alg_GCSE_WND

August 22, 2023

[ ]:
```
import matplotlib.pyplot as plt
import numpy as np
import networkx as nx
from tqdm import tqdm
from tool_box.utils import NMSE

plt.rcParams['figure.dpi'] = 300
N_RF = 1000
SNR = 5 # dB
```

Read Generated Channel Data

[ ]:
```
# read data from \generate_channel_data\generate_channel_data.mat
import scipy.io as sio
generate_channel_data = sio.loadmat('./data/generate_channel_data.mat')
cluster_para = generate_channel_data['meta_data']['cluster_para']
# xi = generate_channel_data['meta_data']['xi']
f_c = generate_channel_data['meta_data']['f_c']
N_x = int(generate_channel_data['meta_data']['N_x'])
N_y = int(generate_channel_data['meta_data']['N_y'])
lambda_c = generate_channel_data['meta_data']['lambda_c']
delta = generate_channel_data['meta_data']['delta']
N = int(generate_channel_data['meta_data']['N'])
L_x = generate_channel_data['meta_data']['L_x']
L_y = generate_channel_data['meta_data']['L_y']

H_channel = np.mat(generate_channel_data['channel']['H_channel'][0][0])
vec_H_a = np.mat(generate_channel_data['channel']['vec_H_a'][0][0])
variance = np.mat(generate_channel_data['channel']['variance'][0][0])
Psi = np.mat(generate_channel_data['channel']['Psi'][0][0])

K = 0
sparseness = 0.1


sparse_power_threshold = 0.9
eta = 0.15
Geo_Approx_para = {
```

```
    'epsilon_1' : 1e4,
    'epsilon_2' : 1e-3,
    'epsilon_3' : 1e3,
    'epsilon_4' : 1e-2,
    'K' : 0,   # the number of nonZero elements in the sparse vector
    'tau' : 0.1,     # the 2K largest entries of the sparse vector
    'alpha' : 5     # the biggest entry of the sparse vector
}
```

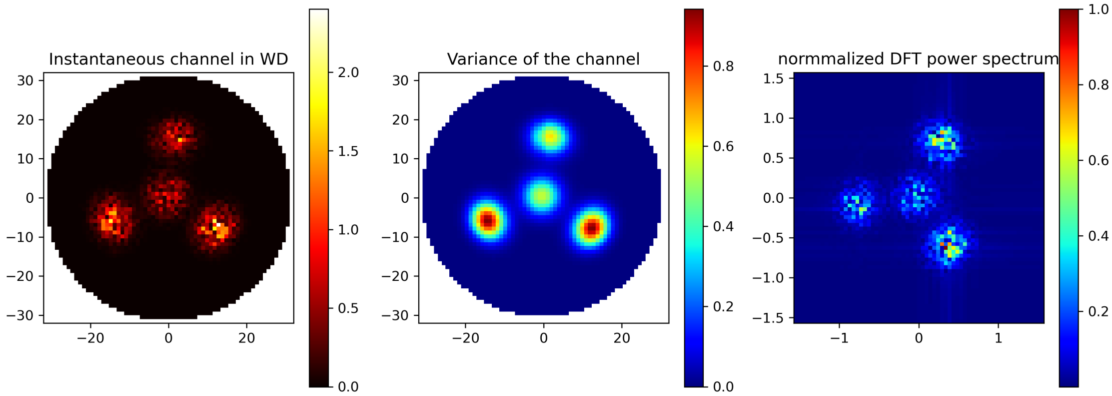
[ ]:
```
xi = []
l_x_abs_max = int(L_x / lambda_c)
l_y_abs_max = int(L_y / lambda_c)
print("l_x_abs_max = ", l_x_abs_max)
print("l_y_abs_max = ", l_y_abs_max)

# for start with -l_x_abs_max to l_x_abs_max
for l_x in range(-l_x_abs_max, l_x_abs_max + 1):
    # for start with -l_y_abs_max to l_y_abs_max
    for l_y in range(-l_y_abs_max, l_y_abs_max + 1):
        k_x = 2 * np.pi * l_x / L_x
        k_y = 2 * np.pi * l_y / L_y
        if k_x ** 2 + k_y ** 2 < (2 * np.pi / lambda_c) ** 2:
            xi.append((l_x, l_y))

M = len(xi)
print("M = ", M)
```

```
l_x_abs_max =  32
l_y_abs_max =  32
M =  3205
```

