Problem Set #1

Serena An and Samyok Nepal–May 27, 2018

1 Introduction

Welcome to the first Brookings Math Circle Summer 2018 Problem Set! **Submissions are due by Sunday 6/10 at 1:00 pm (the day of the first BMC class).**

The Summer 2018 Problem Sets will run over 14 weeks, 6/3 to 9/9 (this first one is being released early). A new problem set and solutions to the previous problem set will be released on our website every Sunday afternoon at approximately 4:00 pm.

We have some basic rules for these problem sets that we hope are very understandable and reasonable. Firstly, no cheating, which includes looking up problems online and asking online for answers. If you cheat, you are really only cheating yourself and making the situation unfair for fellow mathletes. For the Puzzle and Bonus Problem Set we will allow collaboration, but we strongly urge all students to try all problems individually first. For the Summer Challenge, we do not allow any collaboration. Unless specified, for all of these problems, you may only use a pencil or pen and paper; in particular, calculators are not allowed and not needed. Also, please don't hesitate to contact Serena or Samyok if you have a question about a problem, want us to check your work, would like help, or anything else (but we won't give hints for the Summer Challenge; that's to test your ability!).

When there is BMC, the first thing we will do in each class is to go over the problems from the previous problem set. Thus, we strongly encourage every student to try to solve as many problems as possible throughout the week, and come to class prepared with solution sketches or progress with every problem. Remember, you have an entire week, and spreading out work time is typically the most effective. At BMC, we will also hand out a paper copy of the next problem set.

Every problem set contains a Puzzle, multiple Bonus Problems, and five Summer Challenge Problems. The theme of each problem set will vary from week to week. This first week, it will include an assortment of topics, but moving forward, every week's problem set will focus on one of the main competition math areas: algebra, geometry, counting, probability, and number theory.

The Puzzle requires careful thinking and problem solving. Being puzzles, they contrast from the more contest-math themed problems in the bonus problem set. For the Puzzle, we want the student to write a solution sketch as he/she solves the problem to show the thought process. Puzzles will usually require more time than problems in the Bonus Problem Set, but none the less, each Puzzle is fun and doable with some effort.

In the Bonus Problem Set, there is an apple point value to the right of each problem. As a general rule of thumb, the more apples a problem is worth, the more difficult it is (and the more accomplished you

should feel after solving it!). Keep in mind though, that these apples are not perfectly objective, especially considering that everyone has different strengths and weaknesses. Feel free to skip around when solving the problems, and definitely don't feel obligated to do problems in order. Try the ones you think are the most interesting first! This summer, we will be doing problems of difficulty from 1 apple to 5 apples, and the overall difficulty of bonus and challenge problems will increase every week. Try to gain as many apples as you can each week! There will be prizes at each BMC class... ;)

Now we come to the part everyone has been waiting for: The Summer Challenge! Every week, the Summer Challenge will consist of 5 original problems written by Serena and Samyok. Each problem will contain a certain number of apples by it, and for each problem you solve, we will reward you that many apple points. These apple points are based on relative difficulty to each other, and may not exactly align to Bonus Problem Set difficulties. There will be 4 short answer problems worth 1, 2, 3, and 4 apples, and one writing problem worth 5 apples. The writing problem will typically be phrased like any other short answer problem, but what we want you to do is to write out your full solution, not just the answer. Partial credit is possible on the writing problem, so even if you don't have a complete solution, submit what you have!

To submit for the Summer Challenge, go to the BMC website and input your short answers. The writing problem may be handwritten or typed. For those unable to attend BMC that week, you must type your solution and submit on the website. For those attending BMC that week, you may hand us a handwritten solution at class or submit online. If you do plan to hand in a handwritten solution, please indicate so on the website in the space of the writing problem submission.

Just as we said that there will be prizes for the Bonus Problems, there will be better prizes for the Summer Challenge! We will split into 2 divisions, novice and advanced, for prizes. All students participating in BMC or competition math for at least the second year must be in the advanced division, although first year students may choose either division. We will keep a running leaderboard of the top 3 in each division over the entire summer, so try to solve as many as you can to be on the leaderboard! Don't be discouraged if you're not on the leaderboard though, if you put in effort, you will be rewarded. :)

