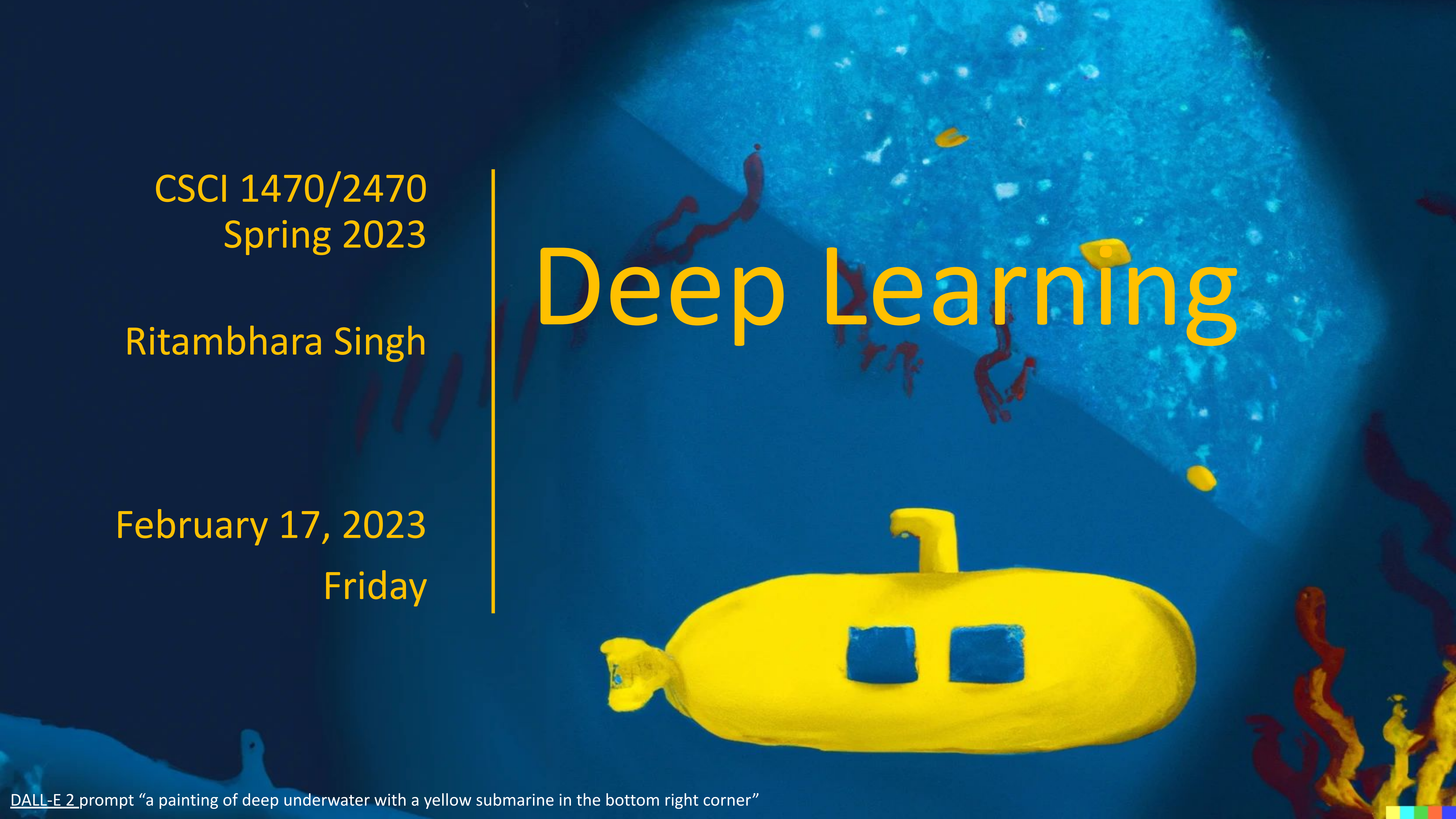
Data & Donuts

Friday, February 17
3:00-4:00 pm
164 Angell Street, 3rd floor

Join us this week with **Peter Hull,** *Professor of Economics*

Organized by the DSI DUG.
Gourmet donuts, fruit, and coffee will be served.



CSCI 1470/2470
Spring 2023

Ritambhara Singh

February 17, 2023
Friday

Deep Learning

DALL-E 2 prompt “a painting of deep underwater with a yellow submarine in the bottom right corner”

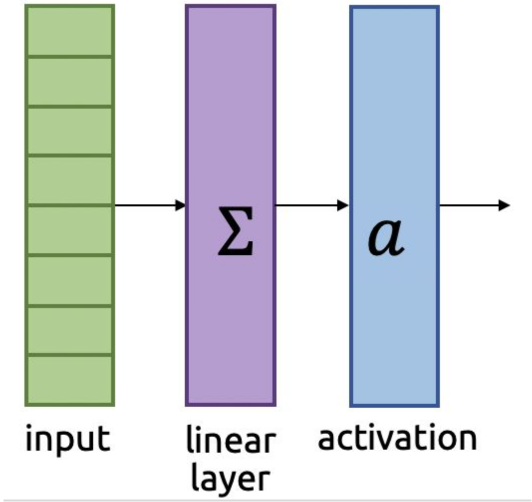
Recap

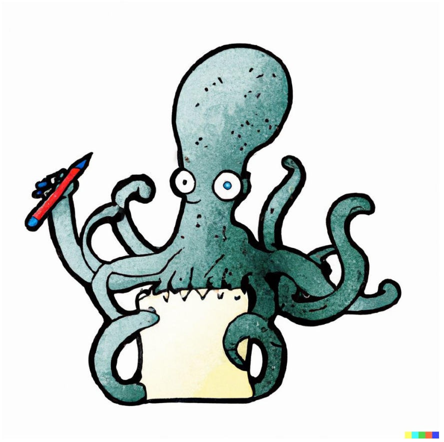
Stacking multiple layers

More layers more complicated function

Linear layers are not sufficient!

Need non-linearity



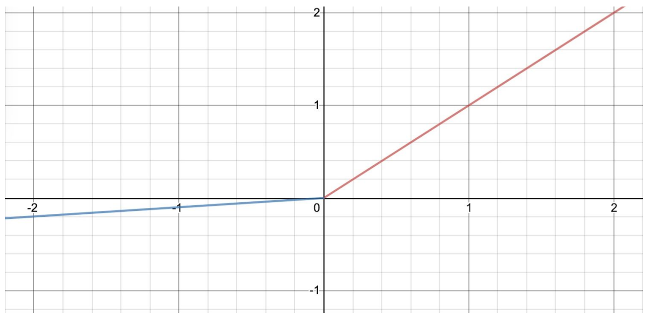


Activation functions

Exploding gradients

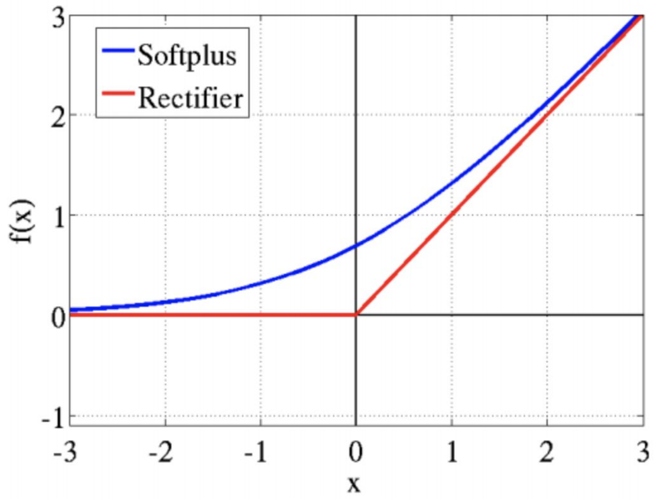
Vanishing gradients

ReLU, Leaky ReLU



Recap: Reasons to use other activation functions

- Bounding network outputs to a particular range
  - Tanh: [-1, 1]
  - Sigmoid: [0,1]
  - Softplus: [0, ∞]



- Example: Predicting a person's age from other biological features
  - Age is a strictly positive quantity
  - We can help our network learn by restricting it to output only positive numbers
  - Use a Softplus activation on the output

Today's goal – continue to learn about multi-layer networks and learn about convolution

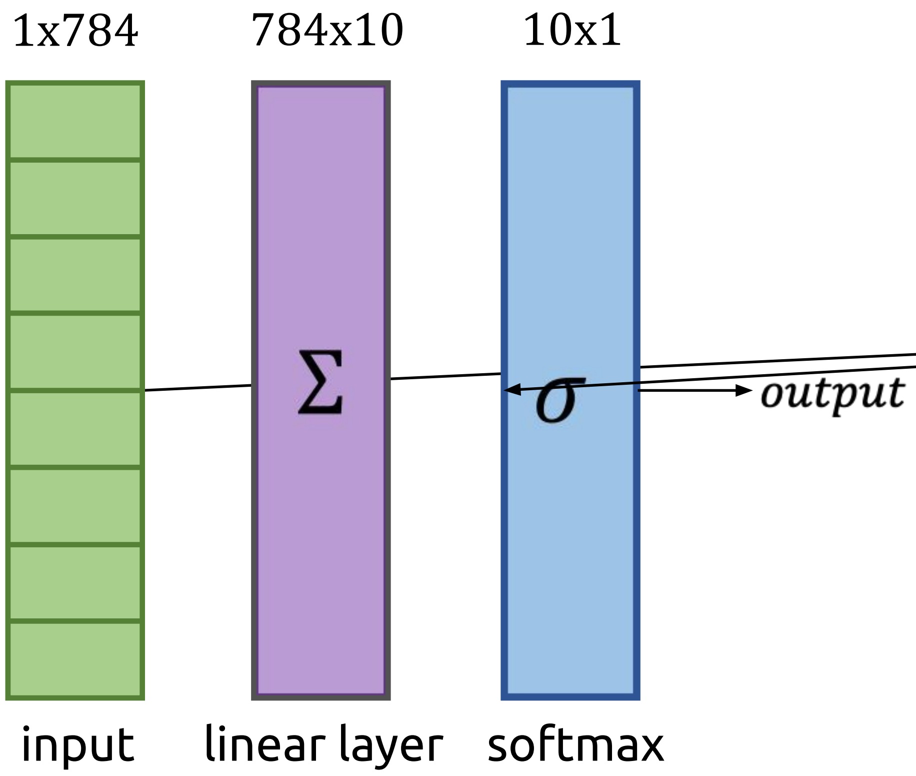
(1) What are hidden layers and hyperparameters?

(2) Universal approximate theorem – what a one-hidden layer network can learn?

