In[219]:=
```
ClearAll["Global`*"]
```

Nuance of the sound

Let's begin by listening to a short piece composed by Hans Zimmer for the movie Dunkirk.

While listening, notice this feeling:
the pitch seems to keep rising and never stops.

HERE

This effect comes from a sound illusion called the Shepard-Risset Glissando.

What is shepherd tone?

A Shepard tone is is an illusion in which the pitch seems to go up (or down) forever, even though it never actually gets higher or lower.

What is shepherd-Risset Glissando?

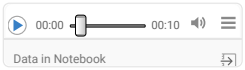
This is the continuous seamless slide (*glissando*) version of shepherd tone.

Frequency *aka* Pitch:

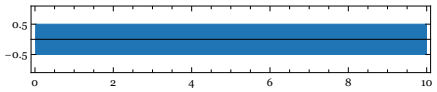
Sound of sine frequency @ 500 Hz with 50% loudness:

In[220]:=
```
ClearAll["Global`*"]

fs = 44100;     (*sample rate*)
dur = 10;      (*seconds*)
t = Range[0, dur, 1 / fs];

signal = 0.50 Sin[2 Pi 500 t];

audio1 = Audio[signal, SampleRate → fs]

AudioPlot[audio1]

(*Export["Sine_500Hz_50Loud.wav", audio1]*)
```

Out[225]=



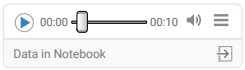
Out[226]=



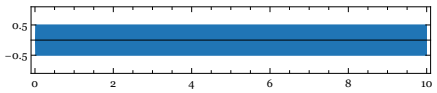
Sound of sine frequency @ 1000 Hz with 50% loudness:

In[227]:=
```
audio2 = Audio[Play[50 Sin[2 π 1000 t], {t, 0, 10}, PlayRange → {-100, 100}]]
AudioPlot[audio2]
(*Export[
 "D:\\Vibration_Isolator_Project\\Shepherd_tone\\sound/Sine_1000Hz_50Loud.
   wav",audio2]*)
```

Out[227]=



Out[228]=

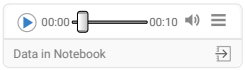


Amplitude *aka* Loudness:

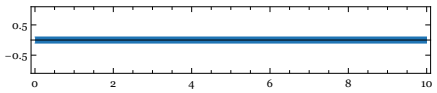
Sound of sine frequency @ 500 Hz with 50% loudness:

In[229]:=
```
audio3 = Audio[Play[10 Sin[2 π 1000 t], {t, 0, 10}, PlayRange → {-100, 100}]]
AudioPlot[audio3]
(*Export["D:\\Vibration_Isolator_Project\\Shepherd_tone\\sound/Sine_1000Hz_10Loud.wav",audio3]*)
```

Out[229]=



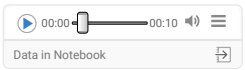
Out[230]=



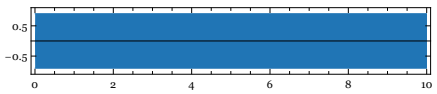
Sound of sine frequency @ 500 Hz with 80% loudness:

In[231]:=
```
audio4 = Audio[Play[90 Sin[2 π 1000 t], {t, 0, 10}, PlayRange → {-100, 100}]]
AudioPlot[audio4]
(*Export["D:\\Vibration_Isolator_Project\\Shepherd_tone\\sound/Sine_1000Hz_90Loud.wav",audio4]*)
```

Out[231]=



Out[232]=

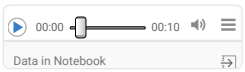


Frequency sweep:

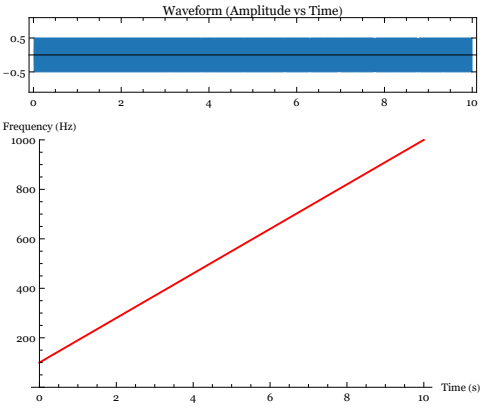
Sound of sine frequency from 100 - 1000 Hz with 50% loudness:

