Challenge #3 - Graphing

Serena An and Samyok Nepal–August 17, 2018

1 Introduction

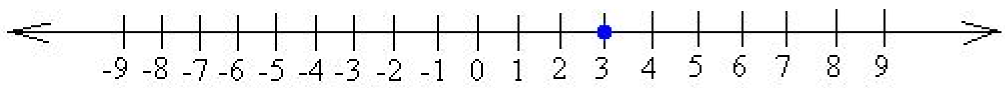
Welcome to the Brookings Math Circle Summer 2018 Problem Sets! **Submissions for this problem set are due by Sunday 6/24 at 1:30 pm (the day of the third BMC class).** For a refresher on submission details or problem set details, see the introduction PDF linked to the Summer Challenge page of our website under Classes.

This handout and problem set will focus on more advanced algebra; in particular, we will cover linear equations and systems of equations. We will build off of the basics that we learned last week. Next week we will learn how to tackle even more advanced and larger systems of equations and how to "cheat the system". Keep calm and do the math!

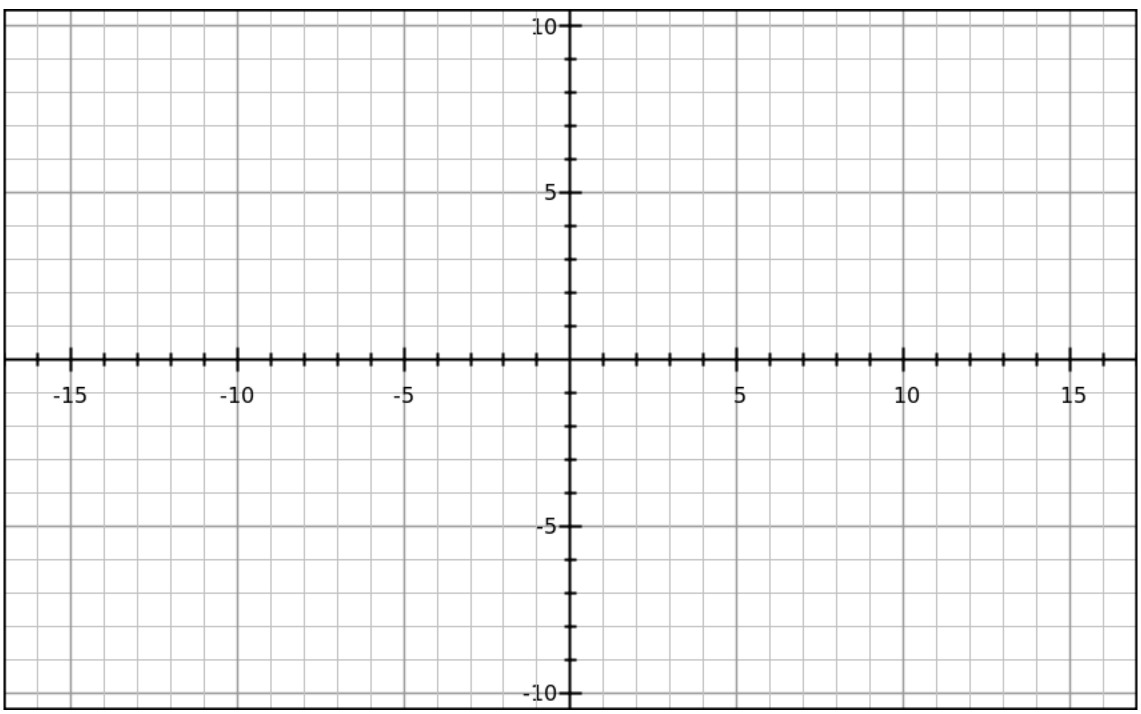
Solutions to problems in the text are at the very back of this packet.

2 Graphing in the Cartesian Coordinate System

Imagine the number line. We can do a lot of things on this number line. For example, the figure below shows the number line for $x = 3$.



Well, after a while, this gets boring. What happens if we stack one number line on top of another with the 0s intersecting? Now we call the number lines **axes** (**axis** is the singular form), and the intersection, where the zeroes are, is called the **origin**. The entire region, or **plane**, containing the two axes is formally called the **Cartesian coordinate system** or the **xy-plane**, but we'll just call it a **graph**.



