

# Vector-Valued Image Regularization with PDEs

## COURSE PROJECT REPORT

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### Fiery Scorpions

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## Introduction

We have tried to implement the paper

[https://tschumperle.users.greyc.fr/publications/tschumperle\\_pami05.pdf](https://tschumperle.users.greyc.fr/publications/tschumperle_pami05.pdf).

We particularly understand and demonstrate that the oriented-Laplacian formalism has an interesting interpretation in terms of local filtering, and represents the right smoothing geometry performed by the PDEs. Thus, it allows us to design an efficient vector-valued regularization approach, respecting desired local smoothing properties.

Finally, we apply our method to solve a wide range of image processing issues, including color image restoration, inpainting, magnification, and flow visualization as mentioned in the paper. We present our results here.

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**Main Formula:**

$$G = \sum_{j=1}^n \nabla I_j \nabla I_j^T$$

$$\frac{\partial I_i}{\partial t} = \text{trace}(TH_i) \quad (i = 1..n)$$

$$T = f_-(\sqrt{\lambda^*_+ + \lambda^*_-})\theta^*_-\theta^{*-T}_- + f_+(\sqrt{\lambda^*_+ + \lambda^*_-})\theta^*_+\theta^{*-T}_+$$

In local areas of a pixel, T remains approximately constant, where we get a solution of the above PDE as follows

$$I_{i(t)} = I_{i(t=0)} * G^{(T,t)}$$

where,

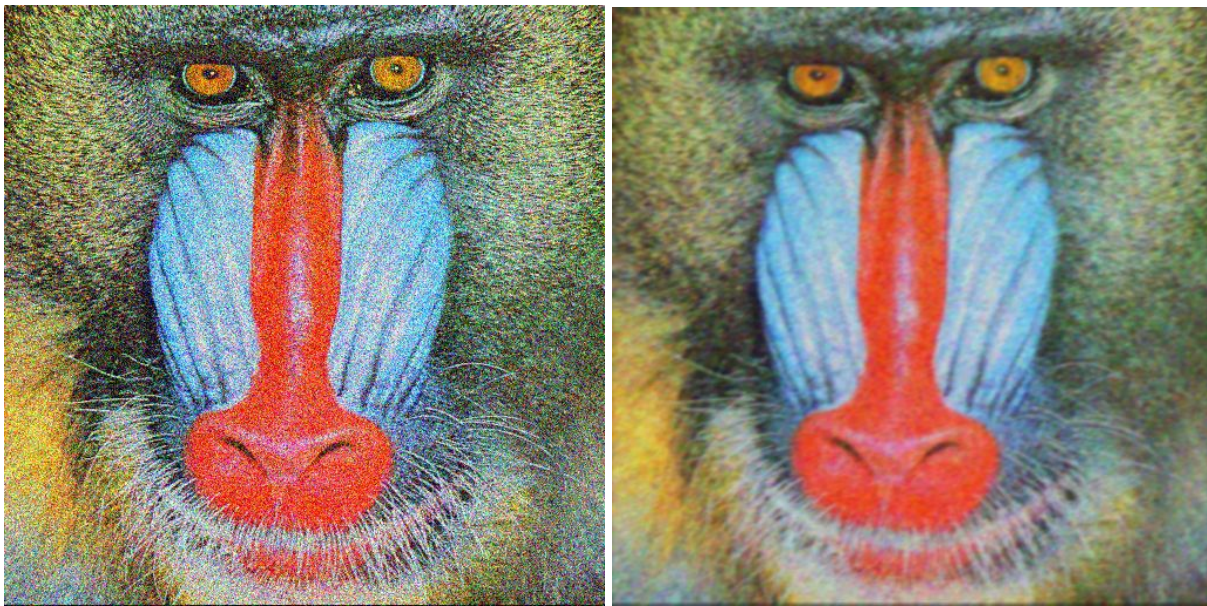
$$G^{(T,t)}(x) = \frac{1}{4\pi t} \exp\left(-\frac{x^T T^{-1} x}{4t}\right) \quad \text{with} \quad X = (x \ y)^T$$

