#### Mud card answers

- What improvements do the algorithms in scikit/ other libraries contain?
  - the learning rate is adaptively determined
  - my algorithm gets stuck in a local minimum, algos in sklearn are more robust (stochastic) and they have a much better chance of finding the global minimum
  - better implementation in cython to optimize for speed
  - etc
- How should we change the loss function using gradient descent for logistic regression
  - the cost function then the logloss metric and you need to update the derivatives in the gradient descent function accordingly
- Do you have recommendations for applying alternative gradient descent techniques?
  - if an ML model has multiple gradient descent algorithms implemented, the sklearn documentation will contain the list with recommendations which one to choose
- The learning rate here is a fixed value, but I think it should decrease as we're approaching the valley. So how can we decide when and how much to decrease our learning rate so that our algorithm won't just leap over the minima?
  - that's called adaptive step size and it's implemented in the opposite way of your description
  - if the gradient is steep, you want to make a small step to avoid numerical instabilities
  - if the gradient is shallow, you can make a larger step
- How would your gradient descent function handle saddles and local minima?
  - it wouldn't, that's why you should only use it for learning purposes :)
- "Can you talk about multi-class logistic regression? What are some typical methods to solve that problem?
  - all sklearn classification models (logistic regression included) handle multiclass
     classification problems natively, no other work or arguments are required on your side
- For the logistic regression, why do we choose logloss metric as the cost function but not others like MSE?
  - MSE is a regression metric because it compares the continuous true labels to the continuous predicted labels
  - for classification, we need a metric that compares the categorical true labels to either the predicted classes (also categorical) or predicted probabilities (continuous)
- I'm confused about how log-loss is defined. MSE and its variants make sense, because we are roughly trying to measure the area between y\_true and y\_pred. What aspect of the two functions is log-loss trying to measure? Is it related to the sigmoid function itself (maybe via derivative?)
- Could you explain the "The logloss metric" more? Like how to use it when we evaluate our prediction model
  - it is not related to the sigmoid function
  - logloss compares the categorical true labels to the predicted probabilities which are continuous
  - read more here

## Regularization

By the end of this lecture, you will be able to

- Describe why regularization is important and what are the two types of regularization
- Describe how regularized linear regression works
- Describe how regularized logistic regression works

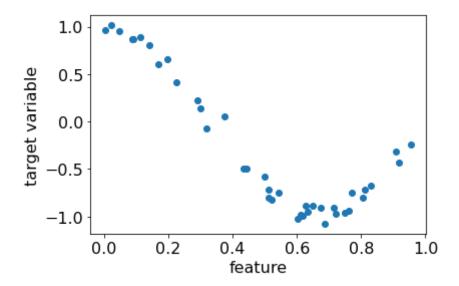
## Regularization

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# Let's work with a new example dataset

```
In [1]:
         # load packages
         import numpy as np
         import matplotlib.pyplot as plt
         import pandas as pd
         from sklearn.preprocessing import PolynomialFeatures
         import matplotlib
         matplotlib.rcParams.update({'font.size': 16})
         df = pd.read csv('data/regularization example.csv')
         X ori = df['x0'].values.reshape(-1, 1)
         y = df['y'].values
         print(np.shape(X ori))
         print(np.shape(y))
         # visualize the data
         plt.scatter(X ori,y)
         plt.xlabel('feature')
         plt.ylabel('target variable')
         plt.show()
        (40, 1)
        (40,)
```



```
In [2]: # lets generate more features because a linear model will obviously be insuffici
    pf = PolynomialFeatures(degree = 20,include_bias=False)
    X = pf.fit_transform(X_ori)
    print(np.shape(X))
    print(pf.get_feature_names())

(40, 20)
['x0', 'x0^2', 'x0^3', 'x0^4', 'x0^5', 'x0^6', 'x0^7', 'x0^8', 'x0^9', 'x0^10',
    'x0^11', 'x0^12', 'x0^13', 'x0^14', 'x0^15', 'x0^16', 'x0^17', 'x0^18', 'x0^19',
    'x0^20']
```