The horizontal axis is called the x **axis**, and the vertical axis is called the y **axis**. With our two-axis graph, we can graph an equation or equations with a maximum of two variables, which we typically will let be x and y.

Head over to `desmos.com/calculator`.

Problem 2.1: Graph Points

Graph points based on the following equations:

1. $y = 2x + 5$
2. $y = -\frac{1}{2}x + 5$
3. $y = 4$
4. $x = 2$

3 Linear Equation Basics

Theorem 3.1: Linear Equations

A linear equation is an equation involving no more than two variables, typically x and y, in which the two variables both have a *maximum degree of* 1. So, both x and y might appear with degree 1, or only one of them will appear with degree 1 (in which case the other has degree 0). The graph of a linear equation is a always straight line in the Cartesian coordinate system. Any two distinct points determine a unique line.

Before we learn the three main ways to write a linear equation, we need two definitions.

Theorem 3.2: Slope

The slope of a line can be represented in many ways. It is denoted usually by the letter m. The slope of the line containing the points with coordinates (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Always remember that with slope, difference between the ys, or the **rise** is in the numerator, and the difference between the xs, or the **run**, is in the denominator.

Problem 3.1

Does it matter which point has the coordinates (x_1, y_1) and which point has the coordinates (x_2, y_2)? Is the slope the same even if the coordinates are reversed?

Lets apply slope to one problem:

Problem 3.2: ***Source: CMMS***

Find the slope of the line passing through the coordinates $(7, -2)$ and $(-4, 1)$.

One more definition:

Theorem 3.3: Intercepts

Intercepts are where the line crosses the two axes. The y **intercept** is the value of y when $x = 0$, or where the line crosses the y *axis*. The x **intercept** is the value of x when $y = 0$, or where the line crosses the x *axis*. With linear equations, the y *intercept* will be used and mentioned more often.

4 Forms of Linear Equations

There are 3 forms of linear equations.

Theorem 4.1: Slope Intercept Form

Slope intercept form is when linear equations are written like

$$y = mx + b.$$

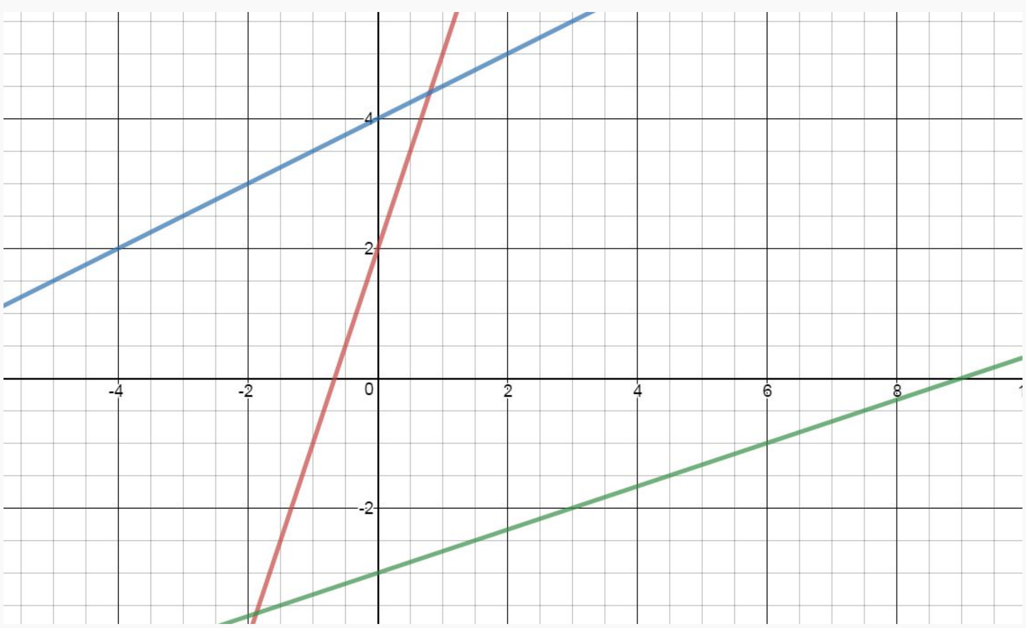
m is the *slope* of the line, and b is the y *intercept*.

Slope intercept form is by far the most common form for a linear equation. With slope intercept form, it is less easy to make mistakes because y is isolated on one side. In addition, it is easy to identify the slope.

