Challenge #6 - Arithmetic

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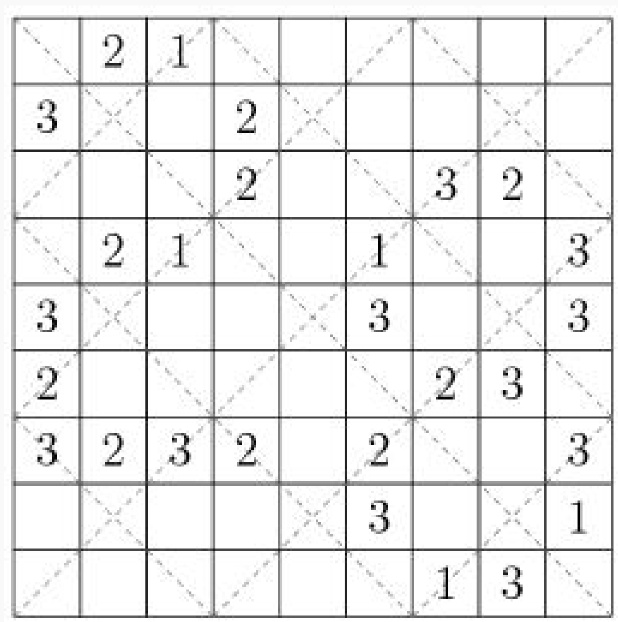
1 Puzzle #6

Problem 1.1: (USAMTS)



Solution 1.1:

1	2	1	3	2	3	3	1	2
3	3	1	2	3	2	1	2	1
2	3	3	2	1	1	3	2	1
1	2	1	3	2	1	2	3	3
3	1	2	1	2	3	2	1	3
2	1	3	3	1	1	2	3	2
3	2	3	2	1	2	1	1	3
2	1	2	1	3	3	3	2	1
1	3	2	1	3	2	1	3	2

2 Summer Challenge #6

Problem 2.1: (Jan Park)

If a is the Least Common Multiple of $(48, 112)$ and b is the Greatest Common Divisor of $(48, 112)$, find $a - 2b$

Solution 2.1: The prime factorizations of 48 and 112 are $2^4 \cdot 3$ and $2^4 \cdot 7$ respectively. The LCM is $2^4 \cdot 3 \cdot 7 = 336$ and the GCD is $2^4 = 16$. Thus, $a - 2b = 336 - 2 \cdot 16 = \boxed{304}$.

Problem 2.2: (Jan Park)

Let $x\#y = x^2 + \frac{y}{x}$ for all positive integers x, y. Find $3\#(4\#8)$.

Solution 2.2: $4\#8 = 4^2 + \frac{8}{4} = 16 + 2 = 18$. $3\#18 = 3^2 + \frac{18}{3} = 9 + 6 = \boxed{15}$.

Problem 2.3: (Jan Park)

How many ways are there to arrange the 26 letters of the alphabet so that the exact substring $EMILY$ is included in the arrangement? (Write in simplest terms of factorials.)

Solution 2.3: Consider the substring $EMILY$ as one letter. There are 21 other letters for a total of 22 elements that we are rearranging. There are $\boxed{22!}$ to do so.

Problem 2.4: (Jan Park)

How many ways are there to distribute 6 candies between Alice, Elena, Rachel, and Susan, where the candies are indistinguishable?

Solution 2.4: Consider 3 dividers and 6 pieces of candy. Alice will get the candies to the left of the first divider, Elena will get the candies in between the first and second dividers, Rachel will get the candies in between the second and third dividers, and Susan will get the candies to the right of the third divider. Thus, there is a 1-1 correspondence between arranging 3 dividers and 6 pieces of candy in a line (where the dividers are indistinguishable and the candies are indistinguishable). The number of ways to arrange 3 dividers and 6 pieces of candy in a line is $\frac{9!}{3! \cdot 6!} = \boxed{84}$.

Problem 2.5: (Serena An)

Alison has an analog clock whose hands have the following lengths: 6 inches (the hour hand), 9 inches (the minute hand), and 12 inches (the second hand). How far does the tip of each hand travel in 1 day?
(Based off of MPfG 2012)

Reminder: Write a full solution to this problem, not just the answer. Even if you do not have a full solution, or a solution for every part, type/write up what you have and you may receive partial credit!

Solution 2.5:

The hour hand makes 2 revolutions in 1 day and the tip of the hand travels 12π inches for 1 revolution, so in 1 day, the tip of the hour hand travels $2 \cdot 12\pi = \boxed{24\pi \text{ inches}}$.

The minute hand makes 1 revolutions in 1 hour, and there are 24 hours in 1 day, so the minute hand makes 24 revolutions in 1 day. The tip of the minute hand travels 18π inches for 1 revolution, so in 1 day, the tip of the minute hand travels $24 \cdot 18\pi = \boxed{432\pi \text{ inches}}$.

The second hand makes 1 revolution in 1 minute, and there are 1440 minutes in 1 day, so the second hand makes 1440 revolutions in 1 day. The tip of the second hand travels 24π inches for 1 revolution, so in 1 day, the tip of the second hand travels $1440 \cdot 24\pi = \boxed{34560\pi \text{ inches}}$.

Brookings Math Circle