

UNIVERSITY OF BIRMINGHAM

School of Computer Science

First Year – MSc Computer Science
Second Year – BSc Artificial Intelligence and Computer Science
First Year – UG Aff Computer Science/Software Engineering
Second Year – BSc Computer Science
Second Year – BEng/MEng Computer Science/Software Engineering
Second Year – MEng Computer Science/Software Engineering
Second Year – BSc Mathematics with Philosophy with Year in Computer Science
Second Year – BSc Physics with Year in Computer Science
Second Year – BSc Accounting and Finance with Year in Computer Science
Second Year – BSc Geography and Economics with Year in Computer Science
Second Year BSc Computer Science with Industrial Year
Second Year – MSci Physics and Particle Cosmology with Year in Computer
Science
Second Year – BSc Political Science and Philosophy with Year in Computer Science

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Models of Computation

Summer Examinations 2013

Time Allowed: 1:30 hours

[Answer ALL Questions]

Turn Over

1. (a) Give one use of regular expressions in programming practice. **[6%]**

For the remainder of the question, the alphabet is $\{a,b\}$.

- (b) Give a regular expression for the language of words in which every a is immediately followed by b . **[7%]**
- (c) Give a deterministic finite automaton for this language. **[7%]**
- (d) Think of a as an open-bracket and b as a close-bracket. So, for example, $aababbab$ is well-bracketed but $abba$ is not. Using the pumping lemma, show that the set of all well-bracketed words is not a regular language. **[6%]**
- (e) Consider the following context-free grammar:

$$V ::= a \mid bVaVb$$

Which of the following words are recognized by the grammar?

- i. $baabb$
- ii. $bbaaabaab$

In each case, either give a derivation or explain why the word cannot be recognized. **[8%]**

- (f) Prove that, in any word recognized by the above grammar, the number of a 's is one greater than the number of b 's. **[7%]**

2. (a) State Rice's Theorem. **[6%]**

- (b) A procedure

```
int MyProc (int x) {
    ...
}
```

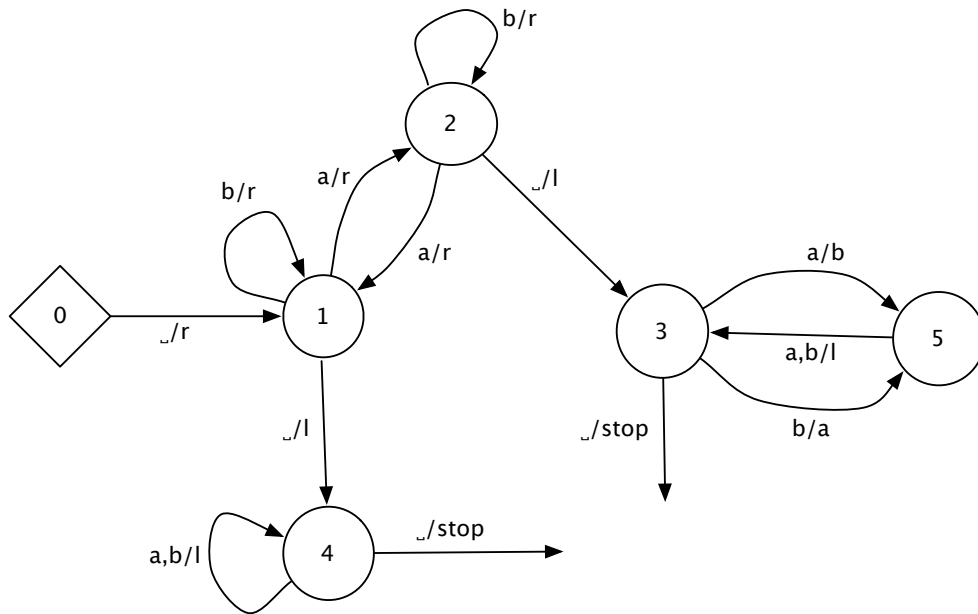
is said to be *red* if the total number of a 's and b 's in the code is prime. Is redness a decidable property? Explain your answer. **[6%]**

- (c) A procedure

```
int MyProc (int x) {
    ...
}
```

is said to be *happy* if, when called with any argument, it does not raise an uncaught exception. Is happiness a decidable property? Explain your answer. **[7%]**

3. Consider the Turing machine depicted as follows:



It starts immediately to the left of a block of a's and b's, and the rest of the tape is blank.

- Execute the above Turing machine on the input aabba, showing each configuration. **[7%]**
- Describe in English what this Turing machine does. **[6%]**
- What is its (time) complexity, in terms of the length n of the block of letters? Explain your answer. **[7%]**

4. Consider the following term of λ -calculus with arithmetic:

$$(\lambda f. \lambda x. f \ f \ x) (\lambda y. y + 3) \ 7$$

- Reduce the above term to normal form. **[7%]**
- State the Church-Rosser theorem. What is its significance for programming languages based on the untyped λ -calculus? **[6%]**
- Annotate the above term with simple types so that it becomes well-typed. **[7%]**