ASTR 5460 Project: Hubble Constant Measurement

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ABSTRACT

I present the measurement of Hubble constant H_0 using the Tip of the Red-Giant Branch (TRGB) method. Data from NASA/IPAC Extragalactic Database (NED) and NED Redshift-Independent Distances are analyzed to find the Hubble constant in the method. Errors and uncertainties for the TRGB method are identified and discussed. Two line fitting algorithms are used to fit Hubble's law, and the Hubble constant from these two fitting algorith are very different. The value of the Hubble constant using χ^2 fitting is $H_0 = 90.61 \pm 3.64$ km s^{-1} Mpc^{-1}, while the Hyperfit algorithm gives $H_0 = 78.14 \pm 2.79$ km s^{-1} Mpc^{-1}. Further investigation, exploration, and calibration are proposed.

1. INTRODUCTION

The Hubble constant (H_0) is an important cosmological parameter that measures the expansion rate of the universe. This constant is first discovered by Edwin Hubble. In Hubble (1929) he discussed the velocity-distance relation, which is now known as Hubble's law. Hubble found a constant K relating the distance of nebulae and their radial velocity. This constant K is now the Hubble constant, denoted as H_0. Hubble (1929) stated the constant K is 500 km s^{-1} Mpc^{-1}. After Hubble's discovery, multiple attempts were made to calibrate H_0. Thanks to the recent development of astronomical instruments, we can now better understand the physics of different astronomical objects, such as supernovae (SNe) and black holes, that can give more accurate measurements of H_0. In Freedman et al. (2001), the measurements of Hubble constant from five secondary methods yield a result of $H_0 = 72 \pm 8$ km s^{-1} Mpc^{-1}. In this work, I reexamine the measurements of H_0 using the Tip of the Red-Giant Branch (TRGB) method. The result from this method is analyzed, and potential errors are discussed.

2. DATA AND SAMPLE SELECTION

NASA/IPAC Extragalactic Database (NED) is used for the data. The distances to each object are from NED Redshift-Independent Distances (NED-D) dataset to ensure all distances are independently measured without redshift (z). The NED-D contains distance modulus measured using 76 methods and has 328317 data points.

For data consistency, I selected the data from a single paper Tully et al. (2013) for the TRGB method. Authors in the paper discussed the uncertainties and different systematic errors, so the data from single paper have better consistency compared to data from a mixture of papers. I discuss this data mixture effect in section 4. I selected this paper based on the size of samples in its category.

The redshifts were acquired using NED's ObjectLook Application Programming Interface (API) endpoint. One query for each interesting data point was sent to this API endpoint. Response from NED contains host galaxy redshift, redshift uncertainties, and redshift reference code for validity check. Data with no host galaxy redshift were removed from the analysis. Compared to the redshift-independent distance, the redshift itself is relatively easier to measure with smaller uncertainties, as it only relies on the spectrum of the object. All redshifts are from NED, and are expected to be highly reliable.

3. METHODS

This section introduces the method used to analyze Hubble's law and measure Hubble's constant. A brief overview of the mechanism of the TRGB method is given. A full in-depth discussion can be found in the source paper cited in each subsection.

Hubble's law is

$$v = H_0 D \tag{1}$$

Where v is the radial velocity, and D is the distance. Typically, the radial velocity is measured using redshift,

$$v = cz \tag{2}$$

Here z is the redshift, c is the speed of light. Therefore, the relation between Hubble constant, distance, and redshift is

$$H_0 = \frac{v}{D} = \frac{cz}{D} \tag{3}$$

This also explains the reason a Redshift-Independent distance is necessary for this paper. If both the numerator and denominator depend on z, the z dependence can be reduced or even cancelled, so this Hubble's law will become another relation that is completely different from the original form, and these factors after the cancellation can lead to different H_0 results. This paper is interested in finding the direct correlation between radial velocity and distance in the original form of Hubble's law.

