Enzyme Kinetics

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$$E + S \underset{k_2}{\overset{k_1}{\rightleftharpoons}} ES \overset{k_3}{\rightarrow} E + P$$

1. Using the law of mass action, write down four equations for the rate of changes of the four species, E, S, ES, and P.

Solution:

Using the law of mass action, the following four equations can be obtained:

$$\begin{cases} \dfrac{d[E]}{dt} = k_2[ES] + k_3[ES] - k_1[E][S] \\ \dfrac{d[S]}{dt} = k_2[ES] - k_1[E][S] \\ \dfrac{d[ES]}{dt} = k_1[E][S] - k_2[ES] - k_3[ES] \\ \dfrac{d[P]}{dt} = k_3[ES] \end{cases}$$

2. Write a code to numerically solve these four equations using the fourth-order Runge-Kutta method. For this exercise, assume that the initial concentration of E is 1 μM, the initial concentration of S is 10 μM, and the initial concentrations of ES and P are both 0. The rate constants are: k1=100μM/min, k2=600μM /min, k3=150μM/min.

Solution:

Based on the four equations for the rate of changes of the four species, implementing the fourth-order Runge-Kutta method to simulate the kinetics of the chemical reaction. Due to the properties of enzyme in this reaction, the total concentration of enzyme will stay the same.

During the reaction, the instantaneous concentrations of the E, S, and ES is expressed in terms of e_T, $s(t)$, $es(t)$ separately. Simplify the equatins as follows:

$$\begin{cases} \dfrac{d[s(t)]}{dt} = k_2[es(t)] - k_1[e_T - es(t)][s(t)] \\ \dfrac{d[es(t)]}{dt} = k_1[e_T - es(t)][s(t)] - k_3[es(t)] - k_2[es(t)] \\ \dfrac{d[P]}{dt} = k_3[es(t)] \end{cases}$$

Write code to implement the fourth-order Runge-Kutta method to simulate the kinetics of the chemical reaction. Define functions to calculate the derivation of $s(t)$ and $es(t)$ and set initial concentrations of each substance. Write loops to perform the simulation to the specified time precision. Use matplotlib to plot the simulation results. Please find the code implementation attached to this test report.

Set time precision as 0.01 second, run the simulation of the chemical reaction.

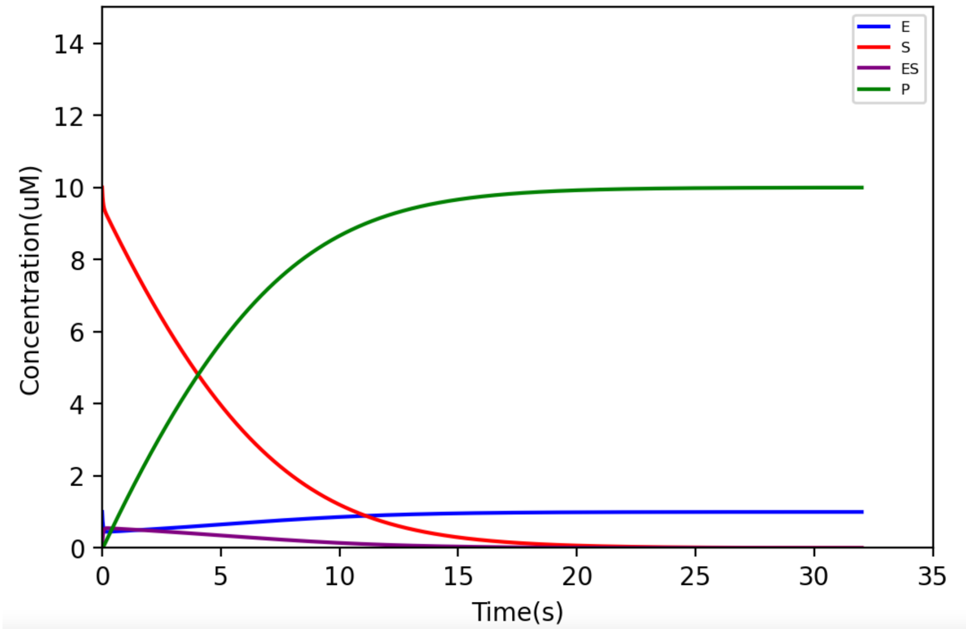

Figure 1 Concentration-Time Plot

The figure above is the simulation result. During the reaction:

a. At the very beginning of the reaction, the concentration of E decrease rapidly as the concentration of ES increase rapidly, indicating that the chemical reactions are taking place violently.

b. The reaction rate gradually decreases between about 0 and 10 seconds. The reaction continuously produces P and consumes S, while the concentrations of ES and E gradually return to their initial values.

c. The reaction rate continues to decrease at around 10 to 20 seconds. The concentrations of E, S, ES and P gradually converge to their final values of approximately 1, 0, 10 and 0 µM respectively, indicating that the reaction is gradually approaching dynamic equilibrium.

3. We define the velocity, V, of the enzymatic reaction to be the rate of change of the product P. Plot the velocity V as a function of the concentration of the substrate S. You should find that, when the concentrations of S are small, the velocity V increases approximately linearly. At large concentrations of S, however, the velocity V saturates to a maximum value, Vm. Find this value Vm from your plot.

Solution:

From the above equation it follows that:

$$\frac{d[V]}{dS} = k_3 \frac{d[es(t)]}{dt}\frac{dt}{dS} = \frac{k_3[k_1 es(t) - k_2 es(t) - k_3 es(t)]}{-k_1 es(t) + k_2 es(t)}$$

Run simulation and plot the figure using matplotlib lib.

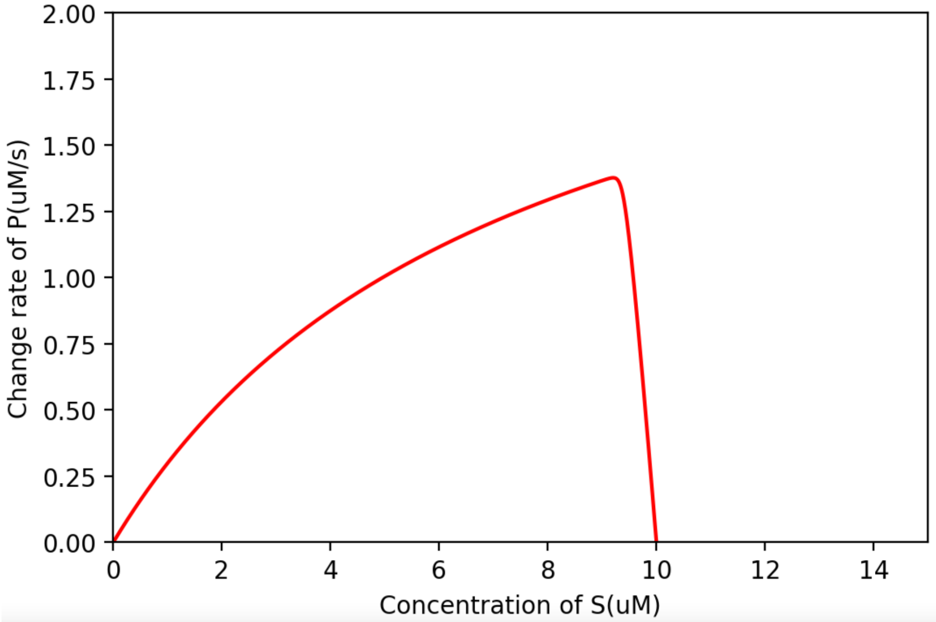

Figure 2 Change Rate (P)-Concentration (S) Plot

The figure above is the change rate of P (μM/s) – concentration of S (μM) plot, indicating that:

a. When S is small, P is positively correlated with S; when S is large, P is negatively correlated with S.

b. The Vm is at the peak point of this figure. At this point the value concentration of S is approximately 9 μM. The value of Vm is calculated to be approximately 1.4 μM.

Thus, $Vm = 1.4\mu M$.