## We split data into train and validation!

```
In [3]:
    from sklearn.model_selection import train_test_split
    X_train, X_val, y_train, y_val = train_test_split(X, y, test_size=0.2, random_st
    print(np.shape(X_train),np.shape(y_train))
    print(np.shape(X_val),np.shape(y_val))

    (32, 20) (32,)
    (8, 20) (8,)
```

## Let's train and validate some linear regression models

#### Use the first feature only

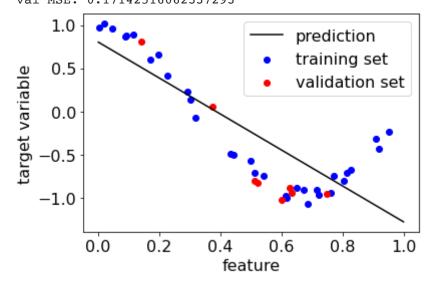
```
In [4]:
    from sklearn.linear_model import LinearRegression
    from sklearn.metrics import mean_squared_error

# let's use only the first feature
    linreg = LinearRegression(fit_intercept=True)
    linreg.fit(X_train[:,:1], y_train)
    print('intercept:',linreg.intercept_)
    print('theta:',linreg.coef_)

train_MSE = mean_squared_error(y_train,linreg.predict(X_train[:,:1]))
    val_MSE = mean_squared_error(y_val,linreg.predict(X_val[:,:1]))
    print('train_MSE:',train_MSE)
    print('val_MSE:',val_MSE)
```

```
# let's visualuze the model
x_model = np.linspace(0,1,100)
plt.scatter(X_train[:,0],y_train,color='b',label='training set')
plt.scatter(X_val[:,0],y_val,color='r',label='validation set')
plt.plot(x_model,linreg.predict(x_model.reshape(-1,1)),color='k',label='predicti
plt.xlabel('feature')
plt.ylabel('target variable')
plt.legend()
plt.show()
```

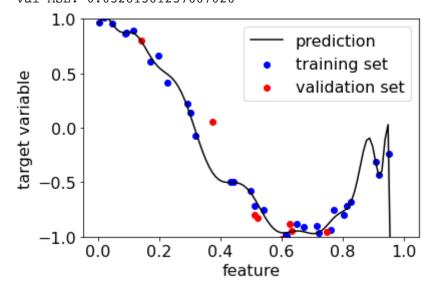
```
intercept: 0.8018842867499771
theta: [-2.08151827]
train MSE: 0.13964692457239292
val MSE: 0.17142516062337293
```



#### Use all features

```
In [5]:
         from sklearn.linear model import LinearRegression
         from sklearn.metrics import mean squared error
         # use all features
         linreg = LinearRegression(fit intercept=True)
         linreg.fit(X_train, y_train)
         print('intercept:',linreg.intercept )
         print('thetas:',linreg.coef )
         train MSE = mean squared error(y train,linreg.predict(X train))
         val_MSE = mean_squared_error(y_val,linreg.predict(X_val))
         print('train MSE:',train MSE)
         print('val MSE:',val MSE)
         # let's visualuze the model
         x \mod = np.linspace(0,1,100)
         plt.scatter(X train[:,0],y train,color='b',label='training set')
         plt.scatter(X val[:,0],y val,color='r',label='validation set')
         plt.plot(x model,linreg.predict(pf.transform(x model.reshape(-1,1))),color='k',1
         plt.ylim([-1,1])
         plt.xlabel('feature')
```

```
plt.ylabel('target variable')
plt.legend()
plt.show()
```



#### What to do?

- the model is visibly performs poorly when only the original feature is used
- the model performs very good on the training set but poorly on the validation set when all features are used
  - the thetas are huge!

