HW: 5.5; 6.1; 6.2

Saturday, September 2, 2023 1:35 PM

5.5: 1, 2, 3, 6, 7
6.1: 2, 6, 5, 8
6.2: 2, 3, 5, 7

V-Sub $v = 1 - e^u$ $\quad \frac{\partial v}{\partial u} = 0 - e^u = -e^u$ $\quad \partial v = -e^u \cdot \partial u$ $\quad \partial u = \frac{\partial v}{-e^u} = -e^{-u}\partial v$

1) $\int \frac{e^u}{(1-e^u)^2}\partial u = \int \frac{e^u}{(v)^2}\partial u = \int \frac{e^u}{v^2} \cdot -e^{-u}\partial v = -\int \frac{\cancel{e^u}}{v^2 \cancel{e^u}}\partial v = -\int \frac{1}{v^2}\partial v$

$= -\int v^{-2}\partial v = \frac{-v^{-1}}{-1} = \frac{1}{v} + C = \boxed{\frac{1}{1-e^u} + C}$

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2) $\int \frac{z^2}{z^3+1}\partial z$ $\quad$ U-Sub $u = z^3 + 1$ $\quad \frac{\partial u}{\partial z} = 3z^2$ $\quad \partial z = \frac{\partial u}{3z^2}$

$\partial z = 3z^{-2}\partial u$

$K = 3 \quad \frac{1}{K} = \frac{1}{3}$

$\frac{1}{K}\int \frac{Kz^2}{u} \cdot 3z^{-2}\partial u = \frac{1}{3}\int \frac{\cancel{3z^2}}{u(\cancel{3z^2})}\partial u = \frac{1}{3}\int \frac{1}{u}\partial u$

$= \frac{1}{3}\ln(u) + C = \boxed{\frac{\ln(z^3+1)}{3} + C} = \boxed{\frac{\ln(|z^3+1|)}{3} + C}$

abs. Val. b/c ln(x) is not defined for x<0

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3) $\int x\sqrt{x+2}\,\partial x$ $\quad$ U-Sub $u = x+2$ $\quad \partial u = (1)\partial x$

$x = u - 2$

$\int x\sqrt{u}\,\partial u = \int (u-2)(u^{\frac{1}{2}})\partial u = \int (u^{\frac{3}{2}} - 2u^{\frac{1}{2}})\partial u$

$= \int u^{\frac{3}{2}} - 2\int u^{\frac{1}{2}} = \frac{u^{\frac{5}{2}}}{\frac{5}{2}} - 2\left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right) = \frac{2u^{\frac{5}{2}}}{5} - \frac{4u^{\frac{3}{2}}}{3} + C$

$= \frac{6u^{\frac{5}{2}}}{15} - \frac{20u^{\frac{3}{2}}}{15} = \frac{6(x+2)^{\frac{5}{2}} - 20(x+2)^{\frac{3}{2}}}{15} = \frac{6(x+2)^{\frac{2}{2}}(x+2)^{\frac{3}{2}} - 20(x+2)^{\frac{3}{2}}}{15}$

$= \frac{2(x+2)^{\frac{3}{2}}(3(x+2)-10)}{15} = \boxed{\frac{2(x+2)^{\frac{3}{2}}(3x-4)}{15} + C}$

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6) $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}\partial x$ $\quad$ U-Sub $u = \frac{1}{x}$ $\quad \frac{\partial u}{\partial x} = -\frac{1}{x^2}$ $\quad \partial x = -x^2\partial u$

$\int_1^2 \frac{e^u}{\cancel{x^2}} \cdot -\cancel{x^2}\partial u = \int_1^3 -e^u\partial u = -e^{\frac{1}{x}}$

$-e^{\frac{1}{2}} + e^{\frac{1}{1}} = \boxed{e - \sqrt{e}}$

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7) $\int_0^a x\sqrt{x^2+a^2}\,\partial x$ $\quad$ U-Sub $u = x^2 + a^2$ $\quad \frac{\partial u}{\partial x} = 2x + 0$