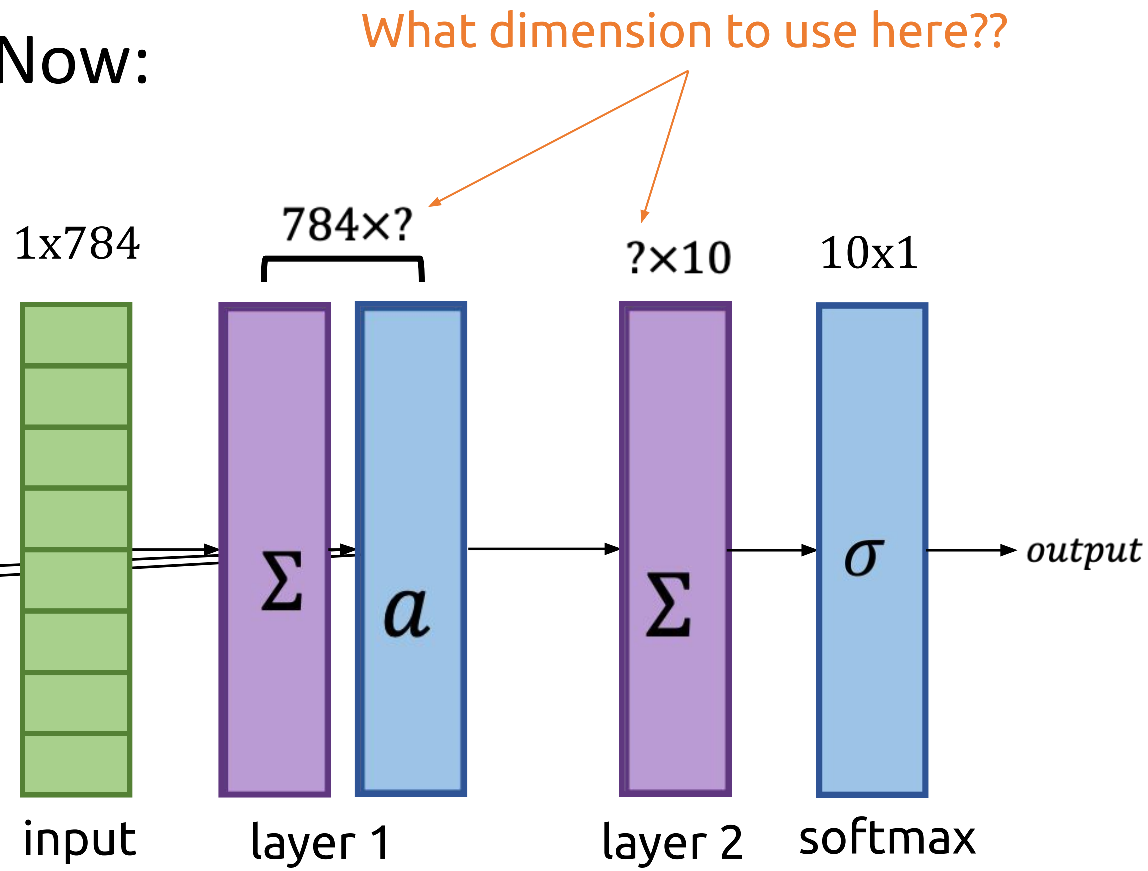
(3) Intro to CNNs – Convolution

Recap: Consequences of adding activation layers

- Previously:

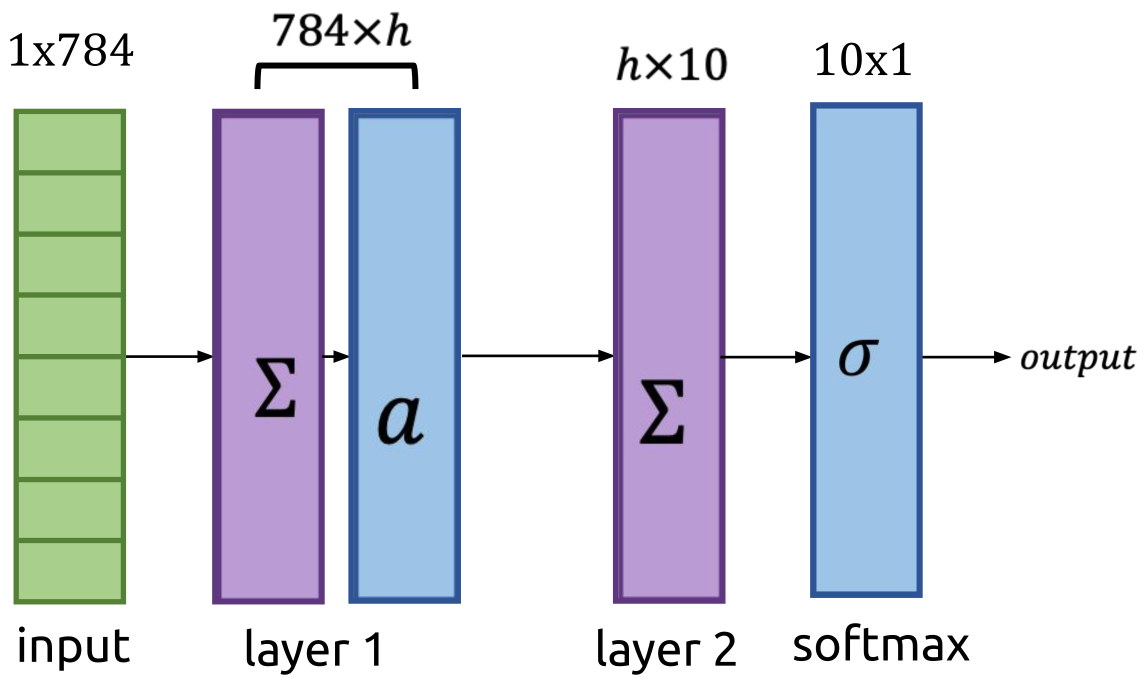


- Now:



"Hidden Layers"

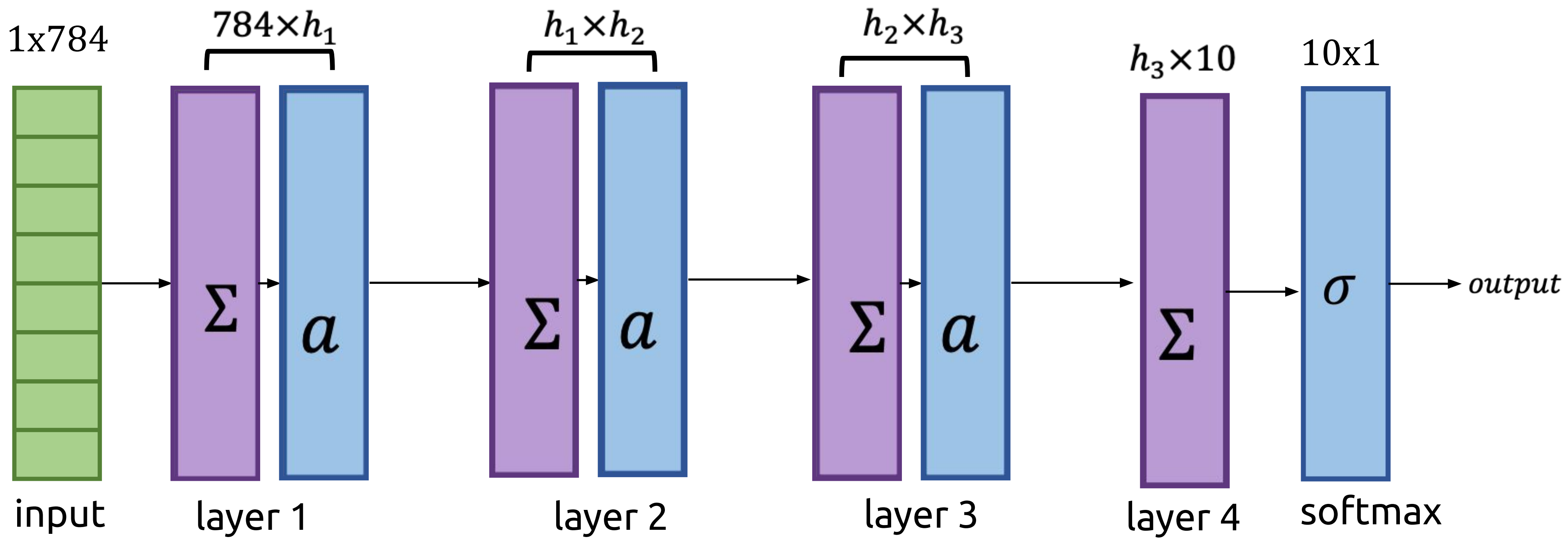
- The output of a function that doesn't feed into the output layer (like softmax) is called a ***hidden layer***
- Have to set the size h of these hidden layers
- More linear units → more hidden layer sizes



Hyperparameters

- Hidden layer sizes are a ***hyperparameter*** — configuration external to model, value usually set before training begins
  - Number of epochs, batch size, etc.
  - Contrast this with a learnable ***parameter***, we keep talking about
- Rule of thumb
  - Start out making hidden layers the same size as the input
  - Then, tweak it to see the effect
- There are more principled (and time-consuming) ways to set them
  - Grid search, random search, Bayesian optimization...
  - See here for an overview and more references

What a multi-layer neural network could look like?



What functions can a one-hidden-layer neural net learn?

Universal Approximation Theorem [Cybenko ‘89]

