Part 1: Local Volatility

1) Introduction:

American style options have been traded on the Chicago Mercantile Exchange's (CME) S&P 500 futures contracts since 28th January 1983. Before Black Monday, October 19th 1987, the curve of the implied volatilities (volatility smile) for options on the S&P 500 was symmetrical. On that day the S&P 500 dropped 20.4 percent (Schwert, 1990) – an event known as a Black Swan. A development from the October 1987 market crash is that almost always today, implied volatilities increase with decreasing strike price i.e. out of the money (OTM) put options have increased in price relative to OTM call options. This is a result of "crashophobia", i.e. investors fearing a reoccurrence of a major stock market crash. OTM puts can be used as insurance against negative stock market shocks. This feature has caused the smile to lose its symmetry and become biased towards the put side. It is often referred to as a "negative" skew (Derman, et al., 1996) and is illustrated in Figure 1.

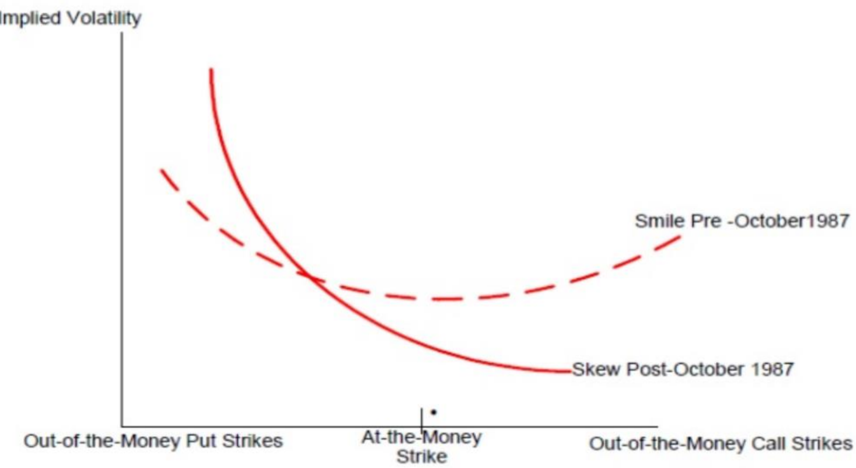


Figure 1: The S&P 500 Implied Volatility Curve Pre- and Post-1987

The Chicago Board Options Exchange (CBOE) Skew Index is an options based indicator that measures the perceived tail risk of the distribution of the S&P 500 log returns at the 30 day horizon. The index measures outlier returns that fall two or more standard deviations away from the mean which are

outside the range measured by the CBOE VIX (CBOE, 2014). Tail risk is the uncertainty associated with the increased probability of outlier returns. SKEW measures this additional risk. Growth in perceived tail risk increases the relative demand for low strike puts, this leads to an overall steepening of the curve of implied volatilities corresponding to increases in SKEW. The returns distribution of the S&P 500 has a fat left tail resulting from a number of negative outliers verifying the non-normal distribution of the log returns (Brown & Warner, 1985) and validating the crash-o-phobia sentiment amongst investors.

CBOE SKEW is derived from the price of the S&P 500 skewness which is calculated in the same way as the S&P 500 VIX and was valued at 125.66 on the 05/11/2014. This was the first reference date of the project; the final project reference date was 09/12/2014. CBOE SKEW calculations are outlined in the CBOE Skew Index-Skew paper (CBOE, 2010) and involve a convoluted methodology that requires the valuation of a SKEW portfolio of both near and next-term option calculations. The following technique offers an alternative proprietary risk analysis tool to determine the tail risk associated with the S&P 500. It involved the use of the Bloomberg local volatility surface to build a 1-month implied price distribution and using this to derive an independent at-risk benchmark to substantiate the corresponding downside tail risk exposure implied by CBOE.

2) Local Volatility Model:

When the Black-Scholes model incorporates a volatility surface, the sum of the volatilities must be entered in order to derive a theoretical option price matching that of the market. Often is the case that this sum figure is used by options market makers to quote prices. One of the primary features of the Black-Scholes (B-S) model as per (Derman, et al., 1996) is that the valuation of such options is preference free as such derivatives can be hedged thereby rendering risk preferences as irrelevant. However, in reality this is not the case. An index option can be valued as though the underlying price is riskless but as mentioned in the introduction, we see risk-preferences present in such option markets in the form of market participants willing to pay a higher premium for out-of-the-money (OTM) puts which highlights a lack of risk neutrality in the marketplace. A second assumption made by the B-S model is that returns on stocks or indices evolve normally with a local volatility which stays constant over all times and levels. This assumption leads to a constant index level spacing similar to that of a binomial tree which contradicts the reality of the marketplace according to (Derman, et al., 1996) due to the volatility skew most certainly resulting in dynamic index level spacing given a rise or fall in volatility. Additionally, (Dupire, 1997) illustrates how poor the B-S model is due to the fact that implied volatility for one option does not translate to marker prices for another option as each has its own implied volatility level as per the Nikkei example of quoted volatilities $\sigma = 20\%$ for where $T = 0.5$ and $\sigma = 18\%$ on the same index where $T = 1$. The fact that both volatilities stay constant is "*worrying*"

The local volatility surface is an extension of the Black-Scholes model allowing for a consistent index market implied volatility surface "*without losing out on the theoretical and practical advantages of the B-S model*". (Derman, et al., 1996), state that rational market makers are likely to base option prices on their *"estimates of future volatility and that the B-S represents an estimated future average future volatility of the index during the options lifetime"*. This implies that such a measure is in fact a "*global measure of volatility as opposed to a local volatility varying at each node*" The variation in the market global measure suggests that the average future volatility in the indices options market is dependent on

the strike (K) and expiration (T) of the option. According to (Derman, et al., 1996) *"A quantity whose average varies with the range over which its calculated must itself vary locally"* which implies a relationship between sigma (σ), future index level (E[St]) and time (t) forming an "*obscure, hidden, local volatility surface*" within the implied volatility surface.

According to (Derman, et al., 1996), once the prices, strikes and expirations are known then we can obtain a local volatility surface whereby $\sigma(S,t)$ can be uniquely determined. The local volatility model assumes that the price of an index option is driven primarily by market views of the local index volatility in the future and time. The local volatility model avoids using implied volatilities to price options, instead it uses the local volatility surface of *"liquid standard options"* in order to deduce future or forward local volatilities that can be used to then price options at time t where $t > 0$. According to (Murphy, 2014), the local volatility function is inferred from the implied volatility surface and must match the market price for the corresponding strike-maturity vanilla option. Once $\sigma(S_t,t)$ is fixed, the evolution of the index becomes known and a market consistent price can be calculated subject to no-arbitrage conditions.

Using the Bloomberg Terminal (OVDV) local volatility function, we retrieved the local volatility levels for the money-ness range 30%-170% in increments of 2.5% from a reference date of 7th of November 2014 to December 5th 2014. We graphed these local volatilities which can be seen below *(Figure 2)*. This graph illustrates the lasting "crashophobia" effect on current market participants with out-of-the-money (OTM) puts experiencing the vast amount of volatility alongside a noticeable plummeting of volatility as the option nears at-the-money (ATM) status interestingly, OTM calls experience relatively low levels of local volatility compared to their put counterparts. This is very much in line with (CBOE, 2010) the "*highly sensitized*" market that exists today with regard to a large downward jump in the SPX and the negative skew associated with it. OTM put volatilities ranging from 60% to 80% at 80% moneyness and the OTM calls reaching a peak of just over 20% volatility for the same level of moneyness.

