Mathematica Project for Calculus 3 (MA2733-02)

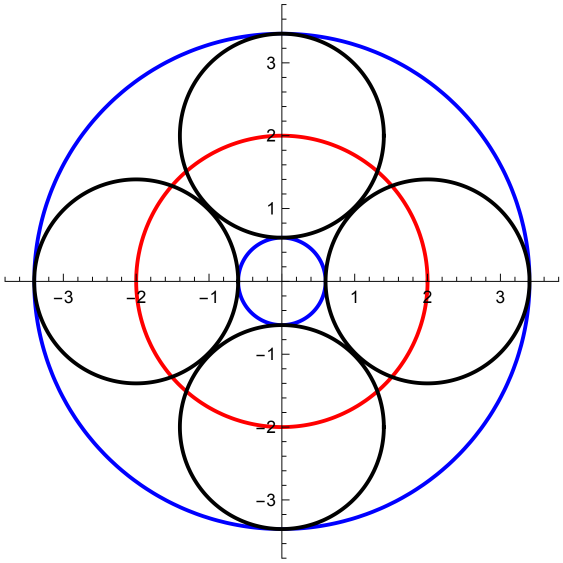
Contributors:
Brody Pennington (BGP99)

Problem 1

```
ParametricPlot[
 {
  {.6 Cos[t], .6 Sin[t]},
  {2 Cos[t], 2 Sin[t]},
  {3.4 Cos[t], 3.4 Sin[t]},

  {2 Cos[0] + 1.4 Cos[t], 2 Sin[0] + 1.4 Sin[t]},
  {2 Cos[π / 2] + 1.4 Cos[t], 2 Sin[π / 2] + 1.4 Sin[t]},
  {2 Cos[π] + 1.4 Cos[t], 2 Sin[π] + 1.4 Sin[t]},
  {2 Cos[3 π / 2] + 1.4 Cos[t], 2 Sin[3 π / 2] + 1.4 Sin[t]}
 },
 {t, 0, 2 π},
 PlotRange → {{-3.8, 3.8}, {-3.8, 3.8}},
 PlotStyle → {
   {Blue},
   {Red},
   {Blue},
   {Black},
   {Black},
   {Black},
   {Black},
  },
 AspectRatio → 1
]
```

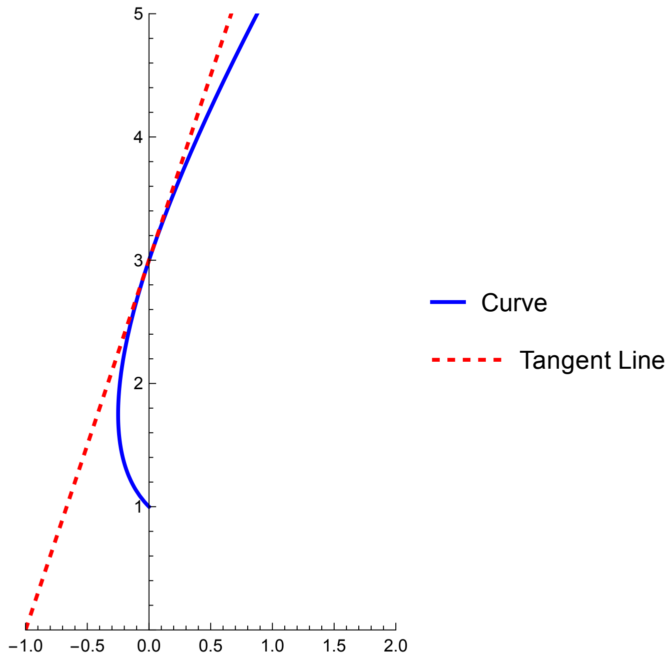
Out[ ]=



Problem 2

```
xExpr = t^2 - t;
yExpr = t^2 + t + 1;
pointX = xExpr /. t → 1;
pointY = yExpr /. t → 1;
dxdt = D[xExpr, t];
dydt = D[yExpr, t];
slope = (dydt / dxdt) /. t → 1;
tangentLineExpr = slope * (x - pointX) + pointY;
Show[
 ParametricPlot[
      {xExpr /. t → t, yExpr /. t → t},
      {t, 0, 2},
      PlotRange → {{-1, 2}, {0, 5}},
      PlotStyle → Blue,
      PlotLegends → {"Curve"}
 ],
 Plot[
      tangentLineExpr,
      {x, -1, 2},
      PlotStyle → {Red, Dashed},
      PlotLegends → {"Tangent Line"}
 ]
]
```

