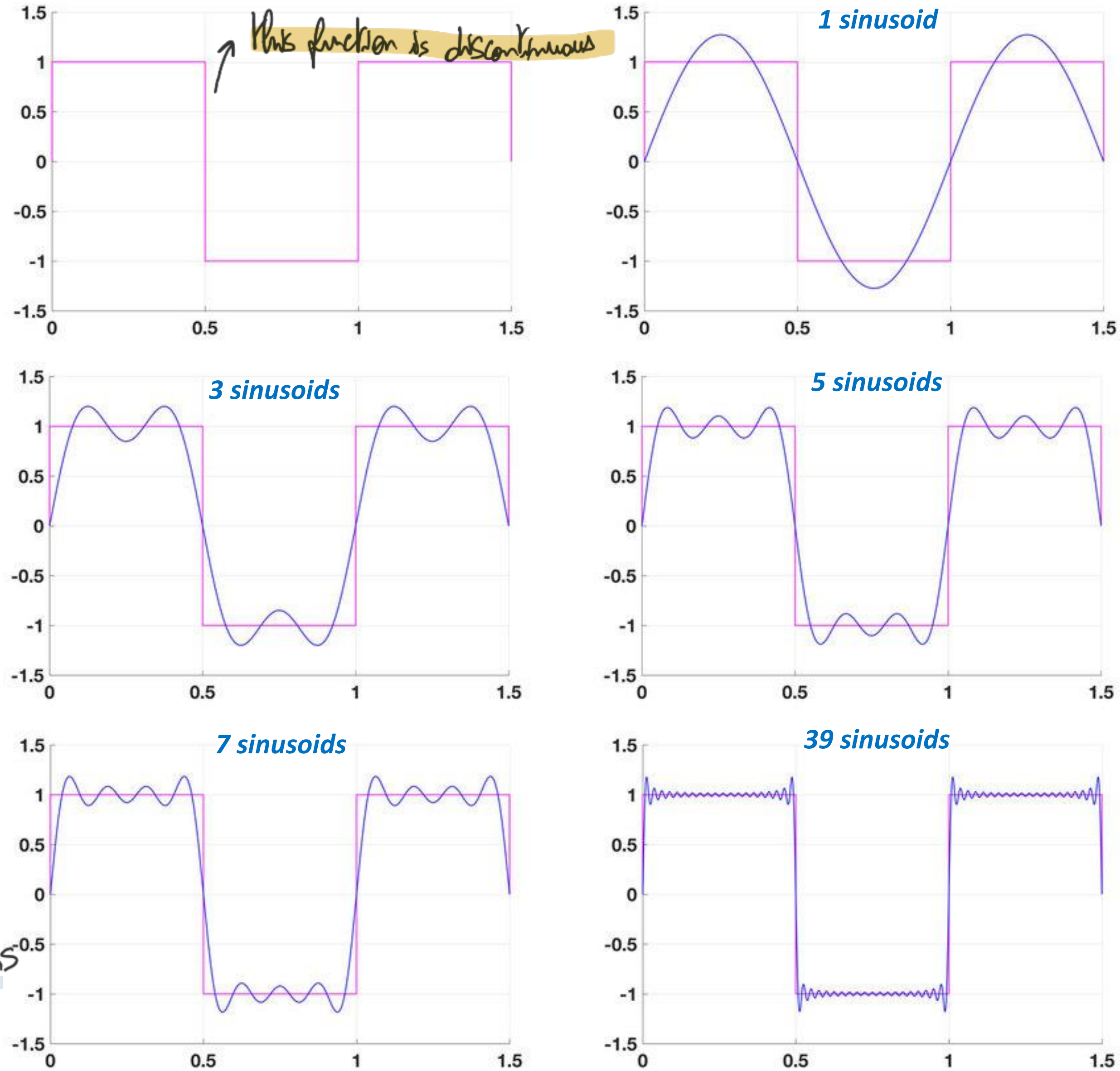
in reality we have fall times and rise times.

… can make a complicated (periodic) signal: the square wave – or: Fourier analysis for dummies

for a clock: $f_0 = 3\,GHz = 3 \cdot 10^9\,Hz$

$T = 0.33 \cdot 10^{-9} = 0.33\,ns$

This function is discontinuous



General Fourier Series

→ In general, they are the harmonics.

Second harmonic = component whose frequency is double the frequency of the original signal ★.

$$x(t) = A_1 \cos(2\pi f_0 t + \theta_1) + A_2 \cos(2\pi 2 f_0 t + \theta_2) + A_3 \cos(2\pi 3 f_0 t + \theta_3) + \ldots + A_k \cos(2\pi k f_0 t + \theta_k) + \ldots$$

→ Synthesised new sounds never heard before. Many analog oscillators generated signals with certain frequencies amplitude and phases.

SYNTHESIS equation

↳ Any signal can be reconstructed ★1

We now do that with digital signal processing.

★ Also called the repetition frequency. (Frequenza fondamentale)

★1 by adding the right harmonics together.

Before doing synthesis, what are the right A_K, θ_K etc.?

General Fourier Series

$$x(t) = A_1 \cos(2\pi f_0 t + \theta_1) + A_2 \cos(2\pi 2 f_0 t + \theta_2) + A_3 \cos(2\pi 3 f_0 t + \theta_3) + \ldots + A_k \cos(2\pi k f_0 t + \theta_k) + \ldots$$

SYNTHESIS equation

General Fourier Series

FAST FOURIER TRANSFORM

$$A_k e^{j\vartheta_k} = \frac{1}{T_0}\int_0^{T_0} x(t)[\cos(2\pi k f_0 t) - j\sin(2\pi k f_0 t)]dt$$

Consider a whole repetition of a signal and find a complex number whose amplitude is the harmonic amplitude and phase is the harmonic phase.

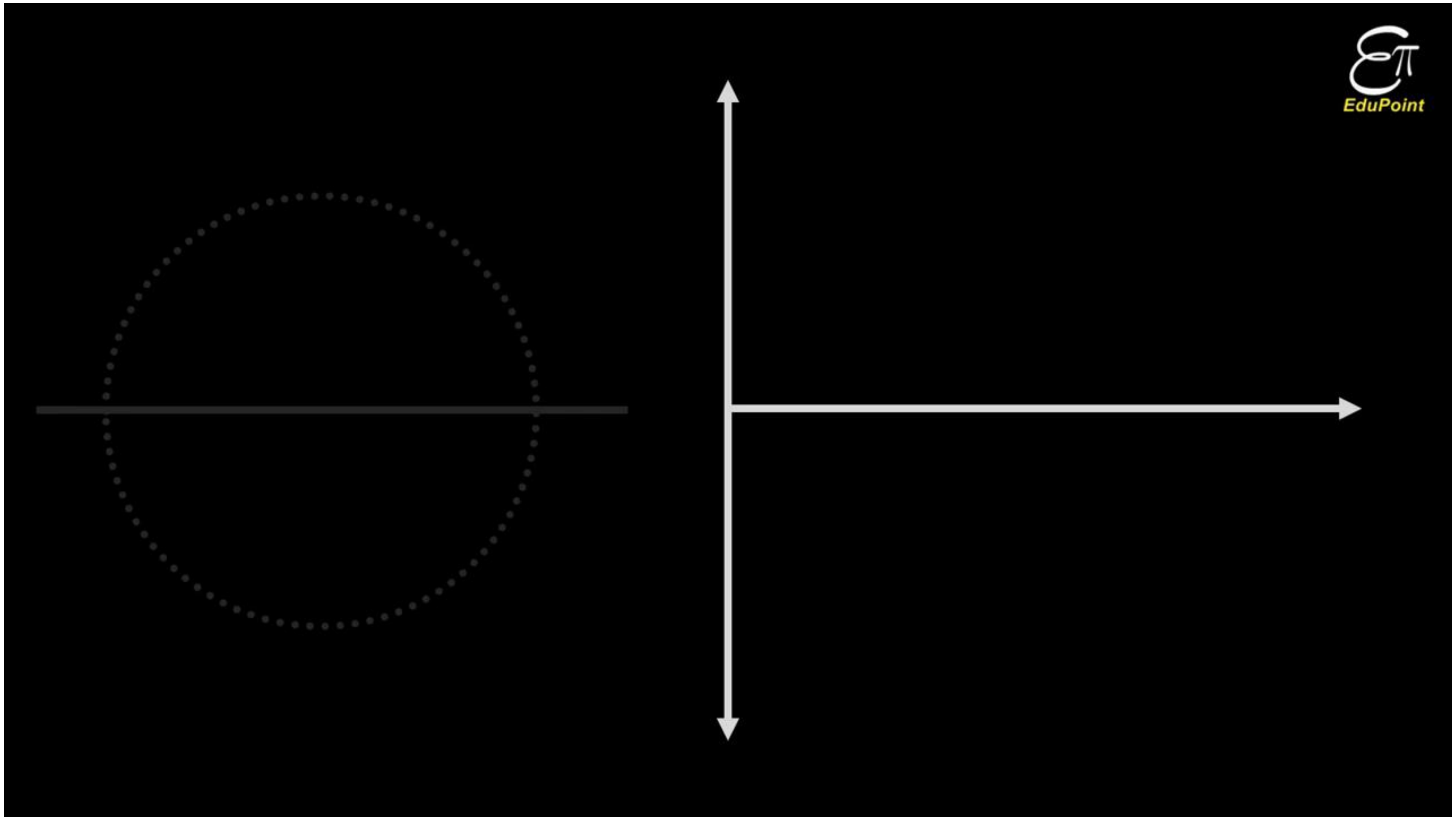
ANALYSIS equation

Spectrum analyzer. We analyze the sound and get amplitude and phases so we can resynthesize the signal.

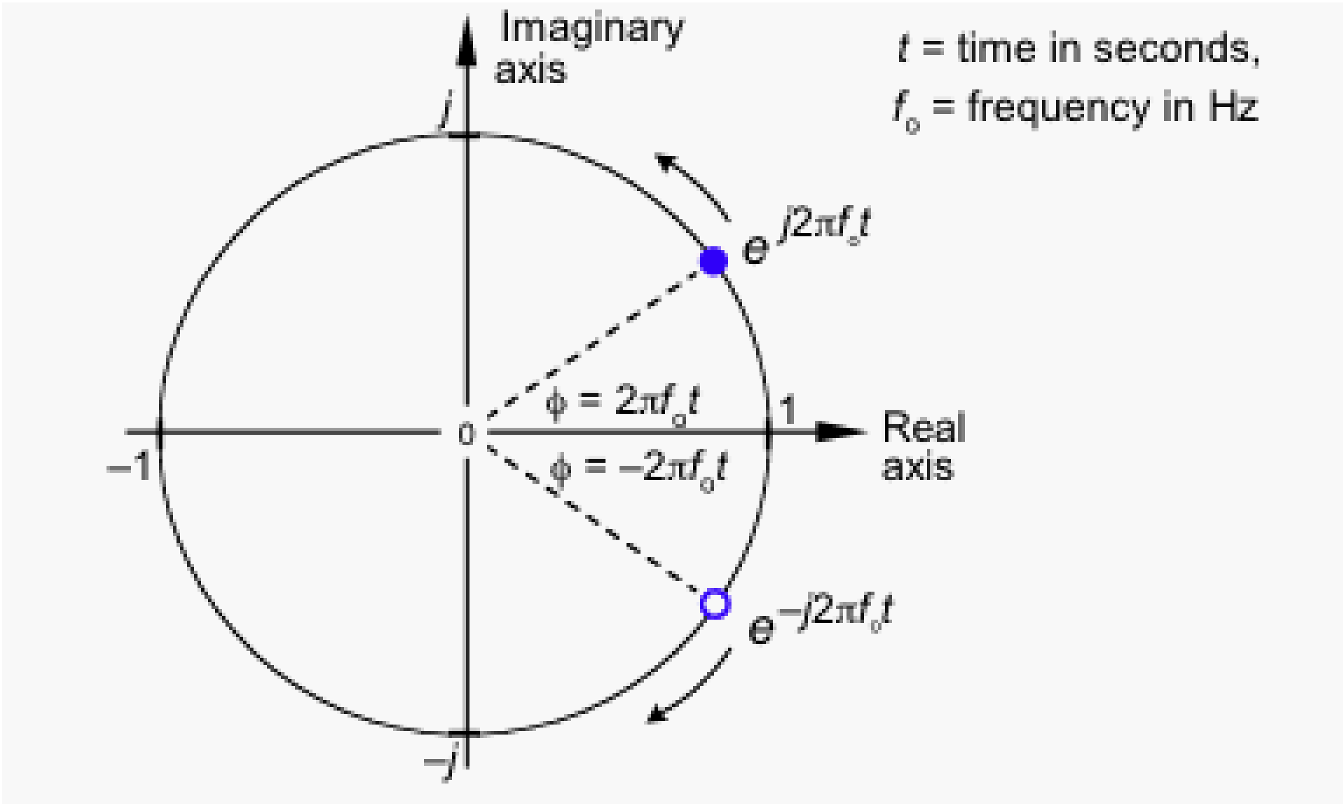
ANALYSIS: I have my signal. Let's see how it is done and its components.

SYNTHESIZE: recreate the signal.

Positive and Negative (?) Frequencies 1/2



Positive and Negative (?) Frequencies 2/2



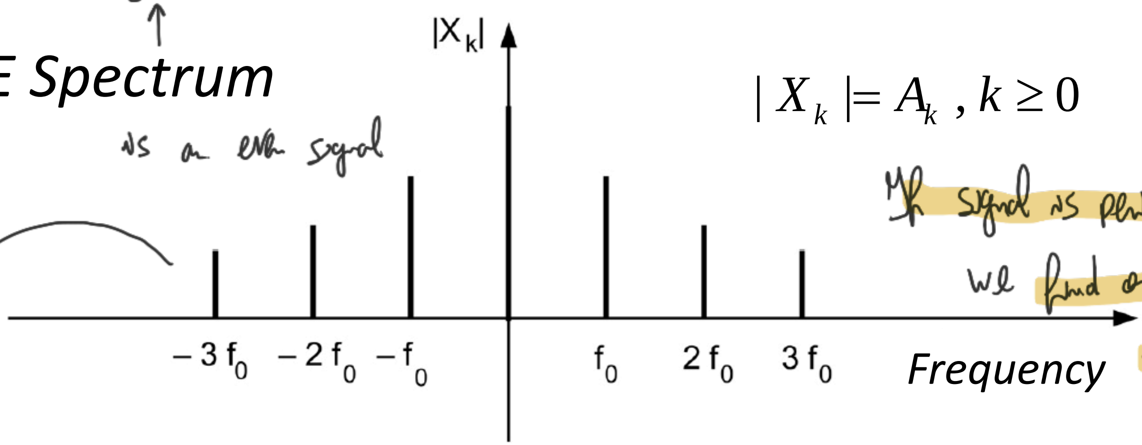
The frequency spectrum: frequency contents of a signal

Range

AMPLITUDE Spectrum

is an even signal

negative frequencies come from the use of complex numbers

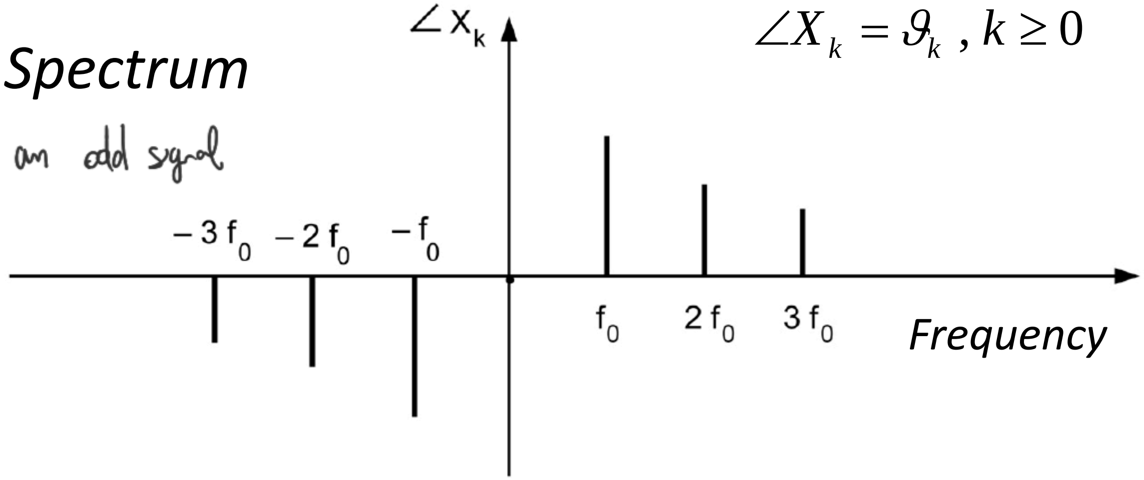


$$|X_k| = A_k \ , k \geq 0$$

If signal is periodic, the only values we find on the spectrum are multiples of the repetition frequency

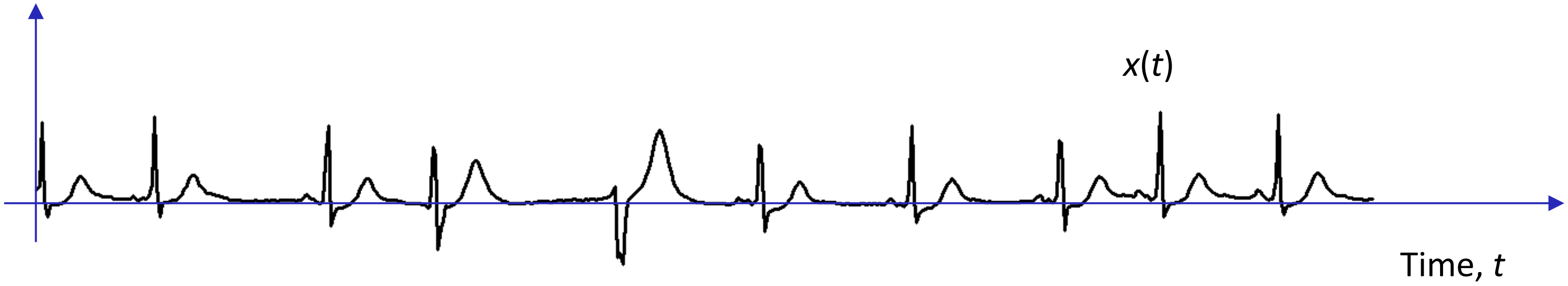
PHASE Spectrum

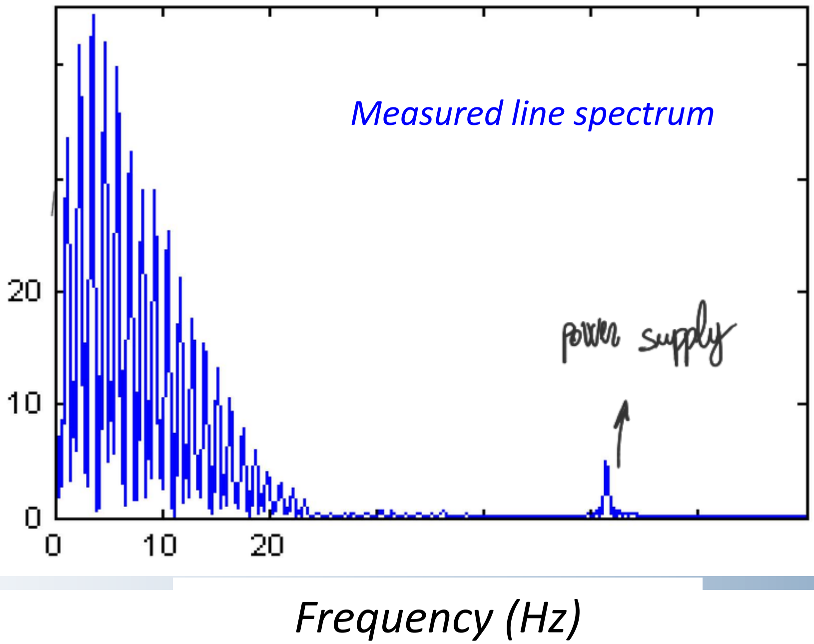
is an odd signal



$$\angle X_k = \vartheta_k \ , k \geq 0$$

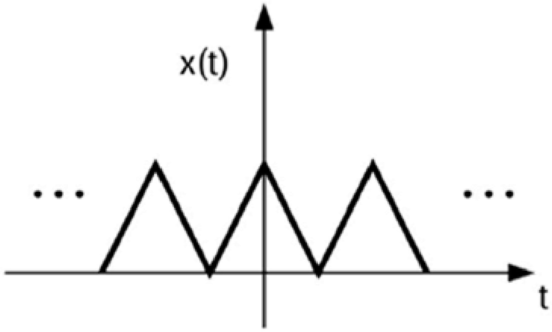
To sum-up: PERIODIC SIGNAL



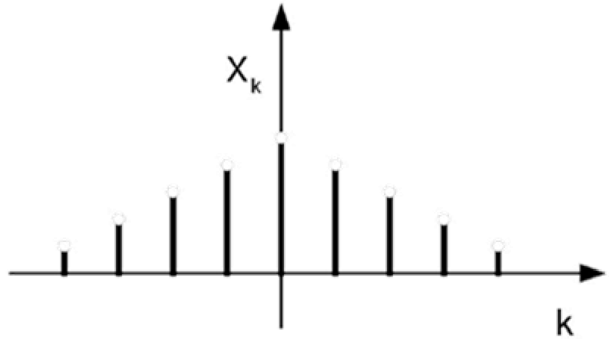


Signals and Spectra

TIME | **FREQUENCY**

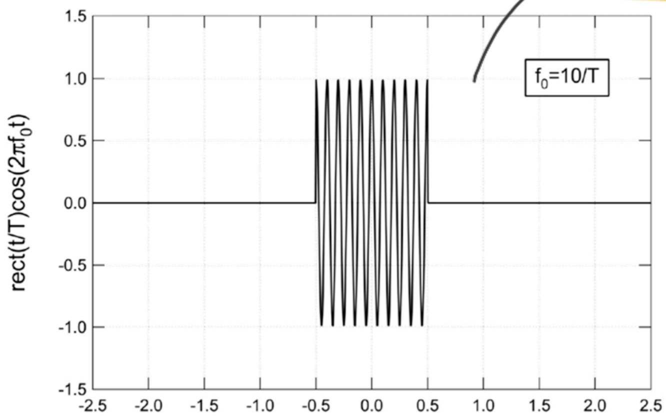


Periodic Signal



Line Spectrum

Pulse with certain oscillations between the pulse.