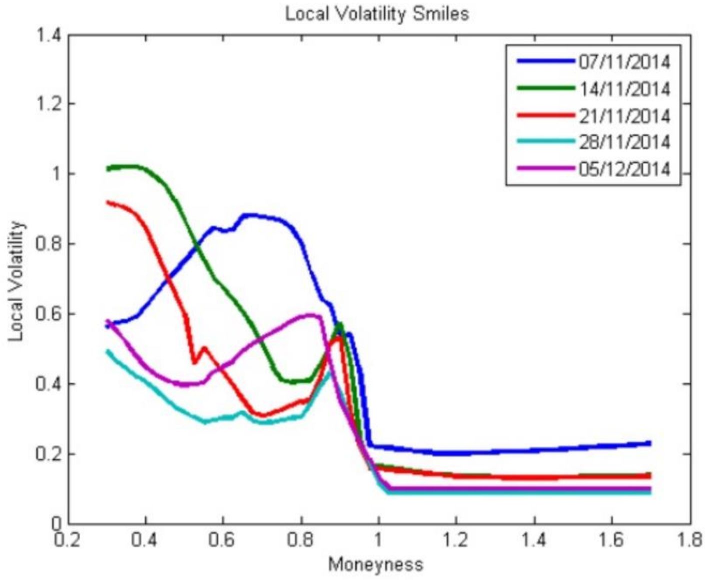


Figure 2: Local volatility plots of our reference period options

By creating a date vector between expiry dates and interpolating between expiration, we were able to create a local volatility surface *(See Figure 3)*. The left-hand surface is our "fine-mesh" which utilizes the spline interpolation smoothing with the right-hand surface using a linear interpolation "coarse-mesh". The use of a spline interpolation in order to generate a smooth surface is a critical component to the model as the accuracy and pricing performance of the local volatility models *"crucially depends on absence of arbitrage in the implied volatility surface"* as per (Fengler, 2005) and that using such smoothing technique eliminates negative transition probabilities, negative local volatilities and reduces mispricing. By doing this, we can consider our local volatility fine-mesh to be arbitrage-free. One can interpret this smooth local volatility surface as the "*collective expectations of option market participants*" assuming the options prices are fair (Derman, et al., 1996).

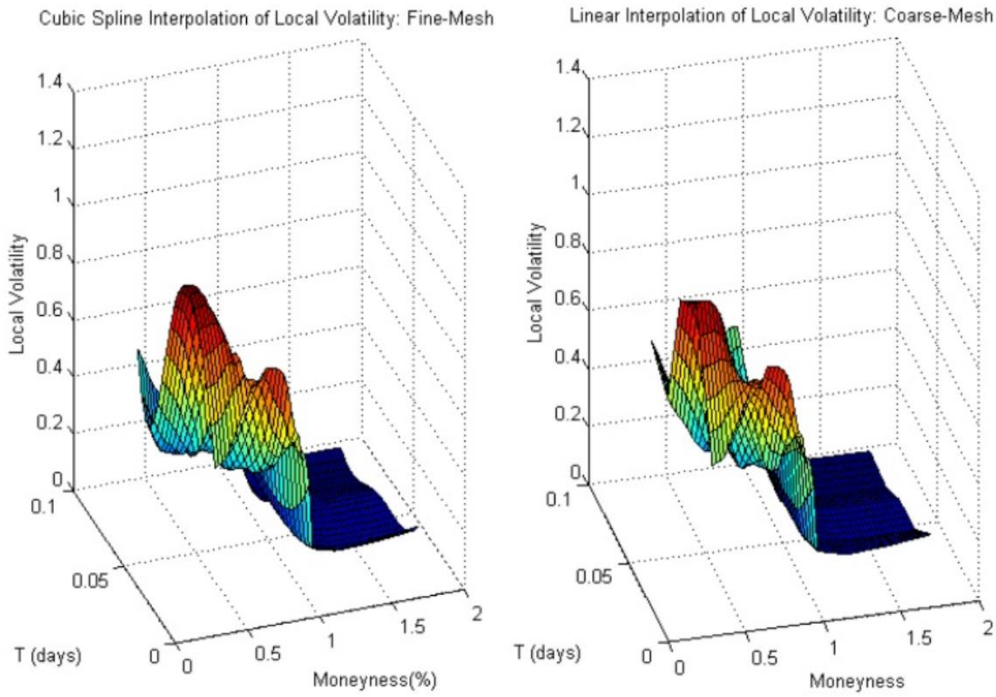


Figure 3: Spline and linear interpolated Local Volatility Surfaces

Using the implied forward prices on Bloomberg terminal to match our expiries, we interpolated the implied forward prices to match our interpolated dates. With our interpolated implied forward prices, local volatility, initial index price of $2023.22 and a μ = 0.567% we ran a Monte Carlo simulation with 10,000 simulations *(See Figure 4)* to simulate our stock out 20 trading days into the future. Our highest level of local volatility was at t = .025 which was a result of our simulated stock price declining at that point in time in line the systematic negative correlation between index level and local volatility as per (Derman, et al., 1996). It should be noted that given the nature of our project, our simulated index levels were dependent on the local volatility which meant that whilst the inverse relationship was present, it was due to the local volatility determining the simulated stock index level, at t = 0.25, our local volatility was at its peak meaning the simulated index level would be fall. The local volatility surface on Bloomberg compared to that of implied volatility for the index highlighted the sensitivity of local volatility to changes in the market level (*See Figure 5*) which is approximately double the sensitivity according to (Murphy, 2014).

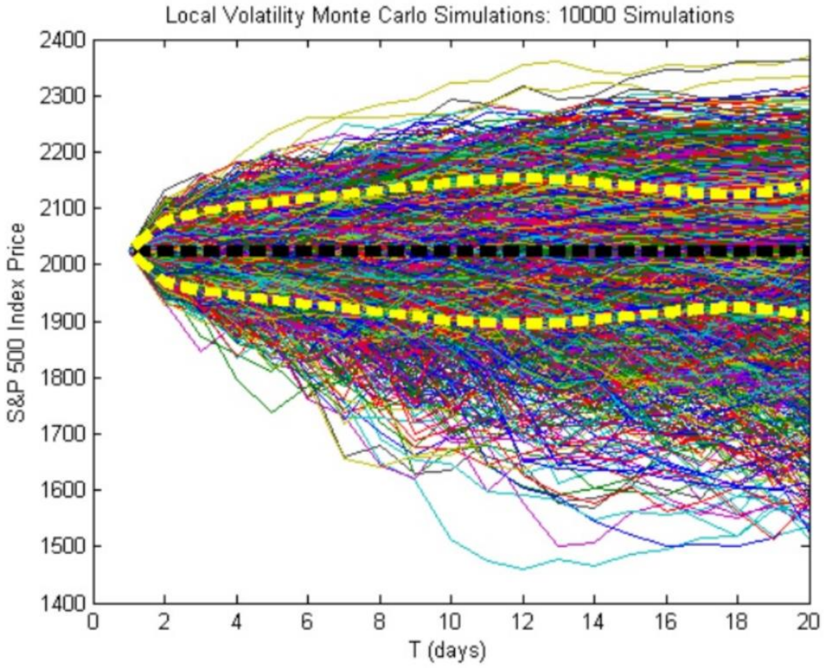
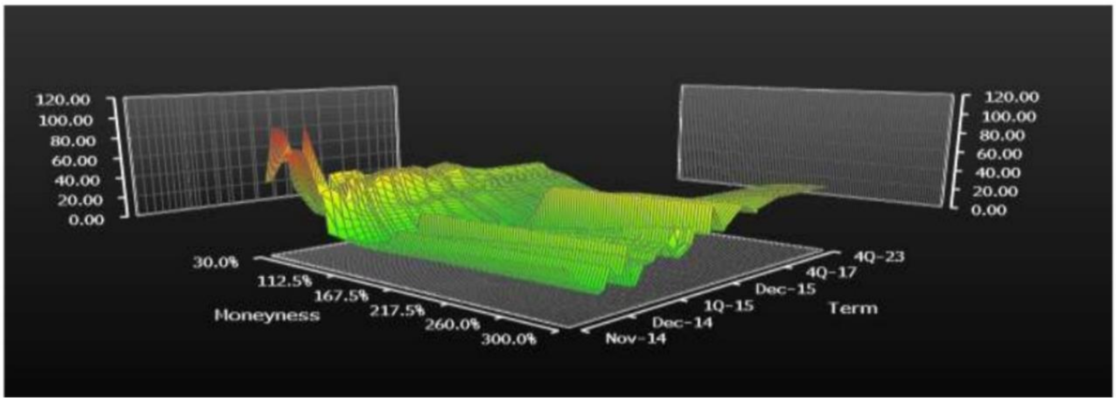


