Direct Localization of Emitters Using Widely Spaced Sensors in Multipath Environments

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***Abstract*—We address the localization of sources with known waveforms in frequency-selective channels. Conventional localization by multilateration is an *indirect* approach that is suboptimal at lower SNR, and breaks down in the presence of multipath. Here, we propose a *direct* localization method (DLM) that exploits the sparsity of the emitters, as well as differences in the properties of the line of sight (LOS) versus multipath components of the signals received at the sensors. It is shown that the proposed method has superior performance relative to other known localization techniques and is robust to sensors with blocked LOS.**

I. Introduction

Traditional time-of-arrival (TOA)-based localization is accomplished through a two-step process. In the first step, sensors measure TOA's from all incoming signals; in the second step, such measurements are transmitted to a central node, that subsequently estimates the location of each emitter by multilateration [1]. In a multipath environment, each sensor receives, in addition to a line-of-sight (LOS) signal, multiple (possibly overlapping) replicas due to non-line-of-sight (NLOS) paths. Due to these multiple arrivals, it is, in general, more challenging to obtain accurate TOA estimates of the LOS components at the sensors. Matched filtering is the most simple and known technique for time delay estimation. However, its performance degrades greatly in the presence of multipath that falls within time resolution equal to the inverse bandwidth of the signal. Moreover, in the case of blockage of the LOS path, the TOA of the first arrival does not correspond to a LOS component anymore, and will corrupt localization. In such a case it is customary to apply various techniques (e.g. [3]) to mitigate the effect of NLOS on the geolocation estimate. An application example of TOA-based localization is found in [5].

Localization in the presence of multipath has received a great deal of attention. However, past and current literature is mostly based on techniques that require first estimating TOA's, and subsequently using these estimates to determine the location of the emitter(s). We refer to these classical localization techniques as *indirect* localization. A better approach than indirect localization is to infer the emitter locations directly from the signal measurements, without estimating the propagation delays first. The concept of direct localization was first introduced by Wax and Kailath [6], but little work was done until the more recent Direct Position Determination (DPD) technique [7]. DPD is designed for multipath-free environments, and outperforms standard indirect localization because it takes into account the fact that signals arriving at different sensors originate from the same location. The improvement in performance is particularly evident at low signal-to-noise ratio (SNR), while at high SNR, DPD and indirect techniques perform similarly. The literature on direct localization in the presence of multipath is scarce. In [8], a maximum likelihood (ML) estimator has been developed for localizing a single emitter assuming a fixed and known number of multipaths, but without providing an efficient way to compute the estimator. In [9], a Direct Positioning Estimation (DPE) technique is proposed for operating in dense multipath environments, but it requires knowledge of the power delay profile, and is limited to localization of a single emitter.

In this paper, we introduce a technique dubbed **d**irect **l**ocalization in **m**ultipath (DLM) for localizing multiple emitters that transmit assumed known waveforms. No prior knowledge is assumed of the statistics of the NLOS paths. Without prior knowledge, NLOS components carry no information, and the best performance is obtained by using only the LOS components [4]. By setting up the problem in a sparse framework, DLM estimates the emitter locations, while treating the NLOS components as interference. Numerical evidence shows that DLM works well in a wide range of multipath scenarios, including sensors with blocked LOS. Moreover DLM requires no channel state information such as number of multipath, sensors with LOS blockage, signal strengths or the power delay profile.

In Section II, we introduce the signal model. Section III presents the DLM. Section IV compares DLM to previous existing techniques. Finally, Section V reports our conclusions.

II. Signal Model

Consider a network composed of L sensors and Q emitters located in a plane. The location of the q-th emitter is defined by two coordinates stacked in a vector $\mathbf{p}_q$. All emitters share the same bandwidth B, and transmit their own signals $\{s_q(t)\}_{q=1}^{Q}$. The number of emitters Q and their waveforms are known. The observation time is T, assumed to be shorter than the

