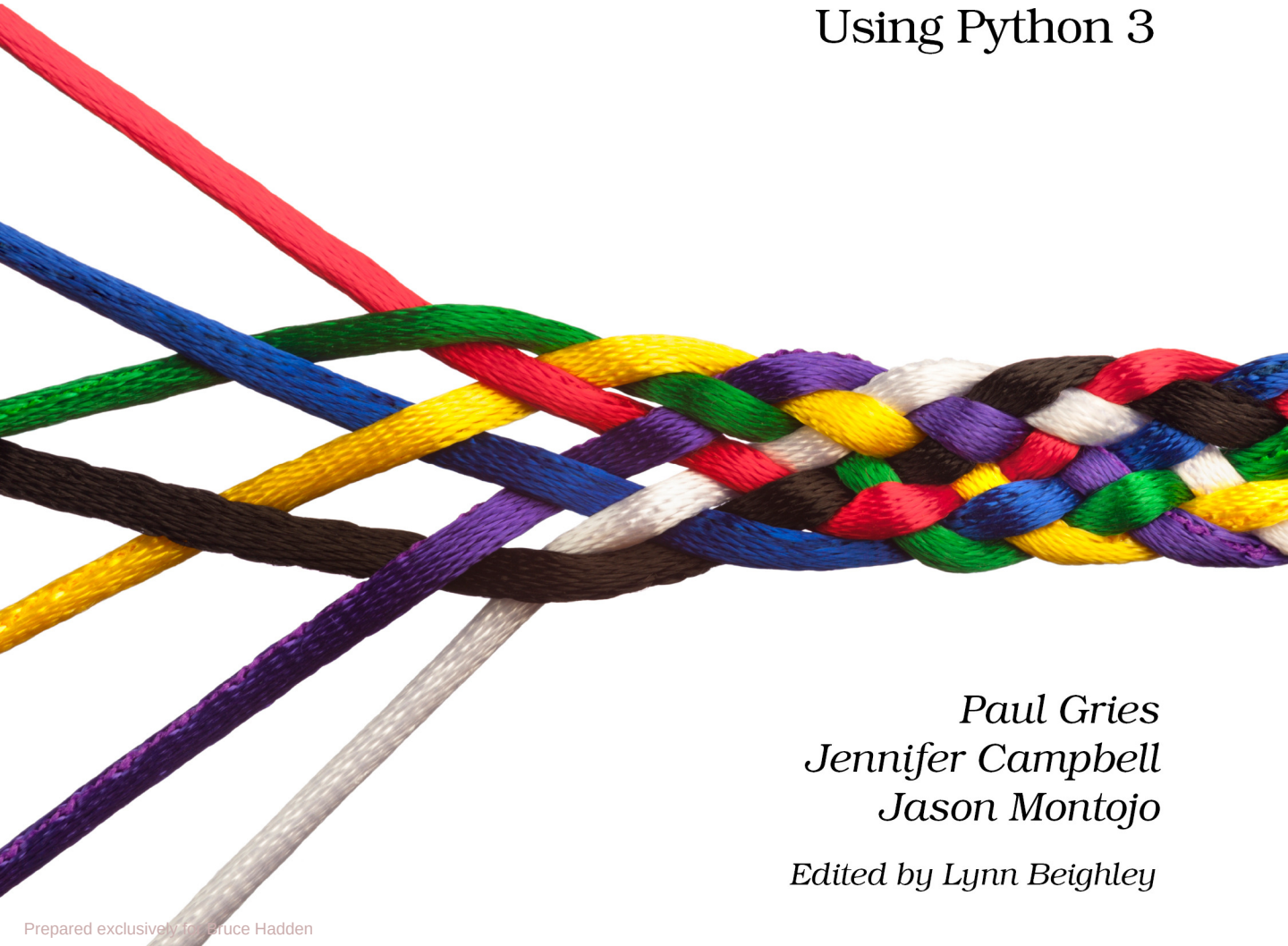
The Pragmatic Programmers

Practical Programming

Second Edition

An Introduction to Computer Science Using Python 3



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Dave & Andy

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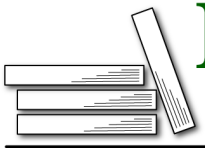
The Pragmatic Bookshelf
Dallas, Texas • Raleigh, North Carolina