Figure 4: Monte Carlo simulations with 95% confidence intervals



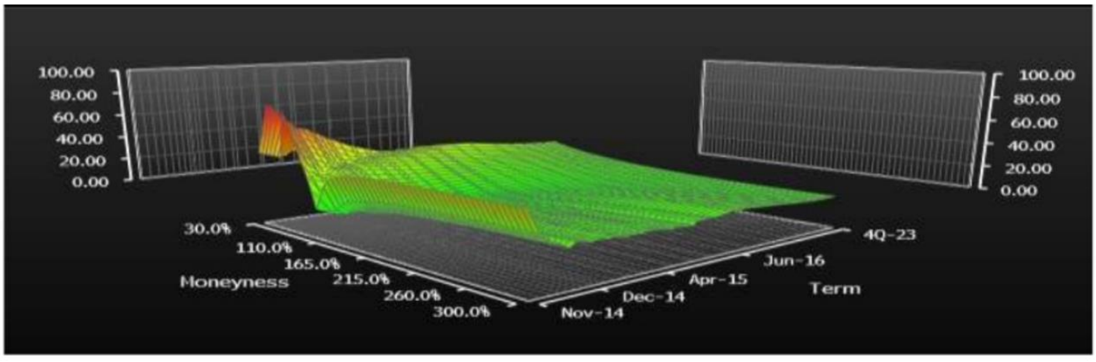
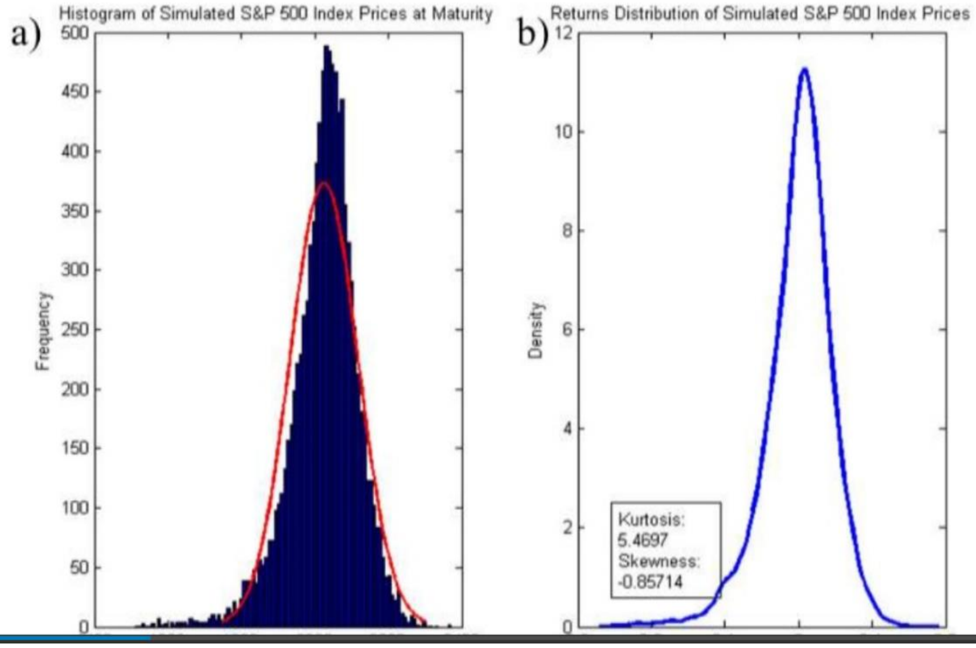


Figure 5: Local Volatility sensitivity (top) versus Implied Volatility (bottom)

3) Analysis of Returns:

A histogram was generated from the simulated S&P 500 stock prices representing the 1 month-implied price distribution for the market index (*see figure 6 (a)*). The data for this was extracted from the local volatility generated S&P 500 stock price after twenty days from our start reference date. A normal distribution curve was plotted over the implied price distribution, represented by the red line in *figure 6 (a)*. As can be seen from the price distribution, the values are leptokurtic. Jumps fatten the weights in the tails. The log returns of the implied price distribution were then generated and are shown in *figure 6 (b)*, with kurtosis of 5.467 and skewness of -0.85714. *Figure 6 (b)* demonstrates a fat left tail with a squeezed right tail.

When SKEW is equal to 100, the distribution of the S&P 500 log-returns is normal and the probability of returns two standard deviations below or above the mean is 4.6% (CBOE, 2010). Above 100 the distribution becomes negatively skewed. The negative returns, shown in the lower left tail in *figure 6 (b)*, demonstrate a negative skewness of the log-returns. This corroborates the 'crash-o-phobia' mentality of investors, who fear a large (greater than 3σ) negative shock to the market.



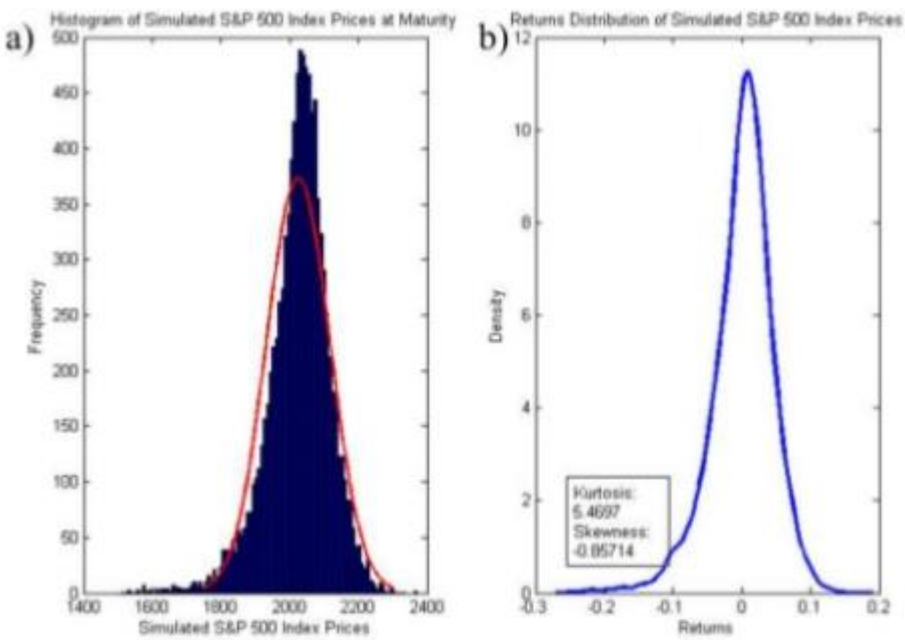


Figure 6: a) Histogram of Simulated S&P 500 Index Prices at Maturity,
b) Histogram of Returns of Simulated S&P 500 Index Prices at Maturity

calculated over the same range of skew for the probabilities 1.63% (125 SKEW) and 1.92% (130 SKEW). An estimate for the skew was calculated to be 116.04. An estimate for the 2 sigma and 3 sigma probabilities were calculated by performing a probability density estimate on the standardised stock price. There is a clear difference between the estimated skew and the actual skew values. Thus, there is a discrepancy between the 2 and 3 standard deviation values for the estimated and actual. There are a number of sources for these discrepancies. The CBOE skew calculation is based on a portfolio that replicates an exposure to 30 day-skewness (CBOE, 2010). The CBOE skew index is calculated with expiry times with increments of one minute, while our calculations were taken with daily time intervals. Another source of error was the interpolation of the local volatility and forward price, which would have compounded errors.

