Language Based Security (LBT)

LIVENESS ANALYSIS

Chiara Bodei, Gian-Luigi Ferrari

Lecture April, 4 2025

Outline

Liveness Analysis

Liveness: Set Theory Approch

Liveness: Monotone Framework approach

Other analyses

Outline

Liveness Analysis

Liveness: previously mentioned

Examples

Code liveness (aka please compiler do not remove it)

The static analysis decides if a given function is ever called at runtime:

- if "*no*", remove the function from the code
- if "*yes*", don't do anything
- but the "*no*" answer *must* always be correct if given

False positives are ok (dead code left in)
False negatives are not ok (removing live code)

Liveness analysis

When a variable is still “needed”,
for a future use?
What is its live range?

Determine the possible live variables of a given program

Track live variables at each point of your program

Liveness analysis

When a variable is still “needed”,
for a future use?
What is its live range?

Determine the possible live variables of a given program

Applications

Compiler optimization: dead code elimination
Register Allocation

If you have dead variables, you can discard them!

Liveness analysis: uses

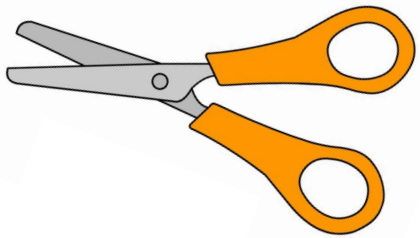
Register allocation

If variable x is live in a basic block, it is a potential candidate for register allocation

Two variables can share the same register if they are not live at the same time

Dead code elimination

If variable x is dead (not live) after an assignment x = …, then the assignment is redundant and can be deleted as dead code



Example 1

Are there any dead variables?

```
x = 2;
x = 5;
<< use x >>
```

Live variables: something important that you cannot remove without impacting overall behavior of program.

First assignment is useless, because first value is not used before update.

Example1

Are there any dead variables?

```
x = 2;
x = 5;
<< use x >>
```

The value of x is not read (used) before the following assignment and therefore, x is dead

Example 2

Are there any dead variables?

```
x = 2;
if (y > 0) {
    x = y+5;
} else {
    x = y+10;
}
<< use x >>
```

First assignment of x is not useful because we rewrite x regardless

Example 2

Are there any dead variables?

```
x = 2;
if (y > 0) {
    x = y+5;
} else {
    x = y+10;
}
<< use x >>
```

The value of x is not read (used) before the following assignment and therefore, x is dead

Example 3

Are there any dead variables?

```
x = 2;
if (y > 50) {
   x = 1;   // x def
} else {
   x = x+2; // x read
}
<< use x >>
```

The value of x is not read (used) before the following assignment and therefore, x is dead

Example 3

Are there any dead variables?

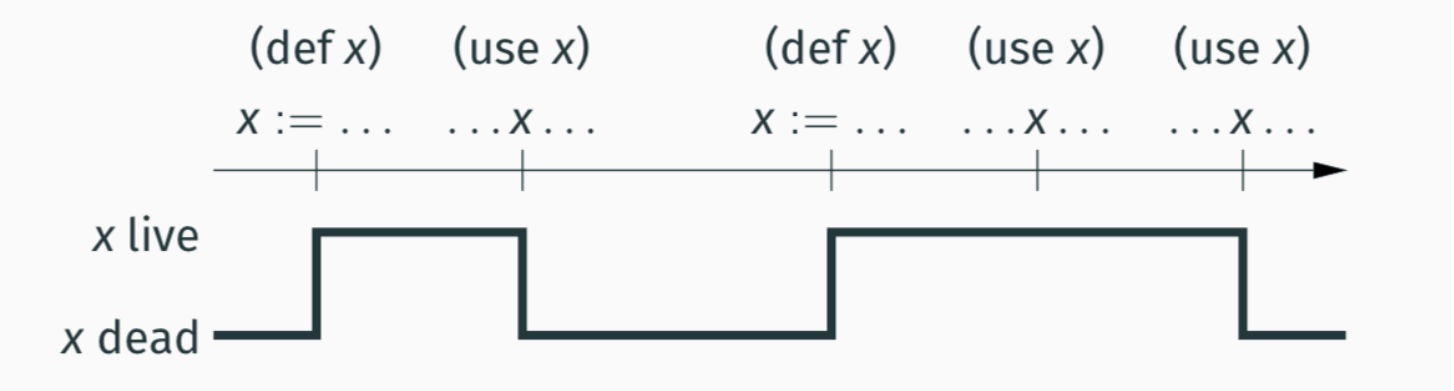
```
x = 2;
if (y > 50) {
   x = 1;   // x def
} else {
   x = x+2; // x read
}
<< use x >>
```

The value of x is read (used) read before one of the following assignments and therefore, x is live

Semantic Liveness

A variable x is “**semantically**”

- **live** at a program point p IFF there exists an **execution path** where its value is read (used) **later** with no writes (re-definitions) to x in between
- **dead** otherwise: its value has **no impact** on the rest of the program execution!

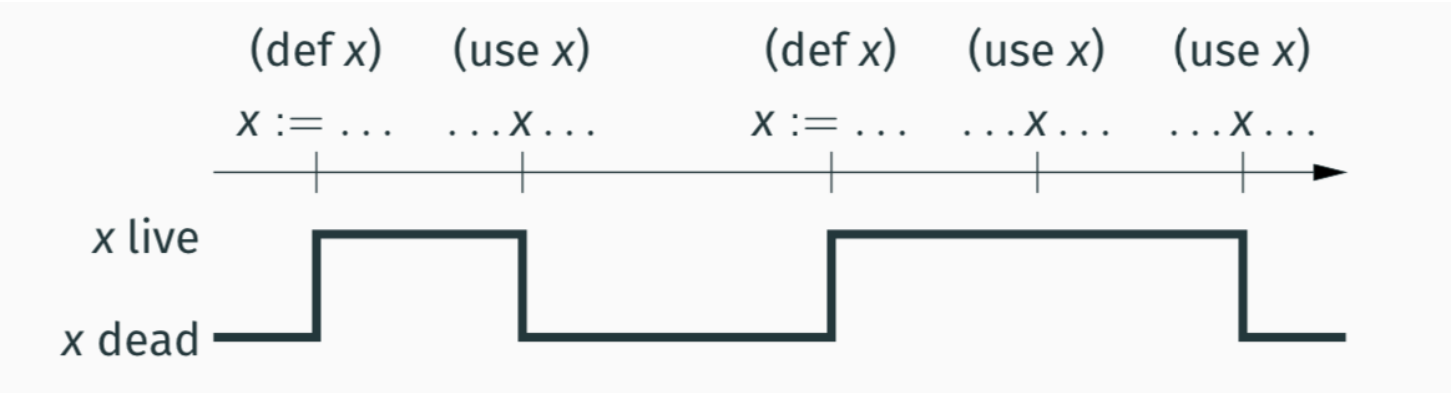


Semantic Liveness

A variable x is "**semantically**"

- **live** at a program point p IFF there exists an **on path** where its value is read (used) **later** with no writes (re-def) to x in between
- **dead** otherwise: its value has **no impact** on the rest of the program execution!

UNDECIDABLE!



?

This is not a trivial property!
Undecidable. Try to approximate with static analysis.

Syntactic liveness

A variable x is “**syntactically**”

- **live** at a program point p IFF if there is a **path** to the exit of the **Control Flow Graph (CFG)** along which its value may be used before it is redefined
- **dead** otherwise

not in general to the execution

Syntactic liveness

A variable x is “**syntactically**”

- **live** at a program point p IFF if there is a **path** to the exit of the **Control Flow Graph (CFG)** along which its value may be used before it is redefined
- **dead** otherwise

Syntactic liveness

- is **decidable** (because it is concerned with properties of the syntactic structure of the program)
- is a **computable approximation** of semantic liveness

How to compute live variables

Naïve solution

Analyse all paths from each use of each variable Look for each variables up and down to see if they are live

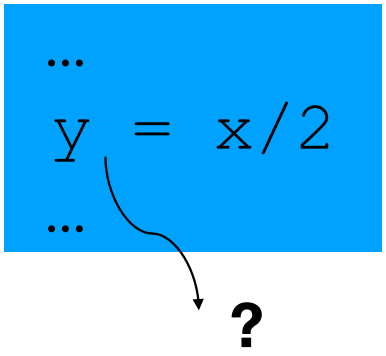
Inefficient: it explores the same paths many times

Liveness static analysis

We focus on a **static analysis** called **liveness analysis**

Liveness is a **data-flow property** of variables

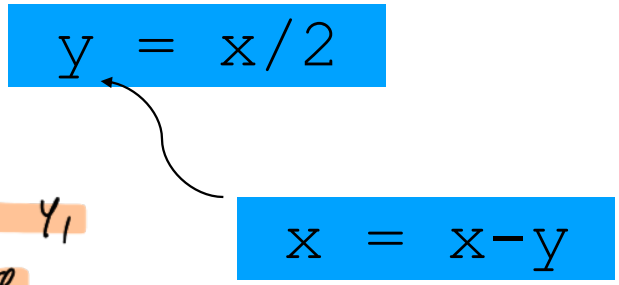
"Is the value of this variable needed **later**?"



Liveness static analysis (cont.)

We focus on a **static analysis** called **liveness analysis**

Liveness is a **data-flow property** of variables:
"Is the value of this variable needed **later**?"



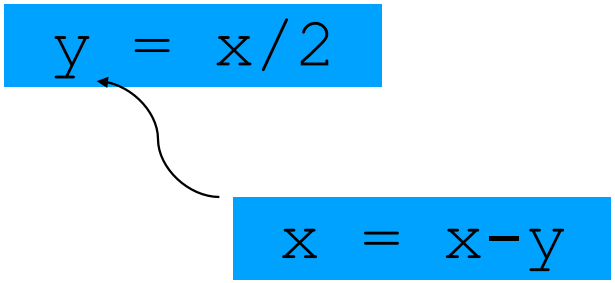
If you find a path in which in CFG you use y, you can say y is alive.

Suppose our analysis associate label for each node with the set of live variables at that point. What can you do? What is important is see to know that something will be used later. You do a backward analysis because you are interested in what comes afterward. You cannot do that by only knowing past.

Liveness static analysis (cont.)

We focus on a **static analysis** called **liveness analysis**

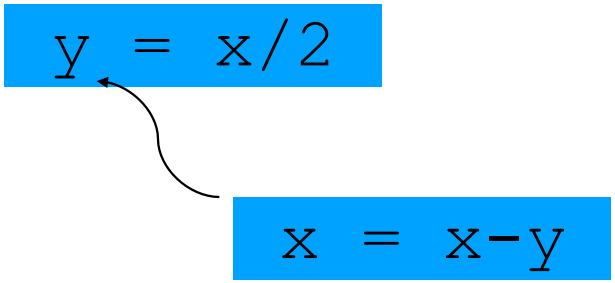
Liveness is a **data-flow property** of variables:
"Is the value of this variable needed **later**?"



Liveness static analysis (cont.)

We focus on a **static analysis** called **liveness analysis**

Liveness is a **data-flow property** of variables:
"Is the value of this variable needed **later**?"

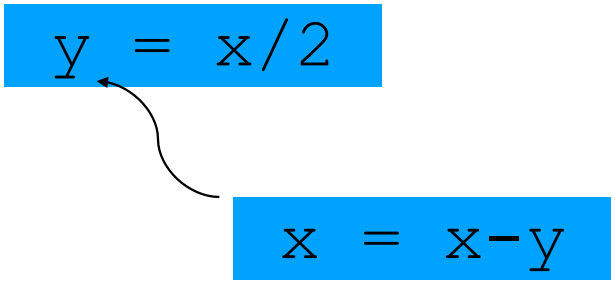


- In which direction the information propagate?

Liveness static analysis (cont.)

We focus on a **static analysis** called **liveness analysis**

Liveness is a **data-flow property** of variables:
"Is the value of this variable needed **later**?"



