TNM087 – Image Processing and Analysis
Lab 3 – Filtering in the frequency domain

TASK 3 - MATLAB code for removing sinusoidal noise

Write a MATLAB code, $RemoveSinusoidalNoise.m$, that removes the most dominant sinusoidal noise by applying a Butterworth Notch Reject filter in the frequency domain. As the input you have the sinusoidal noise corrupted image and the bandreject width of the bandreject filter, $D0$, and the output is the noise "free" image.
As discussed in the book (and lecture notes for Chapter 5), the sinusoidal noise in the spatial domain results in a pair of peaks in the frequency domain. In this task, you are supposed to remove **only** the most dominant sinusoidal noise, even if there are more than one sinusoidal noise corrupting the image.
As discussed in the course, the most dominant peak in the spectrum of an image is commonly the dc-term. Therefore, if you find the position of the maximum of the spectrum, you most probably find the position of the zero-frequency (which is at the center of the spectrum). The second next maximum would either be at a position very close to the center of the spectrum or at the frequency pair of the most dominant sinusoidal noise. **Notice:** It is very important that you read the book **or** the lecture notes for Chapter 5 (**FÖ** 7 **in the course**) on **pages 25-32** before you start doing this task.

Therefore, in order to find the position of the most dominant sinusoidal noise, we propose the following strategy, (but you can of course implement your own idea).

- Find the 2D Fourier transform of the input image and shift the zero-frequency to the center, call it for example F.
- Find the **spectrum/magnitude** of F, call it for example $F2$.
- Set the dc-term and a neighborhood around it in $F2$ to, for example, 0, in order to prevent them being found as the maximum. Use, for example, a 5×5 neighborhood. **Notice:** the center of a $P \times Q$ image is at $(\frac{P}{2}, \frac{Q}{2})$. (**Notice** that since in MATLAB the positions start from 1 and not 0, the center will be at position $(\frac{P}{2} + 1, \frac{Q}{2} + 1)$).
- If you now find the maximum of $F2$, and locate its positions, you will find a pair of positions corresponding to the most dominant frequency peak pair. The MATLAB function max could be used to find the maximum of an image and its position. It is also possible to use the function max and the MATLAB function $find$ to locate the maximum. **Notice:** Since there is a pair of positions corresponding to the most dominant frequency peak pair, you probably find the position of two maxima, so make sure to only use one of them in the following steps.

Now when you have localized the position(s) of the most dominant noise, you can construct the appropriate Butterworth Notch Reject filter transfer function, H (See again Chapter 5 (**FÖ** 7 **in the course**) on **pages 25-32** for help)
Apply now the filter transfer function, H, on the Fourier transform of the noisy image, F, in the frequency domain to eliminate the frequency peak pair corresponding to the most

dominant sinusoidal noise. Taking the $ifftshift$, followed by the inverse Fourier transform, followed by the $real$ part of the image, give you the noise "free" image.

Notice: Your code should work even if the number of rows and/or number of columns of the input image are not **even**.

Test your code on the following five images. In all your tests you can have the order of the Butterworth filter equal to 2, i.e. $n = 2$.

astronaut-interference.tif: Your code should remove the noise completely by using $D0 = 1$ as the second argument in your code.

Einstein_odd_sinus.tif: Your code should remove the noise completely by using $D0 = 1$ as the second argument in your code. Notice that the number of rows and columns in this image are odd and your code is supposed to work properly.

Einstein_sinus_1.tif: Try your code using this test image. As you will notice, you are not able to remove the sinusoidal noise completely by using $D0 = 1$. Increase $D0$ until you are satisfied with your result. Zoom in your result to make sure that the sinusoidal noises are almost gone. What is the smallest $D0$ that removes the noise almost completely? (**answer that in your MATLAB code**).

Einstein_sinus_2.tif: Try your code using this test image. This image is corrupted by **two** sinusoidal noises and your code should be able to remove the most dominant one by $D0 = 1$. Compare the input image with your result (zoom in to see the details). Increasing $D0$ will not remove the other sinusoidal noise, and your code is not supposed to handle that either.
If you want (**not mandatory**) you can improve your code to remove even the second most dominant sinusoidal noise. **However**, the second dominant noise can easily be removed by your code if you run it twice. This means that, after your first call of the function, you apply your code on the result from the first call, then the second dominant noise is also gone. Try this!

car-moire-pattern.tif: Try your code using this test image. Look at the lecture notes for Chapter 5 (**FÖ** $\boxed{7}$ **in the course**) on **page 30**. On that page, you can see that there are four pairs of dominant peaks and your code only eliminates one pair. Your result is somewhat better than the input image, but it is not completely noise free because there are still three pairs to be eliminated.
If you want (**not mandatory**) you can improve your code to remove even the other dominant peaks and thereby removing the moiré pattern from the input image.