$\partial x = \frac{1}{2x}\partial u$

$\int_0^a \cancel{x}\sqrt{u}\,\frac{1}{2\cancel{x}}\partial u = \frac{1}{2}\int_0^a \sqrt{u}\,\partial u = \frac{1}{\cancel{2}}\frac{\cancel{2}u^{\frac{3}{2}}}{3} = \frac{u^{\frac{3}{2}}}{3}$

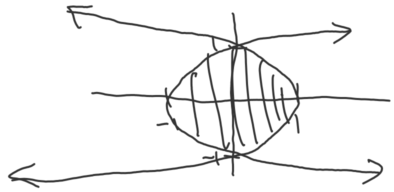
$$= \frac{(x^2+a^2)^{\frac{3}{2}}}{3}\Big|_0^a = \frac{(a^2+a^2)^{\frac{3}{2}}}{3} - \frac{(0+a^2)^{\frac{3}{2}}}{3} = \frac{(2a^2)^{\frac{3}{2}} - (a^2)^{\frac{3}{2}}}{3}$$

$$= \frac{2^{\frac{3}{2}}a^3 - a^3}{3} = \boxed{\frac{a^3\left(2^{\frac{3}{2}}-1\right)}{3}}$$

Chapter 6.1: 2, 6, 5, 8

(2) $x = 1 - y^2$

$x = y^2 - 1$

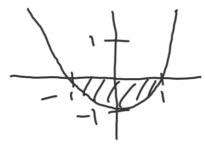


$1 - y^2 = y^2 - 1$

$\frac{2 = 2y^2}{2} = 1 = y^2$

$y = \pm 1$

$$-2\int_{-1}^{1} y^2 - 1 \, dy \implies$$



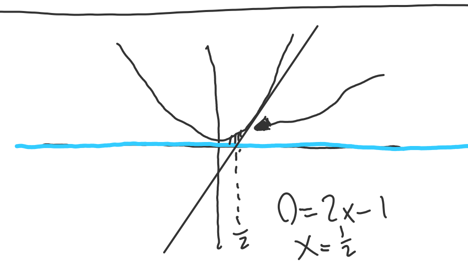
$*(-2) \leftarrow$ opposite b/c negative area shouldn't used abs val...

$$-2\left(\frac{y^3}{3} - y\right)\Big|_{-1}^{1} = -2\left(\left[\frac{1}{3} - \frac{3}{3}\right] - \left[\frac{-1}{3} + \frac{3}{3}\right]\right) = -2\left(-\frac{2}{3} - \frac{2}{3}\right) = -2\left(-\frac{4}{3}\right) = \boxed{\frac{8}{3}}$$

(6) Tangent line of $y = x^2$ @ $(1,1)$

$y - 1 = 2(1)(x-1)$

$\left.\begin{array}{l} y = 2x - 1 \\ y = x^2 \\ y = 0 \end{array}\right\}$



$$\int_0^{\frac{1}{2}} (x^2) + \int_{\frac{1}{2}}^{1} (x^2 - (2x-1))$$

$$\frac{x^3}{3}\Big|_0^{\frac{1}{2}} + \frac{x^3}{3} - (x^2 - x)\Big|_{\frac{1}{2}}^{1}$$

$$\left[\frac{\frac{1}{8}}{3} - 0\right] + \left[\left(\frac{1}{3} - (1 - 1)\right) - \left(\frac{\frac{1}{8}}{3} - \left(\frac{1}{4} - \frac{1}{2}\right)\right)\right]$$

$$= \frac{1}{24} + \left[\frac{1}{3} - \left(\frac{1}{24} - \left(-\frac{1}{4}\right)\right)\right] = \frac{1}{24} + \left[\frac{1}{3} - \frac{7}{24}\right]$$

