Homework 3 Written Questions

CSCI 1430

Template Instructions

This document is a template with specific answer regions and a fixed number of pages. Given large class sizes and limited TA time, the template helps the course staff to grade efficiently and still focus on the content of your submissions. Please help us in this task:

- Make this document anonymous.
- Questions are in the orange boxes. Provide answers in the green boxes.
- Use the footer to check for correct page alignment.
- **Do NOT remove the answer box.**
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- **Extra pages are not permitted unless otherwise specified.**
- **Template edits or page misalignment will lead to a 10 point deduction.**

Gradescope Submission

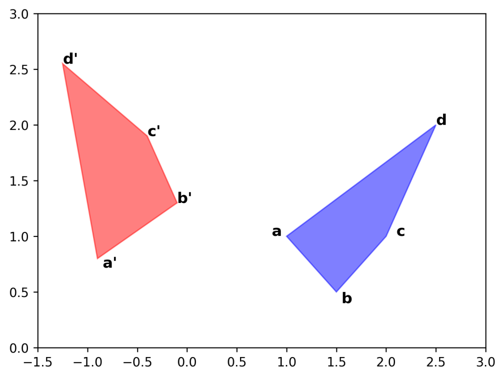
- Compile this document to a PDF and submit it to Gradescope.
- Pages will be automatically assigned to the right questions on Gradescope.

This Homework

- 6 questions **[10 + 8 + 4 + 6 + 3 + 4 = 35 points]**.
- Include code, images, and equations where appropriate.

Q1 — [10 points]

Suppose we have a two quadrilaterals $\mathbf{abcd}$ and $\mathbf{a'b'c'd'}$:



$$\mathbf{a} = (1, 1) \qquad \mathbf{a'} = (-0.9, 0.8)$$
$$\mathbf{b} = (1.5, 0.5) \qquad \mathbf{b'} = (-0.1, 1.3)$$
$$\mathbf{c} = (2, 1) \qquad \mathbf{c'} = (-0.4, 1.9)$$
$$\mathbf{d} = (2.5, 2) \qquad \mathbf{d'} = (-1.25, 2.55)$$

They look like they are related by a rotation and a non-uniform scale transformation. So, let's assume that each point in $\mathbf{abcd}$ could be mapped to its corresponding point in $\mathbf{a'b'c'd'}$ by a 2×2 transformation matrix $\mathbf{M}$, as these can represent non-linear scaling and rotation.

e.g., if $\mathbf{p} = \begin{bmatrix} x \\ y \end{bmatrix}$, $\mathbf{p'} = \begin{bmatrix} x' \\ y' \end{bmatrix}$, and $\mathbf{M} = \begin{bmatrix} m_{1,1} & m_{1,2} \\ m_{2,1} & m_{2,2} \end{bmatrix}$,

then $\begin{bmatrix} m_{1,1} & m_{1,2} \\ m_{2,1} & m_{2,2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$

Q1.1 — [1 point]

Rewrite the equation $\mathbf{Mp} = \mathbf{p'}$ into a pair of linear equations by expanding the matrix multiplication symbolically.

TODO: Replace each of the '__' below with x, y, x', y', or 0.

$$\begin{cases} \_\_m_{1,1} + \_\_m_{1,2} + \_\_m_{2,1} + \_\_m_{2,2} = \_\_ \\ \_\_m_{1,1} + \_\_m_{1,2} + \_\_m_{2,1} + \_\_m_{2,2} = \_\_ \end{cases}$$

Q1.2 — [2 points]

We would like to estimate $\mathbf{M}$ using least squares linear regression. For this, we form a system of linear equations, typically written in the form $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{x}$ contains the parameters of $\mathbf{M}$, matrix $\mathbf{A}$ contains the coordinates of $\mathbf{p}$ and column vector $\mathbf{b}$ contains the corresponding coordinates of $\mathbf{p}'$.

$$\mathbf{Ax} = \mathbf{b} \quad \text{where} \quad \mathbf{A} \times \begin{bmatrix} m_{1,1} \\ m_{1,2} \\ m_{2,1} \\ m_{2,2} \end{bmatrix} = \mathbf{b}$$

As $\mathbf{M}$ has four parameters, we need four equations to *determine* an $\mathbf{M}$. As each point pair provides two equations, that $\mathbf{M}$ will exactly transform those two points $\mathbf{p}$ onto their pairs $\mathbf{p}'$. But, as our quadrilaterals have four points, we could create eight such equations to estimate $\mathbf{M}$—$\mathbf{M}$ is now said to be *overdetermined*. If all point pairs *do not* exactly transform by some $\mathbf{M}$, then using an overdetermined system will find an $\mathbf{M}$ that minimize the distance (or residual error) between all estimated values for $\mathbf{p}'$ and the real values for $\mathbf{p}'$, i.e., that minimize the Euclidean norm (or 2-norm) $||\mathbf{Ax} - \mathbf{b}||_2$.