After deriving the relationship between the redshift and Hubble's law, I will briefly introduce the Tip of the Red-Giant Branch (TRGB) method. The TRGB method was first developed while measuring the distance to Large Magellanic Cloud (LMC). Sakai et al. (2000) found the apparent magnitude of the TRGB peaked at 14.54 mag. The calibration is done by Lee, Freedman, and Madore (Lee et al. (1993)) shows that the absolute magnitude of TRGB is $M = -4.05 \pm 0.04$ mag. Therefore, the distance modulus to LMC is then derived. By using the LMC calibration, TRGB can be another standard candle to measure the cosmic distance. As long as the apparent magnitude for a TRGB star is measured, the distance modulus can be calculated to get the distance to the star.

The radial velocity in this project is calculated using $v = cz$, so all the velocity referred in this work is the heliocentric velocity. Although there are many types of velocity based on different references, the heliocentric velocity is chosen based on two considerations. First, the heliocentric velocity is very easy and straightforward to acquire. As long as the heliocentric redshift of the host galaxy is obtained, the heliocentric radial velocity can be calculated as well. Second, in addition to the simplicity of obtain heliocentric redshift, the query sent to the NED automated ObjectLookup service directly returned redshift, so it is nature to use heliocentric velocity directly in this work. The use of heliocentric velocity should not affect the derivation of Hubble constant, as the velocity used by Hubble in his origin work is heliocentric as well. As long as the objects within the gravitational sphere of influence of Milky Way are removed, the analysis should give us good results.

4. RESULTS AND DISCUSSIONS

NED-D dataset contains Redshift-Independent distances but without errors corresponding to these distances. Since only the distance modulus in the dataset has a corresponding error, all distances are recalculated using distance modulus with uncertainty. A signal-to-noise (SNR) cut is implemented on both distance and velocity to exclude noisy data points with high uncertainty. I used a 3σ SNR rejection For TRGB.

The gravitational interactions between galaxies in Local Group (LG) significantly influence the accuracy of H_0 on the cosmic scale. Therefore, I removed all data within 2 times of Milky Way–Andromeda distance ($0.765\ kpc \times 2 \approx 1.5\ Mpc$).

These data selections may result in selection bias (Malmquist bias), which I will address in section 4.3.

Two different fitting algorithms are discussed and compared here.

4.1. *Using χ^2 fitting*

The χ^2 fitting tries to minimize the residual square divided by the variance of the observation. The objective function is

$$\chi^2 = \sum_i \frac{(E_i - O_i)^2}{\sigma_i^2}$$

$$\min \chi^2$$

Where O is the observed value and E is the expected value as calculated using the fitted line. In χ^2 fitting, I assume the uncertainty of distance will be much larger than the uncertainty of velocity. Therefore, the uncertainty (σ) here is used as the uncertainty of velocity σ_v. This method is very intuitive to use, as in most of the cases the heliocentric velocity are based on the redshift of the host galaxies, which typically have smaller uncertainties. The Hubble constant calculated in this case is $H_0 = 90.61 \pm 3.64$ km s^{-1} Mpc^{-1}.

From figure 1, due to the intrinsic scattering of the data, the fitting range cannot well describe the data spread. Furthermore, this method fail to include the uncertainty on velocity, which derived from redshift. Thus, it is likely that H_0 calculated using this method might be incomprehensive, as the error on distance is not taken into account.

