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Non-Pauli Effects from Noncommutative Spacetimes

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Abstract: Noncommutative spacetimes lead to nonlocal quantum field theories (qft's) where spin-statistics theorems cannot be proved. For this reason, and also backed by detailed arguments, it has been suggested that they get corrected on such spacetimes leading to small violations of the Pauli principle. In a recent paper [1], Pauli-forbidden transitions from spacetime noncommutativity were calculated and confronted with experiments. Here we give details of the computation missing from this paper. The latter was based on a spacetime $\mathcal{B}_{\chi\vec{n}}$ different from the Moyal plane. We argue that it quantizes time in units of χ. Energy is then conserved only mod $\frac{2\pi}{\chi}$. Issues related to superselection rules raised by non-Pauli effects are also discussed in a preliminary manner.

Keywords: Non-Commutative Geometry, Space-Time Symmetries.

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The spin-statistics theorem in three or more dimensions has been proved in many ways in local relativistic qft's. It assumes its comprehensive form in the work of Doplicher and Roberts [2, 3]. It states that identical tensorial particles are bosons and identical spinorial particles are fermions. The proofs of this theorem require the axioms of local relativistic qft's. Deep extensions of the theorem to qft's on gravitational backgrounds exist [4, 5], but they too require spacetime commutativity and a form of locality.

It is reasonable to expect that the spin-statistics connection and its emergent physics can get modified in models where spacetime commutativity and locality do not hold. We made a suggestion along these lines for qft's on the Moyal plane [6]. A subsequent paper by Chakraborty et al [7] developed this idea and showed in a striking calculation that the Pauli repulsion between fermions, infinite for zero separation on commutative spacetimes, softens to a finite value on the Moyal plane. Applications of this effect to statistical mechanics, superconductivity, and Chandrasekhar limit either exist or are in progress [8].

But these papers do not explicitly consider Pauli-forbidden transitions.

With precision experiments at increasingly shorter length and time scales, it is now timely to question principles of local qft's such as Lorentz invariance, CPT theorem and the spin-statistics connection. As regards the last, there exist excellent experiments on Pauli-forbidden transitions, but there is a scarcity of good models to confront data, those of Greenberg and coworkers being among the exceptions. These are reported or reviewed in [9, 10, 11] where also much existing information is surveyed. A desirable model will have a small parameter χ, $\chi = 0$ giving back the standard treatment. Here we develop such an approach adapted to treat Pauli-violating atomic and nuclear transitions.

Our model is based on a spacetime $\mathcal{B}_{\chi\vec{n}}$ different from the Moyal plane $\mathcal{A}_\theta$. The latter also seems to predict the exotic effects we look for, but the calculations get complicated. Just as in the case of $\mathcal{A}_\theta$, $\mathcal{B}_{\chi\vec{n}}$ too can be described in terms of a Drinfel'd twist element $F_{\chi\vec{n}}$(defined in Eq.(1.4)). So the Poincaré group algebra $\mathbb{C}\mathcal{P}$ can act on $\mathcal{B}_{\chi\vec{n}}$ as a Hopf algebra if its coproduct is deformed. Compatibility with this action requires that we deform the standard symmetrization or flip operator τ_0 to

$$\tau_{\chi\vec{n}} = F_{\chi\vec{n}}^{-1}\tau_0 F_{\chi\vec{n}}. \tag{1}$$

That changes the symmetrization and anti-symmetrization of wave functions and leads to novel physics. The details we need about the modified flip $\tau_{\chi\vec{n}}$ and the deformed Hopf algebra of $\mathbb{C}\mathcal{P}$ are in Sec.1.

A typical Pauli-forbidden transition can occur in neutral beryllium with two electrons in the ground state and the remaining two electrons in the excited state: the transition of the excited electrons to the ground state is Pauli-forbidden on the commutative spacetime $\mathcal{B}_0$. But it occurs on $\mathcal{B}_{\chi\vec{n}}$ and we calculate its rate. It involves new physics, relying on the fact that the direction of the unit vector $\vec{n}$ effectively changes with earth's rotation and movements. These are very swift events for noncommutative corrections induced by χ, so that the sudden approximation is appropriate to treat χ-dependent atomic or nuclear phenomena.(We do not consider TeV scale gravity [12].)

When $\vec{n}$ changes to $\vec{m}$ by earth's fast motions, twisted fermions with $\tau_{\chi\vec{n}} = -1$ in the sudden approximation become superpositions of both twisted fermions and twisted bosons ($\tau_{\chi\vec{m}} = \mp 1$) leading to the above process.

We are looking for violations of transitions which are strictly forbidden in standard quantum theory. The calculations are greatly simplified if spin-orbit coupling is neglected. The latter will of course give corrections to our final answer. But in this paper, our focus is on establishing that such violations exist and *estimating* their magnitudes. The corrections due to spin-orbit coupling will not rule out these violations or significantly affect these estimates. For these reasons, we will ignore spin-orbit coupling.

Noncommutative spacetimes emerge from quantum gravity and Planck- scale physics. Thus we are using atomic and nuclear phenomena to probe very high energy physics. Our results are not expected to have much bearing on low energy phenomena.

Section 2 describes the two-electron energy eigenstates on $\mathcal{B}_{\chi\vec{n}}$.

In section 3, we calculate what becomes of these state vectors when $\vec{n}$ rapidly changes to $\vec{m}$. We explicitly find the twisted Bose components induced in certain twisted Fermi levels of $\mathcal{B}_{\chi\vec{n}}$. This enables us to calculate the rate R of transition of the excited electrons to the fully occupied ground level for a sufficiently generic perturbation. R depends on $\vec{n}.\vec{m}$, but since $\vec{m}$ and $\vec{n}$ keep changing, we average them to get an average rate $\langle R\rangle$.

Comparison with experiments are best done by developing a formula for a branching ratio B where the effects not specific to noncommutativity may largely cancel. So we divide $\langle R\rangle$ by a typical rate for an allowed atomic or nuclear transition and find a B. It is $O((\chi\Delta E)^2)$ where ΔE is a suitable energy difference.

The expression for B and the available atomic and nuclear experiments give bounds on χ. The use of B away from its original context is justified as remarked above, B being a ratio. In any case, our bounds are rough. They are reported in Sec.4. The best ones come from neutrino signals of forbidden processes [13, 14, 18] and give $\chi \gtrsim 10^{24}$TeV. This does seem an excessively stringent bound suggesting further checks on its validity. As it stands, it suggests an energy scale beyond Planck scale.

The focus of section 5 shifts away from Pauli principle and probes other features of $\mathcal{B}_{\chi\vec{n}}$. We show that time translation gets quantized on $\mathcal{B}_{\chi\vec{n}}$ in units of χ. Elsewhere this effect has been discussed in detail [24, 25, 26] and it has been proved that energy is conserved only mod $\frac{2\pi}{\chi}$ in scattering processes. A formal scattering theory has also been developed. Thus $\mathcal{B}_{\chi\vec{n}}$ predicts much new physics. Its potential applications to higher dimensional models is also pointed out in section 6.

In the final section 6, we briefly consider the Moyal plane $\mathcal{A}_\theta$ and argue that Pauli-forbidden transitions exist there as well although computations become more involved. We also comment on superselection rules and how they get violated in our models. Their description using qft's is commented on as well.