## Regulazation solves this problem!

## Regularization

By the end of this lecture, you will be able to

- Describe why regularization is important and what are the two types of regularization
- Describe how regularized linear regression works
- Describe how regularized logistic regression works

## Regularization to the rescue!

- let's change the cost function and add a penalty term for large thetas
- Lasso regression: regularize using the l1 norm of theta:

$$L(\theta) = \frac{1}{n} \sum_{i=1}^{n} [(\theta_0 + \sum_{j=1}^{m} \theta_j x_{ij} - y_i)^2] + \frac{\alpha}{m} \sum_{j=0}^{m} |\theta_j|$$

• Ridge regression: regularize using the I2 norm of theta:

$$L(\theta) = \frac{1}{n} \sum_{i=1}^{n} [(\theta_0 + \sum_{j=1}^{m} \theta_j x_{ij} - y_i)^2] + \frac{\alpha}{m} \sum_{j=0}^{m} \theta_j^2$$

- $oldsymbol{lpha}$  is the regularization parameter (positive number), it describes how much we penalize large thetas
- With the cost function changed, the derivatives in gradient descent need to be updated too!

## Feature selection with Lasso regularization

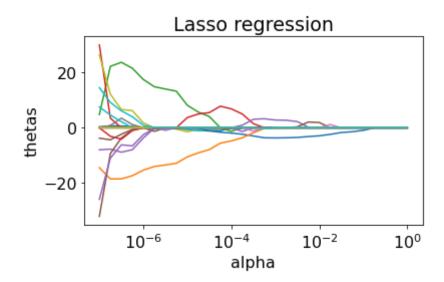
- Least Absolute Shrinkage and Selection Operator
- cost = MSE +  $\alpha$  \* I1 norm of  $\theta$

$$L(\theta) = \frac{1}{n} \sum_{i=1}^{n} [(\theta_0 + \sum_{j=1}^{m} \theta_j x_{ij} - y_i)^2] + \frac{\alpha}{m} \sum_{j=0}^{m} |\theta_j|$$

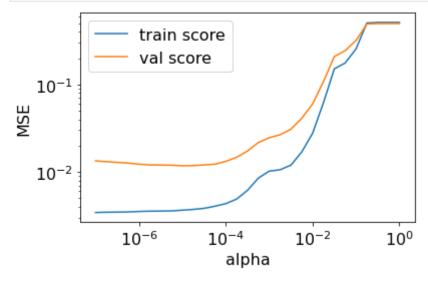
- ideal for feature selection
- as  $\alpha$  increases, more and more feature weights are reduced to 0.

```
In [6]:
         from sklearn.linear_model import Lasso
         from sklearn.metrics import mean squared error
         alpha = np.logspace(-7,0,29)
         thetas = []
         models = []
         train MSE = np.zeros(len(alpha))
         val MSE = np.zeros(len(alpha))
         # do the fit
         for i in range(len(alpha)):
             # load the linear regression model
             lin reg = Lasso(alpha=alpha[i], max iter=100000000)
             lin reg.fit(X train, y train)
             thetas.append(lin reg.coef)
             models.append(lin reg)
             train MSE[i] = mean squared error(y train,lin reg.predict(X train))
             val MSE[i] = mean squared error(y val,lin reg.predict(X val))
```

```
In [7]:
    plt.plot(alpha, thetas)
    plt.semilogx()
    plt.xlabel('alpha')
    plt.ylabel('thetas')
    plt.title('Lasso regression')
    plt.tight_layout()
    plt.savefig('figures/lasso_coefs.png',dpi=300)
    plt.show()
```



```
In [8]: plt.plot(alpha,train_MSE,label='train score')
   plt.plot(alpha,val_MSE,label='val score')
   plt.semilogy()
   plt.semilogx()
   plt.xlabel('alpha')
   plt.ylabel('MSE')
   plt.legend()
   plt.tight_layout()
   plt.savefig('figures/train_val_MSE_lasso.png',dpi=300)
   plt.show()
```



## Bias vs variance

- Bias: the model performs poorly on both the train and validation sets
  - high alpha in our example
- the model performs very well on the training set but it performs poorly on the validation set
  - low alpha in our example
  - lowering the alpha further would improve the train score but the validation score would increase
  - we don't do it because of convergence issues

## The bias-variance trade off

- the curve of the validation score as a function of a hyper-parameter usually has a U shape if evaluation metric needs to be minimized, or an inverted U if the metric needs to be maximized
- choose the hyper-parameter value that gives you the best validation score

#### Quiz

Which alpha value gives the best validation score? Visualize that model!

```
In []:
```