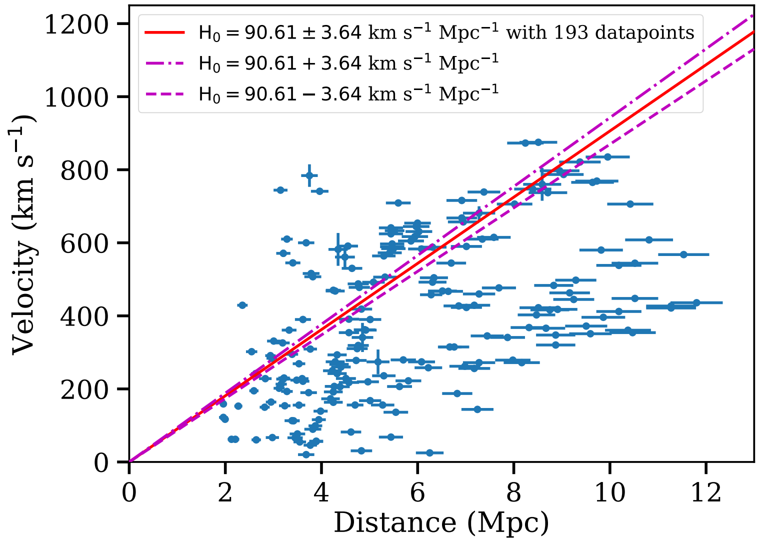

Figure 1. Redshift-Independent distance and Velocity of TRGB method sample. 193 TRGB stars selected from Tully et al. (2013) paper are all within 12 Mpc range of Milky Way. χ^2 fitting is applied. The red line shows the average value of 90.61, while the magenta dot-dash and dashed line represents the 1σ upper and lower limit of the fitting range respectively.

4.2. *Using `Hyperfit`*

The second fitting method I used is `Hyperfit`, as described in Robotham & Obreschkow (2015). The package tried to maximize the log-likelihood function described below. The objective function is

$$\mathcal{O} = \ln\mathcal{L} = \frac{1}{2}\sum_{i=1}^{N}\left[\ln\left(\sigma_\perp + \frac{n^T \mathsf{C}_i n}{n^T n}\right) + \frac{(n^T[x_i - n])^2}{\sigma_\perp^2 n^T n + n^T \mathsf{C}_i n}\right]$$

$$\max \ln\mathcal{L}$$

Where the C_i is the covariance matrix of variables, and the n is the normal vector to the hyperplane defined by parameters of the function we want to fit. In this case, the function is

$$v = H_0 D + b$$

where b is supposed to be 0, since there is no velocity when the distance is 0. However, in reality, `Hypefit` gives $b = -0.3908$, which can be a defect in the fitting, as it is nonphysical. $\sigma_\perp$ is the intrinsic scattering of data in the direction perpendicular to the hyperplane. By maximizing the log-likelihood, a best fitting curve can be found. From 2, the data scattering is very large, as indicated by the green lines. The Hubble constant in this case is $H_0 = 78.14 \pm 2.79$ km s^{-1} Mpc^{-1}. Comparing with the χ^2 fitting, this method includes both error in distance and velocity, so it should be more comprehensive than the previous method. However, the maximum log-likelihood given by the algorithm is -286, which is very small. Therefore, the result using `Hyperfit` can be inconclusive.

Besides the data fitting, I found that there are significantly many outliers in the range 3.35 Mpc to 3.9 Mpc, as shown in the figure 3. These TRGB stars have peculiar velocities much higher than the universe expansion. Although these stars have high peculiar velocities, the distance and velocity uncertainties are quite small. Hawkins et al. (2015) studied hypervelocity stars (HVS) in the solar neighborhood and found the highest Galactic rest-frame velocity for a star is $V_{GFR} = 807$ km s^{-1}. This means that the velocity dispersion for a star is very unlikely higher than 800 km s^{-1} of the local cluster velocity. Therefore, I remove data in the sample where the velocity of the TRGB star is more than 800 km s^{-1} than the local cluster velocity, as shown in figure 3.

4.3. *Error and uncertainties*

Due to the data selection imposed on the different samples, selection bias (Malmquist bias) may be introduced in the analysis. There are two data selections. All data with a distance closer than 1.5 Mpc are removed. Data with SNR $\leq$ 3 (or 10 in the SNe Ia sample) are removed. For the first method, the selection bias is very minimal, as samples with

