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COT 4210

Homework 1
Section 0001

Problem 1:

Prove that $\sum_{i=1}^{n} i = \frac{n^2+n}{2}$

Proof by induction:

Base Case:

$n = 1$

LHS: $\sum_{i=1}^{1} i = 1$ RHS: $\frac{1^2+1}{2} = \frac{2}{2} = 1$

Thus, $\sum_{i=1}^{n} i = \frac{n^2+n}{2}$ holds for $n = 1$

Induction:

Assume that for an arbitrary positive integer k that $\sum_{i=1}^{k} i = \frac{k^2+k}{2}$

Now prove true for $n = k + 1$ that $\sum_{i=1}^{k+1} i = \frac{(k+1)^2+(k+1)}{2}$

$\sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + (k+1)$, since we assumed that $\sum_{i=1}^{k} i = \frac{k^2+k}{2}$ we can rewrite the LHS as $\frac{k^2+k}{2} + (k+1)$

Thus, $\frac{k^2+k}{2} + (k+1) = \frac{(k+1)^2+(k+1)}{2}$

$\frac{k^2+k}{2} + \frac{2(k+1)}{2} = \frac{(k+1)^2+(k+1)}{2}$ Create common Denominator

$\frac{k^2+k}{2} + \frac{2k+2}{2} = \frac{k^2+2k+1+k+1}{2}$ Distribute

$\frac{k^2+3k+2}{2} = \frac{k^2+3k+2}{2}$ Combine like terms

Since $\frac{k^2+3k+2}{2} = \frac{k^2+3k+2}{2}$ is true it is proven that $\sum_{i=1}^{k+1} i = \frac{(k+1)^2+(k+1)}{2}$

Thus, $\sum_{i=1}^{n} i = \frac{n^2+n}{2}$ holds true for all positive integers n.

Problem 2:

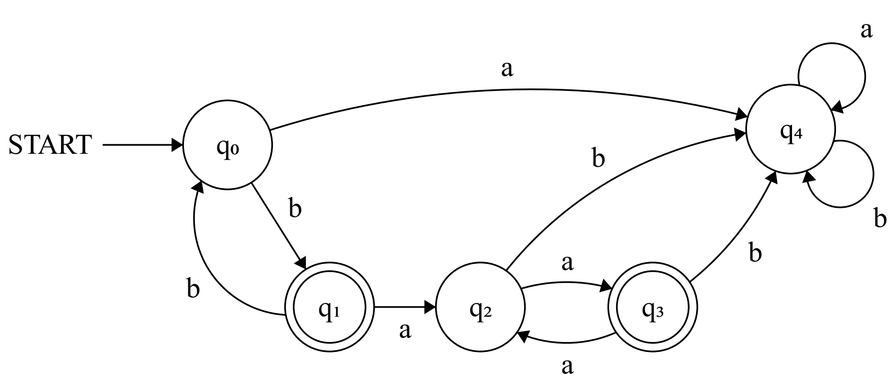
Consider a set H of 4 horses 2 brown and two black, this already contradicts the assumption. However, let's continue the example by removing two horses into new sets as was done in the fallacious proof. Remove one black horse and call this new set H'. Now remove a brown horse and consider it H". We now have a set with one black horse, a set with one brown horse and a set with a brown and black horse. Thus, again breaking the assumption that a set of h horses must all be the same color. Horses can be a large variety of colors and any set of horses greater than one will always have the possibility of being a different color.

Problem 3:

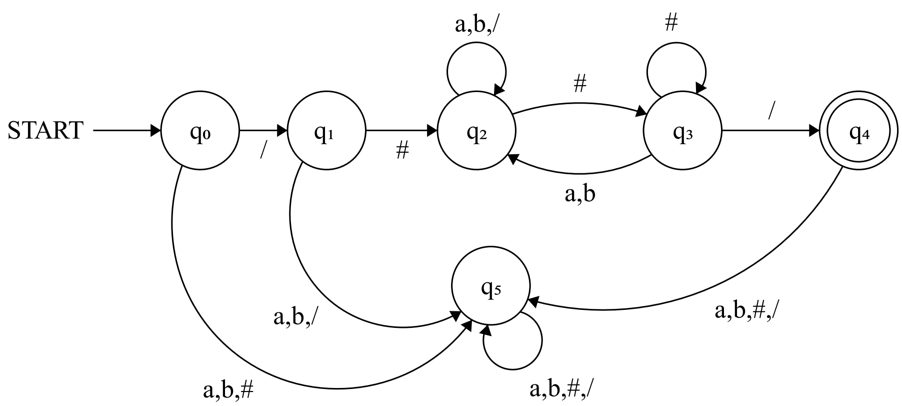
Consider a graph with n nodes where $n \geq 2$ and there are no self-loops. The possible degrees a node can be connected to no nodes and up to all the other nodes but never itself. So, its degrees range from 0 to n-1. Now consider each node to have a different degree starting with 0 and up to n-1. However, there is now a contradiction for a node to have a degree of n-1. This means that it must have an edge between every other node except itself. But we already determined that another node has a degree of zero which means that it isn't connected to any other nodes. Thus, proving that all graphs with with n nodes where $n \geq 2$ and there are no self-loops must have two nodes with equal degrees.

Problem 4

4a.



4b.



Problem 5.

5a. To obtain M' you must change all the accept states of M to non-accept states and all of the non-accept states of M accept states.

5b. The complement of a language is defined as the set of all the possible strings in an alphabet that are not in a language which uses that alphabet. For this example, the DFA M recognizes the language B . Consider B to be a language over the alphabet $\sum$ so the complement of B is the set of all strings over $\sum$ that do not belong to B. The DFA M recognizes all strings over $\sum$ that belong to B and reject all strings over $\sum$ that do not belong to B. By flipping all non-accept states of $M$to accept states and all accept states of M to non-accept states to create M' we can conclude that M' will reject exactly all strings accepted by M and accept exactly all strings rejected by M . Thus, recognizing the complement of B.

5c. A language is considered regular if it can be recognized by a DFA. To prove regular languages are closed under complement it must be true that if a language is regular its complement is also regular (e.g. Its complement can be recognized by a DFA). In this example we know that there exists a DFA M tat recognizes a language L which implies L is a regular language. We also proved you can create a DFA M' that recognizes the compliment of L. Since the compliment of L is recognized by a DFA then it must also be regular. Thus, proving regular languages are closed under complement.