Visualize Channel

[ ]:
```
# visualize the channel
def visualize_wavenumber_domain_channel(vec_H_a):
    mat_H_a = np.mat(np.zeros((2*l_x_abs_max+1, 2*l_y_abs_max+1),␣
 ↪dtype=complex))
    # all set to np.nan in mat_H_a
    mat_H_a[:,:] = np.nan
    for l, l_pair in enumerate(xi):
        l_x = l_pair[0]
        l_y = l_pair[1]
        mat_H_a[l_x + l_x_abs_max, l_y + l_y_abs_max] = vec_H_a[l]

    # plt.figure(figsize=(10, 10))
    plt.imshow(np.abs(mat_H_a), extent=[-l_x_abs_max, l_x_abs_max,␣
 ↪-l_y_abs_max, l_y_abs_max], cmap='hot')
    plt.colorbar()
```

```
    plt.title('Instantaneous channel in WD')

def visualize_wavenumber_variance(variance):
    # plt.figure(figsize=(8, 6))
    plt.imshow(variance, cmap='jet', extent=[-l_x_abs_max, l_x_abs_max,␣
 ↪-l_y_abs_max, l_y_abs_max])
    plt.colorbar()
    plt.title('Variance of the channel')

def DFT_power_spectrum(H_channel):
    # DFT power spectrum of the channel
    DFT_power = np.abs(np.fft.fft2(np.reshape(H_channel, (N_x, N_y))))

    # element shift according to the DFT power spectrum
    DFT_power = np.fft.fftshift(DFT_power)

    # normalize the DFT power spectrum
    DFT_power = DFT_power / np.max(DFT_power)

    # visualize the DFT power spectrum
    # plt.figure(figsize=(10, 10))
    plt.imshow(DFT_power, cmap='jet', extent=[-np.pi/2, np.pi/2, -np.pi/2, np.
 ↪pi/2])
    plt.colorbar()
    plt.title('normmalized DFT power spectrum')
    return DFT_power

plt.figure(figsize=(14, 5))
# sub out of 3 in one row
plt.subplot(131)
visualize_wavenumber_domain_channel(vec_H_a)

plt.subplot(132)
visualize_wavenumber_variance(variance)

plt.subplot(133)
mat_H_DFT = DFT_power_spectrum(H_channel)
vec_H_DFT = np.reshape(
    np.array(
        mat_H_DFT
    ), (int(mat_H_DFT.shape[0] * mat_H_DFT.shape[1]), )
)
```



Observation Model

$$\mathbf{y} = \mathbf{C}\,\mathbf{h}_a + \mathbf{n}$$

[ ]:
```
# Measurement matrxi for compressed sensing, {N}_{RF} \times N
measurement_matrix = np.random.randn(N_RF, N) * 1 / np.sqrt(N)

y = np.dot(
    measurement_matrix,
    np.dot(
        Psi, vec_H_a
    )
)
Phi = np.dot(measurement_matrix, Psi)
```

Graph Generatioon

[ ]:
```

```

Index-Wavenumber Remapping

[ ]:
```
def from_idx_to_l(idx):
    return (idx[0] - l_x_abs_max, idx[1] - l_y_abs_max)

def from_l_to_idx(l):
    return (l[0] + l_x_abs_max, l[1] + l_y_abs_max)
```

Neighbor Remapping

[ ]:
```

```

Graph Edge Value Calculation

$$\eta_{l,l'} = 0.5\,\arcsin\left(\left(1 - m^8\right)^{-0.25}\right)$$

$$m = \frac{(+1) \times \tilde{K} + (-1) \times (N - \tilde{K})}{N}$$

[ ]:
```python
# Useless function
def get_eta(sparseness):
    m = 1 * sparseness + (-1) * (1 - sparseness)
    eta = 0.5 * np.arcsin(
        np.power(
            1 - np.power(m, 8), -0.25
        )
    )
    return eta
# eta = get_eta(sparseness = 0.2)
```

Graph 'Energy Function' Model

$$\begin{aligned} V_{l,l'}\left(s_l, s_{l'}\right) &= -\eta_{l,l'} \cdot \left(s_l s_{l'} - 1\right) \\ &= \eta_{l,l'} \cdot \left(1 - s_l s_{l'}\right) \end{aligned}$$

[ ]:
```python
def func_V(s_l, s_l_prime):
    return eta * (1 - s_l * s_l_prime)
```

$$D_l\left(s_l\right) = -\log\left(\hat{p}\left(h_{s,l}^{(j-1)}\right)\right)$$

[ ]:
```python
def func_D(h_sl, s_l, Geo_Approx_para):
    tau = Geo_Approx_para['tau']
    if s_l == 1:
        if h_sl <= tau:
            epsilon_4 = Geo_Approx_para['epsilon_4']
            return np.log10(epsilon_4)
        else:
            epsilon_3 = Geo_Approx_para['epsilon_3']
            return np.log10(epsilon_3)
    elif s_l == -1:
        if h_sl <= tau:
            epsilon_1 = Geo_Approx_para['epsilon_1']
            return np.log10(epsilon_1)
        else:
            epsilon_2 = Geo_Approx_para['epsilon_2']
            return np.log10(epsilon_2)
    else:
        raise ValueError('s_l must be 1 or -1')
```