Estimated Risk Adjusted Probability			
	SKEW	***2 Std. Dev***	***3 Std. Dev***
Actual	125.66	9.23%	1.67%
Estimate	116.04	3.87%	1.10%

Table 1: Downside risk for 2 and 3 standard deviation events with actual and estimated

The CBOE skew provides some critical information as to what market participants are predicting in relation to a negative outlier and should be incorporated into ones risk analysis. In table 1, we see that both the actual and our estimate SKEW lie above the risk neutral 100 figure indicating that the market remains sensitized to a negative outlier "black-swan" type event to this day. Therefore, as the SKEW increases over 100, market participants are willing to pay an increasing amount for OTM puts in order to minimize their risk, this therefore suggests that investors in the real world are in fact risk-averse, and are willing to pay a higher premium to reduce exposure to a potential "black-swan" event. The SKEW also provides somewhat of a lighthouse function an equally important element to the risk analysis tool.

When the markets are in a steady and calm state with the VIX at a moderate to high level, a rise in the SKEW means that despite the market currently being in a relatively stable state, market participants fears towards a potential downside move is growing as the negative outlier has yet to occur, once the VIX increases significantly it can be safe to assume the market is already in a free fall with the SKEW now falling as the probability of such an event happening going forward has now reduced. This is similar to the earthquake scientist's analogy given by (Murphy, 2014), by monitoring the SKEW alongside the VIX one can monitor risk in the markets in a practical yet effective manner. When it comes to a negative "black-swan" type outlier, one can view the SKEW as the risk tool providing A sense of foresight towards an outlier with the VIX providing the hindsight (*See Figure 7*).

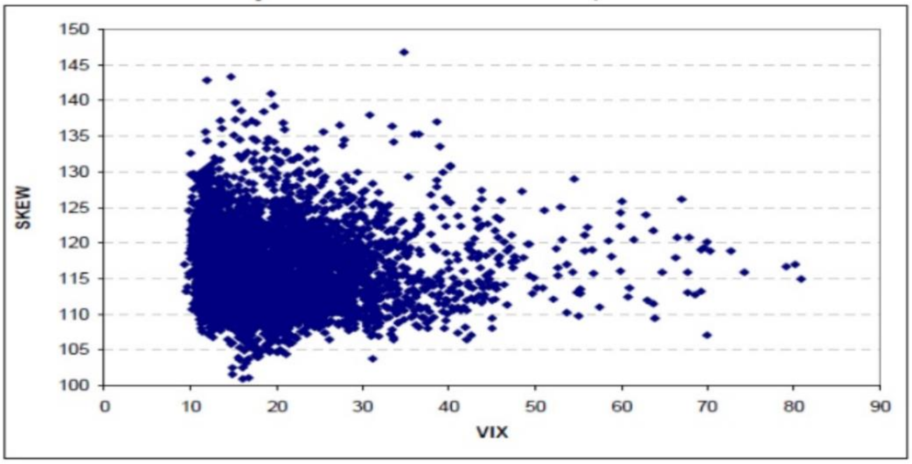


Figure 7: Scatter plot of SKEW and VIX, 1990-2010

Part 2: Heston Model

1) Introduction:

The Heston Stochastic Volatility (HSV) model emanates from a number of other option pricing models including (Black & Scholes, 1973), (Melino & Turnbull, 1990) and (Stein & Stein, 1991). (Black & Scholes, 1973) makes the assumption that stock returns are normally distributed with a known mean and variance. B-S does not depend on mean spot returns and so cannot be generalised by allowing the mean to vary. (Melino & Turnbull, 1990) (M-T) utilise a stochastic volatility model and report its success in pricing currency options. Although successful in this regard the M-T model has the disadvantage of not having a closed form solution. Later the (Stein & Stein, 1991) (S-S) approach used an average of the B-S formula values over different volatility paths and assumed volatility was uncorrelated with spot returns, however since the S-S approach is not correlated with spot returns it cannot capture important skewness effects. The HSV model attempts to improve on these by relating the distribution of spot returns to the cross sectional properties of option prices in order to capture these skewness effects as well as provide a closed form solution for the price of a European call option when the spot asset is correlated with volatility.

2) Heston Stochastic Volatility Model:

The HSV model can conveniently explain properties of option prices in terms of the underlying distribution of spot returns and produce a rich variety of effects compared to other models (Heston, 1993). Heston Stochastic Volatility Model:

$$\boldsymbol{S_t = S_{t-1} + rS_{t-1}dt + \sqrt{V_{t-1}}\, S_{t-1}\sqrt{dt}\, Z_t^1} \qquad \textbf{Equation 1}$$

$$\boldsymbol{V_t = V_{t-1} + \kappa(\theta - V_{t-1})dt + \sigma\sqrt{V_{t-1}}\, \sqrt{dt}\, Z_t^2} \qquad \textbf{Equation 2}$$

The ability for the HSV model to account for the implied volatility skew in the SPX options market stems from the models correlation ρ between volatility and the spot assets price. This correlation factor produces skewness and without it the stochastic volatility only changes kurtosis i.e. it would only affect the pricing of near the money versus far-from-the-money options which is a discrepancy with B-S that

the HSV model proposes to resolve. The model has two important features in that V_t cannot be zero as long as $2\kappa\theta > \sigma$ *i.e.* zero is a reflecting barrier (Murphy, 2014).

The Heston parameters were acquired through Bloomberg using the option valuation (OVME) function. In Equation 1 and Equation 2 above Z^1 and Z^2 are Brownian motion processes correlated by ρ, a value -0.7396. When accounting for the implied volatility skew negative correlation parameters negatively affect the skewness of spot returns. This creates a fat left tail and thin right tail in the distribution of continuously compounded spot returns (Jarrow & Rudd, 1982) and decreases the price of OTM options relative to ITM options (Heston, 1993). The negative correlation parameter observed in this case should decrease the price of OTM options relative to ITM options because the resulting fat left tail of the returns distribution implies a higher chance that the contract will expire out-of-the-money making these types of options less valuable.

The σ parameter indicates the volatility of volatility. When σ is equal to zero volatility is deterministic and spot returns will follow a normal distribution. The σ parameter measures the volatility of volatility and had a value of 76.171%. θ represents the long run variance and had a value of 0.051. This parameter is stochastic and will drift above and below a long run mean θ^* (Heston, 1993). κ is the mean reversion parameter and had a value of 2.4033. The mean reversion determines the relative weights of the current and long run variance of option prices, i.e. larger value of κ will restrict option prices from varying too far from the mean. The fact that we have a large value for κ would suggest that only a small number of the Monte Carlo simulated stock prices would be expected to cross the 130% barrier over the course of the 90 day simulation.

In a stochastic volatility model the σ increases the kurtosis of spot returns and higher values of σ raise far ITM and far OTM option prices and lower near-the-money option prices. In a stochastic volatility model with no correlation between volatility and spot price σ has little effect on skewness. This observation highlights the distinction between the effects of stochastic volatility per se and the effects of a correlation between volatility and spot return as per the HSV model (Heston, 1993). This correlation of volatility with the spot return in the HSV model produces skewness in the prices of OTM options relative to ITM options and more closely matches observed option prices that are far OTM and far ITM when compared to B-S.

3) Analysis of Results:

Actual/365	Vanilla		Knockout	
30 Day	Model	Bloomberg	Model	Bloomberg
HSV	30.8594	30.23	30.8594	30.22
LV	28.1933	27.53	28.1869	27.53
B-S	27.986	28.11	27.986	27.66
90 Day				
HSV	56.3779	55.73	56.3779	55.69
LV	-	55.21	-	55.2
B-S	56.5897	56.46	56.5897	55.47

Table 2: Prices of Vanilla and Knockout options for Heston, Local Volatility and Black Scholes Model

Originally there was a discrepancy between model and Bloomberg prices which arose from choosing the wrong day convention for the HSV and B-S models. The original day convention was Actual/252 and was updated to Actual/365. This resolved a problem surrounding which volatility to choose for the B-S simulations. Originally we tested a number of B-S constant volatilities in order to find which volatility level allowed the model produce the most accurate B-S vanilla and knockout option prices. After the day convention was changed we observed that using the constant B-S volatility level as quoted by Bloomberg produced the most consistently accurate results from the model, i.e. we modelled a number of barrier levels for different time to maturities and found the B-S volatility level as quoted by Bloomberg to work best for the model each time.