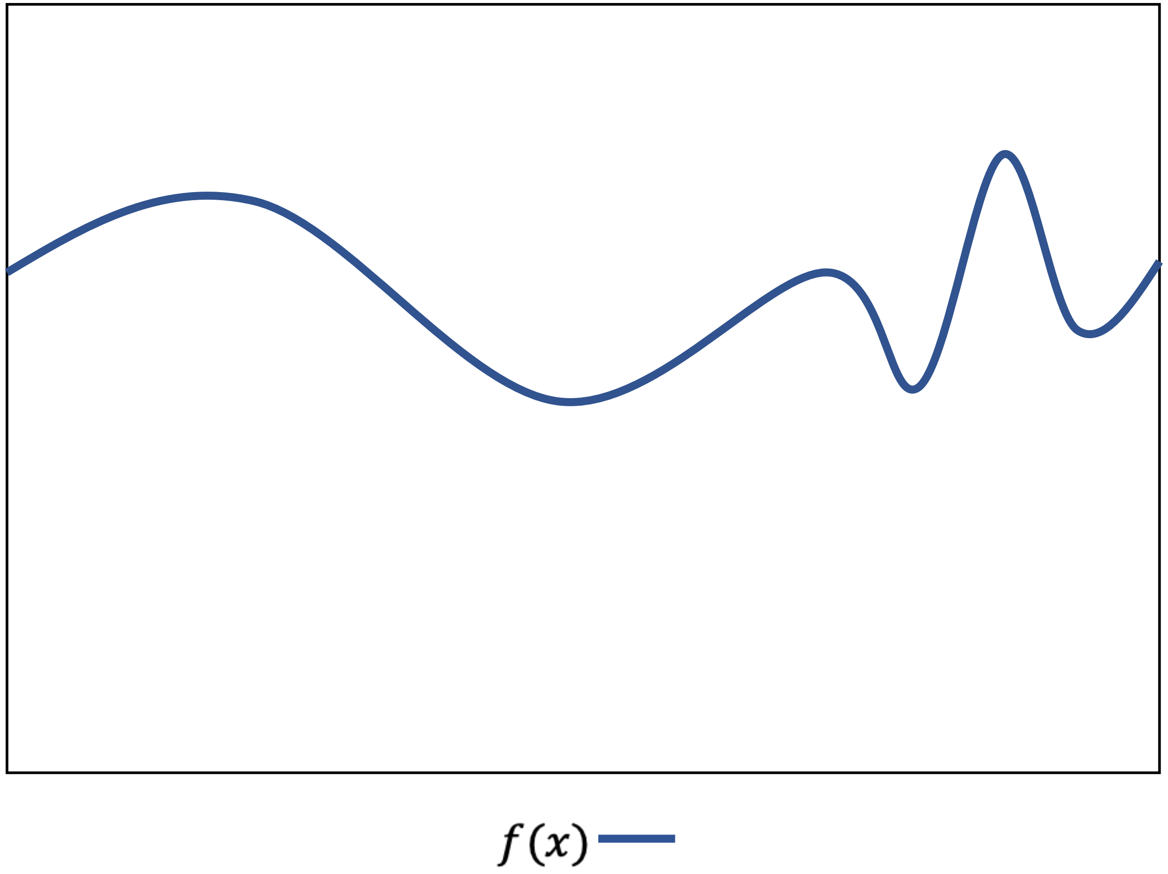
- Remarkably, a one-hidden-layer network can actually represent ***any*** function (under the following assumptions):
  - Function is continuous
  - We are modeling the function over a closed, bounded subset of $\mathbb{R}^n$
  - Activation function is sigmoidal (i.e. bounded and monotonic)

- The proof of this theorem is an existence proof
  - i.e. it tells us that a network exists which can approximate any function, not how to actually learn it

A “Proof By Picture”

Universal Approximation Theorem "Proof"

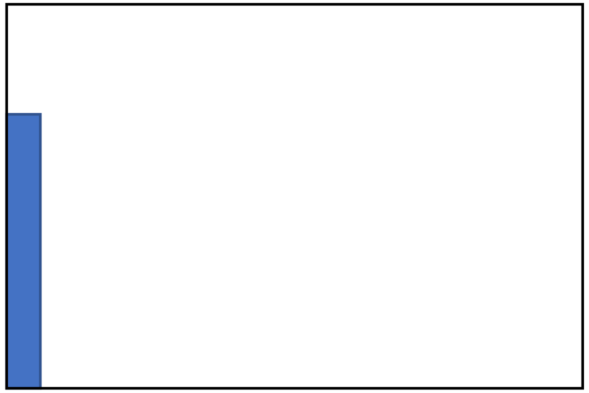
- Start with a complex one dimensional function that relates some input x to some output y
- We don't know what the function that relates x and y is



https://hackernoon.com/illustrative-proof-of-universal-approximation-theorem-5845c02822f6

Universal Approximation Theorem “Proof”

- We can build up this function using simpler functions, i.e. box functions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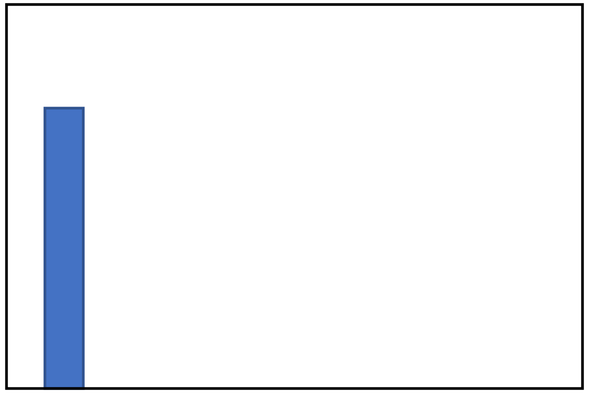
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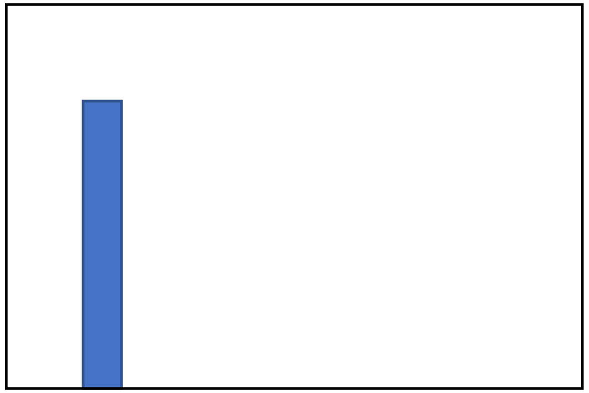
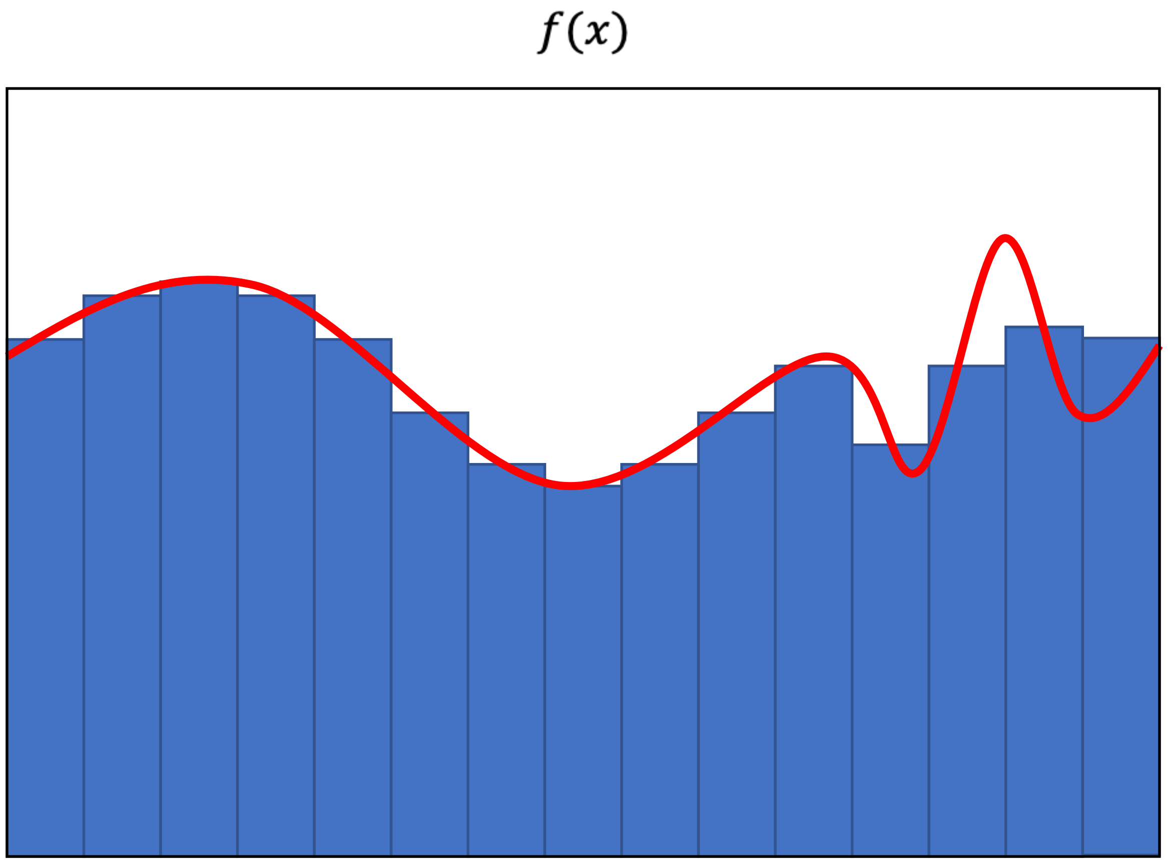
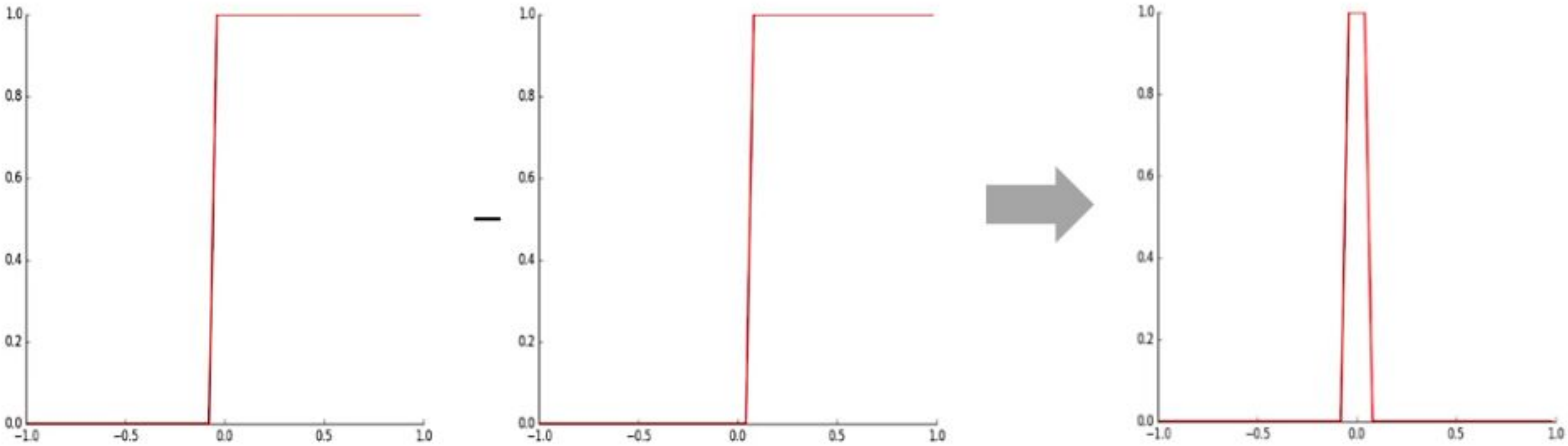


Universal Approximation Theorem "Proof"