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time coherence of the channel, therefore, the channel is time-invariant. The complex-valued baseband signal at the l-th sensor is

$$r_l(t) = r_l^{\mathrm{LOS}}(t) + r_l^{\mathrm{NLOS}}(t) + n_l(t) \qquad 0 \le t \le T, \tag{1}$$

where $n_l(t)$ is white Gaussian noise with known variance σ_n^2. The term $r_l^{\mathrm{LOS}}(t)$ is the sum of all LOS components:

$$r_l^{\mathrm{LOS}}(t) = \sum_{q=1}^{Q} a_{lq} s_q \left(t - \tau_l(\mathbf{p}_q)\right), \tag{2}$$

where a_{lq} is an unknown complex scalar representing the amplitude of the LOS path between the q-th emitter and l-th sensor, and $\tau_l(\mathbf{p})$ is the delay of a signal originating at $\mathbf{p}$ and reaching the l-th sensor:

$$\tau_l(\mathbf{p}) = \left\|\mathbf{p} - \mathbf{p}_l'\right\| / c. \tag{3}$$

In (3), $\mathbf{p}_l'$ is the location of the l-th sensor, c is the speed of light and $\|\cdot\|$ denotes the standard Euclidean norm. The term $r_l^{\mathrm{NLOS}}(t)$ in (1) aggregates all NLOS arrivals:

$$r_l^{\mathrm{NLOS}}(t) = \sum_{q=1}^{Q} \sum_{m=1}^{M_{lq}} a_{lq}^{(m)} s_q \left(t - \tau_{lq}^{(m)}\right), \tag{4}$$

where M_{lq} denotes an unknown number of NLOS paths between the q-th emitter and the l-th sensor, $a_{lq}^{(m)}$ is an unknown complex scalar representing the amplitude of the m-th NLOS path between the q-th emitter and l-th sensor, and $\tau_{lq}^{(m)}$ is the delay of the NLOS component. Note that NLOS components have delays that are specific to the sensor and the emitter. The received signal (1) is sampled at a frequency f_s satisfying the Nyquist sampling criterion: $f_s \ge B$, where B is the passband bandwidth. Let N be the number of time samples collected at each sensor. By stacking the N acquired samples, the received signal $\mathbf{r}_l = [r_l(0), \ldots, r_l((N-1)/f_s)]^T$ at the l-th sensor can be written in the following vector form

$$\mathbf{r}_l = \sum_{q=1}^{Q} \left(\tilde{a}_{lq} \mathbf{s}_q \left(\tau_l(\mathbf{p}_q)\right) + \sum_{m=1}^{M_{lq}} \tilde{a}_{lq}^{(m)} \mathbf{s}_q \left(\tau_{lq}^{(m)}\right) \right) + \mathbf{n}_l, \tag{5}$$

where $\mathbf{s}_q(\tau)$ is a normalized vector of the N received samples from the q-th emitter waveform with delay τ:

$$\mathbf{s}_q(\tau) = \frac{1}{\sqrt{\sum\limits_{n=0}^{N-1} \left|s_q(n/f_s - \tau)\right|^2}} \begin{bmatrix} s_q(0-\tau) \\ \vdots \\ s_q((N-1)/f_s - \tau) \end{bmatrix}, \tag{6}$$

and $\tilde{a}_{lq}$ and $\tilde{a}_{lq}^{(m)}$ are unknown complex constants.

In the next section, we introduce DLM.

III. Proposed Localization Technique

In order to develop a localization technique, it is necessary to understand what parameters of the received signals depend on the emitters' locations. In the signal model introduced in the previous section, we defined the propagation delays of NLOS components (4) as unknown and arbitrary because we assumed no prior statistical knowledge of the channel. Thus, information on the emitter locations is carried only by the LOS components (2). This claim is supported by the analysis in [4], which showed that the CRB increases when NLOS components are present. Thus, without a priori knowledge, the optimal strategy is to reject NLOS components as much as possible, and rely on the LOS components to infer the emitters' locations.

In the case of indirect techniques, the TOA's of the LOS components are estimated, and then used to localize the emitters by multilateration. However, indirect techniques are suboptimal because they estimate TOA's independently at each sensor, instead of taking into account that all the LOS components originate from a single emitter location. In this section, we propose a direct localization technique that relies on the fact that all LOS components must originate from the same location. Under the Gaussian assumption, the maximum likelihood estimator (MLE) is the solution to the following fitting problem

$$\min_{\substack{\mathbf{p}_1,\ldots,\mathbf{p}_Q \\ \tilde{a}_{11},\ldots,\tilde{a}_{LQ} \\ M_{11},\ldots,M_{LQ} \\ \tau_{11}^{(1)},\ldots,\tau_{LQ}^{(M_{LQ})} \\ \tilde{a}_{11}^{1},\ldots,\tilde{a}_{LQ}^{M_{LQ}}}} \sum_{l=1}^{L} \left\| \mathbf{r}_l - \sum_{q=1}^{Q} \tilde{a}_{lq} \mathbf{s}_q \left(\tau_l(\mathbf{p}_q)\right) - \sum_{q=1}^{Q} \sum_{m=1}^{M_{lq}} \tilde{a}_{lq}^{(m)} \mathbf{s}_q \left(\tau_{lq}^{(m)}\right) \right\|^2 \tag{7}$$