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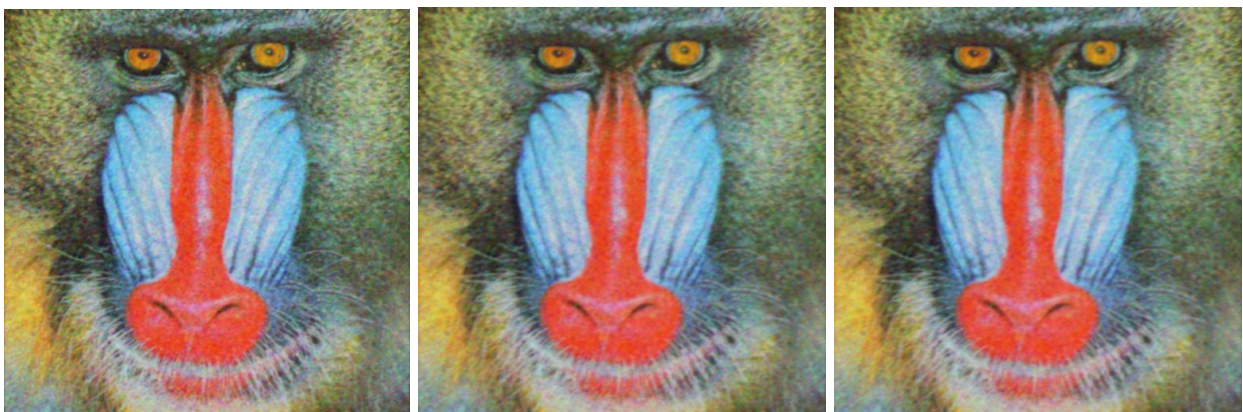
## Image Denoising / Restoration

Best results were obtained for  $t = 5$ .

- Baboon image.



