UNIVERSITY OF BIRMINGHAM

School of Computer Science

Second Year – BSc Artificial Intelligence and Computer Science
First Year – UG Computer Science/Software Engineering
Second Year – BSc Computer Science
Second Year – MSci Computer Science
Second Year – MEng Computer Science/Software Engineering
Second Year – MSci Computer Science with Study Abroad
Second Year – BSc Mathematics and Computer Science with Industrial Year
Second Year – BSc Computer Science with Industrial Year
Second Year – MEng Computer Science/Software Engineering with Industrial Year
Second Year - MSci Computer Science with Industrial Year

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Models of Computation

Summer Examinations 2015

Time allowed: 1 hour 30 minutes

[Answer ALL Questions]

In all questions, assume integers are unlimited.

1. **[21%]** This question is about regular languages and finite-state automata.

   (a) Give a regular expression for the language consisting of all words over the alphabet a, b, c that contain cbb.

   [7%]

   (b) Give a deterministic finite state automaton for this language. **[7%]**

   (c) Using the pumping lemma, show that the language consisting of words $\text{ab}^n\text{a}^{n+1}$, for any natural number n, is not regular. **[7%]**

2. **[15%]** This question is about context-free languages.

   (a) Give an example of how context-free languages (that are not regular) are used in computing practice. **[7%]**

   (b) Here is a grammar over the alphabet $1, 3, 7, +, -$, with start symbol M.

   $$M ::= \quad 1 \mid 3 \mid 7 \mid M + M \mid M - M$$

   Give two derivation trees for $7 - 3 - 1$. For one of these trees—indicating which one—give the corresponding leftmost derivation.

   [8%]

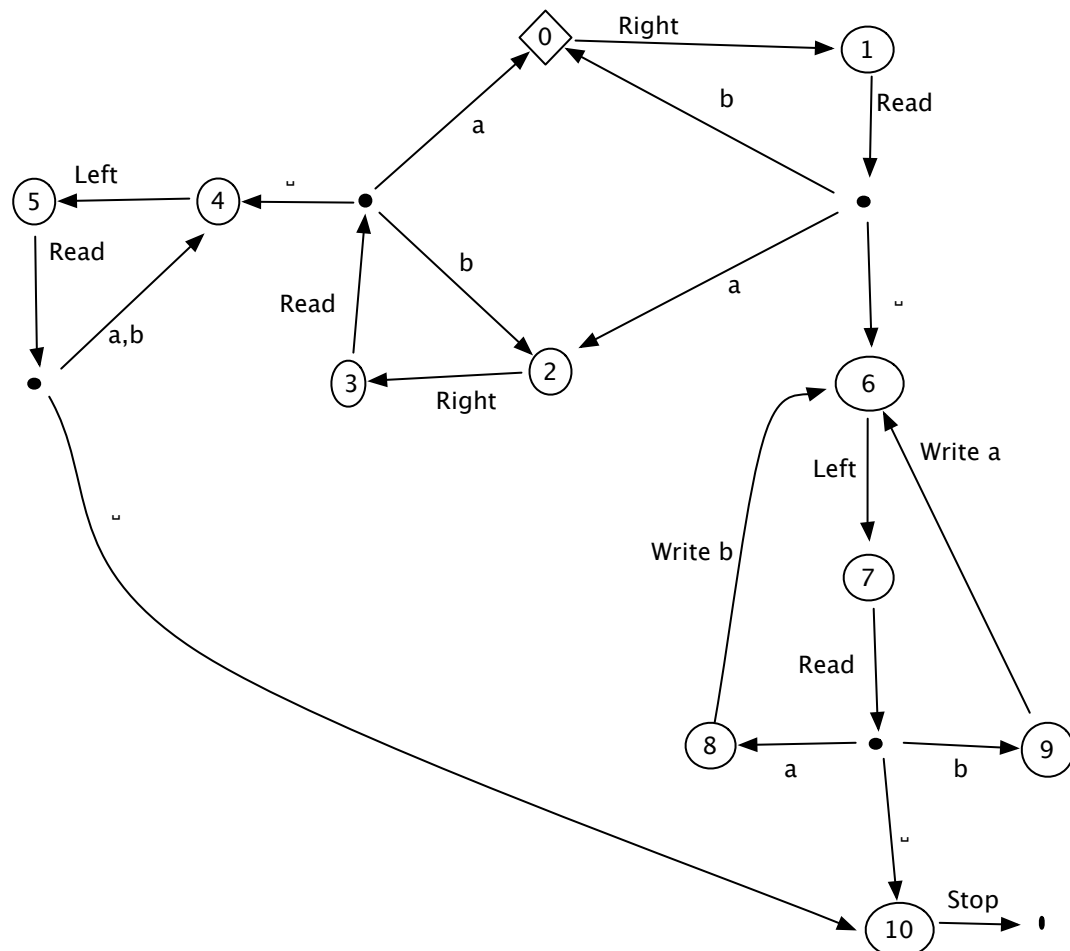3. **[21%]** This question is about decidability.

   (a) What does it mean to say that a property of strings is *decidable*?

   [7%]

   (b) State Church's thesis and explain its relevance to part 3a. **[7%]**

   (c) A Java method `int f (int x)` is said to be *blue* when, if called with any even argument, it returns an even number. Is blueness a decidable property of methods? Explain your answer. **[7%]**

4. **[14%]** This question is about Turing machines.

Consider the Turing machine shown:

The head starts on the blank space to the left of a block of a's and b's, and the rest of the tape is blank.

(a) Execute it on the string aba. **[7%]**

(b) Say in English what this machine does. **[7%]**

5. **[15%]** This question is about satisfiability of propositional formulas (SAT).

    (a) A language school offers evening classes in Arabic, Bengali and Chinese There are two students.

    - Mary takes Arabic and Bengali.
    - Jim takes Arabic and Chinese.

    The administrator has to schedule a class for each language on either Monday or Tuesday. No student should have two classes on the same evening.
    Explain how the administrator's task may be reduced to SAT, by giving a suitable propositional formula.

    [8%]

    (b) SAT is an example of a problem in $\mathcal{NP}$. Explain what it means for a problem to be in $\mathcal{NP}$.

    [7%]

6. **[14%]** This question is about λ-calculus.

    (a) What is the β-reduction rule of λ-calculus?

    [7%]

    (b) Here is a term of λ-calculus with arithmetic:

    $$(\lambda x.\ \lambda y.\ y\,(y\,x))\ 3\ (\lambda x.\ x + 2)$$

    Reduce this term to normal form.

    [7%]