# Week 1: Patterns

Samyok Nepal–8/3/19

1 Exercises

If a problem doesn't have a source, it's from the Patterns Stretch in the 2017-18 MATHCOUNTS Handbook (Warm-up Problems 11-20). The apples are our own rating system of the "difficulty" – attempt all of them though! 3 apples is 'average'.

None of these problems require the use of a calculator! Email us if you attempt any of these throughout the week and get stuck.
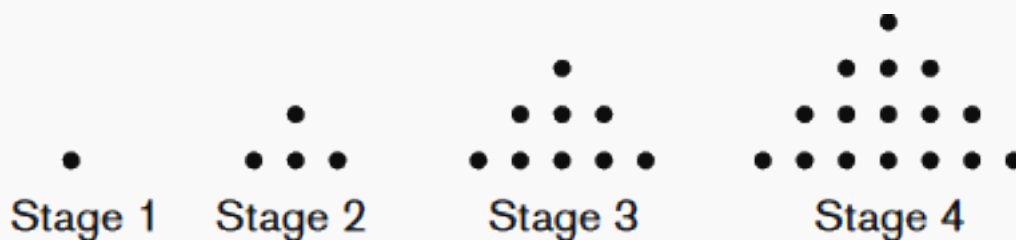

Problem 1.1:

If the pattern continues, what is the value of x in the third figure?
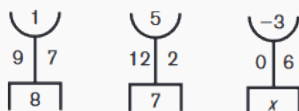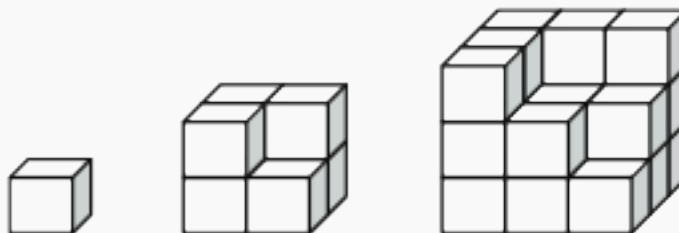

Problem 1.2:

The first four stages of a dot pattern are shown. How many more dots are in the figure at Stage 47 than in the figure at Stage 27?

Problem 1.3: MATHCOUNTS 15-16 Warm-up #152

If this pattern continues, how many cubes will be in the next figure?

Problem 1.4:

The first three terms of a sequence are 1, 2 and 3. Each subsequent term is the sum of the three previous terms. What is the 11th term of this sequence?

Problem 1.5:

What is the sum of the terms in the arithmetic series 2 + 5 + 8 + 11 + 14 + ... + 89 + 92?

Problem 1.6:

Three consecutive terms in an arithmetic sequence are x, 2x + 11 and 4x - 3. What is the constant difference between consecutive terms in this sequence?

Problem 1.7:

What is the sum of the terms in the geometric series 1 + 4 + 16 + ... + 1024?

Problem 1.8:

What is the sum of the first 51 consecutive odd positive integers?

Problem 1.9:

What is the sum of the terms in the infinite series

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$$

Problem 1.10:

What is the sum in the infinite series

$$1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots$$

Problem 1.11: MATHCOUNTS 2016-17 Warm-Up #11

What digit is in the units place in the product (don't calculate either term!)

$$3^{17} \cdot 7^{23}?$$

Problem 1.12:

Let $f(x) = 2x + 3$ and $f^2(x) = f(f(x)) = f(2x + 3) = 2(2x + 3) + 3 = 4x + 9$. If $f^5(x) = ax + b$, what is the value of $a + b$?

Problem 1.13:

The degree measures of the interior angles of a quadrilateral form a geometric sequence whose terms have integer values and are all integer multiples of the first term. What is the largest possible degree measure of an angle in this quadrilateral?

Problem 1.14: MATHCOUNTS 2016-17 Warm-up #98

What is the maximum number of distinct intersections of 30 different coplanar[a] circles?

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[a]Coplanar means in the same plane, if you don't know what it means ignore it – you'll get the same answer regardless