# Hardware-Accelerated Kálmán Filter

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***Abstract*—The Kálmán filter, or linear quadratic estimation (LQE), is frequently used both as a filter and a predictor of future states, and it does this very well. Unfortunately, its predictive and estimative capacity is constrained to its ability to execute both an entire estimation and prediction loop in each sample cycle. The Kálmán filter will be optimized in software and then hardware to compare computational cost, time efficiency, turnaround time, and scalability.**

***Index Terms*—Kálmán Filter, Linear Quadratic Estimation (LQE), FPGA, Embedded, Optimization, Machine Learning, Filtering, Location Tracking**

## I. Introduction

The Kálmán filter, or linear quadratic estimation (LQE), was developed in the late 1950's and early 1960's by Swerling, Kálmán, and Bucy. The filter uses time-dependent measurements of inter-related data to estimate an unknown variable; one of its first implementations was done by Stanley F, Schmidt, who integrated the filter with Apollo's navigation system to best estimate spacecraft trajectory in the Apollo program. True to its initial implementation, it continues to be used determine location in navigation systems for missiles and spacecraft, and has even been used in financial analysis and noise reduction. As useful as the Kálmán filter is for prediction and reaction, each recursive step of the filter must be completed before the next sample of measurements is received. The time constraints presented require either slower sampling rates, which loses accuracy and data, or an optimized solution. Research here will be directed toward the hardware optimization of the Kálmán filter.

## II. Paradigm Overview

For the sake of simplicity, the following three input variables, $x$ (position), $v$ (velocity), and $a$, (acceleration), will be considered in the following paradigm. Note that while the aforementioned variables deal specifically with position, they are more universally-applicable as information regarding $f(x)$, $\frac{df(x)}{dx}$, and $\frac{df(x)^2}{d^2x}$, or any given function and its respective derivatives.

Ideally, measurement of position alone would be enough to generate accurate, reliable estimations of prediction, but due to noise and error they are unreliable. Thus, velocity and acceleration are estimated, then integrated and compared to the position estimation.

### *A. Position Estimation from Position Data, Pros and Cons*

Position data are instantaneous; they know nothing of past points or trends. According to the data, the current position datum has nothing do do with past information, and future inputs will have little to do with the current datum. This means that the information cannot experience drift, present inaccuracies due to error in past estimations. On the other hand, the limited amount of information creates a high level of energy, or variance, in the data. Thus measurement of position alone yields 'jumpiness' in the readings, where the position seems to instantaneously change with little pattern.

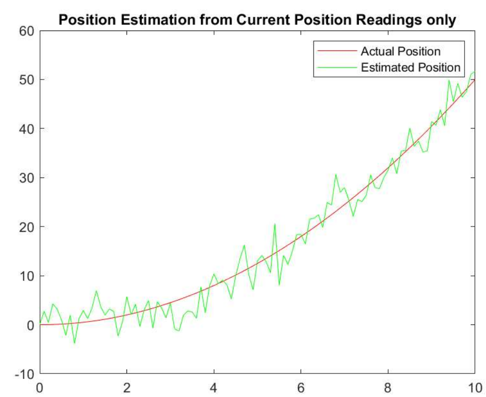


Fig. 1. Note the jumpiness of position readings.

This problem can be addressed by assuming that the object follows the natural laws of physics, namely, that the object has a velocity contributing to its change in position, and an acceleration contributing to its velocity, which then contributes to its position.

### *B. Position Estimation from Velocity Data, Pros and Cons*

While the reception of an object's velocity is instantaneous, its effect on the position of said object is integral; the sum of velocities creates the change in position, as in the following ideal equation:

$$\int_{(\tau-\Delta\tau)}^{\tau} v(t)\,dt = \Delta x$$

Because the velocity-based position estimation is based on the current input and all previous inputs, the location estimation contains much less energy, or is less variant, than the position data alone. Additionally, because position based on velocity is time-dependent, future position may be predicted as well by shifting the above integral by $\Delta t$, resulting in the following equation.

$$\int_{\tau}^{(\tau+\Delta\tau)} v(t)\, dt = \Delta x_{\tau+\Delta\tau}$$

Because estimations of the actual velocity would also have noise, however, the true integrated estimation and prediction would result as follows:

$$\int_{(\tau-\Delta\tau)}^{\tau} v(t) + v_{err}\, dt = \Delta x + v_{err} t$$

As seen above, the velocity data integrate well into a smooth, consistent position function, but result in an error term that grows proportionally with time. The error in each measurement of velocity compounds as time progresses. That time-proportional growth in error is called drift. Additionally, while the velocity data may integrate well to position, the velocity measured may vary between samples. The variance in velocity may be solved by adding yet another derivative, acceleration.