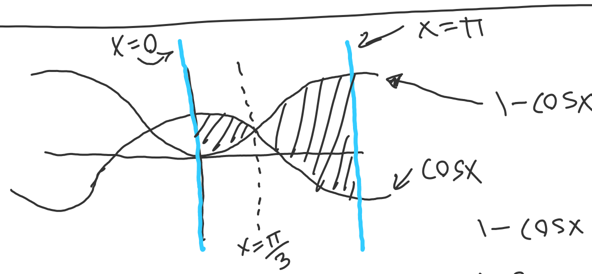
$$= \frac{1}{24} + \frac{1}{24} = \boxed{\frac{1}{12}}$$

(5) $y = \cos x$

$y = 1 - \cos x$

$x = 0$

$x = \pi$



$1 - \cos x = \cos x$

$1 - 2\cos x = 0$

$-2\cos x = -1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$

$$\int_0^{\frac{\pi}{3}} [\cos x - (1 - \cos x)] + \int_{\frac{\pi}{3}}^{\pi} [(1 - \cos x) - \cos x]$$

$\sin x - (x - \sin x)$ $\quad$ $(x - \sin x) - \sin x$

$$2\sin x - x\Big|_0^{\frac{\pi}{3}} + x - 2\sin x\Big|_{\frac{\pi}{3}}^{\pi}$$

$\left[2\sin(\frac{\pi}{3})-\frac{\pi}{3}\right]+\left[(\pi-2\sin(\pi))-(\frac{\pi}{3}-2\sin(\frac{\pi}{3}))\right]$

$(\frac{2\sqrt{3}}{2}-\frac{\pi}{3})+\left[(\pi-0)-(\frac{\pi}{3}-\frac{2\sqrt{3}}{2})\right]$

$\frac{2\sqrt{3}}{2}-\frac{\pi}{3}+\pi-\frac{\pi}{3}+\frac{2\sqrt{3}}{2}=\frac{4\sqrt{3}}{2}+\pi-\frac{2\pi}{3}=\boxed{2\sqrt{3}+\frac{\pi}{3}}$

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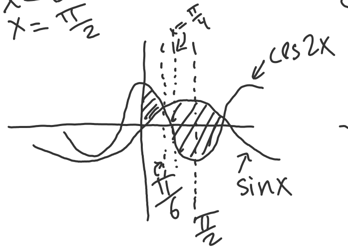
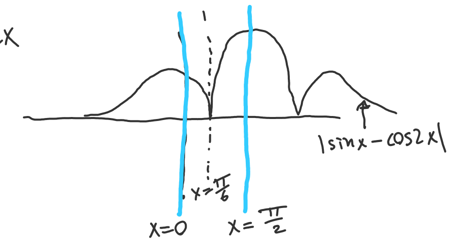
8 $y=|\sin x-\cos 2x| = \begin{cases}-\sin x+\cos 2x & \text{if } \sin x<\cos 2x\\ \sin x-\cos 2x & \text{if } \cos 2x\ge \sin x\end{cases}$

$x=0$
$x=\frac{\pi}{2}$

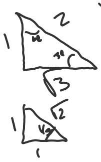




$\sin x=\cos 2x$ $\quad u=\sin x$

$\sin x-\cos 2x=0$
$\sin x-(1-2\sin^2 x)=0$
$\sin x-1+2\sin^2 x=0$
$2\sin^2 x+\sin x-1=0$
$2u^2+u-1=0$
$(2u-1)(u+1)=0$
$u=-1,\ \frac{1}{2}$
~~$\sin x=-1;\ x=\frac{3\pi}{2}>\frac{\pi}{2}$~~
$\sin x=\frac{1}{2}$
$x=\frac{\pi}{6}$

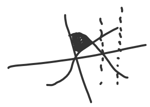
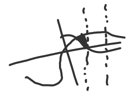
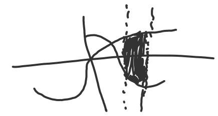
$\int_0^{\frac{\pi}{6}}\cos 2x-\sin x+\int_{\frac{\pi}{6}}^{\frac{\pi}{4}}\sin x-\cos 2x+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\sin x+\cos 2x$



$\int\cos 2x\,dx=\frac{1}{2}\int\cos(u)\,du=\frac{1}{2}\sin u=\frac{\sin 2x}{2}=\frac{2\sin x\cos x}{2}=\sin x\cos x$

