Unitary transformation

In mathematics, a **unitary transformation** is a transformation that preserves the inner product: the inner product of two vectors before the transformation is equal to their inner product after the transformation.

Formal definition

More precisely, a **unitary transformation** is an isomorphism between two Hilbert spaces. In other words, a *unitary transformation* is a bijective function

$$U : H_1 \to H_2$$

where H_1 and H_2 are Hilbert spaces, such that

$$\langle Ux, Uy \rangle_{H_2} = \langle x, y \rangle_{H_1}$$

for all x and y in H_1.

Properties

A unitary transformation is an isometry, as one can see by setting $x = y$ in this formula.

Unitary operator

In the case when H_1 and H_2 are the same space, a unitary transformation is an automorphism of that Hilbert space, and then it is also called a unitary operator.

Antiunitary transformation

A closely related notion is that of **antiunitary transformation**, which is a bijective function

$$U : H_1 \to H_2$$

between two complex Hilbert spaces such that

$$\langle Ux, Uy \rangle = \overline{\langle x, y \rangle} = \langle y, x \rangle$$

for all x and y in H_1, where the horizontal bar represents the complex conjugate.

See also

- Antiunitary
- Orthogonal transformation
- Time reversal
- Unitary group
- Unitary operator
- Unitary matrix
- Wigner's Theorem
- Unitary transformations in quantum mechanics

Retrieved from "https://en.wikipedia.org/w/index.php?title=Unitary_transformation&oldid=879589293"