Time for a little practice:

Problem 4.1

Find the equations of these lines in slope intercept form.



Theorem 4.2: Point Slope Form

The form

$$y - y_1 = m(x - x_1)$$

is known as **Point Slope Form**. m is the slope of the line, and (x_1, y_1) is a point on the line.

Just like how two distinct points determine a unique line, a slope and a point also determine a unique line. Point slope form is typically used when you know (or can easily figure out) the slope, and you know a point. Any point will do, which sometimes makes point slope form the most convenient way to write a linear equation. If you need to write the equation of a line in no particular way and the numbers are ugly, just use point slope form. If permissible, leave it in point slope form and *do not* try to convert it to slope intercept form, as it not only takes extra work and time, it's easy to make a small computation error.

Let's practice:

Problem 4.2: Point Slope

Find the equation of the line that passes through $(10, 3)$ and $(-5, 9)$ in point slope form.

One more:

Theorem 4.3: Standard Form

Standard Form is a linear equation in the form

$$Ax + By = C$$

where A is positive, and A, B, and C are integers. Converting this to slope intercept form, we get $y = -\frac{A}{B}x + \frac{C}{B}$, so

- Slope: $m = -\frac{A}{B}$
- Y Intercept: $b = \frac{C}{B}$

There's no need to memorize that though. First of all, standard form is quite rarely seen in questions, but sometimes questions require you to put your answer in standard form. Second of all, it's very quickly derivable when you need it; just convert it to slope intercept form.

5 Parallel and Perpendicular Lines

Theorem 5.1: Parallel and Perpendicular Lines

Parallel lines always have the same slope.

The slopes of two **perpendicular lines** are *negative reciprocals* of each other. For example, if the slope of a line is $\frac{5}{4}$, the slope of a perpendicular line is $-\frac{4}{5}$. The slopes of two perpendicular lines always multiply to -1.

This problem was solved under 10 seconds. You try! (Samyok changed the numbers *slightly*)

Problem 5.1: 2017 MATHCOUNTS National Countdown Round

Triangle PQR has vertices at $P(1294, 104)$ and $R(-1394, 1123)$. If $\angle$ Q $= 90°$, what is the product of the slopes of the legs?
Original Problem: `https://youtu.be/vFTeN17Z4rc?t=1759`.

6 Systems of Equations

By now you're probably wondering "Ok, cool, I just learned a lot about linear equations. But why are they important?"

So far we've only looked at single equations, but what if we add more equations? Now we're going to look at the big picture, starting with two equations.

Theorem 6.1: Systems of Equations

Two or more equations that represent the relationship between multiple variables is called a **system of equations**. For example,

$$2x + y = 7$$

$$-5x + 8y = 1$$

would be considered a system of equations in x and y. The purpose of a system of equations is to solve for the variables.

The two common techniques for solving a system are **substitution** and **elimination**.

Theorem 6.2: Substitution

Substitution involves plugging in a value for a variable. This value may include variables, or it may just be a constant.

You already experienced the most basic of substitutions last week:

Problem 6.1

Evaluate $6f - g + h^i$ for $f = 2$, $g = 8$, $h = -4$, and $i = 3$.

Another basic case of substitution is to substitute a constant for a variable and solve for the other.

Problem 6.2

What is the value of x in the equation $y = 11x - 8$ when $y = 3$?

Now we will try substituting a value with a variable.

Problem 6.3

What is the value of x in the equation $y = 23x - 108$ if $y = -x + 12$?

Typically, systems of equations problems, or problems involving more than one equation in general, won't be so nice to give you exactly what to plug in for a variable. You will usually have to manipulate the equation a bit. Also remember that it doesn't matter which variable you substitute in for, so try to choose the one that will be less work to get alone.

Exercise 6.1

Solve the following system for x and y.

$$4x + 7y = -40$$

$$x + 2y = 100$$

Solution 6.1: We isolate x in the second equation to get $x = -2y + 100$. Now we substitute that in for x in the first equation to get $4(-2y + 100) + 7y = -40$, or $-8y + 400 + 7y = -40$. Combining constants on the right side and variables on the left, we get $-y = -440$ so $\boxed{y = 440}$. Plugging 440 in for y in $x = -2y + 100$, we get that $\boxed{x = -780}$.

Let's do one more example.