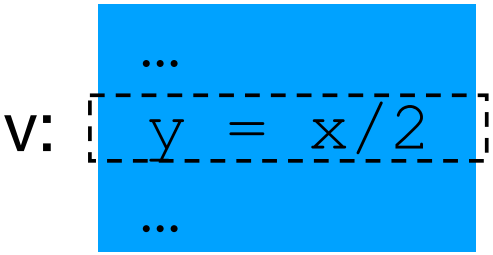
- In which direction the information propagate? **Backward**

Liveness static analysis

- At each instruction, each variable in the program is either **live** or **dead**
- Each instruction, or node of the Control Flow Graph (CFG), has an associated set of **live variables**

Idea

- An instruction makes a variable
  - **live** when it references (uses) it
  - **dead** when it defines (assigns to) it



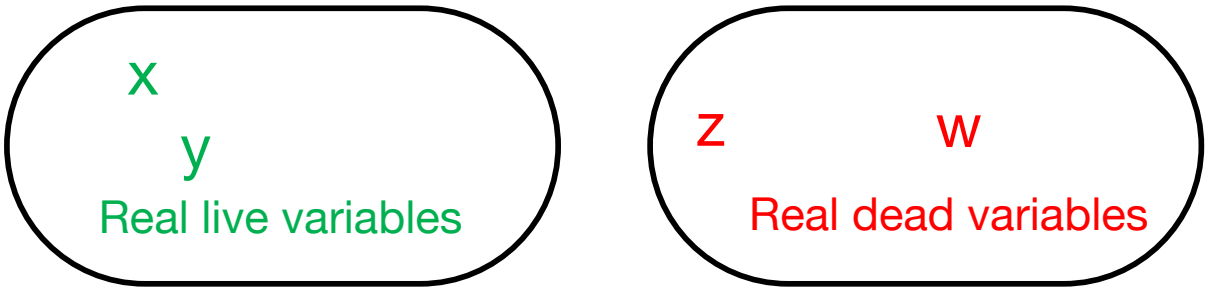
- Variables **live** at v: **[[v]] = {x}**
- Variables **dead** at v: **{y}**

Liveness analysis

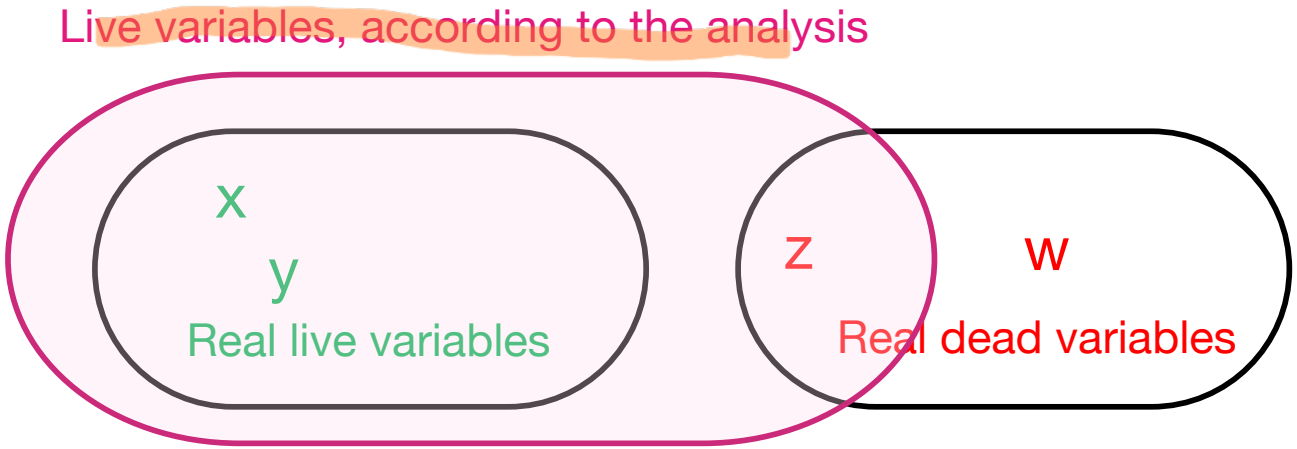
- We want a **conservative analysis**: it must only answer "dead" if the variable is really dead
- Optimization: no need to store the values of dead variables
- It is possible to compute **a safe over-approximation** of liveness
- It is a particular case of the previously mentioned code liveness

- Why is this not a perfect analysis if CFG can show all executive paths?
  Because you lose something with over approximation

Conservative analysis

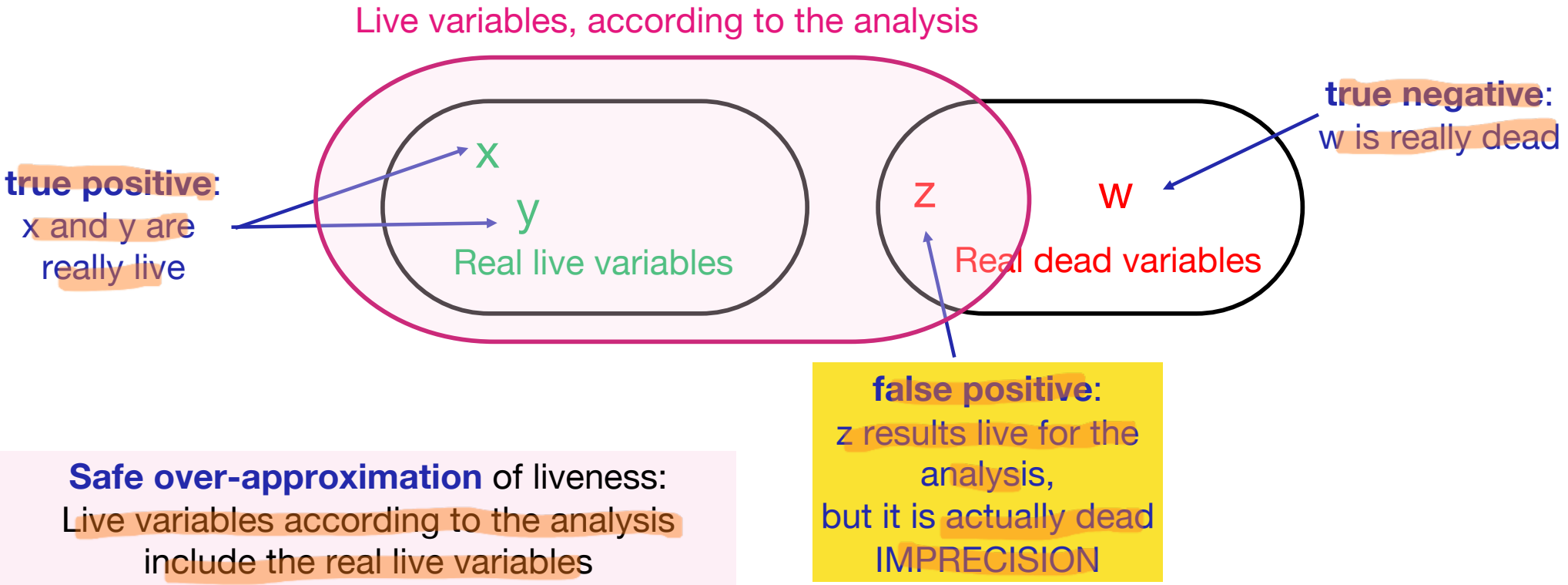


Conservative analysis



Safe over-approximation of liveness

Conservative analysis



Safe over-approximation of liveness:
Live variables according to the analysis
include the real live variables

Outline

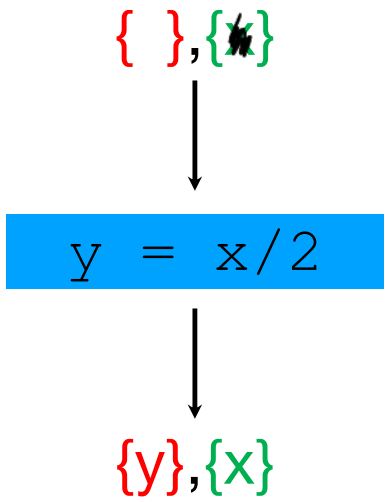
Liveness Analysis

Liveness: Set Theory Approch

Liveness analysis: set theory approach

(Also called Bit Vector Framework)

Intuition



An instruction makes a variable:

- live when it references /uses it
- dead when it defines/assigns to it

Liveness information at each node is updated by:

- adding any variables that become live and
- removing the variables that become dead
- we say that y = x/2 kills y and generates x

y has been redefined so it is dead.

Between reassignment and first use variables are dead [?] We'll see

Liveness analysis: set theory approach

How the statement affects the analysis state

Analysis state = live variables

Sets as analysis information

Key questions:

- In which **direction** the information propagate?

Liveness analysis: set theory approach

How the statement affects the analysis state

Analysis state = live variables

Sets as analysis information

Key questions:

- In which **direction** the information propagate? Backward or forward?

Liveness analysis: set theory approach

How the statement affects the analysis state

Analysis state = live variables

Sets as analysis information

Key questions:

- In which **direction** the information propagate? Backward or forward?
- In which way information coming from other nodes can be **merged**?

Liveness analysis: set theory approach

How the statement affects the analysis state

Analysis state = live variables

Sets as analysis information

Key questions:

- In which **direction** the information propagate? Backward or forward?
- In which way information coming from other nodes can be **merged**? Using union or intersection?

Set theory approach: local data flow

Two functions are used for **local data flow** properties

- gen computes live variables generated by a statement
- kill computes live variables killed by a statement (stop the propagation)

A statement generates a live variable x

- if the statement uses x, e.g.,

  gen(`y = x/2`) = {x}

A statement kills a live variable x

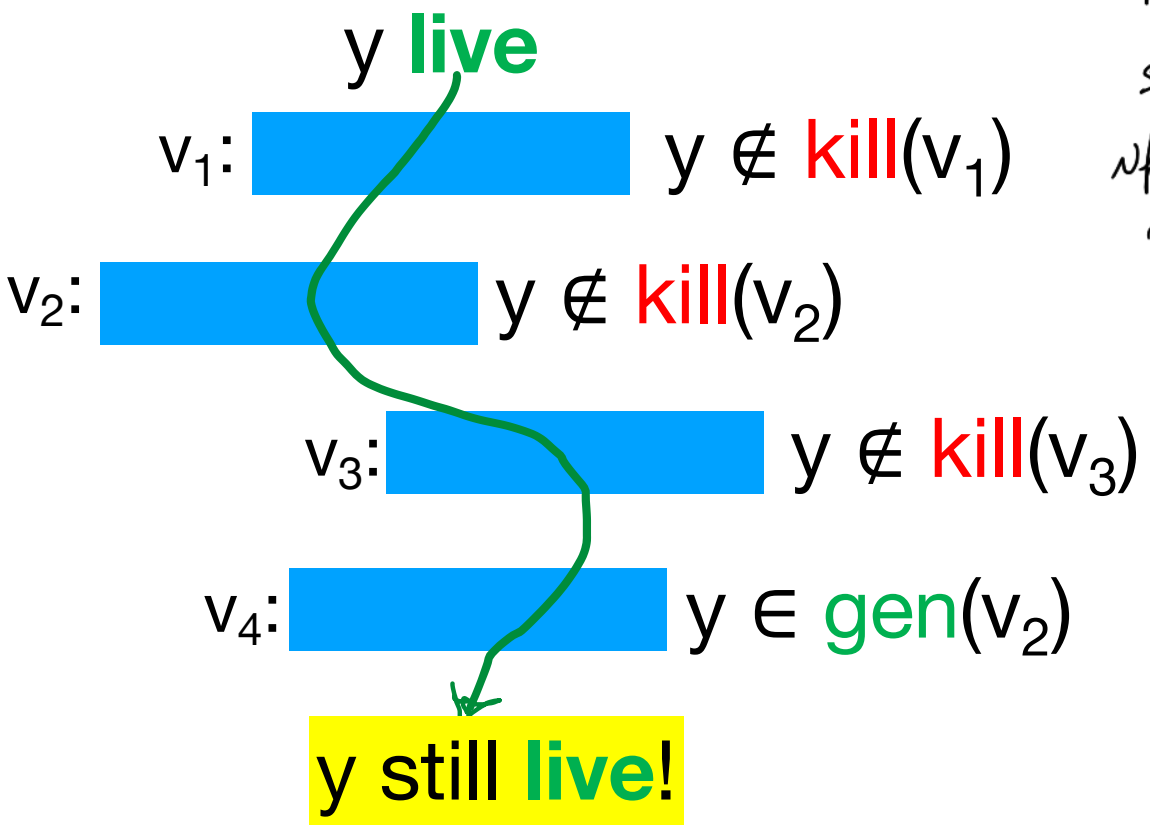
- if the statement defines x, e.g.,

  kill(`y = x/2`) = {y}

An instruction makes a variable:

- live when it references /uses it
- dead when it defines/assigns to it

Liveness



Rough idea:
start with y live and if y has not been redefined, it is still alive

Set theory approach: gen and kill definitions

- gen computes live variables generated by a statement
- kill computes live variables killed by a statement (stop the propagation)