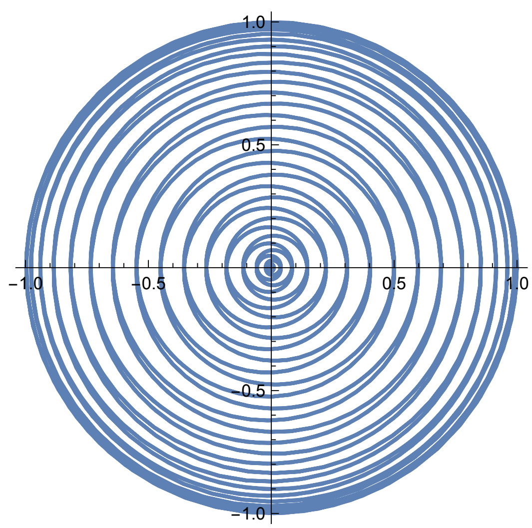
Out[ ]=



Problem 3

In[ ]:=
```
PolarPlot[
 Cos[θ/64]^2, {θ, 0, 128 π},
 PlotRange → All
]
```

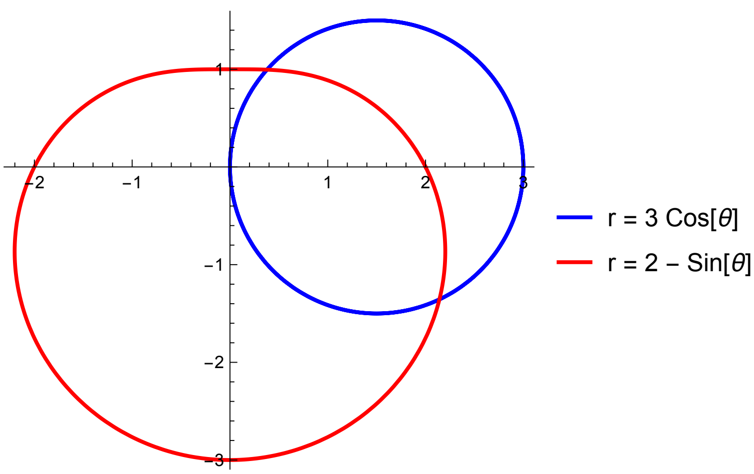
Out[ ]=



Problem 4

```
PolarPlot[
 {3 Cos[θ], 2 - Sin[θ]},
 {θ, 0, 2 Pi},
 PlotLegends → {"r = 3 Cos[θ]", "r = 2 - Sin[θ]"},
 PlotStyle → {Blue, Red},
 PlotRange → All
]
r1[θ_] := 3 Cos[θ];
r2[θ_] := 2 - Sin[θ];
solutions = Solve[3 Cos[θ] == 2 - Sin[θ] && 0 ≤ θ ≤ 2 Pi, θ];
a = θ /. solutions[[1]];
b = θ /. solutions[[2]];
{a, b}
area = (1 / 2) Integrate[(r1[θ] ^ 2 - r2[θ] ^ 2), {θ, a, b}]
```

Out[ ]=



Out[ ]=

$$\left\{2\pi + 2\,\text{ArcTan}\left[\frac{1}{5}\left(1-\sqrt{6}\right)\right],\ 2\,\text{ArcTan}\left[\frac{1}{5}\left(1+\sqrt{6}\right)\right]\right\}$$

