Lecture notes – Sept 15th, 2020

Constrained Optimization Example

x, y – goods

P_x, P_y – prices

I – income

$$\max_{x,y} U(x,y)$$

$$\text{s.t.} \quad P_x \cdot x + P_y \cdot y \le I \quad (\lambda)$$

$\uparrow$ Lagrange multiplier

$$\mathcal{L} = U(x,y) + \lambda \cdot (P_x \cdot x + P_y \cdot y - I)$$

$$\frac{\partial \mathcal{L}}{\partial x} = U_x + \lambda \cdot P_x \qquad \frac{\partial \mathcal{L}}{\partial y} = U_y + \lambda \cdot P_y$$

1. $\lambda = 0$, constraint not binding $\quad P_x \cdot x + P_y \cdot y < I$

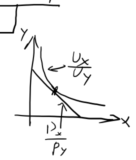
$$U_x = \frac{\partial \mathcal{L}}{\partial x} = 0 \qquad U_x = U_y = 0$$

2. constraint binding. $\quad P_x \cdot x + P_y \cdot y = I$

$$\underline{U_x + \lambda \cdot P_x} = \frac{\partial \mathcal{L}}{\partial x} = 0 \qquad \underline{U_y + \lambda \cdot P_y} = 0$$

$$\downarrow$$

$$MRS_{x,y} = \frac{U_x}{U_y} = \frac{P_x}{P_y}$$



Expectation Example

$$X = \begin{matrix} 1 & 2 & 3 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{matrix}$$

$$\mathbb{E}(X) = \sum_{s=1}^{3} x_s \cdot \mathbb{P}(x = x_s)$$

$$= 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{4}$$

$$= 2$$

$\mathbb{E}(\cdot)$ – function

$$\mathbb{E}(f(x)) = \sum_{s=1}^{3} f(x_s) \cdot \mathbb{P}(x = x_s)$$

example

$$\mathbb{E}(X^2) = \sum_{s=1}^{3} x^2 \cdot P(x) = 1 \cdot \frac{1}{4} + 4 \cdot \frac{1}{2} + 9 \cdot \frac{1}{4}$$

$$= 4.5$$

$$= \begin{pmatrix} 1 \\ 4 \\ 9 \end{pmatrix} \begin{pmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix}$$

Joint Distribution Example

		Alice: Zoo	Alice: Tennis
Bob	Zoo	$\frac{1}{6}$	$\frac{1}{3}$
	Tennis	$\frac{1}{3}$	$\frac{1}{6}$

$$P(A = Z, B = Z) = \frac{1}{6}$$

$$P(A = Z, B = T) = \frac{1}{3}$$

marginal dist: Bob $\to$ Zoo? $\quad P(B=Z) = P(A=Z, B=Z) + P(A=T, B=Z)$

$$= \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

conditional dist: $\quad P(B=Z \mid A=Z) = \dfrac{P(B=Z, A=Z)}{P(A=Z)}$

$$= \frac{1/6}{1/2} = \frac{1}{3}$$

Boolean Operator Rules

True = 1 False = 0

And:

1	1	$\to$	1
1	0	$\to$	0
0	1	$\to$	0
0	0	$\to$	0

Or:

1	1	$\to$	1
1	0		1
0	1		1
0	0		0