subject to $\tau_{lq}^{(m)} > \tau_l(\mathbf{p}_q)$ for all l, q and m. The parameters of interest are the emitter locations $\{\mathbf{p}_q\}_{q=1}^{Q}$, while the rest act as nuisance parameters. Besides the fact that it is an enormous challenge to find an efficient technique for minimizing this objective function, the ML criterion does not even lead to a satisfactory solution. The reason is that M_{lq}, for all l and q, are hyperparameters that control the number of NLOS paths in our model. It is known that increasing the values of hyperparameters always leads to a better fitting error [10], and in our case it would lead to the erroneous conclusion that there are an infinite number of NLOS arrivals. Instead, we assume that the number of NLOS arrivals and the number of emitters are low. This assumption enables the formulation of a feasible solution to the ML multipath estimation problem by means of a sparse recovery technique.

Our technique is divided in two phases which are explained in detail below. In the first phase, NLOS components are canceled out from the received signals by exploiting the fact that NLOS components arrive with a longer delay than LOS components. In the second phase, we use the cleaned version of the received signals to localize the emitters. It is in this phase that the emitter locations are estimated by solving an optimization problem. The key idea is a solution designed to exploit the fact that LOS components must originate from the same location.

A. First Phase: NLOS Interference Mitigation

First, the multipath arrivals are detected at each sensor and their TOA is estimated using a technique like the one in [2]. Obviously, more accurate delay estimation techniques will result in more accurate NLOS interference mitigation. The goal in this phase is not to precisely estimate the LOS TOA's, but to estimate the NLOS TOA's and cancel them out. Assume P_{lq} arrivals are detected at sensor l for waveform q and the corresponding TOA measurements are $\mathcal{T}_{lq} = \left\{\hat{\tau}_{lq}^{1}, \ldots, \hat{\tau}_{lq}^{P_{lq}}\right\}$. Then, the amplitudes of such arrivals can also be estimated by solving a least squares fit (see [2])

$$\underset{\left\{a_{lq}^{p}\right\}}{\arg\min} \left\| \mathbf{r}_l - \sum_{q=1}^{Q} \sum_{p=1}^{P_{lq}} a_{lq}^{p} \mathbf{s}_q \left(\hat{\tau}_{lq}^{p}\right) \right\|^2 . \tag{8}$$

Assuming the set of TOA measurements are ranked in increasing order, i.e. $\hat{\tau}_{lq}^{1} < \ldots < \hat{\tau}_{lq}^{P_{lq}}$ for any l, q, we cancel all arrivals from the received signals except the first one:

$$\hat{\mathbf{r}}_l = \mathbf{r}_l - \sum_{q=1}^{Q} \sum_{p=2}^{P_{lq}} \hat{a}_{lq}^{p} \mathbf{s}_q \left(\hat{\tau}_{lq}^{p}\right), \tag{9}$$

where $\hat{a}_{lq}^{p}$ are the amplitude estimates. If all NLOS arrivals were removed, we could simply use a direct localization technique in the absence of multipath, like DPD. However, the NLOS interference mitigation is not guaranteed to remove all NLOS components for two reasons. First, if the LOS path between a sensor and emitter is blocked, then the first arrival corresponds to a NLOS component, which will not be removed. Also, it is possible that the chosen delay estimation technique misses some arrivals or detects some false ones, resulting in errors in the interference mitigation process. In next section, we present a localization technique designed to work in the presence of LOS and NLOS components. In short, this phase reduces the multipath components but does not necessarily remove them completely.

B. Second Phase: Sparsity-based Direct Localization

In this phase, we find a sparse representation of the received signal provided by the NLOS interference mitigation phase, and show how a variation of the Group Lasso with Overlaps technique proposed in [11] may solve problem (7). In order to find a sparse representation, we first discretize the continuous set of all possible signal propagation TOA's $[0, \tau_{\max}]$, where $\tau_{\max}$ is the largest signal propagation delay allowed at any sensor, into a discrete, finite list

$$\mathcal{D} = \{0, \tau_{\text{res}}, \ldots, (D-1)\tau_{\text{res}}\}. \tag{10}$$

