generate_channel

August 22, 2023

```
[ ]: import matplotlib.pyplot as plt
     import numpy as np
     # import networkx as nx
```

Parameters Initialization

$$\mathbf{y} = \mathbf{C}\ \mathbf{h}_s$$

```
[ ]: f_c = 7e9 # carrier frequency
     N_x = 65 # number of antennas in x direction
     N_y = 65 # number of antennas in y direction
     lambda_c = 3e8/f_c # wavelength
     k_c = 2*np.pi/lambda_c # wave number
     delta = lambda_c/2 # antenna spacing

     N = int(N_x*N_y) # total number of antennas

     L_x = (N_x-1) * delta # length of antenna array in x direction
     L_y = (N_y-1) * delta # length of antenna array in y direction
```

wavenumber domain

$$\xi = \left\{ (l_x, l_y) \mid \left(\frac{2\pi l_x}{L_x}\right)^2 + \left(\frac{2\pi l_y}{L_y}\right)^2 \leq \left(\frac{2\pi}{\lambda}\right)^2 \right\}$$

```
[ ]: xi = []
     l_x_abs_max = int(L_x / lambda_c)
     l_y_abs_max = int(L_y / lambda_c)
     print("l_x_abs_max = ", l_x_abs_max)
     print("l_y_abs_max = ", l_y_abs_max)

     # for start with -l_x_abs_max to l_x_abs_max
     for l_x in range(-l_x_abs_max, l_x_abs_max + 1):
         # for start with -l_y_abs_max to l_y_abs_max
         for l_y in range(-l_y_abs_max, l_y_abs_max + 1):
             k_x = 2 * np.pi * l_x / L_x
             k_y = 2 * np.pi * l_y / L_y
             if k_x ** 2 + k_y ** 2 < (2 * np.pi / lambda_c) ** 2:
```

```
            xi.append((l_x, l_y))
```

```
l_x_abs_max =  32
l_y_abs_max =  32
```

From wavenumber to angles

$$k_x = k_c \sin\theta \cos\phi \xrightarrow{\text{yields}} \phi = \operatorname{sgn}\left(k_y\right) \arccos \frac{k_x}{\sqrt{k_x^2 + k_y^2}}$$

$$k_y = k_c \sin\theta \sin\phi \xrightarrow{\text{yields}} \theta = \arccos \frac{\sqrt{k_c^2 - k_x^2 - k_y^2}}{k_c}$$

[ ]:
```
def from_wavenumber_to_angles(k_x, k_y):
    phi = 0 if np.sqrt(k_x**2 + k_y**2) == 0 else np.sign(k_y) * np.arccos(k_x /
 ↪ np.sqrt(k_x**2 + k_y**2))
    theta = np.arccos(
        np.sqrt(k_c**2 - k_x**2 - k_y**2) / k_c
    )
    return (theta, phi)
```

$\Omega\left(l_x, l_y\right)$

$$\Omega\left(l_x, l_y\right) = \left\{(r, \theta, \phi) \mid \frac{2\pi l_x}{L_x} \leq k_x \leq \frac{2\pi\left(l_x + 1\right)}{L_x}, \frac{2\pi l_y}{L_y} \leq k_y \leq \frac{2\pi\left(l_y + 1\right)}{L_y}, r = 1\right\}$$

Angular Power PDF:

$$p_i(\theta, \phi) = \frac{\alpha_i}{4\pi \sinh \alpha_i} \cdot \exp \alpha \left\{\sin\theta \sin\theta_i \cos\left(\phi - \phi_i\right) + \cos\theta \cos\theta_i\right\}$$

$$A^2\left(\theta, \phi\right) = \sum_{i \in \{1, \ldots, N_c\}} \omega_i p_i\left(\theta, \phi\right)$$