In[233]:=
```
f1 = 100;
f2 = 1000;
T = 10;
ampFreq = 50;
freq[t_] := f1 + (f2 - f1) t / T;
phase[t_] := 2 π ∫_0^t freq[τ] dτ;
audio5 = Audio[Play[ampFreq Sin[phase[t]], {t, 0, T}, PlayRange → {-100, 100}]]
freqPlot =
  Plot[freq[t], {t, 0, T}, PlotRange → {0, f2}, AxesLabel → {"Time (s)", "Frequency (Hz)"}, PlotStyle → Red, ImageSize → 400];
Column[{AudioPlot[audio5, PlotLabel → "Waveform (Amplitude vs Time)", ImageSize → 400], freqPlot}]
(*Export["D:\\Vibration_Isolator_Project\\Shepherd_tone\\sound/Sine_100-1000Hz_50Loud.wav",audio5]*)
```

Out[239]=



Out[241]=

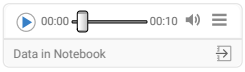


Loudness sweep:

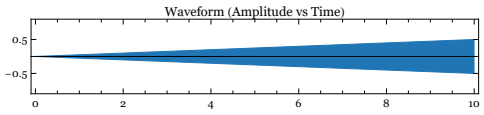
Sound of sine frequency from @500 Hz with 0-100% loudness:

In[242]:=
```
f = 500;
T = 10;
ampMax = 50;
amp[t_] := ampMax t / T;
phase[t_] := 2 π f t;
audio6 = Audio[Play[amp[t] Sin[phase[t]], {t, 0, T}, PlayRange → {-100, 100}]]
ampPlot =
  Plot[amp[t], {t, 0, T}, PlotRange → {0, ampMax}, AxesLabel → {"Time (s)", "Amplitude"}, PlotStyle → Blue, ImageSize → 400];
Column[{AudioPlot[audio6, PlotLabel → "Waveform (Amplitude vs Time)", ImageSize → 400]}]
(*Export["D:\\Vibration_Isolator_Project\\Shepherd_tone\\sound/Sine_500Hz_0-100Loud.wav",audio6]*)
```

Out[247]=



Out[249]=

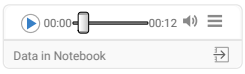


Toward Shephard Tone:

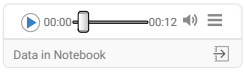
Discreet sweep of sine frequency from 176.8-687.1 Hz with 100% loudness:

In[250]:=
```
ClearAll["Global`*"];
fs = 44100;
Tmax = 12.`;
n = Round[Tmax fs];
line1 = ConstantArray[0.`, n];
line2 = ConstantArray[0.`, n];
amp = 1;
fadeTime = 0.03`;
fadeN = Round[fadeTime fs];
ampWindow[len_] := Module[{f = Min[fadeN, Floor[len/2]], flat},
  flat = Max[len - 2 f, 0];
  Join[Table[0.5` (1 - Cos[π i/f]), {i, 0, f - 1}], ConstantArray[1.`, flat], Table[0.5` (1 + Cos[π i/f]), {i, 0, f - 1}]]];
data = {{0.`, 0.5`, 176.8`, 353.6`}, {0.5`, 1.`, 182.`, 364.`}, {1.`, 1.5`, 187.3`, 374.6`}, {1.5`, 2.`, 192.8`, 385.6`}, {2.`, 2.5`, 198.5`, 396.9`}, {2.5`, 3.`, 204.3`, 408.5`},
   {3.`, 3.5`, 210.3`, 420.5`}, {3.5`, 4.`, 216.4`, 432.8`}, {4.`, 4.5`, 222.8`, 445.5`}, {4.5`, 5.`, 229.3`, 458.6`}, {5.`, 5.5`, 236.`, 472.`}, {5.5`, 6.`, 242.9`, 485.8`},
   {6.`, 6.5`, 250.`, 500.1`}, {6.5`, 7.`, 257.4`, 514.7`}, {7.`, 7.5`, 264.9`, 529.8`}, {7.5`, 8.`, 272.7`, 545.3`}, {8.`, 8.5`, 280.7`, 561.3`}, {8.5`, 9.`, 288.9`, 577.8`},
   {9.`, 9.5`, 297.3`, 594.7`}, {9.5`, 10.`, 306.1`, 612.1`}, {10.`, 10.5`, 315.`, 630.`}, {10.5`, 11.`, 324.3`, 648.5`}, {11.`, 11.5`, 333.8`, 667.5`}, {11.5`, 12.`, 343.5`, 687.1`}};
ph1 = 0.`;
ph2 = 0.`;
Do[{ts, te, f1, f2} = row;
  i1 = Floor[ts fs] + 1;
  i2 = Min[Floor[te fs], n];
  len = i2 - i1 + 1;
  t = Range[0, len - 1]/fs;
  w = ampWindow[len];
  p1 = ph1 + 2 π f1 t;
  p2 = ph2 + 2 π f2 t;
  line1[[i1 ;; i2]] = amp w Sin[p1];
  line2[[i1 ;; i2]] = amp w Sin[p2];
  ph1 = Last[p1] + 2 π f1/fs;
  ph2 = Last[p2] + 2 π f2/fs;, {row, data}];
audioLine1Constantloud = Audio[{line1}, SampleRate → fs]
audioLine2Constantloud = Audio[{line1}, SampleRate → fs]
audioPlot1 = AudioPlot[audioLine1Constantloud, PlotLabel → "Line 1: Waveform (Amplitude vs Time)", ImageSize → 400];
audioPlot2 = AudioPlot[audioLine2Constantloud, PlotLabel → "Line 2: Waveform (Amplitude vs Time)", ImageSize → 400];
freqLine1 = data[[All, {1, 3}]];
freqLine2 = data[[All, {1, 4}]];
freqPlot1 = ListStepPlot[freqLine1, PlotRange → All, AxesLabel → {"Time (s)", "Frequency (Hz)"}, PlotStyle → Red, ImageSize → 400, PlotLabel → "Line 1: Frequency vs Time"];
freqPlot2 = ListStepPlot[freqLine2, PlotRange → All, AxesLabel → {"Time (s)", "Frequency (Hz)"}, PlotStyle → Blue, ImageSize → 400, PlotLabel → "Line 2: Frequency vs Time"];
Row[{audioPlot1, freqPlot1}]
Row[{audioPlot2, freqPlot2}]
(*Export["D:\\Vibration_Isolator_Project\\Shepherd_tone\\sound/Sine_177-343Hz_100Loud.wav",audioLine1Constantloud]
 Export["D:\\Vibration_Isolator_Project\\Shepherd_tone\\sound/Sine_343-687Hz_100Loud.wav",audioLine2Constantloud]*)
```