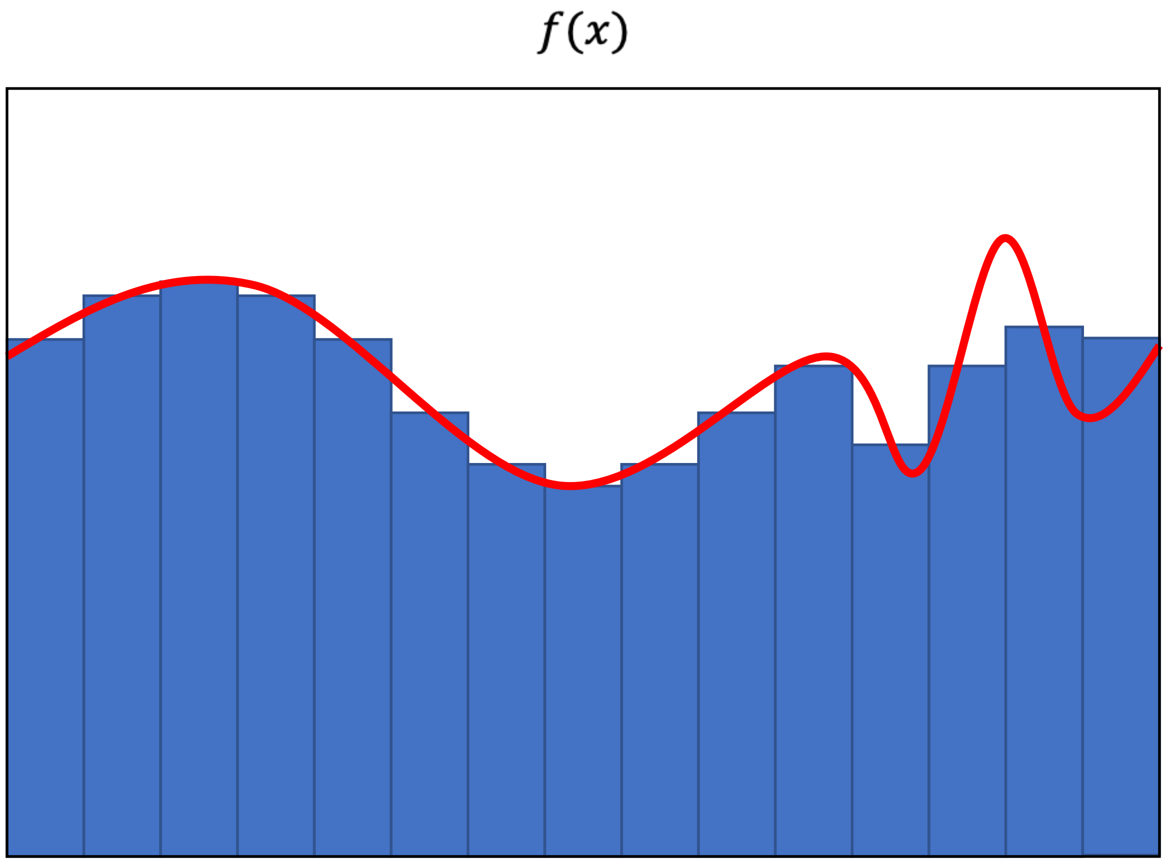
How does this relate to activation functions?

- We can subtract two sigmoids to create these box functions



Universal Approximation Theorem "Proof"

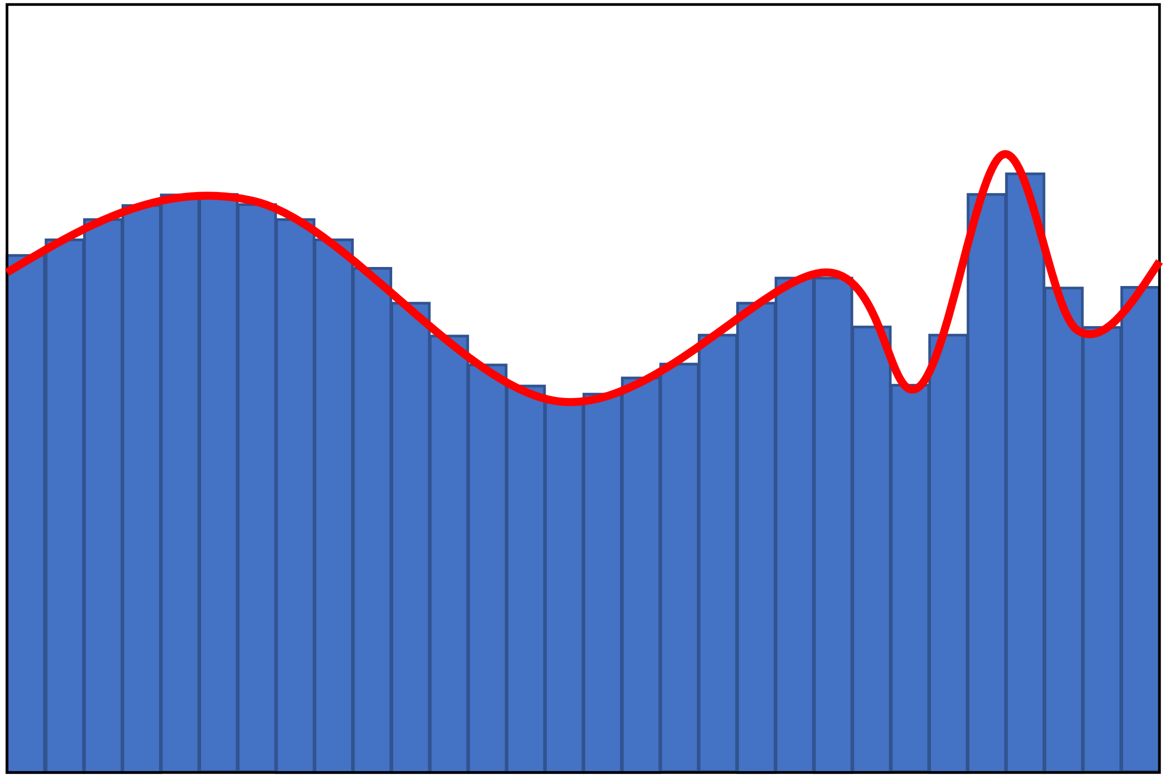
- Summing up these simpler functions can do a pretty good job of approximating the actual function



Universal Approximation Theorem “Proof”

- Using more functions lets us model a complex function more accurately
    - Up to an arbitrary degree of accuracy, if we want

$f(x)$



Any questions?

Universal Approximation Theorem “Proof”

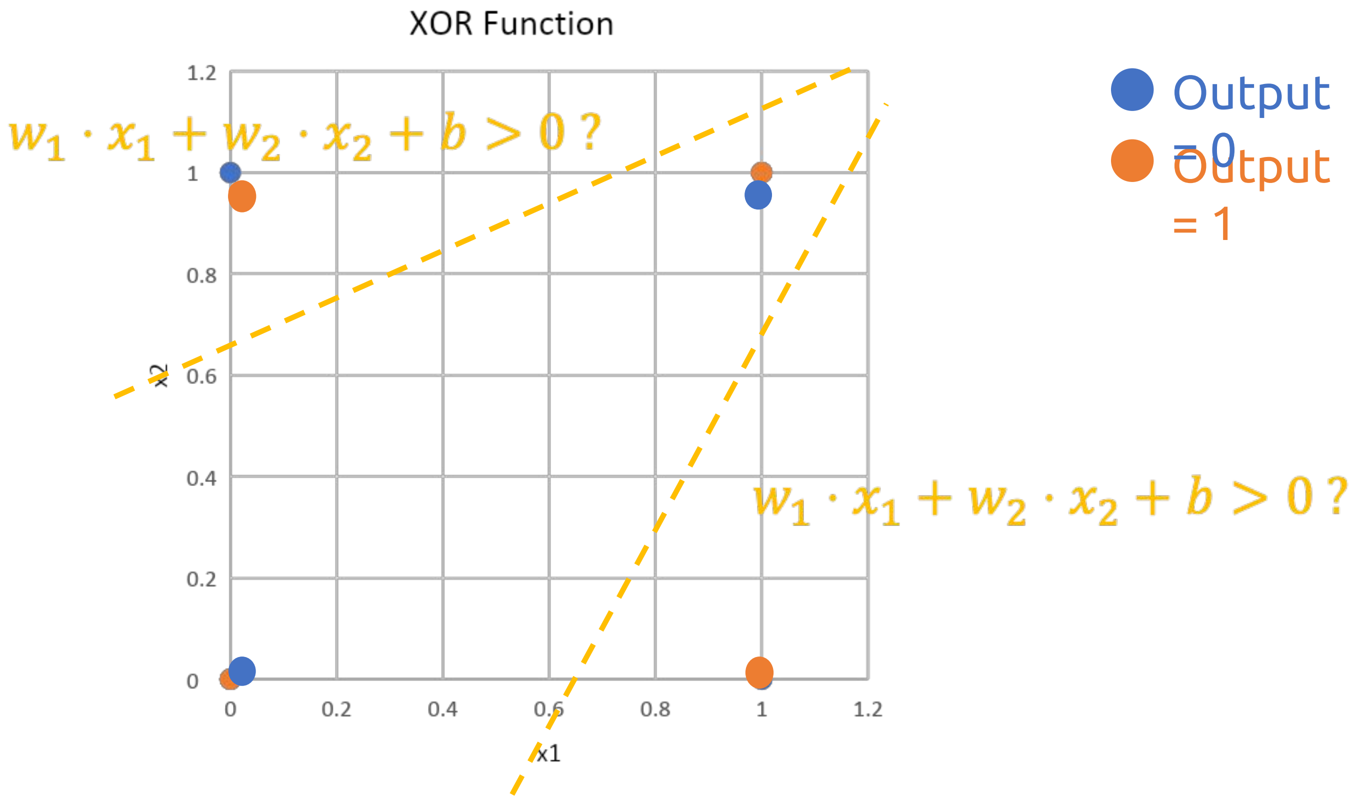
- ***Very*** inefficient way to approximate
  - Need ***lots*** of box functions □ ***lots*** of sigmoids □ very large hidden layer

- Real networks trained with gradient descent can’t even learn these kinds of approximations
  - They **find smooth approximations**, require more hidden layers to get this same level of complexity.

- Nevertheless, the theorem is often cited to back up claims that a sufficiently complex neural net “can learn any function”

Do you remember what function a perceptron could not learn?

Can a multi-layer network learn XOR?



