Number Sequences

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1 Problems

A **sequence** is an enumerated collection of objects, usually numbers. For our purposes, we will only consider sequences of integers, or **integer sequences**. The goal of this handout is the present some beautiful and intriguing number sequences through problem solving and trial and error.

In the following problems, try to figure out the next term in the sequence.

Problem 1.1

2, 3, 5, 7, 11, 13, ...

Solution 1.1: These are prime numbers, and the next prime number is $\boxed{17}$.

Problem 1.2

1, 3, 6, 10, 15, ...

Solution 1.2: The differences between two consecutive terms are 2, 3, 4, and 5, which leads us to believe that the next term is 6 greater than 15, or $\boxed{21}$. Note that because the second differences are equal (all 1), the nth term of this sequence can be expressed by a quadratic in n, namely $\frac{n(n+1)}{2}$. Setting $n = 6$ indeed gets us $\boxed{21}$. These number are known as **triangular numbers** (why?).

Problem 1.3

1, 1, 2, 3, 5, 8, ...

Solution 1.3: Each term past the first two is the sum of the previous two terms. The next term is $8 + 5 = \boxed{13}$. This famous sequence is the **Fibonacci sequence**, which has connections with the golden ratio ϕ and naturally occurring spirals.

Problem 1.4

1, 4, 27, 256, ...

Solution 1.4: We notice that $1 = 1^1$, $4 = 2^2$, $27 = 3^3$, and $256 = 4^4$. The next number in the sequence is thus $5^5 = \boxed{3125}$.

Problem 1.5

77, 49, 36, 18, ...

Solution 1.5: Each term after the first term is the product of the digits of the previous term. The next (and final) term is $1 \cdot 8 = \boxed{8}$. These sequences are called **sequences of persistence**.

Problem 1.6

3, 3, 5, 4, 4, 3, 5, 5, 4, 3, ...

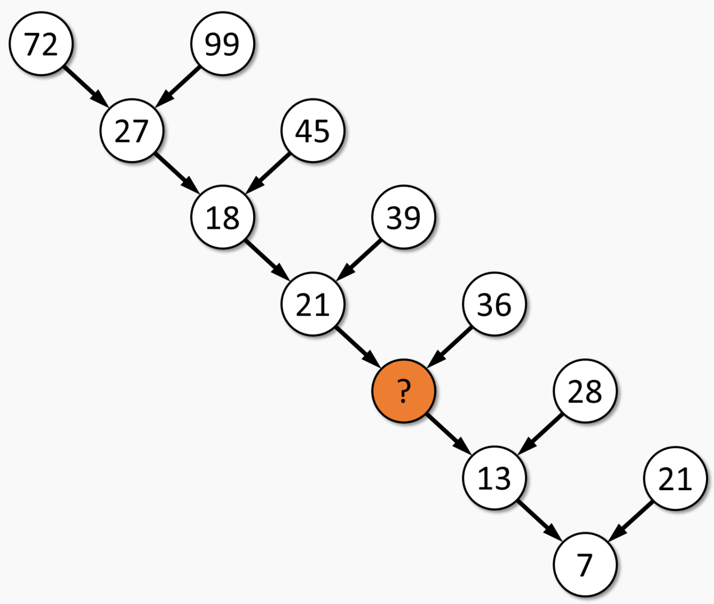
Solution 1.6: There does not appear to be a pattern to the numbers that can be resolved solely by arithmetic. Instead, notice that these numbers are the number of letters in the words "one", "two", ..., "ten". Since "eleven" has 6 letters, the next number is $\boxed{6}$.

Problem 1.7

1, 11, 21, 1211, 111221, 312211, 13112221, ...

Solution 1.7: Each term is obtained by reading aloud the blocks of numbers in the previous term. This is known as a **look-and-say sequence**. The next term is $\boxed{1113213211}$.

Problem 1.8



Note: There are no typos. The last number is indeed 7, not 8.

Solution 1.8: At first glance, it appears that each number in the left column is the difference between the two numbers above it. However, this fails for 21, 13, and 7. Instead, another way to get each number in the left column is to add the digits of the two numbers above. Thus, the missing number is $2+1+3+6 = \boxed{12}$, which checks, as $1+2+2+8 = 13$. This beautiful sequence was discovered by **Nob Yoshigahara**.