Parameter τ_{res} defines the resolution of the delay grid, and D is the number of discrete TOA's computed as $D = \lfloor \tau_{\max}/\tau_{\text{res}} \rfloor$, where operator $\lfloor \cdot \rfloor$ denotes the 'nearest integer'. The NLOS interference mitigation phase is not perfect, thus the expression for $\mathbf{r}_l$ in (5), which includes LOS and NLOS components is still valid for $\hat{\mathbf{r}}_l$. Given these definitions, the received signal at sensor l (5) results in the following alternative sparse representation:

$$\hat{\mathbf{r}}_l \approx \sum_{q=1}^{Q} \mathbf{A}_q \mathbf{x}_{lq} + \mathbf{n}_l, \tag{11}$$

where matrix $\mathbf{A}_q$ is a basis of emitter q's waveforms for all TOA's in $\mathcal{D}$

$$\mathbf{A}_q = \begin{bmatrix} \mathbf{s}_q(0) & \cdots & \mathbf{s}_q((D-1)\tau_{\text{res}}) \end{bmatrix}, \tag{12}$$

and the d-th component of the sparse vector $\mathbf{x}_{lq}$ is

$$\left\{\mathbf{x}_{lq}\right\}_d = \begin{cases} \tilde{a}_{lq} & \text{if } (d-1)\tau_{\text{res}} \approx \tau_l\left(\mathbf{p}_q\right) \\ \tilde{a}_{lq}^{(m)} & \text{if } (d-1)\tau_{\text{res}} \approx \tau_{lq}^{(m)} \\ 0 & \text{else.} \end{cases} \tag{13}$$

Vector $\mathbf{x}_{lq}$ stacks the amplitudes of all multipath arrivals between emitter q and sensor l. Vector $\mathbf{x}_{lq}$ is considered sparse because the number of possible propagation delays is much larger than the actual number of paths: $D >> M_{lq}$ for any l and q. Vectors $\mathbf{x}_{lq}$ could be recovered by a sparse recovery technique, and the TOA's recovered from the locations of the non-zeros. However, contrary to indirect techniques, we are interested in retrieving emitter locations, not in estimating TOA's. Thus we need to find a sparse representation of the received signals (11) that depends on the emitter locations instead of TOA's. To this purpose, discretize the plane where the emitters are to be located into a grid of G squared cells with center points

$$\mathcal{G} = \{\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_G\}. \tag{14}$$

Now we can decompose $\mathbf{x}_{lq}$ into

$$\left\{\mathbf{x}_{lq}\right\}_d = \sum_{\substack{g \text{ such that} \\ \tilde{\tau}_l(\boldsymbol{\theta}_g)=(d-1)\tau_{\text{res}}}} \left\{\mathbf{y}_{lq}\right\}_g + \left\{\mathbf{z}_{lq}\right\}_d , \tag{15}$$

where vectors $\mathbf{y}_{lq} \in \mathbb{C}^G$ and $\mathbf{z}_{lq} \in \mathbb{C}^D$ are sparse representations of the LOS and NLOS components between emitter q and sensor l respectively, and are defined as

$$\left\{\mathbf{y}_{lq}\right\}_g = \begin{cases} \tilde{a}_{lq} & \text{if } \boldsymbol{\theta}_g \approx \mathbf{p}_q \\ 0 & \text{else.} \end{cases} \tag{16a}$$

$$\left\{\mathbf{z}_{lq}\right\}_d = \begin{cases} \tilde{a}_{lq}^{(m)} & \text{if } (d-1)\tau_{\text{res}} \approx \tau_{lq}^{(m)} \\ 0 & \text{else,} \end{cases} \tag{16b}$$

and $\tilde{\tau}_l(\boldsymbol{\theta})$ is a function that approximates $\tau_l(\boldsymbol{\theta})$ with the closest discrete delay from $\mathcal{D}$:

$$\tilde{\tau}_l(\boldsymbol{\theta}) = \underset{\tau \in \mathcal{D}}{\arg\min} |\tau_l(\boldsymbol{\theta}) - \tau|. \tag{17}$$

Thus we moved from a sparse representation based on delays (13) to one based on locations on a grid (16a) and NLOS delays (16b). The locations of the emitters are the parameters of interest, while the NLOS components are nuisance parameters. There is a one-to-one mapping between entries in vectors (16a) and grid locations, and also, between entries in vectors (16b) and discrete delays. The complex values in such vectors represent the complex signal amplitudes. In order to distinguish LOS from NLOS components, we use the fact that LOS components originate from the same location. Due to this property, vectors $\mathbf{y}_{lq}$, for all l and fixed q, have at most one non-zero at the entry associated with position $\mathbf{p}_q$, or in other words, they are jointly sparse. Thus we need a sparse recovery technique that induces *joint sparsity* among vectors $\mathbf{y}_{1q}, \ldots, \mathbf{y}_{Lq}$ for each q, and sparsity of the vectors $\mathbf{z}_{lq}$ for all l and q. According to Obozinsky and Bach [11], these two different types of sparsity can be induced jointly by minimizing the following convex optimization problem