Graph init

[ ]:
```
myGraph = nx.grid_2d_graph(int(2*l_x_abs_max+1), int(2*l_y_abs_max+1))

# traverse graph nodes, minus the absolute max of x and y
node_set = myGraph.nodes()

for l_x in range(-l_x_abs_max, l_x_abs_max+1):
    for l_y in range(-l_y_abs_max, l_y_abs_max+1):
        node_idx = from_l_to_idx((l_x, l_y))

        if (l_x, l_y) not in xi:
            myGraph.remove_node(node_idx)
            continue
        else:
            myGraph.nodes[node_idx]['l'] = (l_x, l_y)
            myGraph.nodes[node_idx]['s'] = -1

# add node 'a' and 'b'
myGraph.add_node('a')   # node alpha
myGraph.add_node('b')   # node beta

# add edge between node and 'a', 'b'
for l in xi:
    node_idx = from_l_to_idx(l)
    myGraph.add_edge('a', node_idx, weight = 0)
    myGraph.add_edge('b', node_idx, weight = 0)
```

neighbor remapping

[ ]:
```
def neighbor_l(Graph, l):
    (l_x, l_y) = l
    node_idx = (l_x + l_x_abs_max, l_y + l_y_abs_max)
    res = []
    for nei_node_idx in Graph.neighbors(node_idx):
        if nei_node_idx == 'a' or nei_node_idx == 'b':
            continue
        else:
            res.append(Graph.nodes[nei_node_idx]['l'])
    return res
print(neighbor_l(myGraph, (0, 0)))
```

```
[(-1, 0), (1, 0), (0, -1), (0, 1)]
```

Find the non Zero enries in the Sparse Channel

[ ]:
```
def find_real_sparseness(vec_H_a, threshold=0.9):
    vec_H_a_abs = np.abs(vec_H_a)
    M = vec_H_a_abs.shape[0]
    # sort the vector in descending order
    vec_H_a_sorted = np.sort(np.reshape(np.array(vec_H_a_abs), (M,)), )
```

```
    vec_H_a_sorted = np.flip(vec_H_a_sorted, axis=0)
    overall_power = np.sum(vec_H_a_sorted)

    K = 0
    for i, val in enumerate(vec_H_a_sorted):
        if np.sum(vec_H_a_sorted[0:i]) >= threshold * overall_power:
            K = i
            break
    tau = 0
    if 2 * K >= M:
        tau = vec_H_a_sorted[-1]
    else:
        tau = vec_H_a_sorted[2 * K]
    alpha = vec_H_a_sorted[0]
    return K, tau, alpha

K, tau, alpha = find_real_sparseness(vec_H_a, threshold=sparse_power_threshold)
sparseness = K / M
print(
    "K = {}, \nN = {}, \n2K = {} \ntau = {}, \nalpha = {}, \nsparseness = {}".
 ↪format(K, N, 2*K, tau, alpha, sparseness)
)
Geo_Approx_para['K'] = K
Geo_Approx_para['tau'] = tau
Geo_Approx_para['alpha'] = alpha    # alpha is the largest value in the vector
```

```
K = 490,
N = 4225,
2K = 980
tau = 0.00839164802111951,
alpha = 2.4020401050588567,
sparseness = 0.15288611544461778
```

[ ]:

```
# print sparseness of vec_H_DFT
K_DFT, tau_DFT, alpha_DFT = find_real_sparseness(vec_H_DFT,␣
 ↪threshold=sparse_power_threshold)
sparseness_DFT = K_DFT / len(vec_H_DFT)
print(
    "K_DFT = {}, \nN_DFT = {}, \n2K_DFT = {} \ntau_DFT = {}, \nalpha_DFT = {},␣
 ↪\nsparseness_DFT = {}".format(K_DFT, len(vec_H_DFT), 2*K_DFT, tau_DFT,␣
 ↪alpha_DFT, sparseness_DFT)
)
```

```
K_DFT = 1102,
N_DFT = 4225,
2K_DFT = 2204
tau_DFT = 0.004134699181134814,
alpha_DFT = 1.0,
```

sparseness_DFT = 0.2608284023668639

Test Region

[ ]:
```python
# print the neighbors of a node
test_nei = myGraph.neighbors((32,32))
for i in test_nei:
    print(myGraph.nodes[i])
```

```
{'l': (-1, 0), 's': -1}
{'l': (1, 0), 's': -1}
{'l': (0, -1), 's': -1}
{'l': (0, 1), 's': -1}
{}
{}
```