## The bias-variance tradeoff with Ridge regularization

• cost = MSE +  $\alpha$  \* I2 norm of  $\theta$ 

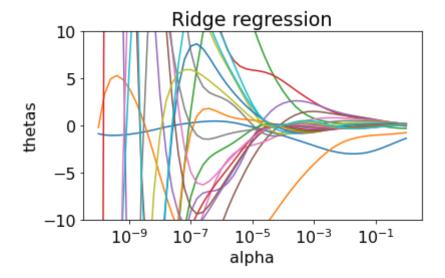
$$L( heta) = rac{1}{n} \sum_{i=1}^n [( heta_0 + \sum_{j=1}^m heta_j x_{ij} - y_i)^2] + rac{lpha}{m} \sum_{j=0}^m heta_j^2$$

- ullet as lpha approaches 0, we reproduce the linear regression weights
- small  $\alpha$  creates high variance
- large  $\alpha$  creates high bias

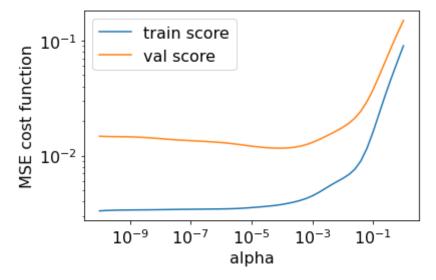
```
In [9]:
         from sklearn.linear model import Ridge
         from sklearn.metrics import mean squared error
         alpha = np.logspace(-10,0,51)
         # arrays to save train and test MSE scores
         train MSE = np.zeros(len(alpha))
         val MSE = np.zeros(len(alpha))
         thetas = []
         # do the fit
         for i in range(len(alpha)):
             # load the linear regression model
             lin reg = Ridge(alpha=alpha[i])
             lin reg.fit(X train, y train)
             thetas.append(lin reg.coef)
             # train and test scores
             train MSE[i] = mean squared error(y train,lin reg.predict(X train))
             val MSE[i] = mean squared error(y val,lin reg.predict(X val))
```

```
In [10]:
    plt.plot(alpha, thetas)
    plt.semilogx()
    plt.ylim([-le1,le1])
    plt.xlabel('alpha')
    plt.ylabel('thetas')
    plt.title('Ridge regression')
```

```
plt.tight_layout()
plt.savefig('figures/ridge_coefs.png',dpi=300)
plt.show()
```



```
In [11]:
    plt.plot(alpha,train_MSE,label='train score')
    plt.plot(alpha,val_MSE,label='val score')
    plt.semilogy()
    plt.semilogx()
    plt.xlabel('alpha')
    plt.ylabel('MSE cost function')
    plt.legend()
    plt.tight_layout()
    plt.savefig('figures/train_val_MSE_ridge.png',dpi=300)
    plt.show()
```



# Quiz

Which  $\alpha$  gives us the best tradeoff between bias and variance?

```
In []:
```

## Regularization

By the end of this lecture, you will be able to

- Describe why regularization is important and what are the two types of regularization
- Describe how regularized linear regression works
- Describe how regularized logistic regression works

## Logistic regression

· Recap: the logloss metric is the cost function

$$L( heta) = -rac{1}{N} \sum_{i=1}^n [y_i \ln(y_i') + (1-y_i) \ln(1-y_i')] \ L( heta) = -rac{1}{N} \sum_{i=1}^n [y_i \ln(rac{1}{1+e^{- heta_0 + \sum_{j=1}^m heta_j x_{ij}}}) + (1-y_i) \ln(1-rac{1}{1+e^{- heta_0 + \sum_{j=1}^m heta_j x_{ij}}})]$$

• the logloss metric with I1 regularization

$$L( heta) = -rac{1}{N} \sum_{i=1}^n [y_i \ln(rac{1}{1+e^{- heta_0 + \sum_{j=1}^m heta_j x_{ij}}}) + (1-y_i) \ln(1-rac{1}{1+e^{- heta_0 + \sum_{j=1}^m heta_j x_{ij}}}))] + rac{lpha}{m} \sum_{j=0}^m | heta_j|$$