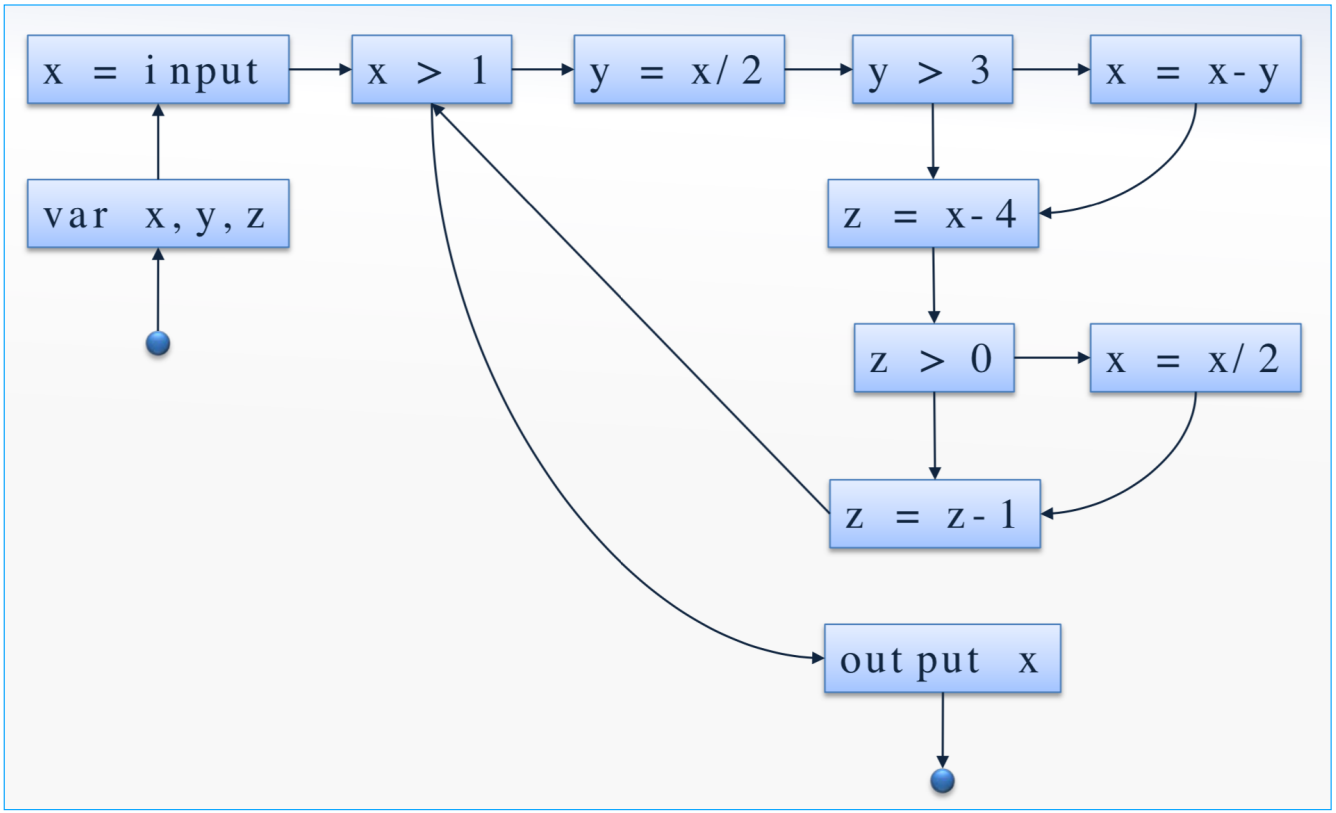
gen ⟦var x1,...,xn⟧= ∅
gen ⟦entry⟧ = ⟦exit⟧= ∅
gen ⟦if (E)⟧= ⟦while (E)⟧ = ⟦output E⟧= U vars(E) (variables occurring in E)
gen ⟦x = E⟧= vars(E) generate every variable included in expression

kill ⟦var x1,...,xn⟧= {x1, ..., xn}
kill ⟦entry⟧ = ⟦exit⟧= ∅
kill ⟦if (E)⟧= ⟦while (E)⟧ = ⟦output E⟧= ∅
kill ⟦x = E⟧= {x}

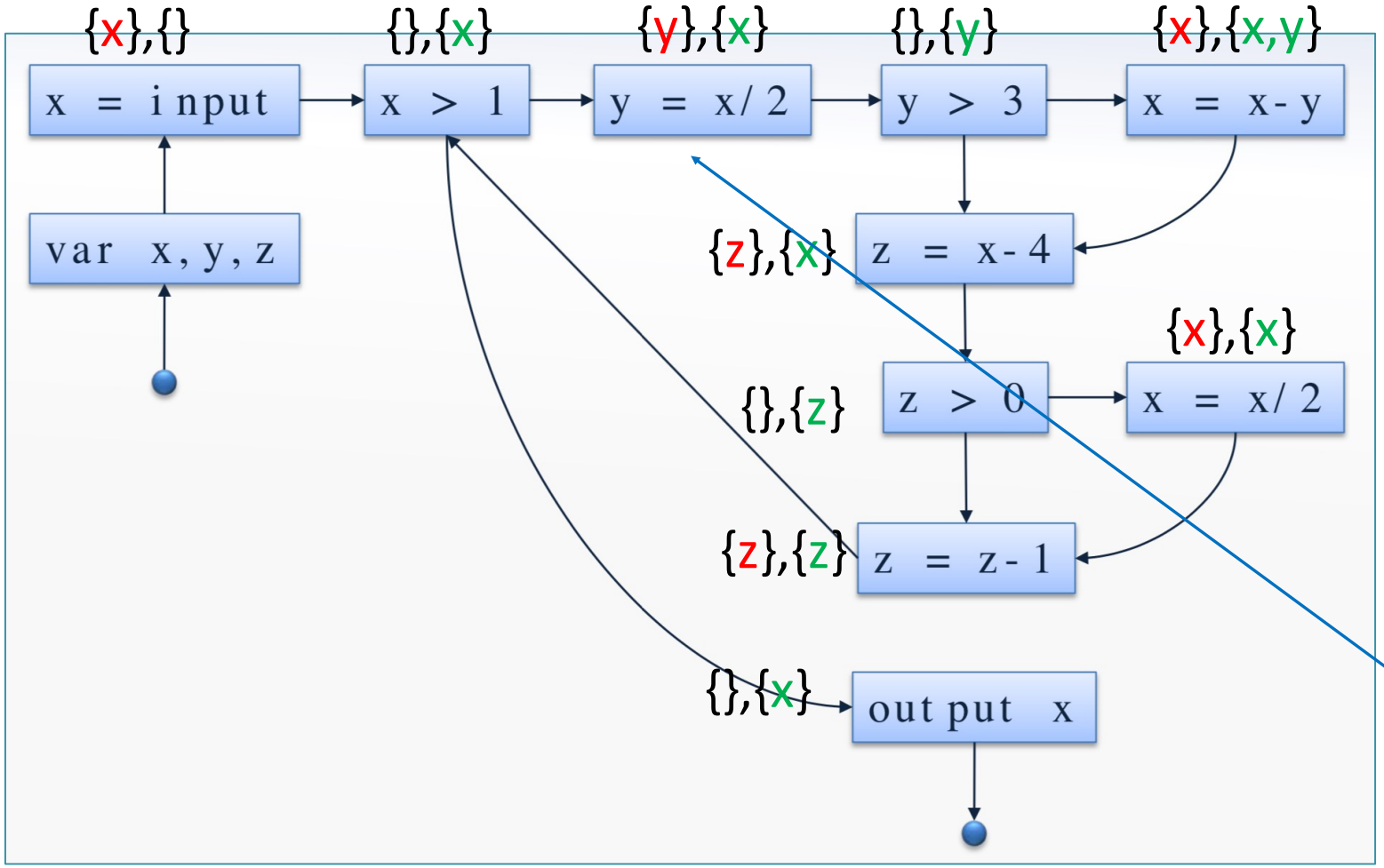
If you do y=y+1, in this definition
gen [y=y+1] = {y}
kill [y=y+1] = {y}

Our running example

```
var x,y,z;
x = input;
while (x>1) {
y = x/2;
if (y>3) x = x-y;
z = x-4;
if (z>0) x = x/2;
z = z-1;
}
output x;
```



Liveness analysis: annotated CFG



An instruction makes a variable:

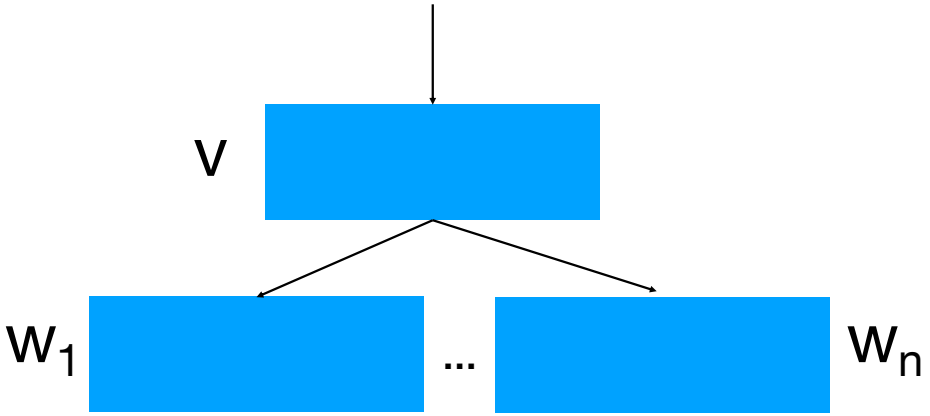
- live when it references /uses it
- dead when it defines/assigns to it
- y = x/2 kills y and generates x

Set theory approach: global data flow

Global data flow properties

We need to know which are the sets of live variables at the entry and at the exit of a node v and how they flow through the graph

- How do we combine information coming from other nodes?



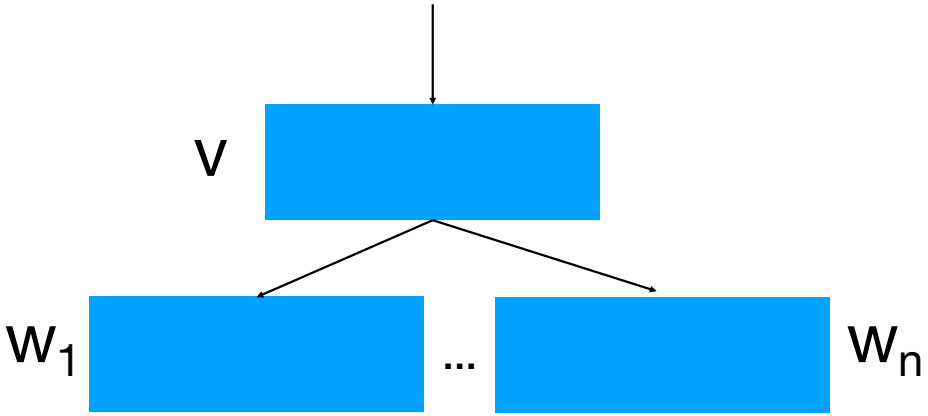
↳ Note that this is a backwards flow.

Take the Union: if at least in one path there is something, you cannot forget it.

Set theory approach: global data flow

Global data flow properties
We need to know which are the sets of live variables at the entry and at the exit of a node v and how they flow through the graph

- How do we combine information coming from other nodes?

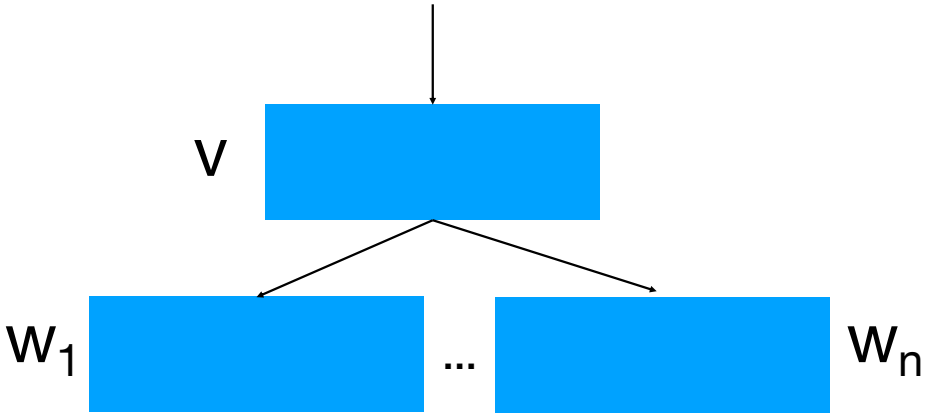


Key questions:
- In which **direction** the information propagate?