$$\min_{\{\mathbf{y}_{lq}\},\{\mathbf{z}_{lq}\}} \sum_{q=1}^{Q} w_q \left\|\mathbf{Y}_q\right\|_{2,1} + \sum_{q=1}^{Q}\sum_{l=1}^{L} \left\|\mathbf{z}_{lq}\right\|_1 \tag{18a}$$

$$\text{s.t.} \sum_{l=1}^{L} \left\| \mathbf{r}_l - \sum_{q=1}^{Q} \mathbf{A}_q \mathbf{x}_{lq} \right\|_2^2 \leq \epsilon \tag{18b}$$

$$\left\{\mathbf{x}_{lq}\right\}_d = \sum_{\substack{g \text{ such that} \\ \tilde{\tau}_l(\boldsymbol{\theta}_g)=(d-1)\tau_{\text{res}}}} \left\{\mathbf{y}_{lq}\right\}_g + \left\{\mathbf{z}_{lq}\right\}_d \quad \text{for all } l \text{ and } q \tag{18c}$$

where $\mathbf{Y}_q = \left[\begin{array}{c|c|c} \mathbf{y}_{1q} & \cdots & \mathbf{y}_{Lq} \end{array}\right]$, the operators in the objective function are the ℓ_2/ℓ_1-norm of a matrix defined as

$$\left\|\mathbf{Y}_q\right\|_{2,1} = \sum_{g=1}^{G} \sqrt{\sum_{l=1}^{L} \left|\left\{\mathbf{y}_{lq}\right\}_g\right|^2} \tag{19}$$

and the ℓ_1-norm of a vector defined as

$$\left\|\mathbf{z}_{lq}\right\|_1 = \sum_{d=1}^{D} \left|\left\{\mathbf{z}_{lq}\right\}_d\right|. \tag{20}$$

In (18a), $\{w_q\}_{q=1}^{Q}$ are weights, and ϵ sets the maximum allowed error between the measurements and the solution. The inputs to this optimization process are the signals $\{\mathbf{r}_l\}_{l=1}^{L}$, and the outputs are the matrices $\{\hat{\mathbf{Y}}_q\}_{q=1}^{Q}$. The estimate of the q-th emitter location is obtained from the non-zero row of $\hat{\mathbf{Y}}_q$. The intuitive idea behind problem (18) is to combine the minimization of the ℓ_2/ℓ_1-norm which is known to induce row sparsity of matrix $\mathbf{Y}_q$, and the minimization of the ℓ_1-norm which is well known to induce sparsity on vectors $\mathbf{z}_{lq}$. Problem (18) falls into the class of second-order cone programming (SOCP) [12], a subfamily of convex problems, for which more efficient methods exist than those used for solving generic convex problems.

Let $\hat{\mathbf{y}}_{lq}$ and $\hat{\mathbf{z}}_{lq}$, for all l and q, be the solutions to problem (11). Parameter ϵ in optimization problem (18) bounds the fitting error between the solution and the received signals. In the absence of noise, the solution should match exactly the received signals, and therefore, we would set $\epsilon = 0$. However, in the presence of noise, from (1) we require

$$\sum_{l=1}^{L} \left\| \hat{\mathbf{r}}_l - \sum_{q=1}^{Q} \mathbf{A}_q \mathbf{x}_{lq} \right\|_2^2 = \sum_{l=1}^{L} \|\mathbf{n}_l\|_2^2 \leq \epsilon. \tag{21}$$

If ϵ is chosen too small, then $\sum_{l=1}^{L} \|\mathbf{n}_l\|_2^2 \nleq \epsilon$, thus making the correct solution infeasible. Because the noise vectors $\{\mathbf{n}_l\}_{l=1}^{L}$ are random independent complex Gaussian vectors of length N, it follows that the error normalized by the noise variance $2\sigma_n^{-2} \sum_{l=1}^{L} \|\mathbf{n}_l\|_2^2$ is a Chi-square random variable with $2NL$ degrees of freedom. Thus parameter ϵ must be set to a large enough value so that $\sum_{l=1}^{L} \|\mathbf{n}_l\|_2^2 \leq \epsilon$ is satisfied with high probability, e.g.

$$\Pr\left(\sum_{l=1}^{L} \|\mathbf{n}_l\|_2^2 \leq \epsilon\right) = 0.99. \tag{22}$$

