BAYESIAN MINIMAX AND ORACLE OPTIMALITY IN AN INVERSE GAUSSIAN SEQUENCE SPACE MODEL

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Joint work with Jan Johannes

Abstract

As Bayesian methods for non-parametric models gather more and more interest, the question of uncertainty quantification and optimality for those methods arose. Admitting existence of a true data-generating distribution, we will recall definitions of contraction rate and uniform contraction rate of a posterior distribution and their frequentist counterparts : convergence rate and uniform convergence rate of an estimator. Given those definitions, from a frequentist point of view, typically, one considers oracle optimal convergence rates (optimality over a class of estimators) and minimax optimal rates (optimal rate over a class of true distributions). Bayesian formulations of those notions still need to be inquired into and, for the moment, contraction rates of Bayesian methods are generally compared to frequentist optimal rates to determine if they are satisfactory. Hence, considering an inverse Gaussian sequence space model, we give a purely Bayesian formulation of oracle optimality and tracks for minimax optimality.

Considering the hierarchical prior in Johannes et al. [2016], we generate a family of fully data-driven prior distributions, meaning that this method does not depend either on the true data-generating distribution or on a class it would belong to. This family is indexed by a so-called "iteration parameter", as, given an element of the family, the subsequent element is obtained by conditioning the posterior by the same data. Interestingly, increasing the value of the iteration parameter gives in some sense more weight to the information contained in the observations than in the hierarchical prior. In particular, for a fixed noise level letting the iteration parameter tend to infinity, the associated posterior distribution shrinks to a point measure. The limite distribution is degenerated on the value of a projection estimator with fully-data driven choice of the dimension parameter using a model selection approach as in Massart [2003] where a penalized contrast criterium is minimized.

In Johannes et al. [2016], the contraction rate of the first element of the family was compared to the frequentist optimal convergence rate. We derive here contraction rate for any element of the family, including the limit case and compare those with our Bayesian oracle optimal rate.

References

J. Johannes, A. Simoni, and R. Schenk. Adaptive bayesian estimation in indirect gaussian sequence space model. 2016.

P. Massart. Concentration inequalities and model selection. In Springer, editor, *Ecole d'Été de Probabilités de Saint-Flour XXXIII*, volume 1869, 2003.