

Unitary transformation

In mathematics, a **unitary transformation** is a transformation that preserves the inner product: the inner product of two vectors before the transformation is equal to their inner product after the transformation.

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Formal definition

More precisely, a **unitary transformation** is an isomorphism between two Hilbert spaces. In other words, a *unitary transformation* is a bijective function

$$U : H_1 \rightarrow H_2$$

where ***H*₁** and ***H*₂** are Hilbert spaces, such that

$$\langle Ux, Uy \rangle_{H_2} = \langle x, y \rangle_{H_1}$$

for all ***x*** and ***y*** in ***H*₁**.

Properties

A unitary transformation is an isometry, as one can see by setting ***x*** = ***y*** in this formula.

Unitary operator

In the case when ***H*₁** and ***H*₂** are the same space, a unitary transformation is an automorphism of that Hilbert space, and then it is also called a unitary operator.

Antiunitary transformation

A closely related notion is that of **antiunitary transformation**, which is a bijective function

$$U : H_1 \rightarrow H_2$$

between two complex Hilbert spaces such that

$$\langle Ux, Uy \rangle = \overline{\langle x, y \rangle} = \langle y, x \rangle$$

for all x and y in H_1 , where the horizontal bar represents the complex conjugate.

See also

- Antiunitary
 - Orthogonal transformation
 - Time reversal
 - Unitary group
 - Unitary operator
 - Unitary matrix
 - Wigner's Theorem
 - Unitary transformations in quantum mechanics
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