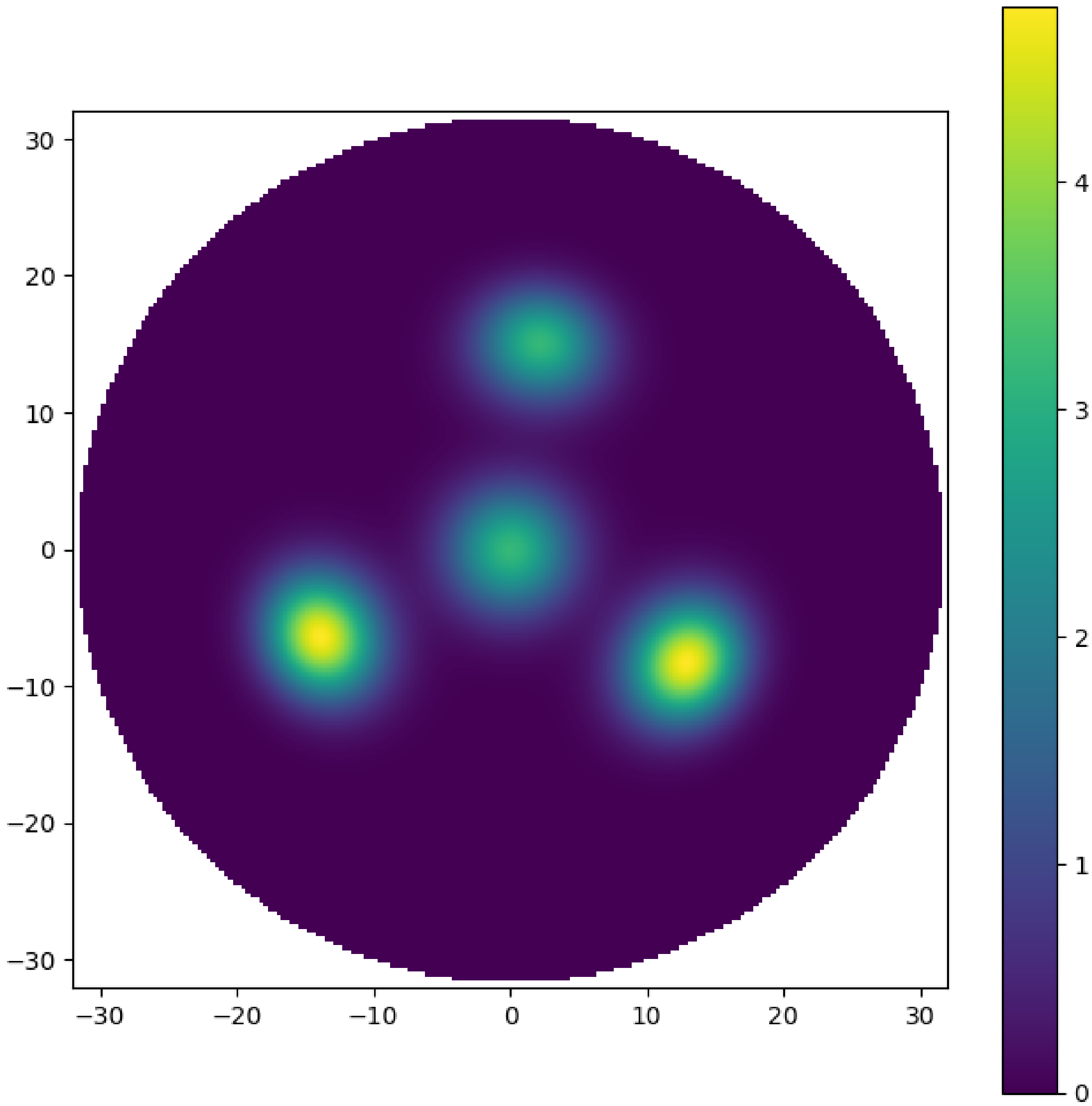
[ ]:
```
cluster_para = {
    # 'alpha': 120,
    'clusters': {
        0 : {
            'alpha': 100,
            'theta': 0.5,
            'phi': 1.,
            'weight': 0.3
        },
        1 : {
            'alpha': 100,
            'theta': -0.5,
```

```
            'phi': 2.,
            'weight': 0.3
        },
        2 : {
            'alpha': 100,
            'theta': 0.5,
            'phi': 3.,
            'weight': 0.2
        },
        3 : {
            'alpha': 100,
            'theta': 0.,
            'phi': 0.,
            'weight': 0.2
        },
    },
}
cluster_para['cluster_num'] = len(cluster_para['clusters'])

def single_cluster_PDF(theta, phi, theta_cen, phi_cen, alpha):
    amp = alpha / (
        4 * np.pi * np.sinh(alpha)
    )
    exp_func = np.exp(
        alpha * (
            np.sin(theta) * np.sin(theta_cen) * np.cos(phi - phi_cen) + np.
↪cos(theta) * np.cos(theta_cen)
        )
    )
    return amp * exp_func

def single_cluster_PDF_wrt_wavenumber(k_x, k_y, theta_cen, phi_cen, alpha):
    if k_x**2 + k_y**2 > k_c**2:
        return 0
    else:
        (theta, phi) = from_wavenumber_to_angles(k_x, k_y)
        return single_cluster_PDF(
            theta=theta,
            phi=phi,
            theta_cen = theta_cen,
            phi_cen = phi_cen,
            alpha = alpha,
        )

def multi_cluster_PDF(k_x, k_y, cluster_para):
    (theta, phi) = from_wavenumber_to_angles(k_x, k_y)
    PDF = 0
```

```python
    for cluster_id in range(cluster_para['cluster_num']):
        PDF += single_cluster_PDF(
            theta=theta,
            phi=phi,
            theta_cen = cluster_para['clusters'][cluster_id]['theta'],