$u=2x \quad \frac{du}{dx}=2 \quad dx=\frac{1}{2}du$

$\sin x\cos x+\cos x\Big|_0^{\frac{\pi}{6}}+(-\cos x-\sin x\cos x\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}}+(-\cos x+\sin x\cos x)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$\cos 2x=0$
$1-2\sin^2 x=0$
$\sin^2 x=\frac{1}{2}$
$\sin x=\frac{1}{\sqrt{2}}$
$x=\frac{\pi}{4}$

$\left[((\frac{1}{2})(\frac{\sqrt{3}}{2})+(\frac{\sqrt{3}}{2}))-(1)\right]+\left[(-\frac{1}{\sqrt{2}}-(\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}))-(-\frac{\sqrt{3}}{2}-(\frac{1}{2})(\frac{\sqrt{3}}{2}))\right]+\left[0-(-\frac{1}{\sqrt{2}}+(\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}))\right]$

$\left[\frac{\sqrt{3}}{4}+\frac{\sqrt{3}}{2}-1\right]+\left[(-\frac{1}{2}-\frac{1}{2})-(-\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{4})\right]+\left[\frac{1}{\sqrt{2}}-\frac{1}{2}\right]$

$\frac{3\sqrt{3}}{4}-1+(-1+\frac{3\sqrt{3}}{4})+\frac{1}{\sqrt{2}}-\frac{1}{2}$

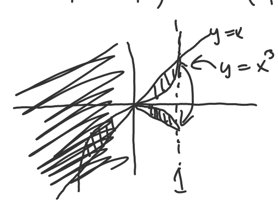
$\frac{6\sqrt{3}}{4}-2+\frac{1}{\sqrt{2}}-\frac{1}{2}=\frac{3\sqrt{3}}{2}-\frac{5}{2}+\frac{1\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}=\boxed{\frac{3\sqrt{3}-5+\sqrt{2}}{2}}$

This ~~approach~~ answer is wrong! I re-tried this question at the bottom... I believe the answer is still wrong but I don't know why?

Chapter 6.2: 2, 3, 5, 7

$A(x)=\pi y^2=\pi(f(x))^2$

2 $y=x^3$ $\quad x\ge 0$ about the x-axis
$y=x$



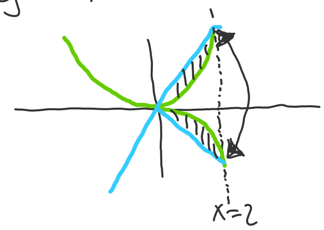
$x^3-x=0$
$x(x^2-1)=0$
$x(x+1)(x-1)=0$
$x=0,$

$A(x)=\pi((x)^2-(x^3)^2)$

$V(x)=\int_0^1\pi(x^2-x^6)\,dx=\pi\left[\frac{x^3}{3}-\frac{x^7}{7}\Big|_0^1\right]=\pi(\frac{1}{3}-\frac{1}{7})$ $\quad \frac{7}{21}-\frac{3}{21}$

$=\boxed{\frac{\pi 4}{21}}$

(3) $x = y^2$, $x = 2y$ about the y-axis $\Big\} =$ $y = x^2$, $y = 2x$ about the x-axis



$x^2 = 2x$
$x^2 - 2x = 0$
$x(x-2) = 0$
$x = 0, 2$

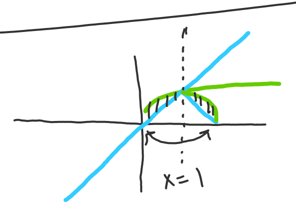
$$A(x) = \int_0^2 \pi\left((2x)^2 - (x^2)^2\right)dx$$

$$\int_0^2 \pi\left(4x^2 - x^4\right)dx$$

$$\pi\left(\frac{4x^3}{3} - \frac{x^5}{5}\Big|_0^2\right) = \pi\left(\frac{4(8)}{3} - \frac{32}{5}\right) = \pi\left(\frac{32}{3} - \frac{32}{5}\right) = \pi\left(\frac{160}{15} - \frac{96}{15}\right) = \pi\left(\frac{64}{15}\right)$$