Note: As systems of linear equations are typically written in the form $\mathbf{Ax} = \mathbf{b}$, we've overloaded the symbol $\mathbf{b}$ here to keep this familiarity. So, be careful—$\mathbf{b}$ is the vector of target x', y' values across all equations, and not the point in the quadrilateral.

Declare $\mathbf{A}$ and $\mathbf{b}$ for all four point pairs.

Replace each '__' below with a 0 or a coordinate value from the quadrilaterals.

$$\begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix} \times \begin{bmatrix} m_{1,1} \\ m_{1,2} \\ m_{2,1} \\ m_{2,2} \end{bmatrix} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

Only Q1.2 should be on this page

Q1.3 — [3 points]
If we use four equations, then $\mathbf{A}$ is square and can be easily inverted to solve for $\mathbf{x}$ by left multiplying the inverse by both sides:

$$\mathbf{A}^{-1}\mathbf{A}\mathbf{x} = \mathbf{A}^{-1}\mathbf{b} \quad \text{so} \quad \mathbf{x} = \mathbf{A}^{-1}\mathbf{b}. \tag{1}$$

If we use more than four equations, then $\mathbf{A}$ is non-square. In this situation, we can use the pseudoinverse of $\mathbf{A}$, written as $\mathbf{A}^{+}$.

$$\mathbf{A}^{+}\mathbf{A}\mathbf{x} = \mathbf{A}^{+}\mathbf{b} \quad \text{so} \quad \mathbf{x} = \mathbf{A}^{+}\mathbf{b}. \tag{2}$$

As long as $\mathbf{A}$ has a pseudoinverse, this solution minimizes $||\mathbf{A}\mathbf{x} - \mathbf{b}||_2$.
This is the closed-form least squares solution, where $\mathbf{x} = (\mathbf{A}^\top\mathbf{A})^{-1}\mathbf{A}^\top\mathbf{b}$ and where $\mathbf{A}^{+} = (\mathbf{A}^\top\mathbf{A})^{-1}\mathbf{A}^\top$.

We can compute the pseudoinverse from the singular value decomposition. In python, `numpy.linalg.lstsq()` will handle this for us, including solving large systems. `lstsq()` takes as input $\mathbf{A}$ and $\mathbf{b}$, and returns a solution for $\mathbf{x}$ along with the residual error. Plug your $\mathbf{A}$,$\mathbf{b}$ values into that function and write the numeric values of the estimated $\mathbf{M}$ matrix below along with the numeric value of the residual error.

Note: We provide a Python script `transformation_viz.py` for you to estimate and visualize the transformation. Within function `estimate_transform()`, declare matrix $\mathbf{A}$ and vector $\mathbf{b}$ and create a function call to `lstsq()` to see how the quadrilaterals match up!

State your $\mathbf{M}$ by replacing each of the '__' below, and state the residual error.

$$\mathbf{M} = \begin{bmatrix} m_{1,1} & m_{1,2} \\ m_{2,1} & m_{2,2} \end{bmatrix} = \begin{bmatrix} \text{--} & \text{--} \\ \text{--} & \text{--} \end{bmatrix}$$

Residual error: xxx

Q1.4 — [4 points]
If the residual is zero (or zero to machine numerical precision), then we can confirm our initial assumption that the transformation is a rotation and a scale. If it is not, then we need a transformation with more degrees of freedom to reduce the residual error.

Determine what kind of transformation it is by forming a system of linear equations and estimating an $\mathbf{M}$ that produces zero residual error (to machine numerical precision).

Use your implementation in `transformation_viz.py` to help. Write out your system's $\mathbf{A}$, $\mathbf{x}$, and $\mathbf{b}$ matrices, state $\mathbf{M}$ and the residual error, and state which kind of transformation it is.

Q2 — [8 points]
In lecture, we've learned that cameras can be represented by intrinsic and extrinsic matrices. These matrices can be used to calculate the projections of points within a 3D world onto 2D image planes. For this, we use *homogeneous coordinates*. The final 3×4 matrix is known as the *camera matrix*.

Recall that the transformation can be represented by the following expression:

$$\begin{bmatrix} f_x & s & 0 \\ 0 & f_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

where f is the focal length, $\mathbf{R}$ is the rotation matrix, $\mathbf{t}$ is the translation vector, w is some weighing/scaling factor, and (u, v) is the position that the point in the real world (x, y, z) projects to on the 2D plane.