• the logloss metric with I2 regularization

$$L( heta) = -rac{1}{N} \sum_{i=1}^n [y_i \ln(rac{1}{1+e^{- heta_0 + \sum_{j=1}^m heta_j x_{ij}}}) + (1-y_i) \ln(1-rac{1}{1+e^{- heta_0 + \sum_{j=1}^m heta_j x_{ij}}}))] + rac{lpha}{m} \sum_{j=0}^m heta_j^2$$

## Logistic regression in sklearn

from sklearn.linear model import LogisticRegression

In [12]:

In the multiclass case, the training algorithm uses the one-vs-rest (OvR)

cross-entropy loss if the 'multi\_class' option is set to 'multinomial'. (Currently the 'multinomial' option is supported only by the 'lbfgs',

scheme if the 'multi class' option is set to 'ovr', and uses the

Logistic Regression (aka logit, MaxEnt) classifier.

'sag', 'saga' and 'newton-cg' solvers.)

This class implements regularized logistic regression using the 'liblinear' library, 'newton-cg', 'sag', 'saga' and 'lbfgs' solvers. \*\*Note that regularization is applied by default\*\*. It can handle both dense and sparse input. Use C-ordered arrays or CSR matrices containing 64-bit floats for optimal performance; any other input format will be converted (and copied).

The 'newton-cg', 'sag', and 'lbfgs' solvers support only L2 regularization with primal formulation, or no regularization. The 'liblinear' solver supports both L1 and L2 regularization, with a dual formulation only for the L2 penalty. The Elastic-Net regularization is only supported by the 'saga' solver.

Read more in the :ref:`User Guide <logistic\_regression>`.

# Parameters

penalty : {'l1', 'l2', 'elasticnet', 'none'}, default='l2'

Used to specify the norm used in the penalization. The 'newton-cg', 'sag' and 'lbfgs' solvers support only 12 penalties. 'elasticnet' is only supported by the 'saga' solver. If 'none' (not supported by the liblinear solver), no regularization is applied.

- .. versionadded:: 0.19
  11 penalty with SAGA solver (allowing 'multinomial' + L1)
- dual : bool, default=False
   Dual or primal formulation. Dual formulation is only implemented for
   l2 penalty with liblinear solver. Prefer dual=False when
   n samples > n features.
- tol : float, default=1e-4
  Tolerance for stopping criteria.
- C : float, default=1.0
   Inverse of regularization strength; must be a positive float.
   Like in support vector machines, smaller values specify stronger regularization.
- fit\_intercept : bool, default=True
   Specifies if a constant (a.k.a. bias or intercept) should be
   added to the decision function.
- intercept\_scaling : float, default=1
   Useful only when the solver 'liblinear' is used
   and self.fit\_intercept is set to True. In this case, x becomes
   [x, self.intercept\_scaling],
   i.e. a "synthetic" feature with constant value equal to
   intercept\_scaling is appended to the instance vector.
   The intercept becomes ``intercept\_scaling \* synthetic\_feature\_weight``.

Note! the synthetic feature weight is subject to 11/12 regularization as all other features. To lessen the effect of regularization on synthetic feature weight

(and therefore on the intercept) intercept\_scaling has to be increased.

class\_weight : dict or 'balanced', default=None
 Weights associated with classes in the form ``{class\_label: weight}``.
 If not given, all classes are supposed to have weight one.

The "balanced" mode uses the values of y to automatically adjust weights inversely proportional to class frequencies in the input data as ``n\_samples / (n\_classes \* np.bincount(y))``.

Note that these weights will be multiplied with sample\_weight (passed through the fit method) if sample\_weight is specified.

.. versionadded:: 0.17
 \*class\_weight='balanced'\*

random\_state : int, RandomState instance, default=None
 Used when ``solver`` == 'sag', 'saga' or 'liblinear' to shuffle the
 data. See :term:`Glossary <random\_state>` for details.

Algorithm to use in the optimization problem.