            phi_cen = cluster_para['clusters'][cluster_id]['phi'],
            alpha = cluster_para['clusters'][cluster_id]['alpha'],
        ) * cluster_para['clusters'][cluster_id]['weight']
    return PDF

def multi_cluster_PDF_wrt_wavenumber(k_x, k_y, cluster_para):
    # (theta, phi) = from_wavenumber_to_angles(k_x, k_y)
    PDF = 0
    for cluster_id in range(cluster_para['cluster_num']):
        PDF += single_cluster_PDF_wrt_wavenumber(
            k_x=k_x,
            k_y=k_y,
            theta_cen = cluster_para['clusters'][cluster_id]['theta'],
            phi_cen = cluster_para['clusters'][cluster_id]['phi'],
            alpha = cluster_para['clusters'][cluster_id]['alpha'],
        ) * cluster_para['clusters'][cluster_id]['weight']
    return PDF

def show_multi_cluster_PDF(cluster_para,resolution=100):
    k_x_test = 2 * np.pi / L_x * np.linspace(-l_x_abs_max, l_x_abs_max,␣
 ↪resolution)
    k_y_test = 2 * np.pi / L_x * np.linspace(-l_y_abs_max, l_y_abs_max,␣
 ↪resolution)
    k_x_mesh, k_y_mesh = np.meshgrid(k_x_test, k_y_test)
    PDF_mesh = np.zeros_like(k_x_mesh)
    for i in range(len(k_x_test)):
        for j in range(len(k_y_test)):
            PDF_mesh[i, j] = multi_cluster_PDF(
                k_x = k_x_test[i],
                k_y = k_y_test[j],
                cluster_para = cluster_para,
            )
    plt.figure(figsize=(8, 8))
    plt.imshow(PDF_mesh, extent=[-l_x_abs_max, l_x_abs_max, -l_y_abs_max,␣
 ↪l_y_abs_max])
    plt.colorbar()
    plt.show()

show_multi_cluster_PDF(cluster_para,resolution=200)
```

```
/var/folders/49/6yz92z2s3plbfs7hpwn91v1c0000gn/T/ipykernel_87673/152206500.py:4:
RuntimeWarning: invalid value encountered in sqrt
```

```
np.sqrt(k_c**2 - k_x**2 - k_y**2) / k_c
```



