Valuation of Commodity Storage

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January 2020

1 Introduction

Define v_i as the decision volume of commodity injected or withdrawn from storage at time t_i. To clarify, positive value of v_i denotes injection into storage, increasing the inventory, where as a negative value denotes the volume withdrawn. The admissible values of v_i are restricted by the minimum and maximum inject/withdraw rate functions v_{min} and v_{max}.

$$v_i \in [v_{min}(t_i, V_i), v_{max}(t_i, V_i)] \tag{1}$$

Note that v_{min} and v_{max} are both functions of time and V_i, which represents the inventory in storage at time t_i. Inventory-varying injection and withdrawal rates are commonly seen in natural gas storage, where higher storage cavern pressue results from higher inventory, which results in a higher maximum withdrawal rate, and lower maximum withdrawal rate.

The inventory V_i can be defined recursively as:

$$V_i = V_{i-1} + v_{i-1} - L(t_{i-1}, V_{i-1}) \tag{2}$$

Where $L(t_{i-1}, V_{i-1})$ is the inventory loss, as a function of time and inventory, evaluated for the previous time period. An alternative representation of inventory is as a summation:

$$V_i = V_{start} + \sum_{j=0}^{i-1} (v_j - L(t_j, V_j)) \tag{3}$$

V_{start} is the inventory at the inception. This is necessary in the case where storage capacity is purchased or leased with some amount of commodity inventory in place.

V_i is itself constrained to be within V_{min} and V_{max}, the minimum and maximum inventory functions.

$$V_i \in [V_{min}(t_i), V_{max}(t_i)] \tag{4}$$

Commonly V_{min} will evaluate to zero for all time periods, but examples where non-zero minimum inventory is needed include when regulations require a minimum level of inventory is held, as is seen for natural gas storage in some European countries. Two possible reasons why V_{max} need to be functions of time are:

- Storage could be leased for consecutive time periods, but for different notional volumes.
- The terms of leased storage commonly stipulate that the storage must be empty at the time that the leased capacity ends.

Mathematical constraints.
- Set of decision times.
- Decision volume (inject/withdraw) for each time.
- Set of admissible decision volumes which adhere to constraints.

2 Cash Flows

Define $p(v_i, t_i)$ as the net present (discounted) cash flows resulting from decision volume v_i at time t_i.

$$p(v_i, t_i) = (\mu(t_i, V_i, v_i) + v_i)s_i d(c(t_i)) - \pi(t_i, V_i, v_i) \tag{5}$$

Where:

- s_i is the spot commodity price at time t_i.
- $c(t_i)$ is the commodity settlement time function which maps from the time that commodity was delivered to the time that payment is made. In European energy markets this is usually a formulaic date in the next month.
- d is the discount factor function. Encorporating current market risk-free interest rates, this function maps from a time of a cash flow to the present value of one unit of money. Any cash flow at future time t can be multiplied by $d(t)$ in order calculate the present value of this cash flow.
- $\mu(t_i, V_i, v_i)$ is the volume of commodity consumed (not added to inventory) by the storage facility, as a function of time, inventory and decision volume. In practice this term is relevant for energy storage where some quantity of the energy commodity is consumed in order to power the motors used to get the commodity into or out of storage.
- $\pi(t_i, V_i, v_i)$ is the NPV of any other costs which are generated. An example of this in practice is the cost of running motors which facilitate injection or withdrawal.

- NPV from inject/withdraw, equals decision volume, adjusted for lost volume, times cmdty forward price, discounted.
- NPV from inject/withdraw cost.
- NPV from inventory cost.
- Terminal cmdty NPV function.

Valuation Theory.
- Optimal value as risk-neutral expectation for NPVs.
- Bellman equation.

Valuation In Practice.
- Bang-bang assumption: reduce the decision volume set.
- Discretise the set of inventories at which the Bellman equation is evaluated.
- Problem then boils down to calculating the expectation.