Verified double-hashing hash map

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Outline

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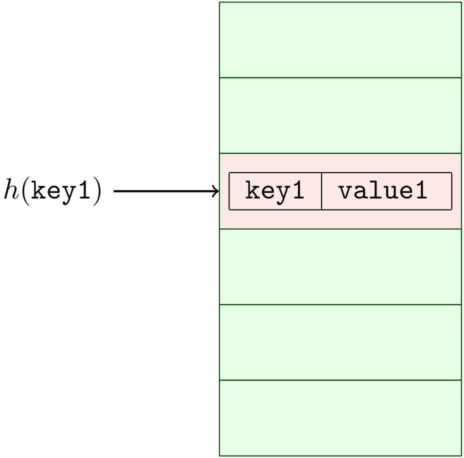
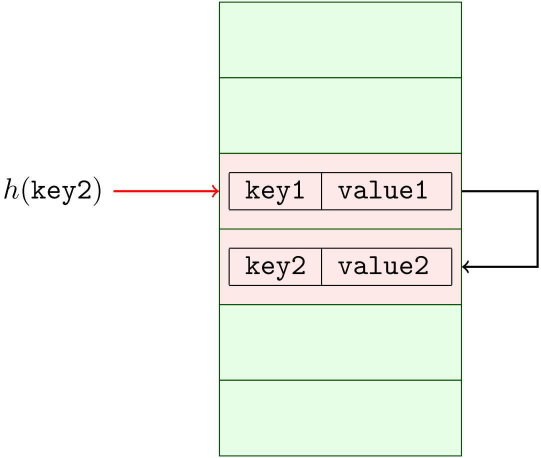
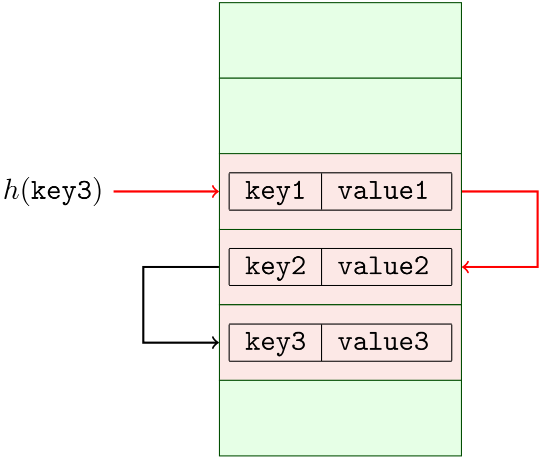
Naive hash table



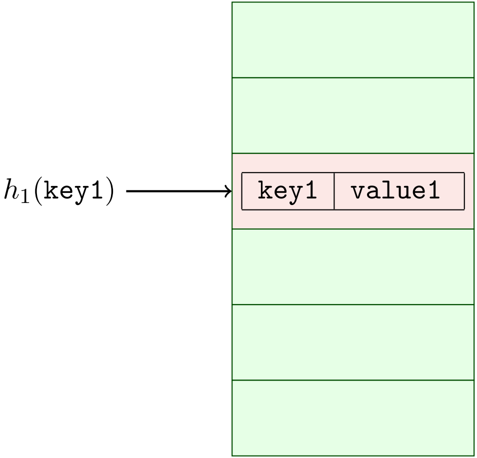
Naive hash table



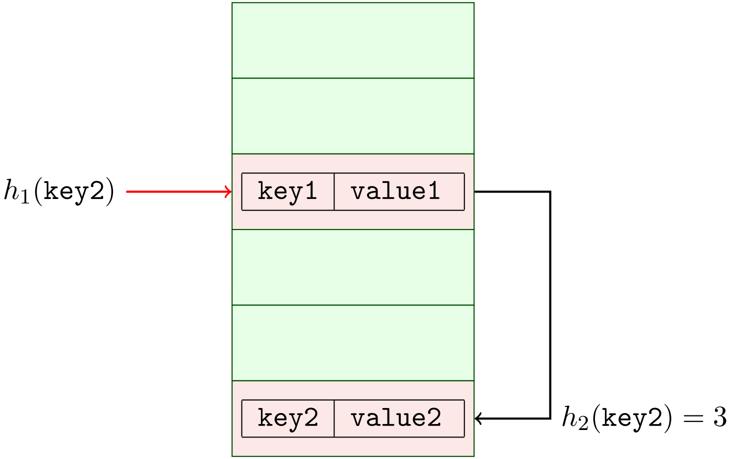
Naive hash table



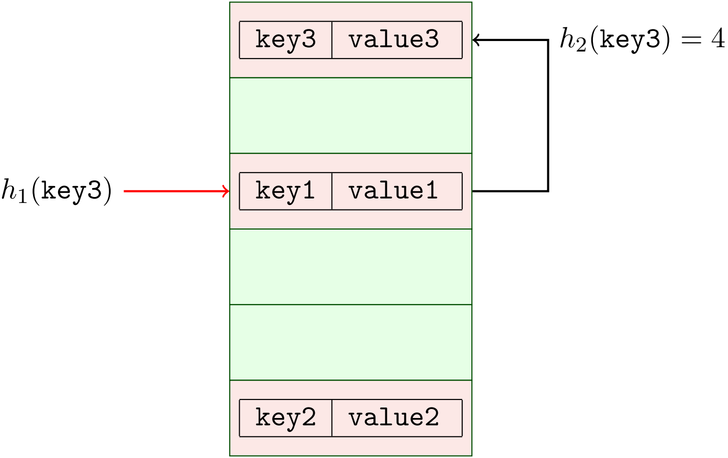
Double hashing



Double hashing



Double hashing

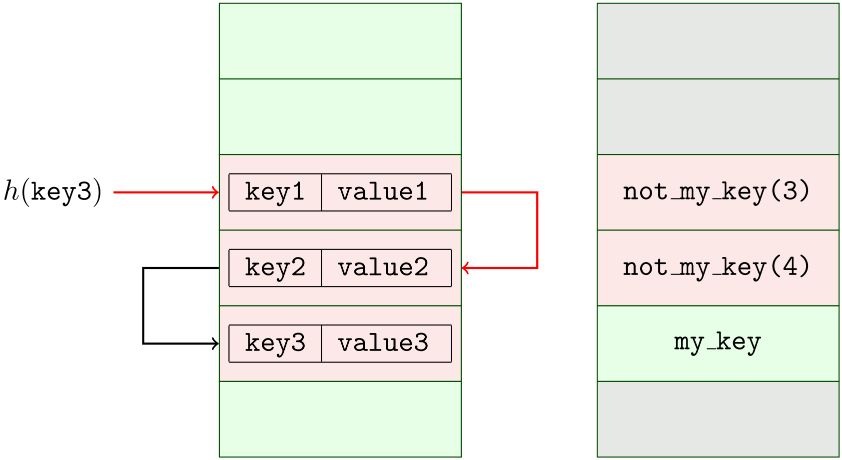


Provided implementation

- A naive implementation
- `findEmpty`, `findKey` perform the loops.

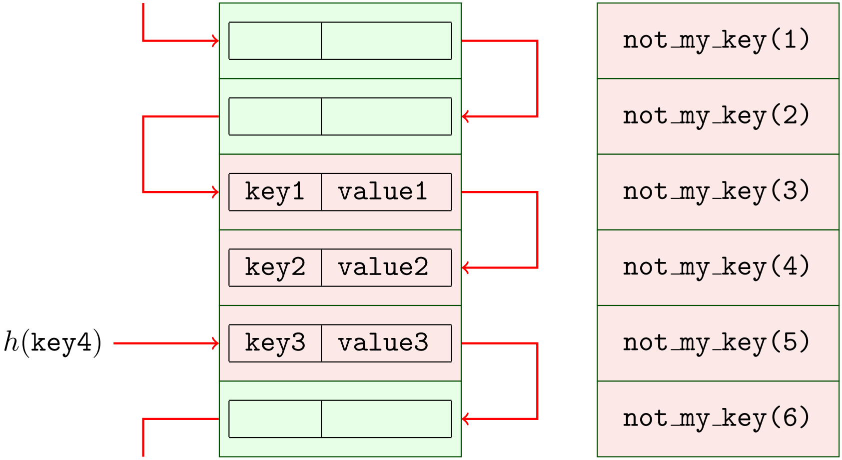
Provided verification

Example: successful search of `key3`



Provided verification

Example: unsuccessful search of `key4`



Provided verification

Part before and after "$\forall i.$`not_my_key(i)` $=$ *true*" provided.

For insertion:

- Same idea
- Property: `findEmpty`

Outline

Modifications

- 64 bits hashes.

offset	entry

Except type changes, only `for` loops modified.

Performance evaluation

- Build a benchmark tool.
- Size, number of accesses, load, read/write ratio, etc. . .
- Converter to C file.
- First warms-up, then measures when target load is reached.

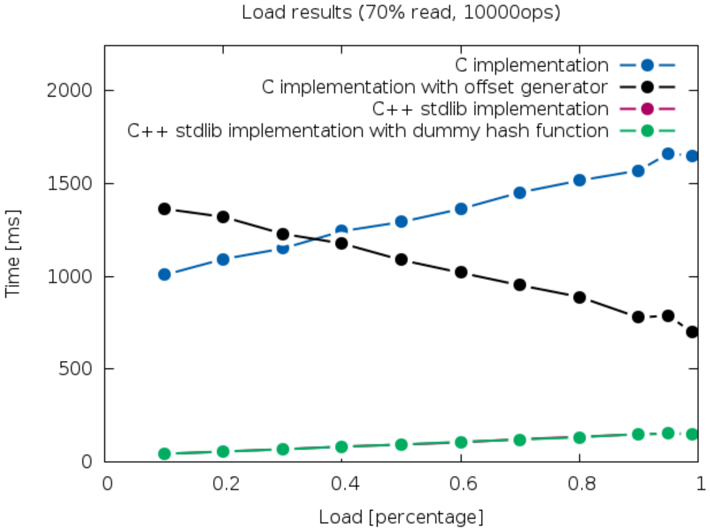
```
test_load.sh length read_ratio load1 [load2...]
```

Evaluation cases

- Worst case: searching a non existing element.