1. The Spacetime $\mathcal{B}_{\chi\vec{n}}$

The elements of $\mathcal{B}_{\chi\vec{n}}$ are functions on the Minkowski space M^4. If x_μ are coordinate functions transforming under the Poincaré group $\mathcal{P}$ in the standard manner, the algebra $\mathcal{B}_{\chi\vec{n}}$ is characterized by the relations

$$[x_0, x_i] = i\chi\epsilon_{kij}n_k x_j, \tag{1.1}$$

$$[x_i, x_j] = 0, \qquad i, j = 1, 2, 3 \tag{1.2}$$

where x_0 is the time function and $\vec{n}$ is a fixed three-dimensional unit vector.

A product map $m_{\chi\vec{n}}$ of two functions f, g, which leads to Eq.(1.1) is given by

$$m_{\chi\vec{n}}(f \otimes g) = f e^{\frac{1}{2}\chi(\overleftarrow{\partial}_t \vec{n}\cdot\vec{L} - \vec{n}\cdot\overleftarrow{L}\vec{\partial}_t)} g \tag{1.3}$$

where $\vec{L} = -i\chi\vec{x}\wedge\vec{\nabla}$ is orbital angular momentum and generates rotations. The product in Eq.(1.3), is associative since $[\partial_t, \vec{n}\cdot\vec{L}] = 0$. Equation (1.3) defines $\mathcal{B}_{\chi\vec{n}}$.

We can write Eq.(1.3) in terms of the twist element

$$F_{\chi\vec{n}} = e^{\frac{1}{2}\chi(\partial_t\otimes\vec{n}\cdot\vec{L} - \vec{n}\cdot\vec{L}\otimes\partial_t)} \tag{1.4}$$

as follows:

$$m_{\chi\vec{n}} = m_0 \cdot F_{\chi\vec{n}}, \tag{1.5}$$

$$m_{\chi\vec{n}}(f \otimes g) = m_0[F_{\chi\vec{n}} f \otimes g] \tag{1.6}$$

where m_0 is point-wise multiplication :

$$m_0(f \otimes g)(p) = f(p)g(p), \quad p = \text{a point of } M^4. \tag{1.7}$$

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The algebra $\mathcal{B}_{\chi\vec{n}}$ is well-suited for deforming dynamics with spherical symmetry as in atomic physics with its central potentials. For the same reason, it is well-adapted to deform quantum fields on black hole backgrounds. The Moyal plane is awkward to deal with in either case (See however [8]).

The form of the twist element in Eq.(1.4) in a generic representation carrying the action of $\mathbb{C}\mathcal{P}$ is known from the general theory of Hopf algebras [20, 21]. Thus in a generic representation carrying the action of $\mathbb{C}\mathcal{P}$, $\vec{L}$ becomes the rotation generator $\vec{J}$ and $i\partial_t$ the translation generator P_0. If $G_{\chi\vec{n}}$ is the generic form of $F_{\chi\vec{n}}$, then

$$G_{\chi\vec{n}} = e^{-\frac{i}{2}\chi(P_0\otimes\vec{n}\cdot\vec{J}-\vec{n}\cdot\vec{J}\otimes P_0)}. \tag{1.8}$$

We note that this form of $G_{\chi\vec{n}}$ is correct for any model which has rotation and time translation symmetry. Relativistic invariance is not called for.

Drinfel'd's original work [19] and subsequent developments by Aschieri et al. [21] and Chaichian et al. [20] show that $\mathbb{C}\mathcal{P}$ acts as a Hopf algebra $H\mathcal{P}$ if its coproduct is modified by the Drinfel'd twist $G_{\chi\vec{n}}$ to $\Delta_{\chi\vec{n}}$:

$$\Delta_{\chi\vec{n}}(g) := G^{-1}_{\chi\vec{n}}(g\otimes g)G_{\chi\vec{n}}, \qquad g\in\mathcal{P}. \tag{1.9}$$

For $\chi=0$, when noncommutativity is absent, symmetrization and anti-symmetrization is achieved using the projectors $\frac{1\pm\tau_0}{2}$. τ_0 here is the flip operator: if $\mathcal{H}$ is a Hilbert space carrying a representation of $\mathcal{P}$ or one of its subgroups, and $\alpha,\,\beta\in\mathcal{H}$, $\tau_0(\alpha\otimes\beta)=\beta\otimes\alpha$. This flip commutes with $\Delta_0(g)$ and is Poincaré invariant for $\chi=0$.

But for $\chi\neq 0$,

$$\tau_0 G_{\chi\vec{n}} = G^{-1}_{\chi\vec{n}}\tau_0 \tag{1.10}$$

and τ_0 fails to commute with $\Delta_{\chi\vec{n}}(g)$: the projectors $\frac{1\pm\tau_0}{2}$ are not Poincaré invariant for $\chi\neq 0$. Hence we must deform τ_0 suitably. Such a deformed flip operator is the twisted flip operator

$$\tau_{\chi\vec{n}} = G^{-1}_{\chi\vec{n}}\tau_0 G_{\chi\vec{n}} = G^{-2}_{\chi\vec{n}}\tau_0,\;\; \tau^2_{\chi\vec{n}}=1. \tag{1.11}$$

Thus if $\mathcal{H}$ is a representation space for $\mathbb{C}\mathcal{P}$ or one of its generic subgroups, and $\alpha\otimes\beta\in\mathcal{H}\otimes\mathcal{H}$, the twisted bosons and fermions are images of $\mathcal{H}\otimes\mathcal{H}$ under the projectors $\frac{\mathbb{I}\pm\tau_{\chi\vec{n}}}{2}$:

$$\text{Twisted Bosons: } \mathcal{H}\otimes_{S_{\chi\vec{n}}}\mathcal{H} := \frac{1+\tau_{\chi\vec{n}}}{2}\mathcal{H}\otimes\mathcal{H} \tag{1.12}$$

$$\text{Twisted Fermions: } \mathcal{H}\otimes_{A_{\chi\vec{n}}}\mathcal{H} := \frac{1-\tau_{\chi\vec{n}}}{2}\mathcal{H}\otimes\mathcal{H}. \tag{1.13}$$

The full justification of Eq.(1.12) and Eq.(1.13) will take us too far into Hopf algebra theory and material which has been extensively treated elsewhere. [See for example [6].]

We note that $\mathcal{H}$ can be the Hilbert space of an electron with spin in the central potential of a nucleus. The single particle symmetry group G we then focus on is $SU(2)\times\mathbb{R}$ where $SU(2)$ is the (two-fold cover of the) rotation group acting also on spin and rotating around the nuclear center, and $\mathbb{R}$ is the time translation group. The generator P_0 of $\mathbb{R}$ is the single-particle Hamiltonian:

$$P_0 \equiv H = \frac{\vec{p}^2}{2\mu} - \frac{Ze^2}{r}, \tag{1.14}$$

$$Z = \text{Nuclear charge},$$

$$\vec{r} = \text{relative coordinates},$$

$$\mu = \text{reduced mass}.$$

For this paper, the Hopf algebra of interest is the group algebra $\mathbb{C}(SU(2) \times \mathbb{R})$ where $\mathbb{R}$ is time translation, along with the coproduct $\Delta_{\chi\vec{n}}$. We are interested in its concrete realization, denoted here as $H_\chi(SU(2) \times \mathbb{R})$, on multi-electron states. We now describe a convenient basis for this Hilbert space and evaluate the coproducts $\Delta_{\chi\vec{n}}(H)$ and $\Delta_{\chi\vec{n}}(J_i)$ of the Hamiltonian and angular momentum in this basis.