Wavenumber Domain Variance

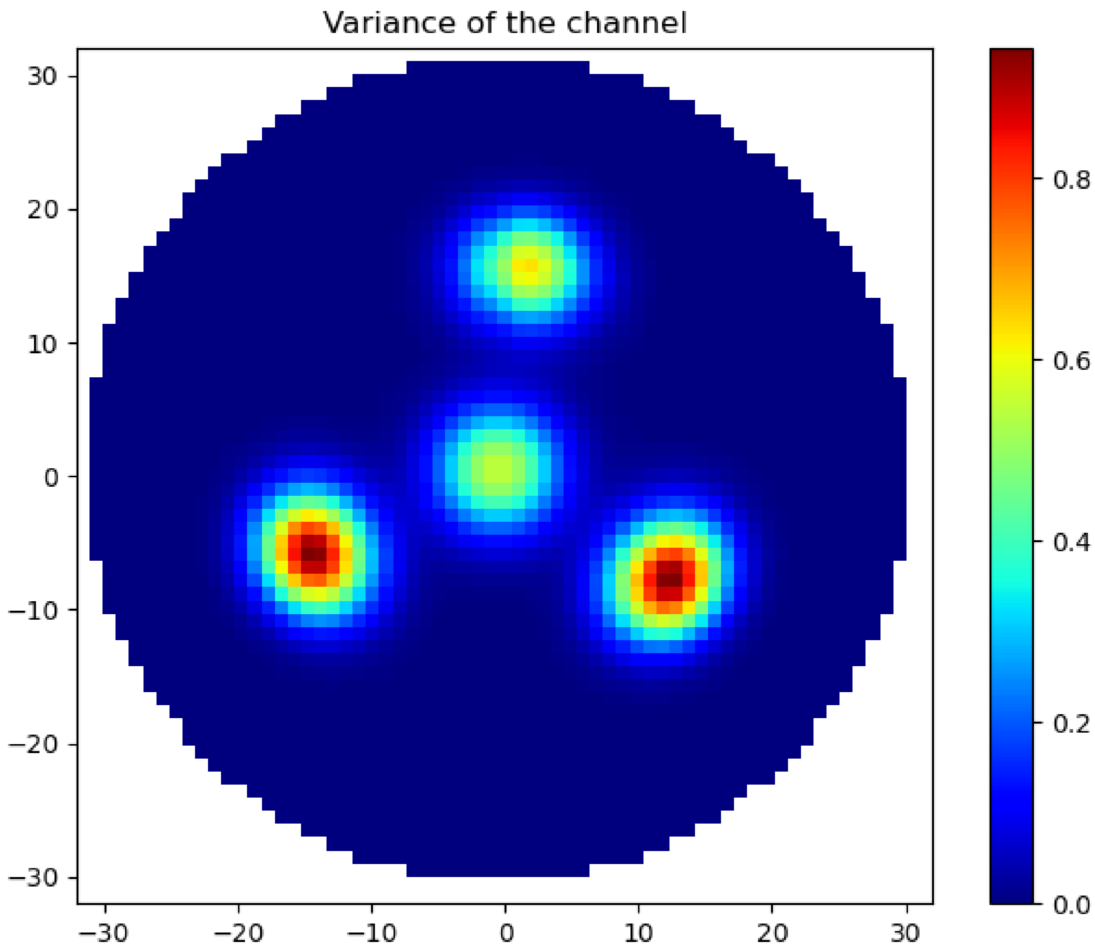
$$\sigma_f^2\left(l_x, l_y\right)=\iint_{\Omega_R\left(l_x, l_y\right)} A^2(\theta, \phi) \sin \theta \mathrm{d} \theta \mathrm{d} \phi$$

$$=\int_{\frac{2\pi l_y}{L_{R,y}}}^{\frac{2\pi\left(l_y+1\right)}{L_{R,y}}} \int_{\frac{2\pi l_x}{L_{R,x}}}^{\frac{2\pi\left(l_x+1\right)}{L_{R,x}}} \frac{A^2\left(k_x, k_y\right)}{\sqrt{k^2-k_x^2-k_y^2}} dk_x dk_y$$