- For small datasets, 'liblinear' is a good choice, whereas 'sag' and 'saga' are faster for large ones.
- For multiclass problems, only 'newton-cg', 'sag', 'saga' and 'lbfgs' handle multinomial loss; 'liblinear' is limited to one-versus-rest schemes.
- 'newton-cg', 'lbfgs', 'sag' and 'saga' handle L2 or no penalty
- 'liblinear' and 'saga' also handle L1 penalty
- 'saga' also supports 'elasticnet' penalty
- 'liblinear' does not support setting ``penalty='none'``

Note that 'sag' and 'saga' fast convergence is only guaranteed on features with approximately the same scale. You can preprocess the data with a scaler from sklearn.preprocessing.

.. versionadded:: 0.17 Stochastic Average Gradient descent solver.

.. versionadded:: 0.19 SAGA solver.

.. versionchanged:: 0.22

The default solver changed from 'liblinear' to 'lbfgs' in 0.22.

max\_iter : int, default=100

Maximum number of iterations taken for the solvers to converge.

multi\_class : {'auto', 'ovr', 'multinomial'}, default='auto'
 If the option chosen is 'ovr', then a binary problem is fit for each
 label. For 'multinomial' the loss minimised is the multinomial loss fit
 across the entire probability distribution, \*even when the data is
 binary\*. 'multinomial' is unavailable when solver='liblinear'.
 'auto' selects 'ovr' if the data is binary, or if solver='liblinear',
 and otherwise selects 'multinomial'.

.. versionadded:: 0.18

Stochastic Average Gradient descent solver for 'multinomial' case.

.. versionchanged:: 0.22

Default changed from 'ovr' to 'auto' in 0.22.

verbose : int, default=0

For the liblinear and lbfgs solvers set verbose to any positive number for verbosity.

warm start : bool, default=False

```
When set to True, reuse the solution of the previous call to fit as
    initialization, otherwise, just erase the previous solution.
    Useless for liblinear solver. See :term:`the Glossary <warm_start>`.
    .. versionadded:: 0.17
       *warm_start* to support *lbfgs*, *newton-cg*, *sag*, *saga* solvers.
n jobs : int, default=None
    Number of CPU cores used when parallelizing over classes if
    multi_class='ovr'". This parameter is ignored when the ``solver`` is
    set to 'liblinear' regardless of whether 'multi_class' is specified or
    not. ``None`` means 1 unless in a :obj: `joblib.parallel backend`
    context. ``-1`` means using all processors.
    See :term:`Glossary <n_jobs>` for more details.
11_ratio : float, default=None
    The Elastic-Net mixing parameter, with ``0 <= 11_ratio <= 1``. Only
    used if ``penalty='elasticnet'``. Setting ``l1_ratio=0`` is equivalent
    to using ``penalty='12'``, while setting ``l1_ratio=1`` is equivalent
    to using ``penalty='l1'``. For ``0 < l1_ratio <1``, the penalty is a
    combination of L1 and L2.
Attributes
_____
classes_ : ndarray of shape (n_classes, )
    A list of class labels known to the classifier.
coef_ : ndarray of shape (1, n_features) or (n_classes, n_features)
    Coefficient of the features in the decision function.
    `coef_` is of shape (1, n_features) when the given problem is binary.
    In particular, when `multi class='multinomial'`, `coef ` corresponds
    to outcome 1 (True) and `-coef ` corresponds to outcome 0 (False).
intercept_ : ndarray of shape (1,) or (n_classes,)
    Intercept (a.k.a. bias) added to the decision function.
    If `fit intercept` is set to False, the intercept is set to zero.
    `intercept_` is of shape (1,) when the given problem is binary.
    In particular, when `multi_class='multinomial'`, `intercept_`
    corresponds to outcome 1 (True) and `-intercept ` corresponds to
    outcome 0 (False).
n iter : ndarray of shape (n classes,) or (1, )
    Actual number of iterations for all classes. If binary or multinomial,
    it returns only 1 element. For liblinear solver, only the maximum
    number of iteration across all classes is given.
    .. versionchanged:: 0.20
        In SciPy <= 1.0.0 the number of lbfgs iterations may exceed
        ``max_iter``. ``n_iter_`` will now report at most ``max_iter``.
See Also
SGDClassifier: Incrementally trained logistic regression (when given
    the parameter ``loss="log"``).
LogisticRegressionCV: Logistic regression with built-in cross validation.
Notes
```

