Famous Proofs

Serena An–7/12/20

1 $\sqrt{2}$ is Irrational

Theorem 1.1

$\sqrt{2}$ is irrational.

Proof. For the sake of contradiction, suppose $\sqrt{2}$ can be written *in lowest terms* as $\frac{p}{q}$ for positive integers p and q. Then squaring both sides,

$$\frac{p^2}{q^2} = 2 \implies p^2 = 2q^2.$$

So p must be even, and we can write $p = 2r$ for some integer r. Then

$$4r^2 = 2q^2 \implies 2r^2 = q^2,$$

which means that q is also even.

However, this shows that p and q are both even, which contradicts the assumption that the fraction $\frac{p}{q}$ was in lowest terms. $\square$

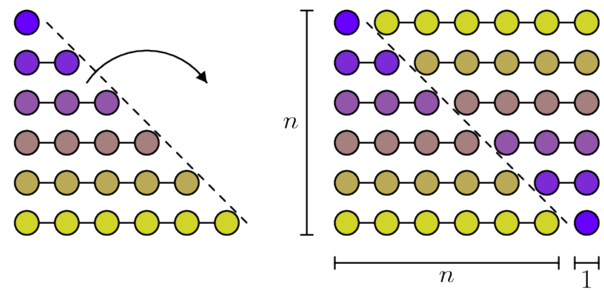
Exercise 1.1

Prove that $\sqrt{p}$ is irrational for all primes p.

2 Number Sums

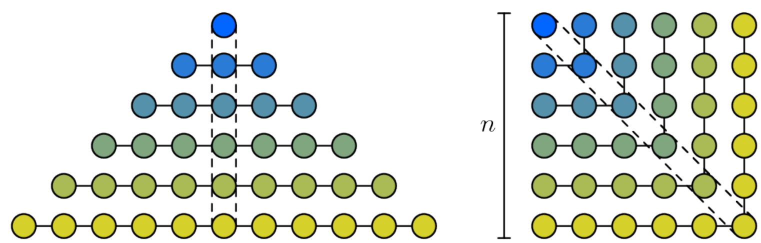
Theorem 2.1

The sum of the first n positive integers is $\frac{n(n+1)}{2}$.



Theorem 2.2

The sum of the first n odd positive integers is n^2.



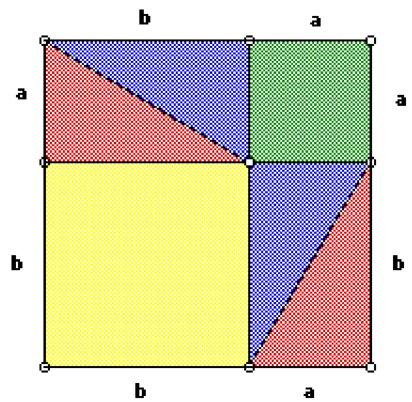
3 Pythagorean Theorem

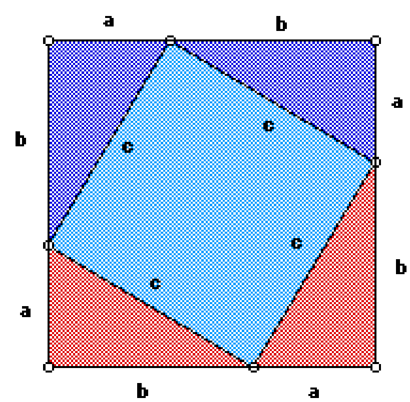
Theorem 3.1

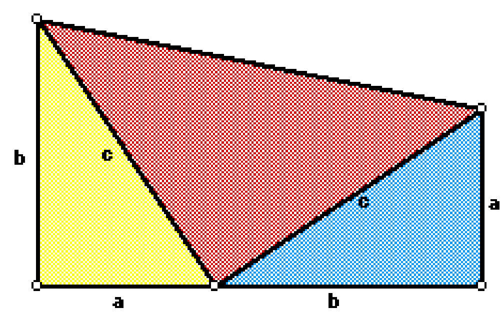
In a right triangle with legs a and b and hypotenuse c,

$$a^2 + b^2 = c^2.$$

We will present two visual proofs.



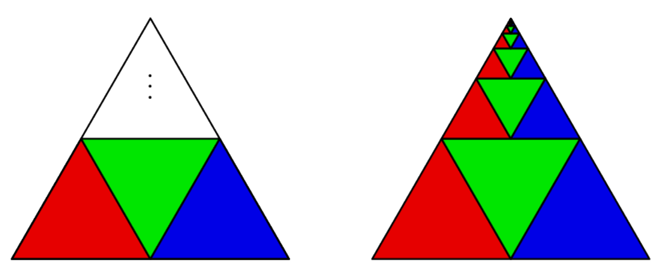


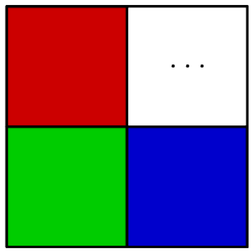


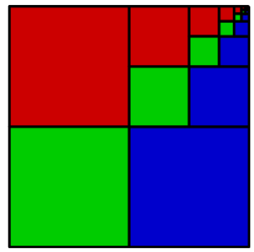
4 Geometric Series

Theorem 4.1

$$\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots = \frac{1}{3}.$$







Theorem 4.2

For $0 < r < 1$,

$$1 + r + r^2 + r^3 + \cdots = \frac{1}{1-r}.$$

(Note that this holds in general for $-1 < r < 1$.)