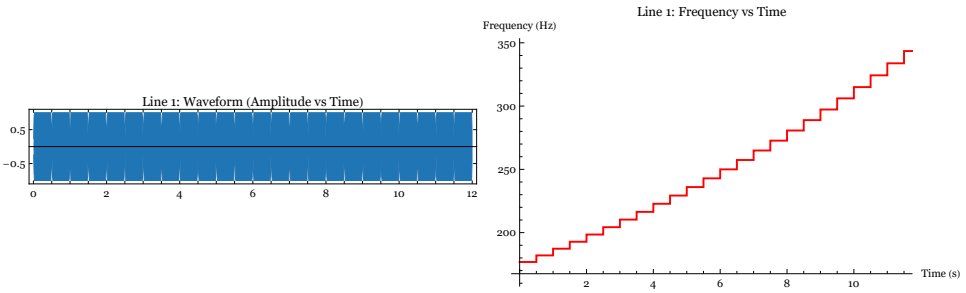
Out[264]=



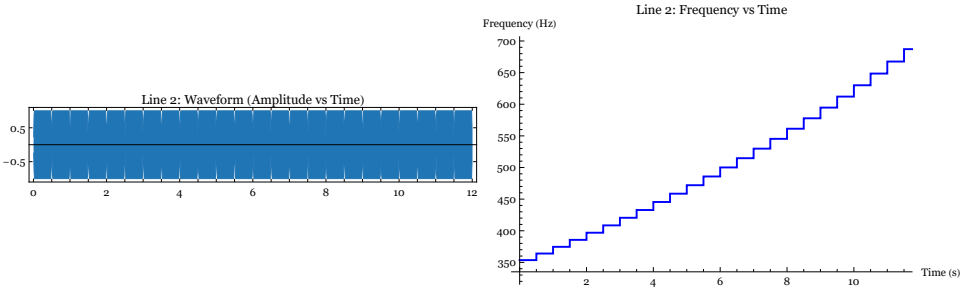
Out[265]=



Out[272]=



Out[273]=



Toward Shephard Tone:

Discreet sweep of sine frequency from 176.8-687.1 Hz with 0-100% loudness:

