Hardware-Accelerated Kálmán Filter

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***Abstract*—The Kálmán filter, or linear quadratic estimation (LQE), was developed in the late 1950's and early 1960's by Swerling, Kalman, and Bucy. The filter uses time-dependent measurements of inter-related data to estimate an unknown variable; one of its first implementations was by**

***Index Terms*—Kálmán Filter, Linear Quadratic Estimation (LQE), FPGA, Embedded, Optimization, Machine Learning, Filtering, Location Tracking**

I. Introduction

The Kálmán filter, or linear quadratic estimation (LQE), was developed in the late 1950's and early 1960's by Swerling, Kálmán, and Bucy. The filter uses time-dependent measurements of inter-related data to estimate an unknown variable; one of its first implementations was done by Stanley F, Schmidt, who integrated the filter with Apollo's navigation system to best estimate spacecraft trajectory in the Apollo program. True to its initial implementation, it continues to be used determine location in navigation systems for missiles and spacecraft, and has even been used in financial analysis and noise reduction. As useful as the Kálmán filter is for prediction and reaction, each recursive step of the filter must be completed before the next sample of measurements is received. The time constraints presented require either slower sampling rates, which loses accuracy and data, or an optimized solution. Research here will be directed toward the hardware optimization of the Kálmán filter.

II. Paradigm Overview

For the sake of simplicity, the following three input variables, x (position), v (velocity), and a, (acceleration), will be considered in the following paradigm. Note that while the aforementioned variables deal specifically with position, they are more universally-applicable as information regarding $f(x)$, $\frac{df(x)}{dx}$, and $\frac{df(x)^2}{d^2x}$, or any given function and its respective derivatives.

Ideally, measurement of position alone would be enough to generate accurate, reliable estimations of prediction, but due to noise and error they are unreliable. Thus, velocity and acceleration are estimated, then integrated and compared to the position estimation.

A. Position Estimation from Position Data, Pros and Cons

Position data are instantaneous; they know nothing of past points or trends. According to the data, the current position datum has nothing do do with past information, and future inputs will have little to do with the current datum. This means that the information cannot experience drift, present inaccuracies due to error in past estimations. On the other hand, the limited amount of information creates a high level of energy, or variance, in the data. Thus measurement of position alone yields 'jumpiness' in the readings, where the position seems to instantaneously change with little pattern.

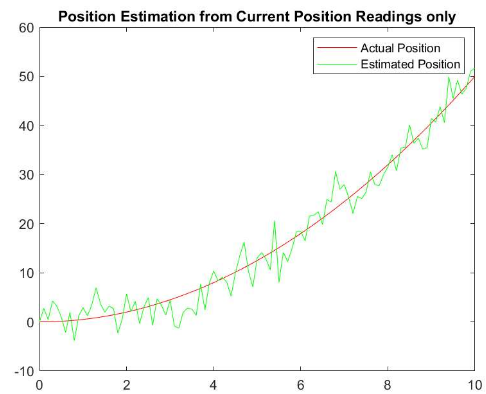

Fig. 1. Note the jumpiness of position readings.

B. Position Estimation from Velocity Data, Pros and Cons

While the reception of an object's velocity is instantaneous, its effect on the position of said object is integral; the sum of velocities creates the change in position, as in the following ideal equation:

$$\int_{(\tau-\Delta\tau)}^{\tau} v(t)\,dt = \Delta x$$

Because the velocity-based position estimation is based on the current input and all previous inputs, the location estimation contains much less energy, or is less variant, than the position data alone. Additionally, because position based on velocity is time-dependent, future position may be predicted as well by shifting the above integral by Δt, resulting in the following equation.

$$\int_{\tau}^{(\tau+\Delta\tau)} v(t)\,dt = \Delta x_{\tau+\Delta\tau}$$

Because estimations of the actual velocity would also have noise, however, the true integrated estimation and prediction would result as follows:

$$\int_{(\tau-\Delta\tau)}^{\tau} v(t) + v_{err}\,dt = \Delta x + v_{err}t$$

As seen above, the velocity data integrate well into a smooth, consistent position function, but result in an error term that grows proportionally with time. The error in each measurement of velocity compounds as time progresses. That time-proportional growth in error is called drift. Additionally, while the velocity data may integrate well to position, the velocity measured may vary between samples. The variance in velocity may be solved by adding yet another derivative, acceleration.

