In [1]:
```python
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import scipy.stats as stats
import pandas as pd
```

In [2]:
```python
temperatures = np.array([953, 955, 948, 951, 957, 949, 954, 950, 959])
```

In [3]:
```python
# 1a
median = np.median(temperatures)
print(f"Sample median: {median}")
```

```
Sample median: 953.0
```

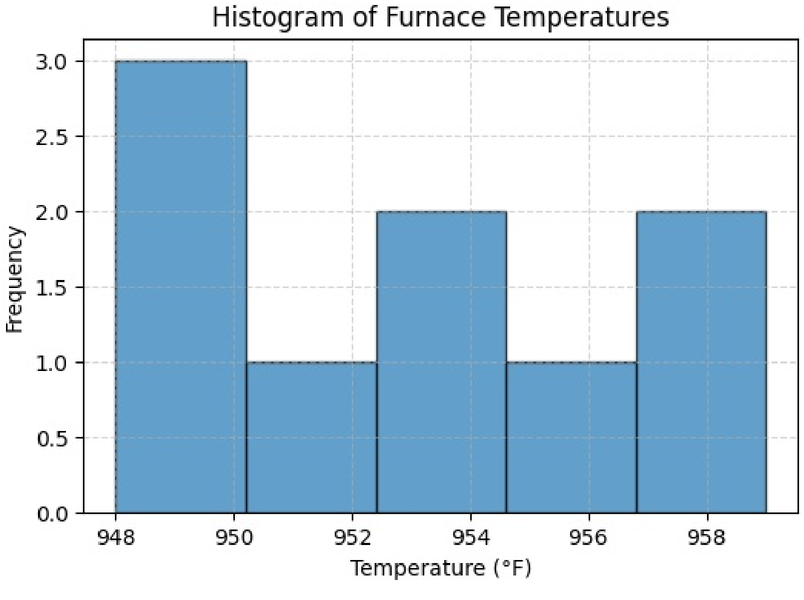
In [4]:
```python
# 1b
sorted_temps = np.sort(temperatures)
current_max = np.max(temperatures)
median_index = len(sorted_temps) // 2

print(f"Largest temperature measurement increase without changing the sample median: {current_max}")
```

```
Largest temperature measurement increase without changing the sample median: 959
```

In [5]:
```python
# 1c
mean = np.mean(temperatures)
std_dev = np.std(temperatures, ddof=1)
print(f"Sample mean: {mean:.2f}")
print(f"Sample standard deviation: {std_dev:.2f}")
```

```
Sample mean: 952.89
Sample standard deviation: 3.72
```

In [6]:
```python
# 1d
plt.figure(figsize=(6, 4))
plt.hist(temperatures, bins='auto', edgecolor='black', alpha=0.7)
plt.title('Histogram of Furnace Temperatures')
plt.xlabel('Temperature (°F)')
plt.ylabel('Frequency')
plt.grid(True, linestyle='--', alpha=0.5)
plt.show()
```



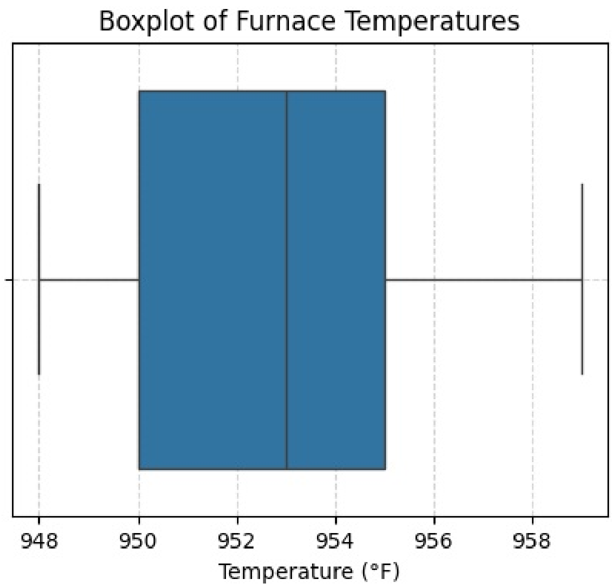
In [7]:
```python
# 1e
def stem_and_leaf(data):
    data_sorted = sorted(data)
    stem_leaf = {}
    for number in data_sorted:
        stem, leaf = divmod(number, 10)
        stem_leaf.setdefault(stem, []).append(leaf)
    print("Stem-and-leaf plot:")
    for stem in sorted(stem_leaf):
        leaves = ' '.join(str(leaf) for leaf in stem_leaf[stem])
        print(f"{stem} | {leaves}")

stem_and_leaf(temperatures)
```

```
Stem-and-leaf plot:
94 | 8 9
95 | 0 1 3 4 5 7 9
```

In [8]:
```python
# 1f
Q1 = np.percentile(temperatures, 25)
Q3 = np.percentile(temperatures, 75)
print(f"Lower quartile (Q1): {Q1}")
print(f"Median (Q2): {median}")
print(f"Upper quartile (Q3): {Q3}")
```

```
Lower quartile (Q1): 950.0
Median (Q2): 953.0
Upper quartile (Q3): 955.0
```

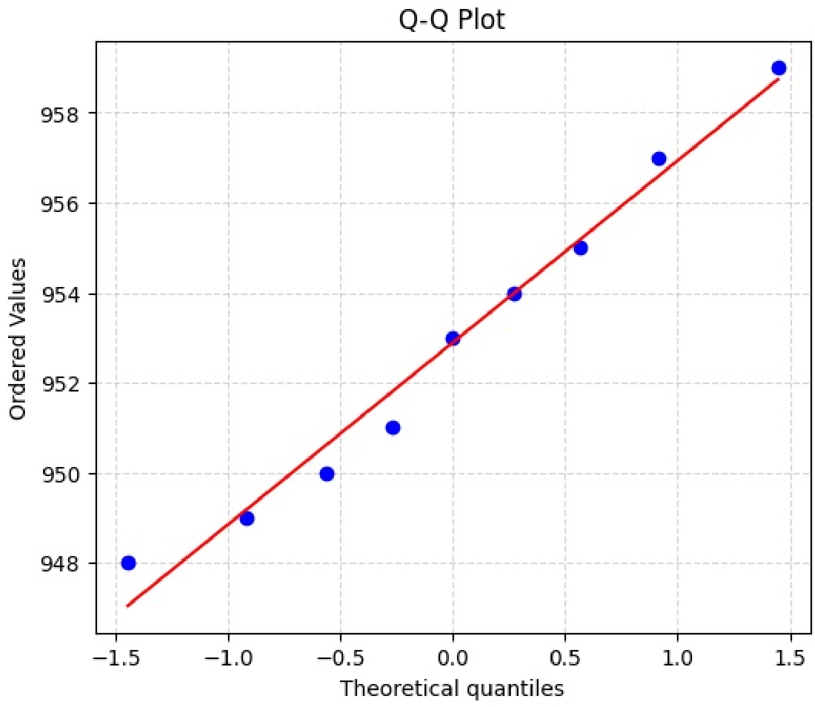
In [9]:
```python
# 1g
plt.figure(figsize=(5, 4))
sns.boxplot(x=temperatures, orient='h')
plt.title('Boxplot of Furnace Temperatures')
plt.xlabel('Temperature (°F)')
plt.grid(True, linestyle='--', alpha=0.5)
plt.show()
```



In [10]:
```python
# 1h
print("Brief description:")
print("Fairly symmetrically distribution.")
print("No big outliers.")
print("No huge standard deviation.")
```

```
Brief description:
Fairly symmetrically distribution.
No big outliers.
No huge standard deviation.
```

In [11]:
```python
# 1i
plt.figure(figsize=(6, 5))
stats.probplot(temperatures, dist="norm", plot=plt)
plt.title('Q-Q Plot')
plt.grid(True, linestyle='--', alpha=0.5)
plt.show()
```



In [12]:
```
print("Normal distribution seems reasonable as data points basically follow a straight line.")
```

```
Normal distribution seems reasonable as data points basically follow a straight line.
```

In [13]:
```
# 2a
from scipy.stats import geom, binom

p = 0.15
# P(X = 3) = (1 - p)^(3 - 1) * p
a_prob = geom.pmf(3, p)
print(f"Probability that the third patient of the day is the first with high blood pressure: {a_prob:.4f}")
```

```
Probability that the third patient of the day is the first with high blood pressure: 0.1084
```

In [14]:
```
# 2b
b_expected = geom.mean(p)
print(f"Average number of patients that must be seen until first patient with high blood pressure: {b_expected:
```

```
Average number of patients that must be seen until first patient with high blood pressure: 6.67
```

In [15]:
```
# 2c
n = 50
k = 10
c_prob = binom.pmf(k, n, p)
print(f"Probability of exactly 10 out of 50 patients having high blood pressure: {c_prob:.4f}")
```

```
Probability of exactly 10 out of 50 patients having high blood pressure: 0.0890
```

In [16]:
```
# 3a
from scipy.stats import hypergeom, binom
import matplotlib.pyplot as plt
```

In [17]:
```
N = 25
K = 2
n = 5
x = 0
```

In [18]:
```
prob_accept_exact = hypergeom.pmf(x, N, K, n)
```

In [19]:
```
print(f"Probability of acceptance: {prob_accept_exact:.5f}")
```

```
Probability of acceptance: 0.63333
```

In [20]:
```
# 3b
p_conforming = (N - K) / N
prob_accept_binom = binom.pmf(n, n, p_conforming)
```

In [21]:
```
print(f"Binomial approximation: {prob_accept_binom:.5f}")
print(f"Difference: {abs(prob_accept_exact - prob_accept_binom):.5f}")
print(f"Binomial approximation not satisfactory, > .10 .")
```

```
Binomial approximation: 0.65908
Difference: 0.02575
Binomial approximation not satisfactory, > .10 .
```

In [22]:
```
# 3c
```

```
N_large = 150
K_large = 2
n_sample = 5
```

In [23]:
```
prob_accept_large_exact = hypergeom.pmf(0, N_large, K_large, n_sample)
```

In [24]:
```
p_conforming_large = (N_large - K_large) / N_large
prob_accept_large_binom = binom.pmf(n_sample, n_sample, p_conforming_large)
```

In [25]:
```
print(f"Binomial approximation: {prob_accept_large_binom:.5f}")
print(f"Difference: {abs(prob_accept_large_exact - prob_accept_large_binom):.5f}")
print(f"Binomial approximation satisfactory, < .10 .")
```

```
Binomial approximation: 0.93509
Difference: 0.00086
Binomial approximation satisfactory, < .10 .
```

In [26]:
```
# 3d
target_accept_prob = 0.05
K_reject = 5
min_n = None
```

In [27]:
```
for n_test in range(1, N + 1):
    accept_prob = hypergeom.pmf(0, N, K_reject, n_test)
    if accept_prob <= target_accept_prob:
        min_n = n_test
        break
```

In [28]:
```
print(f"Minimum sample size to ensure rejection with 0.95 probability when K=5: {min_n}")
```

```
Minimum sample size to ensure rejection with 0.95 probability when K=5: 11
```

In [29]:
```
# 4
from scipy.stats import poisson
```

In [30]:
```
lambda_ = 0.1
```

In [31]:
```
p_at_least_one = 1 - poisson.pmf(0, mu=lambda_)
```

In [32]:
```
print(f"Probability of rsandom unit with at least one surface-finish defect: {p_at_least_one:.4f}")
```

```
Probability of rsandom unit with at least one surface-finish defect: 0.0952
```

In [33]:
```
# 5
from scipy.stats import norm
```

In [34]:
```
mean = 5000
std_dev = 50
percent_below = 0.005
```

In [35]:
```
lsl = norm.ppf(percent_below, loc=mean, scale=std_dev)
```

In [36]:
```
print(f"Lower specification limit such that only 0.5% of the bulbs will not exceed this limit: {lsl:.2f} ")
```

```
Lower specification limit such that only 0.5% of the bulbs will not exceed this limit: 4871.21
```

In [37]:
```
# 6
mu = 100
sigma = 2
lsl = 97
usl = 102
```

In [38]:
```
p_within_spec = norm.cdf(usl, mu, sigma) - norm.cdf(lsl, mu, sigma)
p_below_lsl = norm.cdf(lsl, mu, sigma)
p_above_usl = 1 - norm.cdf(usl, mu, sigma)
```

In [39]:
```
print(f"Proportion of process output within specifications: {p_within_spec:.4f}")
```

```
Proportion of process output within specifications: 0.7745
```

In [40]:
```
rework_cost = 1
scrap_cost = 5
```

In [41]:
```
mean_range = np.linspace(97, 103, 200)
costs = []

for mu_shifted in mean_range:
    p_below = norm.cdf(lsl, mu_shifted, sigma)
    p_above = 1 - norm.cdf(usl, mu_shifted, sigma)
    expected_cost = (p_below * rework_cost) + (p_above * scrap_cost)
    costs.append(expected_cost)
```

In [42]:
```
min_cost = min(costs)
optimal_mean = mean_range[np.argmin(costs)]

print(f"Make the mean {optimal_mean:.2f}")
```

```
Make the mean 98.21
```

In [43]:
```
#  6
n = 50
p = 12 / 38
mu = n * p
sigma = np.sqrt(n * p * (1 - p))
```

In [44]:
```
x = 11.5  # continuity correction for P(X >= 12)
prob = 1 - stats.norm.cdf(x, loc=mu, scale=sigma)
```

In [45]:
```
print(f"Probability that Laura wins at least 12 times: {prob:.4f}")
```

```
Probability that Laura wins at least 12 times: 0.9041
```

In [ ]:
```

```