Next, we discuss what is the purpose of the weights $\{w_q\}_{q=1}^{Q}$ and how to tune them. Denote S_q the number of sensors that receive a LOS path from emitter q, and assume it is known. In such case, there are S_q LOS components whose propagation delays are described by (3). We can show theoretically that w_q must satisfy

$$\sqrt{S_q - 1} < w_q < \sqrt{S_q}. \tag{23}$$

Due to space limitations the proof of (23) will appear in a future publication. We found empirically that a reasonable value satisfying (23) is $w_q = \sqrt{S_q - 0.2}$. Since S_q is unknown a priori, we propose to solve (18) multiple times for different values of S_q. The idea is to start by assuming all sensors are LOS with all emitters: $\hat{S}_q = L$ for all q. Then problem (18) is solved. For all emitters with no location estimate, $\hat{S}_q$ is decreased by one, and problem (18) is solved again. This process is repeated until problem (18) outputs location estimates for all emitters. These steps are explained in more detail in Section III-C.

In Section III-B, a grid of delays $\mathcal{D}$ and a grid of locations $\mathcal{G}$ were introduced that enabled sparse representations of the NLOS and LOS components (16). It is expected that, by decreasing the resolution of the grids, the estimation errors due to off-grid delays and locations become small compared to the signal noise. Denote τ_{res} the time difference between two subsequent discrete delays, and recall d_{res} is the distance between the center points of two adjacent cells in $\mathcal{G}$. We found empirically that a reasonable ratio between both grids is of the order of $2d_{res} = c\tau_{res}$ where c is the speed of light.

C. Algorithm

The inputs to the DLM algorithm are the received signals $\{\mathbf{r}_l\}_{l=1}^{L}$, where L is the number of sensors. The outputs are the emitter locations estimates $\{\hat{\mathbf{p}}_q\}_{q=1}^{Q}$, where Q is the number of emitters. The summary of DLM is as follows:

Input: Received signals $\{\mathbf{r}\}_{l=1}^{L}$, noise variance σ_n^2, the grid of delays $\mathcal{D}$ (10), and the grid of locations $\mathcal{G}$ (14).

Output: Emitter locations estimates $\{\hat{\mathbf{p}}_q\}_{q=1}^{Q}$.

For sensor l **do**
  Estimate multipath TOAs using [2], or any other delay estimation technique of choice.
  Estimate multipath amplitudes by (8).
  For emitter q **do**
    Cancel out all arrivals associated to the q-th emitter waveform, except the first one, using (9).
Initialize noise threshold ϵ by (22), $\hat{\mathbf{p}}_q = \emptyset$ and $\hat{S}_q = L$ for $q = 1, \ldots, Q$.
While $\hat{\mathbf{p}}_q = \emptyset$ for any q **do**
  Set weights to $w_q = \sqrt{\hat{S}_q - 0.2}$ for all q.
  Solve problem (18) numerically. Output: $\hat{\mathbf{Y}}_q$ for all q.
  For emitter q **do**
    If $\hat{\mathbf{Y}}_q = \mathbf{0}$ **then**
      $\hat{\mathbf{p}}_q = \emptyset$ and $\hat{S}_q \leftarrow \hat{S}_q - 1$
    else
      $\hat{\mathbf{p}}_q = \boldsymbol{\theta}_s$, where s is the index of the non-zero

row in $\hat{\mathbf{Y}}_q$, and $\boldsymbol{\theta}_s$ the location mapped to such index.

IV. Numerical Results

In this section, we illustrate the performance of the DLM by numerical examples, and compare it to other existing techniques via Monte Carlo simulations. In all examples, the emitters and sensors are positioned within a square area of $2\,\text{km} \times 2\,\text{km}$, which is divided into a grid of $10\,\text{m} \times 10\,\text{m}$ cells, thus resulting in 40000 cells. The emitters transmit signals that occupy a bandwidth of 1 MHz and are each drawn from a frequency-flat Gaussian stochastic process. All sensors are time-synchronized and sample the received signal at a frequency of 2 MHz for 50 µs, thus each sensor observes 100 samples. For each emitter, we define the SNR per observation time as

$$\text{SNR} = 10\log_{10}\left(\frac{N\sum_{l=1}^{L} P_l}{\sigma_n^2}\right), \tag{24}$$