The single particle basis we choose consists of eigenstates of H and is

$$|N,\ l\rangle \otimes |\alpha\rangle_{\vec{n}} \equiv |N,\ l,\ \alpha\rangle_{\vec{n}}, \quad \alpha = \pm 1 \tag{1.15}$$

where N and l are the principal quantum number and orbital angular momentum and $|\alpha\rangle_{\vec{n}}$ denotes the eigenstates of $\vec{\sigma} \cdot \vec{n}$ (σ_i being Pauli matrices) with eigenvalues α:

$$H|N,\ l,\ \alpha\rangle_{\vec{n}} = E_N|N,\ l,\ \alpha\rangle_{\vec{n}}, \tag{1.16}$$

$$E_N = -\frac{Z \times 13.6}{N^2}\text{eV} = \text{energy for principal quantum number } N$$

$$\vec{\sigma} \cdot \vec{n}|N,\ l,\ \alpha\rangle_{\vec{n}} = \alpha|N,\ l,\ \alpha\rangle_{\vec{n}}. \tag{1.17}$$

The state vector $|N,\ l,\ \alpha\rangle_{\vec{n}}$ is $|N,\ l\rangle \otimes |\alpha\rangle_{\vec{n}}$ where the spin vector $|\alpha\rangle_{\vec{n}}$ can be constructed as follows. Let $g(\vec{n}) \in SU(2)$ (in its defining representation) such that [23, 22]

$$g(\vec{n})\sigma_3 g(\vec{n})^\dagger = \vec{\sigma} \cdot \vec{n} \tag{1.18}$$

and let

$$\sigma_3|\alpha\rangle_{\hat{k}} = \alpha|\alpha\rangle_{\hat{k}}, \quad \hat{k} = \begin{pmatrix} 0, & 0, & 1 \end{pmatrix} \tag{1.19}$$

so that

$$|+\rangle_{\hat{k}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |-\rangle_{\hat{k}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{1.20}$$

Then

$$g(\vec{n})|\alpha\rangle_{\hat{k}} = |\alpha\rangle_{\vec{n}}\ . \tag{1.21}$$

Note that $g(\vec{n})$ is not unique as both $g(\vec{n})$ and $g(\vec{n})e^{i\sigma_3\theta}$ rotate σ_3 to $\vec{\sigma} \cdot \vec{n}$. This ambiguity will disappear when we compute rates. We also do not need an explicit choice of $g(\vec{n})$ to calculate rates.

Next we calculate $\Delta_{\chi\vec{n}}(H)$ and $\Delta_{\chi\vec{n}}(\vec{J})$.

As for $\Delta_{\chi\vec{n}}(H)$ and $\Delta_{\chi\vec{n}}(\vec{n} \cdot \vec{J})$, they are not affected by χ since H and $\vec{n} \cdot \vec{J}$ commute and $G_{\chi\vec{n}}$ contains only these operators. (Hereafter $G_{\chi\vec{n}}$ denotes Eq.(1.8) on the electronic states with spin included.) Thus

$$\Delta_{\chi\vec{n}}(H) = H \otimes 1 + 1 \otimes H, \tag{1.22}$$

$$\Delta_{\chi\vec{n}}(\vec{n} \cdot \vec{J}) = \vec{n} \cdot \vec{J} \otimes 1 + 1 \otimes \vec{n} \cdot \vec{J}. \tag{1.23}$$

The coproduct for the remaining components of $\vec{J}$ can be evaluated as follows. Let $\vec{n}^a$, $(a = 1, 2)$, $\vec{n}$ be an orthonormal positively oriented coordinate system so that $\vec{n}^1 \wedge \vec{n}^2 = \vec{n}$, and let

$$\vec{n}^{\pm} \cdot \vec{J} = (\vec{n}^1 \pm i\vec{n}^2) \cdot \vec{J}.$$

Then

$$\left[\vec{n} \cdot \vec{J},\ \vec{n}^{(\pm)} \cdot \vec{J}\right] = \pm \vec{n}^{(\pm)} \cdot \vec{J}, \tag{1.24}$$

$$\left[\vec{n}^{(+)} \cdot \vec{J},\ \vec{n}^{(-)} \cdot \vec{J}\right] = 2\ \vec{n} \cdot \vec{J}. \tag{1.25}$$

From, this it follows that

$$\Delta_{\chi\vec{n}}(\vec{n}^{(\pm)} \cdot \vec{J}) = \vec{n}^{(\pm)} \cdot \vec{J} \otimes e^{\mp\frac{i}{2}\chi P_0} + e^{\pm\frac{i}{2}\chi P_0} \otimes \vec{n}^{(\pm)} \cdot \vec{J}. \tag{1.26}$$

2. The Electronic States of Be

The nucleus of Be has $Z = 4$. We put two of the four electrons of neutral Be in the $N = 1$ level. The remaining two are put in the $N = 2, l = 0$ level. The choice $l = 0$ for all these levels is deliberate as it greatly simplifies the calculations.

The equations Eq.(1.22), Eq.(1.23) show that energy and $\vec{n} \cdot \vec{J}$ are additive in the twist antisymmetrized levels $\frac{1-\tau_{\chi\vec{n}}}{2}\left(|N,\ l,\ \alpha\rangle_{\vec{n}} \otimes |N',\ l',\ \alpha'\rangle_{\vec{n}}\right)$. We have

$$\Delta_{\chi\vec{n}}(H)\frac{1-\tau_{\chi\vec{n}}}{2}|N,\ 0,\ \alpha\rangle_{\vec{n}} \otimes |N',\ 0,\ \alpha'\rangle_{\vec{n}} = (E_N + E_{N'})\frac{1-\tau_{\chi\vec{n}}}{2}|N,\ 0,\ \alpha\rangle_{\vec{n}} \otimes |N',\ 0,\ \alpha'\rangle_{\vec{n}}, \tag{2.1}$$

$$\Delta_{\chi\vec{n}}(\vec{n} \cdot \vec{J})\frac{1-\tau_{\chi\vec{n}}}{2}|N,\ 0,\ \alpha\rangle_{\vec{n}} \otimes |N',\ 0,\ \alpha'\rangle_{\vec{n}} = \frac{1}{2}(\alpha + \alpha')\frac{1-\tau_{\chi\vec{n}}}{2}|N,\ 0,\ \alpha\rangle_{\vec{n}} \otimes |N',\ 0,\ \alpha'\rangle_{\vec{n}}. \tag{2.2}$$