[ ]:
```python
vec_H_a_abs = np.abs(vec_H_a)
N = vec_H_a_abs.shape[0]
# sort the vector in descending order
vec_H_a_sortd = np.sort(np.reshape(np.array(vec_H_a_abs), (N,)), )
vec_H_a_sortd = np.flip(vec_H_a_sortd, axis=0)
overall_power = np.sum(vec_H_a_sortd)
for i, val in enumerate(vec_H_a_sortd):
    if np.sum(vec_H_a_sortd[0:i]) >= 0.95 * overall_power:
        break
```

Graph Example

[ ]:
```python
G = nx.Graph()
G.add_edge("x", "a", capacity=3.0)
G.add_edge("x", "b", capacity=0.5)
G.add_edge("a", "c", capacity=3.0)
G.add_edge("b", "c", capacity=5.0)
G.add_edge("b", "d", capacity=4.0)
G.add_edge("d", "e", capacity=2.0)
G.add_edge("c", "y", capacity=2.0)
G.add_edge("e", "y", capacity=3.0)
```

[ ]:
```python
cut_value, partition = nx.minimum_cut(G, "x", "y")
reachable, non_reachable = partition

cutset = set()
for u, nbrs in ((n, G[n]) for n in reachable):
    cutset.update((u, v) for v in nbrs if v in non_reachable)
print(sorted(cutset))
```

```
[('a', 'c'), ('x', 'b')]
```

Test the NumPy Matrix Indexing

[ ]:
```
test_mat = np.array([[1,2,3],[4,5,6],[7,8,9]])
test_mat[0,0]
```

[ ]: 1

[ ]:
```
# modify the edge weight between 'a' to (0,0)
myGraph['a'][(32,32)]['weight'] = 2

# read the edge weight between 'a' to (0,0)
test_1 = myGraph.get_edge_data((32,31), 'a')['weight']

# read the edge weight between 'a' to (0,1)
test_2 = myGraph.get_edge_data((32,32), 'a')['weight']

print(test_1, test_2)
```

```
0 2
```

[ ]:
```
myGraph.nodes[(32,32)]
```

[ ]: {'l': (0, 0), 's': -1}

Test EMRF.py

[ ]:
```
from tool_box.EMRF import EMRF
# add noise
noise = np.random.randn(*y.shape) + 1j * np.random.randn(*y.shape)
noise = noise / np.linalg.norm(noise) * np.linalg.norm(y) / 10 ** (SNR / 20)
y = y + noise

test_EMRF = EMRF(
    Graph = myGraph,
    y = y,
    Phi = Phi,
    vec_H_a=vec_H_a,
    xi = xi,
    l_x_abs_max=l_x_abs_max,
    l_y_abs_max=l_y_abs_max,
    Geo_Approx_para=Geo_Approx_para,
)
```

[ ]:
```
def test_visual():
    # subplots 1 row 2 columns
    plt.figure(figsize=(14, 5))

    # subplot 1
    plt.subplot(1, 2, 1)
    # visualize the support variable
    visualize_wavenumber_domain_channel(test_EMRF.get_support() + 1)
```

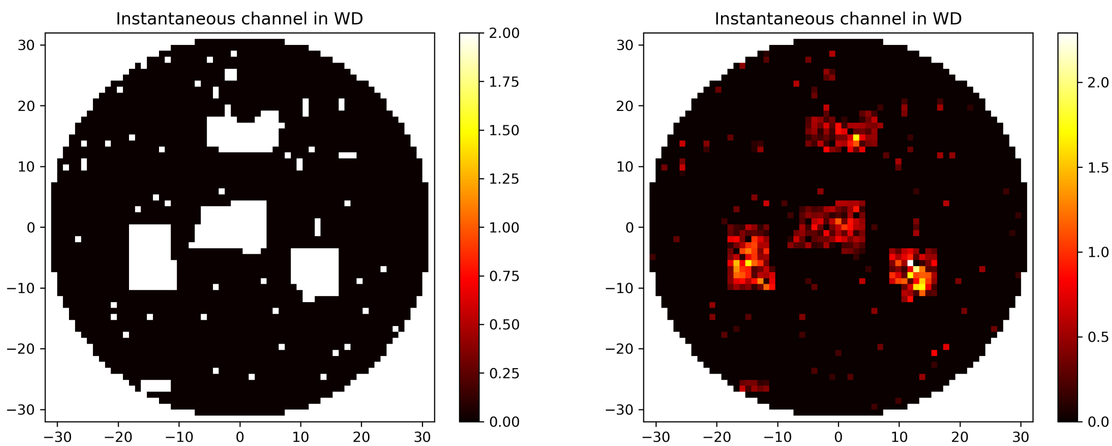
```
    # subplot 2
    plt.subplot(1, 2, 2)
    # visualize the hat_h
    visualize_wavenumber_domain_channel(test_EMRF.hat_h_sl)
    print(test_EMRF.Geo_Approx_para["K"])
```

