Quiz 8

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Section: 412

Started @ 1:00 PM

① Prove that the P-series with $p=4$ given below converges using the integral test.

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90} \approx 1.0823$$

a) $f(x) = \frac{1}{x^4}$ if $\int_1^\infty \frac{1}{x^4}dx$ converges, then $\sum_{n=1}^{\infty} \frac{1}{n^4}$ converges

$$\int_1^\infty \frac{1}{x^4}dx = \lim_{t\to\infty}\left(\int_1^t \frac{1}{x^4}dx\right) = \lim_{t\to\infty}\left(\left[-\frac{1}{3x^3}\right]_1^t\right) = \lim_{t\to\infty}\left(\left(\overset{\nearrow 0}{\cancel{-\frac{1}{3t^3}}}\right) - \left(-\frac{1}{3}\right)\right) = \boxed{\frac{1}{3}}$$

<u>Integral test</u>: $f(x) = \frac{1}{x^4}$

$f(x) = \frac{1}{x^4} \geq 0$ on $[1, \infty)$ and $a_n = f(n)$

$f'(x) = -\frac{4}{x^5}$ $\therefore$ $f(x)$ decreasing on $[1, \infty)$

Because two above assumptions are true, and $\int_1^\infty f(x)dx$ converges to $\frac{1}{3}$ We can conclude that $\sum_{n=1}^{\infty} \frac{1}{n^4}$ also <u>converges</u>, but NOT to $\frac{1}{3}$

b) Approx. the sum of the series using the estimate for the integral test and the first 3 terms. (i.e. give interval)

$$\begin{array}{r} .0625 \\ +.01235 \\ \hline .07485 \end{array}$$

First 3 terms $= \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} = 1 + .0625 + .01235 = 1.07485 = S_3$

$S_3 \leq S \leq S_3 + \int_3^\infty \frac{1}{x^4}dx$

$$\int_3^\infty \frac{1}{x^4} = \lim_{t\to\infty}\left(-\frac{1}{3x^3}\Big|_3^t\right) = \lim_{t\to\infty}\left(\overset{\nearrow 0}{\cancel{\frac{-1}{3t^3}}} - \left(-\frac{1}{3(3)^3}\right)\right) = .01235$$

$$\begin{array}{r} 1.07485 \\ +.01235 \\ \hline 1.08720 \end{array}$$

$$\boxed{1.07485 \leq S \leq 1.08720}$$