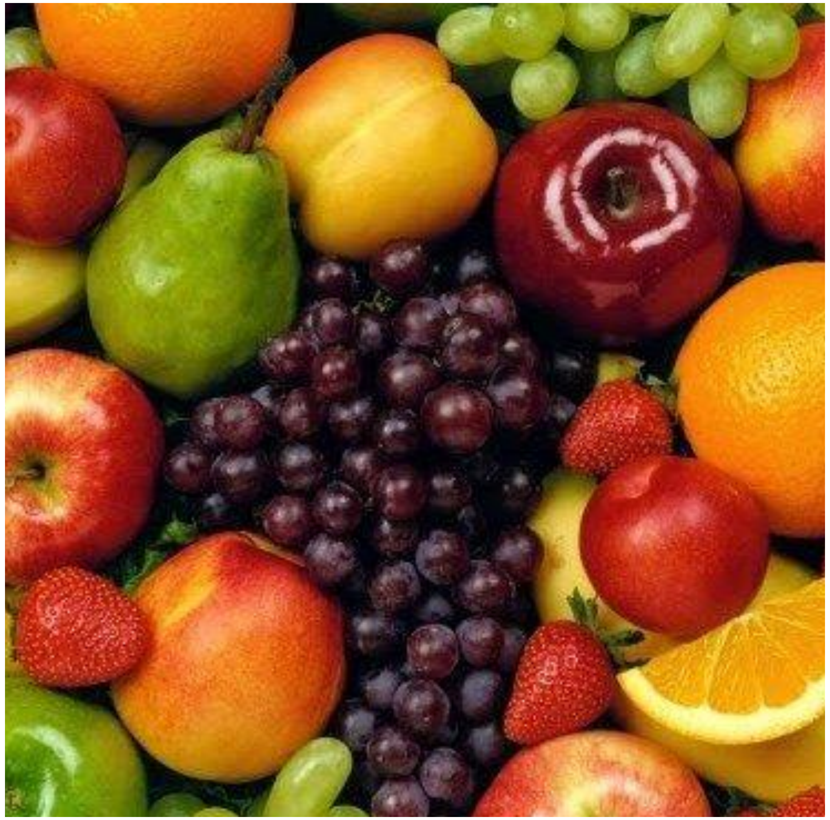
Let’s find out

Google Tensorflow Playground

What kind of datasets CNNs are popularly applied to?

Convolution and CNNs

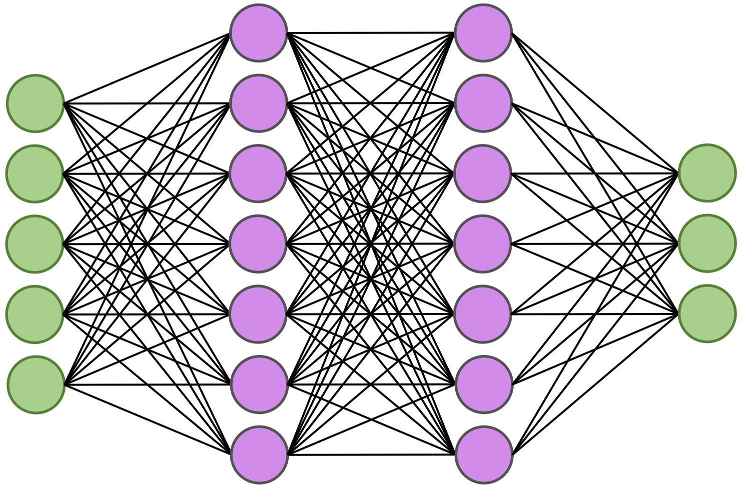




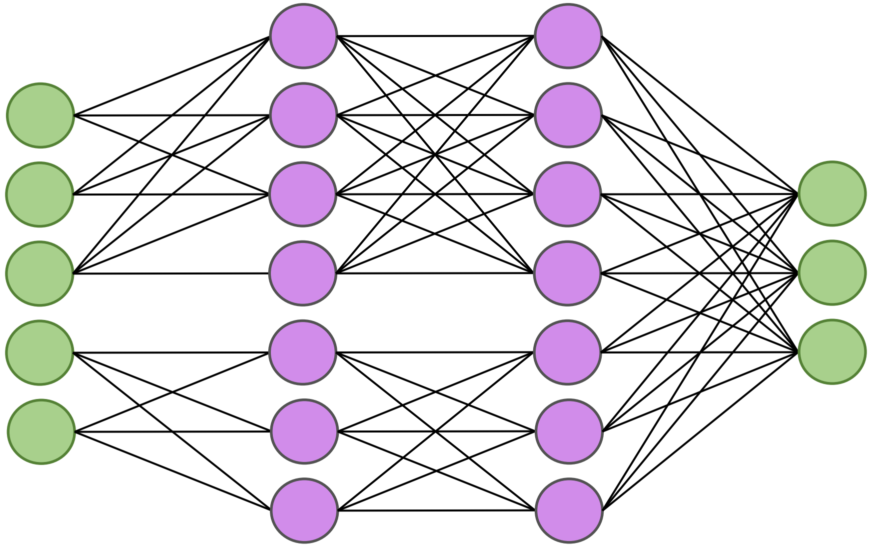
Images!

Does a network have to be fully connected?

Fully Connected



Partially Connected?



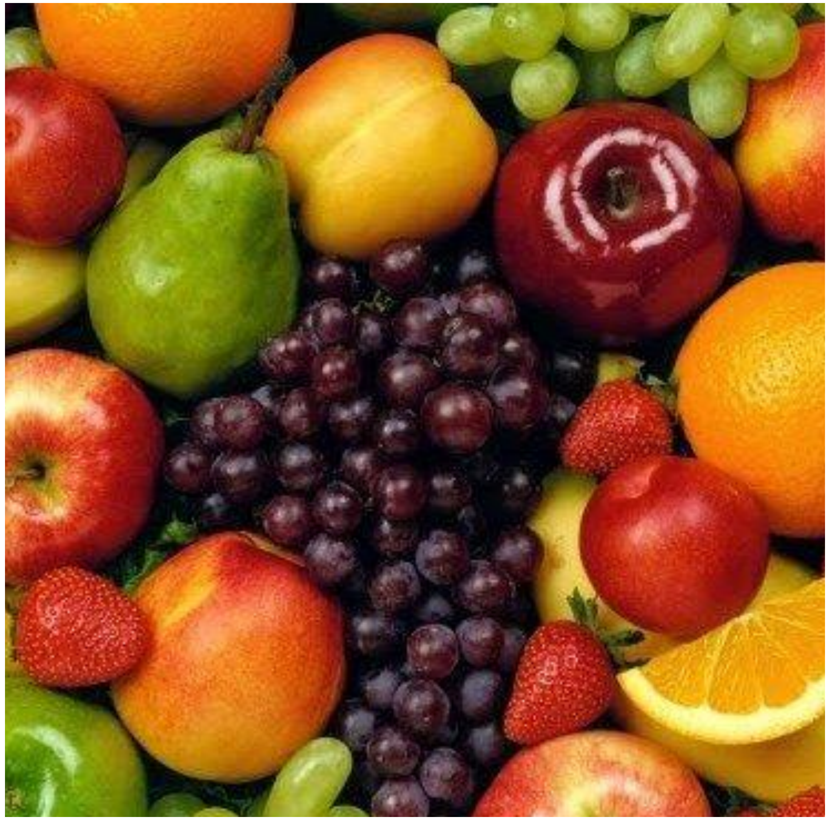
Why would you ever want to do this?

Partially Connected Networks?

- Fewer connections == Worse results? ...right?

- Advantages of Partial Connections
  - Fewer connections □ fewer weights to learn
    - Faster training; more compact models; better generalization performance
  - Can design connectivity pattern that exploits knowledge of the data (like connecting patterns in features)

What's a data type where we can do this?

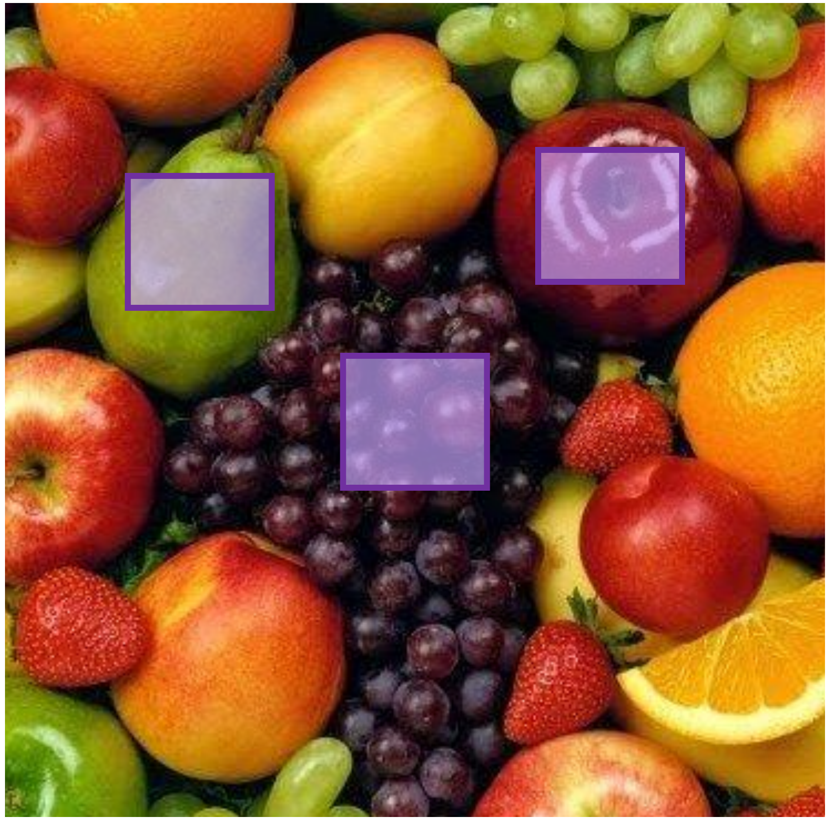
  

Images!

When partially connected networks are useful

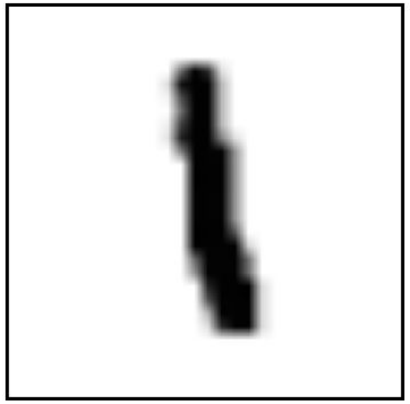
- **Observation:** Nearby pixels are more likely to be related

- **Assumption:** It is okay to only connect the nearby pixels



Limitations of Full Connections for MNIST

Suppose we’ve got a well-trained MNIST classifier…



$$\simeq \begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&.6&.8&0&0&0&0&0&0\\
0&0&0&0&0&0&.7&1&0&0&0&0&0&0\\
0&0&0&0&0&0&.7&1&0&0&0&0&0&0\\
0&0&0&0&0&0&.5&1&.4&0&0&0&0&0\\
0&0&0&0&0&0&0&1&.4&0&0&0&0&0\\
0&0&0&0&0&0&0&1&.4&0&0&0&0&0\\
0&0&0&0&0&0&0&1&.7&0&0&0&0&0\\
0&0&0&0&0&0&0&1&1&0&0&0&0&0\\
0&0&0&0&0&0&0&.9&1&.1&0&0&0&0\\
0&0&0&0&0&0&0&.3&1&.1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}$$