Set theory approach: global data flow

Global data flow properties
We need to know which are the sets of live variables at the entry and at the exit of a node v and how they flow through the graph

- How do we combine information coming from other nodes?

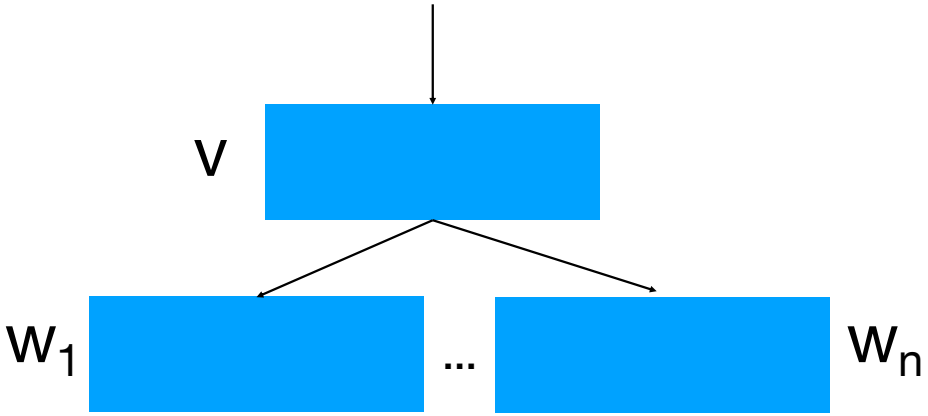


Key questions:
- In which **direction** the information propagate?
  Backward

Set theory approach: global data flow

Global data flow properties
We need to know which are the sets of live variables at the entry and at the exit of a node v and how they flow through the graph

- How do we combine information coming from other nodes?



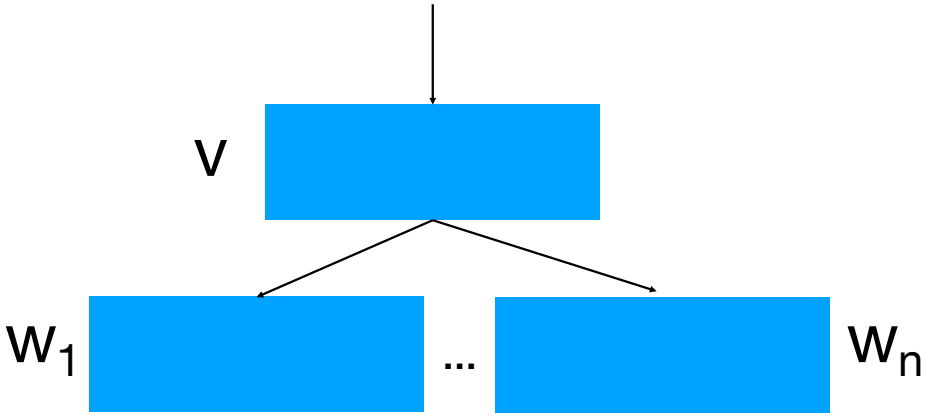
Key questions:
- In which **direction** the information propagate?
  Backward

- In which way information coming from other nodes can be **merged**?

Set theory approach: global data flow

Global data flow properties
We need to know which are the sets of live variables at the entry and at the exit of a node v and how they flow through the graph

- How do we combine information coming from other nodes?



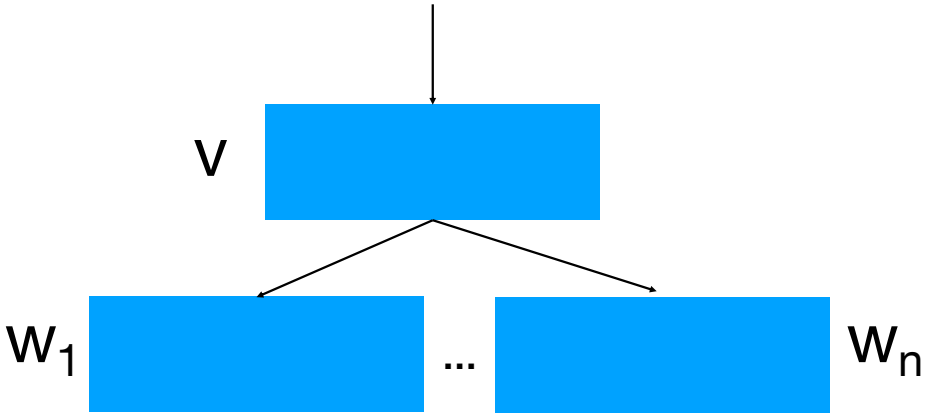
Key questions:
- In which **direction** the information propagate?
Backward

- In which way information coming from other nodes can be **merged**?
Union

Set theory approach: global data flow

Global data flow properties
We need to know which are the sets of live variables at the entry and at the exit of a node v and how they flow through the graph

- How do we combine information coming from other nodes?



Key questions:
- In which **direction** the information propagate?
  Backward
- In which way information coming from other nodes can be **merged**?
  Union

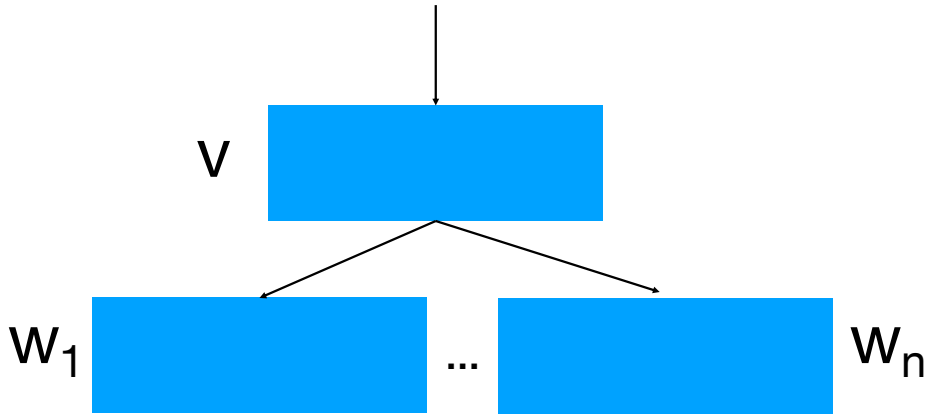
- This is because a variable is live if it **may** be used **later** (at least in a path)

↳ there is at least one path in which we use it!

Set theory approach: flow equations

Given outgoing variables, compute ingoing ones

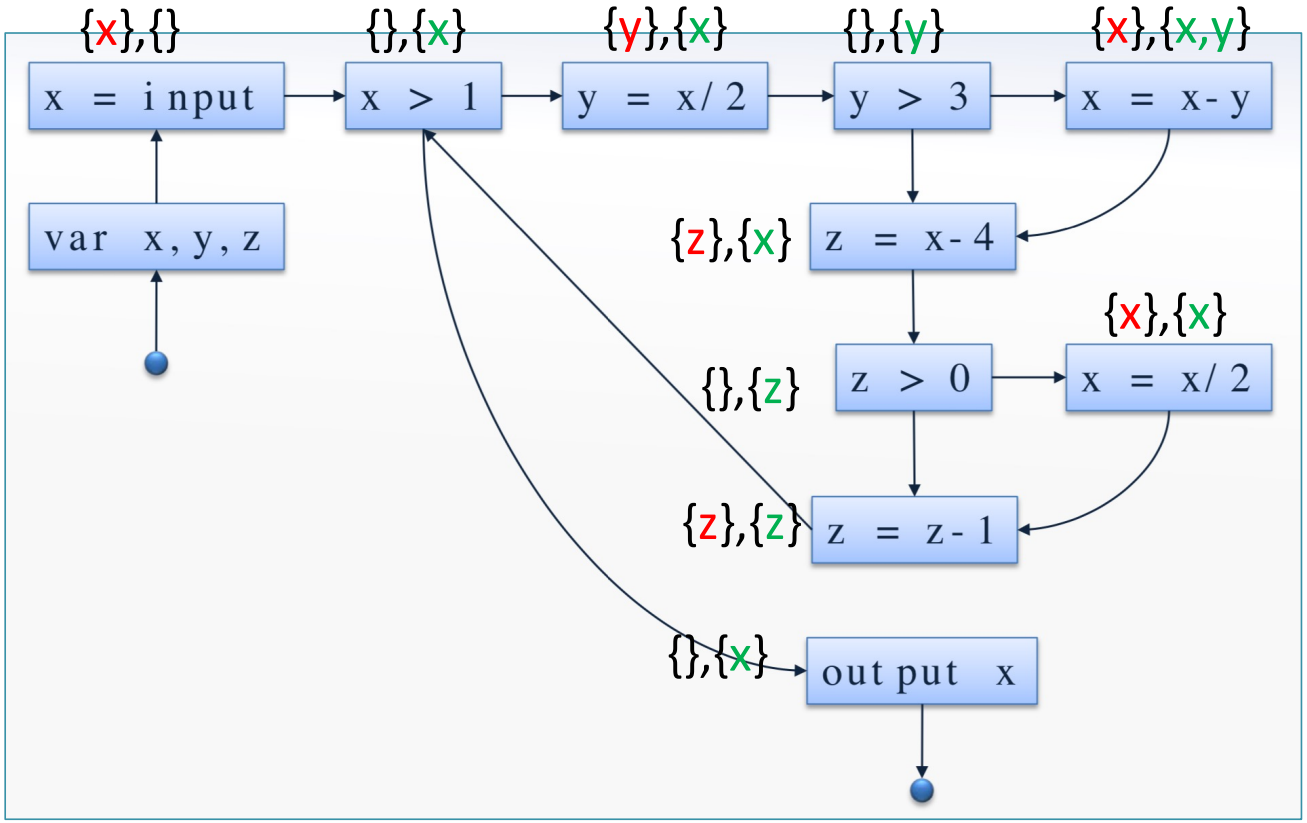
- $Out(v) = \bigcup_{w \in succ(v)} In(w)$
- $In(v) = Gen\ (v) \cup (Out\ (v) \setminus Kill\ (v))$



Liveness analysis: set theory approach

- Initially: **no** live variables (∅)
- **Backward** analysis: propagate live variables in the opposite direction of control flow
- Compute ingoing and outgoing live variables for each node v
- When control flow splits, propagate live variables both ways
- **May** analysis: when control flows merge, joins with union the incoming live variables
- Solve the equations, iterating until fix-point is reached

Liveness analysis: annotated CFG

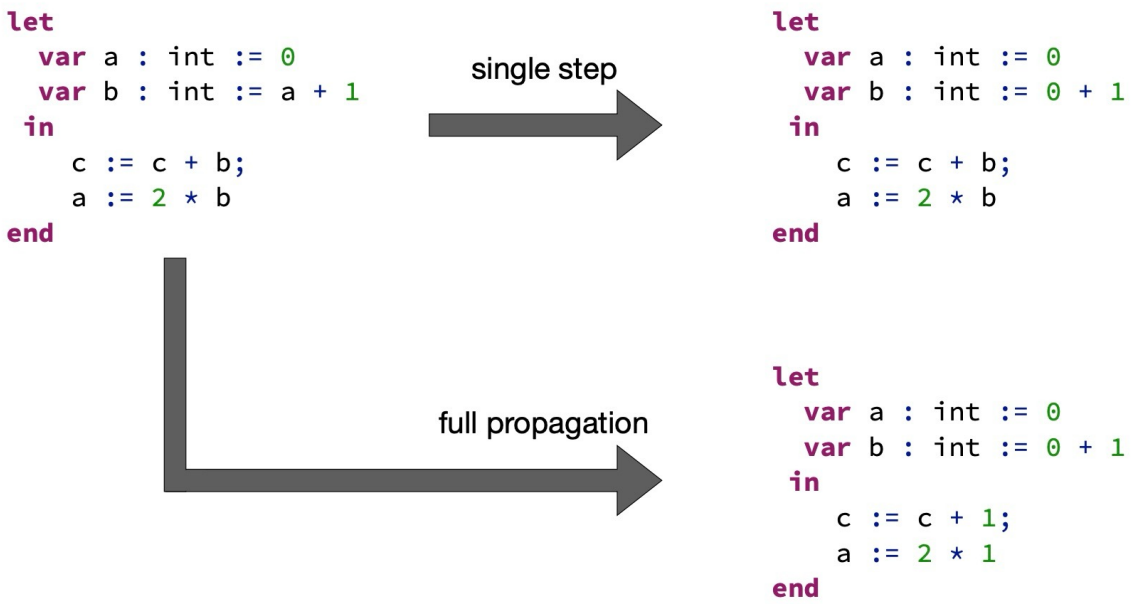


OUT([[var x,y,z]])= IN([[var x,y,z]])= Ø
OUT[[x=input]] = IN[[x>1]]
… (TO BE CONTINUED)