There is a close correlation between the model prices and the quoted Bloomberg prices for all three models at maturities of both 60 days and 90 days. The small differences observed could be explained by lambda as per (Moodley, 2005) whereby the market price of volatility risk is almost impossible to observe. In most cases the calculated model price for both the vanilla and knockout option are very similar if not the same, this is due to a number of factors. Firstly the short time periods involved don't allow for enough time for the Monte Carlo simulations to reach the strike.

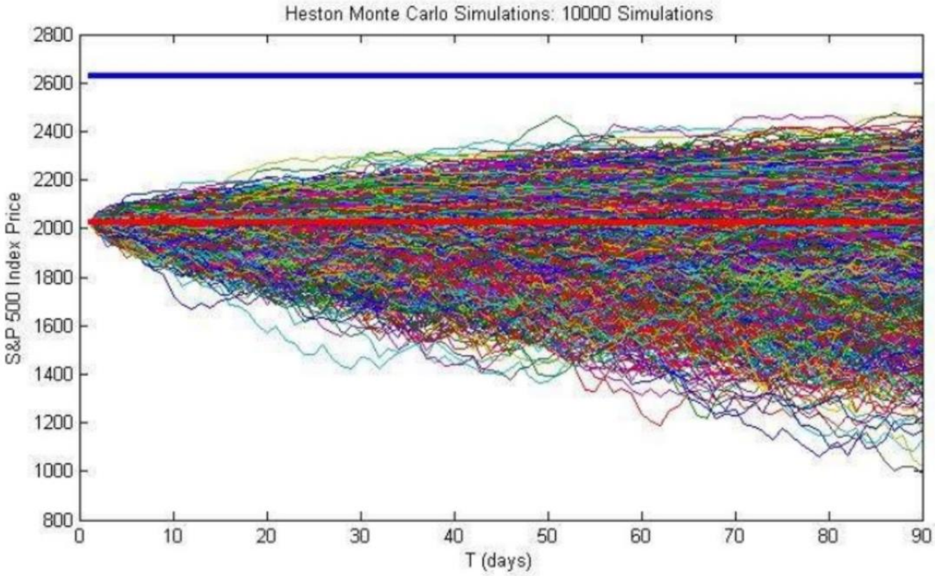


Figure 8: Monte Carlo simulations for HSV model with barrier 130% (Blue Line)

The stochastic model for the variance evolves as a square root diffusion process (Mikhailov & Nögel, 2004) and the low long run variance of the model coupled with the mean reversion factor held back the simulations from deviating too far from the mean over the short time periods. The stochastic volatility associated with the HSV model can only increase gradually via a sequence of small normally distributed increments (Eraker, et al., 2003). The lack of a jump parameter meant that volatility could not rapidly change and cause a jump in stock price over the life of the option. Higher volatilities as well as longer times to maturity would have led to greater price discrepancies between the vanilla and knockout options as illustrated in *(Figure 9)*.

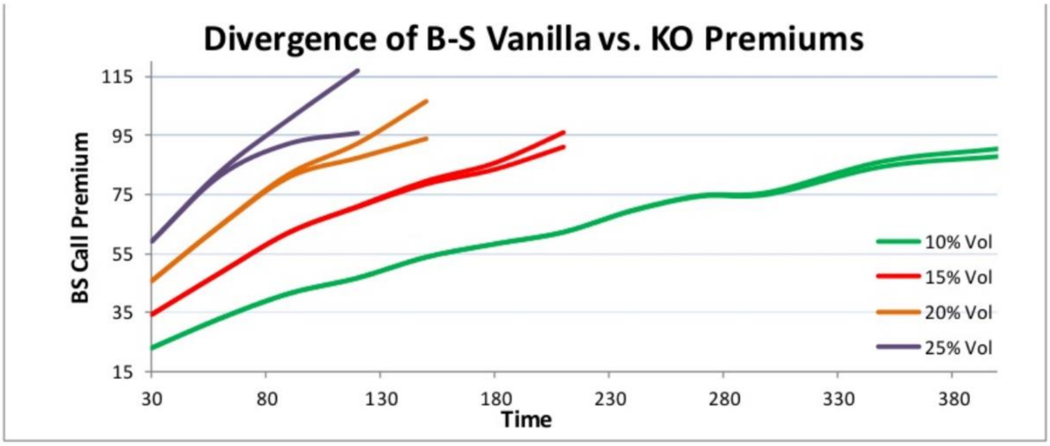


Figure 9: Divergence of B-S Vanilla vs. Knockout Premium with varying volatilities

Figure 9: Divergence of B-S Vanilla vs. Knockout Premium with varying volatilities

The Heston model prices are more expensive than the corresponding B-S prices due to a number of reasons. Firstly the negative correlation factor associated with the Heston Model increasing the prices of the ITM calls for both the vanilla and knockout options as illustrated in figure 10. There's a larger discrepancy between the 30 day HSV and B-S prices versus the 90 day HSV and B-S prices due to the 30 day Monte Carlo simulations being closer to the strike at maturity than the 90 day simulations i.e. the 90 day simulations lie further along the curve in Figure 10 resulting in a smaller price increase.

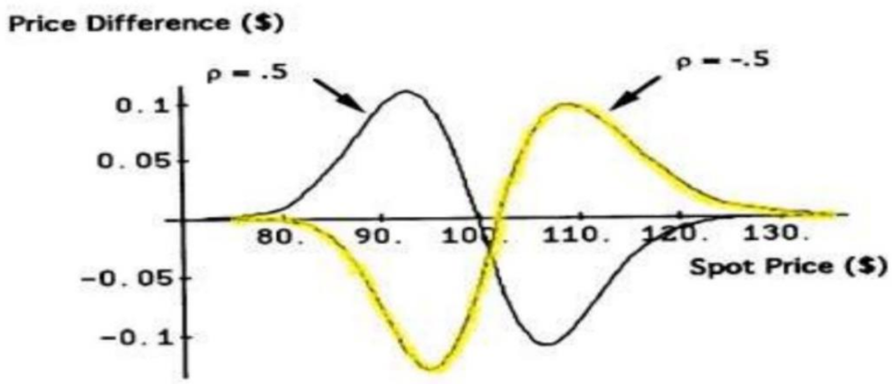


Figure 20: Option prices from the stochastic volatility model minus Black-Scholes values with equal volatility to option maturity

Another reason for the discrepancy in prices is that knockout call options are long gamma in the region near the 100% strike and show higher local volatilities in this region compared to the B-S tree valuation resulting the B-S binomial tree under-pricing the knockout call options (Murphy, 2014).

KO prices increase up until an approximate volatility of 30% after which point the number of Monte Carlo simulations hitting the barrier becomes greater than the number remaining between the money and the KO barrier resulting in a drop off in call premium for higher volatilities. The price rises sharply approaching this point as low numbers of the increasingly valuable ITM simulations have hit the barrier. The price falls off less steeply afterwards as a steadily increasing number of ITM simulations gradually begin to hit the barrier.