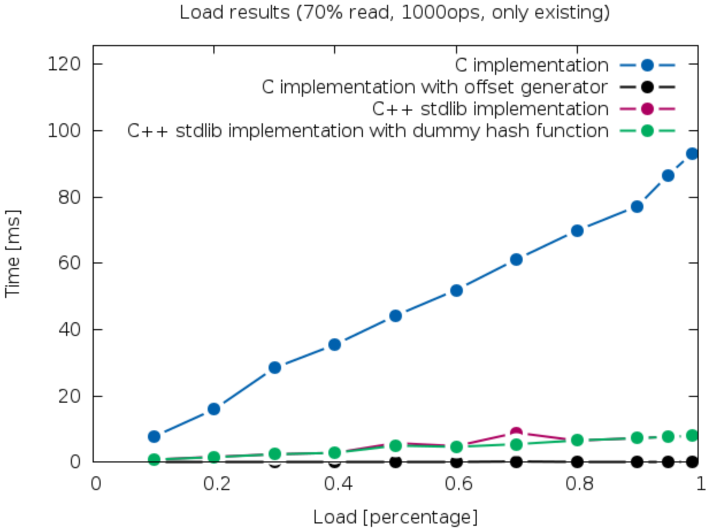
1. Allow searching non existing element.
2. Search only existing element.

Result



Load results (70% read, 10000ops)
C implementation
C implementation with offset generator
C++ stdlib implementation
C++ stdlib implementation with dummy hash function
Time [ms]
2000
1500
1000
500
0
0
0.2
0.4
0.6
0.8
1
Load [percentage]

Result – only existing



Outline

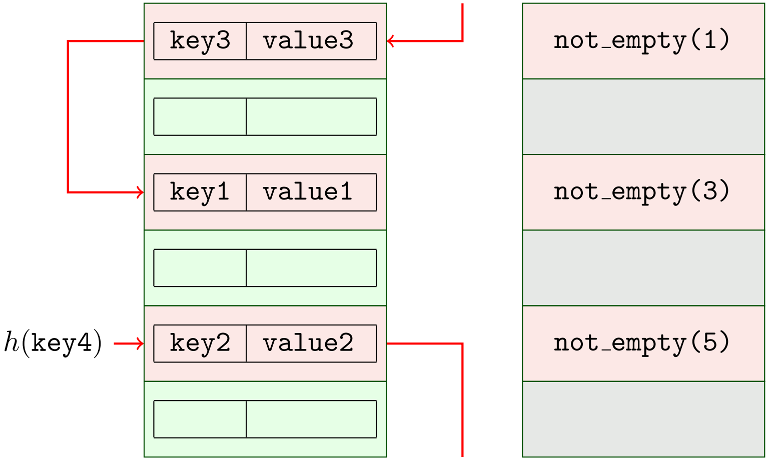
What to prove ?

Goal: show that increment by offset covers all the map.

- Not always true (chinese remainder theorem).
- Requires: `offset` and `capacity` coprime ($gcd = 1$) (necessary and sufficient).

What to prove ?

Insert `key4`: $h_1(\texttt{key4}) = 5$, $h_2(\texttt{key4}) = 2$: search empty ?



Proof steps

If the number of iteration is less than the capacity:

- Build and updated a `list<option<nat>>` with the same pattern.
- Each cell is:
  - `some(n)` if accessed after n iterations.
  - `none` if not accessed.
- Apply Chinese Remainder Theorem.
- Deduce that only `none` are updated to `some`.
- Hence, the number of `some` is the number of iteration.
- For `capacity` iteration, all cells are `some`.

Proof steps

If `some(n)`, then `prop(start+offset*n % capa)`.

`stripe(capacity=7, offset=2, iter=7)`

`none`	
`some(4)`	`prop(2)` holds
`some(1)`	`prop(3)` holds
`some(5)`	`prop(4)` holds
`some(2)`	`prop(5)` holds
`none`	
`some(3)`	`prop(7)` holds

Proof steps

`stripe(capacity=7, offset=2, iter=7)`

`some(7)`
`some(4)`
`some(1)`
`some(5)`
`some(2)`
`some(6)`
`some(3)`

$\Rightarrow$ `count_some` $=$ `iter` $= 7$

$\Rightarrow$ All cells are `some`.

Chinese Remainder Theorem

- Requires to compute *gcd* and prove its properties.
- Almost proved.
- Last assumption: Coprime factorization lemma $(a \perp c \wedge b \perp c \Rightarrow (a \cdot b) \perp c)$.

Outline

Hash-Table software

- Efficient (when key is present).
- Formally verified.
- Requires `capacity` and `offset` coprime.

Remaining work

- Coprime factorization lemma: $a \perp c \wedge b \perp c \Rightarrow (a \cdot b) \perp c$
- Logical operations (`shift` and `bitwise_and`)
- Some typing errors

Side effects

- 6 commits in Verifast tree (`long long` support).
- 9 issues on Verifast.
- A random access sequence generator & benchmark.

Q&A