[ ]:
```python
def multi_cluster_integral_func(k_x, k_y):
    if k_x**2 + k_y**2 > k_c**2:
        return np.nan
    else:
        res = multi_cluster_PDF(
            k_x=k_x,
            k_y=k_y,
            cluster_para=cluster_para,
        ) / np.sqrt(
            k_c**2 - k_x**2 - k_y**2
        )
        return res

def single_variance(l_x, l_y, resolution=100):
    # integral region
    # k_x from 2*np.pi * l_x/ L_x to 2*np.pi * (l_x+1)/ L_x
    # k_y from 2*np.pi * l_y/ L_y to 2*np.pi * (l_y+1)/ L_y
    k_x_region = np.linspace(2*np.pi * l_x/ L_x, 2*np.pi * (l_x+1)/ L_x,␣
 ↪resolution)
    k_y_region = np.linspace(2*np.pi * l_y/ L_y, 2*np.pi * (l_y+1)/ L_y,␣
 ↪resolution)

    # mesh grid
    k_mesh = np.meshgrid(k_x_region, k_y_region)

    # grid size
    dk = (k_x_region[1] - k_x_region[0]) * (k_y_region[1] - k_y_region[0])

    integral_res = 0
    for k_x in k_x_region:
        for k_y in k_y_region:
            integral_res += multi_cluster_integral_func(k_x, k_y)

    return integral_res * dk

l_x_grid = np.arange(-l_x_abs_max, l_x_abs_max+1)
l_y_grid = np.arange(-l_y_abs_max, l_y_abs_max+1)
l_mesh = np.meshgrid(l_x_grid, l_y_grid)

variance = np.zeros((2*l_x_abs_max+1, 2*l_y_abs_max+1))

for l_x in range(-l_x_abs_max, l_x_abs_max+1):
    for l_y in range(-l_y_abs_max, l_y_abs_max+1):
        l_x_idx = l_x + l_x_abs_max
        l_y_idx = l_y + l_y_abs_max
        variance[l_x_idx, l_y_idx] = single_variance(l_x, l_y, resolution=10)
```

```python
# plot

def visualize_wavenumber_variance(variance):
    plt.figure(figsize=(8, 6))
    plt.imshow(variance, cmap='jet', extent=[-l_x_abs_max, l_x_abs_max,␣
 ↪-l_y_abs_max, l_y_abs_max])
    plt.colorbar()
    plt.title('Variance of the channel')

visualize_wavenumber_variance(variance)
```

```
/var/folders/49/6yz92z2s3plbfs7hpwn91v1c0000gn/T/ipykernel_87673/1157091051.py:5
: RuntimeWarning: divide by zero encountered in double_scalars
  res = multi_cluster_PDF(
```