As for $\Delta_{\chi\vec{n}}(\vec{n}^{(\pm)} \cdot \vec{J})$, we find,

$$\Delta_{\chi\vec{n}}(\vec{n}^{(+)} \cdot \vec{J})\left\{\begin{array}{c} |N,\ 0,\ +1\rangle_{\vec{n}} \otimes |N',\ 0,\ -1\rangle_{\vec{n}}, \\ |N,\ 0,\ -1\rangle_{\vec{n}} \otimes |N',\ 0,\ +1\rangle_{\vec{n}} \end{array}\right\} = \left\{\begin{array}{c} e^{\frac{i}{2}\chi E_N}|N,\ 0,\ +1\rangle_{\vec{n}} \otimes |N',\ 0,\ +1\rangle_{\vec{n}}, \\ e^{-\frac{i}{2}\chi E_{N'}}|N,\ 0,\ +1\rangle_{\vec{n}} \otimes |N',\ 0,\ +1\rangle_{\vec{n}} \end{array}\right\}; \tag{2.3}$$

$$\Delta_{\chi\vec{n}}(\vec{n}^{(+)} \cdot \vec{J})\left\{\begin{array}{c} |N,\ 0,\ +1\rangle_{\vec{n}} \otimes |N',\ 0,\ +1\rangle_{\vec{n}}, \\ |N,\ 0,\ -1\rangle_{\vec{n}} \otimes |N',\ 0,\ -1\rangle_{\vec{n}} \end{array}\right\} = \left\{\begin{array}{c} 0, \\ e^{-\frac{i}{2}\chi E_{N'}}|N,\ 0,\ +1\rangle_{\vec{n}} \otimes |N',\ 0,\ -1\rangle_{\vec{n}} \\ +\,e^{\frac{i}{2}\chi E_N}|N,\ 0,\ -1\rangle_{\vec{n}} \otimes |N',\ 0,\ +1\rangle_{\vec{n}} \end{array}\right\}; \tag{2.4}$$

$$\Delta_{\chi\vec{n}}(\vec{n}^{(-)} \cdot \vec{J})\left\{\begin{array}{c} |N,\ 0,\ +1\rangle_{\vec{n}} \otimes |N',\ 0,\ -1\rangle_{\vec{n}}, \\ |N,\ 0,\ -1\rangle_{\vec{n}} \otimes |N',\ 0,\ +1\rangle_{\vec{n}} \end{array}\right\} = \left\{\begin{array}{c} e^{\frac{i}{2}\chi E_{N'}}|N,\ 0,\ -1\rangle_{\vec{n}} \otimes |N',\ 0,\ -1\rangle_{\vec{n}}, \\ e^{-\frac{i}{2}\chi E_N}|N,\ 0,\ -1\rangle_{\vec{n}} \otimes |N',\ 0,\ -1\rangle_{\vec{n}} \end{array}\right\}; \tag{2.5}$$

$$\Delta_{\chi\vec{n}}(\vec{n}^{(-)} \cdot \vec{J})\left\{\begin{array}{c} |N,\ 0,\ +1\rangle_{\vec{n}} \otimes |N',\ 0,\ +1\rangle_{\vec{n}}, \\ |N,\ 0,\ -1\rangle_{\vec{n}} \otimes |N',\ 0,\ -1\rangle_{\vec{n}} \end{array}\right\} = \left\{\begin{array}{c} e^{\frac{i}{2}\chi E_{N'}}|N,\ 0,\ -1\rangle_{\vec{n}} \otimes |N',\ 0,\ +1\rangle_{\vec{n}} \\ +\,e^{-\frac{i}{2}\chi E_N}|N,\ 0,\ +1\rangle_{\vec{n}} \otimes |N',\ 0,\ -1\rangle_{\vec{n}}, \\ 0 \end{array}\right\}. \tag{2.6}$$

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2.1 *The two-electron ground state*

For $\chi = 0$, it is unique, being the (untwisted) spin-singlet state,

$$\frac{1-\tau_0}{\sqrt{2}}|1,\ 0,\ +1\rangle_{\vec{n}} \otimes |1,\ 0,\ -1\rangle_{\vec{n}} = \frac{1}{\sqrt{2}}\left(|1,\ 0,\ +1\rangle_{\vec{n}} \otimes |1,\ 0,\ -1\rangle_{\vec{n}} - |1,\ 0,\ -1\rangle_{\vec{n}} \otimes |1,\ 0,\ +1\rangle_{\vec{n}}\right) \tag{2.7}$$

with energy $2E_{10}$.

As χ is changed away from 0, thus vector is deformed to

$$\begin{aligned} |1\ 1\rangle_{\chi\vec{n}} &= \frac{1-\tau_{\chi\vec{n}}}{\sqrt{2}}|1,\ 0,\ +1\rangle_{\vec{n}}\ |1,\ 0,\ -1\rangle_{\vec{n}} \\ &= \frac{1}{\sqrt{2}}\left[|1,\ 0,\ +1\rangle_{\vec{n}}\ |1,\ 0,\ -1\rangle_{\vec{n}} - e^{i\chi E_1}|1,\ 0,\ -1\rangle_{\vec{n}}\ |1,\ 0,\ +1\rangle_{\vec{n}}\right]. \end{aligned} \tag{2.8}$$

Its energy still remains $2E_{10}$ in view of Eq.(2.1).

No new linearly independent state appears by continuity: if they had appeared, then as $\chi \to 0$, the ground state would not be unique. We can verify this assertion by calculating $\frac{1-\tau_{\chi\vec{n}}}{\sqrt{2}}|1,\ 0,\ \alpha\rangle_{\vec{n}}\otimes|1,\ 0,\ \alpha'\rangle_{\vec{n}}$ for any choice of α, α' and verifying that it is either proportional to Eq.(2.8) or zero.

The values of $\Delta_{\chi\vec{n}}(\vec{n}\cdot\vec{J})$ and $\Delta_{\chi\vec{n}}(\vec{n}^{(\pm)}\cdot\vec{J})$ on $|1\ 1\rangle_{\chi\vec{n}}$ are also zero from Eq.(2.2), Eq.(2.3) and Eq.(2.5). So it is a twisted spin-singlet with zero (twisted) value for total angular momentum.

2.2 *The two-electron excited state*

The actual Pauli-forbidden transition we will calculate will use the excited state

$$\frac{1-\tau_{\chi\vec{n}}}{\sqrt{2}}\left[|2,\ 0,\ +1\rangle_{\vec{n}}\ |3,\ 0,\ +1\rangle_{\vec{n}}\right] \tag{2.9}$$

which is part of a (twisted!) spin triplet with orbital angular momentum 0 and energy $E_2 + E_3$.

For completeness, we here list all the spin triplet and singlet components of the states with energy $E_2 + E_3$.

The triplet vectors

$$\Delta_{\chi\vec{n}}(\vec{n}\cdot\vec{J}) = 1 : \frac{1}{\sqrt{2}}\left[|2,\ 0,\ +1\rangle_{\vec{n}}\ |3,\ 0,\ +1\rangle_{\vec{n}} - e^{\frac{i}{2}\chi(E_3-E_2)}|3,\ 0,\ +1\rangle_{\vec{n}}\ |2,\ 0,\ +1\rangle_{\vec{n}}\right]. \tag{2.10}$$

$$\Delta_{\chi\vec{n}}(\vec{n}\cdot\vec{J}) = 0 : \frac{1}{2}\left[\begin{array}{l} e^{\frac{i}{2}\chi E_3}|2,\ 0,\ -1\rangle_{\vec{n}}\ |3,\ 0,\ +1\rangle_{\vec{n}} - e^{-\frac{i}{2}\chi E_2}|3,\ 0,\ +1\rangle_{\vec{n}}\ |2,\ 0,\ -1\rangle_{\vec{n}} \\ +\, e^{-\frac{i}{2}\chi E_2}|2,\ 0,\ +1\rangle_{\vec{n}}\ |3,\ 0,\ -1\rangle_{\vec{n}} - e^{\frac{i}{2}\chi E_3}|3,\ 0,\ -1\rangle_{\vec{n}}\ |2,\ 0,\ +1\rangle_{\vec{n}} \end{array}\right]. \tag{2.11}$$

$$\Delta_{\chi\vec{n}}(\vec{n}\cdot\vec{J}) = -1 : \frac{1}{\sqrt{2}}\left[|2,\ 0,\ -1\rangle_{\vec{n}}\ |3,\ 0,\ -1\rangle_{\vec{n}} - e^{-\frac{i}{2}\chi(E_3-E_2)}|3,\ 0,\ -1\rangle_{\vec{n}}\ |2,\ 0,\ -1\rangle_{\vec{n}}\right]. \tag{2.12}$$

The singlet vector

$$\Delta_{\chi\vec{n}}(\vec{n}\cdot\vec{J}) = \Delta_{\chi\vec{n}}(\vec{n}^{(\pm)}\cdot\vec{J}) = 0 : \frac{1}{2}\left[\begin{array}{l} e^{\frac{i}{2}\chi E_3}|2,\ 0,\ -1\rangle_{\vec{n}}\ |3,\ 0,\ +1\rangle_{\vec{n}} - e^{-\frac{i}{2}\chi E_2}|3,\ 0,\ +1\rangle_{\vec{n}}\ |2,\ 0,\ -1\rangle_{\vec{n}} \\ -\ e^{-\frac{i}{2}\chi E_2}|2,\ 0,\ +1\rangle_{\vec{n}}\ |3,\ 0,\ -1\rangle_{\vec{n}} + e^{\frac{i}{2}\chi E_3}|3,\ 0,\ -1\rangle_{\vec{n}}\ |2,\ 0,\ +1\rangle_{\vec{n}} \end{array}\right]. \tag{2.13}$$

In the above equations for the triplet and singlet states, the values of the $\vec{n}\cdot\vec{J}$ components of the angular momenta are specified next to each of the states by specifying the values of $\Delta_{\chi\vec{n}}(\vec{n}\cdot\vec{J})$ on each of these states. This is similar to specifying the values of the third component of angular momentum.

3. The Non-Pauli Rate

This section contains the formula Eq.(3.15) for confrontation with experiments. The rest of this section is a derivation of this formula.

3.1 *Spin overlaps*

The basic transition we focus on is from the triplet excited state Eq.(2.10) for twist $G_{\chi\vec{n}}$ to the ground state levels for twist $G_{\chi\vec{m}}$. That involves the calculation of the overlap ${}_{\vec{m}}\langle\alpha'|\alpha\rangle_{\vec{n}}$ which follows from Eq.(1.21):

$${}_{\vec{m}}\langle\alpha'|\alpha\rangle_{\vec{n}} = \Big(g(\vec{m})^{\dagger}g(\vec{n})\Big)_{\alpha'\alpha}. \tag{3.1}$$

This expression depends on the choice of $g(\vec{n})$, $g(\vec{m})$. But in rates, we get its squared modulus. That depends only on $\vec{m}\cdot\vec{n}$:

$$|{}_{\vec{m}}\langle\alpha'|\alpha\rangle_{\vec{n}}|^2 = \frac{1}{2}\left[1 + (-1)^{\frac{(\alpha'-\alpha)}{2}}\vec{m}\cdot\vec{n}\right]. \tag{3.2}$$

Here is a simple proof of Eq.(3.2). The R.H.S is

$$g(\vec{m})^{\dagger}_{\alpha'\rho}g(\vec{n})_{\rho\alpha}g(\vec{n})^{\dagger}_{\alpha\lambda}g(\vec{m})_{\lambda\alpha'}$$

for fixed α, α' and summed ρ, λ. Consider $\alpha = \alpha' = 1$:

$$\begin{aligned} |{}_{\vec{m}}\langle +1| + 1\rangle_{\vec{n}}|^2 &= Tr\left(g(\vec{n})\frac{1+\tau_3}{2}g(\vec{n})^{\dagger}\right)\left(g(\vec{m})\frac{1+\tau_3}{2}g(\vec{m})^{\dagger}\right) \\ &= \frac{1}{4}Tr\,[1+\vec{n}\cdot\vec{\tau}]\,[1+\vec{m}\cdot\vec{\tau}] \\ &= \frac{1}{2}\,[1+\vec{m}\cdot\vec{n}] \end{aligned}.$$

In a similar way we can establish Eq.(3.2) for any α, α'.

We can now see the root of the non-Pauli transition. consider the *twist symmetrized* ground state for twist along $\vec{m}$:

$$\frac{1+\tau_{\chi\vec{m}}}{\sqrt{2}}|1,\ 0,\ \alpha\rangle_{\vec{m}}\ |1,\ 0,\ \beta\rangle_{\vec{m}} = \frac{1}{\sqrt{2}}\left[|1,\ 0,\ \alpha\rangle_{\vec{m}}\ |1,\ 0,\ \beta\rangle_{\vec{m}} + e^{\frac{i}{2}\chi E_1(\alpha-\beta)}|1,\ 0,\ \beta\rangle_{\vec{m}}\ |1,\ 0,\ \alpha\rangle_{\vec{m}}\right]. \tag{3.3}$$

It is part of the spin triplet which with the twist antisymmetrized singlet gives the four two-electron ground states.

The normalized radial wave function for principal quantum number N can be denoted by $|N\rangle$. It is independent of the twist direction. The tensor product $|N\rangle \otimes |M\rangle$ can then be written as $|N, M\rangle$.

Now a generic perturbation, call it V_0, will have a non-zero radial matrix element $\langle 1\ 1|V_0|2\ 3\rangle$ where V_0 is regarded as spin-independent for illustration. Then the Pauli-forbidden amplitudes are roughly proportional to this factor multiplied by spin overlaps ${}_{\vec{m}}\langle\alpha\ \beta|\frac{(1+\tau_{\chi\vec{m}})}{\sqrt{2}}|+1\ \ +1\rangle_{\vec{n}}$: the spin-statistics connection does not permit $\frac{(1+\tau_{\chi\vec{m}})}{\sqrt{2}}|\alpha\ \beta\rangle_{\vec{m}}$. But we will see that these overlaps are not zero. So there are Pauli-forbidden transitions.

For $\chi = 0$, let V_0 be a generic spin-independent perturbation of the two-electron Hamiltonian. We do not show its dependence on electron coordinates, but we can assume it to be symmetric in them as it preserves statistics:

$$[V_0,\ \tau_0] = 0. \tag{3.4}$$

For $\chi \neq 0$, we have to modify V_0 to $V_{\chi\vec{n}}$:

$$V_{\chi\vec{n}} = \frac{1}{2}\left[V_0 + \tau_{\chi\vec{n}} V_0 \tau_{\chi\vec{n}}\right] \tag{3.5}$$

so that it preserves the twisted statistics. As V_0 is an external perturbation which causes transitions between levels, it can be time-dependent. The perturbation has additional time dependence as $\vec{n}$ changes with time.

The perturbed two-electron Hamiltonian is

$$H' = \Delta_{\chi\vec{n}}(H) + V_{\chi\vec{n}}. \tag{3.6}$$

Let $\vec{\rho}(t)$ be a time-dependent unit vector which at $t = t_i$ is $\vec{n}$ and at time $t = t_f$ is $\vec{m}$. To leading order in $V_{\chi\vec{n}}$, the transition matrix element from an initial state $|I\rangle$ of energy E_I at time t_i to an orthogonal final state $|F\rangle$ of energy E_F at time t_f is

$$-ie^{-i(t_f - t_i)E_f}\langle F|\int_{t_i}^{t_f} d\tau e^{i\tau H} V_{\chi\vec{\rho}(\tau)} e^{-i\tau H}|I\rangle.$$

For us

$$|I\rangle = \frac{1-\tau_{\chi\vec{n}}}{\sqrt{2}}|2,\ 0,\ \ +1\rangle_{\vec{n}}\ |3,\ 0,\ \ +1\rangle_{\vec{n}}, \tag{3.7}$$

with $E_I = E_2 + E_3$.

For $|F\rangle$, we choose a *Pauli-forbidden* ground state

$$\frac{1+\tau_{\chi\vec{m}}}{\sqrt{2}}|1,\ 0,\ \alpha\rangle_{\vec{m}}\ |1,\ 0,\ \alpha'\rangle_{\vec{m}}$$

(This vector is not normalized if $\alpha = \alpha'$. We will fix that problem later.)

From Eq.(3.5), we can see that $V_{\chi\vec{n}} = V_0 + O(\chi)$. The explicit calculations below show that the amplitude is $O(\chi)$ if $V_{\chi\vec{n}}$ is approximated by V_0. So we approximate $V_{\chi\vec{n}}$ by V_0 in Eq.(3.5) neglecting terms of $O(\chi)$.

As V_0 is symmetric in electron coordinates, for the radial matrix element, $\langle 1\ 1|V_0|2,\ 3\rangle = \langle 1\ 1|V_0|3,\ 2\rangle$.

We now use this identity to simplify the probability for transition P_χ to any Pauli-forbidden ground state. That is obtained from modulus squared of the amplitude by summing over $|F\rangle$ after normalizing them. But the projector to the Pauli-forbidden ground states is

$$Q = |1,\ 1\rangle\langle 1,\ 1|\mathbb{I}_{\text{spin}} - |1,\ 1\rangle_{\chi\vec{m}\ \chi\vec{m}}\langle 1,\ 1| \tag{3.8}$$

where $\mathbb{I}_{\text{spin}}$ is the unit operator on spin space.

Thus the probability of interest is

$$P_\chi = \langle I| \left(\int_{t_i}^{t_f} d\tau e^{i\tau 2E_1} V_0(\tau) e^{-i\tau(E_2+E_3)} \right)^* Q \left(\int_{t_i}^{t_f} e^{i\tau 2E_1} V_0(\tau) e^{-i\tau(E_2+E_3)} \right) |I\rangle. \tag{3.9}$$

This simplifies to the following on using the symmetry of V_0:

$$P_\chi = |\langle 1\ 1| \int_{t_i}^{t_f} d\tau e^{i\tau 2E_1} V_0(\tau) e^{-i\tau(E_2+E_3)} |2\ 3\rangle|^2 \times P^\chi_{\text{SPIN}} \tag{3.10}$$

where

$$P^\chi_{\text{SPIN}} = \frac{1}{2}| \left(1 - e^{\frac{i}{2}\chi(E_3-E_2)}\right)|^2 \left[1 - \frac{1}{2}| \left({}_{\vec{m}}\langle +\ -| - e^{-i\chi E_1}\ {}_{\vec{m}}\langle -\ +|\right) |+\ +\rangle_{\vec{n}}|^2 \right]. \tag{3.11}$$

As claimed, P_χ is $O(\chi^2)$.

P^χ_{SPIN} can be evaluated using Eq.(3.2). The result is

$$P^\chi_{\text{SPIN}} = 2\sin^2(\frac{\chi}{4}\Delta E) \left[1 - \frac{1}{4}(1-(\vec{m}\cdot\vec{n})^2)(1-\cos(\chi E_1))\right] \tag{3.12}$$

where $\Delta E = E_3 - E_2$.

Here since $\vec{n}$ and $\vec{m}$ vary, it is best to average over them using the rotationally invariant measure. We first average over $\vec{m}$ by integrating over its polar and azimuthal angles θ_m, ϕ_m using the standard measure

$$\frac{d\omega_m}{4\pi}, \quad d\omega_m = d\cos\theta_m d\phi_m.$$

Then

$$\int \frac{d\omega_m}{4\pi}\mathbb{I} = 1, \quad \int \frac{d\omega_m}{4\pi} m_i = 0, \quad \int \frac{d\omega_m}{4\pi} m_i m_j = \frac{1}{3}\delta_{ij} \tag{3.13}$$

giving for the average $\langle P_\chi\rangle$ of P_χ,

$$\langle P_\chi\rangle = \left\{ |\langle 1\ 1| \int_{t_i}^{t_f} e^{i\tau 2E_1} V_0(\tau) e^{-i\tau(E_2+E_3)} |2\ 3\rangle|^2 \right\} \times \left\{ \frac{1}{3}(5+\cos(\chi E_1))\sin^2(\frac{\chi}{4}\Delta E) \right\}. \tag{3.14}$$

There is no need to average over $\vec{n}$ as this is $\vec{n}$-independent.

The magnitude of the prefactor in braces is that of a typical probability for a Pauli-allowed process. Thus the branching ratio of a Pauli-forbidden to a Pauli-allowed process is

$$B_\chi = \frac{1}{3}(5+\cos(\chi E_1))\sin^2(\frac{\chi}{4}\Delta E), \quad \Delta E = E_3 - E_2. \tag{3.15}$$

It is independent of t_i, t_f. It is this expression we use to confront experiments as it is a ratio and may not be sensitive to the details of its derivation.

4. Experiments and Bounds on χ

The experiments searching for Pauli-forbidden transitions can be broadly classified into atomic and nuclear experiments. Here we discuss each experiment separately.

Some of the above experiments give only lifetimes for the forbidden processes. To obtain the branching ratios in such cases we multiply the given rate with the typical lifetimes for such processes. In the case of an atomic process, we use the number 10^{-16} seconds and for a nuclear process we use 10^{-23} seconds for typical lifetimes.

Bounds from The Borexino Experiment

The Borexino collaboration has used its counting test facility to obtain limits on the violation of the Pauli exclusion principle (PEP) using nuclear transitions in ^{12}C and ^{16}O nuclei. The method is to search for γ, n, p and/or α emitted in a non-Paulian transition of $1P$ shell nucleons to the filled $1S_{1/2}$ shell in nuclei. Various stringent bounds were obtained as a result.

We use the following result from the Borexino experiment [13]:

$$\tau(^{12}C \rightarrow^{12} \widetilde{C} + \gamma) \geq 5.0 \times 10^{31} \text{years.} \tag{4.1}$$

In the above process, $^{12}\widetilde{C}$ denotes an anomalous carbon nucleus with an extra nucleon in the filled K shell of ^{12}C. This corresponds to a branching ratio of the order of 10^{-62}. We take ΔE for this process to be of the order of 1MeV to get a bound on χ.

Bounds from The Kamiokande Detector

In this experiment searches were made for forbidden transitions in ^{16}O nuclei and they obtain a bound on the ratio of forbidden transitions to normal transitions. The bound for this ratio is $< 2.3 \times 10^{-57}$ [14]. Again for this process ΔE is assumed to be of the order of 1MeV.

Bounds from The NEMO Experiment

Similar to nucleon transitions, experiments searching for Pauli-forbidden atomic transitions have also been performed. The NEMO collaboration [15] searches for anomalous $^{12}\widehat{C}$ atoms which are those with 3 K-shell electrons. The method used is the γ ray activation analysis in a sample of boron where the impurity carbon has been removed radiochemically. The bound on the existence of such atoms is given by the ratio of abundances of $^{12}\widehat{C}$ to ^{12}C: it is $< 2.5 \times 10^{-12}$. It corresponds to a limit on the lifetime with respect to violation of the Pauli principle by electrons in a carbon atom of $\tau \geq 2 \times 10^{21}$years. We take ΔE for this process to be 272 eV to calculate a bound on χ.

The NEMO-2 collaboration has also performed nucleon transition experiments [16] and the limit obtained is

$$\tau(^{12}C \rightarrow^{12} \widetilde{C} + \gamma) \geq 4.2 \times 10^{24} \text{years.} \tag{4.2}$$

This corresponds to a branching ratio of the order $< 10^{-55}$ if we assume ΔE for this process to be of the order of 1MeV.

Experiment	Type	Bound on χ (Length scales)	Bound on χ (Energy scales)
Borexino	Nuclear	$\lesssim 10^{-47}$ m	$\gtrsim 10^{28}$ TeV
Kamiokande	Nuclear	10^{-42} m	10^{23} TeV
NEMO	Atomic	10^{-12} m	10^{5} eV
NEMO-2	Nuclear	10^{-41} m	10^{22} TeV
Maryland	Atomic	10^{-20} m	10 TeV
VIP	Atomic	10^{-21} m	100 TeV

Table 1: Bounds on the noncommutativity parameter χ

Bounds from experiments at Maryland

Atomic transition experiments have been conducted by Ramberg and Snow in Maryland using copper (Cu) atoms. The idea here is to introduce new electrons into a copper strip and to look for the K X-rays that would be emitted if one of these electrons were to be captured by a Cu atom and cascade down to the $1S$ state despite the fact that the $1S$ level was already filled with two electrons. The probability for this to occur was found to be less than 1.76×10^{-26} [17]. This corresponds to a lifetime of $\tau > 8.36 \times 10^{3}$years. We assume ΔE for this process to be of the order of 1.5KeV.

Bounds from the VIP experiment

An improved version of the experiment at Maryland has been performed by the VIP collaboration [18]. They improved the limit obtained by Ramberg and Snow at Maryland by a factor of about 40. The limit on the probability of PEP violating interactions between external electrons and copper is found to be less than 4.5×10^{-28}. Here again we take ΔE to be of the order of 1.5KeV.

The bounds are summarized in Table (1).

5. Time Quantization

The algebra $B_{\chi\vec{n}}$ leads to time-quantization in units of χ and therefore [24, 25] energy nonconservation: it is conserved only mod $\frac{2\pi}{\chi}$. An effect of this sort was first discovered by Chaichian [26] for a cylindrical noncommutative spacetime. Quantum physics on such spacetime including scattering theory was later developed in [25].

Time quantization comes about as follows. From Eq.(1.1), one sees that x_0 generates rotations around $\vec{n}$ and that $e^{i\frac{2\pi}{\chi}x_0}$, being 2π rotation, acts as identity on x_i. Being a time exponential, it also commutes with momentum operators. Thus it is in the center of the algebra generated by $B_{\chi\vec{n}}$ and by its momentum operators. Hence it is a multiple of the identity in an irreducible representation of the latter:

$$e^{i\frac{2\pi}{\chi}x_0} = e^{i\phi}\mathbb{I}, \tag{5.1}$$

$e^{i\phi}$ being characteristic of the representation.

A consequence of Eq.(5.1) is that the spectrum spec x_0 of x_0 is quantized:

$$\text{Spec } x_0 = \chi\left(\mathbb{Z} + \frac{\phi}{2\pi}\right). \tag{5.2}$$

As explained in [25, 24], a quantum field ψ is defined only on the spectrum of time operator x_0. Time translations are from one point of this spectrum to another, so that only the time translations

$$(e^{i\chi P_0})^N, \quad N \in \mathbb{Z}$$

exist on quantum fields.

But then P_0 and $P_0 + \frac{2\pi}{\chi}M$, $M \in \mathbb{Z}$ generate the same time translation. Due to this we can anticipate energy conservation only mod $\frac{2\pi}{\chi}$ in scattering processes. This anticipation is correct. In [24], scattering theory with time quantization has been developed and energy is found to be conserved only mod $\frac{2\pi}{\chi}$.

An interesting application of such time quantization is to extra-dimensional models. Thus for example if spacetime is $M^4 \times S^1$ where M^4 is our four-dimensional spacetime, and the time operator x_0 fails to commute with the $e^{i\phi}$ which generates the algebra of functions on S^1,

$$x_0 e^{i\phi} = e^{i\phi} x_0 + \chi e^{i\phi}, \tag{5.3}$$

then scattering theory on M^4 will conserve energy only mod χ. No further interaction is needed for this energy nonconservation to occur.

Such energy nonconservation can be tested by experiments. Unfortunately, we know of no recent experiment to test energy conservation.

6. Final Remarks

Non-Pauli transitions are expected to occur on the Moyal plane $\mathcal{A}_\theta$ as well. But while the Moyal plane has the defining relations

$$[x_\mu,\ x_\nu] = i\theta_{\mu\nu} \tag{6.1}$$

$$\theta_{\mu\nu} = -\theta_{\nu\mu} = \text{constant} \tag{6.2}$$

which are manifestly invariant under translations for the coordinates $x_\mu \to x_\mu + a_\mu$ (a_μ=constant), they are not invariant under the naive rotation of coordinates. So the Moyal plane is not adapted to discuss atomic processes where rotational invariance plays a crucial role. That makes the calculations complicated.

The Bose and Fermi sectors of a local quantum field theory are superselected. But here we find transitions between these sectors.

There is no necessary contradiction due to the fact that the models of this paper seem to violate a superselection rule which has been proved from general principles of quantum theory [27]. According to Greenberg and Messiah [27], this rule does not require even the principles of local quantum field theories for its validity. The reason for this apparent violation is as follows. In the model considered here, the flip operator $\tau_{\chi\vec{n}}$ and hence what is meant by twisted Bose and Fermi particles itself changes with time. This situation does not occur in standard quantum physics and is in fact the source of non-Pauli effects. Also for fixed $\vec{n}$, matrix elements of observables between twisted Bose and twisted Fermi states are zero so that in this sense there is no violation of superselection rules.