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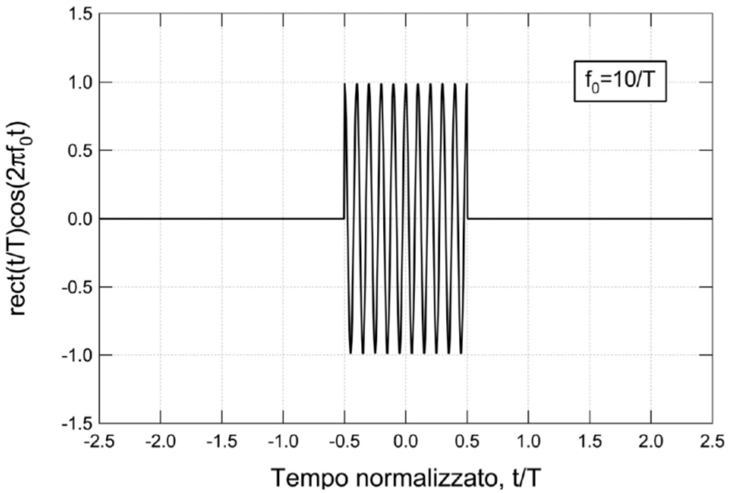
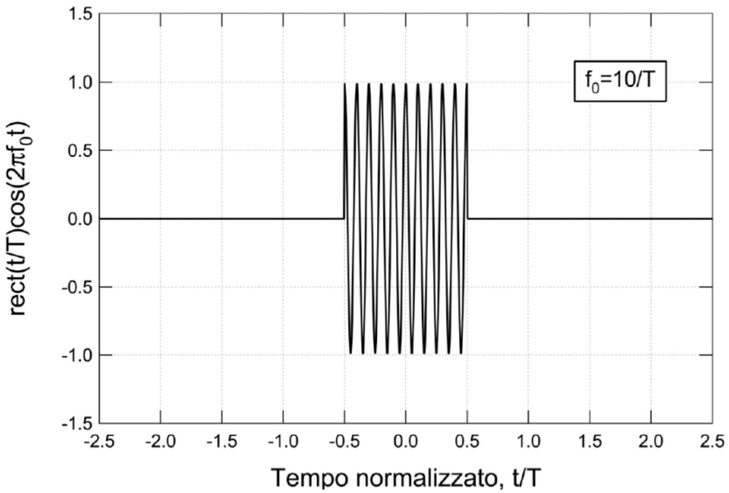
Aperiodic Signal or Non periodic Signal

Time and Frequency

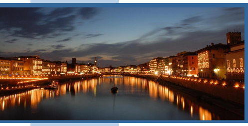


Time and Frequency

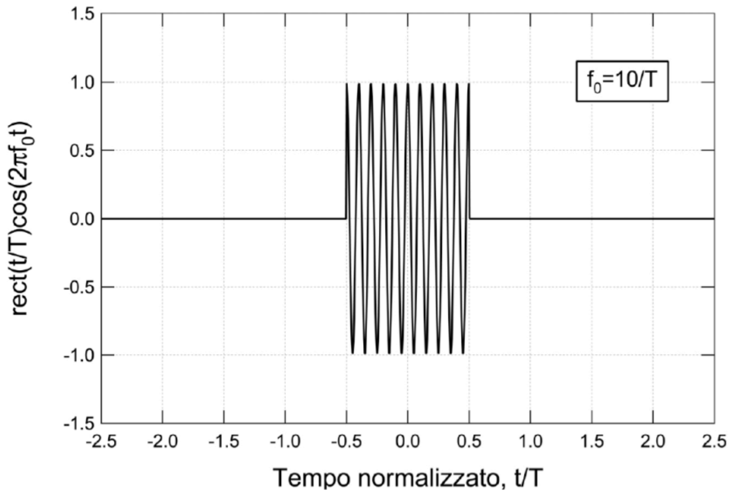


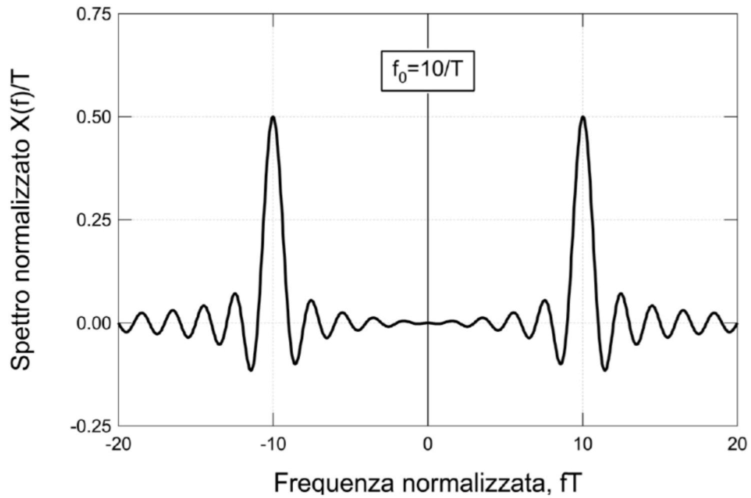
$$X(f)=\int_{-\infty}^{+\infty} x(t)\,e^{-j2\pi ft}dt$$

ANALISYS *Equation*



Time and Frequency





With an aperiodic signal we have a continuous spectrum not discrete (with finite frequencies). But the frequency of 10/T which takes control during the pulse has a strong influence on the spectrum.

$$X(f) = \int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi f t}\, dt$$

ANALISYS *Equation*

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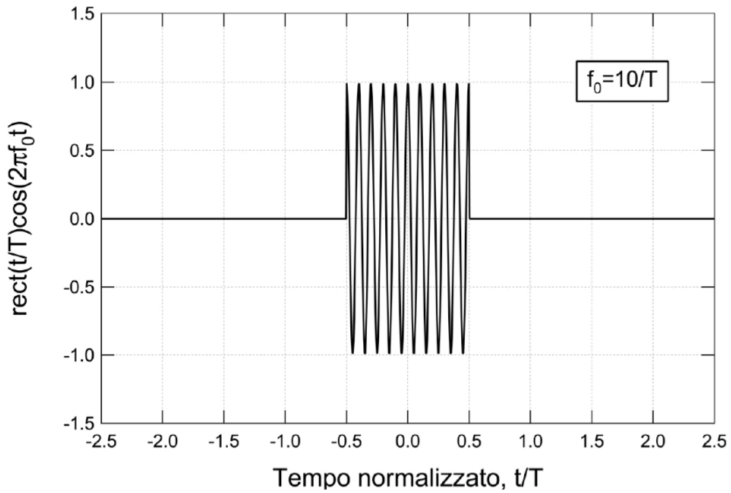
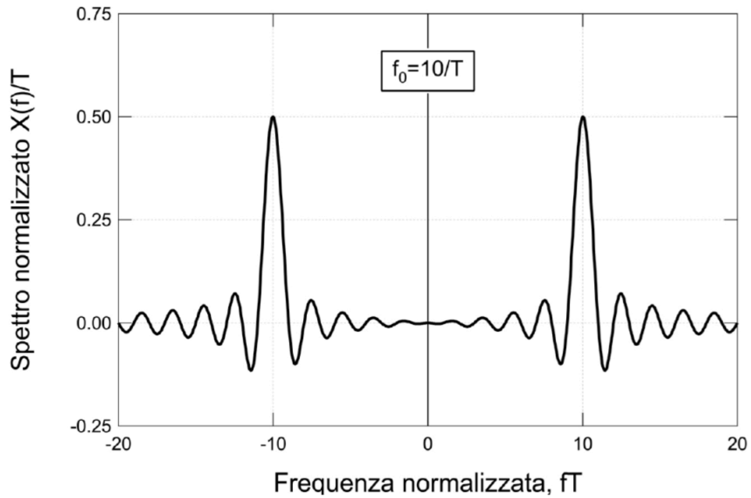
Time and Frequency





INVERSE FFT

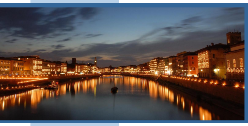
$$x(t)=\int_{-\infty}^{+\infty} X(f)\, e^{j2\pi ft} df$$

***SYNTHESIS** Equation*

$$X(f)=\int_{-\infty}^{+\infty} x(t)\, e^{-j2\pi ft} dt$$

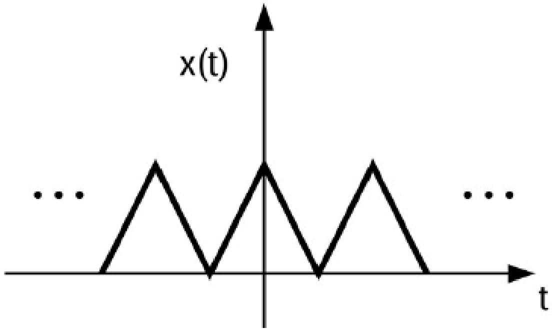
FFT

***ANALISYS** Equation*

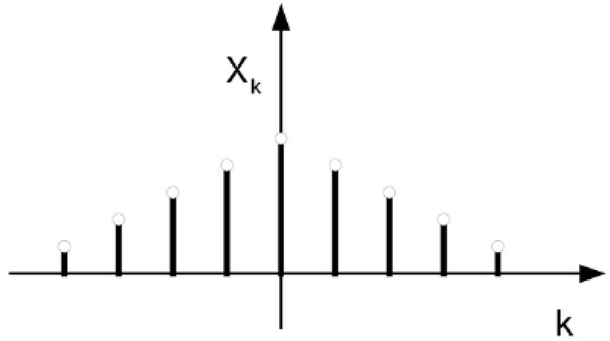


Different kind of Spectra

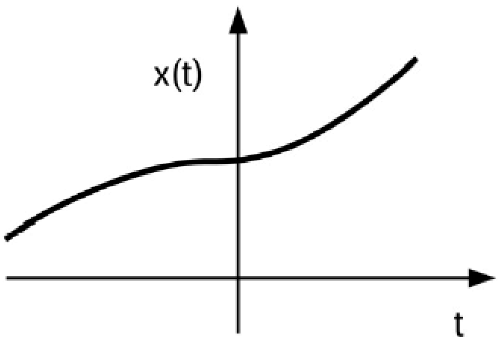
Time	**Frequency**



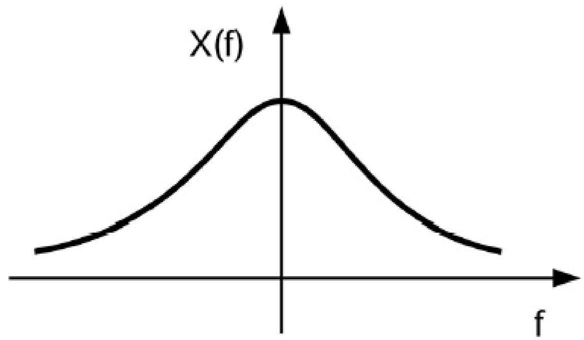
Periodic signal



Line (discrete) spectrum



Non-periodic signal



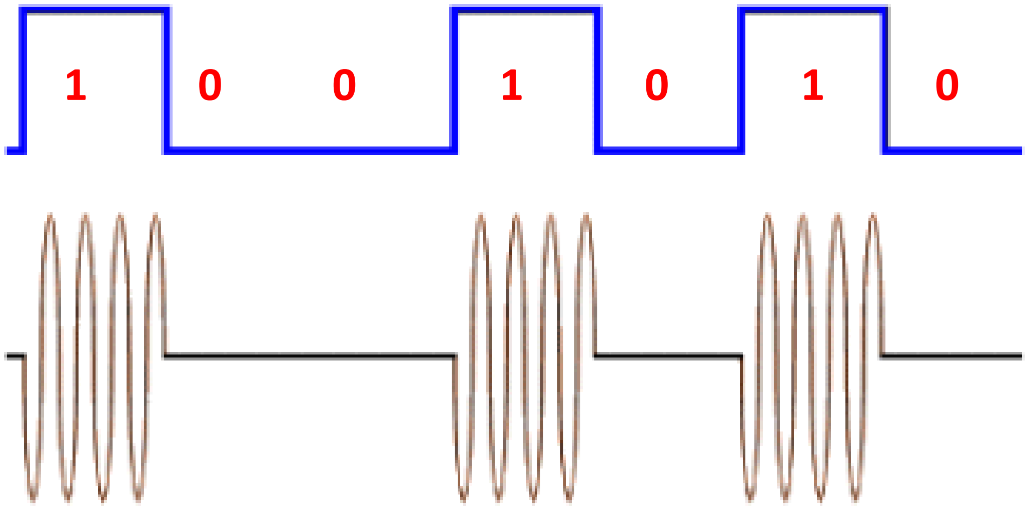
Continuous spectrum

Non.-periodic, RANDOM Signals

SEGNALE ALEATORIO

(LIVELLO BANDA BASE, FISICO)

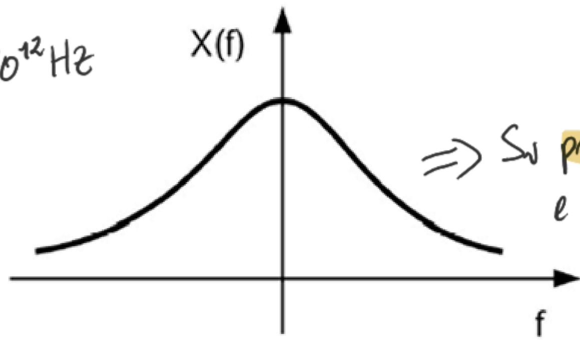
(SEGNALE MODULATO FISICO INTERNET)



Quando trasmetto un flusso di bit su linea fisica mando un treno di impulsi sinusoidali.

$194\,THz = 194 \cdot 10^{12}\,Hz$

SPETTRO DI POTENZA DEL RANDOM SIGNAL



⇒ Si prende lo spettro di questi segnali e si analizza.

Continuous POWER spectrum as well !

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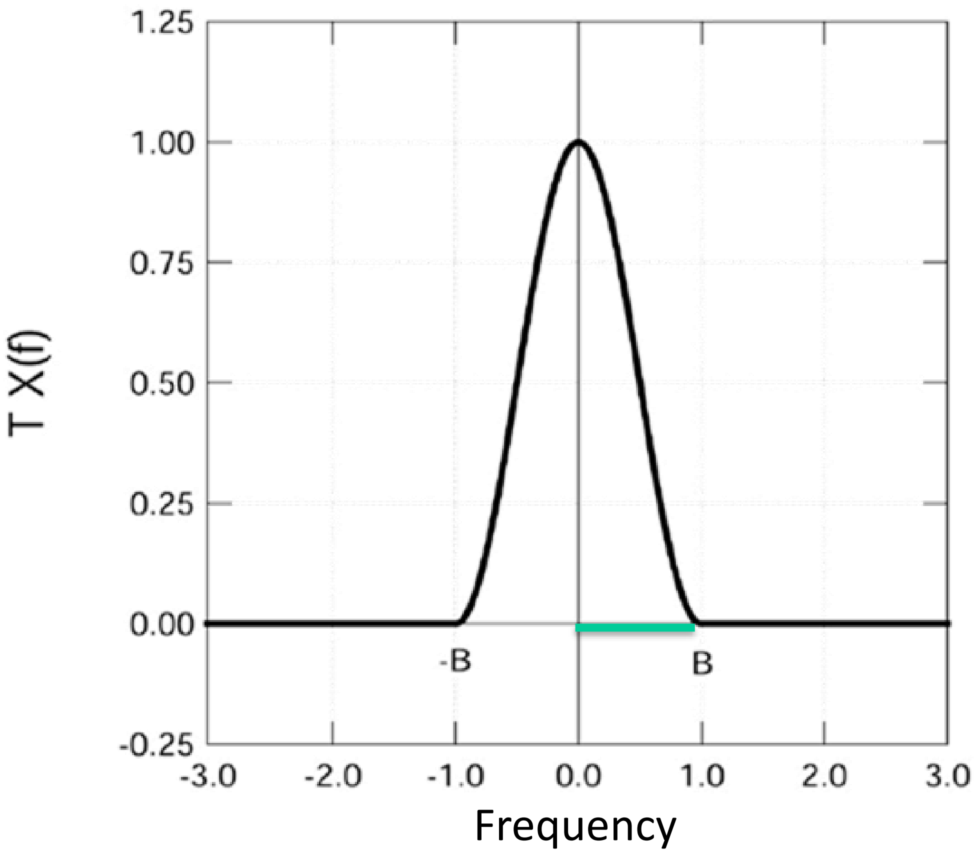
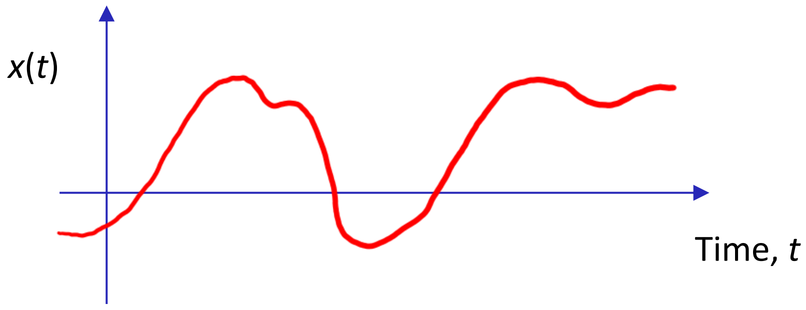
The BANDWIDTH of a signal

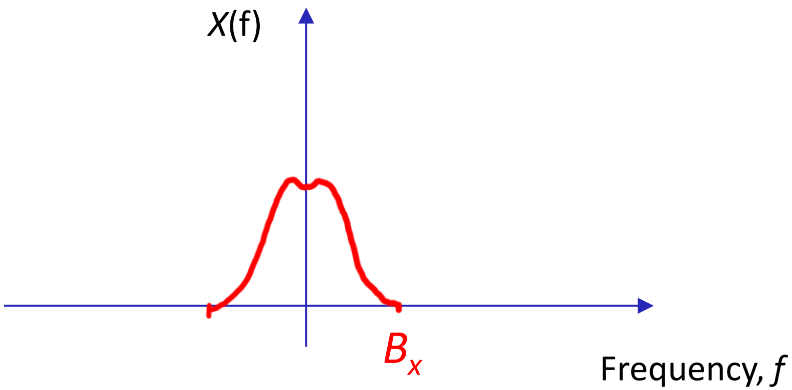
The bandwidth B is the width of the frequency interval (on positive frequencies) on which the signal spectrum is different from 0



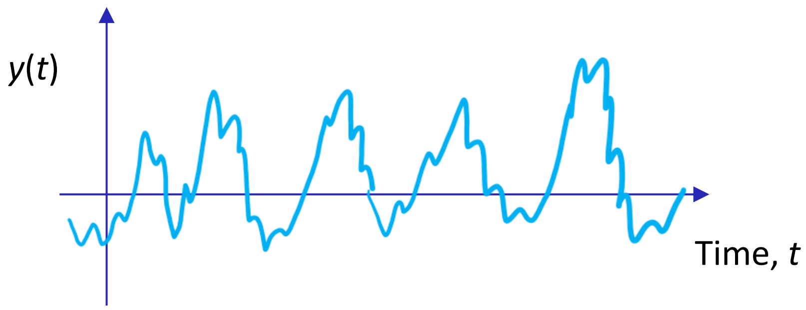
Wideband and Narrowband

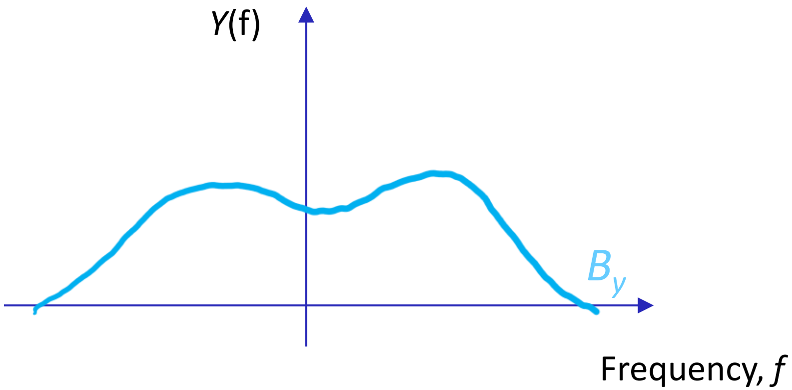
"Slow", narrowband signal and its spectrum





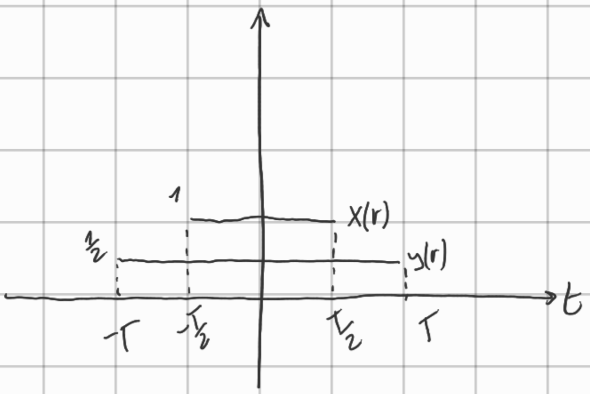
"Fast", wideband signal and its spectrum

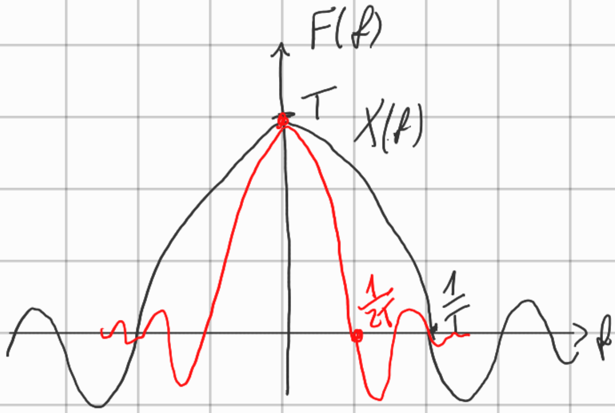




$B_x < B_y$

Immaginiamo di avere un segnale che è un singolo impulso del clock e un altro di metà ampiezza e durata doppia.