Dictionary Matrix

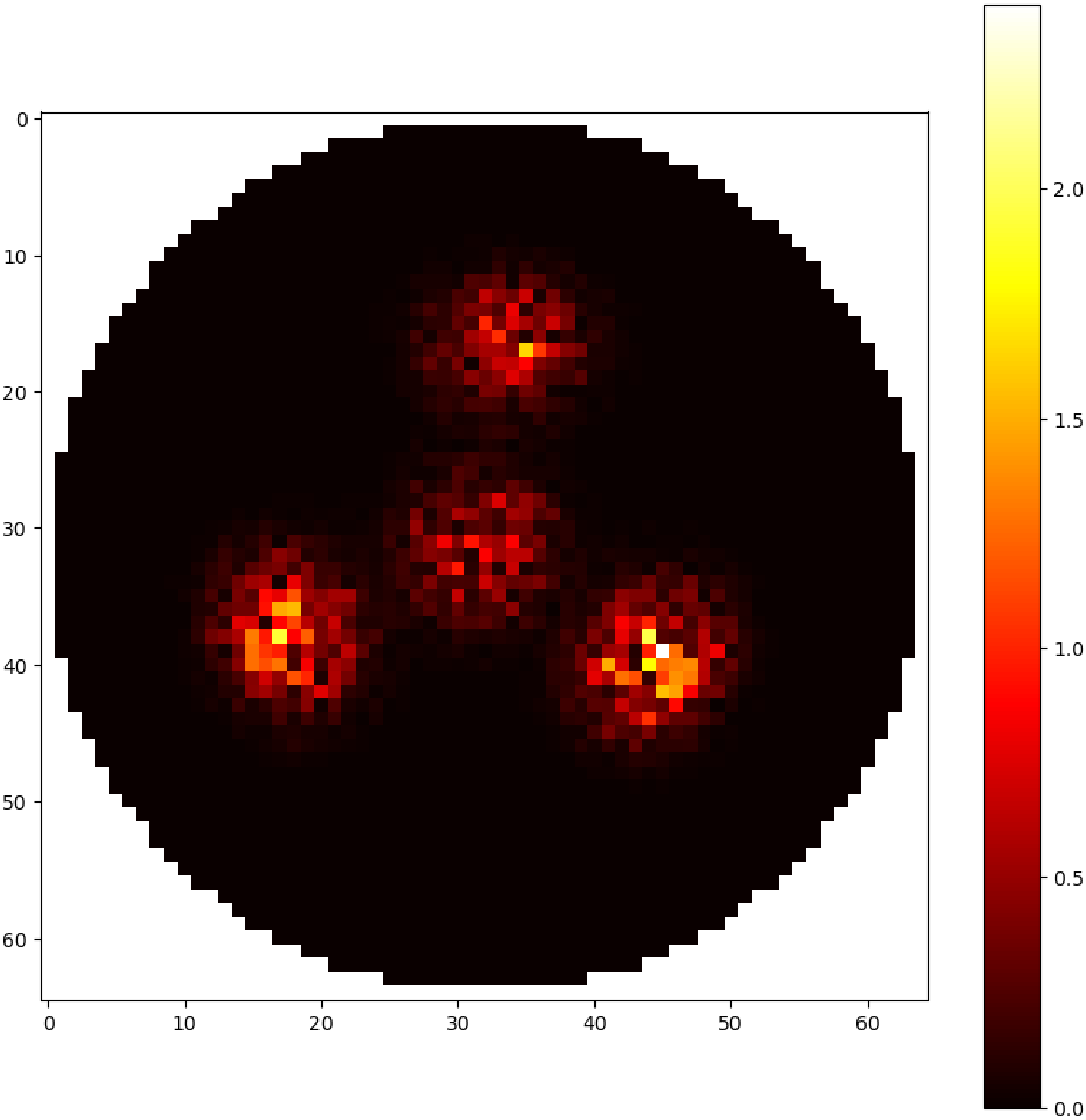
$$[\,]_{n,l} = \frac{1}{\sqrt{N}} \exp j \left\{ \frac{2\pi l_x}{L_x} \delta \cdot n_x + \frac{2\pi l_y}{L_y} \delta \cdot n_y \right\}$$

[ ]:
```
def Psi_col(l_pair):
    l_x = l_pair[0]
    l_y = l_pair[1]
    amp = 1 / np.sqrt(N)
    exp_func = np.exp(
        1j * (
            2*np.pi * l_x / L_x * delta * (np.arange(N_x) - (N_x - 1) / 2) + \
                2*np.pi * l_y / L_y * delta * (np.arange(N_y) - (N_y - 1) / 2)[:
↪, np.newaxis]
        )
    )
    vec = np.reshape(exp_func, (N_x*N_y, ))
    return amp * vec

# a = Psi_col((1, 1))
# np.shape(a)

xi_norm = len(xi)
Psi = np.zeros((N, xi_norm), dtype=np.complex_)

for l, l_pair in enumerate(xi):
    Psi[:, l] = Psi_col(l_pair)
    # print(l_pair)
```

$$[\mathbf{H}_a]_{l_x,l_y} \sim \mathcal{CN}\left(0, \sigma_f^2\left(l_x, l_y\right)\right)$$