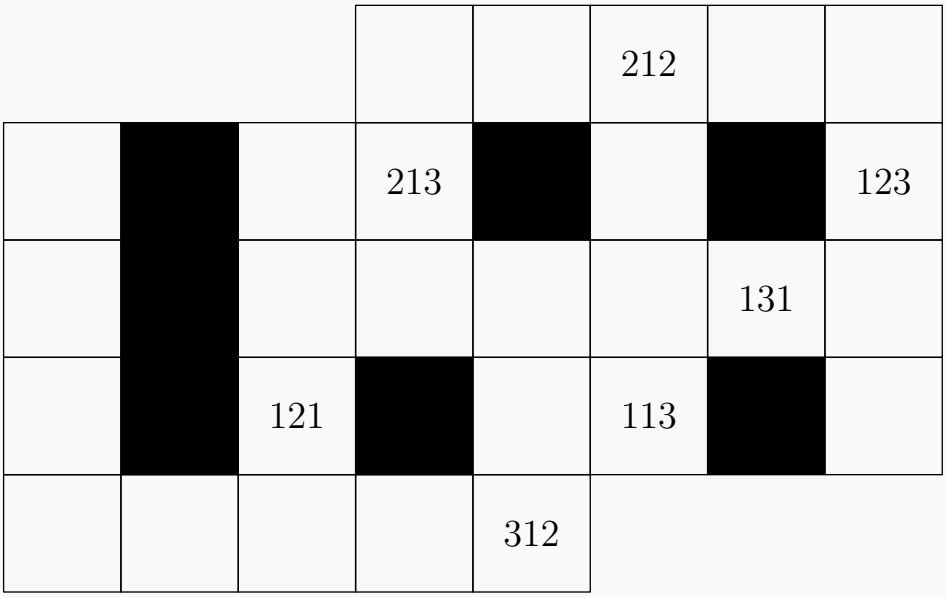
Finally, we are not expecting every student to solve every problem, as the problems, overall, are quite challenging. However, we do expect all BMC students to at least try every problem and still spend some time on a problem even if you don't see the solution immediately. Only with time, effort, and patience will your problem solving skills improve, and trust us, the journey is worth it.

We hope you enjoy the weekly Summer 2018 Problem Sets! Keep calm and do the math!

2 Puzzle #1

Problem 2.1: (USAMTS)

Fill each white square in with a number so that each of the 27 three-digit numbers whose digits are all 1, 2, or 3 is used exactly once. For each pair of white squares sharing a side, the two numbers must have equal digits in exactly two of the three positions (ones, tens, hundreds). Some numbers have been given to you.



111	112	~~113~~	211	~~212~~	~~213~~	311	~~312~~	313
~~121~~	122	~~123~~	221	222	223	321	322	323
~~131~~	132	133	231	232	233	331	332	333

There is only one correct solution. It may be helpful to copy the diagram on paper. Bring your finished diagram or progress to the first Brookings Math Circle class. We will explain the solution and discuss! Good luck; this is the first step of the Brookings Math Circle Summer 2018 Challenge.

3 Bonus Problem Set #1

Problem 3.1: (AMC 8)

The sum of two prime numbers is 85. What is the product of these two prime numbers?

Problem 3.2: (MATHCOUNTS)

A hexagon and pentagon share the property that the side lengths of each are consecutive integers, and the perimeter of each is 45 cm. What is the difference in length between the shortest side of the pentagon and the shortest side of the hexagon?

Problem 3.3: (AMC 8)

How many integers between 1000 and 9999 have four distinct digits?

Problem 3.4: (MATHCOUNTS)

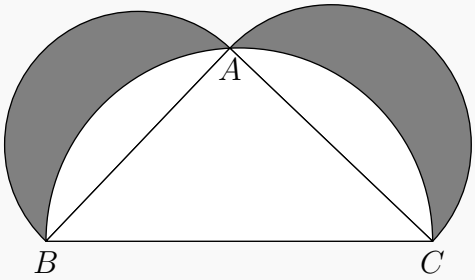
What is the number of degrees of the acute angle formed by the minute and hour hands of a clock at 11:10 PM?

Problem 3.5: (CMMS)

How many squares can be formed using four of the dots in the unit grid as vertices?

Problem 3.6: (MPfG)

The figure below shows a triangle ABC with a semicircle on each of its three sides.



If $AB = 20$, $AC = 21$, and $BC = 29$, what is the area of the shaded region?

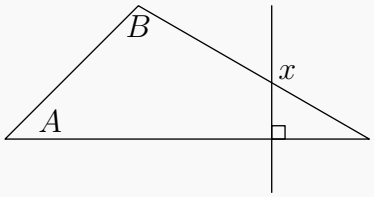
Problem 3.7: (AIME)

The AIME Triathlon consists of a half-mile swim, a 30-mile bicycle ride, and an eight-mile run. Tom swims, bicycles, and runs at constant rates. He runs fives times as fast as he swims, and he bicycles twice as fast as he runs. Tom completes the AIME Triathlon in four and a quarter hours. How many minutes does he spend bicycling?

4 Summer Challenge #1

Problem 4.1: (Samyok Nepal)

What is angle x if $\angle A$ is $45°$ and $\angle B$ is $105°$?



Problem 4.2: (Serena An)

What is the units digit of

$$(12^2 + 23^2 + 34^2 + 45^2 + 56^2 + 67^2 + 78^2 + 89^2)^2?$$

Problem 4.3: (Serena An)

Serena flips 1000 fair coins while Samyok flips 999 fair coins. What is the probability that Serena flips more heads than Samyok? Express your answer as a common fraction.

Problem 4.4: (Serena An)

If $x^{x^{x^{\cdots}}} = 2$, what is x^4?

Problem 4.5: (Samyok Nepal)

Samyok has a special spinner. On this spinner, there are 4 numbers: 1, 2, 3, and 4. However, the probability of getting a number is proportional to that number. For example, the probability of getting a 2 is half of the probability of getting a 4. Samyok spins the spinner twice. What is the probability that the sum of the two spins is 4?

Reminder: Write a full solution to this problem, not just the answer. Even if you do not have a full solution, type/write up what you have and you may receive partial credit!