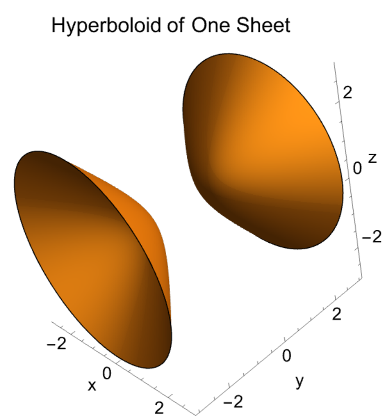
Out[ ]=

$$\frac{6\sqrt{6}}{5}$$

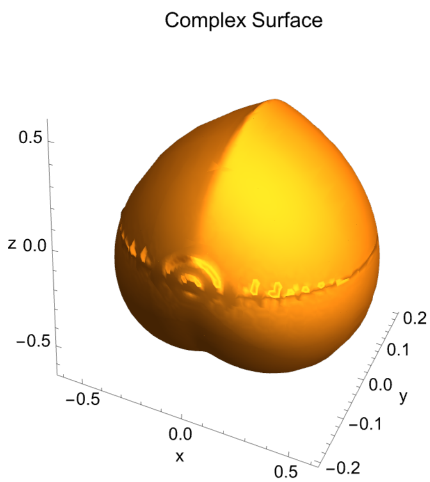
Problem 5

In[ ]:=
```
ContourPlot3D[
 -x^2 + y^2 - z^2 == 1,
 {x, -3, 3}, {y, -3, 3}, {z, -3, 3},
 PlotRange → All,
 Mesh → None,
 PlotPoints → 50,
 AxesLabel → {"x", "y", "z"},
 Boxed → False,
 PlotLabel → "Hyperboloid of One Sheet"
]
ContourPlot3D[
 (3 x^2 + 3 z^2 + 25 y^2 - 1)^3 + 4 x^2 z^3 + (6 / 25) y^2 z^3 == 0,
 {x, -2, 2}, {y, -2, 2}, {z, -2, 2},
 PlotRange → All,
 Mesh → None,
 PlotPoints → 50,
 AxesLabel → {"x", "y", "z"},
 Boxed → False,
 PlotLabel → "Complex Surface"
]
```

Out[ ]=



Out[ ]=



Problem 6

```
ClearAll["Global`*"]
r[t_] := {t / (t^2 + 1), Sqrt[t - 3], 3 Cos[t/2]}
rPrime = D[r[t], t]
rDoublePrime = D[r[t], {t, 2}]
T = FullSimplify[rPrime / Sqrt[rPrime.rPrime]]
crossProduct = FullSimplify[Cross[rPrime, rDoublePrime]]
```

Out[ ]=

$$\left\{-\frac{2 t^2}{\left(1+t^2\right)^2}+\frac{1}{1+t^2}, \frac{1}{2 \sqrt{-3+t}}, -\frac{3}{2} \operatorname{Sin}\left[\frac{t}{2}\right]\right\}$$

Out[ ]=

$$\left\{-\frac{4 t}{\left(1+t^2\right)^2}+t\left(\frac{8 t^2}{\left(1+t^2\right)^3}-\frac{2}{\left(1+t^2\right)^2}\right), -\frac{1}{4(-3+t)^{3/2}}, -\frac{3}{4} \operatorname{Cos}\left[\frac{t}{2}\right]\right\}$$

Out[ ]=

$$\left\{-\frac{2\left(-1+t^2\right)}{\left(1+t^2\right)^2 \sqrt{\frac{1}{-3+t}+\frac{4\left(-1+t^2\right)^2}{\left(1+t^2\right)^4}+9 \operatorname{Sin}\left[\frac{t}{2}\right]^2}}, \frac{1}{\sqrt{-3+t} \sqrt{\frac{1}{-3+t}+\frac{4\left(-1+t^2\right)^2}{\left(1+t^2\right)^4}+9 \operatorname{Sin}\left[\frac{t}{2}\right]^2}}, -\frac{3 \operatorname{Sin}\left[\frac{t}{2}\right]}{\sqrt{\frac{1}{-3+t}+\frac{4\left(-1+t^2\right)^2}{\left(1+t^2\right)^4}+9 \operatorname{Sin}\left[\frac{t}{2}\right]^2}}\right\}$$