In[274]:=
```
ClearAll["Global`*"];
fs = 44100;
Tmax = 12.0;
n = Round[Tmax fs];
line1 = ConstantArray[0.`, n];
line2 = ConstantArray[0.`, n];
loudMax = 240.`;
ampMax = 1.`;
ampScale[l_] := ampMax Clip[1/loudMax, {0, 1}];
fadeTime = 0.03`;
fadeN = Round[fadeTime fs];
ampWindow[len_] := Module[{f = Min[fadeN, Floor[len/2]], flat},
  flat = Max[len - 2 f, 0];
  Join[Table[0.5` (1 - Cos[π i/f]), {i, 0, f - 1}], ConstantArray[1.`, flat], Table[0.5` (1 + Cos[π i/f]), {i, 0, f - 1}]]];
freqData = {{0.`, 0.5`, 176.8`, 353.6`}, {0.5`, 1.`, 182.`, 364.`}, {1.`, 1.5`, 187.3`, 374.6`}, {1.5`, 2.`, 192.8`, 385.6`}, {2.`, 2.5`, 198.5`, 396.9`},
   {2.5`, 3.`, 204.3`, 408.5`}, {3.`, 3.5`, 210.3`, 420.5`}, {3.5`, 4.`, 216.4`, 432.8`}, {4.`, 4.5`, 222.8`, 445.5`}, {4.5`, 5.`, 229.3`, 458.6`},
   {5.`, 5.5`, 236.`, 472.`}, {5.5`, 6.`, 242.9`, 485.8`}, {6.`, 6.5`, 250.`, 500.1`}, {6.5`, 7.`, 257.4`, 514.7`}, {7.`, 7.5`, 264.9`, 529.8`},
   {7.5`, 8.`, 272.7`, 545.3`}, {8.`, 8.5`, 280.7`, 561.3`}, {8.5`, 9.`, 288.9`, 577.8`}, {9.`, 9.5`, 297.3`, 594.7`}, {9.5`, 10.`, 306.1`, 612.1`},
   {10.`, 10.5`, 315.`, 630.`}, {10.5`, 11.`, 324.3`, 648.5`}, {11.`, 11.5`, 333.8`, 667.5`}, {11.5`, 12.`, 343.5`, 687.1`}, {12.0, 12.5, 353.6`, 707.2`}};

loudData = {{0, 240}, {10, 230}, {20, 220}, {30, 210}, {40, 200}, {50, 190}, {60, 180}, {70, 170}, {80, 160}, {90, 150}, {100, 140}, {110, 130},
   {120, 120}, {130, 110}, {140, 100}, {150, 90}, {160, 80}, {170, 70}, {180, 60}, {190, 50}, {200, 40}, {210, 30}, {220, 20}, {230, 10}, {0, 240}};
ph1 = 0.`;
ph2 = 0.`;
Do[{ts, te, f1, f2} = freqData[[k]];
  {l1, l2} = loudData[[k]];
  i1 = Floor[ts fs] + 1;
  i2 = Min[Floor[te fs], n];
  len = i2 - i1 + 1;
  t = Range[0, len - 1]/fs;
  w = ampWindow[len];
  a1 = ampScale[l1];
  a2 = ampScale[l2];
  p1 = ph1 + 2 π f1 t;
  p2 = ph2 + 2 π f2 t;
  line1[[i1 ;; i2]] = a1 w Sin[p1];
  line2[[i1 ;; i2]] = a2 w Sin[p2];
  ph1 = Last[p1] + 2 π f1/fs;
  ph2 = Last[p2] + 2 π f2/fs;, {k, Length[freqData]}];
audioLine1 = Audio[{line1}, SampleRate → fs]
audioLine2 = Audio[{line2}, SampleRate → fs]
audioPlot1 = AudioPlot[audioLine1, PlotLabel → "Line 1: Waveform (Amplitude vs Time)", ImageSize → 400];
audioPlot2 = AudioPlot[audioLine2, PlotLabel → "Line 2: Waveform (Amplitude vs Time)", ImageSize → 400];
freqPlot1 = ListStepPlot[freqData[[All, {1, 3}]], AxesLabel → {"Time (s)", "Frequency (Hz)"}, PlotStyle → Red, ImageSize → 400, PlotLabel → "Line 1: Frequency vs Time"];
freqPlot2 = ListStepPlot[freqData[[All, {1, 4}]], AxesLabel → {"Time (s)", "Frequency (Hz)"}, PlotStyle → Blue, ImageSize → 400, PlotLabel → "Line 2: Frequency vs Time"];
Grid[{{audioPlot1, freqPlot1}, {audioPlot2, freqPlot2}}, Alignment → Center, Spacings → {2, 2}]
(*Export["D:\\Vibration_Isolator_Project\\Shepherd_tone\\sound/Sine_177-343Hz_0-100Loud.wav",audioLine1]
 Export["D:\\Vibration_Isolator_Project\\Shepherd_tone\\sound/Sine_343-687Hz_100-0Loud.wav",audioLine2]*)
```

Last: {} has zero length and no last element.