Note that the noncommutative models being discussed here are designed to probe energy scales vastly higher than those where local quantum theories have been tested. But certainly a deeper study of this violation is important.

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References

[1] A. P. Balachandran, Anosh Joseph, Pramod Padmanabhan, " Non-Pauli Transitions from Spacetime Nonommutativity", Phys. Rev. Lett. 105, 051601 (2010) and arXiv:1003.2250v3 [hep-th].

[2] S. Doplicher, J. E. Roberts, "Why there is a field algebra with a compact gauge group describing the superselection structure in particle physics", Commun. Math. Phys. 131: No. 1 51-107(1990).

[3] S. Doplicher, R. Haag and J. E. Roberts,"Local observables and particle statistics. II", Commun. Math. Phys. 35: No. 1, 49-85(1974); "Local Observables And Particle Statistics. I.", Commun. Math. Phys. 23: No. 3, 199-230(1971).

[4] R. Brunetti, K. Fredenhagen, R. Verch, "The generally covariant locality principle – A new paradigm for local quantum physics", Commun.Math.Phys. 237 (2003) 31-68, arXiv:math-ph/0112041v1.

[5] R. Verch, "A spin-statistics theorem for quantum fields on curved spacetime manifolds in a generally covariant framework", Commun.Math.Phys.223:261-288,2001, arXiv:math-ph/0102035v2.

[6] A. P. Balachandran, G. Mangano, A. Pinzul, S. Vaidya, "Spin and Statistics on the Groenewold-Moyal Plane: Pauli-Forbidden Levels and Transitions", Int.J.Mod.Phys. A21 (2006) 3111-3126, arXiv:hep-th/0508002v2; A. P. Balachandran, T. R. Govindarajan, G. Mangano, A. Pinzul, B. A. Qureshi and S. Vaidya, "Statistics and UV-IR mixing with twisted Poincare invariance", Phys. Rev. D **75**, 045009 (2007), [arXiv:hep-th/0608179].

[7] B. Chakraborty, S. Gangopadhyay, A. G. Hazra and F. G. Scholtz, "Twisted Galilean symmetry and the Pauli principle at low energies", J. Phys. A **39**, 9557 (2006), [arXiv:hep-th/0601121].

[8] Prasad Basu, Rahul Srivastava, Sachindeo Vaidya, "Thermal Correlation Functions of Twisted Quantum Fields", Phys. Rev. D 82 (2010) 025005, arXiv:1003.4069 [hep-th]; Prasad Basu, Biswajit Chakraborty, Sachindeo Vaidya, "Fate of the Superconducting Ground State on the Moyal Plane", Phys. Lett. B 690 (2010) 431, arXiv:0911.4581 [hep-th]; Nirmalendu Acharyya, Sachindeo Vaidya "Accelerated Observer in Noncommutative Spacetime", JHEP 09 (2010) 045, arXiv:1005.4666v2 [hep-th].

[9] O. W. Greenberg, R. N. Mohapatra, "Local Quantum Field Theory of Possible Violation of the Pauli Principle", Phys. Rev. Lett. 59, 25072510 (1987); "Difficulties with a Local Quantum Field Theory of Possible Violation of the Pauli Principle", Phys. Rev. Lett. 62, 712714 (1989).

[10] O. W. Greenberg, R. N. Mohapatra, "Phenomenology of small violations of Fermi and Bose statistics", Phys. Rev. D 39, 20322038 (1989).

[11] O. W. Greenberg, "Particles with small violations of Fermi or Bose statistics", Phys. Rev. D 43, 41114120 (1991); "Theories of violation of statistics", arXiv:hep-th/0007054v2.

[12] L. Randall and R. Sundrum, "A large mass hierarchy from a small extra dimension," Phys. Rev. Lett.83, 3370 (1999), [arXiv:hep-ph/9905221].

[13] The Borexino Collaboration, "New experimental limits on the Pauli forbidden transitions in ^{12}C nuclei obtained with 485 days Borexino data," Phys.Rev.C81:034317,2010, arXiv:0911.0548v1 [hep-ex].

[14] Y. Suzuki *et al.* [Kamiokande Collaboration], "Study of invisible nucleon decay, N $\rightarrow$ neutrino neutrino anti-neutrino, and a forbidden nuclear transition in the Kamiokande detector," Phys. Lett. B **311**, 357 (1993).

[15] A. S. Barabash *et al.*, "Search for anomalous carbon atoms evidence of violation of the Pauli principle during the period of nucleosynthesis", JETP Lett. **68** (1998) 112

[16] R. Arnold *et al.*, "Testing the Pauli exclusion principle with the NEMO-2 detector", Eur. Phys. J. A **6**, 361 (1999).

[17] E. Ramberg and G. A. Snow, "A new experimental limit on small violation of the Pauli principle", Phys. Lett. B **238**, 438 (1990).

[18] S. Bartalucci *et al.*, "New experimental limit on the Pauli exclusion principle violation by electrons", Phys. Lett. B **641**, 18 (2006).

[19] V. G. Drinfel'd, "Almost cocommutative Hopf algebras", Leningrad Math. J. 1 (1990), 321-332; V. G. Drinfel'd, "Quasi-Hopf algebras", Leningrad Math. J. **1**, 1419-1457 (1990).

[20] M. Chaichian *et al.*, "On a Lorentz invariant interpretation of noncommutative space-time and its implications on noncommutative QFT", Phys. Lett. **B 604**, 98 (2004), [arXiv:hep-th/0408069], M. Chaichian, P. Presnajder, A. Tureanu, "New concept of relativistic invariance in NC space-time: Twisted Poincaré symmetry and its implications", Phys. Rev. Lett. 94, 151602 (2005), [arXiv:hep-th/0409096].

[21] P. Aschieri *et al.*, "A gravity theory on noncommutative spaces", Class. Quant. Grav. **22**, 3511 (2005), [arXiv:hep-th/0504183].

[22] A. P. Balachandran, G. Marmo, B. S. Skagerstam, A. Stern, "Gauge symmetries and fiber bundles : applications to particle dynamics", Springer-Verlag, 1983.

[23] A. P. Balachandran, G. Marmo, B. S. Skagerstam, A. Stern, "Classical Topology and Quantum States", World Scientific Publishing Co. Pte. Ltd. , Singapore, 1991.

[24] A. P. Balachandran, T. R. Govindarajan, A. G. Martins, Paulo Teotonio-Sobrinho, "Time-space noncommutativity: quantised evolutions", JHEP11(2004) 068.

[25] A. P. Balachandran, A. G. Martins, Paulo Teotonio-Sobrinho, "Discrete time evolution and energy nonconservation in noncommutative physics", JHEP05 (2007) 066.

[26] M. Chaichian, A. Demichev, P. Presnajder, "Field Theory on Noncommutative Space-Time and the Deformed Virasoro Algebra", arXiv:hep-th/0003270v2.

[27] A. M. L. Messiah and O. W. Greenberg, Phys. Rev. 136, B248 (1964)