ECMT6006 2020S1 Assignment 2

Due: 23:59 on Monday 25 May 2020

Please submit your solutions via Turnitin in Canvas. Except for special circumstances, I DO NOT accept late submission.

Problem 1. Consider the following ARMA(1,1)-GARCH(1,1) process

$$
\begin{aligned}
Y_t &= \phi Y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1} \\
\varepsilon_t | \mathcal{F}_{t-1} &\sim N(0, \sigma_t^2) \\
\sigma_t^2 &= \omega + \beta \sigma_{t-1}^2 + \alpha \varepsilon_{t-1}^2
\end{aligned}
$$

where $|\phi| < 1$ and $\alpha + \beta < 1$. Assume $\{Y_t\}$ is weakly stationary. Answer the following.

(i) Find $E_t(Y_{t+1})$

(ii) Find $E(Y_t)$

(iii) Find $\mathrm{Var}_t(Y_{t+1})$

(iv) Find $\mathrm{Var}(Y_{t+1})$

(v) Find $E_t(Y_{t+1}^2)$

(vi) Find $E(\sigma_t^2)$.

Problem 2. Download the S&P500 index[1] over the period 2010–2015. Convert the prices into continuously compounded returns.

(i) Consider a ARMA(p, q) conditional mean model (allowing for the constant term) for the returns with $p = 0, 1, \ldots, 5$ and $q = 0, 1, \ldots, 5$. Use the three information criteria discussed in class (AIC, HQIC, BIC) to select the best model. Report your results.

(ii) Obtain the residuals from the conditional mean model selected by BIC, and estimate a GARCH(1,1) model using the residuals. Report the estimated parameters in this conditional mean and conditional variance model.

[1] You can download the S&P500 index daily price from Yahoo Finance: `https://finance.yahoo.com/quote/%5EGSPC/history?p=%5EGSPC`. When constructing the daily returns, you can use either open price or adjusted close price as the index price on that day. Or alternatively, you may construct the daily index return by the adjusted close (end of the day) and open (beginning of the day) prices on the same day. Just keep a note on what you do.

(iii) Plot the estimated conditional volatility in *annualized standard deviation*[2].

Problem 3. (Forecasting Evaluation and Comparison) The file "`term_premium.xlsx`" contains monthly term premium on a 10-year zero coupon bond from April 1953 to December 2013 with $T = 729$ observations. In this exercise, you will use this dataset to (a) generate forecasts using an ARMA(1,1) and a "Random Walk" model, (b) evaluate the optimality of the model forecasts, and (c) compare these two model forecasts.

(i) Generate a time-series plot of the term premium data in the data file `term_premium.xlsx`, with a proper x-label and y-label.

(ii) Use the first half of the sample (first $R = 364$ observations from April 1953 to July 1983) to estimate a stationary ARMA(1,1) model

$$Y_t = \phi_0 + \phi Y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1}, \quad \varepsilon_t \sim i.i.d.\ WN(0, \sigma^2),$$

and obtain the parameter estimates $\hat{\phi}_0, \hat{\phi}, \hat{\theta}$.

(iii) What is the feasible *one-step-ahead* forecast $\hat{Y}_{t+1|t}$?

(iv) Let $\hat{\varepsilon}_0 = 0$ and $Y_0 = \hat{\phi}_0/(1 - \hat{\phi})$,[3] compute $\hat{Y}_{t+1|t}$ for $t = 0, 1, \ldots, T - 1$ and plot them together with the real data Y_t for $t = 1, \ldots, T$.

(v) Note that $\hat{Y}_{t+1|t}$ for $t = R, \ldots, T-1$ are *fixed window forecasts* in the out-of-sample analysis of the above ARMA(1,1) model. Compute the forecast errors $e_{t+1|t} = Y_{t+1} - \hat{Y}_{t+1|t}$ for $t = R, \ldots, T-1$, and generate a time-series plot of them.

(vi) What would you expect for the serial correlation of the forecast errors if the above forecasts are optimal? Plot the sample ACF of the forecast error series $\{e_{t+1|t}, t = R, \ldots, T-1\}$, and conduct bot Ljung-Box test and robust test on the joint serial correlation with lages $L = 5, 10, 20$. What is your conclusion by examining the ACF plot and the test results?

(vii) The "Mincer-Zarnowitz" (MZ) regression[4] regresses the realized data on its forecast and a constant. The null hypothesis is that the intercept is 0 and the slope is 1. Now consider an alternative and equivalent formulation:

$$e_{t+1|t} = \alpha_0 + \alpha_1 \hat{Y}_{t+1|t} + u_{t+1}. \tag{1}$$

What parameter restrictions would you test in this regression to examine the optimality of $\hat{Y}_{t+1|t}$? Run regression (1) using your data on $e_{t+1|t}$ and $\hat{Y}_{t+1|t}$ for $t = R, \ldots, T - 1$, and conduct a test to draw conclusion on the optimality of your forecasts.

[2] The annualized standard deviation is computed as $\sqrt{252}\times$ daily standard deviation, because there are normally 252 trading days in a year.

[3] Note that $E(\varepsilon_t) = 0$ and $E(Y_t) = \phi_0/(1 - \phi)$ in this model. Here we just set the initial values of ε and Y to be their unconditional means.

[4] See Section 6.2 on Page 192–195 in the textbook.

(viii) Now consider an alternative model

$$Y_t = Y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim i.i.d.\ WN(0, \sigma^2).$$

which is a "Random Walk". What is the best one-step-ahead forecast $\hat{Y}_{t+1|t}$ for this model?

(ix) Repeat steps (v), (vi), (vii) for the Random Walk model.

(x) Compare the forecasts from the ARMA(1,1) model and Random Walk model by a "Diebold-Mariano" (DM)[5] test with a squared error loss function. Specifically, the difference of the two losses is

$$d_t = (e^a_{t+1|t})^2 - (e^b_{t+1|t})^2, \quad t = R, \ldots, T-1$$

where $e^a_{t+1|t}$ and $e^b_{t+1|t}$ denote the forecast errors from the ARMA(1,1) model and Random Walk model respectively. Conduct a DM test by testing the zero mean of d_t using a robust t-test with both White and Newey-West robust errors. Draw your conclusion on which model is better in terms of forecasting power.

Problem 4. Question 1 in Section 9.10.2 on Page 331 in the textbook

Problem 5. Question 2 in Section 9.10.2 on Page 333 in the textbook

Problem 6. Use the 2010–2015 daily S&P 500 index returns that you downloaded for Assignment 3 Problem 4, estimate the conditional 5%-VaR using the below two methods.

(i) (Historical Simulation) Use the daily data in the whole year 2010 to obtain the empirical distribution of the return on the first day of 2011, and compute the 5%-VaR on the first day of 2011. Next, repeat this day after day using a one-year rolling window, and obtain the daily VaR forecast from 2011 to 2015.

(ii) (Parametric GARCH Model) Assume the daily returns follow the below model with constant conditional mean and GARCH(1,1) conditional variance:

$$\begin{aligned} r_t &= \mu + \varepsilon_t, \\ \varepsilon_t &= \sigma_t \nu_t, \quad \nu_t \sim i.i.d.\ N(0,1), \\ \sigma_t^2 &= \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2. \end{aligned}$$

Estimate the daily conditional 5%-VaR process and and make a time-series plot of it.

[5] See Section 6.3 on Page 201–205 in the textbook.