Exercise 6.2: (CMMS)

Find $m + n$ if $6m = -n + 2$ and $6n = 4m - 1$.

Solution 6.2: From rearranging the first equation, we get $n = -6m + 2$. Plugging that in for n in the second equation, we get $6(-6m + 2) = 4m - 1$, or $-36m + 12 = 4m - 1$. Moving the ms to one side and constants to another, we get $-40m = -13$, so $m = \frac{13}{40}$. Plugging that in for m in $n = -6m + 2$, we get $n = -6(\frac{13}{40}) + 2 = \frac{-39}{20} + 2 = \frac{1}{20}$. Thus, $m + n = \frac{13}{40} + \frac{1}{20} = \frac{15}{40} = \boxed{\frac{3}{8}}$.

Hopefully you see the pattern when it comes to substitution: rearrange, plug in a value for the first variable, solve for the second one, and plug in the second variable to solve for the first one. No need to commit this to memory, you'll know what to do at every step by intuition as long as you understand what substitution is.

Theorem 6.3: Elimination

Elimination occurs when we add or subtract equations in a system to eliminate one variable.

Just like substitution, you have to choose which variable to eliminate, so decide based on which one would be easier and require less manipulation.

Exercise 6.3: (CMMS)

Solve the following system for x and y.

$$2x + 5y = -4$$

$$3x - 5y = 19$$

Solution 6.3: We add the equations so the $+5y$ and $-5y$ cancel and get $5x = 15$, so $x = 3$. Plugging in $\boxed{x = 3}$ into either equation and solving gives $\boxed{y = -2}$

Typically, the numbers won't be that nice and allow for instant addition or subtraction. Normally, you will need to multiply the equations by a constant.

Exercise 6.4

Solve the following system for x and y.

$$2x + 3y = 6$$

$$7x - 5y = -10$$

Solution 6.4: First we need to decide which variable to eliminate. We will eliminate x and solve for y first, although you can do it the other way around. We multiply both sides of the first equation by 7 to get $14x + 21y = 42$. Next we multiply both sides of the second equation by 2 to get $14x - 10y = -20$. The reason why we did that is so the coefficient of the xs in the two equations are the same. Now we subtract the second equation from the first one to get $31y = 62$, so $\boxed{y = 2}$. Plugging that value in for x in any of our previous equations yields $\boxed{x = 0}$.

7 Graphical Connections

How does this relate to the graphs and lines we learned about earlier? **The solution of a system of equations is the point of intersection for the graphs of the two equations.**

Theorem 7.1: Special Cases

Linear equations that are *parallel* have **no solutions**.
Linear equations that *coincide* (are the same line) have **infinite solutions**.
Otherwise, two lines that are not the same line and not parallel have exactly **one** solution.

This should make intuitive sense. Parallel lines never meet so there will never be an ordered pair that satisfies both linear equations. Lines that coincide share infinitely many points with each other, so there are infinitely many ordered pairs that satisfy both linear equations.

8 Word problems

So far, we've been giving you the two equations, but oftentimes, you will need to write the equations yourself. Remember the basics of defining variables and forming equations. Let's try two examples.

Exercise 8.1

The sum of two numbers is 56, and their difference is 4. What is the smaller of the two numbers?

Solution 8.1: Using elimination: Let the the larger number be l and the smaller number be s. We have the equations $l + s = 56$ and $l - s = 4$. Subtracting, we get $2s = 52$, so $\boxed{s = 26}$.

Exercise 8.2: (CMMS)

Shannon has a pocket full of quarters and dimes. If she has 18 coins for a total of \$2.40, how many quarters does she have?