Following are denoised images for For  $t=1$ , 5 and 10 respectively



- 
- Child image



Following are denoised images for For  $t=1, 5$  and  $10$  respectively





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- Desert image



Following are denoised images for For  $t=1, 5$  and  $10$  respectively



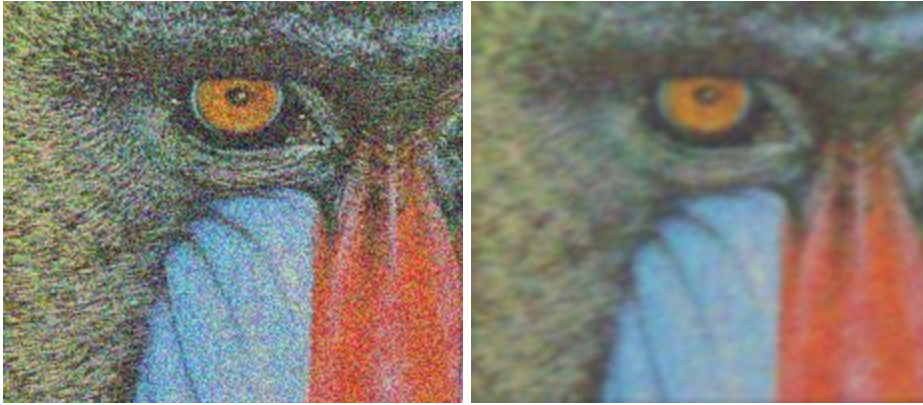
- 
- Face image



Following are denoised images for For  $t=1, 5$  and  $10$  respectively



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- Monkey image



Following are denoised images for For  $t=1, 5$  and 10 respectively





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## Image Inpainting

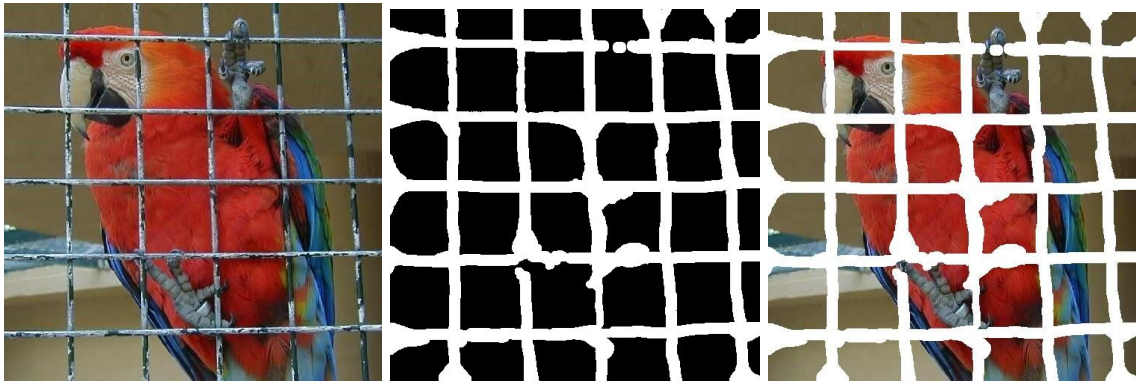
Showing results of 150 iterations which took about 7 min for each image