$$= \boxed{\frac{\pi 64}{15}}$$

(5) $y = \sqrt[4]{x}$, $y = x$ about $x = 1$

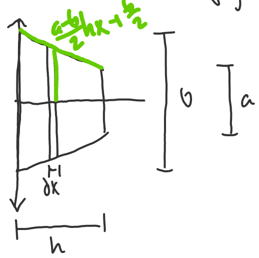


$$\int_0^1 \pi\left((\sqrt[4]{x})^2 - (x)^2\right)dx$$

$$\pi\int_0^1 \left(\sqrt[2]{x} - x^2\right)dx$$

$$\pi\left(\frac{2x^{\frac{3}{2}}}{3} - \frac{x^3}{3}\Big|_0^1\right) = \pi\left(\frac{2(1)^{\frac{3}{2}}}{3} - \frac{1}{3}\right) = \pi\left(\frac{2}{3} - \frac{1}{3}\right) = \boxed{\frac{\pi}{3}}$$

(7) Frustrum of a pyramid (pyramid with square as peak instead of point)



$y = \frac{a-b}{2h}x + b$ $\quad$ $(0, \frac{b}{2}), (h, \frac{a}{2})$

$y = \frac{a-b}{2h}x + \frac{b}{2}$ $\quad$ $y - \frac{b}{2} = m(x - 0)$

$\frac{a}{2} - \frac{b}{2} = m(h)$

$m = \frac{a-b}{2h}$

$$V = \int_0^h \left(2\left(\frac{a-b}{2h}x + \frac{b}{2}\right)\right)^2 dx = \int_0^h \left(\frac{a-b}{h}x + b\right)^2 dx$$

$u = \frac{a-b}{h}x + b$
$du = \frac{a-b}{h}dx$
$dx = \frac{h}{a-b}du$

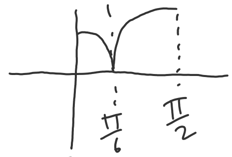
$$= \int_0^h u^2\, du \cdot \frac{h}{a-b} = \frac{h}{a-b}\cdot\left(\frac{1}{3}u^3\Big|_0^h\right) = \frac{h}{a-b}\cdot\left(\frac{1}{3}\left(\frac{a-b}{h}x + b\right)^3\Big|_0^h\right)$$

$$= \frac{h}{a-b}\cdot\left(\frac{1}{3}(a-b+b)^3\right) - \left(\frac{1}{3}(b^3)\right) = \frac{h}{a-b}\cdot\left(\frac{a^3}{3} - \frac{b^3}{3}\right) = \frac{h}{a-b}\cdot\frac{a^3-b^3}{3}$$

$$= \frac{h(a^3-b^3)}{3(a-b)} = \frac{h\cancel{(a-b)}(a^2+ab+b^2)}{3\cancel{(a-b)}} = \boxed{\frac{h(a^2+ab+b^2)}{3}}$$

Re-Trying 6.1: 8

(8) $\int_0^{\frac{\pi}{2}} |\sin x - \cos 2x| \, dx$



$\sin x = \cos 2x$

$x = \frac{\pi}{6}$

$$\int_0^{\frac{\pi}{6}} (\cos 2x - \sin x)\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin x - \cos 2x)\,dx$$

$$\left(\frac{\sin 2x}{2} + \cos x \Big|_0^{\frac{\pi}{6}}\right) + \left(-\cos x - \frac{\sin 2x}{2} \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}\right) = \left(\left(\frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2}\right) - (0-1)\right) + \left((0-0) - \left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4}\right)\right)$$

$$= \left(\frac{\sqrt{3} + 2\sqrt{3}}{4} + 1\right) + \left(0 + \frac{3\sqrt{3}}{4}\right) = \boxed{\frac{3\sqrt{3}}{2} + 1}$$