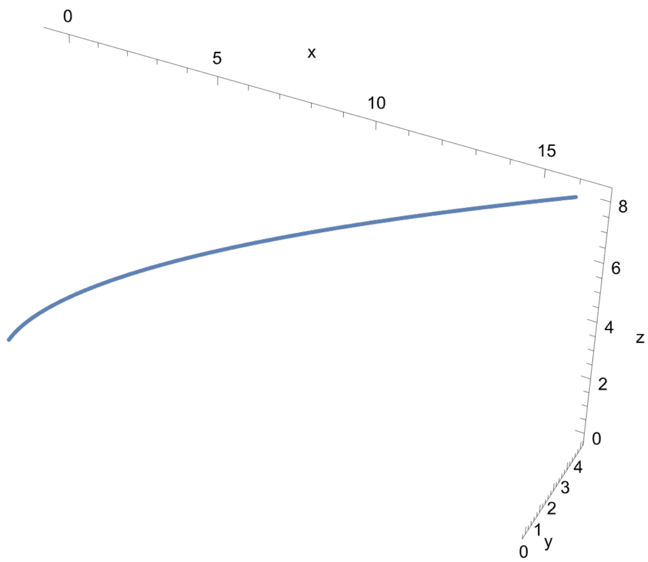
Out[ ]=

$$\left\{-\frac{3\left((-3+t) \operatorname{Cos}\left[\frac{t}{2}\right]+\operatorname{Sin}\left[\frac{t}{2}\right]\right)}{8(-3+t)^{3/2}}, -\frac{3\left(-1+t^4\right) \operatorname{Cos}\left[\frac{t}{2}\right]+12 t\left(-3+t^2\right) \operatorname{Sin}\left[\frac{t}{2}\right]}{4\left(1+t^2\right)^3}, \frac{-1-3 t(12+t(-4+(-4+t) t))}{4(-3+t)^{3/2}\left(1+t^2\right)^3}\right\}$$

Problem 7

```
ClearAll["Global`*"]
r[t_] := {t^4, t^2, t^3}
ParametricPlot3D[
 r[t], {t, 0, 2},
 AxesLabel → {"x", "y", "z"},
 Boxed → False
]
rPrime[t_] := D[r[t], t]
magR[t_] := Sqrt[rPrime[t].rPrime[t]]
lengthOfCurve = NIntegrate[magR[t], {t, 0, 2}, PrecisionGoal → 6]
```

Out[ ]=



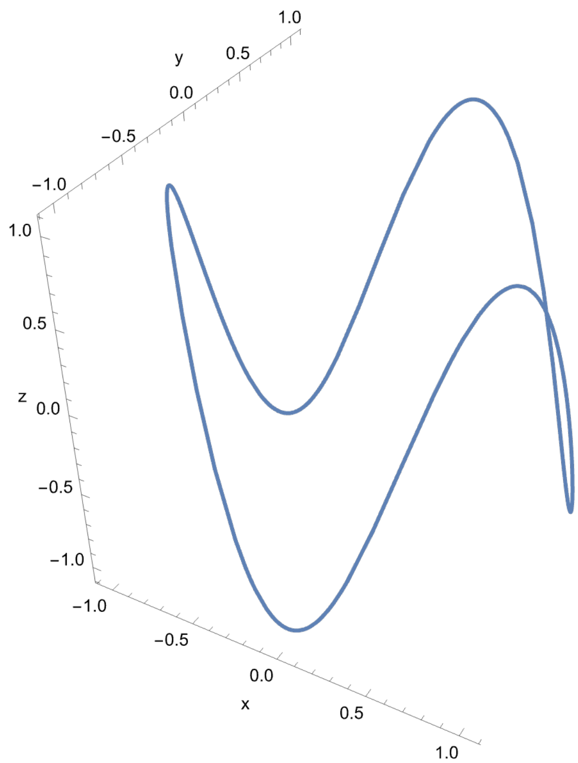
Out[ ]=

```
18.6833
```

Problem 8

```
ClearAll["Global`*"]
r[t_] := {Sin[t], Cos[t], Cos[3 t]}
ParametricPlot3D[r[t], {t, 0, 2 Pi},
 AxesLabel → {"x", "y", "z"}, Boxed → False, PlotRange → All]
rPrime[t_] := Evaluate[D[r[t], t]]
rPrime2[t_] := Evaluate[D[rPrime[t], t]]
magNum[t_] := Norm[Cross[rPrime[t], rPrime2[t]]]
magDenom[t_] := Norm[rPrime[t]] ^ 3
curve[t_] := magNum[t] / magDenom[t]
(*sin(t)=1=pi/2*)
curveAtPoint = curve[Pi / 2]
```

Out[ ]=



Out[ ]=

$$\frac{1}{10}$$

Problem 9

```
ClearAll["Global`*"]
r[t_] := {t^2, Log[t], 2 t}
(*velocity vector→ (dr/dt)*)
velocity = Simplify[D[r[t], t]]
(*T vector*)
Tmag = Simplify[Sqrt[velocity.velocity]]
T[t_] := Simplify[velocity / Tmag]
(*acceleration vector→ (d²r/dt²)*)
acceleration = Simplify[D[velocity, t]]
(*N vector*)
N1[t_] := Simplify[acceleration - (acceleration.T[t]) * T[t]]
Nmag = Simplify[Sqrt[N1[1].N1[1]]]
N2[t_] := Simplify[N1[t] / Nmag]
(*B vector*)
B[t_] := Simplify[Cross[T[t], N2[t]]]
(*t=1*)
t0 = 1;
Print[""]
Print["At t = 1:"]
Print["T = ", Simplify[T[t0]]]
Print["N = ", Simplify[N2[t0]]]
Print["B = ", Simplify[B[t0]]]
```

