Valuation of Commodity Storage

Jake C. Fowler

January 2020

1 Introduction

Define v_i as the decision volume of commodity injected or withdrawn from storage at time t_i. To clarify, positive value of v_i denotes injection into storage, increasing the inventory, where as a negative value denotes the volume withdrawn. The admissible values of v_i are restricted by the minimum and maximum inject/withdraw rate functions v_{min} and v_{max}.

$$v_i \in [v_{min}(t_i, V_i), v_{max}(t_i, V_i)] \tag{1}$$

Note that v_{min} and v_{max} are both functions of time and V_i, which represents the inventory in storage at time t_i. Inventory-varying injection and withdrawal rates are commonly seen in natural gas storage, where higher storage cavern pressue results from higher inventory, which results in a higher maximum withdrawal rate, and lower maximum withdrawal rate.

The inventory V_i can be defined recursively as:

$$V_i = V_{i-1} + v_{i-1} - L(t_{i-1}, V_{i-1}) \tag{2}$$

Where $L(t_{i-1}, V_{i-1})$ is the inventory loss, as a function of time and inventory, evaluated for the previous time period. An alternative representation of inventory is as a summation:

$$V_i = V_{start} + \sum_{j=0}^{i-1} (v_j - L(t_j, V_j)) \tag{3}$$

V_{start} is the inventory at the inception. This is necessary in the case where storage capacity is purchased or leased with some amount of commodity inventory in place.

V_i is itself constrained to be within V_{min} and V_{max}, the minimum and maximum inventory functions.

$$V_i \in [V_{min}(t_i), V_{max}(t_i)] \tag{4}$$

Commonly V_{min} will evaluate to zero for all time periods, but examples where non-zero minimum inventory is needed include when regulations require a minimum level of inventory is held, as is seen for natural gas storage in some European countries. Two possible reasons why V_{max} need to be functions of time are:

- Storage could be leased for consecutive time periods, but for different notional volumes.
- The terms of leased storage commonly stipulate that the storage must be empty at the time that the leased capacity ends.

Mathematical constraints.
- Set of decision times.
- Decision volume (inject/withdraw) for each time.
- Set of admissible decision volumes which adhere to constraints.

Cash flows.
- NPV from inject/withdraw, equals decision volume, adjusted for lost volume, times cmdty forward price, discounted.
- NPV from inject/withdraw cost.
- NPV from inventory cost.
- Terminal cmdty NPV function.

Valuation Theory.
- Optimal value as risk-neutral expectation for NPVs.
- Bellman equation.

Valuation In Practice.
- Bang-bang assumption: reduce the decision volume set.
- Discretise the set of inventories at which the Bellman equation is evaluated.
- Problem then boils down to calculating the expectation.