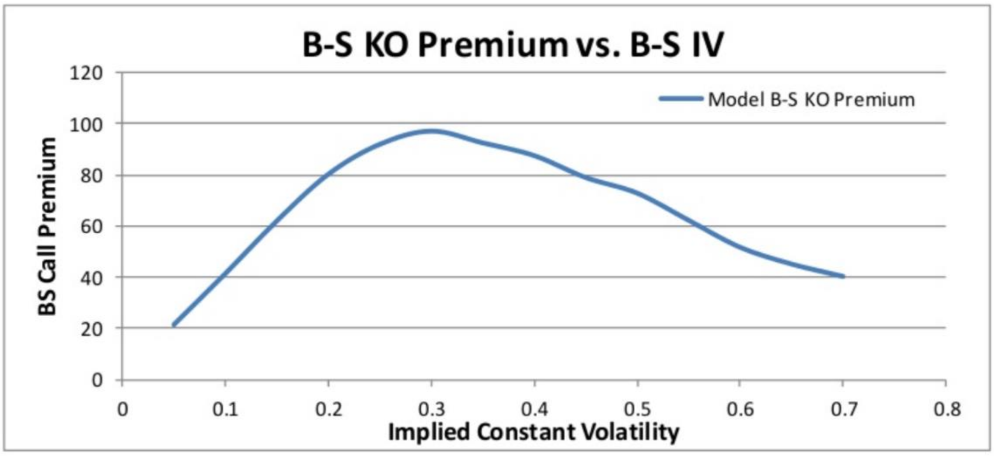


Figure 31:

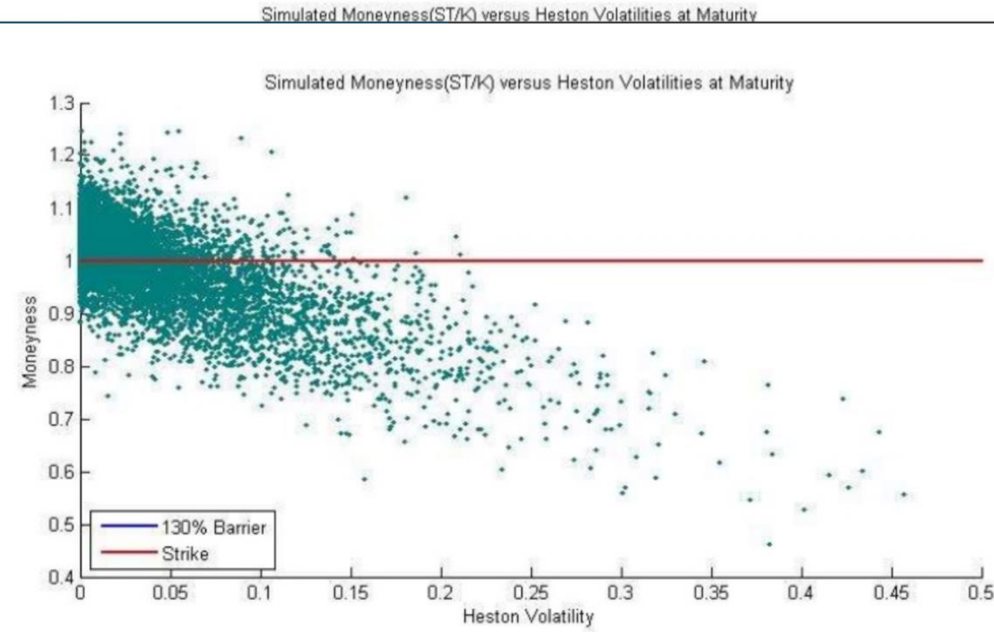


Figure 42:

Figure 42:

Figure 12 illustrates the negative correlation parameter between volatility and spot price. It is observed that simulations lower moneyness achieve the highest levels of Heston volatility while simulations maturing near the money have consistently low levels of volatility.

Overall all three models performed well in calculating the prices of vanilla and knockout options with The Heston Stochastic Volatility pricing model outperforming the Local volatility and Black-Scholes pricing models due to its incorporation of stochastic volatility and its relation of spot returns to the cross-sectional properties of option prices (Heston, 1993). The HSV model although proving the most optimum of the three still has certain limitations. Periods of market stress are characterised by a short time period with multiple large movements that the HSV model is not capable of generating. This is due to the diffusive nature stochastic volatility not being able to increase rapidly enough to generate these shocks (Eraker, et al., 2003).

Bibliography

Bampou, D., 2008. *Numerical Methods for Pricing Exotic Options,* London: Imperial College London.

Black, F. & Scholes, M., 1973. The pricing of options and corporate liabilities. *The journal of political economy,* pp. 637-654.

Black, F. & Scholes, M., 1973. The pricing of options and corporate liabilities. *The journal of political economy,* pp. 637-654.

Brown, S. & Warner, J., 1985. Using daily stock returns: The case of event studies. *Journal of financial economics,* 1(14), pp. 3-31.

CBOE, 2010. *CBOE White Paper - SKEW.* [Online]
Available at: www.cboe.com/micro/skew/documents/SKEWwhitepaperjan2011.pdf

Cvitanic, J. & Zapatero, F., 2004. *Introduction to the Economics and Mathematics of Financial Markets.* Massachusetts: Massachusetts Institute of Technology.

Dempster, M. A. H. & Pliska, S. R., 2000. *Mathematics of Derivative Securities.* Cambridge, United Kingdom: Cambridge University Press.

Derman, E. & Kani, I., 1994. Riding on a smile. *Risk,* 7(2), pp. 32-39.

Derman, E. & Kani, I., 1994. The Volatility Smile and Its Implied Tree. *Quantitative Strategies Research Notes.*

Derman, E., Kani, I. & Chriss, N., 1996. Implied Trinomial Trees of the Volatility Smile. *Quantitative Strategies Research Notes.*

Derman, E., Kani, I. & Zou, J. Z., 1996. The Local Volatility Surface: Unlocking the Information in Index Option Prices. *Financial Analysts Journal,* 52(4), pp. 26-36.

Dupire, B., 1994. Pricing with a Smile. *RISK,* 7(1), pp. 18-20.

Dupire, B., 1997. Pricing and hedging with smiles. In: *Mathematics of derivative securities..* Cambridge: Cambridge Uni. Press, pp. 103-112.

Eraker, B., Johannes, M. & Polson, N., 2003. The Impact of Jumps inVolatility and Returns. *The Journal of Finance,* 58(3), pp. 1296-3000.

Fengler, M. R., 2005. *SFB 649 Discussion Paper 2005-019,* Berlin: Deutsche Forschungsgemeinschaft.

Hagen, P., Kumar, D., Andrew, L. & Woodward, D., 2002. Managing Smile Risk. *Willmot Magazine,* Volume September, pp. 84-108.

Heston, S., 1993. A closed-form solution for options with stochastic volatility with applications to bond and currency options.. *Review of Financial Studies,* 6(2), pp. 327-343.

Hull, J. C., 2012. *Options, Futures, and Other Derivatives.* 8 ed. Boston: Pearson.

Jarrow, R. & Rudd, A., 1982. Approximate option valuation for arbitrary stochastic processes. *Journal of Financial Econometrics,* 10(3), pp. 347-369.

McLaughlin, P., 2012. *SABR Interpolated Implied Volatility Surfaces for Local Volatility Models: Theory and Applications to Pricing and Hedging of Exotic Equity Options.* Limerick: University of Limerick.

Melino, A. & Turnbull, S., 1990. Pricing foreign currency options with stochastic volatility. *Journal of Econometrics,* 1(45), pp. 239-265.

Mikhailov, S. & Nögel, U., 2004. Heston's Stochastic Volatility Model Implementation, Calibration and Some Extensions. *Wilmott Magazine,* 4(1), pp. 74-79.

Moodley, N., 2005. *The Heston Model: A Practical Approach,* Johannesburg: s.n.

Murphy, B., 2014. Heston SV model two important features. *FI6071:Dynamic Asset Pricing Theory,* University of Limerick(unpublished), p. 23 Oct.

Murphy, B., 2014. Heston SV model two important features. *FI6071:Dynamic Asset Pricing Theory,* University of Limerick(unpublished), p. 23 Oct.

Rubinstein, M., 1994. Implied Binomial Trees. *The Journal of Finance,* 49(3), pp. 771-818.

Schwert, G. W., 1990. Stock Volatility and the Crash of '87. *The Review of Financial Studies,* Volume 3, pp. 77-102.

Stein, E. & Stein, J., 1991. Stock price distributions with stochastic volatility: an analytic approach. *Review of Financial Studies,* 4(4), pp. 727-752.

Valente, M. & Crane, A., 2010. Public Responsibility and Private Enterprise in Developing Countries. *California MAnagement Review,* pp. 52-78.

Appendix

1. Matlab Code: Part 1

```
% FI6081: Dynamic Asset Pricing Theory - Assignment 1
% Local Volatility & Heston Model Project
% Group B: Barry Sheehan 0854867, Garry Lynch 0871117,
%          Niall Gilbride 09008201, Evan Ryan 14106523
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 1) Create Date vector between expiry dates.

SPXTradingDaysperYear=252;
S0=2023.22; % SPX Index Price as at 05/11/2014 08:15 (EST)
```

```
daysbetween(1,:)=sum(isbusday(busdays('11/07/14','11/14/14')));
daysbetween(2,:)=sum(isbusday(busdays('11/15/14','11/21/14')));
daysbetween(3,:)=sum(isbusday(busdays('11/22/14','11/28/14')));
daysbetween(4,:)=sum(isbusday(busdays('11/29/14','12/05/14')));

dates(1,:)=0; % Initialise date matrix
for i = 2:(length(daysbetween)+1)
  dates(i,:)=dates(i-1,:)+daysbetween(i-1,:);
end
dates(1,:)=1;
T=dates(:)/SPXTradingDaysperYear;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 2) Interpolate between expirations to create fine-mesh local volatility
%    surface. Compare to coarse-mesh local volatility surface.
```

```
interpDates=(1/SPXTradingDaysperYear):(1/SPXTradingDaysperYear):...
    (max(dates(:)/SPXTradingDaysperYear));

% Input Local Volatility data from FI6071 Project Vols spreadsheet.xlsx
lvmatrix = xlsread('FI6071 Project Vols spreadsheet.xlsx','Matlab');
```

```
interpLVspline=interp1(T,lvmatrix,interpDates,'spline');

interpLVlinear=interp1(T,lvmatrix,interpDates,'linear');

LocalMoneyness=(0.30:0.025:1.70)';


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% 3) Interpolate current SPX implied forward prices:


ImpFwd=[2022.04; 2021.19; 2020.21; 2019.66; 2019.00]; %Market ImpFwd prices

interpImpFwd=interp1(T,ImpFwd',interpDates,'linear');


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% 4) Monte Carlo Simulations


dt=1/SPXTradingDaysperYear;
```

```
N = 10000; %Number of Simulations

numberOfTimeSteps = max(dates);


E = randn(numberOfTimeSteps,N);


St = [S0*ones(1,N); zeros(numberOfTimeSteps-1,N)];

ESt=[S0 zeros(1,numberOfTimeSteps-1)];    % Initialize forecast schedule

UprSt=[S0 zeros(1,numberOfTimeSteps-1)]; % Initialize Upper 95% Boundary

LwrSt=[S0 zeros(1,numberOfTimeSteps-1)];  % Initialize Lower 95% Boundary



lv = zeros(numberOfTimeSteps,N); % Initialize Local Volatility

lv(1,:)=interp1(LocalMoneyness,interpLVspline(1,:),...

  St(1,:)./interpImpFwd(1),'spline','extrap'); % ATM Day 1 Local Volatility
```

```

mu=0.00567; %Annualised daily returns (Garry what is this exactly agan?)


for i=1:1:N %simulation steps

    for j=2:1:numberOfTimeSteps %time steps

        St(j,i) = St(j-1,i)*exp((mu-0.5*lv(j-1,i)^2)*dt+...
```

```
            lv(j-1,i)*sqrt(dt)*E(j,i));

        lv(j,:)=interp1(LocalMoneyness,interpLVspline(j,:)...

            ,St(j,:)./interpImpFwd(j),'spline','extrap');


    end

end


lv2(1,1)=interp1(LocalMoneyness,interpLVspline(1,:)...

,ESt(1,1)./interpImpFwd(1),'spline','extrap'); %ATM Day 1 Local Volatility


for i=1:1:numberOfTimeSteps-1 %95% confidence intervals

    ESt(i+1)=St(1,1)*exp(mu*i*dt);

    UprSt(i+1)=ESt(i+1)+1.96*lv2(1,i)*St(1,1)*sqrt(i*dt);

    LwrSt(i+1)=ESt(i+1)-1.96*lv2(1,i)*St(1,1)*sqrt(i*dt);

    lv2(1,i+1)=interp1(LocalMoneyness,interpLVspline(i+1,:)...

        ,ESt(1,i+1)./interpImpFwd(i+1),'spline','extrap');

end


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% 5) Analyse Returns Distribution


returns=(St(end,:)-St(1,:))./St(1,:);

[f,xi]=ksdensity(returns);


MeanReturn=mean(returns);

StdDevReturn=std(returns)/2;


% Downside event Probability Estimates:

SkewActual=125.66; % CBOE SKEW as at 05/11/2014 08:15 (EST)


standardised_sims=(St(end,:)-mean(St(end,:)))/std(St(end,:));


TwoSigmaEventActual=interp1([125 130],[0.0905 0.1040],SkewActual,'spline');

TwoSigmaEventEstimate=ksdensity(standardised_sims,-2,...
```

```
    'function','cdf');% Probability of 2-Sigma event occuring


ThreeSigmaEventActual=interp1([125 130],[0.0163 0.0192]...
    ,SkewActual,'spline');
ThreeSigmaEventEstimate=ksdensity(standardised_sims,-3,...
    'function','cdf'); % Probability of 3-Sigma event occuring


CBOESkewEstimate=interp1([0.0104 0.0133],[115 120],...
    ThreeSigmaEventEstimate,'linear'); % Estimate of CBOE SKEW

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 6) Plot results


% i) Plot Fine-mesh and Coarse-mesh Local Vloatility Matrices
```

```
subplot(3,3,1)
surf(LocalMoneyness,interpDates,interpLVspline)
title('Cubic Spline Interpolation of Local Volatility: Fine-Mesh')
xlabel('Moneyness(%)')
ylabel('T (days)')
zlabel('Local Volatility')


subplot(3,3,2)
surf(LocalMoneyness,interpDates,interpLVlinear)
title('Linear Interpolation of Local Volatility: Coarse-Mesh')
xlabel('Moneyness')
ylabel('T (days)')
zlabel('Local Volatility')
```