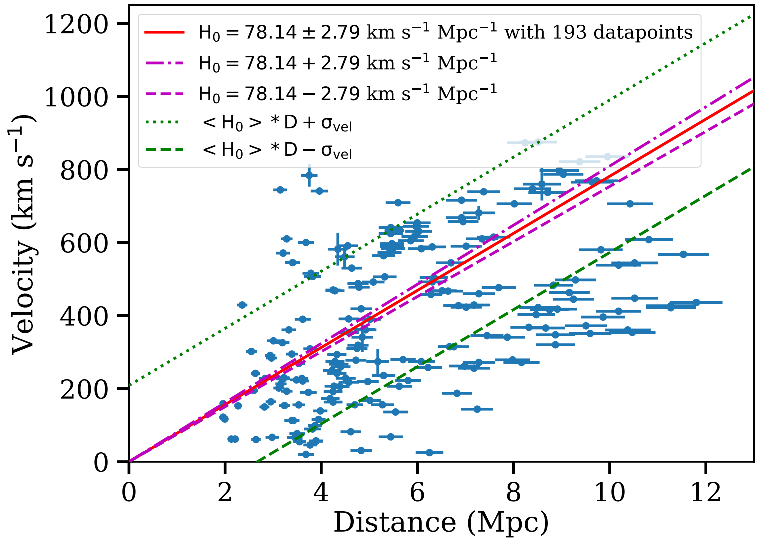

Figure 2. Same TRGB sample as used in section 4.1. The fitting package is `Hyperfit`. Red solid line shows the average value of H_0, and the magenta shows the upper and lower range. Th green lines show the 1σ spread of velocity $\sigma_{vel} = 208.78$.

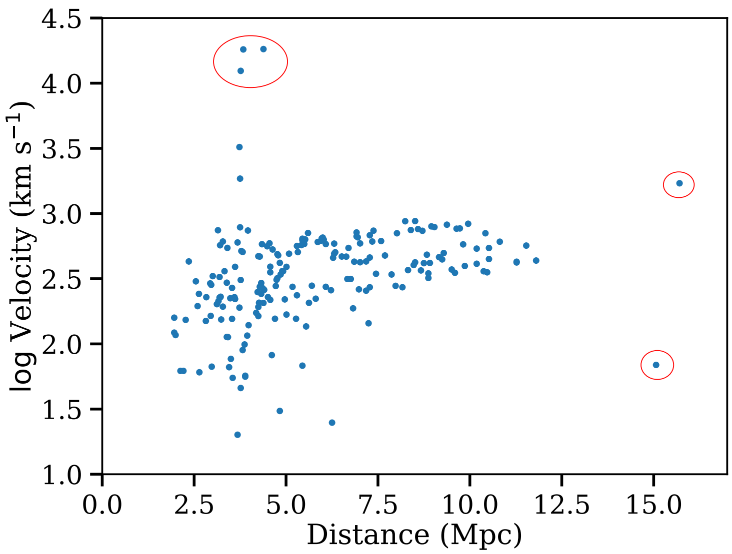

Figure 3. Log velocity vs. distance, illustrating the spread of the data. Points in the red circles are removed. In the upper red ellipse, the stars are excluded because of the unusual velocities. In the right two small circles, data are removed because there are only two stars in that distance, so these stars cannot be comprehensive representatives for the mean motion of stars in that distance.

distance closer than 1.5 Mpc are very likely to be influenced by the LG, and therefore the Hubble constant calculation is not an accurate reflection of the general expansion of the universe if those data are taken into account. Only if data with an error that spread across 1.5 Mpc will likely be most biased because I might remove data that are supposed to be included or vice-versa. However, the amount of data in this region is very small compared to the large sample sizes.

3σ data rejection will not be causing significant bias because I am only decreasing the ratio of nominal value to errors. All noisy data are removed. That is, I am restricting generally combined errors. Therefore, these two data selections should not introduce a large Malmquist bias.