For each following question, you are given the camera specifications and a sample 3D point from the real world. Fill in the camera's intrinsic $\mathbf{K}$ and extrinsic $[\mathbf{Rt}]$ matrices; then, perform the multiplications and perspective division (unhomogenize) to find the 2D coordinate of the projected point.

Q2.1.1 — [1 point]

A camera with a focal length of 1 in both the u and v directions, a translation of 5 along the x-axis, and no skew or rotation.

TODO: Fill in the __ entries.

$$\mathbf{K} \quad \times \quad [\mathbf{Rt}] \quad \times \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \_\_ & \_\_ & 0 \\ 0 & \_\_ & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix} \times \begin{bmatrix} 30 \\ -20 \\ 10 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

$$= \_\_ \times \begin{bmatrix} \_\_ \\ \_\_ \\ 1 \end{bmatrix}$$

Only Q2 and Q2.1.1 should be on this page

Q2.1.2 — [1 point]

A camera with focal length of 2 in both the u and v directions, a translation of 5 along the x-axis, and no skew or rotation.

$$
\begin{aligned}
&= \begin{bmatrix} \text{--} & \text{--} & 0 \\ 0 & \text{--} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \text{--} & \text{--} & \text{--} & \text{--} \\ \text{--} & \text{--} & \text{--} & \text{--} \\ \text{--} & \text{--} & \text{--} & \text{--} \end{bmatrix} \times \begin{bmatrix} 30 \\ \text{-20} \\ 10 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} \text{--} \\ \text{--} \\ \text{--} \end{bmatrix} \\
&= \text{--} \times \begin{bmatrix} \text{--} \\ \text{--} \\ 1 \end{bmatrix}
\end{aligned}
$$

Q2.2 — [1 point]

Compare the two image coordinates you've calculated in parts a and b. What visual effect might we say was induced by this change in intrinsic parameters? **[2–3 sentences]**

Q2.3 — [5 points]
We provide stencil code for a camera simulation in `camera_simulation.py`. Given a camera matrix, the simulator visualizes an image that a camera would produce.

Please implement `calculate_camera_matrix()` by calculating the camera matrix using the parameters given in the code. When successful, you will see a bunny rendered as dots (as below). Paste your code for this function and attach a screenshot of the working demo once you finish. Play around with the sliders to see how different parameters affect the projection!

```
def calculate_camera_matrix(tx, ty, tz, alpha, beta, gamma, fx,
                                fy, skew, u, v):
    ########################
    # TODO: Your calculate_camera_matrix() code here #
    # Hint: Calculate the rotation matrices for the x, y, and z
                                axes separately.
    # Then multiply them to get the rotational part of the
                                extrinsic matrix.
    ########################

    return (initial_camera_matrix_to_replace,
    initial_intrinsic_matrix_to_replace,
    initial_extrinsic_matrix_to_replace)
```

```
################################################
# YOU MAY USE THIS ADDITIONAL PAGE

# WARNING: IF YOU DON'T END UP USING THIS PAGE
# KEEP THESE COMMENTS TO MAINTAIN PAGE ALIGNMENT
################################################
```

Q3 — [4 points]
Let's consider two-view geometry.

Q3.1 — [2 points]

Given a stereo pair of calibrated cameras, briefly describe depth estimation by triangulation from disparity. Describe the inputs and outputs of the process. You may wish to use a diagram. **[3–4 sentences]**

Q3.2 — [2 points]

We wish to estimate depth in real-world units (e.g., meters) by triangulation. This is sometimes called ‘absolute depth.’ What parameters or measurements of my stereo pair camera system do I need to know to estimate absolute depth? Provide units. **[3–4 sentences]**

Q4 — [6 points]
Given the algorithms that we've learned in computer vision, we know that whether we can estimate the essential matrix, the fundamental matrix, or both depends on the setup of the cameras and images. Suppose we have three captures of an object of unknown size, without any other metadata (e.g., no Exif metadata):

(i) A video of a camera circling the object;
(ii) A stereo pair of calibrated cameras that captured two images of the object; and
(iii) Two images on the Web captured at different times, with different cameras, and from different locations, with overlap in their views (e.g., Colosseum in Rome).

Q4.1 — [3 points]

For each of the above setups, what can we recover?
Provide brief explanations to support your choices. **[1–2 sentences]**

(i) Setup 1

☐ Essential Matrix
☐ Fundamental Matrix
☐ Both

TODO Why?

(ii) Setup 2

☐ Essential Matrix
☐ Fundamental Matrix
☐ Both

TODO Why?