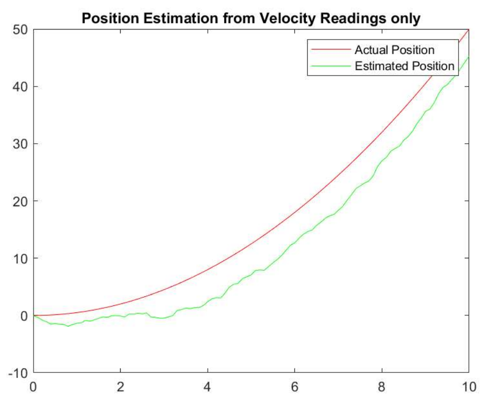


Fig. 2. Note how the the velocity decreases jumpiness, but also skews the estimations.

### *C. Position and Velocity Estimation from Acceleration Data, Pros and Cons*

Acceleration data is used to estimate and predict velocity and, as a result, position. Its effects are slower to be seen, but just as present, as is seen in the following equation:

$$\int_{(\tau-\Delta\tau)}^{\tau} [\int_{(\tau-2\Delta\tau)}^{(\tau-\Delta\tau)} a(t)\, dt]\, dt = \int_{(\tau-\Delta\tau)}^{\tau} v(t)+a(t)t\ dt = \Delta x$$

Because the acceleration- and velocity-based position estimation is based on the current input and all previous inputs of both acceleration and velocity, the location prediction contains even less energy, or is less variant, than both the position data alone and the velocity-position analysis, creating an even smoother position estimation graph. However, because both acceleration and velocity measurements have noise now, the integration would result as follows:

$$\int_{(\tau-\Delta\tau)}^{\tau} [\int_{(\tau-2\Delta\tau)}^{(\tau-\Delta\tau)} a(t)dt]dt = \int_{(\tau-\Delta\tau)}^{\tau} v(t)+[a(t)+a_{err}]t\, dt$$
$$= \Delta x + v_{err} t + \frac{a_{err}}{2} t^2$$

As seen above, the acceleration integrates well into a smooth, consistent position function, but results in an error term that now grows proportionally with the square of time, which, if used alone creates even greater drift from the actual values. The drift can be addressed, as will be seen shortly. First, however, the added predictive benefits of acceleration should be noted. Where before, the effects of velocity could be seen $\Delta\tau$ seconds in advance, Acceleration measurements provide a relatively reliable prediction $2\Delta\tau$ seconds in advance. The prediction window for a model using acceleration data is twice that of one using velocity alone.

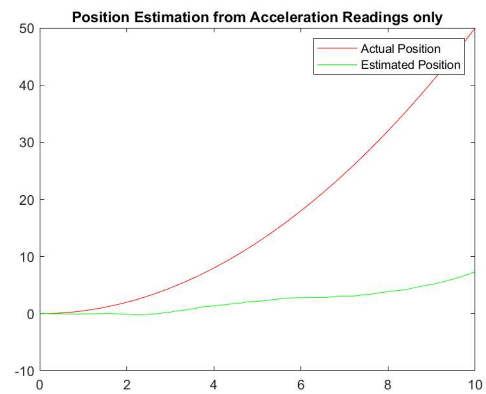


Fig. 3. Note the large drift in acceleration.

### *D. The Statistical Combination of Position, Velocity, and Acceleration Data to Increase Accuracy of Estimations and Predictions*

The power of each data type is realized when the Kálmán filter combines the three together. Position data experience a high variance, but no drift, and no inherent predictive capabilities. Velocity has a lower variance, a linear drift of position estimation which can be minimized using acceleration, and an inherent predictive window of $\Delta\tau$, or the period between samples. While acceleration minimizes the positional drift due to velocity error and has an inherent prediction window of $2\Delta\tau$, its position estimation tends to drift on the order of $t^2$.

The Kálmán filter is powerful because it capitalizes on the strengths of all three data types, creating a very accurate estimation and prediction of the object's position, or $f(x)$. It does so by using the acceleration and velocity as a predictor of position, then comparing that predicted position to the position recorded by the positional tracking unit. The result of the mathematical comparison yields the estimation of current position, which is then used to iteratively predict the next position. The following graph shows an example of its implementation:

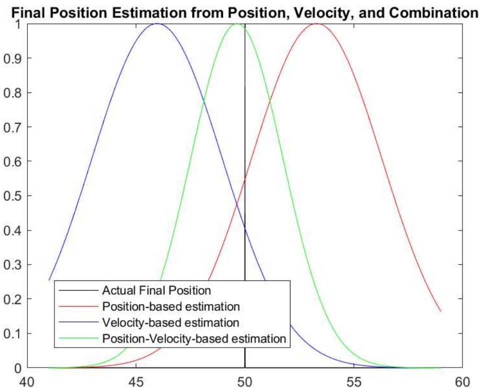


Fig. 4. Note the way position and velocity estimations balance each other out.