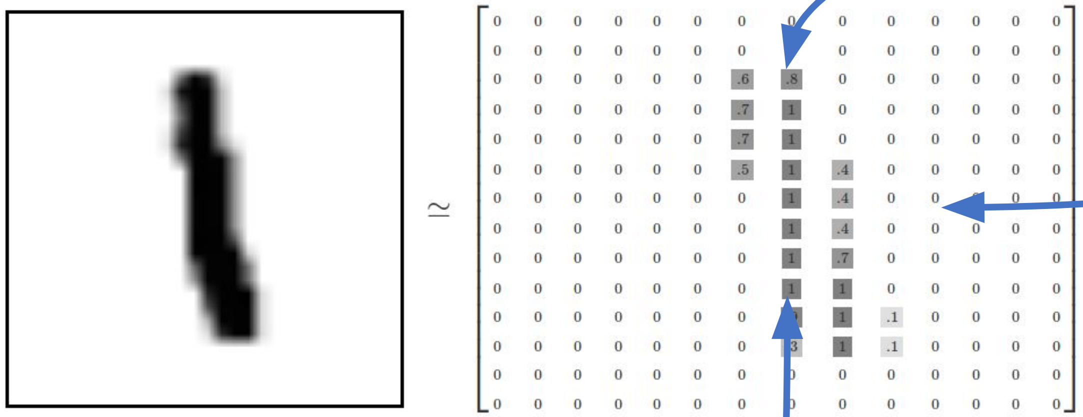
#1 encoded as

Limitations of Full Connections for MNIST

Suppose we've got a well-trained MNIST classifier…

this pixel gets weight 0.6

this pixel gets weight 0.1

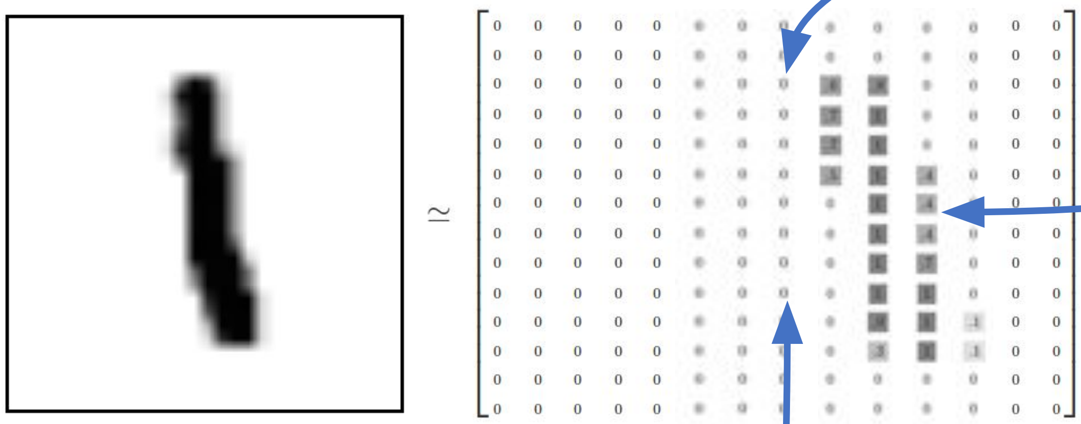


#1 encoded as □

this pixel gets weight 0.9

Limitations of Full Connections for MNIST

If we shift the digit to the right, then a different set of weights becomes relevant □ network might have trouble classifying this as a 1…



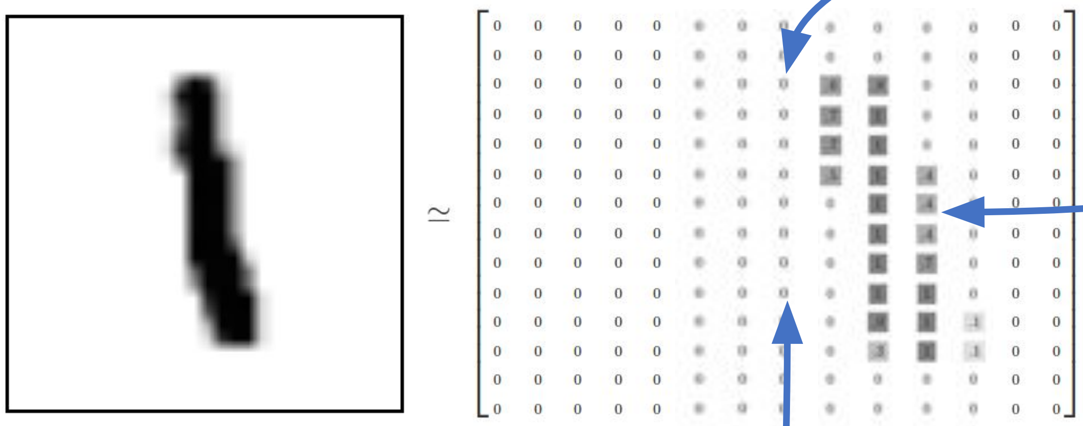
#1 encoded as □

Can you tell this is a 1?

This would ***not*** be a problem for the human visual system

Our eyes don't look at absolute intensity values…

this pixel has a low intensity



this pixel has a high intensity

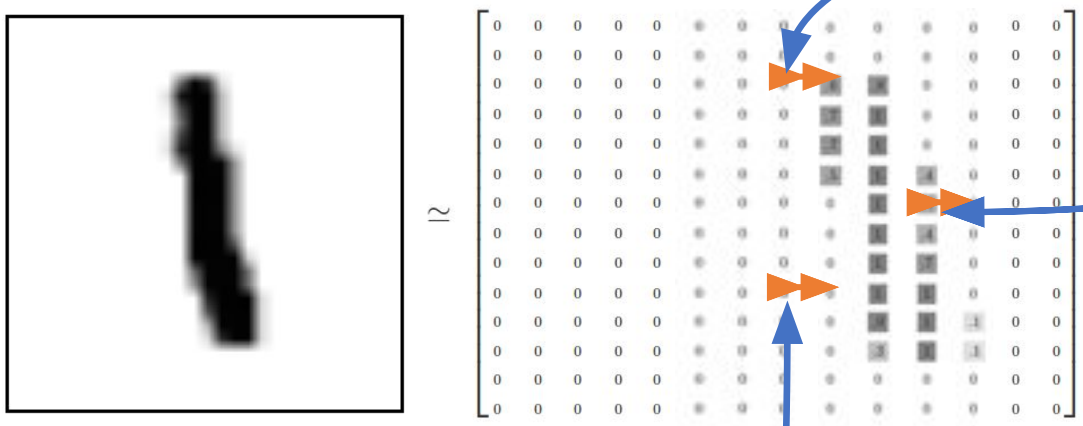
#1 encoded as □

this pixel has a low intensity

This would ***not*** be a problem for the human visual system

...but rather ***local differences*** in intensities

this intensity difference is large



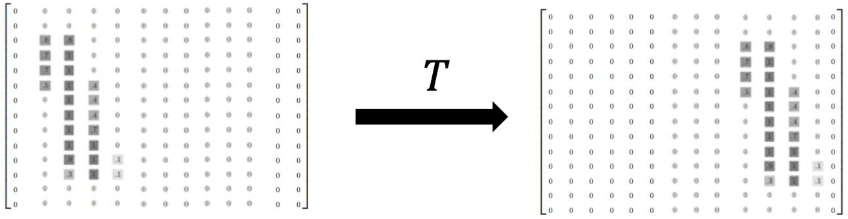
this intensity difference is large

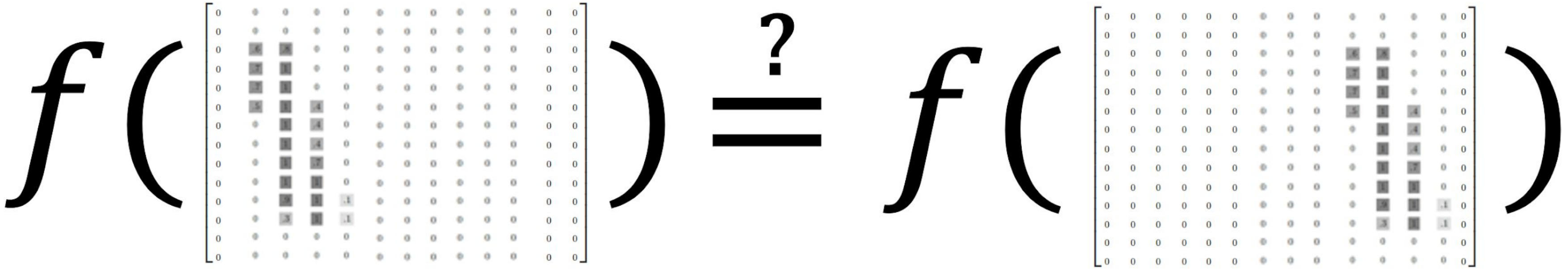
#1 encoded as □

this intensity difference is zero

Translational Invariance

- To make a neural net f robust in this same way, it should ideally satisfy ***translational invariance***: $f\big(T(x)\big) = f(x)$, where
  - x is the input image
  - T is a translation (i.e. a horizonal and/or vertical shift)

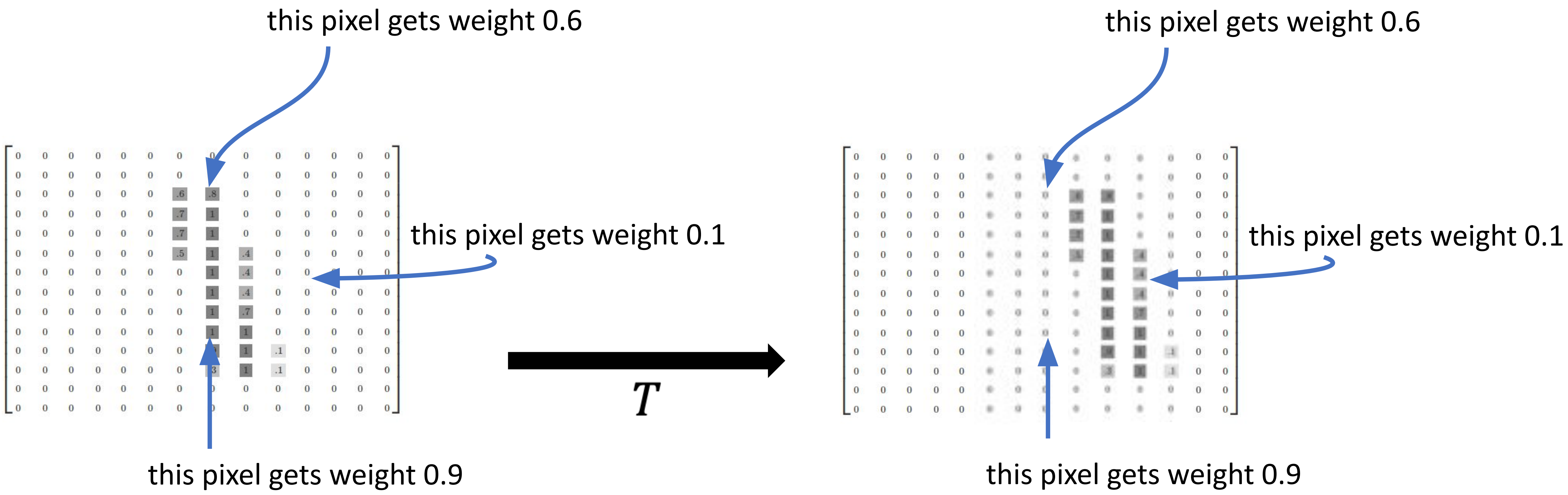




Fully Connected Nets are ***not*** Translationally Invariant

How to make the network translationally invariant?

Focus on local differences/patterns

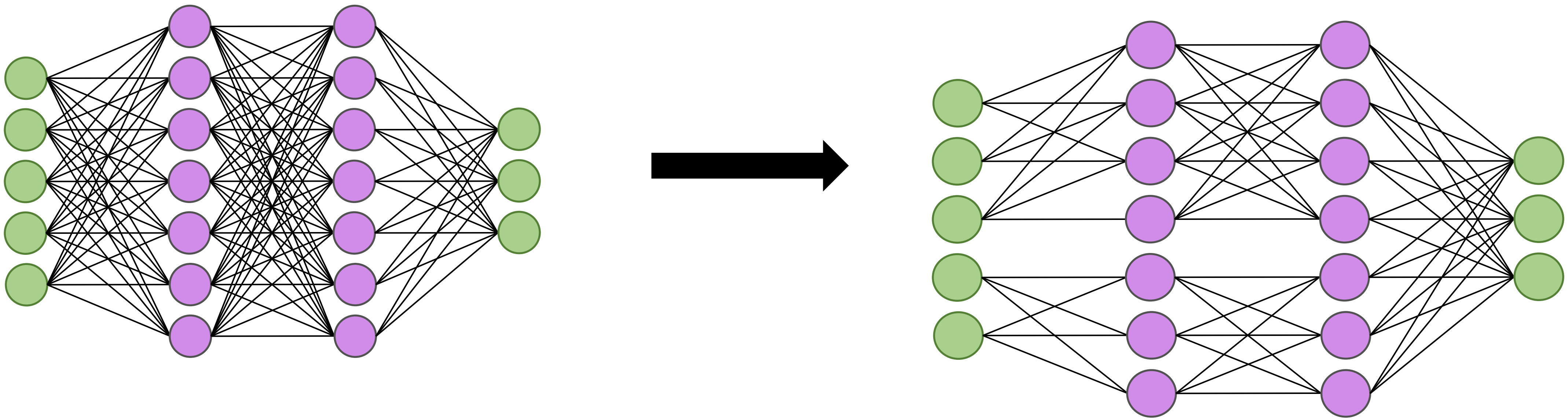


Sum of these three: $0.6 \cdot 0.8 + 0.1 \cdot 0 + 0.9 \cdot 1 = 1.38$

Sum of these three: $0.6 \cdot 0 + 0.1 \cdot 0.4 + 0.9 \cdot 0 = 0.4$

Focusing on local patterns = partial connections

Any questions?



How do we do that?

The Main Building Block: Convolution

Convolution is an operation that takes two inputs:

(1) An image (2D – B/W)

(2) A filter (also called a kernel)

1	1	1
0	0	0
−1	−1	−1

2D array of numbers; could be any values

What Convolution Does (Visually)

image

2	0	1	3
7	1	1	0
0	2	5	0
0	5	1	4

$\otimes$

filter/kernel

1	1	1
0	0	0
-1	-1	-1

(We use this symbol for convolution)
(The verb form is “convolve”)

What Convolution Does (Visually)

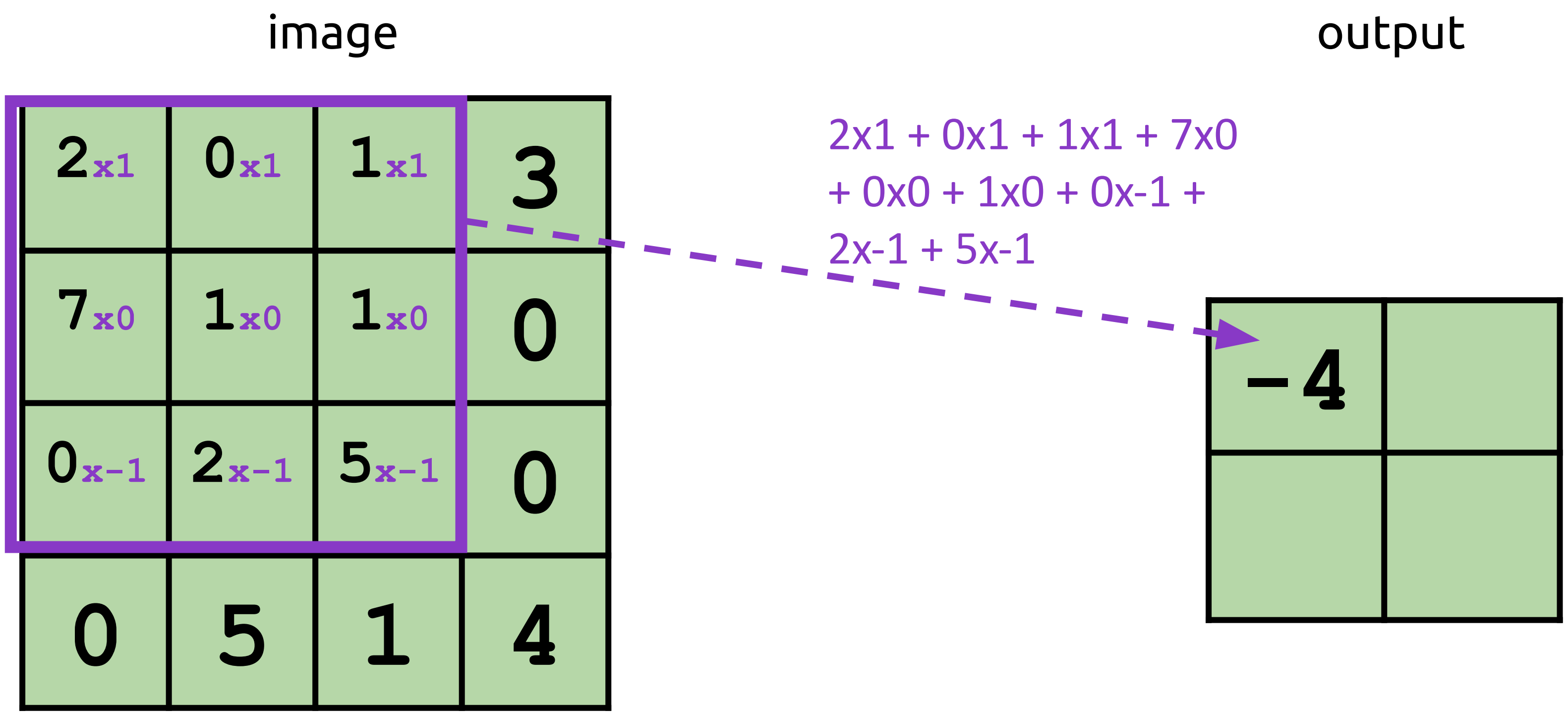
Overlay the filter on the image

image

1	1	1	3
0	0	0	0
-1	-1	-1	0
0	5	1	4

What Convolution Does (Visually)

Sum up multiplied values to produce output value



What Convolution Does (Visually)

Move the filter over by one pixel

image

1	1	1	3
0	0	0	0
-1	-1	-1	0
0	5	1	4

output

-4	

What Convolution Does (Visually)

Move the filter over by one pixel

image

2	1	1	1
7	0	0	0
0	-1	-1	-1
0	5	1	4

output

-4	

What Convolution Does (Visually)

Repeat (multiply, sum up)

image

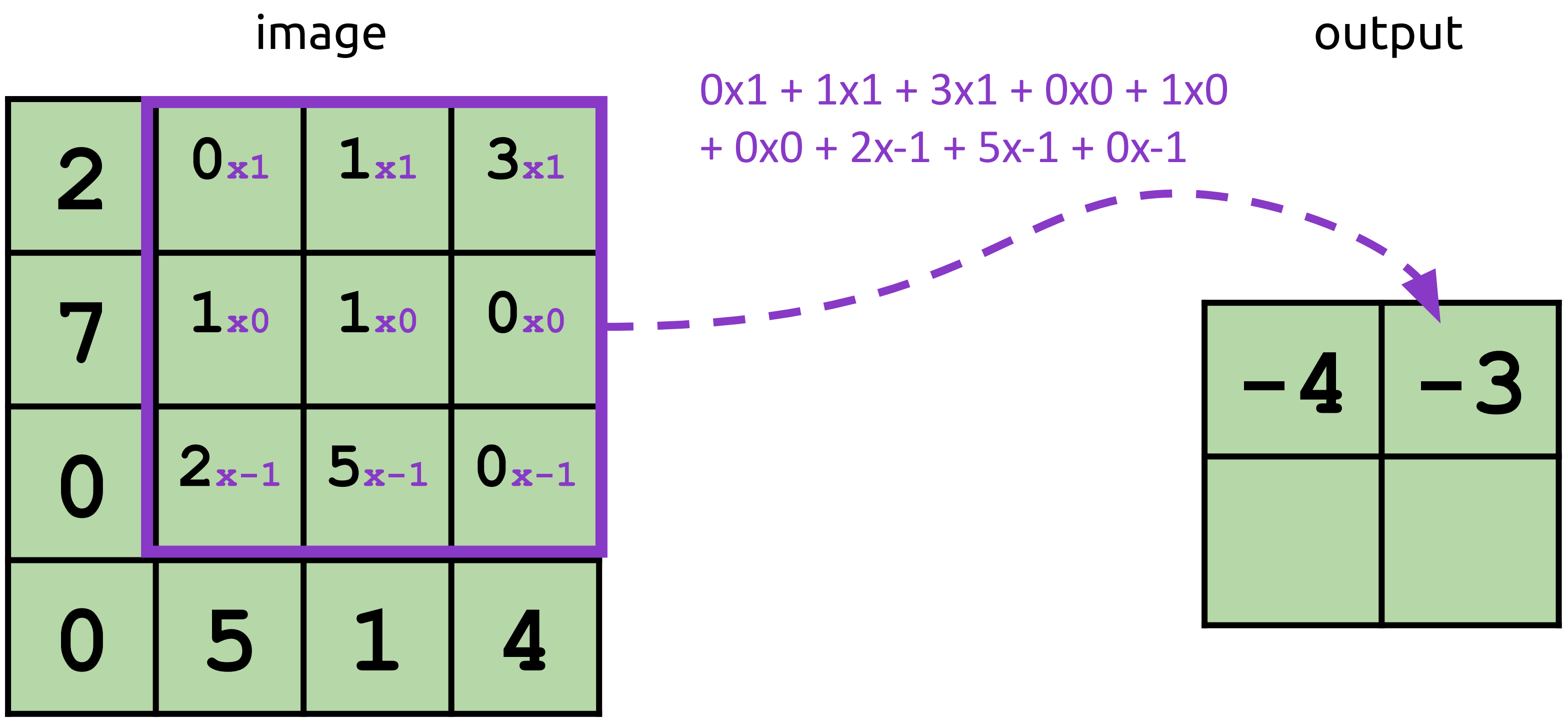
2	0 x1	1 x1	3 x1
7	1 x0	1 x0	0 x0
0	2 x-1	5 x-1	0 x-1
0	5	1	4

output

-4	

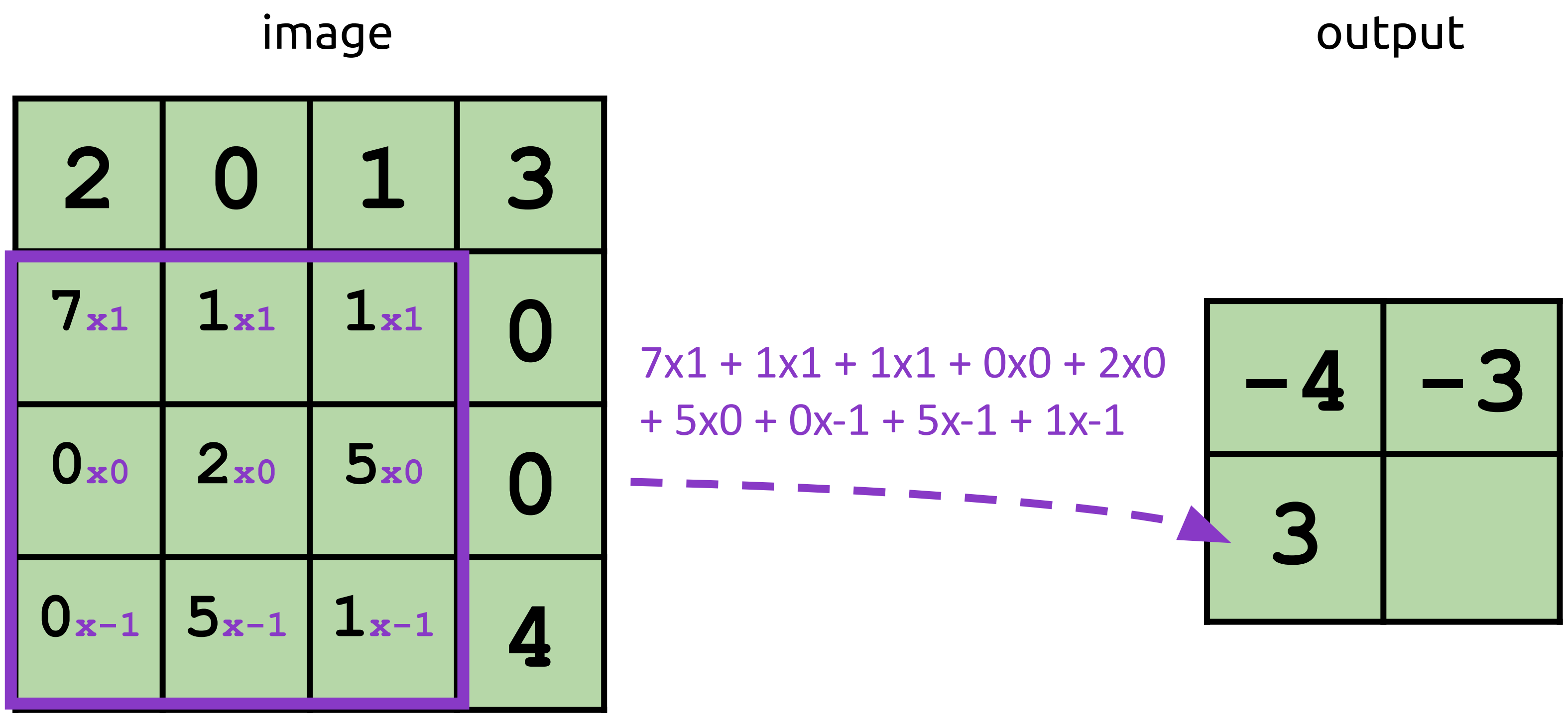
What Convolution Does (Visually)

Repeat (multiply, sum up)



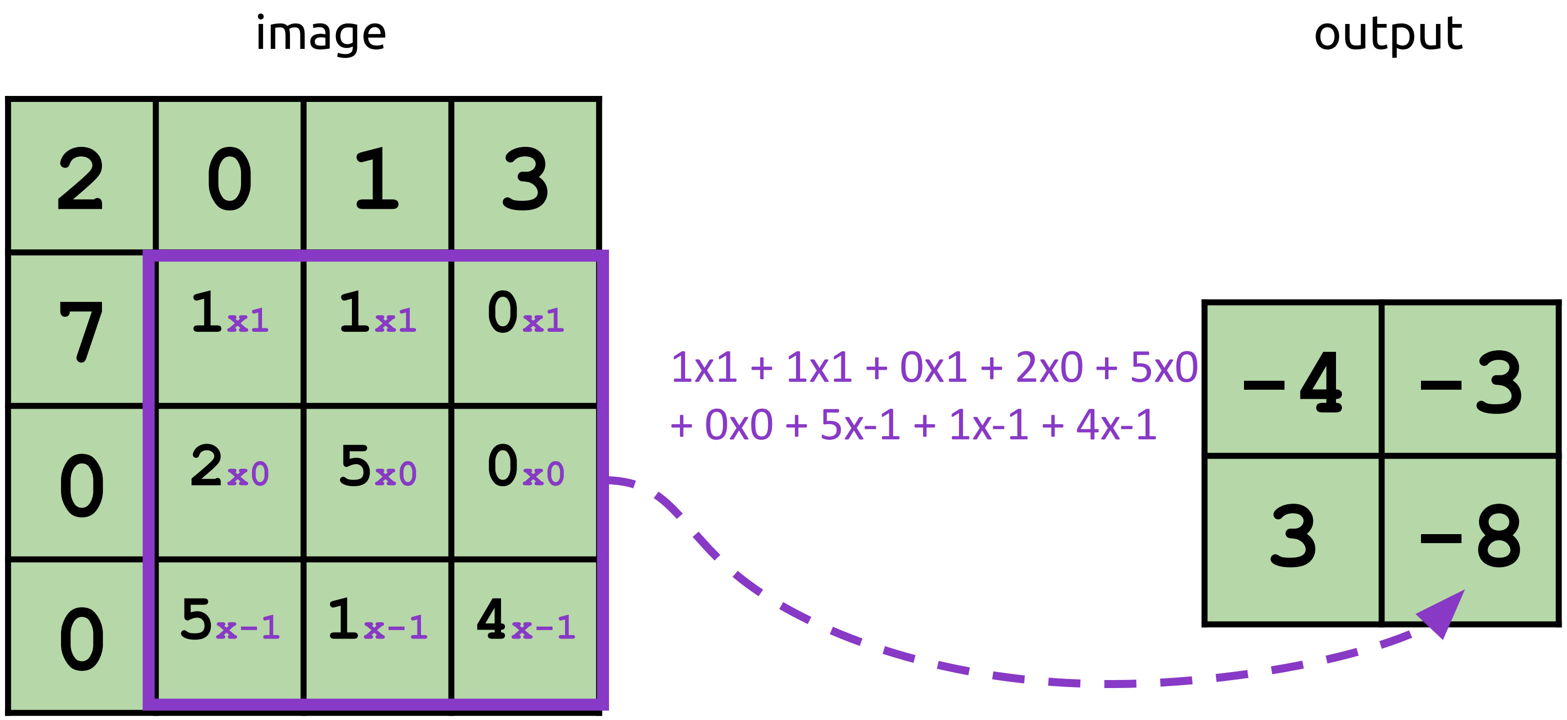
What Convolution Does (Visually)

Repeat...



What Convolution Does (Visually)

Repeat…



What Convolution Does (Visually)

In summary:

$$\underset{\text{image}}{\begin{bmatrix} 2 & 0 & 1 & 3 \\ 7 & 1 & 1 & 0 \\ 0 & 2 & 5 & 0 \\ 0 & 5 & 1 & 4 \end{bmatrix}} \otimes \underset{\text{filter/kernel}}{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}} = \underset{\text{output}}{\begin{bmatrix} -4 & -3 \\ 3 & -8 \end{bmatrix}}$$

Try it out yourself!

Convolve this image

2	0	3	1
1	1	0	0
1	0	2	0
1	0	1	2

$\otimes$

With this filter

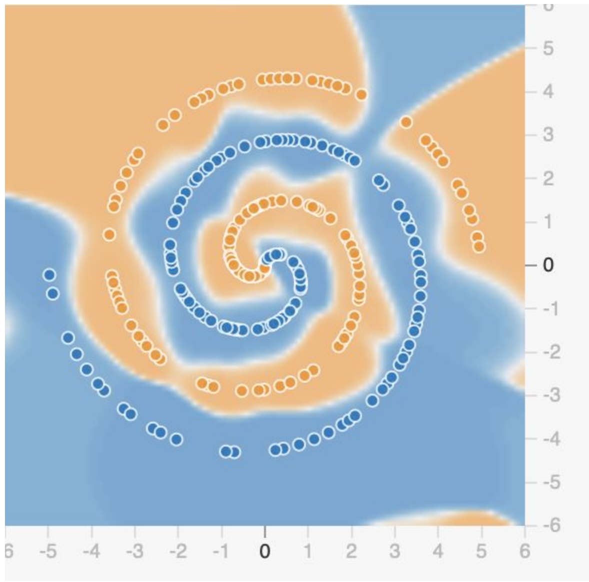
1	0	-1
2	0	-2
1	0	-1

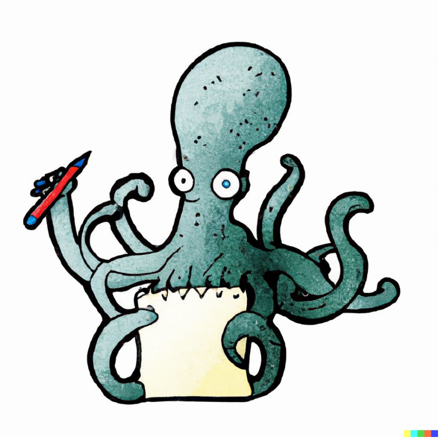
$$\begin{bmatrix} 2 & 0 & 3 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 1 & 2 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

Recap

Building multi-layer neural networks

- Hidden layers
- What a one-hidden layer network can learn
- What a multi-layer network can learn





Introduction to CNNs

- Partially connected networks are useful (e.g., for images!)
- Fully connected networks are not transitionally invariant
- Convolutional filter