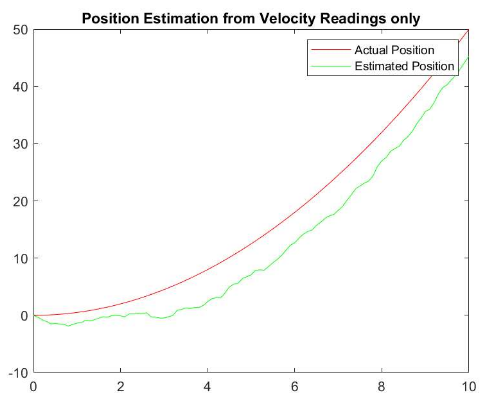

Fig. 2. Note how the the velocity decreases jumpiness, but also skews the estimations.

C. Position and Velocity Estimation from Acceleration Data, Pros and Cons

Acceleration data is used to estimate and predict velocity and, as a result, position. Its effects are slower to be seen, but just as present, as is seen in the following equation:

$$\int_{(\tau-\Delta\tau)}^{\tau}[\int_{(\tau-2\Delta\tau)}^{(\tau-\Delta\tau)} a(t)\,dt]\,dt = \int_{(\tau-\Delta\tau)}^{\tau} v(t)+a(t)t\ dt = \Delta x$$

Because the acceleration- and velocity-based position estimation is based on the current input and all previous inputs of both acceleration and velocity, the location prediction contains even less energy, or is less variant, than both the position data alone and the velocity-position analysis, creating an even smoother position estimation graph. However, because both acceleration and velocity measurements have noise now, the integration would result as follows:

$$\int_{(\tau-\Delta\tau)}^{\tau}[\int_{(\tau-2\Delta\tau)}^{(\tau-\Delta\tau)} a(t)dt]dt = \int_{(\tau-\Delta\tau)}^{\tau} v(t)+[a(t)+a_{err}]t\,dt$$
$$= \Delta x + v_{err}t + \frac{a_{err}}{2}t^2$$

As seen above, the acceleration integrates well into a smooth, consistent position function, but results in an error term that now grows proportionally with the square of time, which, if used alone creates even greater drift from the actual values. The drift can be addressed, as will be seen shortly. First, however, the added predictive benefits of acceleration should be noted. Where before, the effects of velocity could be seen $\Delta\tau$ seconds in advance, Acceleration measurements provide a relatively reliable prediction $2\Delta\tau$ seconds in advance. The prediction window for a model using acceleration data is twice that of one using velocity alone.

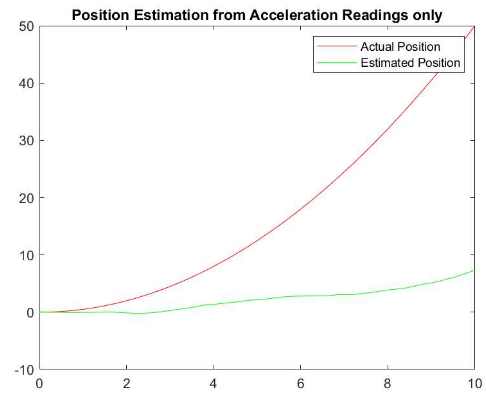

Fig. 3. Note the large drift in acceleration.

D. The Statistical Combination of Position, Velocity, and Acceleration Data to Increase Accuracy of Estimations and Predictions

The power of using the Kálmán filter becomes evident when combining the three data together. Position data experience a high variance, but no drift, and no inherent predictive capabilities. Velocity has a lower variance, a linear drift of position estimation which can be minimized using acceleration, and an inherent predictive window of $\Delta\tau$, or the period between samples. While acceleration minimizes the positional drift due to velocity error and has an inherent prediction window of $2\Delta\tau$, its position estimation tends to drift on the order of t^2.

The obvious question is "How can they be combined to best estimate current position and predict future positions?"