[ ]:
```
vec_sigma_f = np.mat(np.zeros((xi_norm, 1)))
for l, l_pair in enumerate(xi):
    l_x = l_pair[0]
    l_y = l_pair[1]
    vec_sigma_f[l] = variance[l_x + l_x_abs_max, l_y + l_y_abs_max]
    # if nan
    if np.isnan(vec_sigma_f[l]):
        vec_sigma_f[l] = 0

np.random.seed(0)
vec_H_a = np.mat(np.random.normal(0, 1, (xi_norm, 1)) + 1j * np.random.
↪normal(0, 1, (xi_norm, 1)))

# Hadamard product of vec_sigma_f and vec_H_a
vec_H_a = np.multiply(vec_sigma_f, vec_H_a)
```

```python
# visualize the channel
def visualize_wavenumber_domain_channel(vec_H_a):
    mat_H_a = np.mat(np.zeros((2*l_x_abs_max+1, 2*l_y_abs_max+1),␣
 ↪dtype=complex))
    # all set to np.nan in mat_H_a
    mat_H_a[:,:] = np.nan
    for l, l_pair in enumerate(xi):
        l_x = l_pair[0]
        l_y = l_pair[1]
        mat_H_a[l_x + l_x_abs_max, l_y + l_y_abs_max] = vec_H_a[l]

    plt.figure(figsize=(10, 10))
    plt.imshow(np.abs(mat_H_a), cmap='hot')
    plt.colorbar()
visualize_wavenumber_domain_channel(vec_H_a)
```