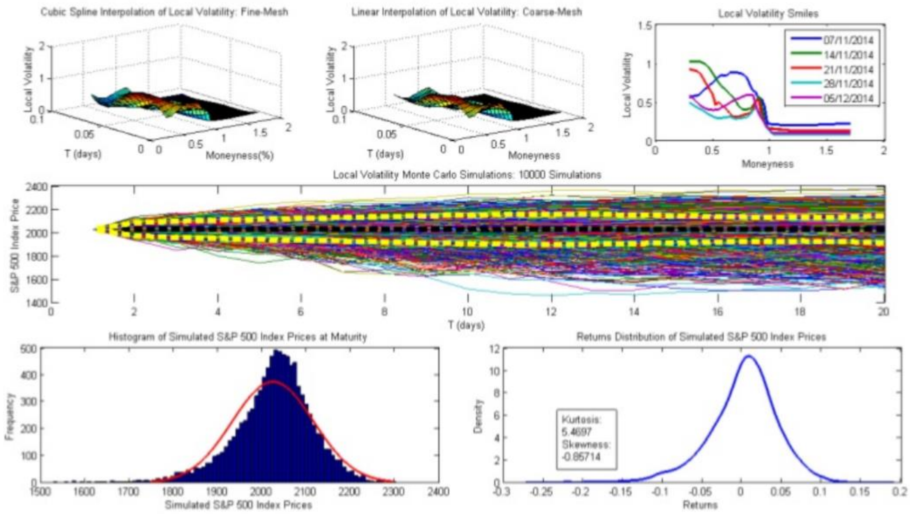
```
% ii) Plot local volatility smiles
subplot(3,3,3)
plot(LocalMoneyness,interpLVlinear(dates,:),'Linewidth',2.5);
title('Local Volatility Smiles')
xlabel('Moneyness')
ylabel('Local Volatility')
legend('07/11/2014','14/11/2014','21/11/2014','28/11/2014','05/12/2014')
```

```
% iii) Plot Monte Carlo Simulation with confidence intervals
subplot(3,3,[4,5,6])
plot(St)
hold all
plot(ESt,'--','Color','black','Linewidth',4.5)
plot(UprSt,'--','Color','yellow','Linewidth',4.5)
plot(LwrSt,'--','Color','yellow','Linewidth',4.5)
title(['Local Volatility Monte Carlo Simulations: '...
    num2str(N),' Simulations'])
xlabel('T (days)')
ylabel('S&P 500 Index Price')%check Legend

% iv) Plot histogram of Simulated S&P 500 Index Prices at Maturity
subplot('Position',[0.12,0.075,0.37,0.25])
histfit(St(end,:))
```

```
title('Histogram of Simulated S&P 500 Index Prices at Maturity')
xlabel('Simulated S&P 500 Index Prices')
ylabel('Frequency')

% v) Plot histogram of Returns Distribution of Simulated S&P 500 Index...
%    Prices at Maturity
subplot('Position',[0.55,0.075,0.37,0.25])
plot(xi,f,'Linewidth',2.5)
title('Returns Distribution of Simulated S&P 500 Index Prices')
xlabel('Returns')
ylabel('Density')
str = {'Kurtosis:',num2str(kurtosis(returns)), 'Skewness: '...
       ,num2str(skewness(returns))};
annotation('textbox', [0.6,0.1,0.055,0.11],'String', str);
```



Matlab Code Part 1 Published Graphs

2. Matlab Code: Part 2

```
% FI6081: Dynamic Asset Pricing Theory - Assignment 1

% Local Volatility & Heston Model Project

% Group B: Barry Sheehan 0854867, Garry Lynch 0871117,

%          Niall Gilbride 09008201, Evan Ryan 14106523

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% 1) Initialise Heston Parameters

S0=2023.22; % SPX Index Price as at 05/11/2014 08:15 (EST)

K=2023.22; % ATM Strike

SPXTradingDaysperYear=365;

numberOfTimeSteps=90; % Days to Maturity

Time=numberOfTimeSteps/SPXTradingDaysperYear;
```

```
delT = 1/365;

rf = 0.00232; % MMkt USD Rate

div=0.0; % Dividend Yield

mu=0.00567; %Annualised daily returns

% Heston Parameters as at 05/11/2014 08:15 (EST)

V0 = .017; % Initial Variance

theta =.051; % Long Run Variance

rho=-0.7396; % Correlation

sigma=0.76171; % Volatility of Volatility

kappa = 2.4033; % Mean Reversion

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% 2) Monte Carlo Simulation with Heston Stochastic Volatility
```

```
numberOfSimulations = 10000;

NormRand1 = randn(numberOfTimeSteps,numberOfSimulations);

NormRand2 = randn(numberOfTimeSteps,numberOfSimulations);

St = [S0*ones(1,numberOfSimulations); zeros(numberOfTimeSteps-1,...

numberOfSimulations)];
```

```
V = [V0*ones(1,numberOfSimulations); zeros(numberOfTimeSteps-1,...

    numberOfSimulations)];

W1=NormRand1;

W2=zeros(numberOfTimeSteps,numberOfSimulations);

for i=1:1:numberOfSimulations

    for j=1:1:numberOfTimeSteps

        W2(j,i)=rho*NormRand1(j,i)+sqrt(1-rho^2)*NormRand2(j,i);

    end

end;

for i=1:1:numberOfSimulations

    for j=2:1:numberOfTimeSteps
```

```
     St(j,i) = St(j-1,i) + (rf-div)*St(j-1,i)*delT + St(j-1,i)*...
       sqrt(V(j-1,i))*W1(j,i)*sqrt(delT);
     V(j,i)=V(j-1,i) + kappa*(theta - V(j-1,i))*delT + sigma*...
            sqrt(V(j-1,i))*W2(j,i)*sqrt(delT);
     V(j,i)= abs(V(j,i)); %prevents negative volatilities in this
    %                      CIR-style model.
   end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 3) Price SPX European Barrier Up-and-Out Call option (K=100%,H=130%)
Barriers=[1.00:0.01:1.3].*S0;
```

```
Heston_KOBarrierCall_Premium=zeros(length(Barriers),1);
for i=1:1:length(Barriers)
KO_Barrier=Barriers(i);
% Barrier set at 130% of current SPX Index Price
% as at 05/11/2014 08:15 (EST)
Call_Payoff=zeros(1,numberOfSimulations);
    for j=1:numberOfSimulations
```

```
        Call_Payoff(1,j)=max(0,St(end,j)-K);

        if max(St(:,j)) >= KO_Barrier == 1

            Call_Payoff(1,j)=0;

        else

            Call_Payoff(1,j)=exp(-rf*Time)*Call_Payoff(1,j);

        end

    end

Heston_KOBarrierCall_Premium(i) = mean(Call_Payoff);

end

Heston_VanillaCall_Premium = exp(-rf*Time)*mean(max(0,St(end,:)-K));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% 4) Plot results

% i) Plot Heston Vol Monte Carlo Simulation
```

```
subplot(1,2,1)

plot(St)

hold all

Barrier1=plot(KO_Barrier*ones(numberOfTimeSteps),'-','Color',...

    'blue','Linewidth',3);

Strike1=plot(K*ones(numberOfTimeSteps),'-','Color','red','Linewidth',3);

title(['Heston Monte Carlo Simulations: '...

    num2str(numberOfSimulations),' Simulations'])

xlabel('T (days)')

ylabel('S&P 500 Index Price')

% ii) Scatterplot of Simulated Moneyness(ST/K) versus Heston Volatilities

%     at Maturity
```

```
scatter(V(end,:),St(end,:)./K,10,'filled','d','MarkerFaceColor',[0 0.5 0.5])

hold all

Strike=refline([0 K/S0]);
```



```
set(Strike,'Color','r','LineWidth',2)

Barrier=refline([0 KO_Barrier/S0]);

set(Barrier,'Color','b','LineWidth',2)

title('Simulated Moneyness(ST/K) versus Heston Volatilities at Maturity')
```

```
xlabel('Heston Volatility')

ylabel('Moneyness')

legend([Barrier Strike],'130% Barrier','Strike','location','SouthWest')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% 5) Monte Carlo Simulation with Black-Scholes Constant Volatility

St2 = [S0*ones(1,numberOfSimulations); zeros(numberOfTimeSteps-1,...

    numberOfSimulations)];

BSsigma=.13728;

% Bloomberg quoted volatility of 13.728%

E = randn(numberOfTimeSteps,numberOfSimulations);

Call_PayoffBS=zeros(1,numberOfSimulations);

BSKO_Barrier=S0*1.3;
```

```
% Barrier set at 130% of current SPX Index Price

% as at 05/11/2014 08:15 (EST)


    for i=1:1:numberOfSimulations %simulation steps

        for j=2:1:numberOfTimeSteps %time steps

            St2(j,i) = St2(j-1,i)*exp((mu-0.5*BSsigma^2)*delT+...

                BSsigma*sqrt(delT)*E(j,i));

            Call_PayoffBS(1,i)=max(0,St2(end,i)-K);

            if max(St2(:,i)) >= BSKO_Barrier == 1

                Call_PayoffBS(1,i)=0;

            else

                Call_PayoffBS(1,i)=exp(-rf*Time)*Call_PayoffBS(1,i);

            end
```



```
        end

        BS_KOBarrierCall_Premium=mean(Call_PayoffBS);

    end

BS_VanillaCall_Premium = exp(-rf*Time)*mean(max(0,St2(end,:)-K));
```