where N is the number of observations per sensor, P_l is the tap power of the LOS component between the emitter and sensor l, and σ_n^2 the variance of the noise after sampling. According to [13], in urban and suburban areas, the signal strengths of LOS and NLOS paths may be modeled as random variables with log-normal distributions. Thus it follows that the tap powers expressed in dB are random variables with normal distribution. For our simulations, we set the standard deviation of the tap powers to 5 dB. Furthermore, we define the path length as the propagation delay of a LOS or NLOS component divided by the speed of light. The excess delays/paths lengths are defined as the difference of propagation delays/path lengths between the NLOS component and LOS component of an emitter-sensor path. All figures compare the performance of two direct localization techniques. The first direct technique is DPD [7], which outputs the maximum likelihood estimate of the locations in absence of frequency-selective multipath. The second one is our proposed method DLM. For reference, performance of two indirect techniques is also shown. Indirect localization comprises a two-step process. For the first step, one indirect technique estimates the TOA's by simple matched filter, and the other, by the super-resolution method in [2] which is specifically designed to work in the presence of multipath. In a second step, both indirect techniques perform multilateration using Chen's method [3] to mitigate the problem of potential LOS blockage on sensors.

The bandwidth of the emitted signals limits the localization accuracy, and it is known that the ranging resolution is approximately

$$r = \frac{c}{B} \tag{25}$$

where c is the speed of light and B is the signal bandwidth. For the particular case of a 1 MHz bandwidth, the waveform ranging resolution is then 300 m. In the following experiments, the excess path length of a NLOS path is referenced to the signal resolution r. Also we define the probability of correct recovery for the case of a single emitter as

$$P_c = \frac{1}{Z}\sum_{z=1}^{Z} \mathbb{1}\left(|\mathbf{p} - \hat{\mathbf{p}}^{(z)}| < \frac{r}{3}\right), \tag{26}$$

where $\mathbf{p}$ is the true emitter's location, Z is the number of times that the experiment is repeated, $\hat{\mathbf{p}}^{(z)}$ is the emitter's location estimate for the z-th repetition and $\mathbb{1}(\cdot)$ is the indicator function. For some of our tests we plot the normalized root mean square error (rMSE) defined as

$$\text{rMSE} = \frac{1}{r}\sqrt{\frac{1}{Z}\sum_{z=1}^{Z}(\mathbf{p} - \hat{\mathbf{p}}^{(z)})^2}. \tag{27}$$

All experiments are repeated 1000 times, i.e. $Z = 1000$.

A. Performance in Multipath Environments

For this experiment we position four sensors at coordinates (500 m,500 m), (500 m,−500 m), (−500 m,500 m) and (−500 m,−500 m) and a single emitter at the origin (0,0). All paths have the same power. All sensors receive a LOS component, except for sensor 4 whose LOS path is blocked. Sensor 3, in addition to the LOS component, also receives a single NLOS component with an excess path length of 200 m. Sensor 4 receives a single NLOS component with an excess path length of 400 m. Fig. 1 plots the probability of correct recovery versus SNR. DLM outperforms all other techniques. Comparing DLM with the indirect technique with super-resolution TOA, it shows how the performance gain is 2 dB at SNR = 20 dB. Fig. 2 plots the rMSE versus SNR. Interestingly, for an SNR in the range 20 dB – 30 dB, DLM shows an rMSE slightly worse than that of the indirect technique with super-resolution TOA. It turns out that in the presence of multipath, DLM can precisely localize emitters with high probability, but when it fails, it can result in arbitrary large errors due to the nature of sparse recovery techniques. Thus in the subsequent experiment, we plot the probability of correct recovery rather than the rMSE, as it is a more representative metric.

B. Probability of Correct Recovery vs. Delay Spread

For this experiment, we simulate Turin's urban channel model [13]. We position 5 sensors at coordinates at coordinates (500 m,500 m), (500 m,−500 m), (−500 m,500 m), (−500 m,−500 m) and (0,0), and a single emitter at (160 m,130 m). All sensors, except for the one on the origin, receive a LOS component. Moreover, all sensors receive NLOS components whose arrival times are modelled by a Poisson process. The mean inter-arrival time is set to 1 µs. The average power $\bar{P}$ of an arrival at sensor l is governed by the power delay profile (PDP)

$$\bar{P}_l(t) = \exp\left(-\frac{t - t_l^{(0)}}{t_{rms}}\right) \tag{28}$$

where t is the arrival time, $t_l^{(0)}$ is the arrival time of the LOS path and t_{rms} is the RMS delay spread. Such PDP assigns smaller power to later arrivals. Fig. 3 shows how the localization