Another Kind of DFA

Beyond states: Constant Propagation analysis

Goal: determine for each node whether or not a variable has a **constant** value whenever execution reaches that point (so we don't have to recompute it)
Optimisation; replacing uses of a variable with its value if that value is guaranteed to be constant



Beyond states: Constant Propagation analysis

```
let
  var a : int := 0
  var b : int := a + 1
 in
    c := c + b;
    a := 2 * b
end
```

single step

```
let
  var a : int := 0
  var b : int := 0 + 1
 in
    c := c + b;
    a := 2 * b
end
```

full propagation

```
let
  var a : int := 0
  var b : int := 0 + 1
 in
    c := c + 1;
    a := 2 * 1
end
```

Goal: determine for each node whether or not a variable has a constant value whenever execution reaches that point

```
let
  var a : int := 0
  var b : int := a + 1
 in
    c := c + b;
    a := 2 * b
end
```

$a \mapsto 0$

$a \mapsto 0, b \mapsto 1$

$a \mapsto 0, b \mapsto 1, c \mapsto ?$

$a \mapsto 2, b \mapsto 1, c \mapsto ?$

Kill/gen do not work here: we need the previous information to compute the current one

Can we use a set for this map? Keys map to single values, so no

But what if we keep multiple values? Analysing loops may not terminate

Beyond states: Constant Propagation analysis

But what if we keep multiple values? Analysing loops may not terminate
The (possibly) infinite kill set makes this impractical

There is impracticality and we need to use monotone FW

```
let
  var a : int := 0
  var b : int := a + 1
 in
    while y > a + b do
        a := a + 1
end
```

a ↦ 0

a ↦ 0; b ↦ 1

a ↦ 0; b ↦ 1 a ↦ 0,1; b ↦ 1

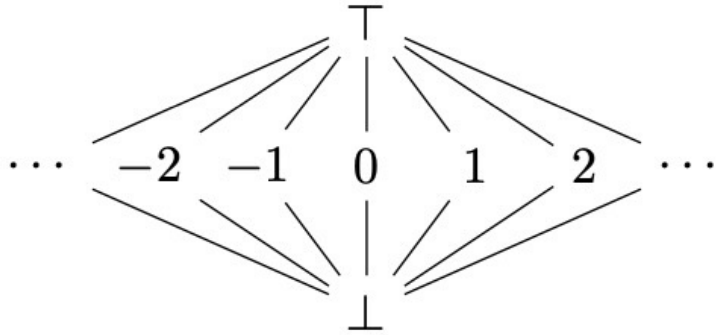
a ↦ 0,1; b ↦ 1 a ↦ 0,1,2; b ↦ 1

Monotone framework to the rescue!

Constant Propagation analysis: Monotone framework

It is a **forward** flow problem

Similar to sign analysis



$$a \,\widehat{op}\, b = \begin{cases} \bot & \text{if } a = \bot \text{ or } b = \bot \\ \top & \text{otherwise, if } a = \top \text{ or } b = \top \\ a \; op \; b & \text{if } a \in \mathbb{Z} \text{ and } b \in \mathbb{Z} \end{cases}$$

…

Outline

Liveness Analysis

Liveness: Set Theory Approch

Liveness: Monotone Framework approach

Dataflow Analysis

Dataflow analysis is a static program analysis technique used to infer properties about program variables (or other program elements) at various points in the control flow

How data is related to each other

The reasoning is about data, and dependencies between data

It systematically tracks how information "flows" through the program based on its structure, without actually executing the code

Dataflow Analysis

Dataflow analysis is a static program analysis technique used to infer properties about program variables (or other program elements) at various points in the control flow

The reasoning is about data, and dependencies between data

It systematically tracks how information "flows" through the program based on its structure, without actually executing the code

Monotone framework: combination of

- a **complete lattice** and
- a **space of monotone functions**

} This is enough to find solutions for our properties

Monotone Framework: Core Concepts

Starting Point

- **CFG** (Control Flow Graph) of the program: analysis information is propagated along CFG edges
- A complete **lattice** of finite height, that describes abstract information to infer at each node (it may be parameterized by the program)

Monotone Framework: Core Concepts

Starting Point

- **CFG** (Control Flow Graph) of the program: analysis information is propagated along CFG edges
- A complete **lattice** of finite height, that describes abstract information to infer at each node (it may be parameterized by the program)

Constraint Variables

- Each CFG node v (representing each program point) gets a variable [[v]]
- Values range over lattice elements

Monotone Framework: Core Concepts

Starting Point

- **CFG** (Control Flow Graph) of the program: analysis information is propagated along CFG edges
- A complete **lattice** of finite height, that describes abstract information to infer at each node (it may be parameterized by the program)

Constraint Variables

- Each CFG node v (representing each program point) gets a variable [[v]]
- Values range over lattice elements

Dataflow Constraints: generation and solving

- Constraints relate values of a node to its neighbors, defining how each statement impacts the analysis state
- Based on language constructs represented in the CFG

Monotone Framework: Core Concepts

Starting Point

- **CFG** (Control Flow Graph) of the program: analysis information is propagated along CFG edges
- A complete **lattice** of finite height, that describes abstract information to infer at each node (it may be parameterized by the program)

Constraint Variables

- Each CFG node v (representing each program point) gets a variable [[v]]
- Values range over lattice elements

Dataflow Constraints: generation and solving

- Constraints relate values of a node to its neighbors, defining how each statement impacts the analysis state
- Based on language constructs represented in the CFG

Fixed-Point Computation

- If constraints are equations with monotone RHS, we can use fixed-point algorithm that iterate until fix-point is reached: this yields the least solution (most precise)

Monotone Framework approach

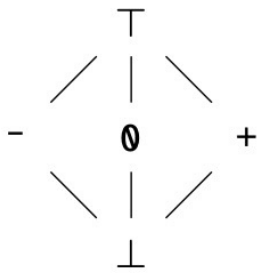
For a given program to be analyzed, a monotone framework can be instantiated by: specifying:

- the CFG and
- the rules for assigning dataflow constraints to its nodes

An analysis is **sound** if all solutions to the constraints correspond to correct information about the program

Sign Analysis in the Monotone Framework

- Lattice



- *States* = *Vars* → *Sign*
- For each CFG node v we assign a constraint variable [[v]] denoting an abstract state that gives the sign values for all variables at the program point immediately after v
- The dataflow constraints model the effects of program execution on the abstract states
- Auxiliary function that combines the abstract states from the predecessors of a node v:

$$JOIN(v) = \bigsqcup_{w \in pred(v)} [\![w]\!]$$

← set of all predecessors of node **v**

X = E [[v]] = JOIN (v)[X ↦ eval(JOIN (v), E)]

eval computes the sign of the expression

Equations

A program with n CFG nodes, $v_1,\ldots, v_n$, is thus represented by n equations:

In general we need a constraint for each node

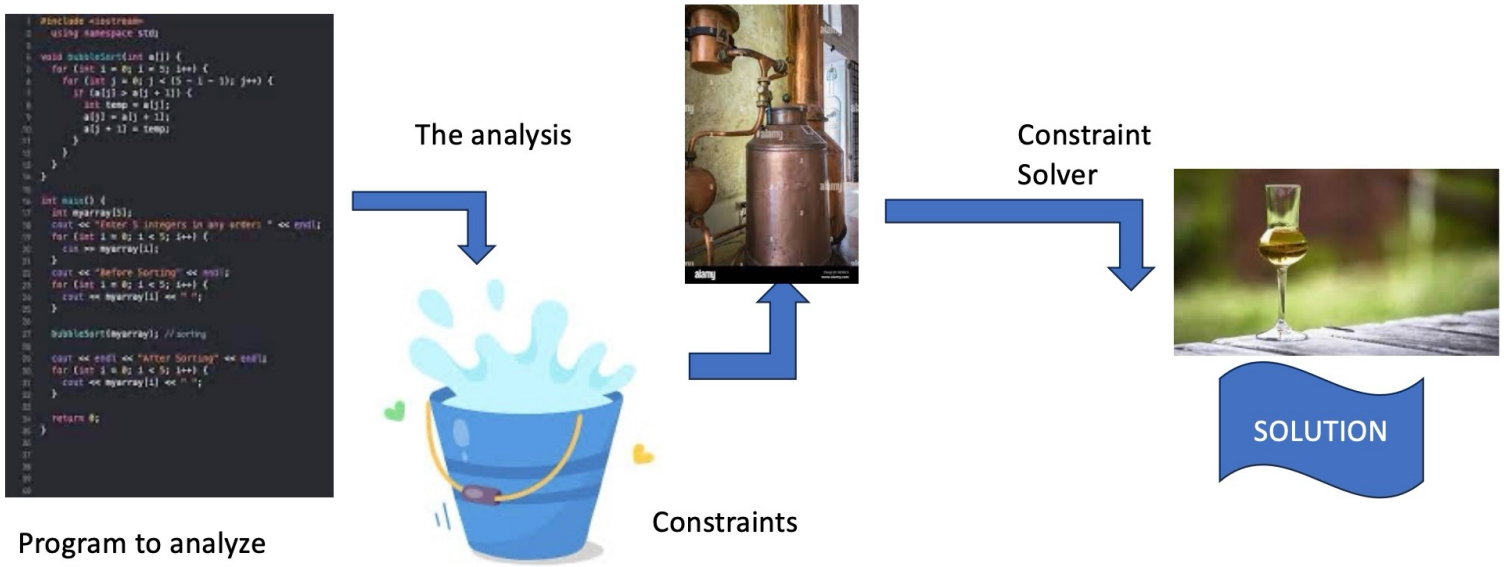
- $[[v_1]] = afv_1\ ([[v_1]], \ldots, [[v_n]])$
- $[[v_2]] = afv_2\ ([[v_1]], \ldots, [[v_n]])$
- ...
- $[[v_n]] = afv_n([[v_1]],\ldots, [[v_n]])$

where afv: $States^n \rightarrow States$ for each CFG node v

The lattice and the constraints form a monotone framework

Note that all the right-hand sides of our constraints correspond to monotone function

The basic idea is to separate the analysis specification from algorithmic aspects and implementation details



Use powerset of sets of variables you have at your disposal as lattice

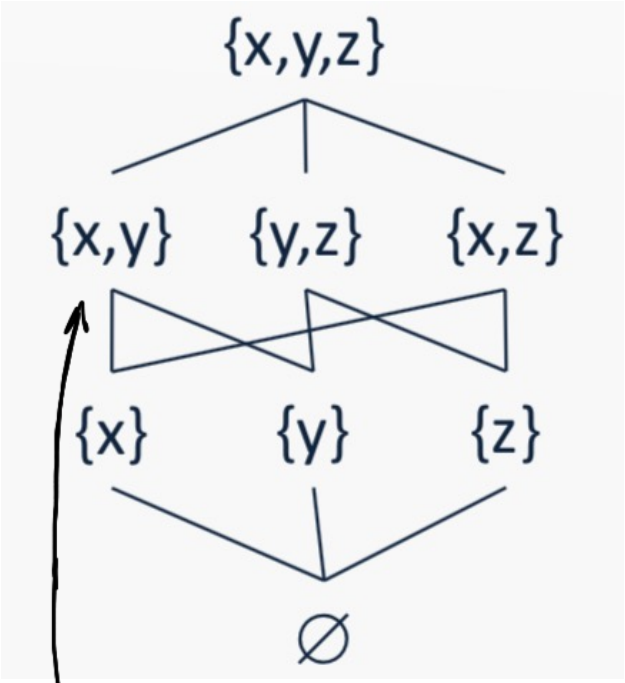
Liveness analysis

- This **data-flow analysis** determines the set of potentially **live variables in each point** of a given program
- Applications: **remove dead stores** and **dead code**