Out[ ]=

$$\left\{2\,t,\ \frac{1}{t},\ 2\right\}$$

Out[ ]=

$$\sqrt{4+\frac{1}{t^2}+4\,t^2}$$

Out[ ]=

$$\left\{2,\ -\frac{1}{t^2},\ 0\right\}$$

Out[ ]=

$$2\,\sqrt{\frac{1}{t^2}}$$

At t = 1:

$$T = \left\{\frac{2\,t}{\sqrt{4+\frac{1}{t^2}+4\,t^2}},\ \frac{1}{t\,\sqrt{4+\frac{1}{t^2}+4\,t^2}},\ \frac{2}{\sqrt{4+\frac{1}{t^2}+4\,t^2}}\right\}$$

$$N = \left\{\frac{2}{\sqrt{\frac{1}{t^2}}\,\left(1+2\,t^2\right)},\ -\frac{2}{\sqrt{\frac{1}{t^2}}\,\left(1+2\,t^2\right)},\ \frac{2-4\,t^2}{2\,\sqrt{\frac{1}{t^2}}\,\left(t+2\,t^3\right)}\right\}$$

$$B = \left\{\frac{\sqrt{\frac{1}{t^2}}}{\sqrt{4+\frac{1}{t^2}+4\,t^2}},\ \frac{2}{\sqrt{\frac{1}{t^2}}\,\sqrt{4+\frac{1}{t^2}+4\,t^2}},\ -\frac{2\,\sqrt{\frac{1}{t^2}}\,t}{\sqrt{4+\frac{1}{t^2}+4\,t^2}}\right\}$$

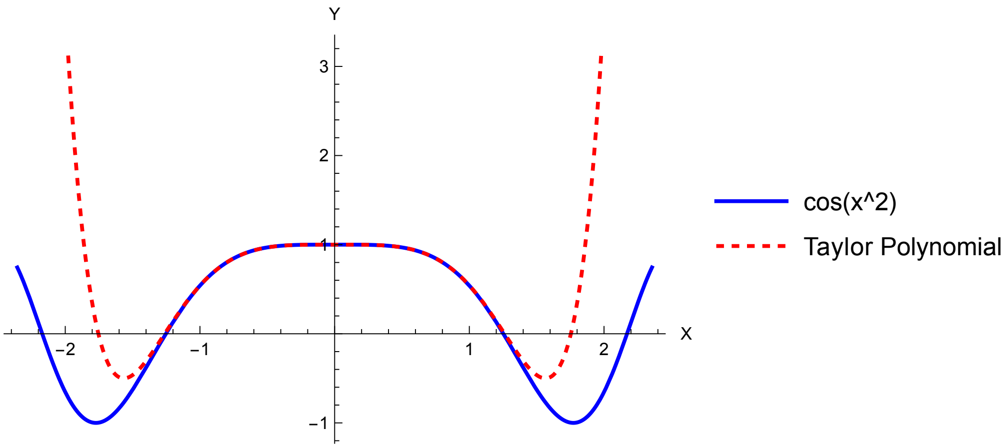
Problem 10

```
ClearAll["Global`*"]
(*Taylor Series for Cos(x^2)*)
series = Normal[Series[Cos[x^2], {x, 0, 10}]]
Plot[{Cos[x^2], series}, {x, -3 Pi / 4, 3 Pi / 4},
 PlotLegends → {"cos(x^2)", "Taylor Polynomial"},
 PlotStyle → {{Blue}, {Red, Dashed}},
 AxesLabel → {"X", "Y"}
]
actualValue = N[Cos[(Pi / 4) ^ 2]]
taylorValue = N[series /. x → Pi / 4]
Print["At x = π/4:"]
Print["Actual value = ", actualValue]
Print["Taylor polynomial value = ", taylorValue]
```

Out[ ]=

$$1 - \frac{x^4}{2} + \frac{x^8}{24}$$

Out[ ]=



Out[ ]=

0.815705

Out[ ]=

0.815781

At x = π/4:

Actual value = 0.815705

Taylor polynomial value = 0.815781