Last: {} has zero length and no last element.

Out[291]=

00:00 00:12

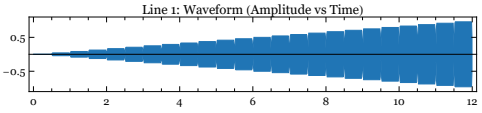
Data in Notebook

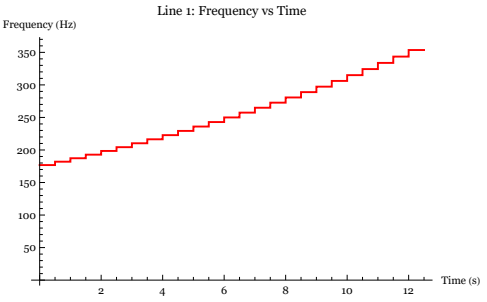
Out[292]=

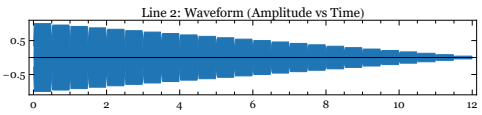
00:00 00:12

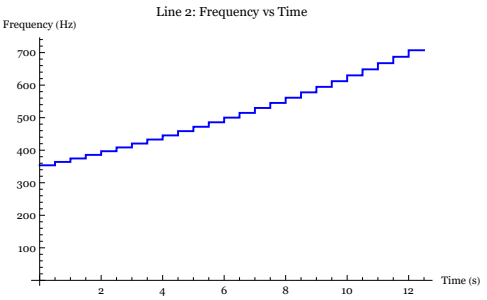
Data in Notebook

Out[297]=


Line 1: Waveform (Amplitude vs Time)
0.5
−0.5
0
2
4
6
8
10
12


Line 1: Frequency vs Time
Frequency (Hz)
350
300
250
200
150
100
50
2
4
6
8
10
12
Time (s)


Line 2: Waveform (Amplitude vs Time)
0.5
−0.5
0
2
4
6
8
10
12

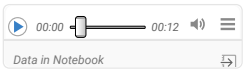

Line 2: Frequency vs Time
Frequency (Hz)
700
600
500
400
300
200
100
2
4
6
8
10
12
Time (s)

Shephard Tone:

Now add the play the both sound together.

In[298]:=
```
audioBoth = Audio[{line1, line2}, SampleRate → fs]
T = 12;
nLoops = 5;
loopsLow = Table[freqData[[All, {1, 3}]] /. {t_, f_} :↦ {t + k T, f}, {k, 0, nLoops - 1}];
loopsHigh = Table[freqData[[All, {1, 4}]] /. {t_, f_} :↦ {t + k T, f}, {k, 0, nLoops - 1}];
freqPlotBoth = ListStepPlot[Join[loopsLow, loopsHigh],
    AxesLabel → {"Time (s)", "Frequency (Hz)"}, PlotStyle → Join[Table[Red, nLoops], Table[Blue, nLoops]],
    (*PlotLabel→"Line 1 and Line 2 Frequency (5 Loops, Broken)"*)ImageSize → 400];
audioLoop4 = AudioJoin[ConstantArray[audioBoth, nLoops]];
audioPlotBoth = AudioPlot[audioLoop4, PlotLayout → {"Overlaid", "Overlaid"},
    PlotRange → All, (*PlotLabel→"Line 1 and Line 2 Waveforms (5 Loops)"*)ImageSize → 400];
Row[{audioPlotBoth, freqPlotBoth}]

freqLow = Flatten[loopsLow, 1];
freqHigh = Flatten[loopsHigh, 1];

freqLowTable = ({#[[1]], #[[2]], 1} & /@ freqLow);
freqHighTable = ({#[[1]], #[[2]], 2} & /@ freqHigh);

freqTable = Join[freqLowTable, freqHighTable];

(*Export["E:\\BrokenFate\\github_files\\Reflections_of_a_Curious_Mind\\graph\\frequency_vs_time.txt",freqTable,"Table"]*)


(*Export[
 "D:\\Vibration_Isolator_Project\\Shepherd_tone\\sound/Shepherd_sine_177-343Hz_0-100Loud_sine_343-687Hz_100-0Loud.wav",
 audioBoth]*)
```

Out[298]=



Out[306]=