Picco spettro = sua area. Passa a 0 all'inverso della durata dell'impulso e ai suoi multipli.

$X(f) \simeq \frac{\sin f}{f}$

Le componenti più grosse sono tra 0 e $\frac{1}{T}$, quello è il range di frequenze più importante ed è la banda del segnale.

Misurare lo spettro ci serve a capire la banda B del segnale. $B_x = \frac{1}{T}$.

y(t) è più lento. La forma dello spettro è la stessa. Lo spettro è più schiacciato.

Il punto in cui va a 0 non è $\frac{1}{T}$ ma $\frac{1}{2T}$.

Banda di $y = \frac{1}{2T}$, cioè $B_y = \frac{B_x}{2}$.

Perché B_y è la metà di B_x? Perché il segnale di y è il doppio più lento del segnale di x.

Se un segnale è il doppio più lento di un altro, allora anche le sue sinusoidi componenti lo sono.

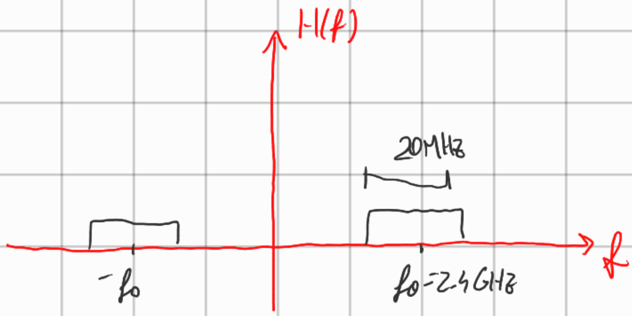
Alta banda = ho bisogno di sinusoidi ad alta frequenza e quindi avrò segnali veloci.

Ciò vale anche per le immagini che posso analizzare con Fourier.

NOTA: Tutto ciò che ha uno spettro che è compreso nei 20KHz è udibile.

I cellulari hanno uno spettro non centrato allo zero ma intorno a una frequenza portante.

Wifi lavora su frequenza portante di 2.4 GHz con banda di 20 MHz.



C'é un dominio diverso da quello del tempo che é quello delle frequenze.

Direct FFT per andare nel dominio della frequenza, Inverse FFT per tornare nel dominio del tempo.

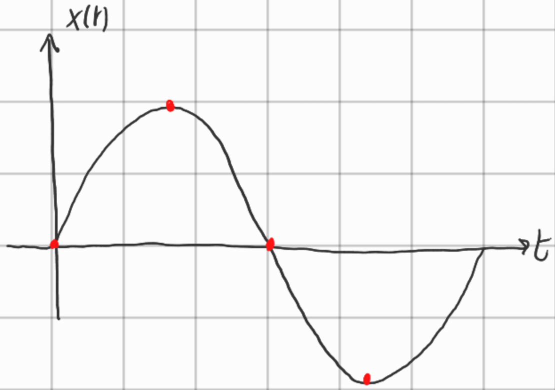
Sono peró lo stesso algoritmo a meno del segno.

"What happens when I do signal sampling?"

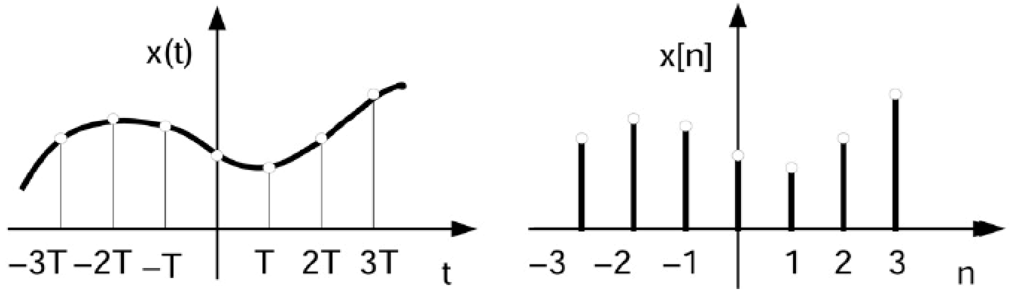
$x(t) = \sin(2\pi f_0 t)$

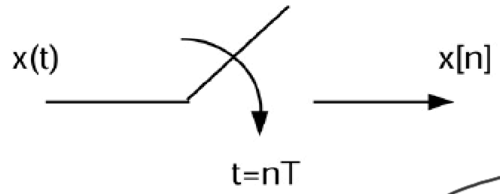
$T_0 = \frac{1}{f_0}$ Sampling a signal = create a specific collection of samples based on a sampling frequency f_c

Sampling period $T_c = \frac{1}{f_c}$. Let's say $f_0 = 1MHz$, $f_c = 4MHz$.



Sampling and Spectra





$\to X[n] = x(nT) = \sin(2\pi f_0 n T) =$

$= \sin\left(2\pi \cdot 1 \cdot n \cdot \frac{1}{4}\right) = \sin\left(\pi \cdot \frac{n}{2}\right)$

Sampling means doing this:

$$x[n] = x(nT)$$

↳ Sample number n is

Poisson's relation that gives the spectrum of the digital signal

What happens to my signal when I do my sampling?

scaling factor = sampling frequency

$$\overline{X}(f) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{T}\right)$$

→ When you do your sampling and start from a spectrum like the one below

GOOD NEWS: even with a sampled signal we will be able to analyze it with Fourier. You will have discrete some components

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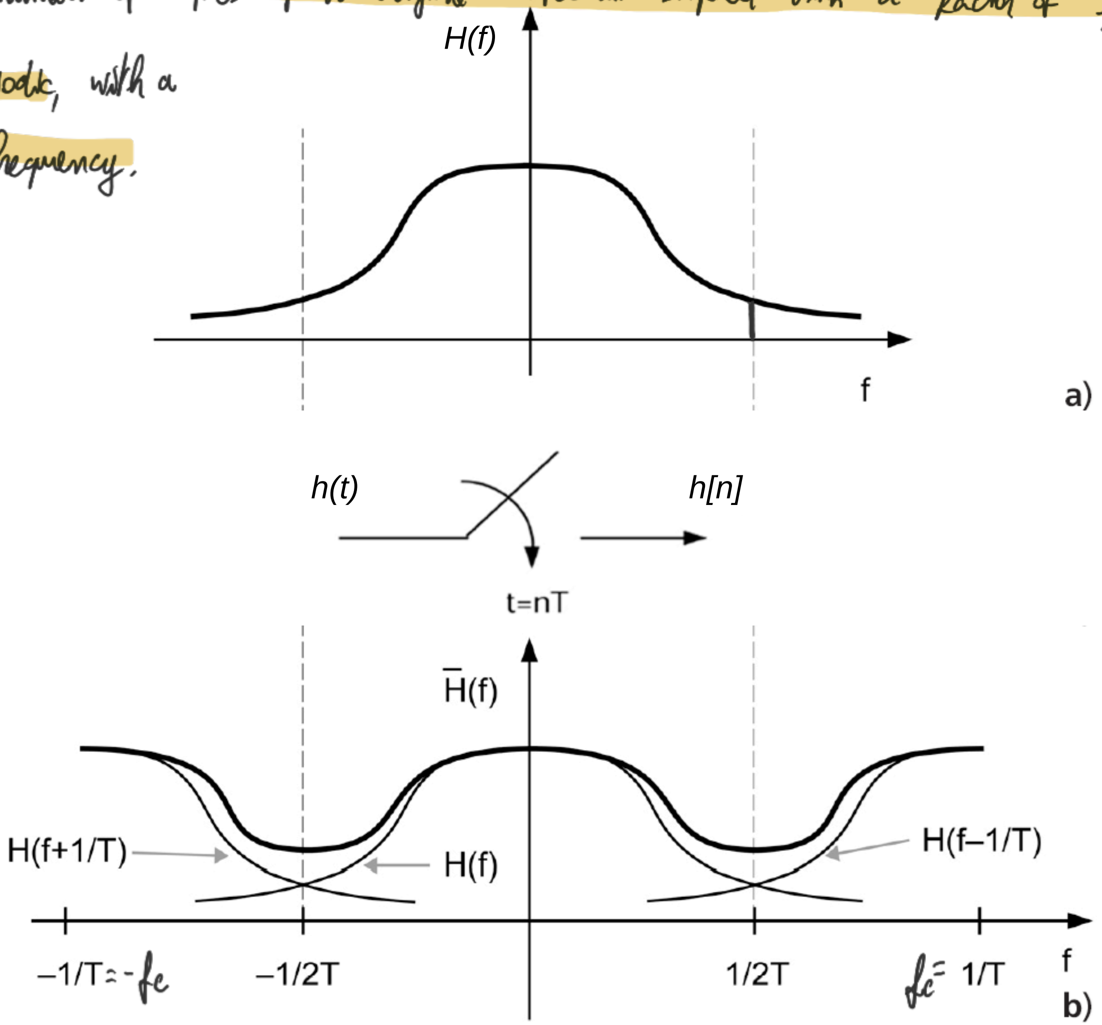
Marco Luise
Digital Signals

* like cos(2π f₀ mT). There will be differences in the samples: you start with one spectrum and end up *2

Periodic spectrum of a sampled signal

*2 with an infinite number of copies of the original spectrum shifted with a factor of $\frac{1}{T}$.

The new spectrum is periodic, with a period is the sample frequency.



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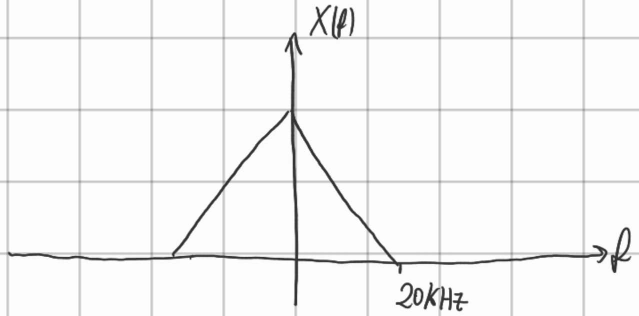
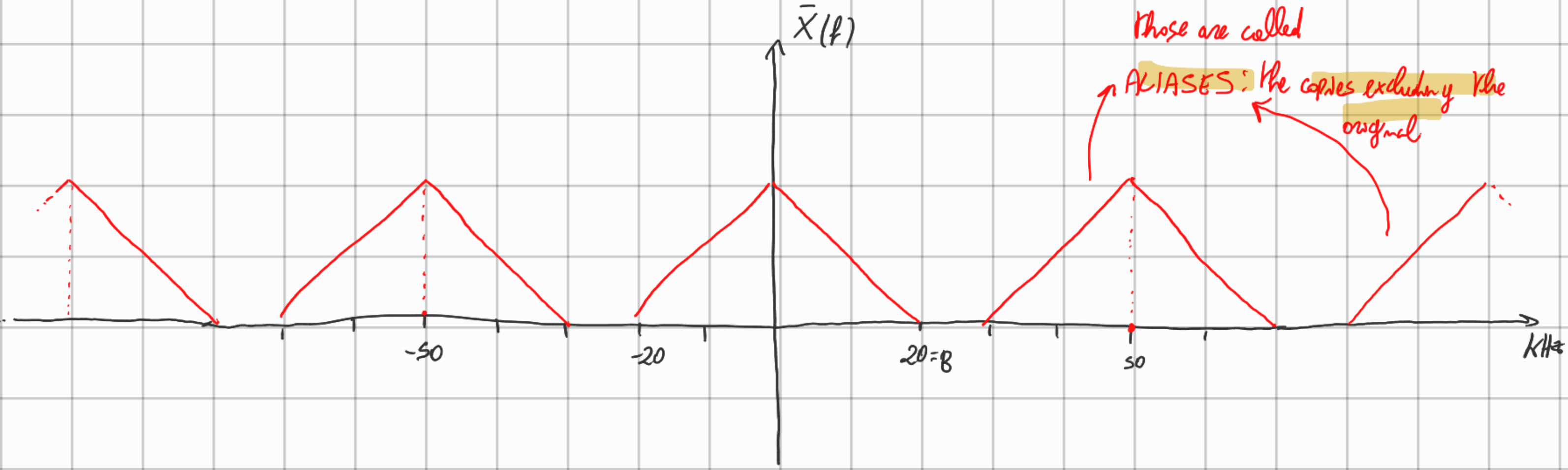
Let's say we have an audio signal of $B = 20\,KHz$:



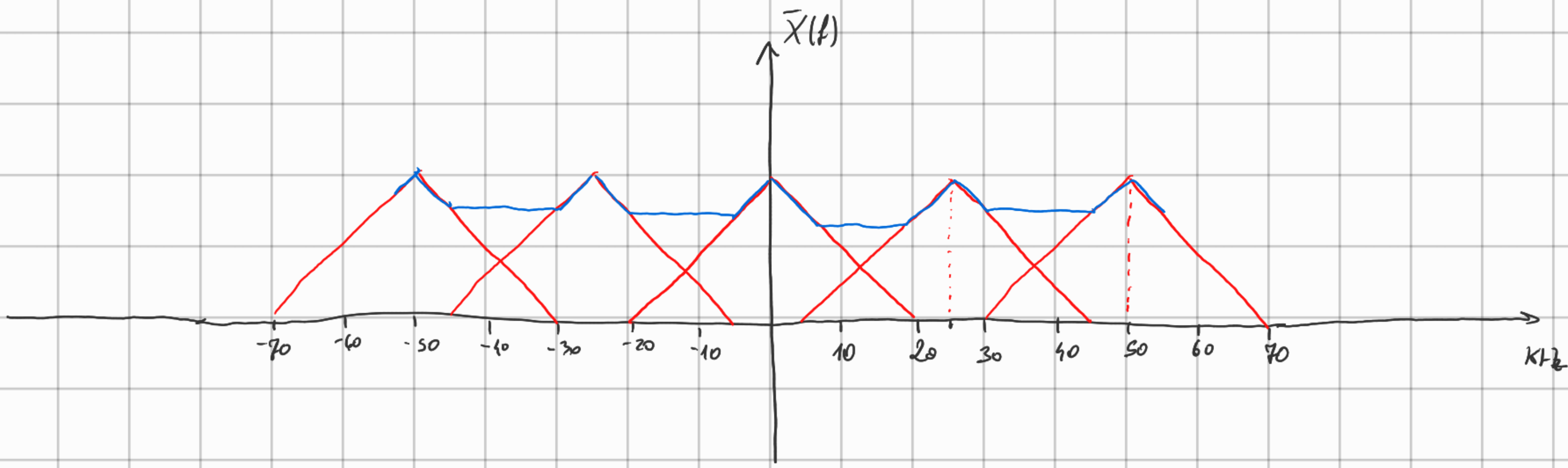
Let's try 25KHz and 50KHz as f_c.

Let's see the sampled spectrum:



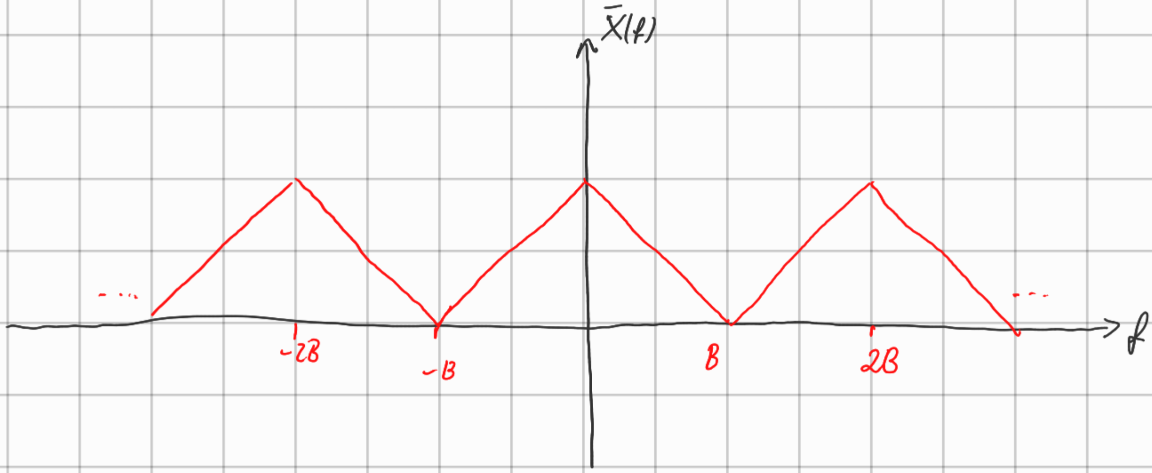
$$\bar{X}(f) = \frac{1}{T}\left(\dots + X(f+2f_c) + X(f+f_c) + X(f) + X(f-f_c) + X(f-2f_c) + \dots\right)$$

I still see the original spectrum! This is a good frequency. What if I use $f_c = 25\,KHz$?
Lower f_c = lower samples = lower space required to store.



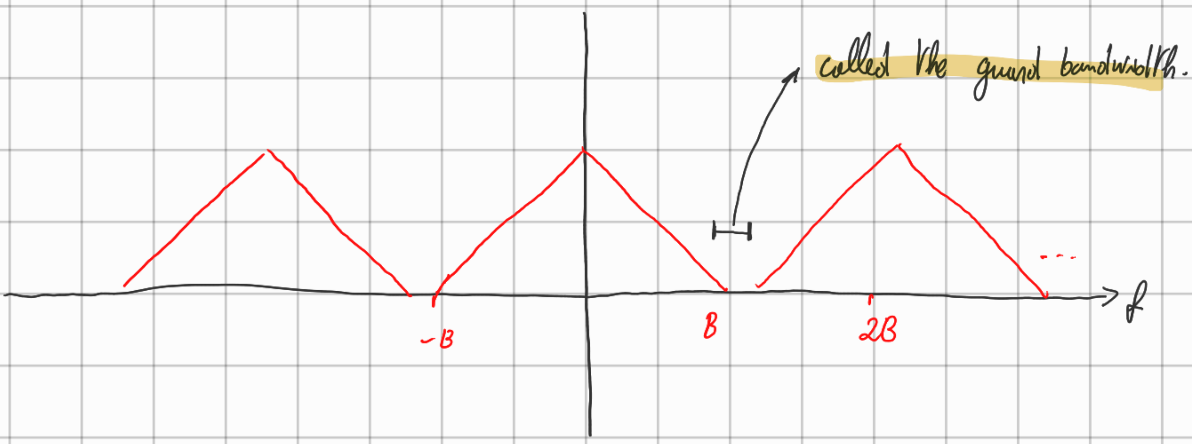
The sum does not have a replica of the original spectrum. The copies interfere and we have created an ALIASING ERROR:

So how can we choose the right sampling frequency? We use the Nyquist rule.
We need to find the boundary. Let's take a band limited signal. What is the limit situation?
Create a spectrum where the first alias starts when the main spectrum ends.