- Parrot image

Original Image

Mask

Masked Image

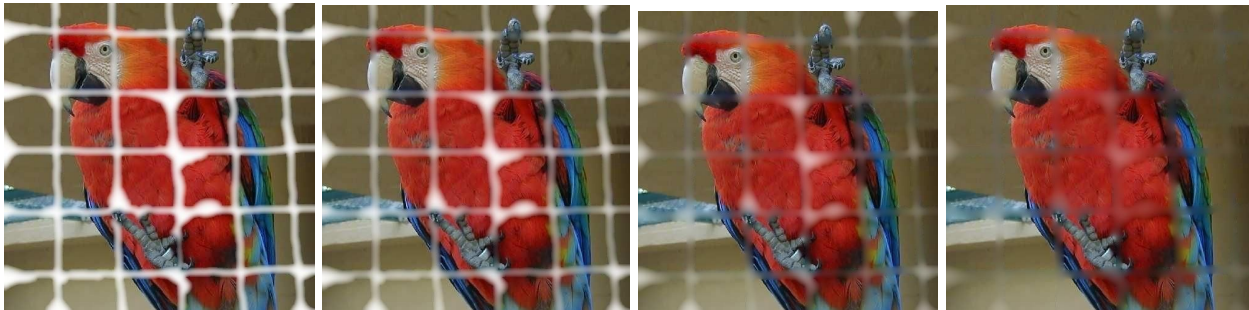


10 Iter

20 Iter

50 Iter

100 Iter







- Text Image

Orig Image

Mask

150 Iter

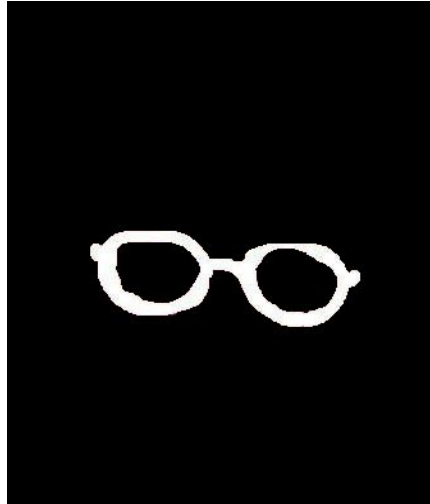


- 
- Person with specs

Orig Image

Mask

150 Iter(some grey effects  
because of unknown reason)

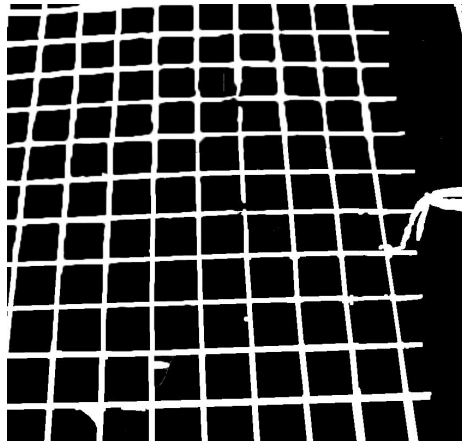
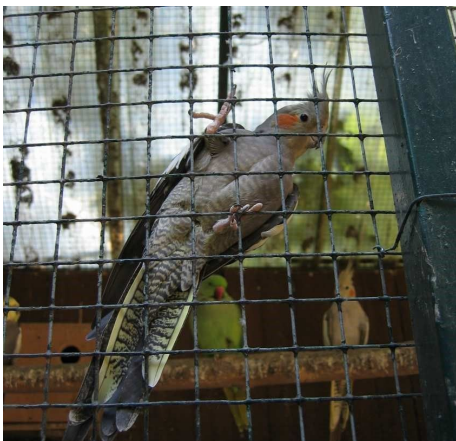


- Bird Cage

Orig Image

Mask

150 Iter

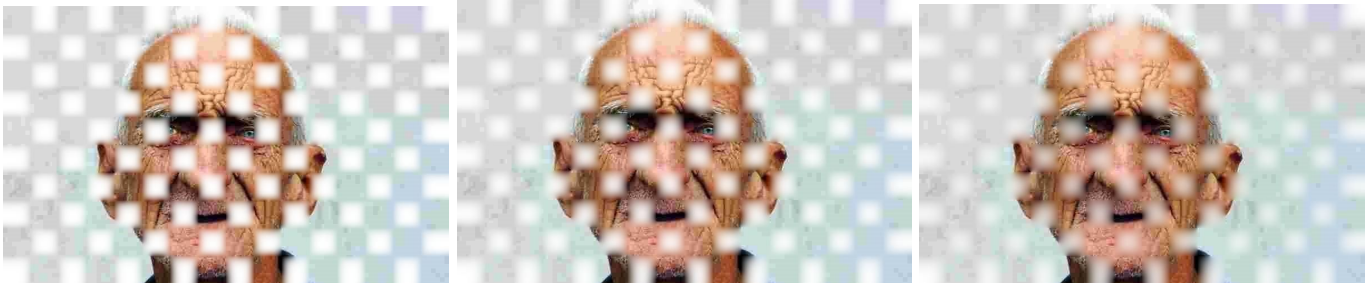
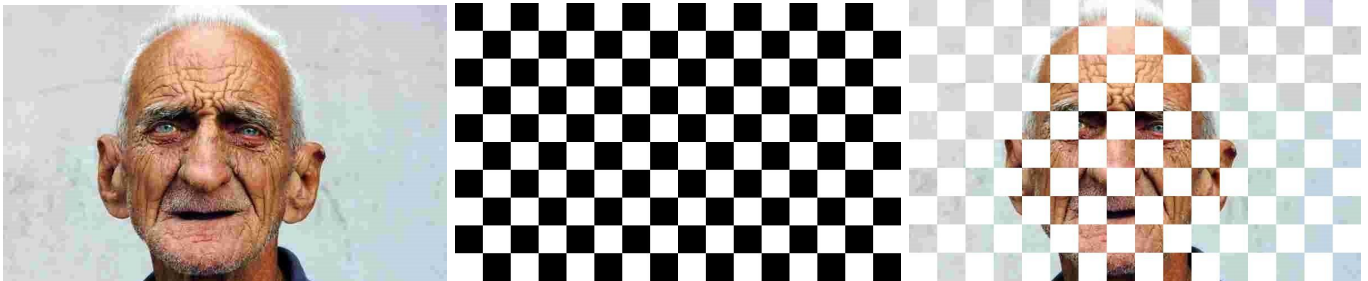


- 
- 50 % Data Removal 50 block size

Orig Image

Mask

150 Iter



After 20 Iter

After 50 Iter

After 70 Iter

Orig Image

After 150 Iter





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- 50 % Data Removal 10 block size

Orig Image

Mask

orig Image with Mask



Inpainted Image (70 Iter)

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## Image Magnification / Supersampling

Original images -



- Bilinear interpolation vs PDE approach.