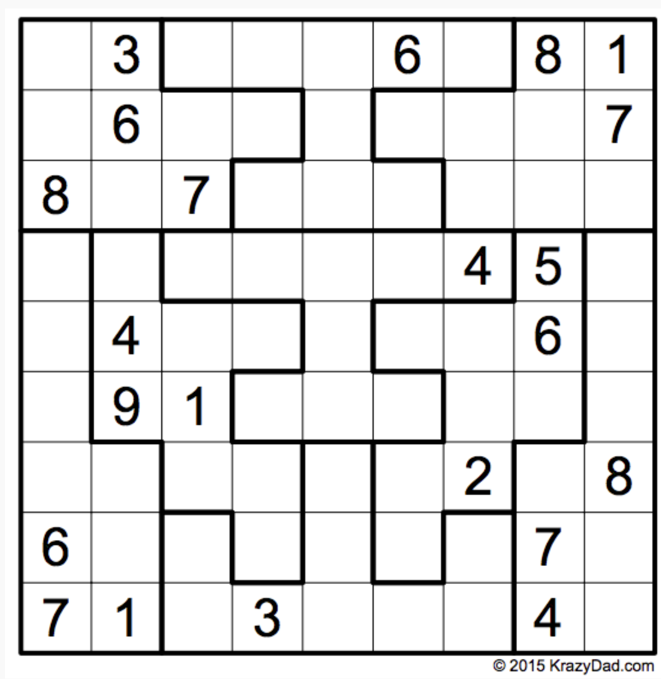
Solution 8.2: Using substitution: Let the number of quarters Shannon has be q, and let the number of dimes she has be d. We have the equations $q + d = 18$ and $0.25q + 0.1d = 2.40$. Rearrange the first equation to get $q = -d+18$, and plug that in for q in the second equation to get $0.25(-d+18)+0.1d = 2.40$, or $-0.15d = -2.10$, so $d = 14$. Plugging in 14 for d in $q = -d + 18$, we get that $\boxed{q = 4}$.

That's the end of today's lesson! That was a lot of information, so it's ok (recommended actually) to have to read over it a few more times. Linear equations and systems of equations show up all the time in math competitions and practice problems, so you will eventually master it!

9 Puzzle #3

Problem 9.1: (krazydad.com)

This is jigsaw sudoku, which has similar rules as regular sudoku. Each row, column, and jigsaw piece contains one of each of the digits from 1 to 9. To help you get more acquainted with jigsaw sudoku, visit `https://krazydad.com/blog/2012/09/07/get-started-with-jigsaw-sudoku/`.



Solution 9.1:

5	3	2	9	4	6	7	8	1
1	6	9	4	8	5	3	2	7
8	2	7	5	3	1	6	9	4
2	7	8	1	6	3	4	5	9
3	4	5	8	9	7	1	6	2
4	9	1	7	5	2	8	3	6
9	5	3	6	7	4	2	1	8
6	8	4	2	1	9	5	7	3
7	1	6	3	2	8	9	4	5

10 Bonus Problem Set #3

Problem 10.1: (Samyok Nepal)

Samyok has 100 boxes. Every day, he buys 12 more boxes. Similarly, Serena has 240 boxes, but she buys only 10 every day. How many days will it take for Samyok and Serena to have an equal number of boxes?

Solution 10.1: We can set this up as a simple system of equations: Let N be the number of boxes Samyok **N**epal has, and A be the number of boxes Serena **A**n has. Additionally, x will be the number of days. We have the following system:

$$N = 100 + 12x$$
$$A = 240 + 10x$$

Because we want $N = A$, we have

$$100 + 12x = 240 + 10x.$$

We can solve this easily to get $\boxed{x = 60}$.

Problem 10.2: (Samyok Nepal)

Let line l be the line that is perpendicular to the line $y = -2x + 8$ and passes through the point (2,4). If line l is expressed as $y = mx + b$, what is the sum of m and b?

Solution 10.2: l has a slope of $m = \frac{1}{2}$. To find the exact line, we plug $(2, 4)$ into $y = \frac{1}{2}m + b$. We now get

$$4 = 2 + b$$

. This means that $m + b = \boxed{\frac{5}{2}}$

Problem 10.3: (Samyok Nepal)

Let B, M, and C be 3 distinct positive integers. The sums of B, M, and C two at a time are 17, 25, or 28, in no particular order. What is the largest number?

Solution 10.3: Since all three numbers are different, no two of B, M, C are equal. Then,

$$B + M = 28$$
$$M + C = 25$$
$$B + C = 17$$
$$B + 2M + C = 53$$

We can immediately subtract $(B+C)=17$ to get $2M=36 \implies M=18$. Plugging this into the other equations then gives us $C=7$ and $B=10$, so our largest is $\boxed{18}$.

Problem 10.4: (Introduction to Algebra/MATHCOUNTS)

Tweedledum says, "The sum of your weight and twice mine is 361 pounds." Tweedledee says, "On the contrary, the sum of your weight and twice mine is 362 pounds!" If they are both correct, how much do they weigh together?