Liveness analysis: the lattice

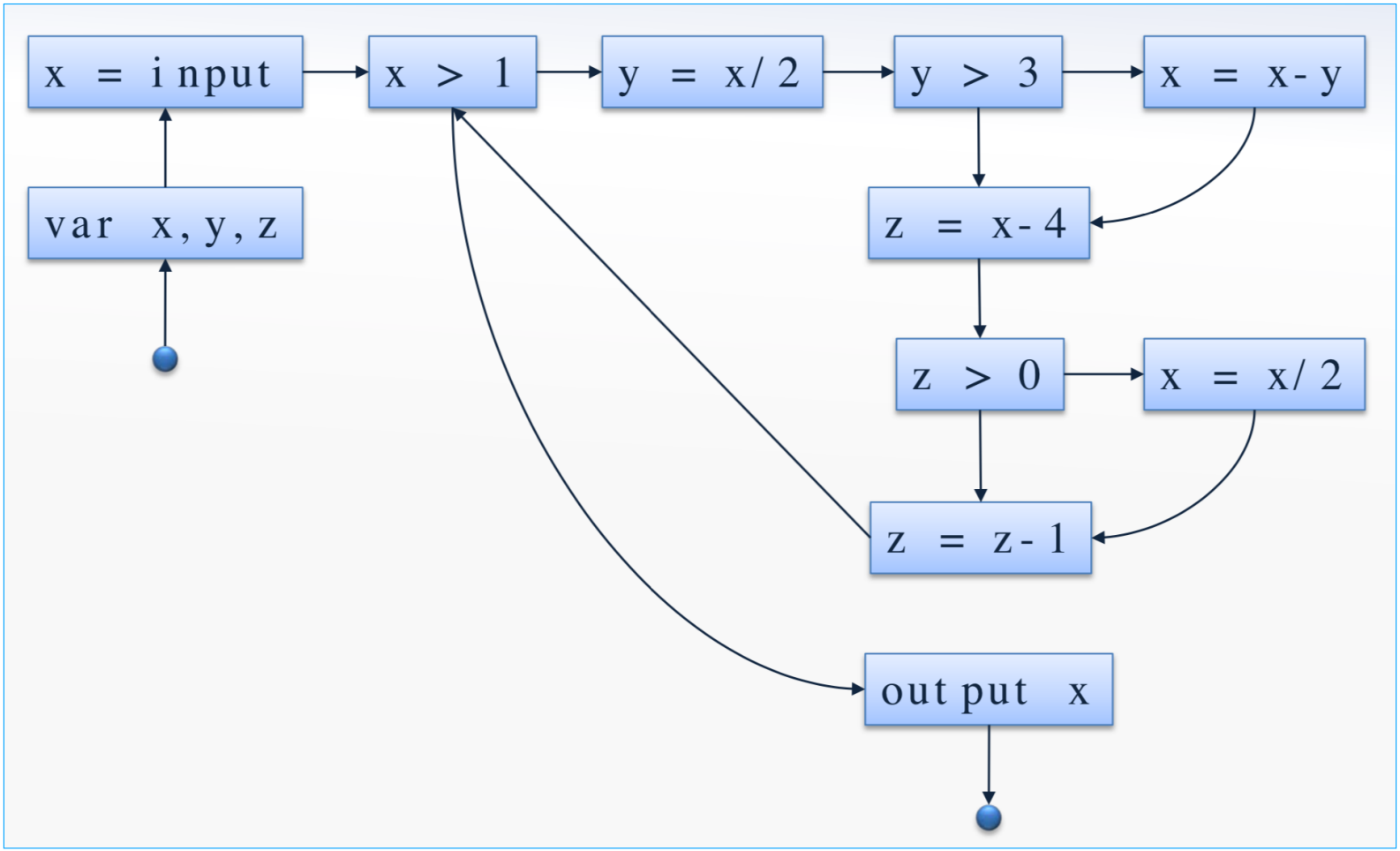
The **lattice** modelling our **abstract states** is **$(\wp(\{Var\});\subseteq)$** with Var, variables in the program (**parameterized** lattice). Again, a powerset for **live variables**

```
var x,y,z;
x = input;
while (x>1) {
y = x/2;
if (y>3) x = x-y;
z = x-4;
if (z>0) x = x/2;
z = z-1;
}
output x;
```



Variables in this state are live.
They correspond to set approach 110 (x, y, z)

Liveness analysis: the Control Flow Graph



Setting up the monotone framework

For every node v of the control flow graph we have a variable yielding the set of variables that are live at the program point after v

$[\![v]\!]$

the set of expressions that are live at the program before v

- In which **direction** the information propagate?

- Potentially live, MAY ANALYSIS

Setting up the monotone framework

For every node v of the control flow graph we have a variable yielding the set of variables that are live at the program point after v

$[\![v]\!]$

the set of expressions that are live at the program before v

- In which **direction** the information propagate? **Backward**

Setting up the monotone framework

For every node v of the control flow graph we have a variable yielding the set of variables that are live at the program point after v

$[\![v]\!]$

the set of expressions that are live at the program before v

- In which **direction** the information propagate? **Backward**
- In which way information coming from other nodes can be **merged**?

This is because a variable is live if it **may** be used **later** (at least in one path)

Setting up the monotone framework

For every node v of the control flow graph we have a variable yielding the set of variables that are live at the program point after v

$[\![v]\!]$

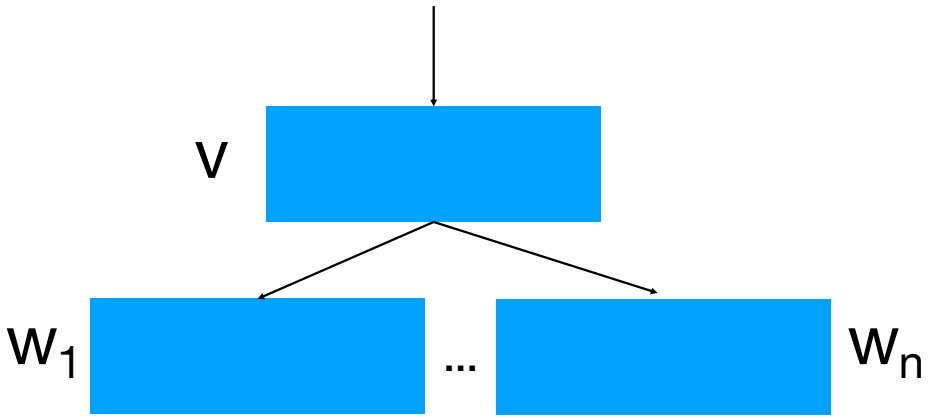
the set of expressions that are live at the program before v

- In which **direction** the information propagate? **Backward**
- In which way information coming from other nodes can be **merged**? **Union**

This is because a variable is live if it **may** be used **later** (at least in one path)

JOIN

$$JOIN(v) = \bigsqcup_{w \in succ(v)} [\![w]\!]$$



Termination

The set of all variables is finite, and

- the JOIN operation (i.e., meet) is **union**, hence
- the set associated with a data flow variable can only **grow**

This implies that termination is guaranteed (Think of monotone framework. To make it understandable, by how we defined it, we never actually shrunk ($\{x\} \cup \{x\}$, so that's true)

As initial value is ∅, live variables analysis converges on the smallest set

- Union: things always grow and you are going to converge
- we know thanks to theorems that we get to least fixed point

Why the Set Only Grows (Monotonicity)

In backward static analysis of liveness:

- We start with the empty set at the end of the control flow.
- At each program point, we compute the live variables set by **taking the union of the successors**.
- If a statement **uses** a variable, that variable must be added to the live set.
- If a statement **defines (kills)** a variable, it is removed from the set.

Now, the key part: since we are solving this as a **fixed-point iteration**, the set of live variables at each program point **only grows across iterations**. The reason is:

1. Initially, every point starts with the empty set.
2. At each iteration, we update the live variable set by taking the **union** of the successors' sets.
3. Since union **only adds elements**, the set never shrinks between iterations.
4. Eventually, we reach a fixed point where further iterations do not change the set.

Why Killing a Variable Doesn’t Contradict Growth

You're right that a variable may be removed (killed) from the live set at a particular program point, but that happens **within one evaluation of a statement**. However, in the fixed-point computation:

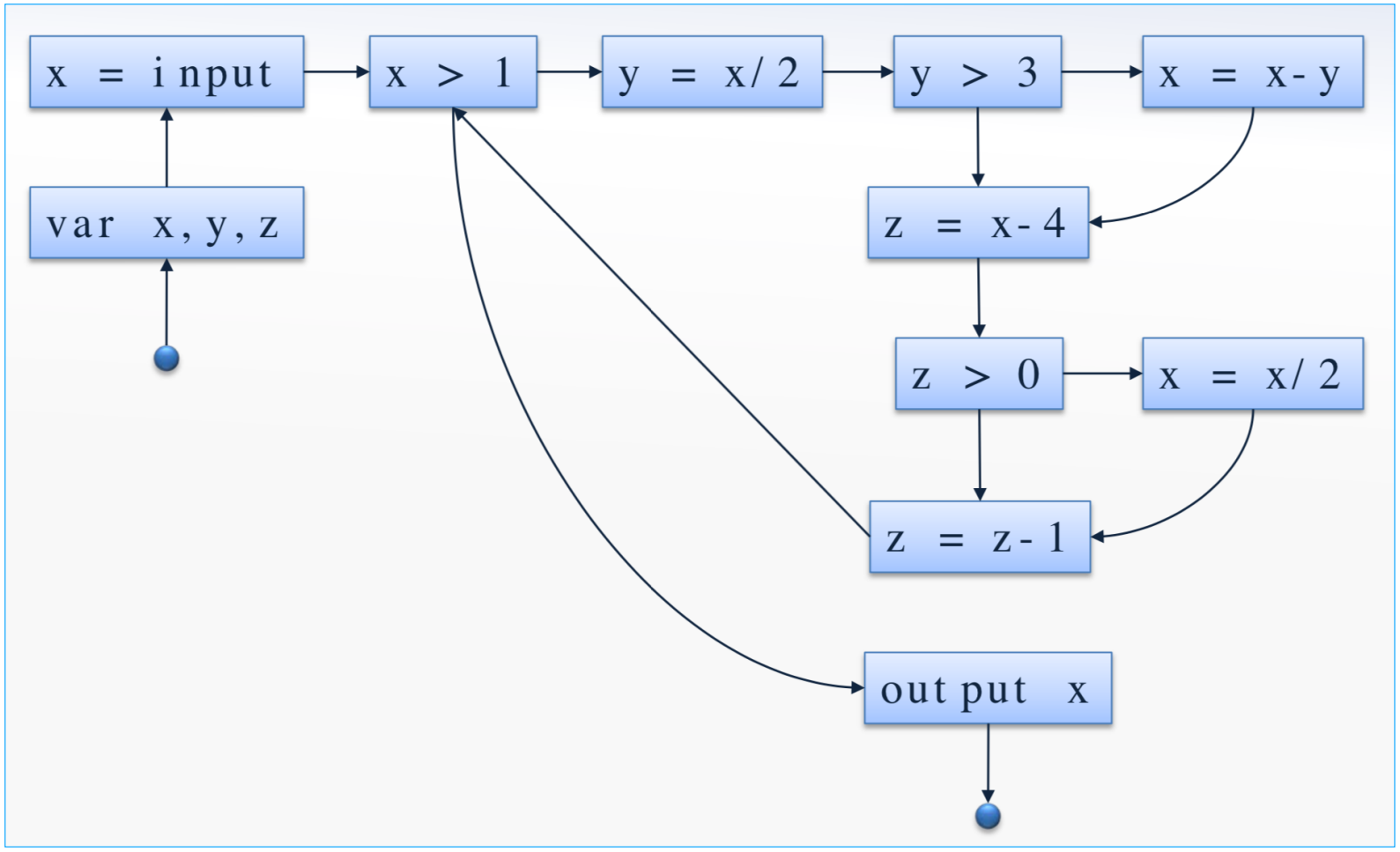
- The next iteration could bring back that variable if it appears in the union of a successor's live set.
- Since every iteration only takes unions, we never "undo" a previous addition in the iterative process.

Why This Guarantees Termination

1. The number of variables in the program is **finite**.
2. The powerset lattice (the structure we are working in) has a **finite height** (at most 2^n elements, where n is the number of variables).
3. The iteration process **only increases sets** and thus moves in one direction (upward in the lattice).
4. Since we can only add elements (never remove from a previous iteration), we must eventually reach a fixed point where further updates do not change anything.

Thus, termination is **guaranteed** because the iterative process cannot continue indefinitely—it must stabilize at some point.

