Week 2: Patterns

Samyok Nepal–6/28/20

1 Warm Ups

Problem 1.1: AoPS

Two cards are dealt from a standard deck of 52 cards. What is the probability that the first card is a red card and the second is a spade?

Problem 1.2: AoPS

Bag A has 3 white marbles and 4 black marbles. Bag B has 6 yellow marbles and 4 blue marbles. Bag C has 2 yellow marbles and 5 blue marbles. A marble is drawn at random from Bag A. If it is white, a marble is drawn at random from Bag B, otherwise – if it is black – a marble is drawn from Bag C. What is the probability that the second marble drawn is yellow?

Problem 1.3: AoPS

Sheila has been invited to a picnic tomorrow. If it rains, there is a 20% probability that Shelia will decide to go, but if it is sunny, there is an 80% probability that Shelia will decide to go. The forecast for tomorrow states that there is a 40% chance of rain. What is the probability that Shelia will attend the picnic?

Problem 1.4: AoPS

Becky lives in a place with a very clear sky. On Friday night, there is a 60% chance that she will see a shooting star on any given hour. If Becky goes outside and watches the sky for two hours, what is the probability that she sees a shooting star?

Problem 1.5: Mandelbrot

On a Saturday night, Becky goes stargazing. There's an 80% chance of seeing a shooting star in any given hour. If the probability is uniform throughout the hour, what is the probability that Becky will see a shooting star in the first 15 minutes?

Problem 1.6: MATHCOUNTS

A six sided dice is weighted such that

$$P(1) = P(2), P(3) = P(4) = P(5), P(4) = 3P(2), \text{ and } P(5) = 2P(6)$$

where $P(n)$ is the probability that n is rolled. What is the probability that a 6 is rolled?

Problem 1.7: 2014 AIME #6

Charles has two six-sided die. One of the die is fair, and the other die is biased so that it comes up six with probability $\frac{2}{3}$ and each of the other five sides has probability $\frac{1}{15}$. Charles chooses one of the two dice at random and rolls it three times. Given that the first two rolls are both sixes, the probability that the third roll will also be a six is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

Problem 1.8:

Point C is chosen at random atop a 5 foot by 5 foot square table. A circular disk with a radius of 1 foot is placed on the table with its center directly on point C. What is the probability for the entire disk to be on top of the table?
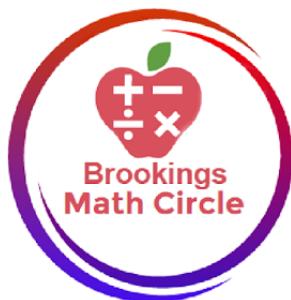
Problem 1.9:

Suppose two numbers x and y are each chosen such that $0 < x < 1$ and $0 < y < 1$. What is the probability such that $x + y > \frac{3}{2}$.

Problem 1.10:

My friend and I are hoping to meet up for lunch. We will each arrive at a random time between noon and 1pm, stay for 15 minutes, then leave. What is the probability that we meet each other while at the restaurant?

Problem 1.11: AMC

If three points are chosen at random on the circumference of a circle, what is the probability that the triangle formed by those points does not have a side greater than the length of the radius of the circle?