3x -



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4x -





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10x -



- 
- Nearest neighbour vs PDE approach

3x -



4x -

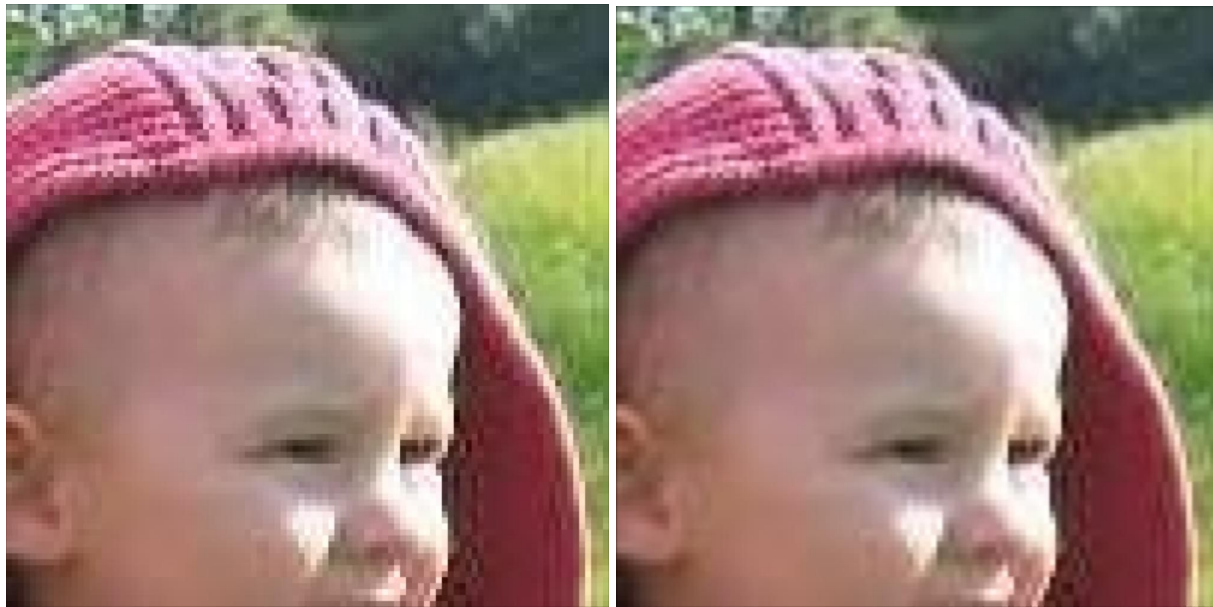




10x -







## Flow Visualization

- Original Image

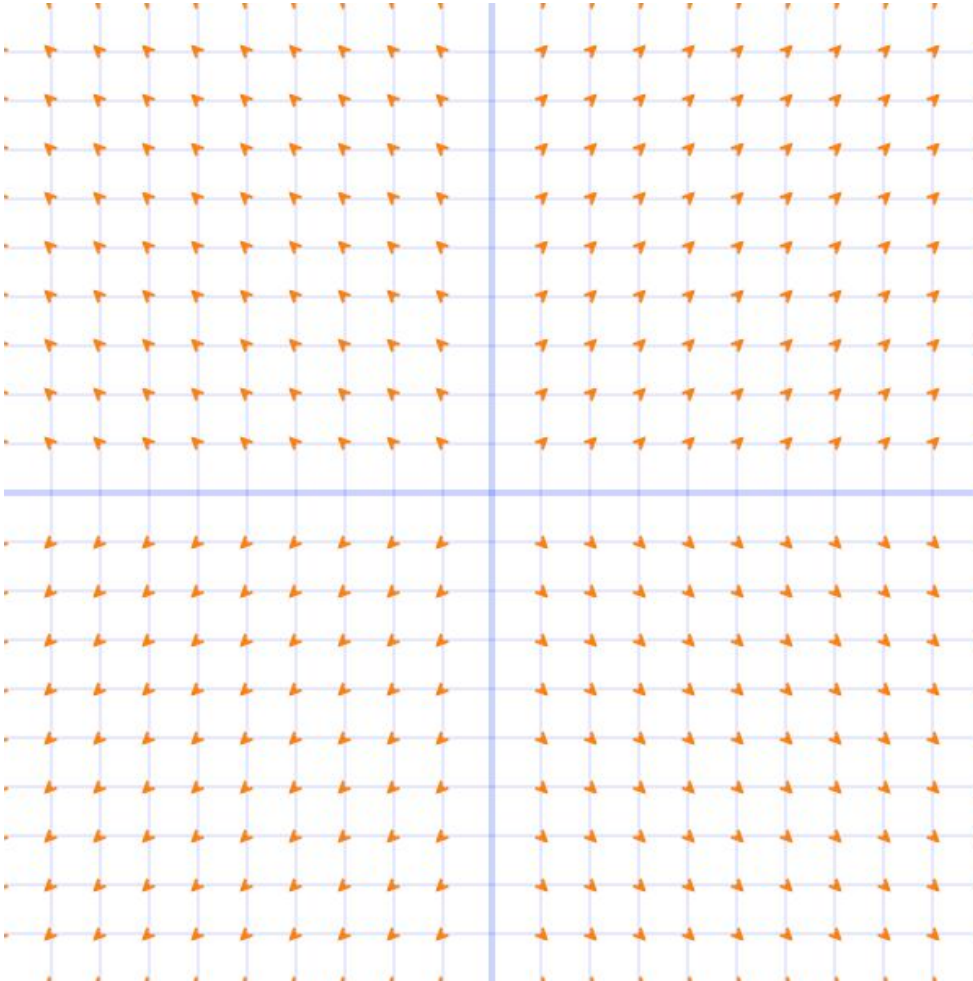


Took around 18 min to produce the below result for 1500 iterations across 3 fields.

Format of display :-

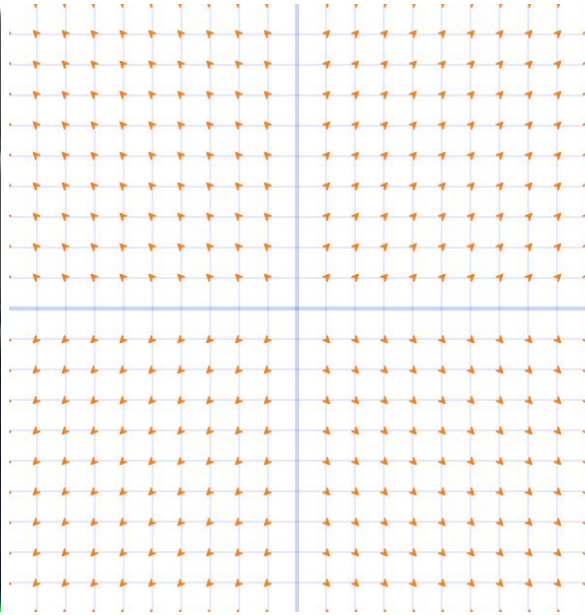
100 iter	200 Iter
300 Iter	400 Iter
500 Iter	Field

- 
- Field 1

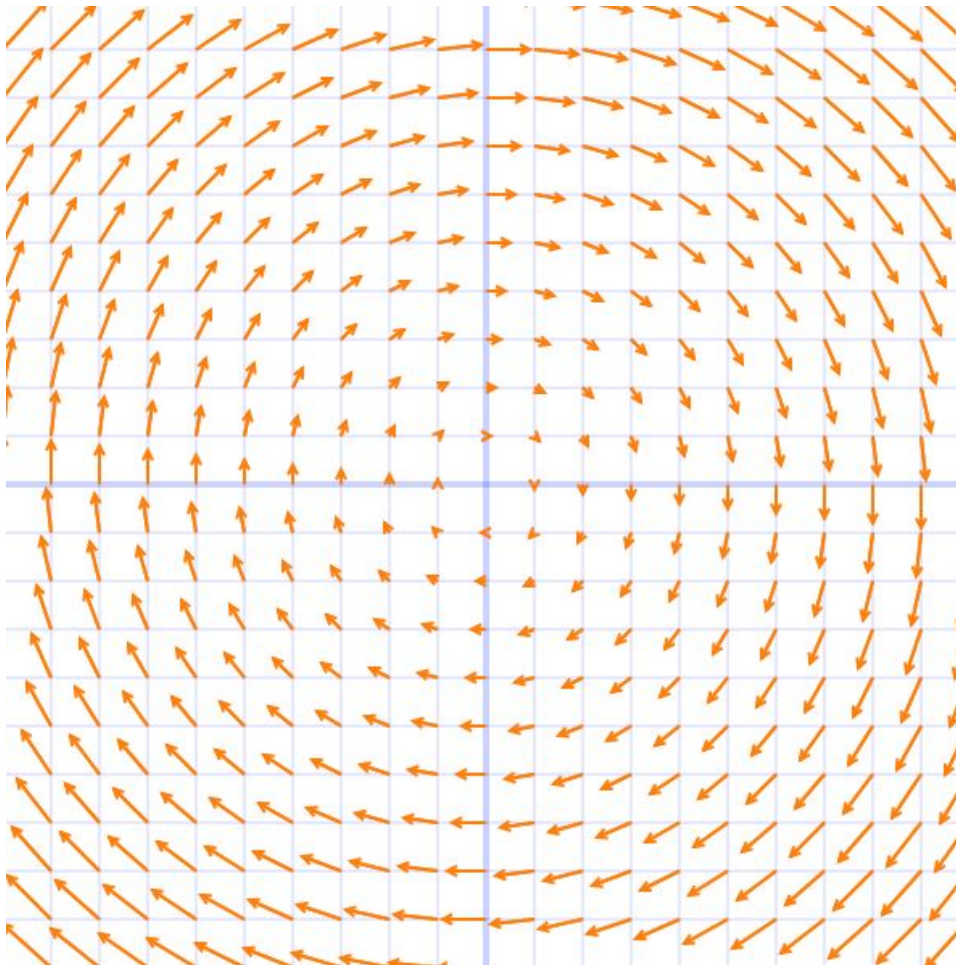






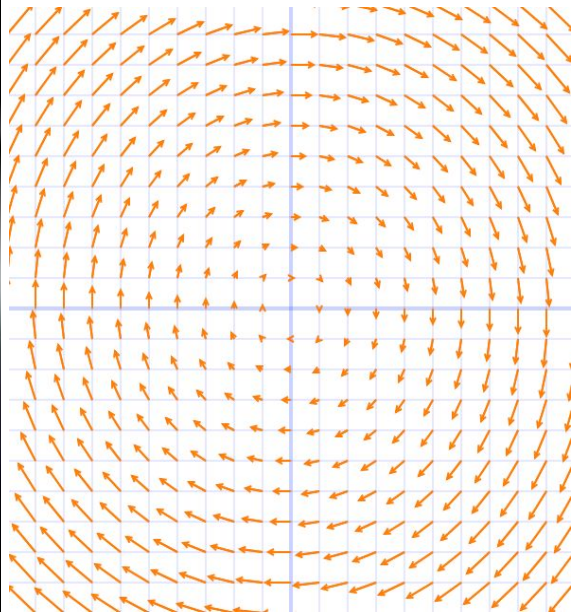


- Field2









- Field3