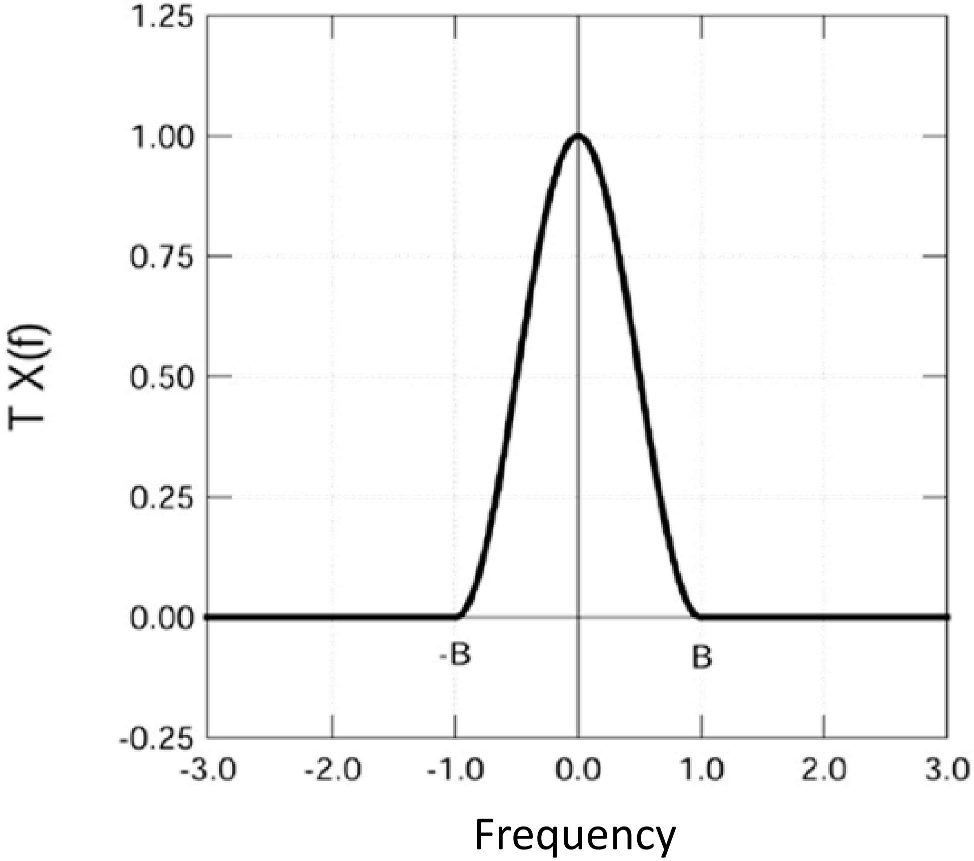
The sampling frequency has to be AT LEAST 2 times the bandwidth of the signal.

$f_c \geq 2B$. Usually we take a bit more as a margin.

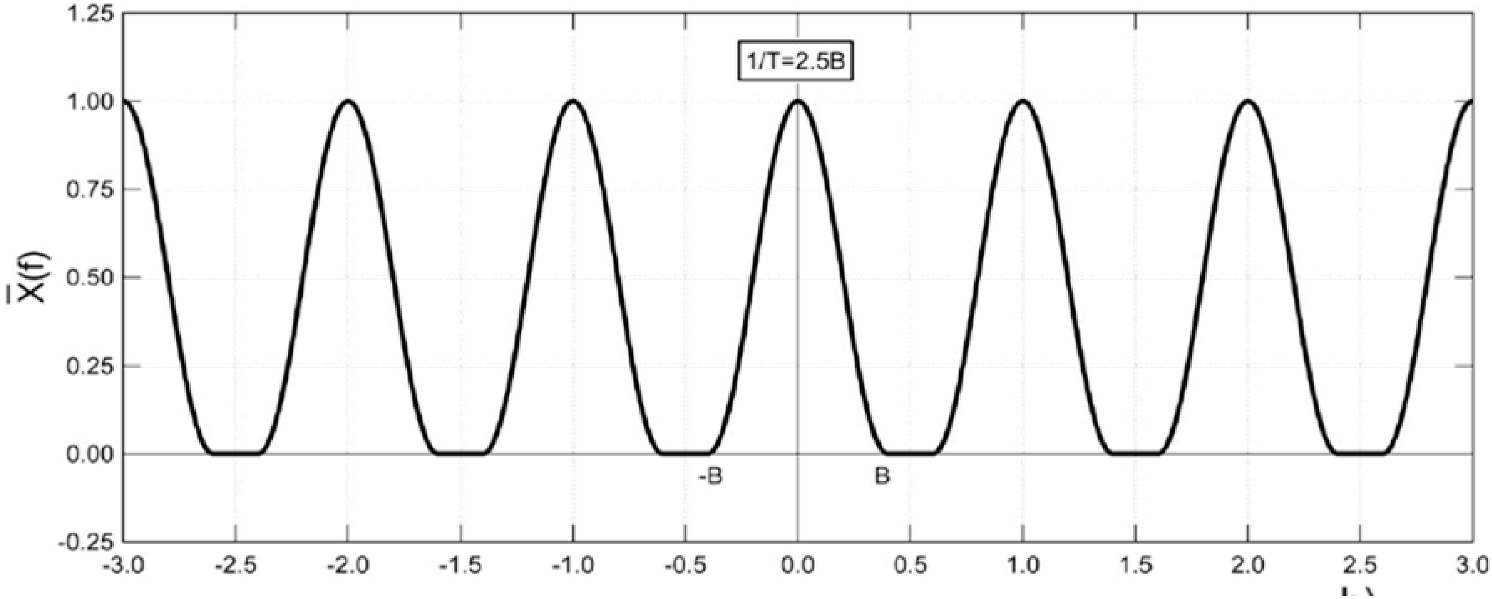


For music the CD standard is 44.1 KHz. In some cases $f_c = 48$ KHz is used. In studio recording for max quality $f_c = 96$ KHz is used.

An example



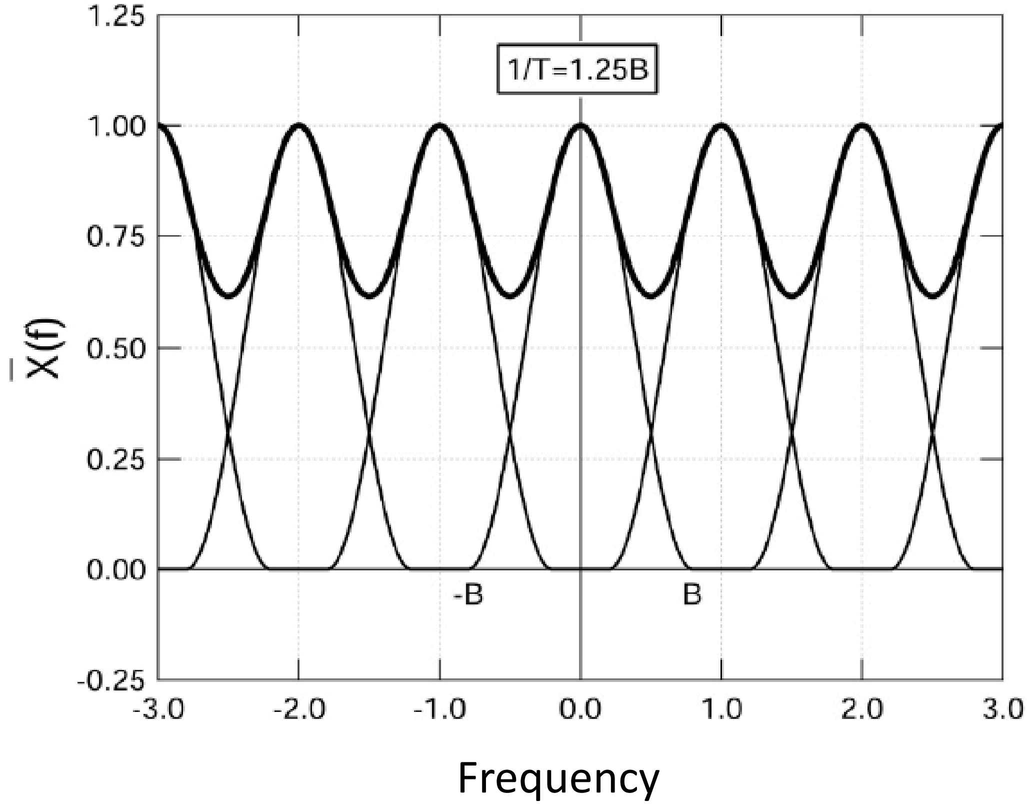
No Aliasing



Frequency

Using a sampling frequency f_c=5B/2

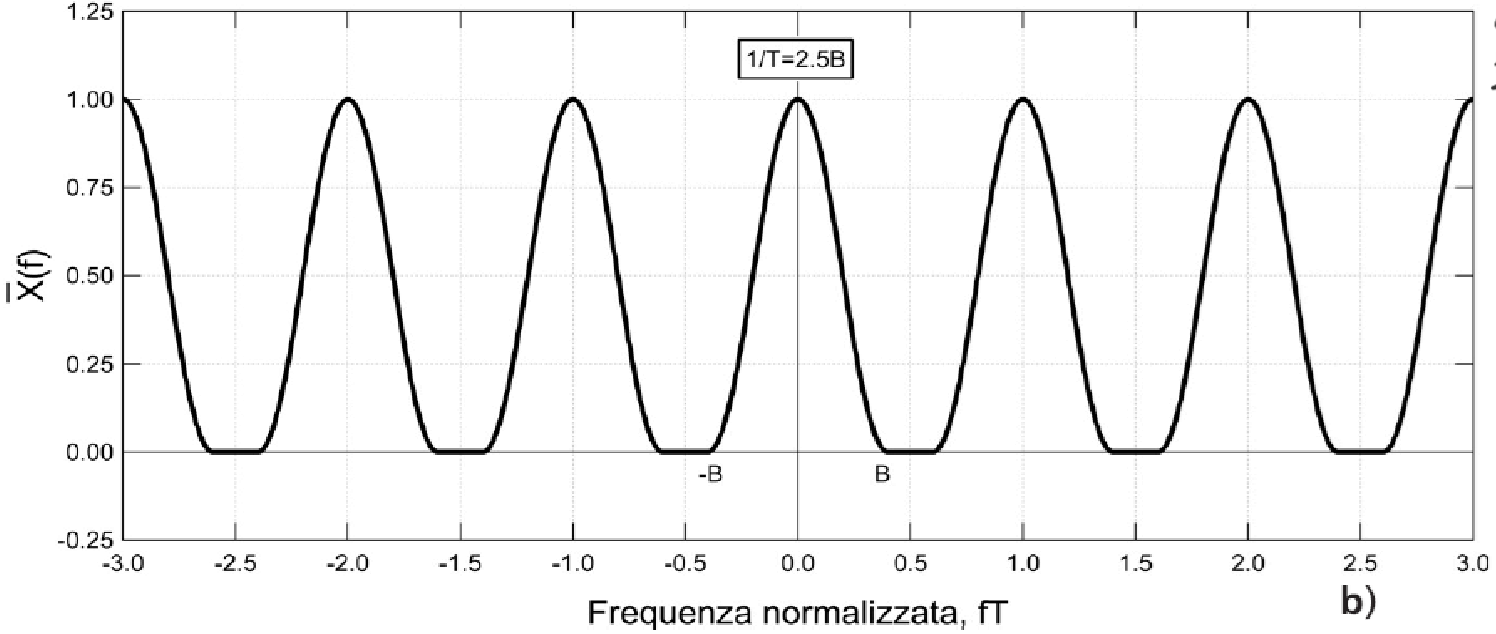
Aliasing Error !



Using a sampling frequency f_c=5B/4

Nyquist rule

Digital to analog converter deve filtrare gli alias (nel campionamento) nella ricostruz. chiamate IMMAGINI (filtro anti immagini)



Not to have aliasing, we have to make sure that

$$f_c \geq 2B$$

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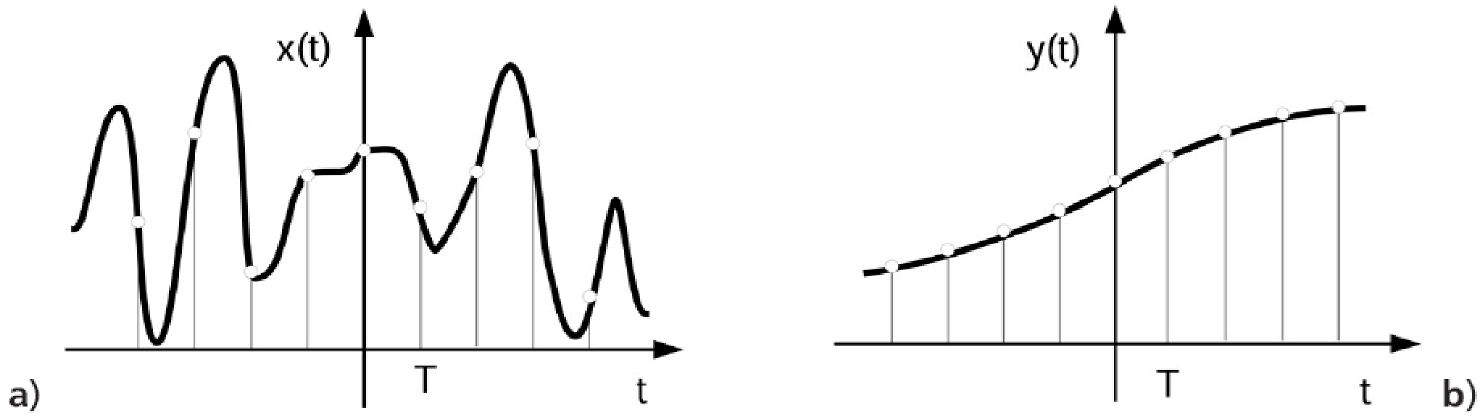
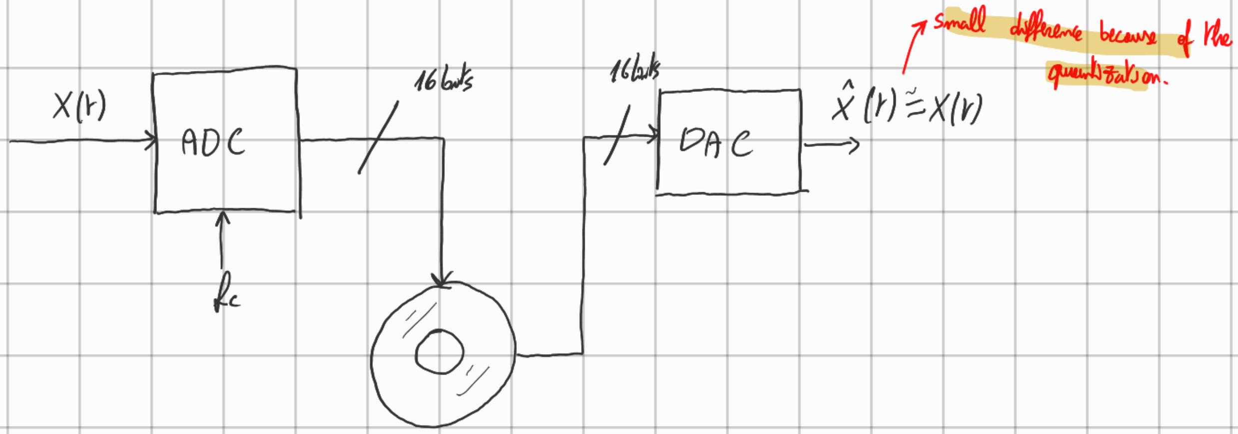
Nyquist rule in the time domain



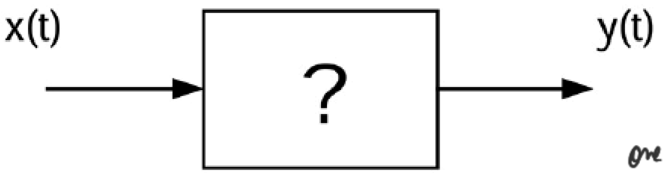
The sampling period is adequate for y(t), but it is clearly too large for x(t). In this sense, the sampling frequency must be commensurate with the signal bandwidth (with the rate of change of the signal), as the Nyquist rule suggests



Problem: we have a digital spectrum when we read the digital signal! We need something that filters out the aliases.

Filtering 1/2

Linear: if $x(t) = x_1(t) + x_2(t) \cdot c$ then $y(t) = y_1(t) + y_2(t) c$, the same you would get if you just inputted each one separately.



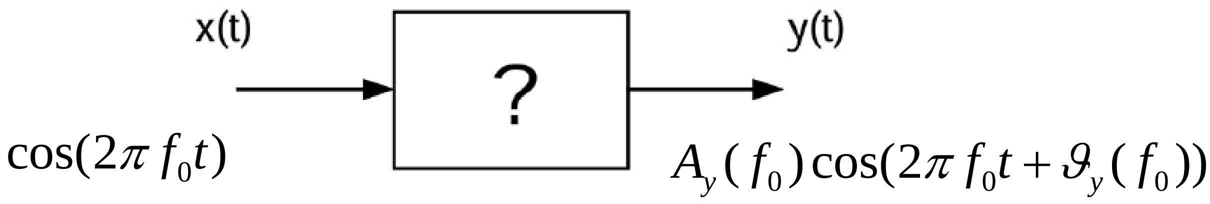
Something you apply to a signal

- A **filter** is a «black box» (HW, SW in an ARM DSP processor) with a linear behavior.
- If $x(t)$ is sinusoidal, then $y(t)$ is sinusoidal at the same frequency: frequencies are not changed
- BUT what is changed is the amplitude/phase of the oscillation:

$$x(t) = \cos(2\pi f_0 t) \quad \Rightarrow \quad y(t) = A_y \cos(2\pi f_0 t + \vartheta_y)$$

So a signal with a certain spectrum gets outputted as another signal whose spectrum was altered frequency by frequency. The black box modified one by one each frequency (one by one because linearity)

Filtering 2/2



- Not all frequencies are the same: according to the properties of the filter systems, some frequencies ar more altered then others (for instance, enhanced/attenuated)
- The filter *responds* differently according to the particular frequency of the input: the *frequency response* of the filter is

$$H(f_0) = A_y(f_0)e^{j\vartheta_y(f_0)}$$

To understand how my filter works theoretically we should test the filter at every frequency. *

* We can do this because the filter is linear.

What's the purpose of Filtering ?

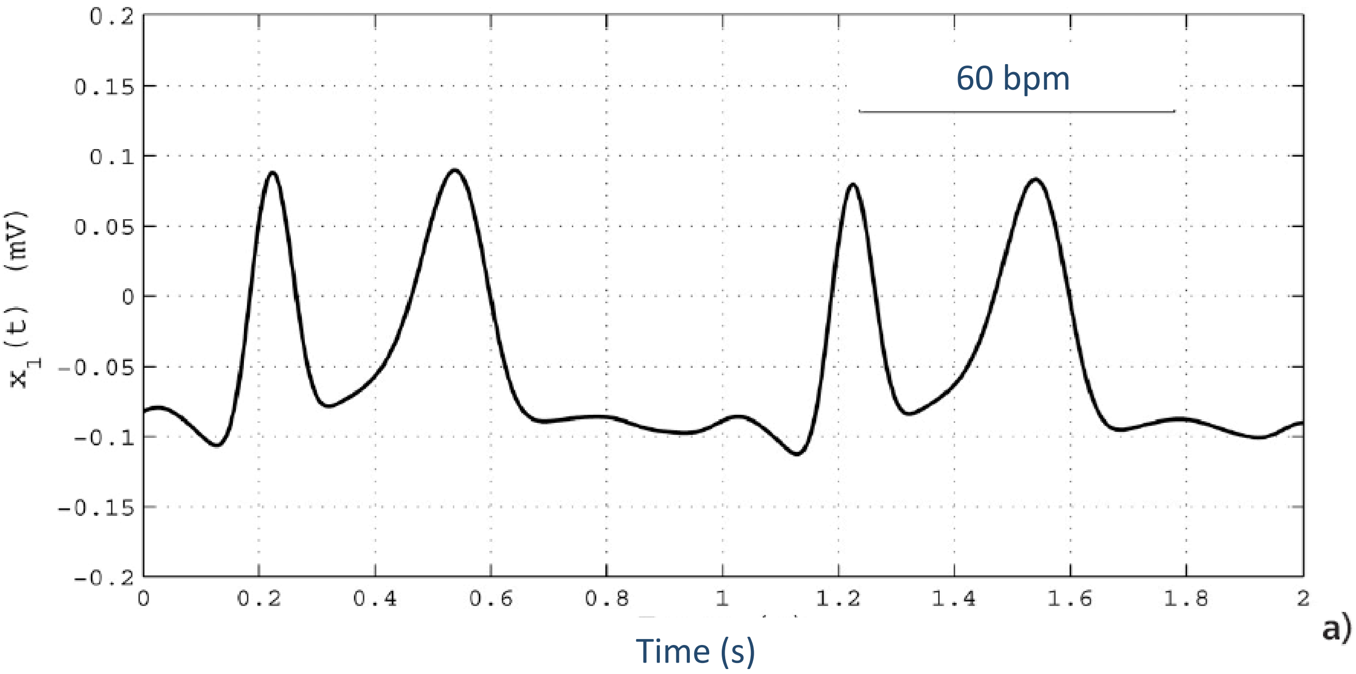
- A certain signal is made of the superposition of two components:

$$x(t) = x_1(t) + x_2(t)$$

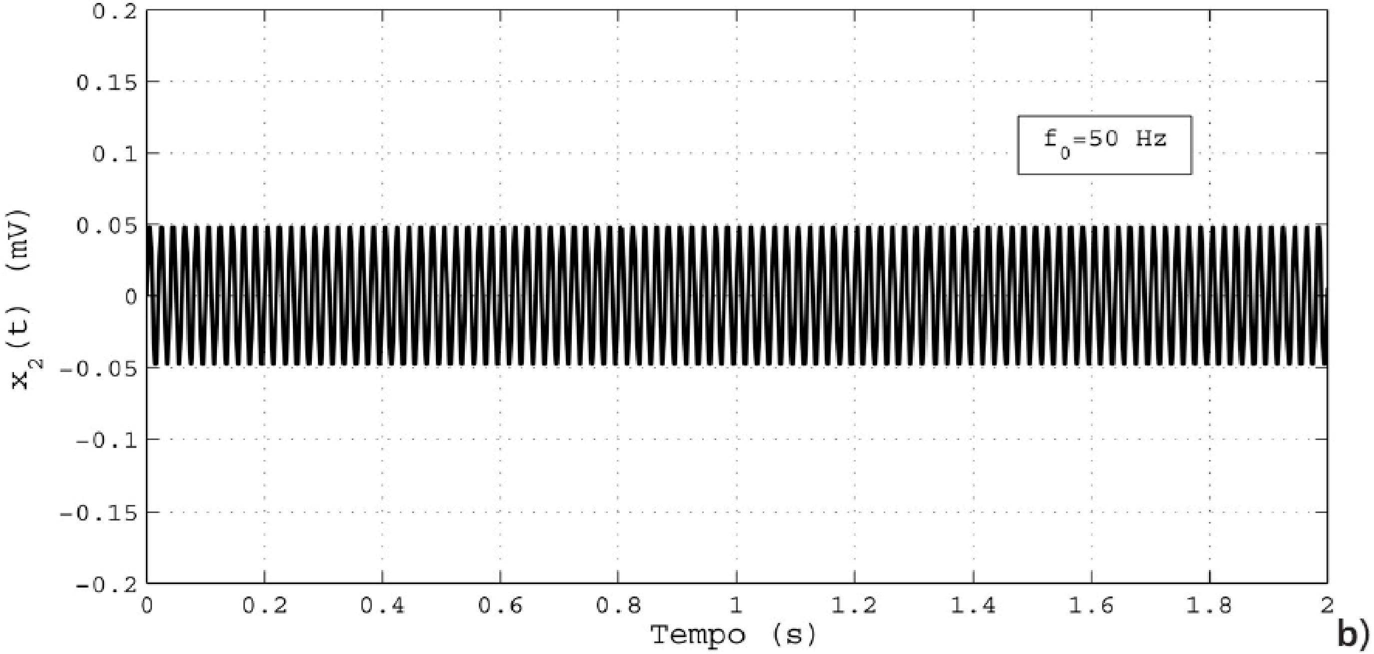
The first component is the useful signal, the second is just *noise* (disturbance), an unwanted component we would like to rid of.