The underlying C implementation uses a random number generator to select features when fitting the model. It is thus not uncommon, to have slightly different results for the same input data. If that happens, try with a smaller tol parameter. Predict output may not match that of standalone liblinear in certain cases. See :ref: differences from liblinear liblinear differences> in the narrative documentation. References L-BFGS-B -- Software for Large-scale Bound-constrained Optimization Ciyou Zhu, Richard Byrd, Jorge Nocedal and Jose Luis Morales. http://users.iems.northwestern.edu/~nocedal/lbfgsb.html LIBLINEAR -- A Library for Large Linear Classification https://www.csie.ntu.edu.tw/~cjlin/liblinear/ SAG -- Mark Schmidt, Nicolas Le Roux, and Francis Bach Minimizing Finite Sums with the Stochastic Average Gradient https://hal.inria.fr/hal-00860051/document SAGA -- Defazio, A., Bach F. & Lacoste-Julien S. (2014). SAGA: A Fast Incremental Gradient Method With Support for Non-Strongly Convex Composite Objectives https://arxiv.org/abs/1407.0202 Hsiang-Fu Yu, Fang-Lan Huang, Chih-Jen Lin (2011). Dual coordinate descent methods for logistic regression and maximum entropy models. Machine Learning 85(1-2):41-75. https://www.csie.ntu.edu.tw/~cjlin/papers/maxent dual.pdf Examples >>> from sklearn.datasets import load iris >>> from sklearn.linear\_model import LogisticRegression >>> X, y = load iris(return X y=True) >>> clf = LogisticRegression(random state=0).fit(X, y) >>> clf.predict(X[:2, :]) array([0, 0]) >>> clf.predict proba(X[:2, :]) array([[9.8...e-01, 1.8...e-02, 1.4...e-08],[9.7...e-01, 2.8...e-02, ...e-08]]) >>> clf.score(X, y) 0.97... Method resolution order: LogisticRegression sklearn.linear model. base.LinearClassifierMixin sklearn.base.ClassifierMixin sklearn.linear model. base.SparseCoefMixin sklearn.base.BaseEstimator builtins.object

\_\_init\_\_(self, penalty='12', \*, dual=False, tol=0.0001, C=1.0, fit\_intercept

Methods defined here:

=True, intercept\_scaling=1, class\_weight=None, random\_state=None, solver='lbfg s', max iter=100, multi class='auto', verbose=0, warm start=False, n jobs=None,

```
11 ratio=None)
        Initialize self. See help(type(self)) for accurate signature.
    fit(self, X, y, sample_weight=None)
       Fit the model according to the given training data.
       Parameters
        _____
       X : {array-like, sparse matrix} of shape (n_samples, n_features)
            Training vector, where n_samples is the number of samples and
            n_features is the number of features.
       y : array-like of shape (n_samples,)
            Target vector relative to X.
        sample_weight : array-like of shape (n_samples,) default=None
            Array of weights that are assigned to individual samples.
            If not provided, then each sample is given unit weight.
            .. versionadded:: 0.17
               *sample_weight* support to LogisticRegression.
       Returns
        _____
        self
           Fitted estimator.
       Notes
       The SAGA solver supports both float64 and float32 bit arrays.
   predict_log_proba(self, X)
       Predict logarithm of probability estimates.
        The returned estimates for all classes are ordered by the
        label of classes.
       Parameters
        _____
       X : array-like of shape (n samples, n features)
           Vector to be scored, where `n samples` is the number of samples and
            `n features` is the number of features.
       Returns
        T : array-like of shape (n samples, n classes)
            Returns the log-probability of the sample for each class in the
            model, where classes are ordered as they are in ``self.classes ``.
   predict proba(self, X)
       Probability estimates.
        The returned estimates for all classes are ordered by the
        label of classes.
       For a multi class problem, if multi class is set to be "multinomial"
       the softmax function is used to find the predicted probability of
        each class.
       Else use a one-vs-rest approach, i.e calculate the probability
       of each class assuming it to be positive using the logistic function.
        and normalize these values across all the classes.
```

```
Parameters
   X : array-like of shape (n_samples, n_features)
       Vector to be scored, where `n_samples` is the number of samples and
        `n_features` is the number of features.
   Returns
    _____
   T : array-like of shape (n_samples, n_classes)
       Returns the probability of the sample for each class in the model,
       where classes are ordered as they are in ``self.classes ``.
Methods inherited from sklearn.linear model. base.LinearClassifierMixin:
decision function(self, X)
   Predict confidence scores for samples.
   The confidence score for a sample is proportional to the signed
   distance of that sample to the hyperplane.
   Parameters
   X : array-like or sparse matrix, shape (n_samples, n_features)
       Samples.
   Returns
   array, shape=(n samples,) if n classes == 2 else (n samples, n classes)
       Confidence scores per (sample, class) combination. In the binary
       case, confidence score for self.classes [1] where >0 means this
       class would be predicted.
predict(self, X)
   Predict class labels for samples in X.
   Parameters
    -----
   X : array-like or sparse matrix, shape (n samples, n features)
   Returns
    _____
   C : array, shape [n samples]
       Predicted class label per sample.
._____
Methods inherited from sklearn.base.ClassifierMixin:
score(self, X, y, sample weight=None)
   Return the mean accuracy on the given test data and labels.
    In multi-label classification, this is the subset accuracy
   which is a harsh metric since you require for each sample that
   each label set be correctly predicted.
   Parameters
```