Solution 10.4: Let m be Tweedledum's weight and e be Tweedledee's weight. We have the equations $2m+e=361$ and $m+2e=362$. Adding the equations, we get $3m+3e=723$. Dividing by 3, we get $m+e=241$. They weigh $\boxed{241}$ pounds together. (Note that we did not need to solve for their individual weights.)

Problem 10.5: (2012 AMC 10A)

The product of two positive numbers is 9. The reciprocal of one of these numbers is 4 times the reciprocal of the other number. What is the sum of the two numbers?

Solution 10.5: We let x and y be the two numbers. From the second equation, we know that $\frac{1}{x}=4\cdot\frac{1}{y}$ (we could interchange x and y in that equation; it doesn't matter), so $y=4x$. Since $xy=9$, substituting $4x$ for y yields $4x^2=9$, or $x^2=\frac{9}{4}$. Since x and y are positive, $x=\frac{3}{2}$ and $y=\frac{3}{2}\cdot 4=6$. Their sum is $\boxed{\frac{15}{2}}$.

Problem 10.6: (Samyok Nepal)

Solve for x and y in the following system of equations.

$$\frac{12}{x}+\frac{10}{y}=5$$
$$\frac{42}{x}-\frac{40}{y}=10$$

Solution 10.6: Multiply the first equation by 4 to get $\frac{48}{x}+\frac{40}{y}=20$. Adding this to the second equation, we get $\frac{90}{x}=30$, so $\boxed{x=3}$. Substituting $x=3$ into either equation, we get $\boxed{y=10}$.

Problem 10.7: (Intermediate Algebra)

Find all ordered pairs (x,y) that satisfy both $\sqrt{x}+\sqrt{y}=7$ and $3\sqrt{x}-4\sqrt{y}=-14$.

Solution 10.7: Multiplying the first equation by 4, we get $4\sqrt{x}+4\sqrt{y}=28$. Adding this to the second equation, we get $7\sqrt{x}=14$, or $\sqrt{x}=2$. This means that $\boxed{x=4}$. Substituting $x=4$ into either equation, we get $\boxed{y=25}$.

Problem 10.8: (Samyok Nepal)

Find all values of a such that $\sqrt[3]{x}+\sqrt[3]{y}=7$ and $a\sqrt[3]{x}-4\sqrt[3]{y}=-14$ has no solutions.

Solution 10.8: Multiply the first equation by 4 to get $4\sqrt[3]{x}+4\sqrt[3]{y}=28$. Adding this to the second equation, we get $(a+4)\sqrt[3]{x}=14$. The only way for this to not have a solution for x is if $a+4=0$, or $\boxed{a=-4}$. For all other a, the solution to the system is $x=(\frac{14}{a+4})^3$ and $y=(7-\frac{14}{a+4})^3$.

11 Summer Challenge #3

Problem 11.1: (Serena An)

Solve for x and y given that $y=-1.2x+6$ and $4y=1.2x-1$.

Solution 11.1: Adding the two equations, we get $5y=5$, so $\boxed{y=1}$. Substituting 1 in for y in either equation, we get that $x=\frac{25}{6}$.

Problem 11.2: (Serena An)

If $17\sqrt{x}+2\sqrt[3]{y}=74$ and $15\sqrt{x}-\sqrt[3]{y}=57$, what is $y-x$?

Solution 11.2: Multiplying the second equation by 2, we get $30\sqrt{x}-2\sqrt[3]{y}=114$. Adding this to the first equation, we get $47\sqrt{x}=188$, so $\sqrt{x}=4$ and $x=16$. Plugging 16 in for x in either equation, we get $\sqrt[3]{y}=3$, so $y=27$. Thus, $y-x=\boxed{11}$.

Problem 11.3: (Serena An)

Normally at the BMC store, you can buy exactly 5 pencils and 4 apples with \$3.75. Today, there is a special sale, and pencils are being sold for one-third off the original price. Now you can buy 5 pencils and 6 apples for \$5.00. What is the original cost of one pencil?

Solution 11.3: Let p be the regular price of one pencil and let a be the price of an apple. The first sentence translates to $5p+4a=3.75$. In the special sale, the new price of one pencil is $\frac{2}{3}p$, so the third sentence translates to $5\cdot\frac{2}{3}p+6a=5$. Multiplying the first equation by 3 and the second equation by 2 (to eliminate the a), we get $15p+12a=11.25$ and $\frac{20}{3}p+12a=10$. Subtracting the second equation from the first equation, we get $\frac{25}{3}p=1.25$, so $\boxed{p=0.15}$.

Problem 11.4: (Serena An)

If $x+y\sqrt{x+y\sqrt{x+y\sqrt{\dots}}}=9$ and $x-y=1$, what is $x+y$?

Solution 11.4: The first equation is equivalent to $x+y\sqrt{9}=9$, or $x+3y=9$. Adding $x+3y=9$ and $x-y=1$, we get $2x+2y=10$, so $\boxed{x+y=5}$. Alternatively, you could solve for x and y individually and then add.

Problem 11.5: (Serena An)

Try to solve both parts. If you can solve only one part, then you will receive partial credit.

(a) Three vertices of a square are at $(20, 5)$, $(2, 3)$, and $(0, 21)$. What are the coordinates of the fourth vertex?

(b) Two *non-adjacent* vertices of a square are at $(3, 4)$ and $(-3, -4)$. What are the coordinates of the other two vertices?

Reminder: Write a full solution to this problem, not just the answer. Even if you do not have a full solution, type/write up what you have and you may receive partial credit!

Solution 11.5:

(a) We present two solutions to this part. With both solutions, we sketch the points $(20, 5)$, $(2, 3)$, and $(0, 21)$ so we have a rough idea of where the other point must be.

Geometric Properties: Let the unknown point be (x, y). The diagonals of a square bisect each other (at right angles), so the center of the square (where the diagonals intersect) is the midpoint of the diagonal connecting $(0, 21)$ and $(20, 5)$, and $(2, 3)$ and (x, y). The midpoint of the segment connecting $(0, 21)$ and $(20, 5)$ is $(\frac{0+20}{2}, \frac{21+5}{2}) = (10, 13)$. This must also be the midpoint of $(2, 3)$ and (x, y), so $\frac{2+x}{2} = 10$ and $\frac{3+y}{2} = 13$, so $(x, y) = \boxed{(18, 23)}$.

Slopes: Remember that the sides of the square form 90° angles with each other, so opposite sides are parallel and the slopes of opposite sides are equal. To get from $(2, 3)$ to $(20, 5)$, we went "across" 18 and "up" 2. So to get from $(0, 21)$ to the fourth vertex, we need to go "across" 18 and "up" 2 again to get $(0 + 18, 21 + 2) = \boxed{(18, 23)}$.

(b) From the given, the segment connecting $(3, 4)$ and $(-3, -4)$ must be a diagonal, and its midpoint is $(0, 0)$, which must be the center of the square and the midpoint of the other two points. Imagine rotating the segment connecting $(3, 4)$ and $(-3, -4)$ 90° clockwise to get the other diagonal. The linear equation of the line containing $(3, 4)$ and $(-3, -4)$ is $y = \frac{4}{3}x$, and the perpendicular line that also passes through the origin is $y = -\frac{3}{4}x$. The other two points must be on this line and be the same distance from the origin as $(3, 4)$ and $(-3, -4)$ are from the origin. We see that letting $x = 4$ in the new equation yields the point $(4, -3)$ and letting $x = -4$ yields the point $(-4, 3)$. Also, these four points are all the same distance from the origin, so the other two vertices are $\boxed{(4, -3) \text{ and } (-4, 3)}$.

12 Solutions to Selected Problems

Problem 12.1

Does it matter which point has the coordinates (x_1, y_1) and which point has the coordinates (x_2, y_2) in the formula for slope? Is the slope the same even if the coordinates are reversed?

Solution 12.1: Fortunately, it doesn't matter! If it did matter, then two points would actually lead to two different linear equations. To show this is true, we will use two arbitrary points, the first one with coordinates (a, b) and the second point with coordinates (c, d). Letting (a, b) be (x_1, y_1) and (c, d) be (x_2, y_2) in our formula for slope, we get that the slope is $\frac{b-d}{a-c}$. Now we will let (c, d) be (x_1, y_1) and (a, b) be (x_2, y_2) in our formula for slope to get a slope of $\frac{d-b}{c-a} = \frac{-b+d}{-a+c} = \frac{-b+d}{-a+c} \cdot \frac{-1}{-1} = \frac{b-d}{a-c}$. Our two values for slope are equal, and we conclude that the order of which we put our two points into the slope formula does not matter.

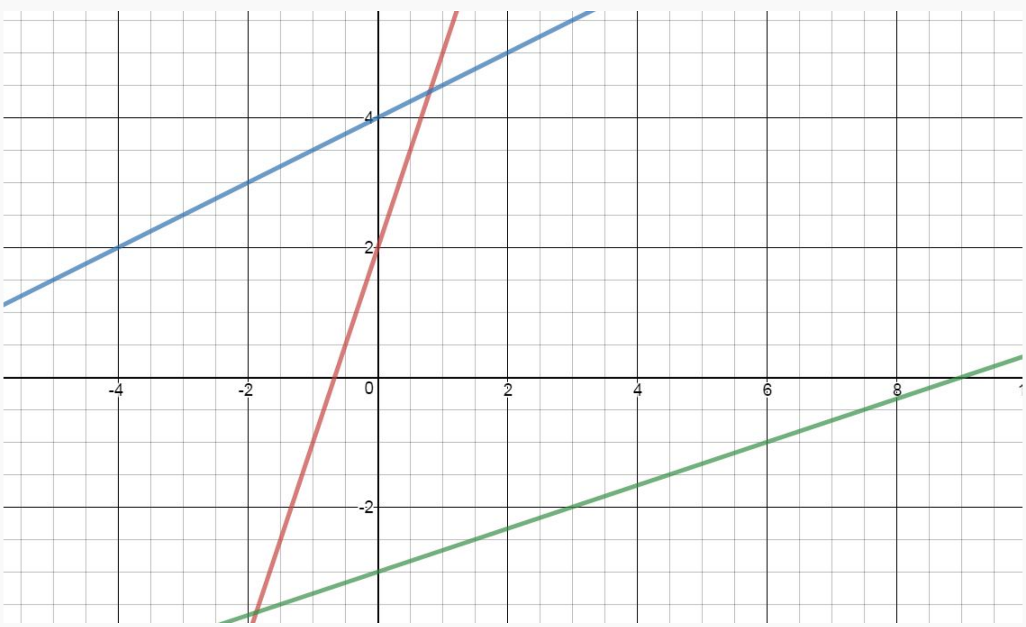
Problem 12.2: ***Source: CMMS***

Find the slope of the line passing through the coordinates $(7, -2)$ and $(-4, 1)$.

Solution 12.2: We use our formula for slope to get $\frac{-2-1}{7-(-4)} = \boxed{-\frac{3}{11}}$.

Problem 12.3

Find the equations of these lines in slope intercept form.



Solution 12.3:

Green: $y = \frac{1}{3}x - 3$ Red: $y = 3x + 2$ Blue: $y = \frac{1}{2}x + 4$

Problem 12.4: Point Slope

Find the equation of the line that passes through $(10, 3)$ and $(-5, 9)$ in point slope form.

Solution 12.4: $m = \frac{3-9}{10-(-5)} = \frac{-6}{15} = -\frac{2}{5}$. Using $(10, 3)$ as our point (or you could use $(-5, 9)$), we get $\boxed{y - 3 = -\frac{2}{5}(x - 10)}$.

Problem 12.5: *Source: 2017 MATHCOUNTS National Countdown Round*

Triangle PQR has vertices at $P(1294, 104)$ and $R(-1394, 1123)$. If $\angle\, \mathrm{Q} = 90°$, what is the product of the slopes of the legs?
Original Problem: `https://youtu.be/vFTeN17Z4rc?t=1759`.

Solution 12.5: The legs of a right triangle are perpendicular, so the product of the slopes of the legs is $\boxed{-1}$.

Problem 12.6

Evaluate $6f - g + h^i$ for $f = 2$, $g = 8$, $h = -4$, and $i = 3$.

Solution 12.6: $6(2) - (8) + (-4)^{(}3) = 12 - 8 - 64 = \boxed{-60}$

Problem 12.7

What is the value of x in the equation $y = 11x - 8$ when $y = 3$?

Solution 12.7: We solve $3 = 11x - 8$ to get $\boxed{x = 1}$.

Problem 12.8

What is the value of x in the equation $y = 23x - 108$ if $y = -x + 12$?

Solution 12.8: Plugging in $-x + 12$ for y, we get the equation $-x + 12 = 23x - 108$. Combining like terms, we get $120 = 24x$, so $\boxed{x = 5}$.