## III. Mathematical Implementation

## IV. Prepare Your Paper Before Styling

Before you begin to format your paper, first write and save the content as a separate text file. Complete all content and organizational editing before formatting. Please note sections IV-A–IV-E below for more information on proofreading, spelling and grammar.

Keep your text and graphic files separate until after the text has been formatted and styled. Do not number text heads—LaTeX will do that for you.

### A. Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, ac, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

### B. Units

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as "3.5-inch disk drive".
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### C. Equations

Number equations consecutively. To make your equations more compact, you may use the solidus ( / ), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \tag{1}$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use "(1)", not "Eq. (1)" or "equation (1)", except at the beginning of a sentence: "Equation (1) is . . ."

### D. LaTeX-Specific Advice

Please use "soft" (e.g., `\eqref{Eq}`) cross references instead of "hard" references (e.g., `(1)`). That will make it possible to combine sections, add equations, or change the order of figures or citations without having to go through the file line by line.

Please don't use the `{eqnarray}` equation environment. Use `{align}` or `{IEEEeqnarray}` instead. The `{eqnarray}` environment leaves unsightly spaces around relation symbols.

Please note that the `{subequations}` environment in LaTeX will increment the main equation counter even when there are no equation numbers displayed. If you forget that, you might write an article in which the equation numbers skip from (17) to (20), causing the copy editors to wonder if you've discovered a new method of counting.

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### E. Some Common Mistakes

- The word "data" is plural, not singular.
- The subscript for the permeability of vacuum $\mu_0$, and other common scientific constants, is zero with subscript formatting, not a lowercase letter "o".
- In American English, commas, semicolons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
- A graph within a graph is an "inset", not an "insert". The word alternatively is preferred to the word "alternately" (unless you really mean something that alternates).
- Do not use the word "essentially" to mean "approximately" or "effectively".
- In your paper title, if the words "that uses" can accurately replace the word "using", capitalize the "u"; if not, keep using lower-cased.
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- The prefix "non" is not a word; it should be joined to the word it modifies, usually without a hyphen.

- There is no period after the "et" in the Latin abbreviation "et al.".
- The abbreviation "i.e." means "that is", and the abbreviation "e.g." means "for example".

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### *G. Identify the Headings*

Headings, or heads, are organizational devices that guide the reader through your paper. There are two types: component heads and text heads.

Component heads identify the different components of your paper and are not topically subordinate to each other. Examples include Acknowledgments and References and, for these, the correct style to use is "Heading 5". Use "figure caption" for your Figure captions, and "table head" for your table title. Run-in heads, such as "Abstract", will require you to apply a style (in this case, italic) in addition to the style provided by the drop down menu to differentiate the head from the text.

Text heads organize the topics on a relational, hierarchical basis. For example, the paper title is the primary text head because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and, conversely, if there are not at least two sub-topics, then no subheads should be introduced.

### *H. Figures and Tables*

*a) Positioning Figures and Tables:* Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation "Fig. 5", even at the beginning of a sentence.

TABLE I
TABLE TYPE STYLES

| Table Head | Table Column Head | | |
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[a]Sample of a Table footnote.

Fig. 5. Example of a figure caption.

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity "Magnetization", or "Magnetization, M", not just "M". If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write "Magnetization (A/m)" or "Magnetization {A[m(1)]}", not just "A/m". Do not label axes with a ratio of quantities and units. For example, write "Temperature (K)", not "Temperature/K".

## ACKNOWLEDGMENT

The preferred spelling of the word "acknowledgment" in America is without an "e" after the "g". Avoid the stilted expression "one of us (R. B. G.) thanks ...". Instead, try "R. B. G. thanks...". Put sponsor acknowledgments in the unnumbered footnote on the first page.

## REFERENCES

Please number citations consecutively within brackets [1]. The sentence punctuation follows the bracket [2]. Refer simply to the reference number, as in [3]—do not use "Ref. [3]" or "reference [3]" except at the beginning of a sentence: "Reference [3] was the first ..."

Number footnotes separately in superscripts. Place the actual footnote at the bottom of the column in which it was cited. Do not put footnotes in the abstract or reference list. Use letters for table footnotes.

Unless there are six authors or more give all authors' names; do not use "et al.". Papers that have not been published, even if they have been submitted for publication, should be cited as "unpublished" [4]. Papers that have been accepted for publication should be cited as "in press" [5]. Capitalize only the first word in a paper title, except for proper nouns and element symbols.

For papers published in translation journals, please give the English citation first, followed by the original foreign-language citation [6].

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