Example: (low-power, low amplitude) ECG signal plus an unwanted component (interference) coming from the 50 Hz AC power supply.

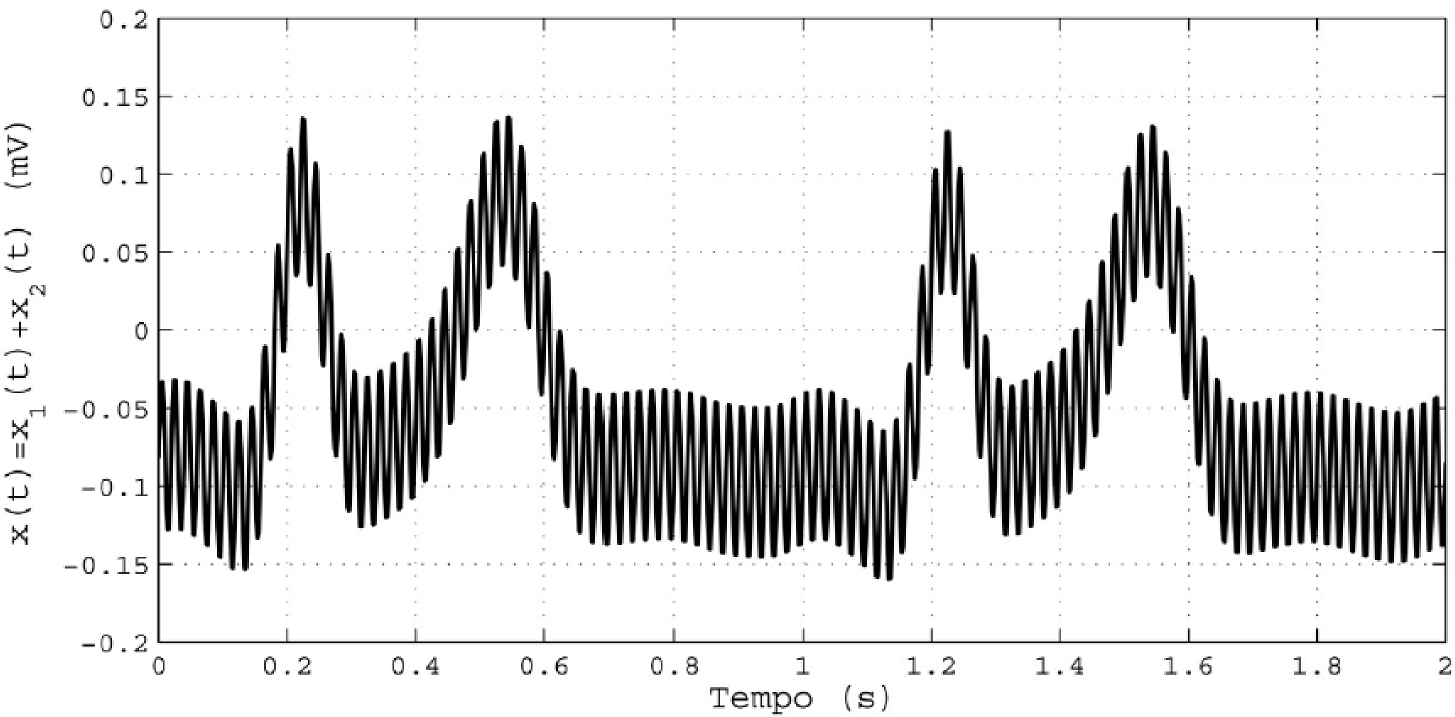
The Notion of FILTER



The Notion of FILTER

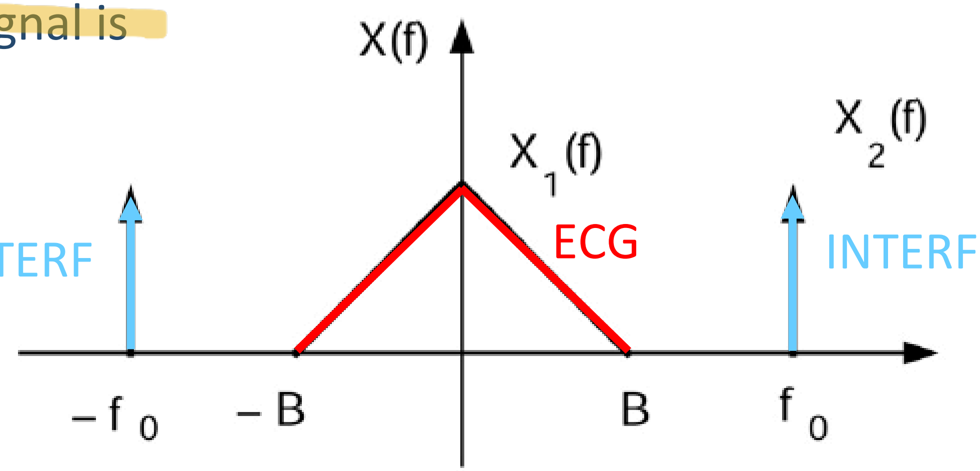


The Notion of FILTER



Filtering !

Is there any hope to *reject* (eliminate) the unwanted component? Let us examine the problem in the *frequency domain*. The spectra of the signal is



Spectrum of an unwanted signal is on another band than the important signal.

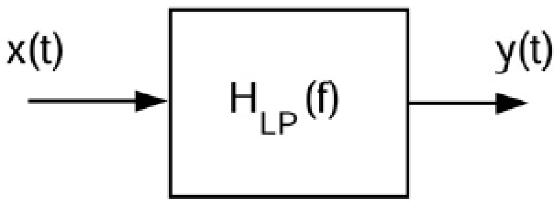
$$X(f) = X_1(f) + X_2(f)$$

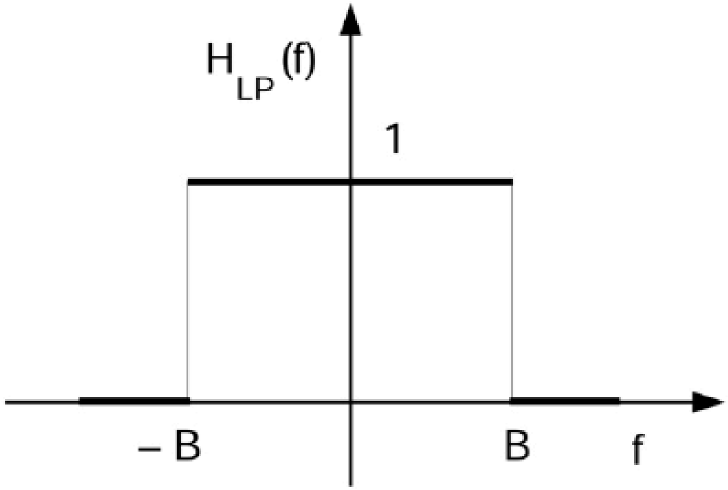
Lucky me!

Whilst the two signals fully overlap in time, their spectra lie in two separate frequency intervals (*bands*) ! We can separate the two (rejecting the interference) with a filter having a appropriate *frequency response* (with unequal treatment of frequency bands)

Low-pass FIlter

We can *filter out* (reject) the higher-frequency, unwanted component $x_2(t)$ applying to the whole signal $x(t)$ an LTI system that *passes* the low-frequency components and *blocks* the high-frequency ones: a low-pass filter with frequency response

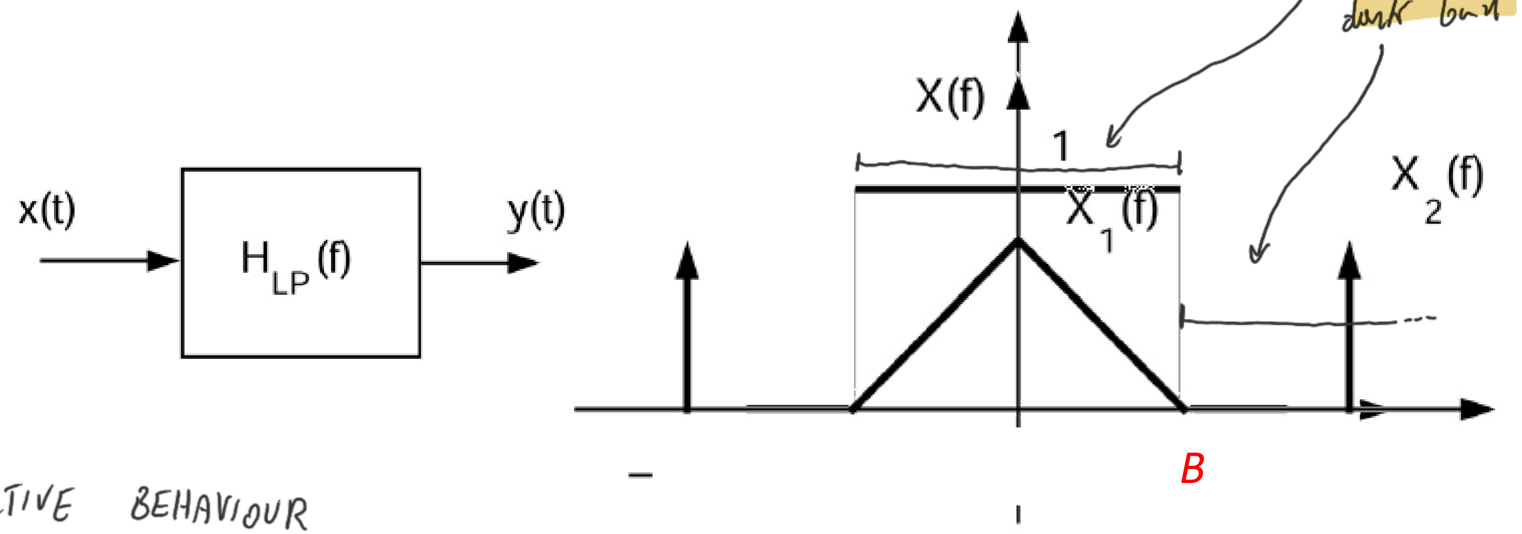




$$Y(f) = X(f)\,H(f)$$

Low-pass Filter

We design a filter so that all the components outside the band are 0. This is a pass band and a dark band



SELECTIVE BEHAVIOUR

B is the value of the filter bandwidth (to be considered on positive frequencies only)

This is a low pass filter with a bandwidth B. It's called like that because it passes low frequencies only.

$$Y(f) = X(f)\,H(f)$$

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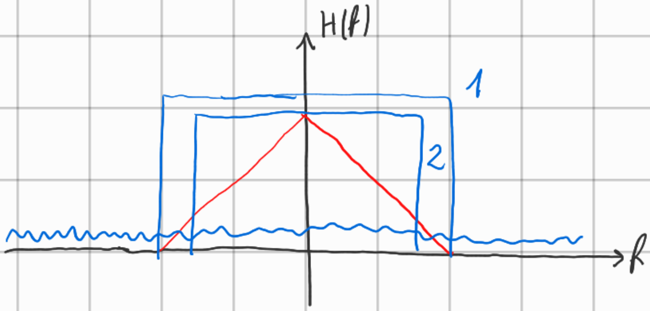
If you have noise in your signal you have two options: either not touch the signal or touch it a little bit.



Filtering Example

a)

Input signal $x\ (t_1,t_2)$

Filtering Example

Blur: lose high frequencies

b)

Low-pass filtered signal $y(t_1,t_2)$

Filtering Example

a)

High-pass filtered signal $y(t_1,t_2)$

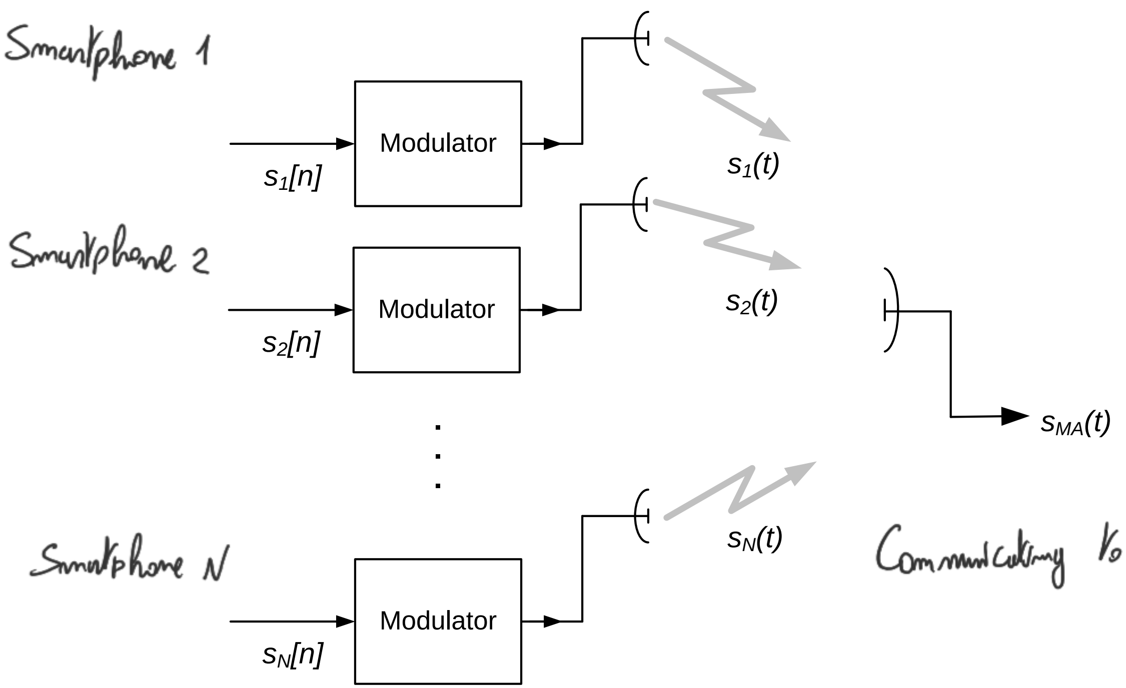
Filtering Example

b)

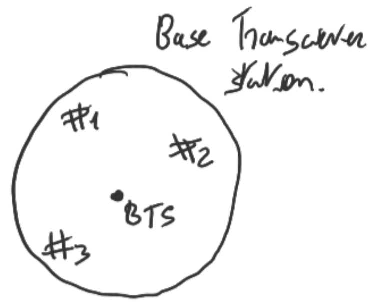
Same, with inverted brightness scale

More on Filtering: Frequency-Division Multiple Access

- **Many users share the same physical medium (for example, the same band of frequency). How can they co-exist ?**

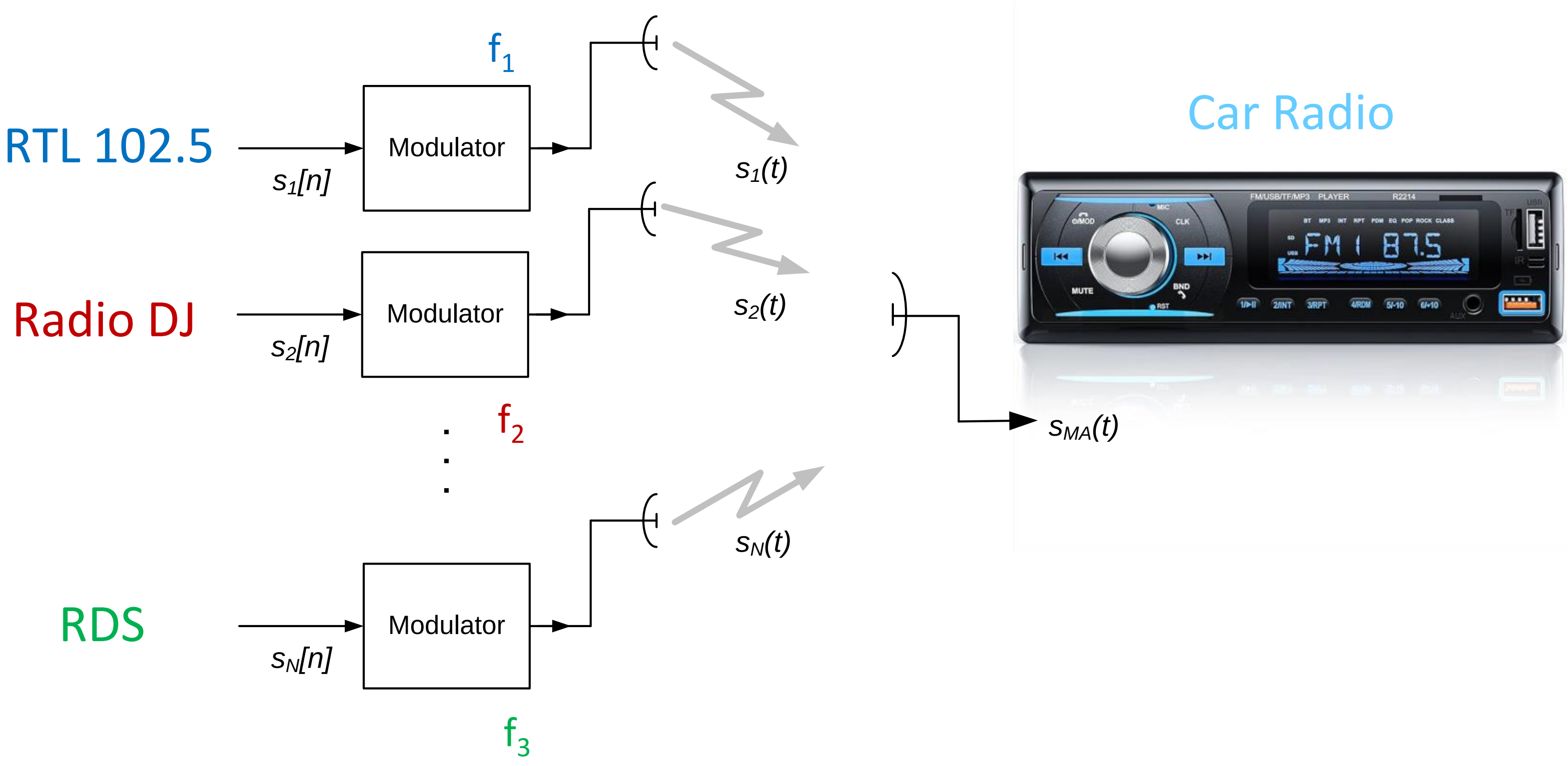


Communicating to a cell:



Example: FM Radio

- Many users share the same physical medium (for example, the same band of frequency). How can they co-exist ?



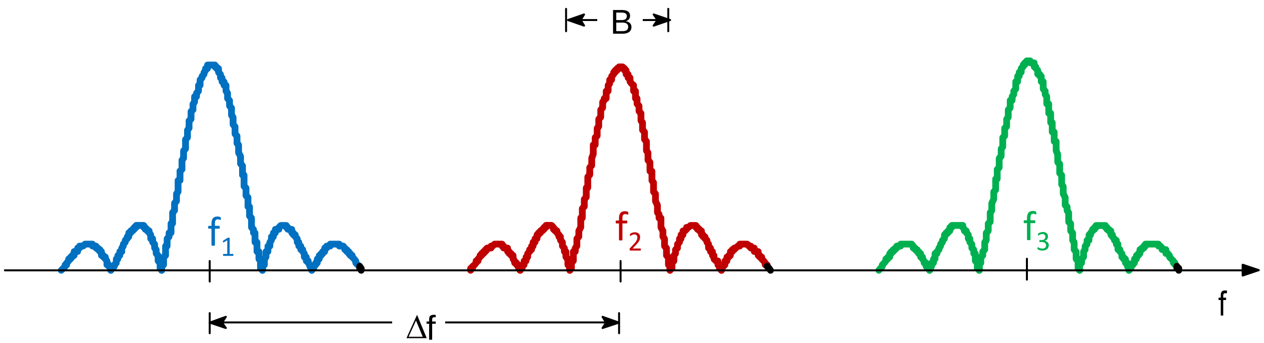
So the signals get one over the other. How to distinguish?

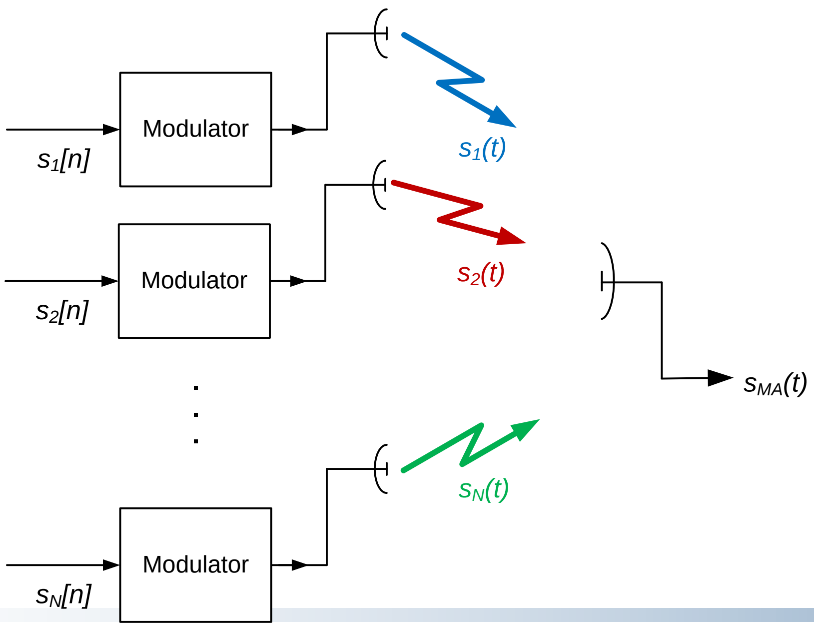
$$S_{MA}(t) = S_1(t) + S_2(t) + \dots + S_N(t).$$

We have different signals and we put them on different frequencies. We call them carrier frequencies. So we filter out the signal we want and we are done.

$S_{MA}(t)$ = MULTIPLEX SIGNAL

Frequency Division Multiple Access





Different radio stations co-exist on the same FM frequency band 88-108 MHz by using separate bandwdiths and different carrier frequencies

B=150 kHz

Δf=250 kHz

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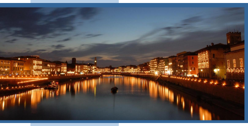
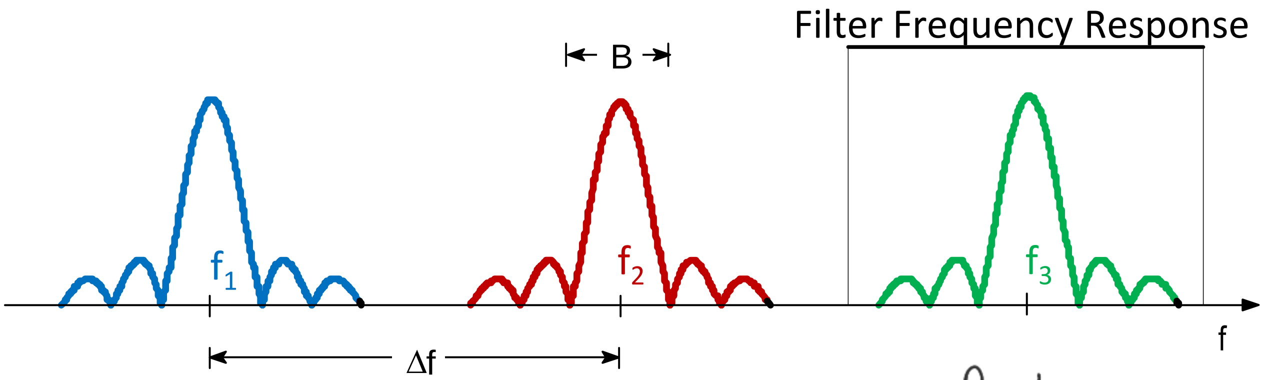
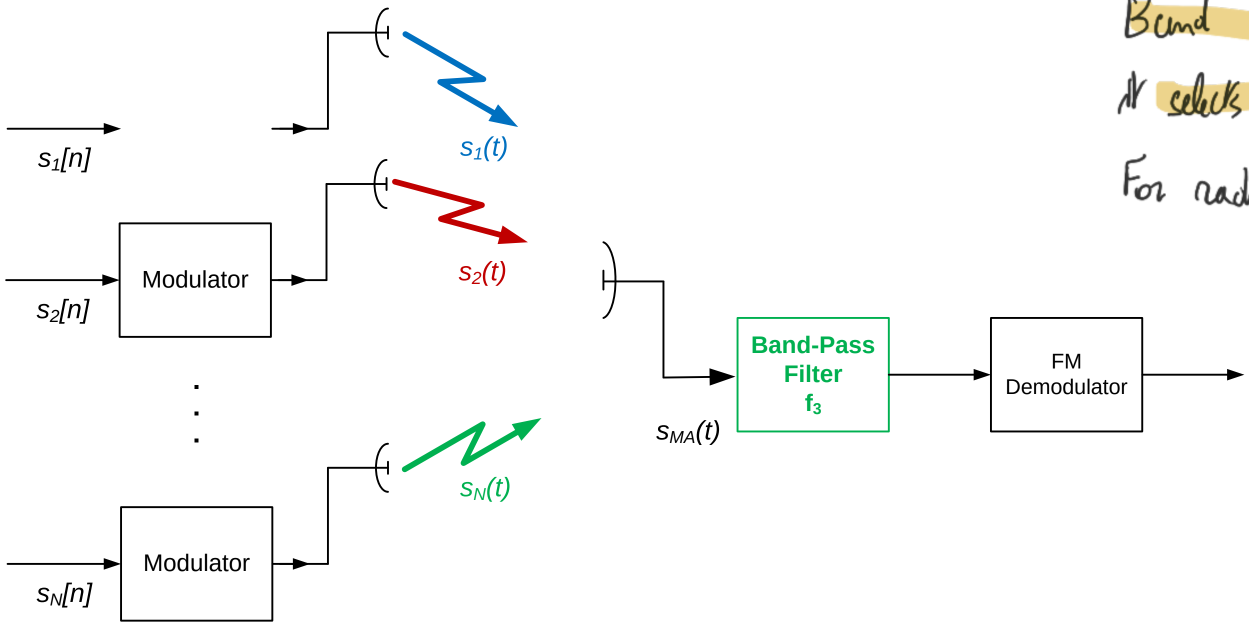


Demultiplexing and.. listening



Band Pass filter, because it selects a specific band to filter.

For radios, the 150kHz is the band.



Un esempio per trasmettere può essere un segnale che mappa livelli logici sul livello fisico +10V con 1, -10 per 0.

Baseband Physical Data Signal

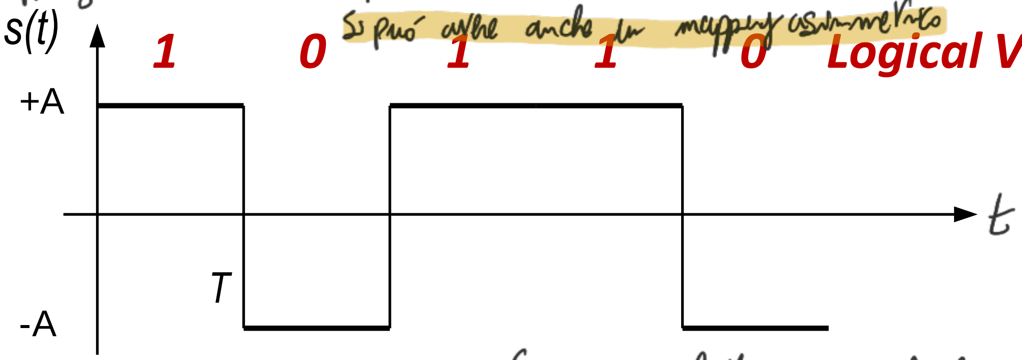
Mapping simmetrico di tipo Non Return to Zero

Si può avere anche un mapping asimmetrico

Logical Values

$\{a_k\} = \pm A$

Physical Values



Come sono fatti i segnali fisici? Durante la trasmissione abbiamo sempre un flusso di bit

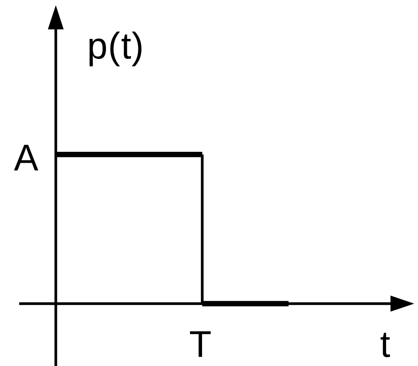
Binary Baseband Digital Signal

Pulse/Symbol Interval: T = Bit Interval: T_b

Pulse/Symbol Rate: R=1/T = Bit Rate: $R_b=1/T_b$

Inverso del tempo di bit

Basic Pulse p(t):



Bipolar Format

$$s(t) = \sum_{k=-\infty}^{\infty} a_k \cdot p(t-kT)$$

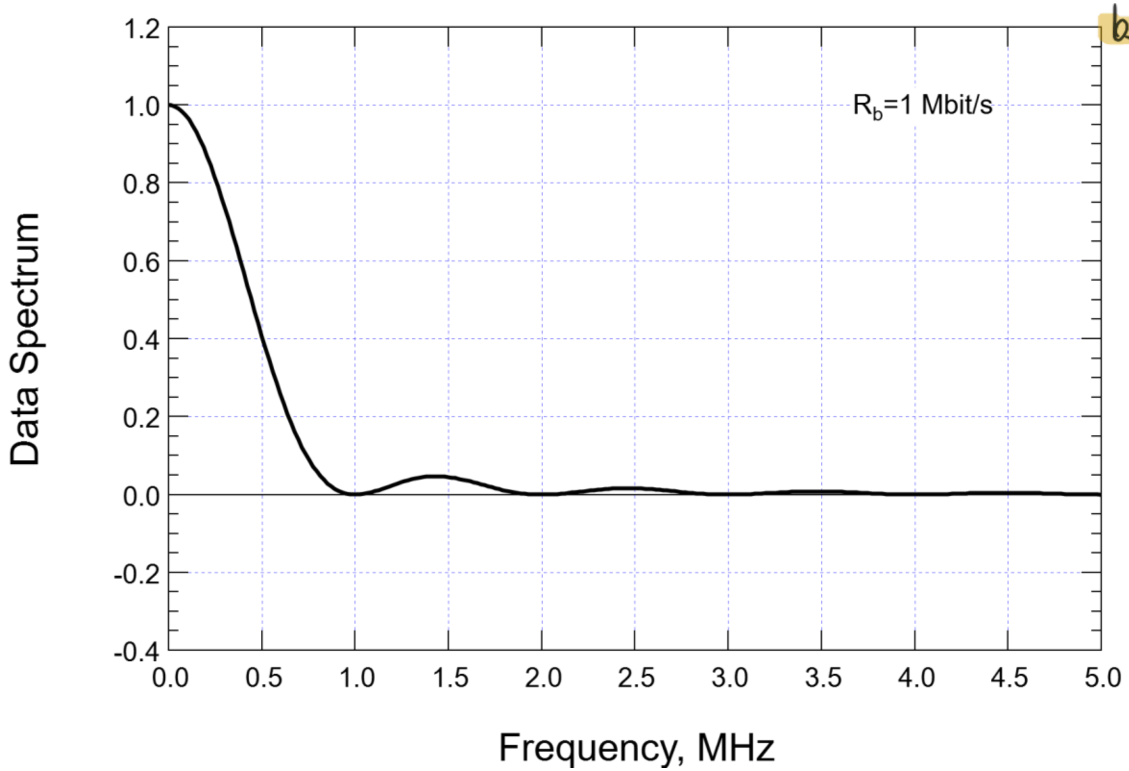
Per il momento gli impulsi sono dei bit, un tempo T_b, che indica la durata di un bit.

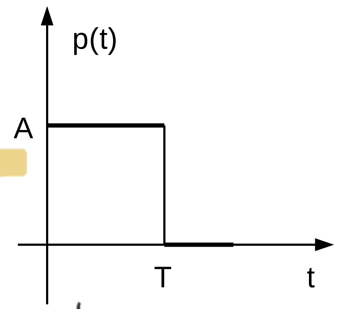
Si nota: il segnale ① non è né periodico né aperiodico (non ha inizio e fine). È un segnale pressocché random. Lo spettro di un segnale random è più complicato da calcolare, ma devo comunque trattare tutte le frequenze.

Power Spectral Density (PSD) of the Baseband Data Signal

$$S(f) = A^2 T^2 \left(\frac{\sin(\pi f T)}{\pi f T} \right)^2$$

⇒ Spettro del segnale che porta i bit.





Pulse: impulso NRZ

Rappresenta come la potenza fisica del segnale è distribuita sulle frequenze ed è sempre positivo.

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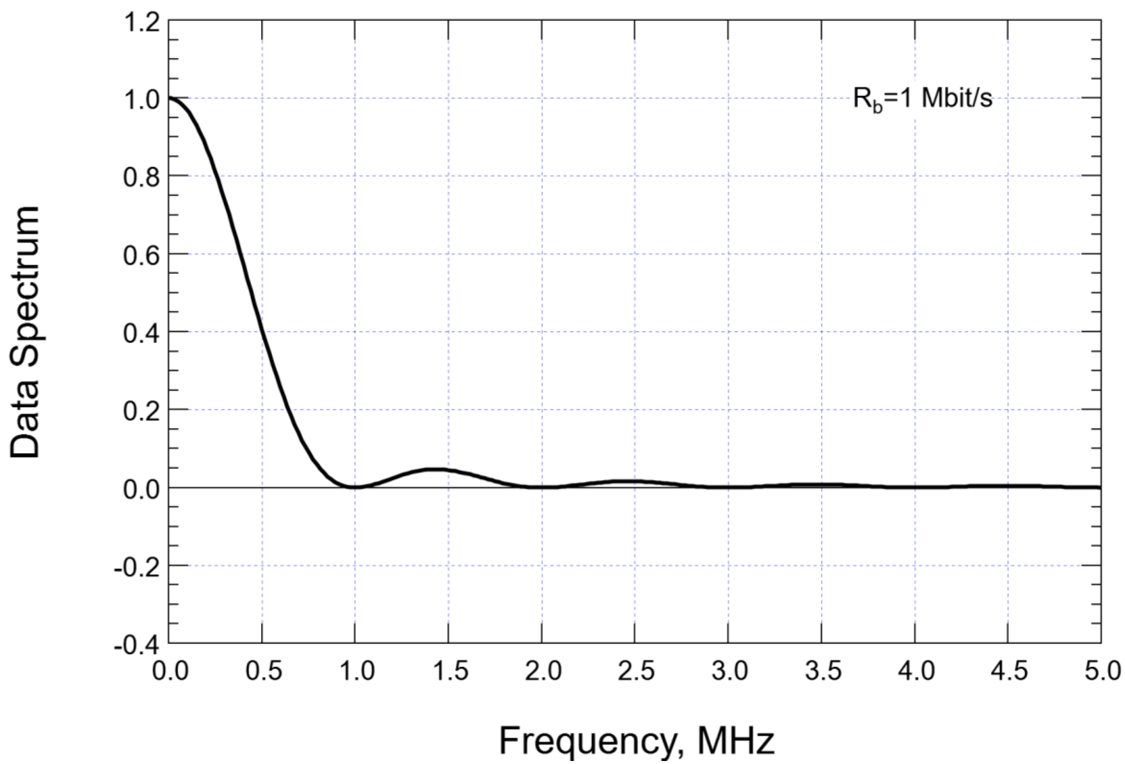
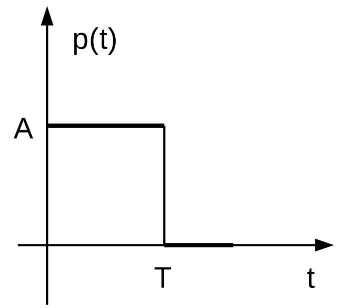
Es: se vado a 1Mbit/s, fino a 1MHz lo spettro è importante, oltre è trascurabile.
Abbiamo individuato la banda del segnale che porta il segnale.
È importante sapere la banda in funzione del bitrate: se esso aumenta, diminuisce T e il segnale varia più velocemente e quindi serve maggiore banda.

Il mezzo fisico ha una limitazione. Regola: banda = bitrate?

Power Spectral Density (PSD) of the Baseband Data Signal

Bandwdith=1/T=1/T_b=R_b





Is it true that the bandwdith (Hz) is the same thing as the bit rate (bit/s) of the link?

La banda del segnale é sempre la stessa, sia che i bit arrivano che non.
Se internet va lento é perché i bit non arrivano.

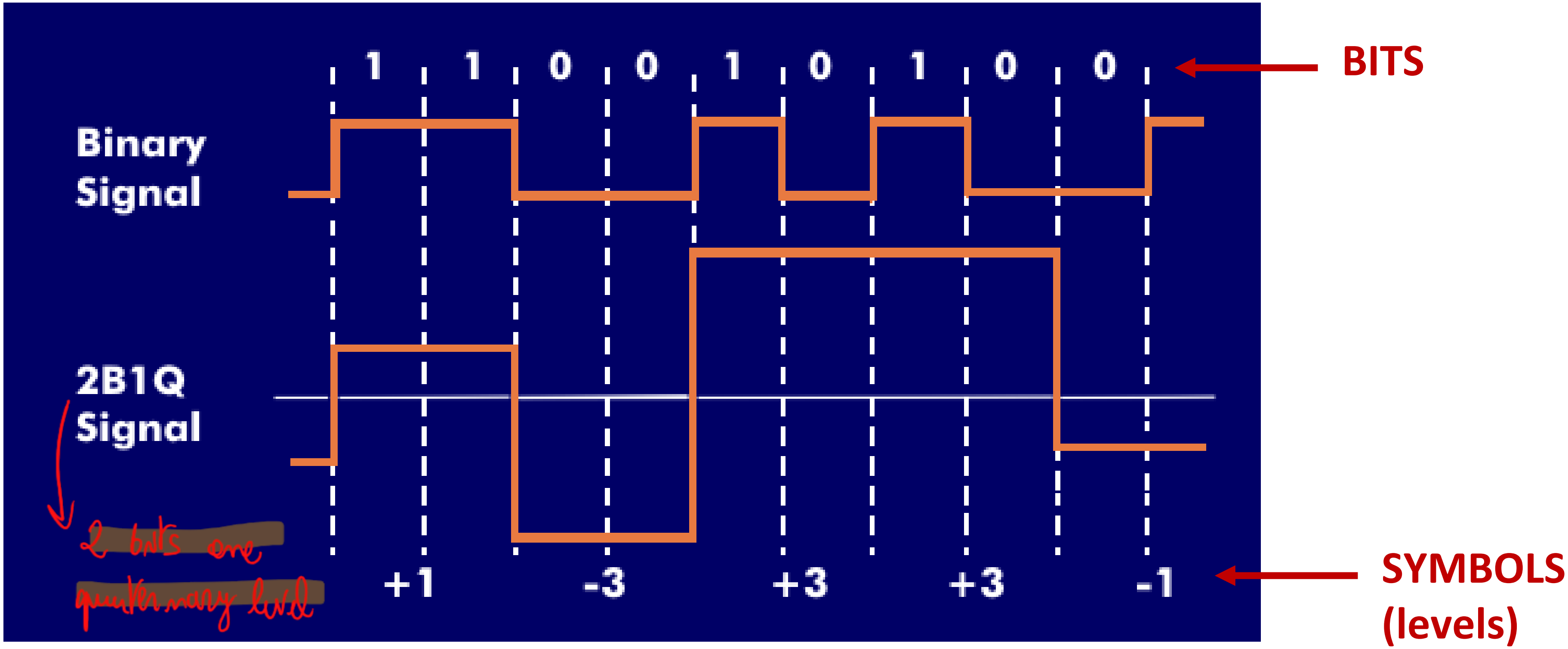
Is it true that the bandwdith (Hz) is the same thing as the bit rate (bit/s) of the link?

Pre-Internet Technology: ISDN

Anni 90, quando c'era poca roba digitale. Servivano interconnessioni tra computer in aziende o università.

- 2B+D digital link on telephone lines @ R_b=2*64+16=144 kbit/s

↳ Canale di servizio



Idea era creare una rete digitale che integra tutti i servizi. Il telefono digitalizzava la voce e la trasferiva. Questa non é decollata peró.

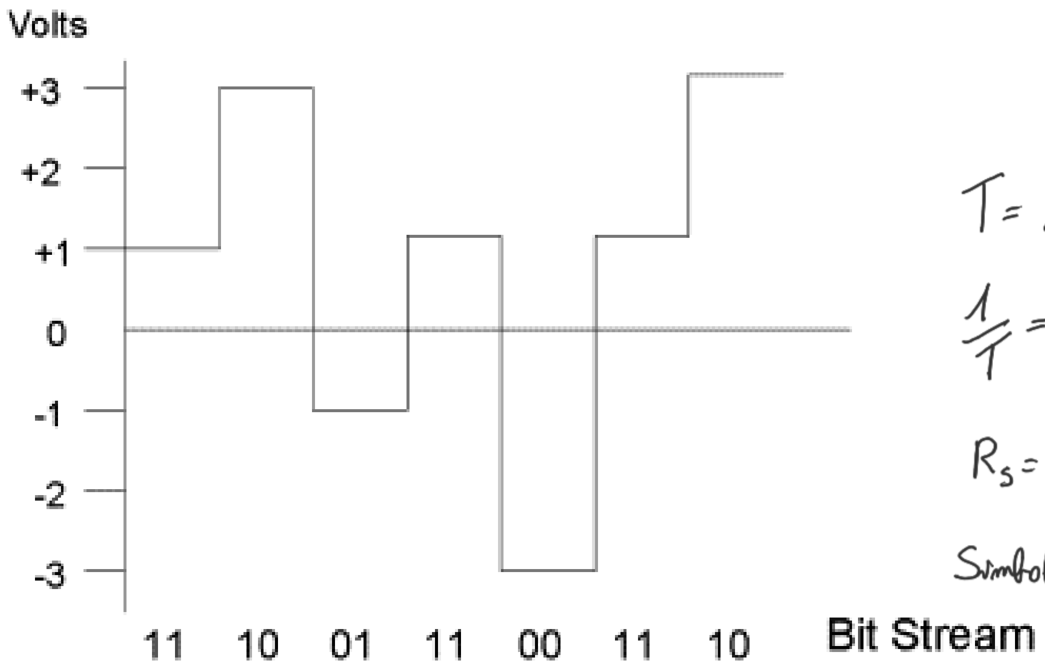
Formato dei tempi: qui avevamo il formato simboli: non ho solo 2 livelli che mappano 1 o 0 digitali, ho 4 livelli possibili: $-1, +1, -3, +3$. Ha alfabeto finito, é sempre digitale e ha il solito impulso rettangolare, ma un impulso rettangolare non vale un bit ma un simbolo. Ci sono info diverse, la durata di un simbolo (time) e il symbol rate.

Qual é il legame tra simboli e bit?

La durata di un bit non é uguale a quella di un simbolo: $T = 2T_B$

Multilevel data signal…

- **There is a clear difference between *bits* and *symbols***
  - *They are both represented in the physical world by rectangular NRZ pulses*
  - *BUT a symbol pulse* is wider than a bit pulse
  - BECAUSE *a symbol carries more than 1 bit…*



$T = 2T_B$

$\frac{1}{T} = \frac{1}{2T_B}$

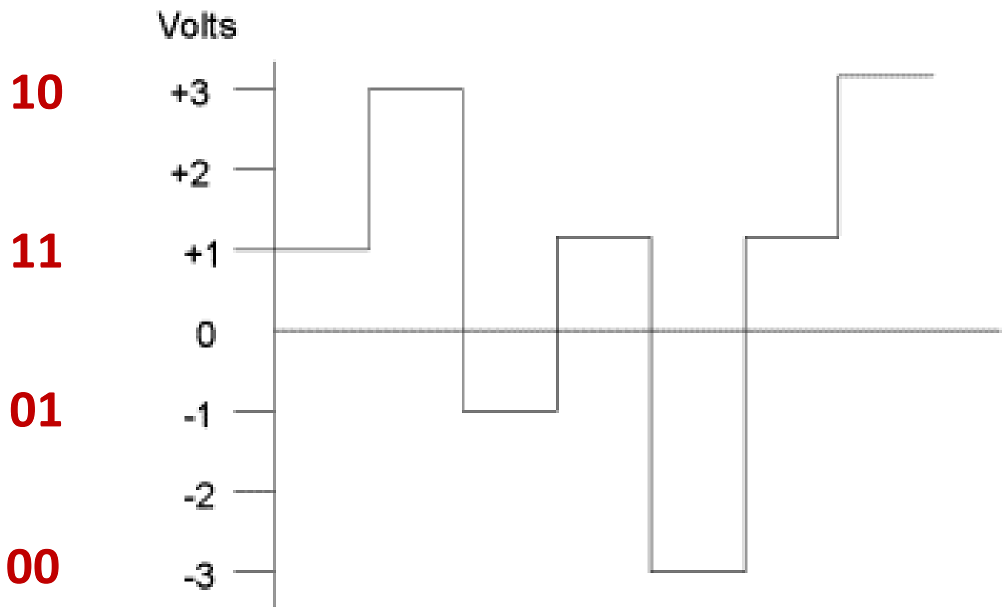
$R_s = \frac{R_b}{2}$

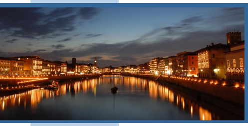
Simboli vanno più lenti

…and its *mapping*

- **Every symbol level is *mapped* to a (small) packet of bits**
  - *The number of bit in a packet is ust equal to the factor by which the symbol is wider in time than a bit*
  - *The receiver uses the reverse mapping to extract logical bits form physical symbol levels*

Segnale digitale multilivello è più lento e ha meno impulsi





Bit Rate and Symbol Rate

In this case,

$T_s = 2T_b$

The symbol rate is

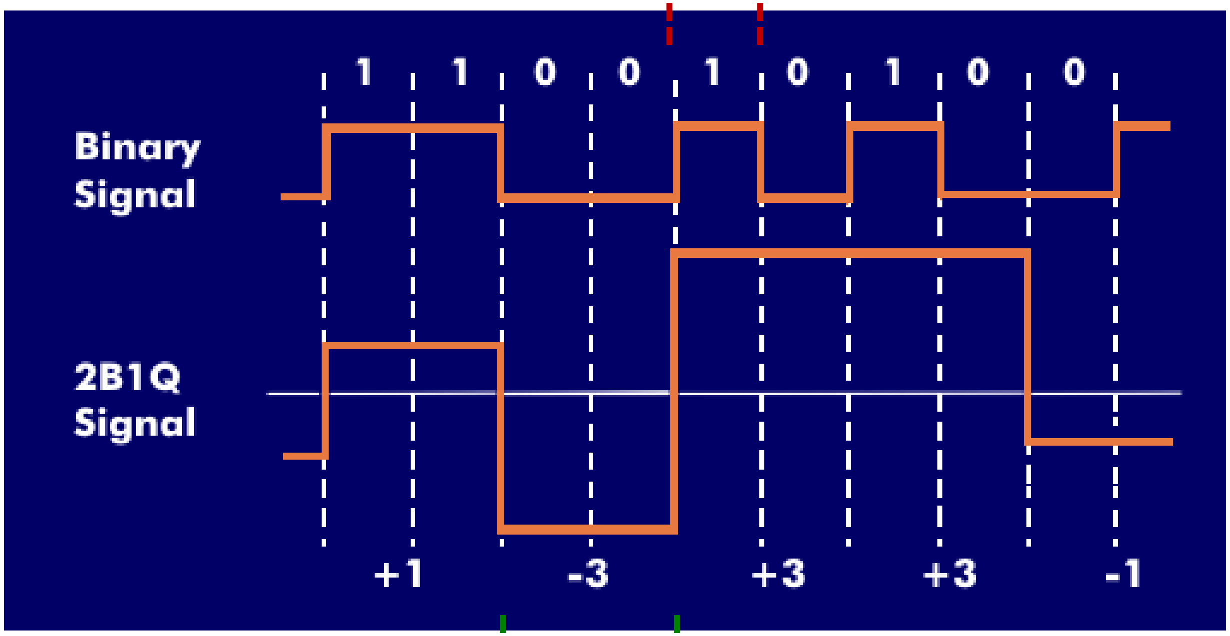
$R_s = 1/T_s$

and the bit rate is

$R_b = 2R_s$

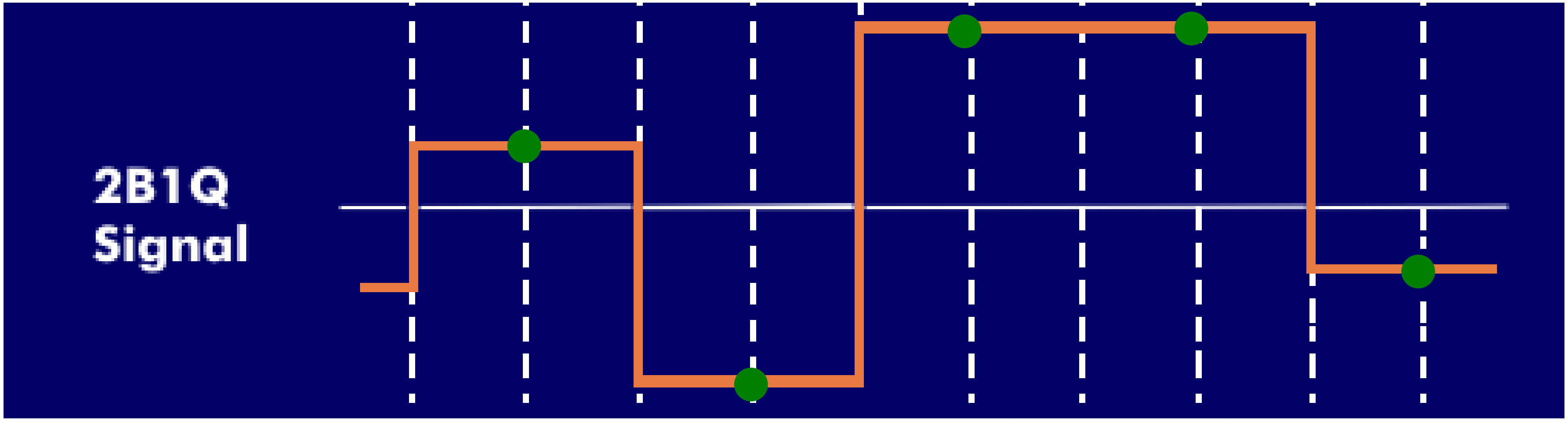
The bit rate is always *larger* than the symbol rate because one symbol "carries" more than one bit

Bit time T_b



Symbol time T_s

Alice & Bob (encoding and decoding)



+1	-3	+3	+3	-1
11	00	10	10	01

At the receiver, Bob re-clocks the signal at the center of each pulse (symbol) and form the observed values re-construct the stream of **symbols** and **bits**

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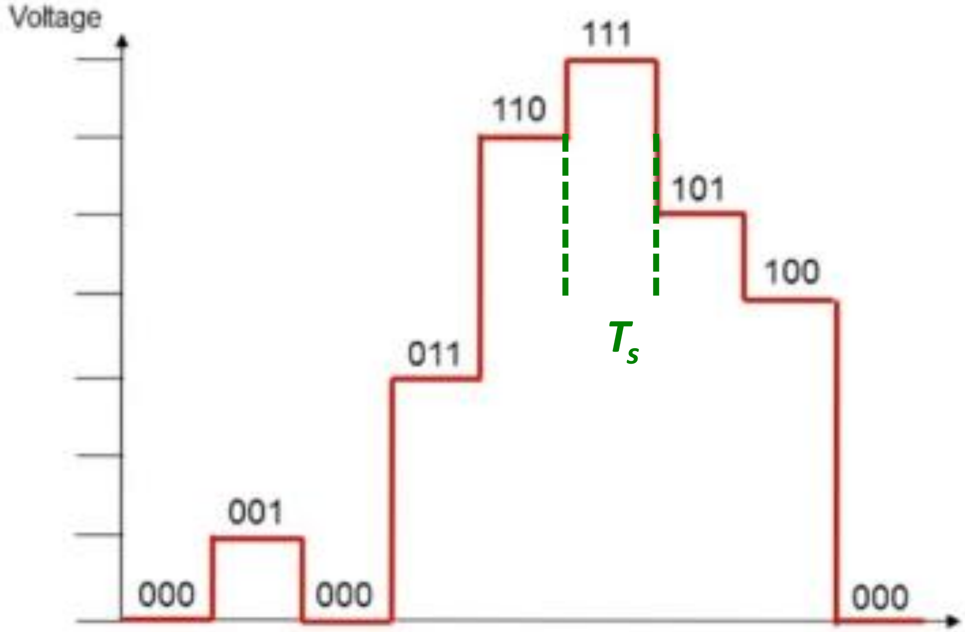
Bit Rate, Symbol Rate, and Bandwidth 1/2

M=8 symbol levels → 3 bits/symbol

- The time width of any pulse carrying a symbol is T_s

- Therefore, the *bandwidth* of the multilevel data signal is $B=1/\ T_s$
- The bandwidth is equal to SYMBOL RATE, not to the bit rate !!!



Possiamo rallentare il segnale e il suo clock!

numero dei livelli

WHAT IS THE PURPOSE OF ALL THIS ?

$\log_2(M)$ [fattore di rallentamento dei livelli] = N_b (n. bit per simbolo)

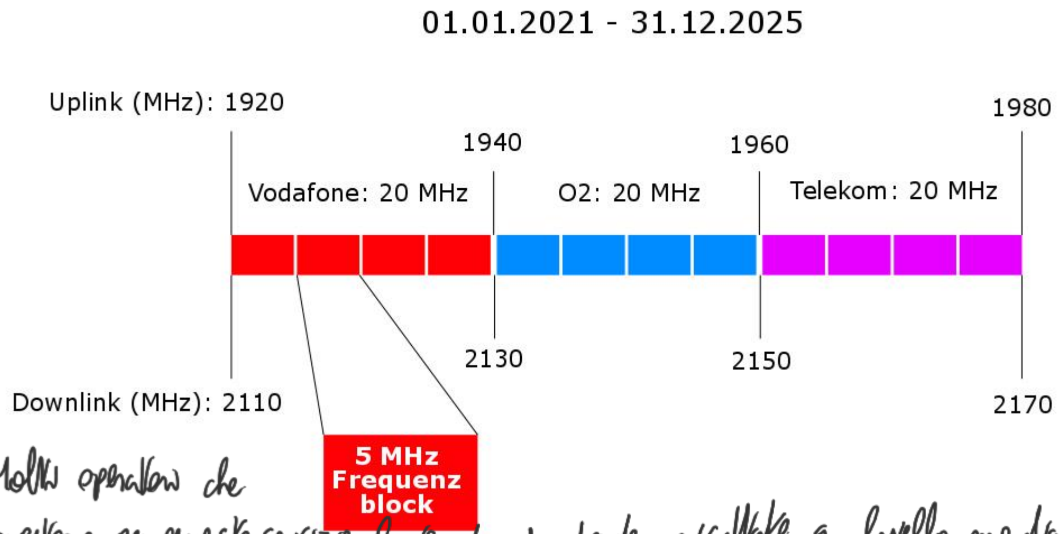
$T_S = N_b T_b \Rightarrow R_S = \frac{R_b}{N_b}$

Motivo è rallentare il segnale, in modo che la banda è più piccola!

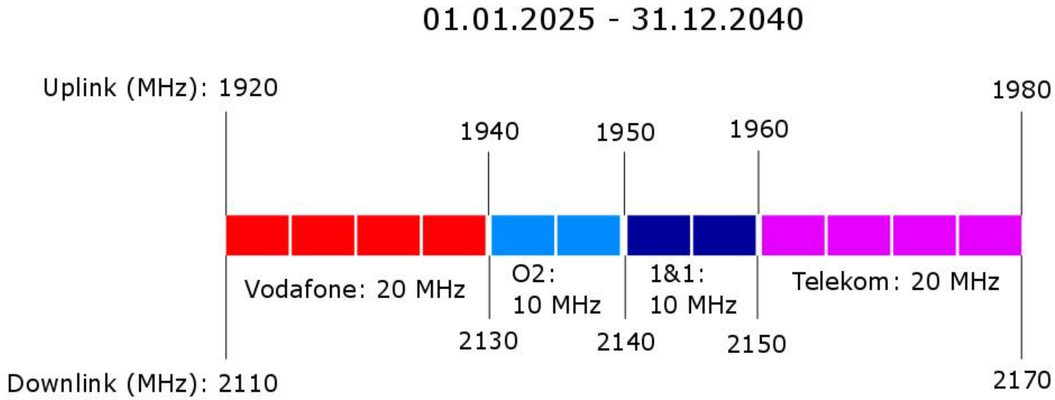
Ora la banda non è il bitrate! Perché la durata dell'impulso è il tempo di simbolo. La banda è il SYMBOL RATE! Più piccola di un fattore N_b.

5G Bandwdith Allocation: a Sample from Germany

- A certain mobile operator is assigned very well defined chunks (blocks) of frequency under the regulation of public bodies (Ministry, Agency etc.)
  - Operators compete for frequencies in an AUCTION
- Total revenue to the Ministry from 5G frequencies auction in Italy (same in Germany): 6 G€
- You strive to squeeze as much bitrate as possible into such a bandwidth to serve as much users as possible with high-bit rate services



Molti operatori che competono per questo servizio, lavorando in bande allocate a livello mondiale.



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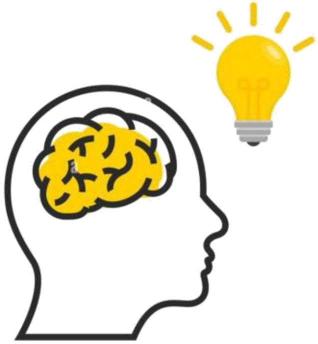
Le bande di frequenza sono messe all'asta!

Data la banda, rialzo il bitrate. Visto che la banda è assegnata, con un segnale binario classico, se ho 20MHz di banda ci schiaffo 20Mbit/s. Ma usando un formato a simboli, posso aumentare il bitrate. Questa cosa si chiama EFFICIENZA SPETTRALE

The idea



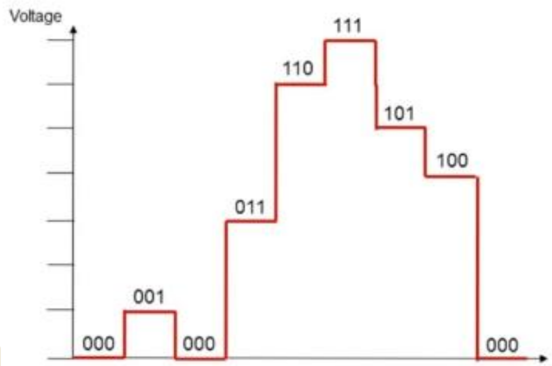
- The bandwidth B is assigned
- $R_s = B$ therefore, the (maximum) symbol rate R_s is assigned as well
- If I want to squeeze as much bitrate as possible into my bandwidth, I have to increase M

Banda = Symbol Rate

Multilevel Technology

$$R_b = \log_2(M) \cdot R_s = \boxed{\log_2(M)} \cdot B$$

Number of bits carried by one symbol



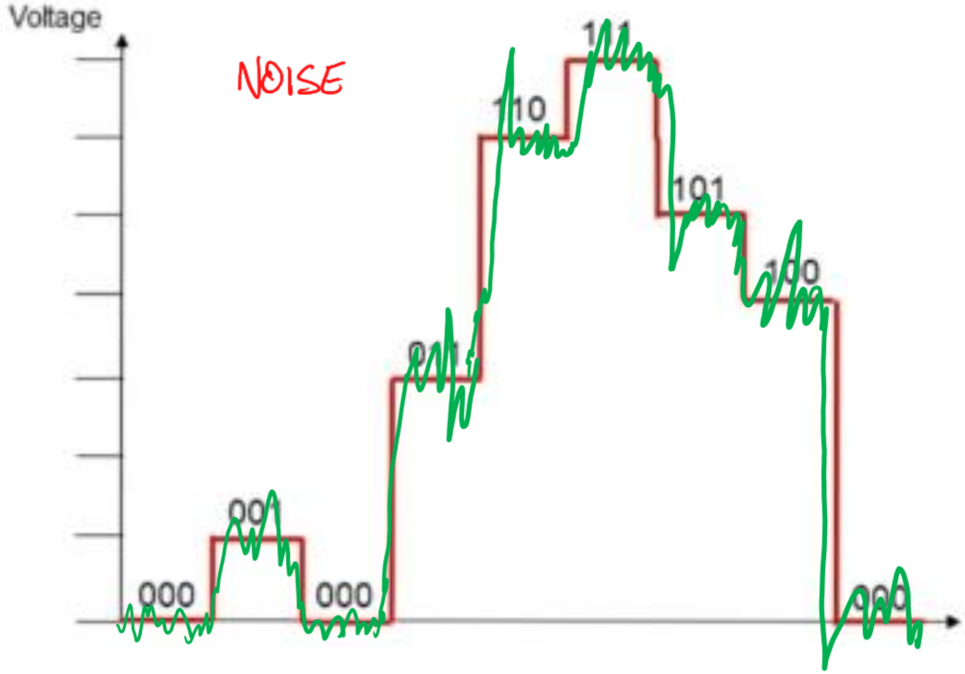
- The bit rate is indeed proportional to the bandwidth, but receives a "boost" from multilevel constellations – **up to *M*=256 levels in 5G** ! Ciò vale anche per la banda dell'ADSL, ma lì la limitazione sta nel filo che fa schifo.

Why not increasing *M* as much as possible?

Limite Rischio:

The actual physical signal received by Bob is corrupted by noise, distortion, interference especially when the distance between Alice & Bob is large and the received signals is weak – we have the green waveform instead of the ideal, theoretical red

M=8 symbol levels → 3 bits/symbol



Then what?

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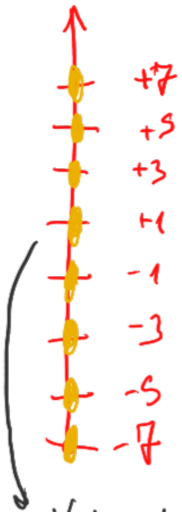
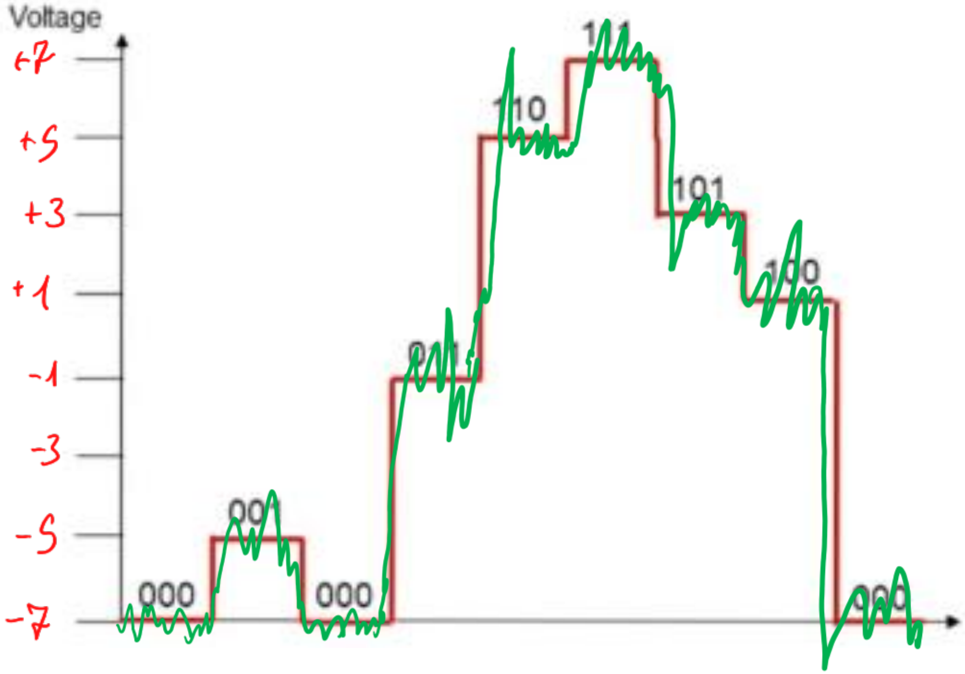
Why not increasing *M* as much as possible?

Effetto del rumore:

M=8 symbol levels → 3 bits/symbol

Look at the skyline from this "side" observation point





Vedo degli intervalli di questo tipo a causa del rumore

Can we compare the "theoretical" (red) view with the "actual" (green) one?

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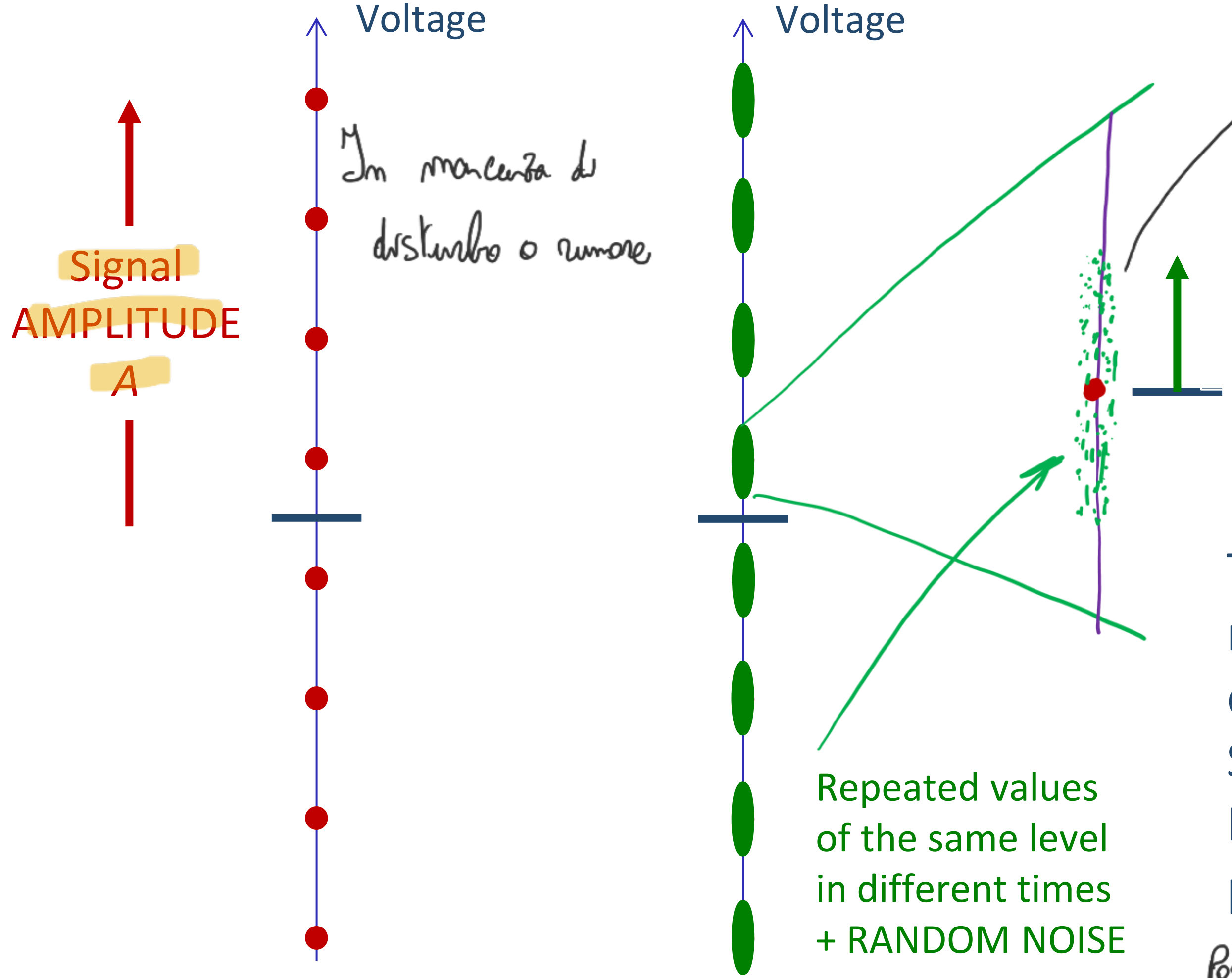
SIGNAL and NOISE (The Skyline)



In mancanza di disturbo o rumore

Valori random più o meno centrati attorno al valore nominale.

Noise STANDARD DEVIATION σ

Come capisco se la presenza di fluttuaz. random dà fastidio?

The "quality" of the received signal is quantified by the SIGNAL TO NOISE RATIO (SNR), proportional to A^2/σ^2

Potenza ($Amp.^2$) del segnale

Rapporto dell[illegible]

Posso confrontare quanto è grande il segnale con quanto è grande il disturbo.

Il riferimento di grandezza di disturbo è da rif. all'ampiezza di segnale.

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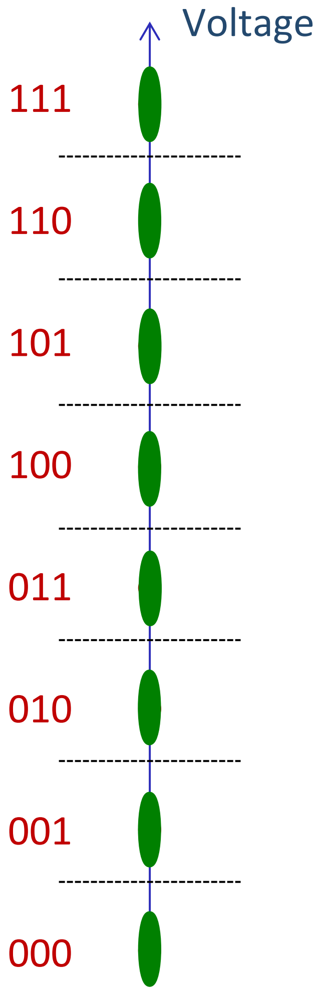
"Quanto segnale sto ricevendo rispetto a quanto disturbo sto ricevendo. Es. ristorante pieno con alto livello di disturbo.

Valori umani che vengono il link attivo é $SNR = 10$. Se $SNR = 100$ il link é ottimo.
In pratica, TV satellitare SNR é debole (36000 km dal satellite). Mi serve una parabola che raccoglie quel tipo di segnale. $SNR \approx 10$ o meno. Per il cellulare il SNR varia a seconda di lontananza dalla torre. Sotto torre $SNR \simeq 1000-10000$. Al chiuso anche $SNR \cong 100-10$

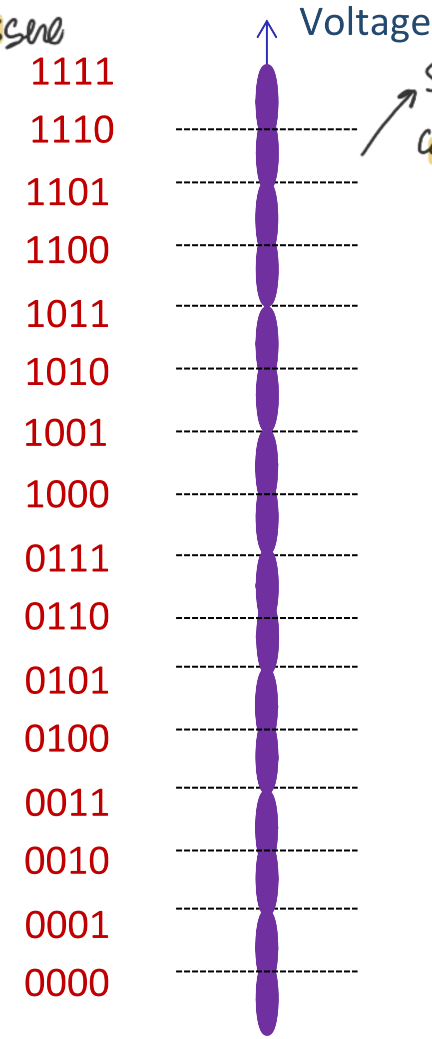
Doubling *M*

Disturbo comincia a essere pericoloso quando: *↓



With this SNR, the "clouds" are still inside their own respective "decision regions" separated by the dashed boundaries

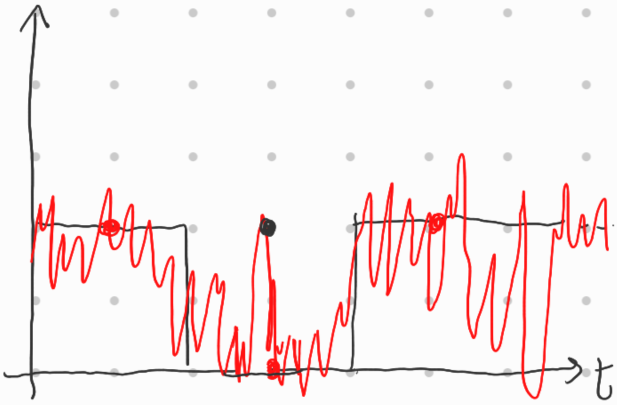
Potremmo aumentare la potenza ma ci sono dei limiti.



Se vedessi una linea continua avrei "line outage"

For the *same* SNR, the "clouds" invade the neighboring decision regions (the levels are *too many*).

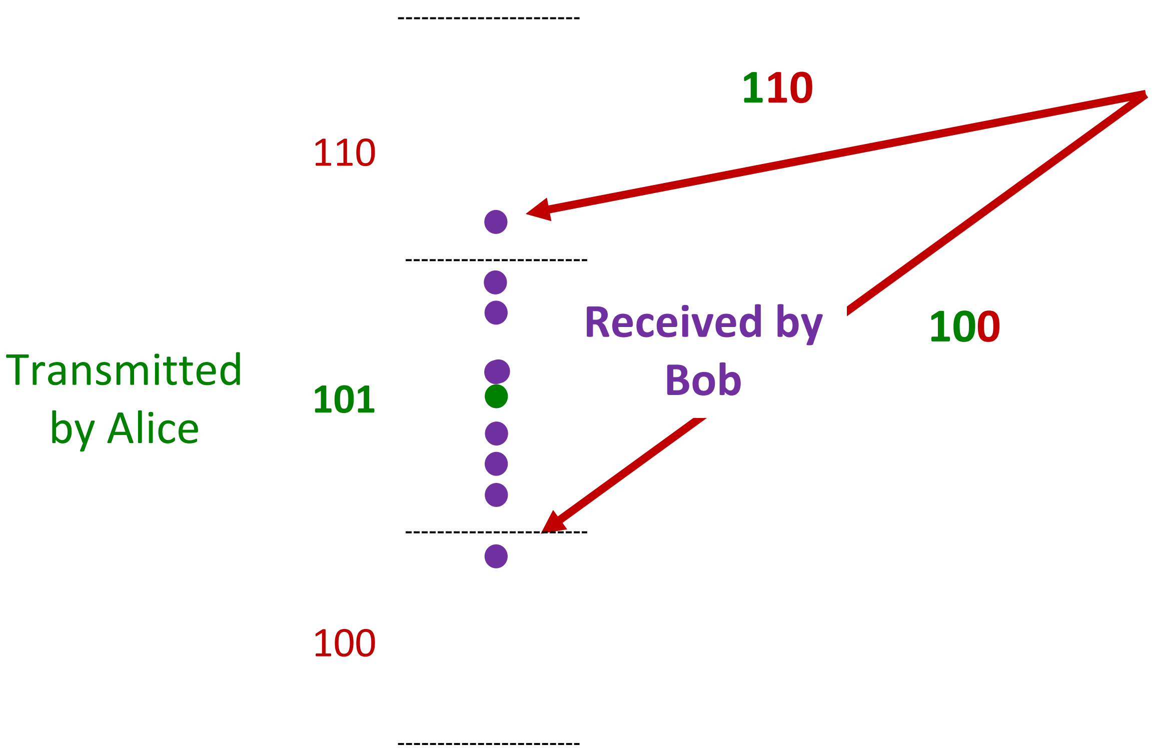
Then what ?



SNR = 4

Se controllando all'istante di reclocking posso incorrere in un errore.
Comincio a sbagliare bit quando ho delle fluttuazioni molto grandi.

The SER / BER



SYMBOL ERRORS lead to BIT ERRORS....

Misura vera e propria della qualità del link.

With a certain frequency, the digital link is affected by bit errors. The BIT ERROR RATE (BER) depends of the SNR on the link

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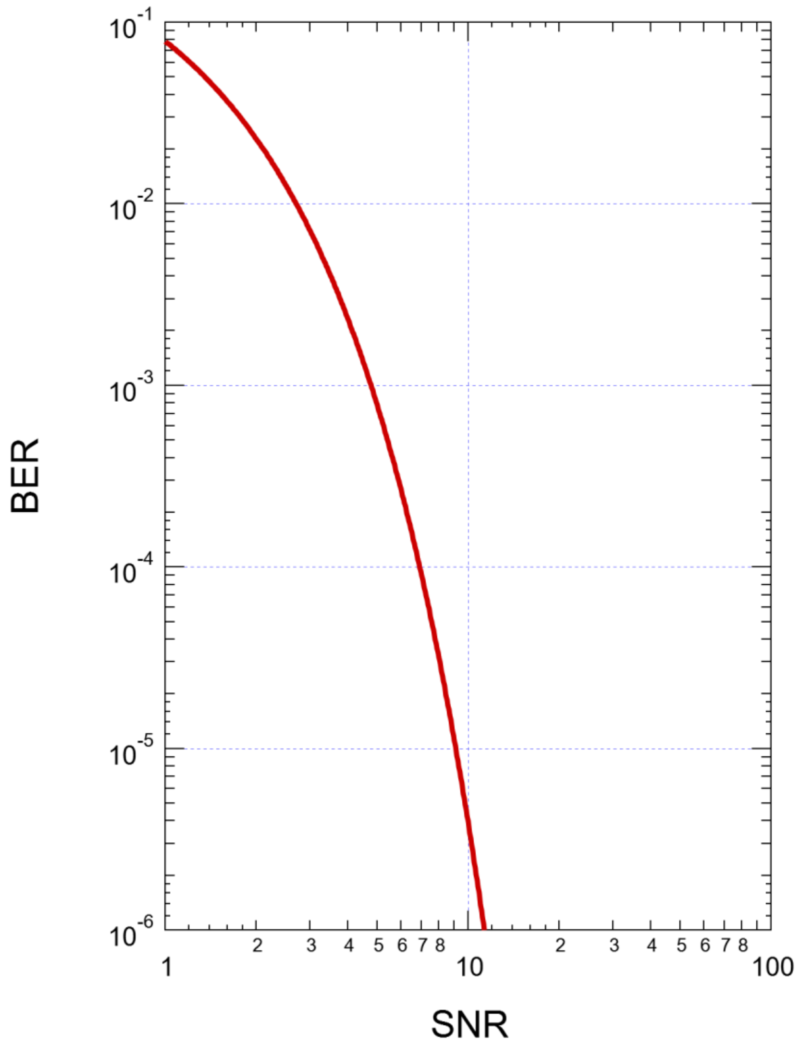
BER vs SNR



LINK OUTAGE se va giù

- The dependence of the BER on the SNR is exponential
- The SNR depends on the application: 100 ÷ 1000 for cellular, 1 ÷ 10 for satellite, up to $10^4 \div 10^5$ for ADSL

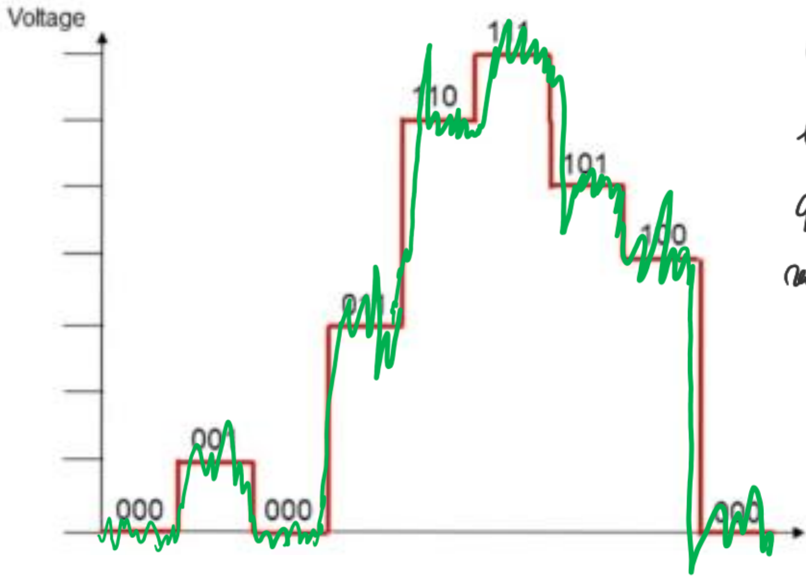
→ l'effetto dipende dall'applicaz

- The effect of the BER strongly depends on the application:
  - 1 error in 1000 bits (10^{-3}) is good for voice, terrible for image/video
  - Commercial TV needs 10^{-10}
  - Bank transfers $<10^{-12}$

BER Sample – 5 wrong pixels in 10^4

Shannon Capacity

Q



Come capisco, data una banda e un rapporto segnale rumore, qual è il massimo bitrate raggiungibile?

The higher the number of levels *M*, the less robust is the signal versus noise. The upper limit of the bit-rate (i.e., of the constellation size) for a certain SNR to attain RELIABLE COMMUNICATIONS is given by Shannon's capacity formula

Banda

è ovvio che esce il rapporto segnale rumore

$$R_b \le B \log_2(1+SNR) \quad (bit/s)$$