As Tully et al. (2013) mentioned, their data catalog has distances with uneven quality. Therefore, the extremely high velocity can be explained for two reasons. The first possibility is that the high peculiar velocity can be due to the systematic errors in the distance measurements because some datasets may be very low quality and thus have very

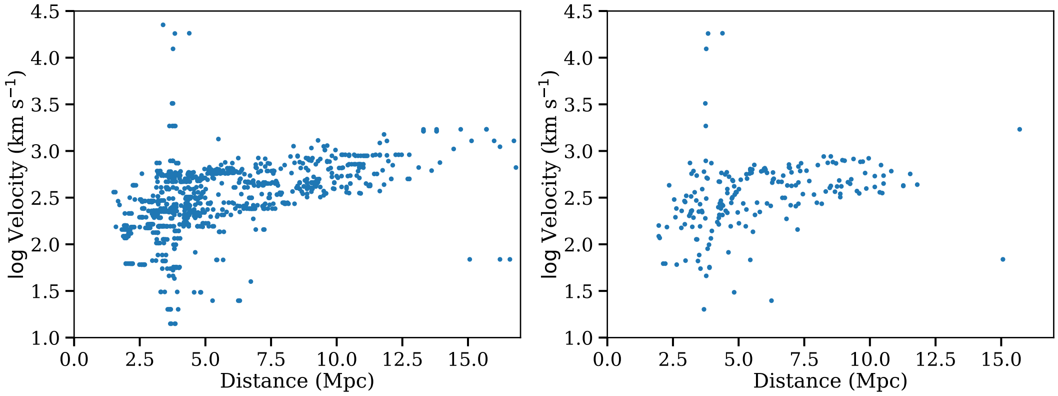

Figure 4. A mixture of dataset vs. single dataset. Data used in the left two figures are from NED-D rows with the "METHOD" column marked as "TRGB". Data used in the right two figures are from a single paper with reference code "2013AJ....146...86T" in addition to the method condition. Data on the left panel are more spread out, and there are several more data points with high peculiar velocities.

large uncertainties. The other reason is that peculiar velocities for the stars in the catalog are spread out, as shown in Figure 16 in Tully et al. (2013). Hence, it is possible that some stars can really have very high peculiar velocities ($\sim 10^4$). Even though this is the case, these high peculiar velocity stars should not be considered. Because Hubble constant measures the general trend for the expansion rate of the universe, based on the assumption of isotropic and homogeneous in large scale. Therefore, the high velocity stars should not be the real focus and fail to be a good representation of the general expansion of the universe.

The effect of a mixture of different papers compared to a single paper is discussed in this paragraph. Data from a single paper tend to have the same systematic error. Therefore, it is not necessary to pay special attention to calibrate different datasets to expose different systematic errors and to think of methods to remedy them. The mixture of data from several papers might have a closer-to-the-consensus result, which is the case shown in 4, but it can be due to the cancellation of different systematic errors. Data are more unreliable in this case before calibrating.

5. CONCLUSION

I explored the measurements of Hubble constant H_0 using the TRGB method. During the fitting process, two different methods, χ^2 and `Hyperfit` are used. The result from the χ^2 fitting method is $H_0 = 90.61 \pm 3.64$ km s^{-1} Mpc^{-1}, which is quite far from the census value in Freedman et al. (2001). The other fitting method `Hyperfit` yields a Hubble constant of $H_0 = 78.14 \pm 2.79$ km s^{-1} Mpc^{-1}, about 8 km s^{-1} Mpc^{-1} away from the consensus value in Freedman et al. (2001). Both methods fail to give a satisfactory result, either lack of comprehensiveness or conclusiveness. This is very likely due to the large scattering of the data. Further investigations, such as using more data, or cross-comparing with other catalog should be performed before reaching a more definitive result. TRGB stars used in this case have relatively low to medium redshift, so it can be a good measurement for the local universe expansion rate. For an earlier universe with higher redshift, measuring the Hubble constant will require adopting a cosmological model.

APPENDIX

A. DATA AND CODE AVAILABILITY

This is an open-source project. The NED-D data are available at https://ned.ipac.caltech.edu/Library/Distances/. The code is solely developed by the author, and it is available at https://github.com/Brookluo/hubble_constant. NED host galaxy query results are in the git repository as well.

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