Test $\alpha-\beta$-swap

[ ]:
```
iter_num = 20
# use tqdm
for i in tqdm(range(iter_num)):
    test_EMRF.alpha_beta_swap()

test_visual()
```

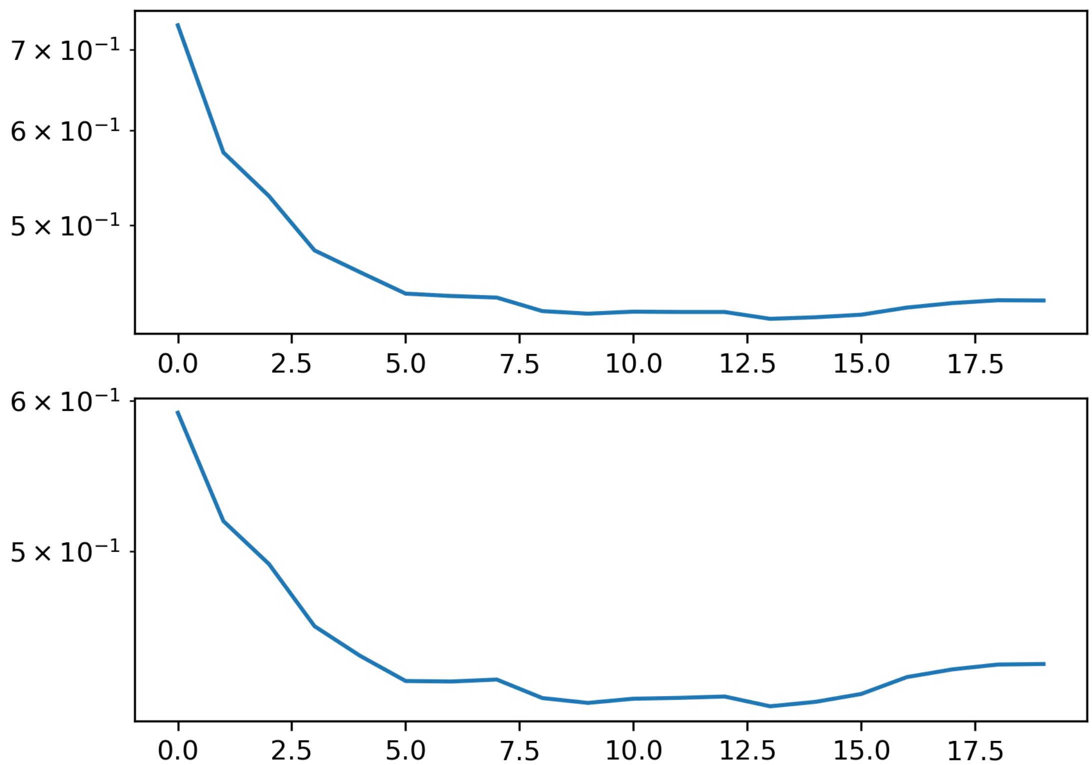
```
  0%|          | 0/20 [00:00<?, ?it/s]  5%|          | 1/20 [00:00<00:08,
2.36it/s]/Users/brook1711/Documents/Documents -  's MacBook Pro/GitHub/WCL-
GXF/code/tool_box/EMRF.py:234: RuntimeWarning: divide by zero encountered in
log10
  return - np.log10(pdf)
100%|     | 20/20 [00:13<00:00,  1.53it/s]
```

490



[ ]:
```
NMSE_list_v2 = test_EMRF.NMSE_list_v2
NMSE_list = test_EMRF.NMSE_list
# plot NMSE_list
plt.figure()
plt.subplot(211)
plt.plot(NMSE_list)
# set log scale
plt.yscale('log')
```

```python
plt.subplot(212)
plt.plot(NMSE_list_v2)
# set log scale
plt.yscale('log')
```




```
[ ]:
```