X : array-like of shape (n\_samples, n\_features)

Test samples.

```
y : array-like of shape (n_samples,) or (n_samples, n_outputs)
       True labels for `X`.
   sample_weight : array-like of shape (n_samples,), default=None
       Sample weights.
   Returns
   _____
   score : float
       Mean accuracy of ``self.predict(X)`` wrt. `y`.
______
Data descriptors inherited from sklearn.base.ClassifierMixin:
__dict
   dictionary for instance variables (if defined)
   list of weak references to the object (if defined)
______
Methods inherited from sklearn.linear model. base.SparseCoefMixin:
densify(self)
   Convert coefficient matrix to dense array format.
   Converts the ``coef_`` member (back) to a numpy.ndarray. This is the
   default format of ``coef_`` and is required for fitting, so calling
   this method is only required on models that have previously been
   sparsified; otherwise, it is a no-op.
   Returns
   _____
   self
       Fitted estimator.
sparsify(self)
   Convert coefficient matrix to sparse format.
   Converts the ``coef `` member to a scipy.sparse matrix, which for
   L1-regularized models can be much more memory- and storage-efficient
   than the usual numpy.ndarray representation.
   The ``intercept_`` member is not converted.
   Returns
   -----
   self
       Fitted estimator.
   Notes
   For non-sparse models, i.e. when there are not many zeros in ``coef_``,
   this may actually *increase* memory usage, so use this method with
   care. A rule of thumb is that the number of zero elements, which can
   be computed with ``(coef_ == 0).sum()``, must be more than 50% for this
   to provide significant benefits.
   After calling this method, further fitting with the partial_fit
   method (if any) will not work until you call densify.
```

```
Methods inherited from sklearn.base.BaseEstimator:
__getstate__(self)
__repr__(self, N_CHAR_MAX=700)
   Return repr(self).
__setstate__(self, state)
get params(self, deep=True)
    Get parameters for this estimator.
    Parameters
    -----
    deep : bool, default=True
        If True, will return the parameters for this estimator and
        contained subobjects that are estimators.
    Returns
    _____
    params : dict
        Parameter names mapped to their values.
set_params(self, **params)
    Set the parameters of this estimator.
    The method works on simple estimators as well as on nested objects
    (such as :class: `~sklearn.pipeline.Pipeline`). The latter have
    parameters of the form ``<component>__<parameter>`` so that it's
    possible to update each component of a nested object.
   Parameters
    -----
    **params : dict
        Estimator parameters.
    Returns
    _____
    self : estimator instance
       Estimator instance.
```

#### Mud card

In []: