A new Lorentzian Geometry

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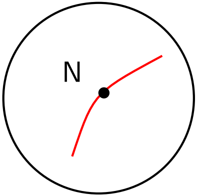
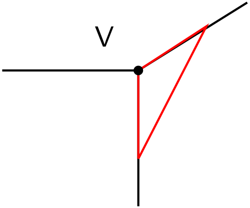
The larger picture: Regularity in geometry

Regularity determines basic geometric features

Example 1: Walking on a sphere vs. walking on a cube.

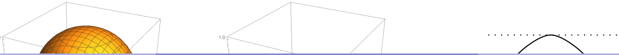




Sphere: Walking along a geodesic/meridian is locally always the shortest path.

Cube: It is always shorter to deviate to the right face than to go along the edge.

Example 2: Squeezing the sphere. [Hartman-Wintner, 52]



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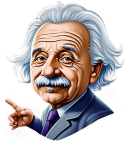
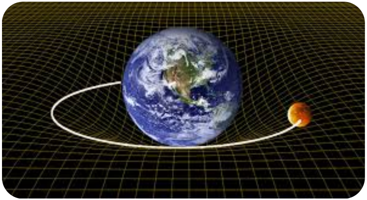
Lorentzian Geometry is the Language of General Relativity

GR, Einstein's theory of space, time & gravity:
Gravity is universal, hence prop. of surrounding spacetime
Curvature proportional to mass/energy content



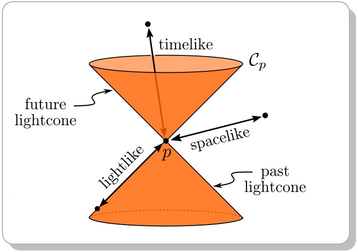
$$Ric - \frac{1}{2} R\, g \;=\; \frac{8\pi G}{c^4}\, T \qquad \text{(E)}$$



Matter tells spacetime how to curve,
spacetime tells matter how to move.

Geometric description via a spacetime manifold (M, g)

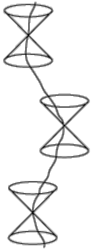
Causality is the fundamental structure of Lor. Geometry



$$v \in T_pM, \quad g_p(v,v) \begin{cases} < 0 & \text{timelike} \\ = 0 & \text{lightlike/null} \\ \leq 0 & \text{causal} \\ > 0 & \text{spacelike} \end{cases}$$

extends naturally to

(loc. Lipschitz) curves



Causal relations between points $p, q \in M$

$p \ll q$:⇔ connected by future timelike curve

$p \leq q$:⇔ connected by future causal curve

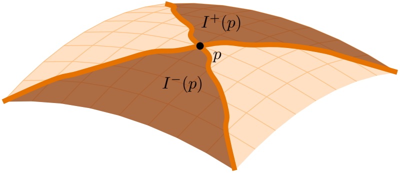
Chronological/causal future/past

$I^+(p) := \{q \in M : \ p \ll q\}$

$J^+(p) := \{q \in M : \ p \leq q\}$

past versions: $I^-(p)$, $J^-(p)$

for $A \subseteq M$: $I^\pm(A)$, $J^\pm(A)$



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Smooth Lorentzian Geometry is not enough

Generally from (E): $Ric - \frac{1}{2} Rg = \frac{8\pi G}{c^4}\, T$; $\quad Ric \propto \partial^2 g + (\partial g)^2$;
$T \notin C^0 \rightsquigarrow g \propto C^{1,1}$; below C^2 (smooth f.a.p.p.) non-smooth:=below C^2

Motivations from physics & mathematics

- *PDE/initial value* point-of-view (regularity in hyperbolic PDE!)
- physically relevant *models*: matched spacetimes

  [Senovilla, Mars, Sánchez-Pérez, Manzano, Ohanyan, S.,...]

  impulsive gravitational waves

  [Barrabés, Griffiths, Hogan, Podolský, Sämann, Švarc, S....]
- approaches to *Quantum Gravity* (causal set theory, CDT,...)

"The discontinuities of the matter [...] which must be allowed [...] pose enormous problems [...]. To avoid this annoying problem though—despite it being completely fundamental!—we will implicitly assume that g is at least C^2." *[Garcia Parrado-Senovilla, CQG, 2005]*

But much has happened since!

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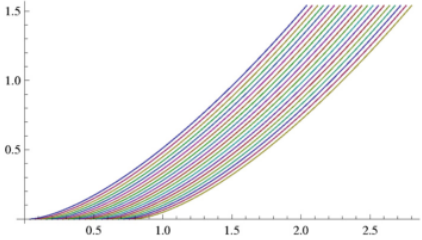
Back to larger picture: Regularity in Lorentzian causality

Regularity determines basic geometric features

Example 3: Lightcones bubble up. [Chruściel-Grant, 12]



Some null geodescis for $\alpha = 1/3$

$M = \mathbb{R}^2$, $g \in C^{0,\alpha}$ $(\alpha < 1)$

$g = -(dt + (1 - |t|^\alpha)dx)^2 + dx^2$

Non-unique null geodesics
$\rightsquigarrow$ null cone has full measure
causal bubble

Ex. 4: The future is not always open. [Grant-Kunzinger-Sämann-S, 20]

What can be done: Approaches to non-smooth LG

Spacetime setting: smooth manifold but metric g below C^2 *(analytic)*

- distributional curvature: $g \in H^1 \cap L^\infty$ maximal w stable $\mathcal{D}'$-curvature
  [Geroch-Traschen, 87], [LeFloch-Mardare, 07], [Vickers-S, 09]
- non-linear distributional geometry (algebras of generalised functions)
  [Farkas, Grosser, Hanel, Hörmann, Kunzinger, Mayerhofer, Nigsch, Oberguggenberger, Spreitzer, Vickers, S. . . 01–]
- advanced regularisation techniques $\check{g}_\varepsilon \prec g \prec \hat{g}_\varepsilon$ [Chruściel-Grant, 12]
- causality for $g \in C^0$ [Sämann, 16], closed cone structures [Minguzzi, 19]

$\rightsquigarrow$ Singularity theorems in low regularity: $g \in C^{1,1}$, $g \in C^1$, $g \in C^{0,1}$
[Graf, Grant, Kunzinger, Ohayan, Schinnerl, Stojkovic, Vickers, S, 15–]
currently: $g \in W^{1,p}$, $R \in L^p$ [Kunzinger-Reintjes-Vega-S, 25–]

- C^0-inextendibility [Sbierski, 15-, Ling, 24, Monsani-Solanki-Prados-Sämann, ong.]

A new geometry: beyond the manifold setting *(synthetic)*

This will be the focus for the rest of the talk.

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How to detect curvature: A glimpse of Riemannian mflds

Riemannian manifold (M, h); curvature $R_{XY}Z = [\nabla_X, \nabla_Y]Z - \nabla_{[X,Y]}Z$

Sectional curvature $\mathrm{Sec}(X, Y) = \dfrac{\langle R_{XY}Y, X\rangle}{\|X\|^2\|Y\|^2 - \langle X, Y\rangle^2}$

Theorem (Toponogov $\triangle$-comparison)
$\mathrm{Sec} \geq 0 \leq 0 \geq K \leq K \iff$

For all (small) geodesic triangles $\triangle abc$ in (M, h) consider a comparison triangle $\triangle\bar{a}\bar{b}\bar{c}$ in the Euclidean Plane $\mathbb{R}^2$2D *model space of constant curvature K*.

Then for all p, q on its sides and corresponding comparison points $\bar{p}$, $\bar{q}$

$$d_h(p, q) \geq\leq\geq\leq d_{\mathbb{R}^2}\bar{d}_K(\bar{p}, \bar{q}).$$

Sectional curvature bounds for metric spaces

Definition (Length space) (X, d) metric space w d intrinsic, i.e.

$$d(x, y) = \inf\{L_d(\gamma) : \gamma \text{ from } x \text{ to } y \text{ continuous}\}$$

Geodesics $\gamma : [0, 1] \to X$ with $L_d(\gamma) = d\big(\gamma(0), \gamma(1)\big)$

Definition (Synthetic curvature bounds)

A length space has curvature bounded belowabove by K if for all (small) *geodesic triangles* $\triangle abc$ and their *comparison triangles* $\triangle \bar{a}\bar{b}\bar{c}$ in the model space of curvature K and all points p, q on its sides and corresponding $\bar{p}$, $\bar{q}$

$$d(p, q) \geq\leq \bar{d}_K(\bar{p}, \bar{q}).$$

Curvature bounded below / above: *Alexandrov spaces* / *CAT(K)-spaces*
Metric geometry: rich theory since the 1980-ies: GH-convergence, preservation of curvature bds., Gromov pre-compactness thm.

How to detect curvature: smooth Lorentzian world

Sectional curvature $\mathrm{Sec}(X,Y) = \dfrac{\langle R(X,Y)Y, X\rangle}{\langle X,X\rangle\langle Y,Y\rangle - \langle X,Y\rangle^2}$

Kulkarni: If Sec is bounded below (above), then it is constant.

Definition ("Correct" curvature bounds) [Andersson-Howard, 98]

A smooth Lorentzian manifold has $\mathrm{Sec} \geq K$ if
spacelike sectional curvatures $\geq K$ and *timelike* sectional curvatures $\leq K$.

How to go beyond Lorentzian manifolds?

	smooth	**synthetic**
pos. def.	Riemannian mflds	metric length spaces
Lorentzian	Lorentzian manifolds	??? Lor. length spaces

What is the analogue of metric (length) spaces in the *Lorentzian setting*?

Serious issue

No natural distance function $\rightsquigarrow$ no direct entry into world of metric spaces

Lorentzian (pre-)length spaces [Kunzinger-Sämann, 18] based on causal spaces [Kronheimer-Penrose, 67] & the use of the time separation

Null distance [Sormani-Vega, 16] uses a time function to define distance function; contributions by [Allen, Burtscher, Galloway, Garcia-Heveling, Sakovich, Kunzinger, S. . .]

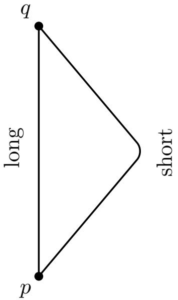
Lorentzian time separation vs. Riemannian distance

Length of causal curves $\gamma: I \to M$: $\quad L_g(\gamma) := \int_I \sqrt{-g(\dot\gamma, \dot\gamma)}$

Null curves have $L_g(\gamma) = 0$, timelike can be arbirarily short!

Time separation function

$\tau(p,q) := \sup\{L_g(\gamma) : \gamma$ future causal from p to $q\}$



Properties:

- lower semi-continuous,
- satisfies reverse Δ-inequality:

$$\tau(p,q) + \tau(q,r) \le \tau(p,r)$$

- τ is not a distance function

Why: implements basic causality already of SRT (twin paradox)

Consequences: No cheap entry into the world of metric geometry!

Causality w/o metric: Lorentzian (pre-)length spaces

Causal space: X (metrizable) topological space with *abstract causality:*

$\leq$ preorder on X, $\ll$ transitive relation contained in $\leq$

Abstract time separation: $\tau \colon X \times X \to [0, \infty]$ lower semicontinuous

Definition [Kunzinger-Sämann, 18]

$(X, \ll, \leq, \tau)$ is a *Lorentzian pre-length space* if for $p \leq q \leq r$

$$\tau(p,r) \geq \tau(p,q) + \tau(q,r) \quad \text{and} \quad \tau(p,q) \begin{cases} = 0 & \text{if } x \not\leq y \\ > 0 & \Leftrightarrow x \ll y \end{cases}$$

It is a *Lorentzian length space* if τ is intrinsic.

Examples

- *smooth spacetimes* (M, g) with usual time separation τ
- *Lorentz(-Finsler)* spacetimes, of *low regularity* $(g \in C^0 + ...)$
- *finite directed graphs* (causal sets)

Lorentzian *causality theory*

variants by [Braun-McCann, 23-], [Minguzzi-Suhr, 24-], [Müller, 24-]

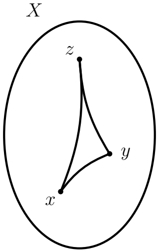
Timelike curvature via triangle comparison

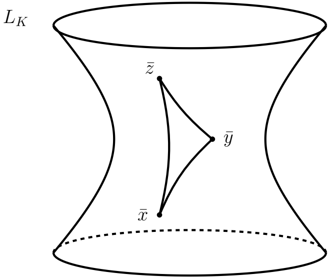
Definition (Synthetic curvature bounds)

$(X, \ll, \leq, \tau)$ has *timelike curvature* $\geq K \leq K$ if

1. some technical conditions hold
2. for all *small timelike triangles* Δxyz and their comparison $\Delta \bar{x}\bar{y}\bar{z}$ in L_K and all p, q resp. $\bar{p}$, $\bar{q}$

$$\tau(p,q) \leq \geq \bar{\tau}_K(\bar{p},\bar{q}).$$





Faithful extension of sectional curvature bounds

Timelike curvature comparison: The whole truth

Theorem (Equivalence of curvature conditions) [Beran-Kunzinger-Rott, 24]

Let X be a chronological LpLS. Recall curvature bds. below/above by K in the following senses:

(i) Timelike triangle comp.
(ii) One-sided timelike
(iii) Causal triangle co
(iv) One-sided causal t
(v) Strict causal trian
(vi) One-sided strict c
(vii) Monotonicity com
(viii) One-sided monoto
(ix) Angle comp.

If X is strongly causal, r
bounds additionally is lo
(8), then *all notions of c*

In general the following relations hold:

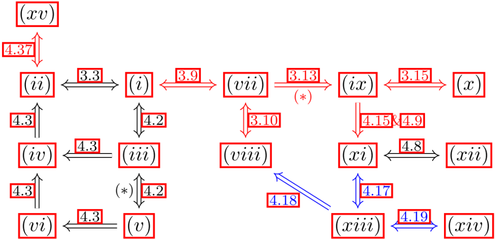


Figure 4: All relations between different formulations of curvature bounds for Lorentzian pre-length space. Black arrows are always valid, red arrows require X to be regular, and blue arrows require X to be strongly causal, regular and locally D_K-geodesic. The two instances of additional assumptions for one direction of curvature bounds are decorated by $(*)$ in the figure.

Selected results (1/3)

Theorem [Kunzinger-Sämann 19, Beran-Sämann, 22]

In a strongly causal Lorentzian pre-length space with *timelike curvature bounded below* timelike geodesics do *not branch*.

Theorem [Grant-Kunzinger-Sämann, 19]

A timelike geodesically complete spacetime (or LLS) is *inextendible as a regular LLS*, i.e., any LLS-extension necessarily has unbounded curvature.

Complements class. [Beem-Ehrlich, 80s], C^0-results [Galloway-Ling-Sbierski, 18] [Minguzzi-Suhr, 19] and relates to *curvature blow up*!

Splitting theorem [Beran-Ohanyan-Rott-Solis, 23]

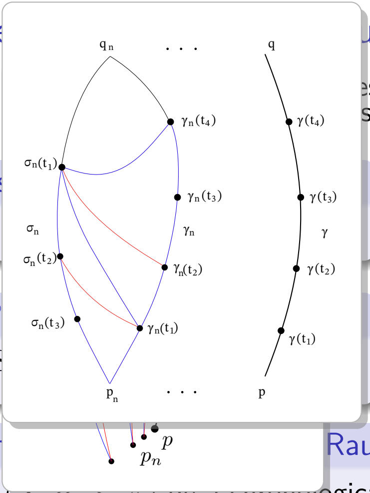
Let X be a globally hyperbolic LLS with global timelike $K \geq 0$. If X contains a complete timelike line (& some technical conditions) then it splits into a product $\mathbb{R} \times S$ with S a metric length space with $K \geq 0$.

Generalises smooth Lorentzian & synthetic Riemannian results.

Sele… …uch comparison Theorem

[Grant-Kunzing…

- …es
- …st conjugate pt.



Clas…

$0 <$ …

Synth. Rier…

$\mathrm{CAT}(K) \Rightarrow$ …

C^∞…

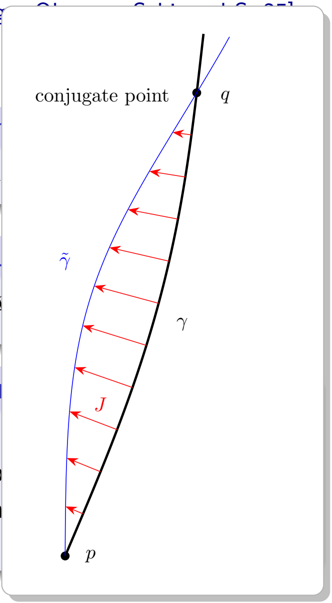
tl. S…

Synthetic L…

tl. Sec. $\leq K$…

Thr… …Rauch; precise for…

Let … be a chronological, regular, local…
and finitely τ-measurable Lorentzian pre-length sp…
curvature $\leq K$ and let $\gamma \in \mathrm{TGeo}(X)$ be such tha…
$q := \gamma(1)$ are symmetric conjugate along γ. Then…



Closely related to

Selected results (3/3)

- *Time functions* [Burtscher-García-Heveling, 21]
- *Null distance & LLS* [Kunzinger-S., 22]
- compatibility w *distr. curv. bds* [Erös-Kunzinger-Ohanyan-Vardabasso, 25]
- Lorentzian Hausdorff *dimension*, *measure* [McCann-Sämann, 22]
- *Gluing* of Lorentzian length spaces [Beran-Rott, 24, Rott, 23]
- *Causal/ideal bdries* [Ake Hau-Burgos-Solis, 23–25], [Burgos-Che-Prados, ong.]
- *sympl./cont. geo.* [Abbondandolo-Benedetti-Polterovich, 22, Hedicke, 24]
- *Sub-Lorentzian* Heisenberg group as LLS [Borza-Rigoni-Zoghlami, 25]

Lorentzian Gromov–Hausdorff convergence

Stability of lower timelike, Geometric precompactness [Mondino-Sämann, 25]
variants by [Minguzzi-Suhr, 22], [Bykov-Minguzzi-Suhr, 24], [Müller, 24–]
[Sakovich–Sormani, 24], [Che-Perales-Sormani, 25]
[Perales-Prados-Sormani-Zoglami, ong.]

- *Machine learning* in spacetimes [Law-Lucas, 23]

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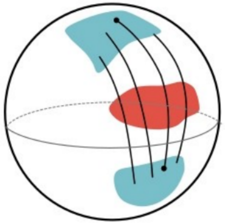
Ricci bounds via optimal transport: the basic idea

- *Optimal Transport*: [Monge, Kantorovich]
  move matter (distribution μ_1) in the cheapest/optimal way (to μ_2)
- *Minimize*

$$\int_{X\times Y} c(x,y)\,\mathrm{d}\pi(x,y) \qquad (c\text{...cost function})$$

  over transport plans $\pi \in \mathcal{P}(X\times Y)$
  w given marginals $(\mathrm{pr}_X)_\sharp\pi = \mu_1$, $(\mathrm{pr}_Y)_\sharp\pi = \mu_2$

- What is *optimal* depends on cost,
  but also on *distances* & *geometry*!
- Turn this on its head:
  define *curvature* by requiring that
  OT behaves as in model spaces
- Riemannian case: cost $c = d$
- Lorentzian case: cost $c = \tau$



Transporting *clouds* of points on the sphere

Ricci Bounds via Optimal Transport: Riemannian case

Thm. (Displacement convexity) [Lott-Villani, Sturm 06-09]

(M, h) complete Riemannian manifold

$$\mathrm{Ric} \geq 0 \iff (M, d_h, \mathrm{vol}_h) \text{ is an } \mathrm{CD}(0, \infty)\text{-space}$$

Definitions. On a metric measure space $(X, d, \mathfrak{m})$ define

- *Wasserstein distance:* $W_2(\mu_1, \mu_2) = \left(\inf\limits_{\pi \in \Pi} \int_{X \times X} d(x, y)^2 \, d\pi(x, y) \right)^{\frac{1}{2}}$
- *Wasserstein geodesic:* continuous curve $(\mu_t)_{0 \leq t \leq 1}$

$$W_2(\mu_s, \mu_t) = |t - s| \cdot W_2(\mu_1, \mu_2)$$

- *Entropy functional:* $E(\mu|\mathfrak{m}) = \int_X \rho \log \rho \, d\mathfrak{m}$ for $\mu = \rho\mathfrak{m}$
- $\mathrm{CD}(0, \infty)$*-space:* E convex along Wasserstein geodesics, i.e.,

$$E(\mu_t|\mathfrak{m}) \leq (1 - t)E(\mu_1|\mathfrak{m}) + tE(\mu_2|\mathfrak{m})$$

Again turn this into definition of synthetic curvature bounds...CD(K,N)
$\rightsquigarrow$ $(R)CD$*-spaces:* stability u. measured GH-conv. [Ambrosio-Gigli-Savare, 14]

Ricci Bounds via Optimal Transport: Lorentzian case

Thm. (Lor. displacement convexity) [McCann, Mondino-Suhr, 20]

(M, g) globally hyperbolic n-dim. spacetime

$$\mathrm{Ric}(X, X) \geq 0 \text{ for } X \text{ timelike} \iff (M, \tau, \mathrm{vol}_g) \text{ is } \mathrm{TCD}(0, n)\text{-space}$$

Definitions. Measured Lorentzian pre-length space $(X, \mathfrak{m}, \ll, \leq, \tau)$

- OT & causality [Eckstein-Miller, 17]: $\pi \in \Pi_{\leq}$ (concentrated on $\ll$)
- *p-Lorentz Wasserstein distance:* $(0 < p \leq 1)$ (for reverse $\triangle$-ineq.)

$$l_p(\mu_1, \mu_2) = \Big(\sup_{\pi \in \Pi_{\leq}} \int_{X \times X} \tau(x, y)^p \, d\pi(x, y) \Big)^{1/p}$$

- *Entropy functional:* $\mathrm{Ent}(\mu|\mathfrak{m}) = \displaystyle\int \rho \log(\rho) d\mathfrak{m}$ for $\mu = \rho\mathfrak{m}$
- $\mathrm{TCD}(K, N)$*:* along l_p-geos μ_t we have for $e(t) := \mathrm{Ent}(\mu_t|\mathfrak{m})$

$$e''(t) - 1/N \, e'(t)^2 \geq K \int_{X \times X} \tau(x, y)^2 \pi(dxdx)$$

Turn into definition: measured Lorentzian pre-length space w TCD(K,N)

Selected results

- *Ricci curvature bounds* via optimal transport on *null hypersurfaces* [McCann, 24, Ketterer, 24, Cavalletti-Manini-Mondino, 24]
- OT for general Lorentzian cost [Calisti-Ohanyan-Sálamo, 25]
- Compatibility with distributional bounds [Braun-Sálamo, ongoing]
- (vacuum) *Einstein equations* [Mondino-Suhr, 23]
- Hawking singularity theorem in TCD/TMCP [Cavaletti-Mondino 24]
- Penrose singularity theorem [Cavaletti-Manini-Mondino, 24]

First order Sobolev calculus on metric measure spacetimes

Infinitesimal Minkowskianity, nonlinear elliptic p-d'Alembertian comparison [Beran-Braun-Calisti-Gigli-McCann-Ohanyan-Rott-Sämann, 24]

- Lorentzian splitting
  - Elliptic proof of class. result [Braun-Gigli-McCann-Ohanyan-Sämann, 24]
  - C^1 (weighted) spacetimes [Braun-Gigli-McCann-Ohanyan-Sämann, 25]

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Summary

A new Lorentzian geometry

(Measured) Lorentzian Length Spaces $(X, \mathfrak{m} \ll, \leq, \tau)$

- provide a general mathematical setting for
  - *sectional* curvature and
  - *Ricci* curvature (bounds)
- that contains
  - *low regularity spacetimes* but also
  - *discrete spaces*

Gives framework for

- approaches to non-smooth spacetime geometry & beyond
- fundamentally discrete approaches to QG

causal set theory, CDT, causal fermion systems, ...

Outlook: Causal set theory [Bombelli-Lee-Meyer-Sorkin, 87]

ingredients: *causal set* $(X, \leq)$, partial order

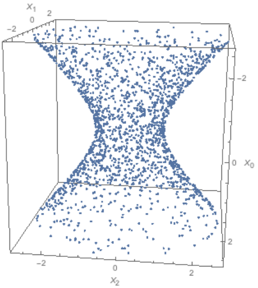
locally finite: $J(x,y) = \{z : x \leq z \leq y\}$ finite

CS hypothesis: QT of causal sets X;

(M,g) approximation of X

$$\mathcal{C}(M,\rho) \ \ni \ X \ \longleftrightarrow (M,g)$$

sprinkling of dS



Hauptvermutung of CST

X can be embedded at density ρ_C into distinct spacetimes iff they are "close".

$(X, \ll, \leq, \tau)$ is a LpLS

- chain: $C := (x_i)_{i=1}^n$: $x_i < x_{i+1}$
- length: $L(C) = n$; $\ll := <$
- $\tau(x,y) := \sup\{L(C) : C$ chain from x to $y\}$

Hauptvermutung translates into statement on convergence of LpLS.

Olivier Ricci curvature and stability [Barton-Borza-Röhrig, ongoing]

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Meet the EF-Geometry Team

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Thank you for your attention

Picture credits

- slide 3, Cartoon-Einstein: https://wallpapers.com/albert-einstein-png (freely downloadable)
- slide 3, Spacetime-curvature: Einstein relatively easy, https://einsteinrelativelyeasy.com/
- slide 4, lightcone : arXiv:1607.04202[gr-qc], Le Tiec, Alexandre and Novak, Jerome, Theory of Gravitational Waves. arXiv:1607.04202, https://doi.org/10.48550/arXiv.1607.04202
- slide 4, cone-trajectory: Stanford Encyclopedia of Philosophy https://plato.stanford.edu/entries/spacetime-singularities/lightcone.html
- slide 11, spacetime bubble: Chruściel, Piotr T., James, D. E., On Lorentzian causality with continuous metrics, arXiv:1111.0400 [gr-qc], https://doi.org/10.48550/arXiv.1111.0400
- slide 32, spacetime-sprinkling: Sumati Surya, The causal set approach to quantum gravity. Living Reviews in Relativity (2019) 22:5 https://doi.org/10.1007/s41114-019-0023-

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