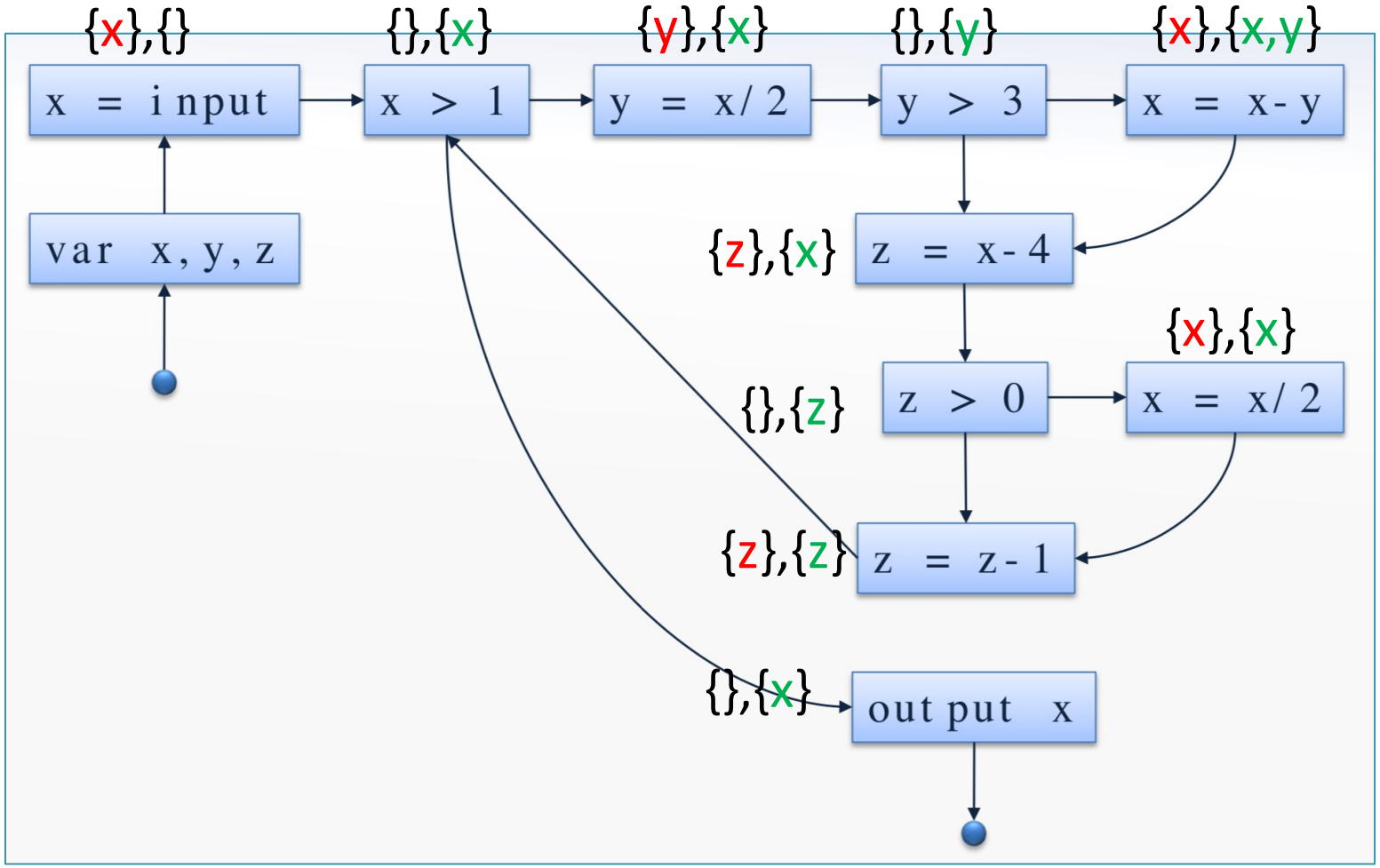
Liveness analysis: annotated CFG



An instruction makes a variable:

- live when it references /uses it
- dead when it defines/assigns to it

Liveness analysis: constraint variables



For every node **v**

- **[[v]]** include the variables that are live at the program point before v
- Liveness flows backwards through the CFG

Live Variables Analysis: dataflow equations

- Dataflow equations (transfer fuctions) tell us how liveness information is propagated
- For the variables declaration node

$$[\![\text{var } x1,\ldots,xn]\!] = \text{JOIN}(v) \setminus \{x1, \ldots, xn\}$$

- For the entry node

$$[\![\text{entry}]\!] = [\![\text{exit}]\!] = \emptyset$$

$$\text{JOIN}(v) = \bigcup_{w \in \text{succ}(v)} [\![w]\!]$$

- For conditions, loops and ouputs

$$[\![\text{if } (E)]\!] = [\![\text{while } (E)]\!] = [\![\text{output } E]\!] = \text{JOIN}(v) \cup \text{vars}(E)$$ (vars occurring in E)

- For assignment

$$[\![x = E]\!] = (\text{JOIN}(v) \setminus \{x\}) \cup \text{vars}(E)$$

killed variable — generated variables

- For any other nodes

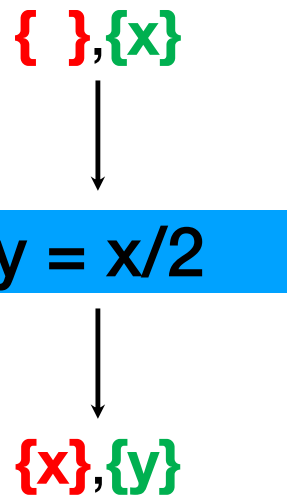
$$[\![v]\!] = \text{JOIN}(v)$$

where right-handsides are **monotone**

Inheritance

Is rhs monotone?

Liveness analysis: annotated CFG



$$[\![y = x/2]\!] = \text{JOIN}(v) \setminus \{x\} \cup \{y\} = \bigcup_{w \in succ(v)} [\![w]\!] = [\![y > 3]\!] \setminus \{x\} \cup \{y\}$$

An instruction makes a variable:

- live when it references /uses it
- dead when it defines/assigns to it
- y = x/2 kills y and generates x

$[\text{var } x,y,z] = [x = \text{input}] \setminus \{x,y,z\}$

$[x = \text{input}] = [x > 1] \setminus \{x\}$

$[x > 1] = ([y = x/2] \cup [\text{output } x]) \cup \{x\}$

$[y = x/2] = \ldots ([y > 3] \setminus \{y\}) \cup \{x\}$

$[y > 3] = ([x = x - y] \cup [z = x - 4]) \cup \{y\}$

$[x = x - y] = ([\ z = x - 4] \setminus \{x\}) \cup \{x, y\}$

$[z = x - 4] = ([z > 0] \setminus \{z\}) \cup \{x\}$

$[z > 0] = ([x = x/2] \cup [z = z - 1]) \cup \{z\}$

$[x = x/2] = ([z = z - 1] \setminus \{x\}) \cup \{x\}$

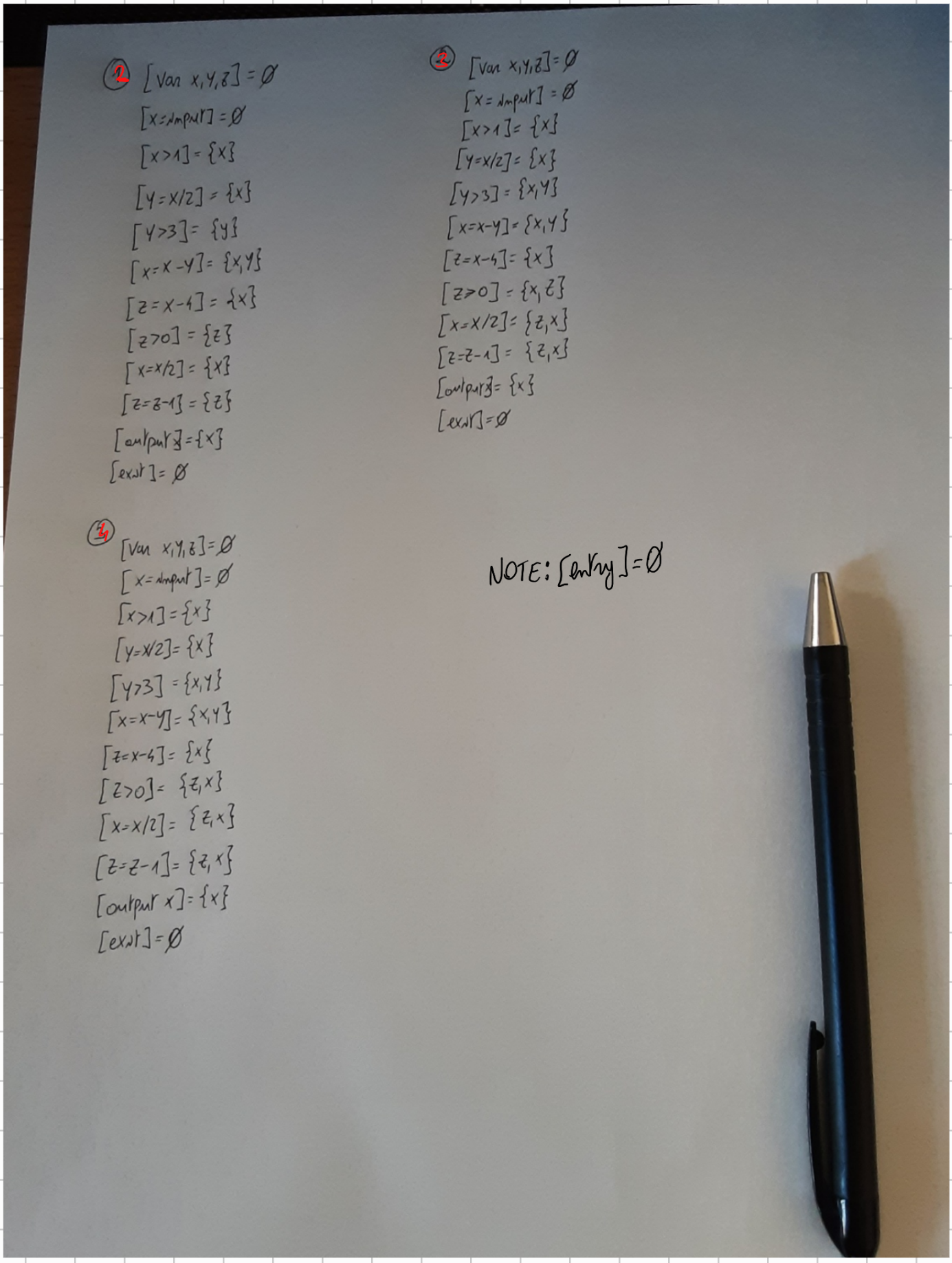
$[z = z - 1] = ([x > 1] \setminus \{z\}) \cup \{z\}$

$[\text{output } x] = [\text{exit}.] \cup \{x\}$

$[\text{exit}] = \emptyset$

Do not forget output and exit!

First iteration is of all empty sets as states



① [Var x,y,z] = ∅
[x=input] = ∅
[x>1] = {x}
[y=x/2] = {x}
[y>3] = {y}
[x=x-y] = {x,y}
[z=x-1] = {x}
[z>0] = {z}
[x=x/2] = {x}
[z=z-1] = {z}
[output x] = {x}
[exit] = ∅

② [Var x,y,z] = ∅
[x=input] = ∅
[x>1] = {x}
[y=x/2] = {x}
[y>3] = {x,y}
[x=x-y] = {x,y}
[z=x-1] = {x}
[z>0] = {x,z}
[x=x/2] = {z,x}
[z=z-1] = {z,x}
[output] = {x}
[exit] = ∅

③ [Var x,y,z] = ∅
[x=input] = ∅
[x>1] = {x}
[y=x/2] = {x}
[y>3] = {x,y}
[x=x-y] = {x,y}
[z=x-1] = {x}
[z>0] = {z,x}
[x=x/2] = {z,x}
[z=z-1] = {z,x}
[output x] = {x}
[exit] = ∅

NOTE: [entry] = ∅

We can make conclusion: y, z are never live at the same time, and z is not read after the assignment z = z-1. It can be stripped. You can re

Example optimization

- y and z are never live at the same time ⇒ they can share the same variable location
- the value assigned in z=z-1 is never read (used) after the assignment ⇒ the assignment can be skipped

```
var x,y,z;
x = input;
while (x>1) {
y = x/2;
if (y>3) x = x-y;
z = x-4;
if (z>0) x = x/2;
z = z-1;
}
output x;
```

```
var x,yz;
x = input;
while (x>1) {
yz = x/2;
if (yz>3) x = x-yz;
yz = x-4;
if (yz>0) x = x/2;
/* z = z-1; */
}
output x;
```

112

Note that there might be consequences: imagine you do key=0; to wipe secrets. But if assignment is removed, that is bad! We need to find balance.