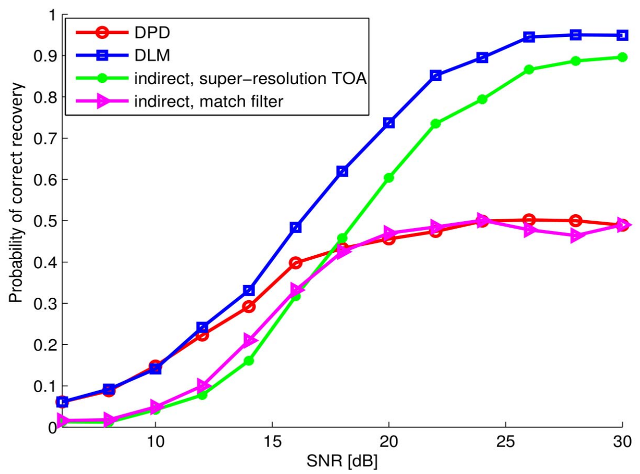

Fig. 1. Probability of correct recovery vs. SNR. The layout consists of four equally spaced sensors and a single emitter in the middle. Sensors 1 and 2 receive a LOS path. Sensors 3 receives a LOS and NLOS path. Sensor 4 received only a NLOS path.

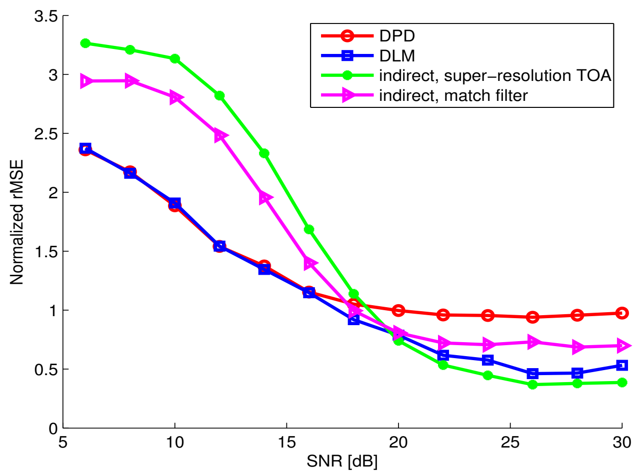

Fig. 2. Root mean square error vs. SNR. The layout consists of four equally spaced sensors and a single emitter in the middle. Sensors 1 and 2 receive a LOS path. Sensors 3 receives a LOS and NLOS path. Sensor 4 received only a NLOS path.

performance of all techniques decreases as the RMS delay spread increases. Again, DLM outperforms all other methods. For RMS delays approaching zero, the NLOS paths are almost non-existent, and therefore; DPD and DLM perform equally. However, as the delay spread starts increasing, DPD drops below the performance of indirect techniques because it is not designed for multipath environments.

V. Conclusions

A novel technique for direct localization of mutliple emitters in the presence of multipath in a sparse framework has been proposed. DLM can precisely recover the emitter locations with higher probability than traditional indirect localization techniques, and also, it is robust to sensors with LOS blockage.

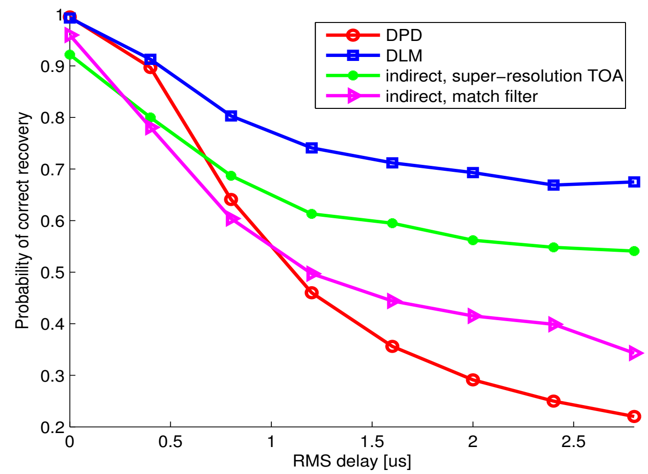

Fig. 3. Probability of correct recovery vs. delay spread for an SNR of 20 dB. The layout consists of five sensors at coordinates (500 m,500 m), (500 m,−500 m), (−500 m,500 m), (−500 m,−500 m) and (0,0), and a single emitter at (160 m,130 m). The sensor on the origin has its LOS blocked. All sensors receive NLOS paths with arrival times following a Poisson distribution. The mean arrival rate is set equal to the bandwidth of the signal.

DLM requires no prior channel knowledge like delay spread or power delay profile of the multipath. The performance gain does not come for free, as DLM requires larger computational resources than other techniques.

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