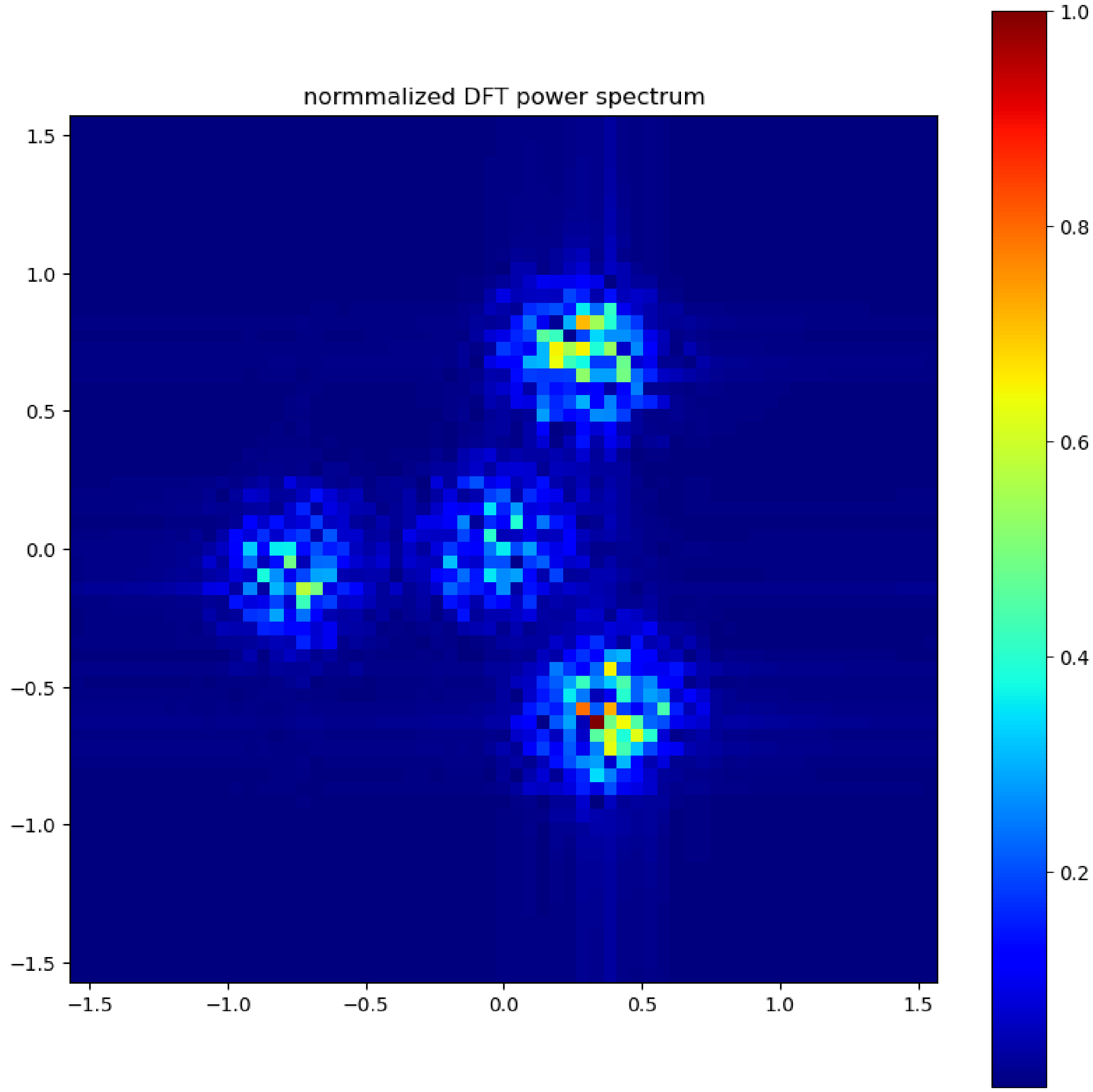
$$\mathbf{H} \in \mathbb{C}^{N\times 1} = \underbrace{}_{\mathbb{C}^{N\times|\xi|}} \mathrm{vec}\left(\mathbf{H}_a\right)$$

[ ]:
```
H_channel = np.zeros((N,1), dtype=np.complex64)
H_channel = Psi * vec_H_a
```

DFT power spectral

[ ]:
```
def DFT_power_spectrum(H_channel):
    # DFT power spectrum of the channel
    DFT_power = np.abs(np.fft.fft2(np.reshape(H_channel, (N_x, N_y))))
```

```
    # element shift according to the DFT power spectrum
    DFT_power = np.fft.fftshift(DFT_power)

    # normalize the DFT power spectrum
    DFT_power = DFT_power / np.max(DFT_power)

    # visualize the DFT power spectrum
    plt.figure(figsize=(10, 10))
    plt.imshow(DFT_power, cmap='jet', extent=[-np.pi/2, np.pi/2, -np.pi/2, np.
 ↪pi/2])
    plt.colorbar()
    plt.title('normmalized DFT power spectrum')

    # return DFT_power
DFT_power_spectrum(H_channel)
```



Save data

[ ]:
```
generate_channel_data = {
    'meta_data' :{
        'cluster_para' : cluster_para,
        'xi' : xi,
        'f_c' : f_c,
        'N_x' : N_x,
        'N_y' : N_y,
        'lambda_c' : lambda_c,
        'delta' : delta,
        'N' : N,
        'L_x' : L_x,
        'L_y' : L_y,
    },
    'channel' : {
        'H_channel' : H_channel,
        'vec_H_a' : vec_H_a,
        'variance' : variance,
        'Psi' : Psi,
    }
}

# save to \generate_channel_data\generate_channel_data.mat
import scipy.io as sio
sio.savemat('data/generate_channel_data.mat', generate_channel_data)
```

[ ]:
```

```