Equations

A program with n CFG nodes, $v_1,\ldots, v_n$, is thus represented by n equations:

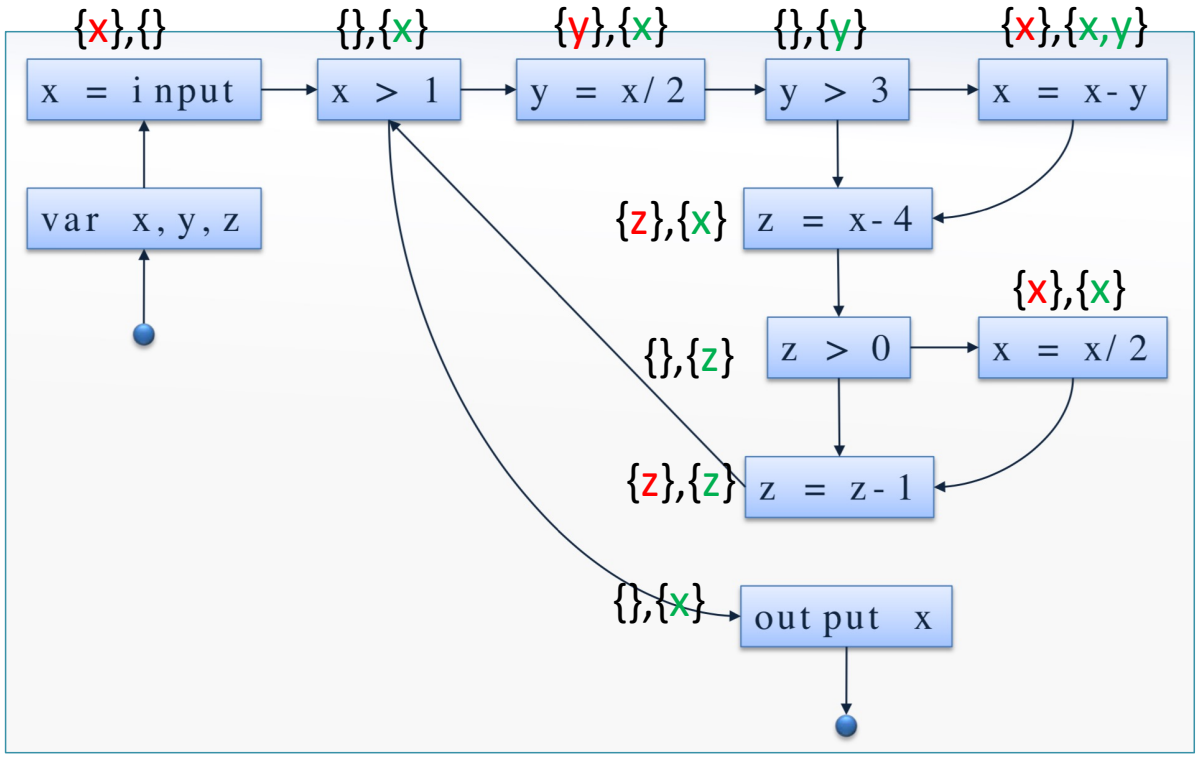
- $[[v_1]] = afv_1\ ([[v_1]], \ldots, [[v_n]])$
- $[[v_2]] = afv_2\ ([[v_1]], \ldots, [[v_n]])$
- ...
- $[[v_n]] = afv_n([[v_1]],\ldots, [[v_n]])$

where afv: States^n → States for each CFG node v

Again, the lattice $(\wp(\{Var\});\subseteq)$ and the constraints extracted, using previously defined rules, form a **monotone framework**

Back to our example

⟦var x1,...,xn⟧= JOIN(v) \{x1, ..., xn}



⟦var x, y, z⟧ = ⟦x=input⟧ \ {x,y,z}

Live Variables Analysis: algorithm

- Analysis: computing the set of live variables for each node n in the CFG such that the liveness data-flow equations are satisfied

- A simple way to solve the data-flow equations is to adopt an iterative strategy, starting from setting **[[v]] =** ∅ for every node

- This algorithm is guaranteed to terminate since there are a finite number of variables in each program and the effect of one iteration is monotonic

Naïve algorithm

```
x = (⊥, ⊥,…, ⊥);
do {
  t = x;
  x = f(x)
while (x ≠ t)
```

Live Variables Analysis: algorithm

- Analysis: computing the set of live variables for each node n in the CFG such that the liveness data-flow equations are satisfied

- A simple way to solve the data-flow equations is to adopt an iterative strategy, starting from setting **[[v]] =** ∅ for every node

- This algorithm is guaranteed to terminate since there are a finite number of variables in each program and the effect of one iteration is monotonic

Naïve algorithm

```
x = (⊥, ⊥,…, ⊥);
do {
  t = x;
  x = f(x)
while (x ≠ t)
```

Example optimization

- y and z are never live at the same time* ⇒ they can share the same variable location
- the value assigned in z=z-1 is never read (used) after the assignment ⇒ the assignment can be skipped

```
var x,y,z;
x = input;
while (x>1) {
y = x/2;
if (y>3) x = x-y;
z = x-4;
if (z>0) x = x/2;
z = z-1;
}
output x;
```

```
var x,yz;
x = input;
while (x>1) {
yz = x/2;
if (yz>3) x = x-yz;
yz = x-4;
if (yz>0) x = x/2;
/* z = z-1; */
}
output x;
```


* You can use the same memory cell

Live Variables Analysis: algorithm

- We can also use the **worklist** algorithm
- The **worklist W** contains nodes from the program graph (the ones we need to reconsider)
- The auxiliary function **dep** that gives the set of nodes w where v occurs on the right-hand side of the constraint for w.
- **dep** v contains the set of nodes that depend on v
- All the nodes provided by dep are inspected
- The analysis assignment will be updated if required: if this happens the target nodes are added to **W**

```
x1 = ⊥; ... xn = ⊥;
W = {v1, ..., vn};
while (W≠∅) {
  vi = W.removeNext();
  y = fi(x1, ..., xn);
  if (y≠xi) {
    for (vj ∈ dep(vi)) W.add(vj);
    xi = y;
  }
}
```

Live Variables Analysis: recap

- Data-flow analysis collects information about how data moves through a program
- Variable liveness is a data-flow property
- Liveness variable analysis is a backwards dataflow analysis for determining variable liveness
- LVA may be expressed in terms of data-flow equations
- A simple iterative algorithm can be used to find the smallest solution to the LVA data-flow equations

Types of Dataflow Analysis: back and forth

We distinguish between **forward** and **backward** analyses based on the **direction** in which information propagates through the control flow graph (CFG)

A **forward** analysis gathers information about what has happened before a program point. It starts at the entry node and moves forward through the CFG. The constraints at each node depend only on its predecessors. Classic examples include:

- **Sign analysis**: tracks the sign of variables
- **Available expressions**: determines which expressions have already been computed and not invalidated

A **backward** analysis collects information about what must hold after a program point. It starts at the exit node and proceeds backward through the CFG. The constraints here depend on successors. Examples include:

- **Live variable analysis**: tracks which variables will be used in the future

Types of Dataflow Analysis: must and may

We distinguish between **may** and **must** analyses based, for analyses that are based on a powerset lattices

A **may** analysis aims to find what may possibly be true at a program point. It computes an over-approximation of the actual behavior and uses the lattice order $\subseteq$ and union ($\cup$) to combine information. These analyses are typically used when missing a possibility could lead to incorrect behavior. Examples include:

- Live variables

Property holds in all paths

A **must** analysis focuses on what must definitely be true. It computes an under-approximation, using the lattice order $\supseteq$ and intersection ($\cap$) to combine information. This kind of analysis is used when precision about guaranteed properties is crucial. Examples include:

- Available expressions

Property holds in some paths

Outline

Liveness Analysis

Liveness: Set Theory Approch

Liveness: Monotone Framework approach

Other analyses

Very busy expressions

An expression e is **very busy** at point p if

- On every path from p, expression e is evaluated before the value of e is changed

```
var x,a,b;
x = input;
a = x-1;
b = x-2;
while (x>0) {
  output a*b-x;
  x = x-1;
}
output a*b;
```

- So expression e is "useful".

Very busy expressions

MUST ANALYSIS

Analysis: same lattice and auxiliary functions as for available expressions

- In which **direction** the information propagate? Backward or forward?

- In which way information coming from other nodes can be **merged**? Union or intersection?

Very busy expressions

Analysis: same lattice and auxiliary functions as for available expressions

- In which **direction** the information propagate? Backward or forward?
  Backward: I should have knowledge of the future to know if they are busy
- In which way information coming from other nodes can be **merged**? Union or intersection? **Intersection** ∀

Very busy expressions

Optimization: useful for code hoisting and saves code space

```
var x,a,b;
x = input;
a = x-1;
b = x-2;
while (x>0) {
  output a*b-x;
  x = x-1;
}
output a*b;
```

```
var x,a,b,atimesb;
x = input;
a = x-1;
b = x-2;
atimesb = a*b;
while (x>0) {
  output atimesb-x;
  x = x-1;
}
output atimesb;
```

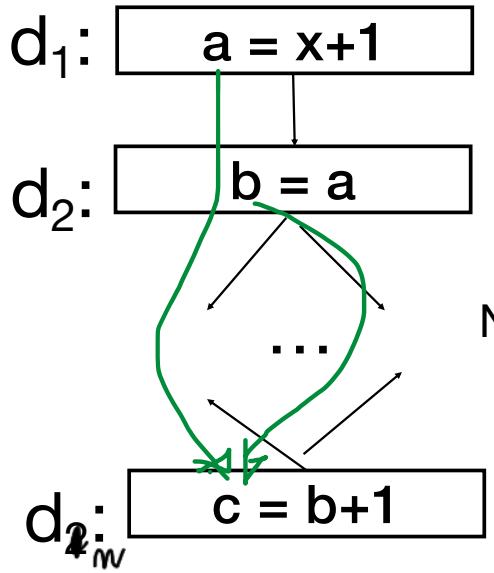
Replace computation that takes once the computation and you use it every time you need it (no recompute)

Here, the compiler can perform code hoisting and move the computation to the earliest program point where it is very busy

Reachable definitions

- A definition of a variable v is an assignment to v
- A definition of variable v **reaches** point p if there is no intervening assignment to v (that contains a definition of v and therefore a kill)

```
void foo(int x) {
  int a, b, c;
  a = x + 1;
  b = a;
  if (…) { … }
  else  { … }
  c = b + 1;
   return c;
}
```



No use of a and b] No redefinition of a, that remains valid

There is no kill until point p

Reachable definitions

Analysis: powerset lattice of all assignments (represented as CFG nodes) occurring in the program

It can relates definitions (i.e. assignments) to it possible uses (def-use graph)

- In which **direction** the information propagate? Backward or forward?
  Backward

- In which way information coming from other nodes can be **merged**? Union or intersection? **Union**

Reachable definitions

Analysis: powerset lattice of all assignments (represented as CFG nodes) occurring in the program

It can relates definitions (i.e. assignments) to it possible uses (def-use graph)

- In which **direction** the information propagate? Backward or forward?
  Forward Need to know if something is useful from the past

- In which way information coming from other nodes can be **merged**? Union or intersection? **Union**

MAY

Reachable definitions

It can relates definitions (i.e. assignments) to it possible uses (def-use graph)

Optimizations:

- dead code elimination and
- code motion Rearrangement

Types of Dataflow Analysis

	Forward	*Backward*
May	Reaching Definitions	Live Variables
Must	Available Expressions	Very Busy Expressions

Types of Dataflow Analysis: back and forth

We distinguish between **forward** and **backward** analyses based on the direction in which information propagates through the control flow graph (CFG)

A **forward** analysis gathers information about what has happened before a program point. It starts at the entry node and moves forward through the CFG. The constraints at each node depend only on its predecessors. Classic examples include:

- **Sign analysis**: tracks the sign of variables
- **Available expressions**: determines which expressions have already been computed and not invalidated

A **backward** analysis collects information about what must hold after a program point. It starts at the exit node and proceeds backward through the CFG. The constraints here depend on successors. Examples include:

- **Live variable analysis**: tracks which variables will be used in the future
- **Very busy expressions**: finds expressions that will definitely be evaluated later

Types of Dataflow Analysis: must and may

We distinguish between **may** and **must** analyses based, for analyses that are based on a powerset lattices

A **may** analysis aims to find what may possibly be true at a program point. It computes an over-approximation of the actual behavior and uses the lattice order $\subseteq$ and union ($\cup$) to combine information. These analyses are typically used when missing a possibility could lead to incorrect behavior. Examples include:

- Live variables
- Reaching definitions

A **must** analysis focuses on what must definitely be true. It computes an under-approximation, using the lattice order $\supseteq$ and intersection ($\cap$) to combine information. This kind of analysis is used when precision about guaranteed properties is crucial. Examples include:

- Available expressions
- Very busy expressions

Bibliography

- Anders Møller & Michael I. Schwartzbach. *Static Program Analysis* https://cs.au.dk/~amoeller/spa/
- F. Nielson, H. R. Nielson, and C. Hankin. *Principles of Program Analysis* Springer-Verlag. 1998.