(iii) Setup 3

☐ Essential Matrix
☐ Fundamental Matrix
☐ Both

TODO Why?

Q4.2 — [3 points]

State one practical advantage and one practical disadvantage of using each setup for depth reconstruction. **[2–3 sentences]**

(i) Setup 1

(ii) Setup 2

(iii) Setup 3

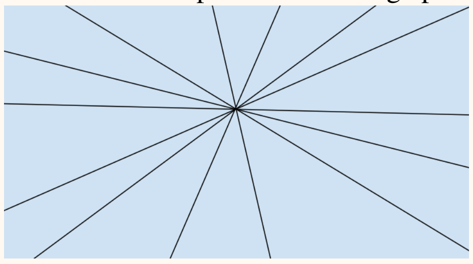
Q5 — [3 points]
In two-view camera geometry, what do the following epipolar lines say about the relative positions of the cameras? **[1–2 sentences each]**

Please draw or describe the relative positions of the two camera planes.

Note: The Spring '22 course staff created an interactive demo to explore the different scenarios. Move the cameras around and look at how the epipolar lines change to gain a better feel for epipolar geometry.

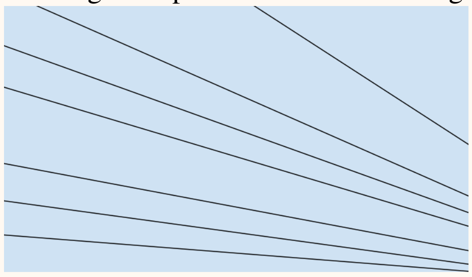
Q5.1 — [1 point]

Radiate out of a point on the image plane.



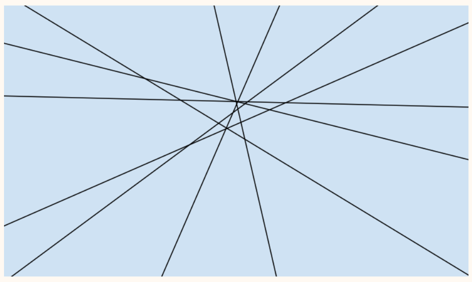
Q5.2 — [1 point]

Converge to a point outside of the image plane.



Q5.3 — [1 point]

Notice the misalignment of the epipolar lines in the image below? What went wrong in the calculation of the fundamental matrix and how can we fix it?
Hint: Check slides from the lecture on stereo geometry.



Q6 — [4 points]

Cameras are used in surveillance systems. One argument in favor of surveillance systems is to deter and solve crime to improve safety. Another is that if you're not doing anything wrong, you don't have anything to worry about. One argument against surveillance systems is that they compromise people's privacy even when no wrongdoing is taking place. Another is that they increase stress and anxiety.

Computer vision allows the *automation* of surveillance. For instance, it lets us find the mathematical relationship between multiple cameras to track objects and people in 3D spaces, or it can reduce the burden upon a human operator who need only respond to detected events rather than actively monitor many cameras. Such functionality makes it easier to scale a surveillance operation.

On Brown's campus, the number of surveillance cameras has increased as technology has advanced: from 60 in 2000 (Brown Daily Herald 2008/01) to 800 by 2020 (BDH 2020/02), including temporarily in residence halls (BDH 2021/07). The City of Providence is investigating installing noise cameras (BDH 2024/03), potentially on Thayer Street (BDH opinion 2024/04).

Suppose Brown both did and did not use computer vision automation. How comfortable are you with Brown's surveillance apparatus in each case? In what circumstances do you believe that the potential benefits of surveillance *automation* outweigh the potential concerns, and why? **[8–10 sentences]**

Q6.1 — [4 points]

Unmanned aerial vehicles—sometimes called drones—often carry cameras. Their cameras can be used for navigation via manually remote control, or for use within computer vision strategies like camera pose estimation and depth estimation to enable assisted or autonomous flying in complex environments.

For your CSCI 1430 final project, you are developing a drone for life-saving organ delivery. You create a successful computer vision algorithm that allows your drone to navigate autonomously. You are approached by several organizations that want to pay you for access to your project, but you are also considering open sourcing your algorithm with a permissive software license.

Please list three organizations that might be interested in acquiring your project for their own purposes. If each of these organizations used your project, who could benefit and how? Who could be harmed and how? Would an open-source license affect this? **[6—9 sentences]**

Feedback? (Optional)

We appreciate your feedback on how to improve the course. You can provide anonymous feedback through this form which can be accessed using your Brown account (your identity will not be collected). If you have urgent non-anonymous comments/questions, please email the instructor.