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Contents

Changes in the Beta Releases

Beta 2—5 March

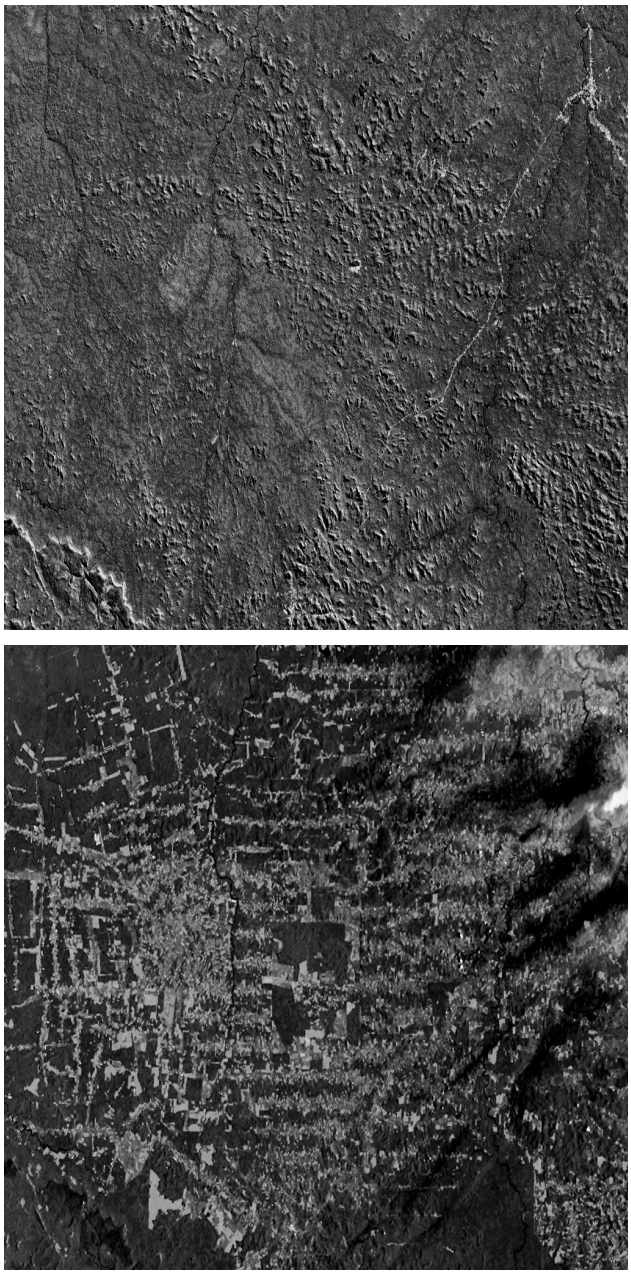
We've added the Testing and Debugging chapter. Once you've written your code, the next step is testing it to make sure that it works correctly. In this chapter, you will learn how to choose good test cases and how to test your code using Python's unittest module.

We'd love you to let us know what you think about the book in the forums or by submitting errata!

CHAPTER 1

What's Programming?

Take a look at the pictures in Figure 1, *The Rainforest Retreats*, on page 2. The first one shows forest cover in the Amazon basin in 1975. The second one shows the same area 26 years later. Anyone can see that much of the rainforest has been destroyed, but how much is "much"?



(Photo credit: NASA/Goddard Space Flight Center Scientific Visualization Studio)

Figure 1—The Rainforest Retreats

Now look at Figure 2, *Healthy blood cells---or are they?*, on page 3.

(Photo credit: CDC)

Figure 2—Healthy blood cells—or are they?

Are these blood cells healthy? Do any of them show signs of leukemia? It would take an expert doctor a few minutes to tell. Multiply those minutes by the number of people who need to be screened. There simply aren't enough human doctors in the world to check everyone.

This is where computers come in. Computer programs can measure the differences between two pictures and count the number of oddly shaped platelets in a blood sample. Geneticists use programs to analyze gene sequences; statisticians, to analyze the spread of diseases; geologists, to predict the effects of earthquakes; economists, to analyze fluctuations in the stock market; and climatologists, to study global warming. More and more scientists are writing programs to help them do their work. In turn, those programs are making entirely new kinds of science possible.

Of course, computers are good for a lot more than just science. We used computers to write this book; you have probably used one today to chat with friends, find out where your lectures are, or look for a restaurant that serves pizza *and* Chinese food. Every day, someone figures out how to make a computer do something that has never been done before. Together, those "somethings" are changing the world.

This book will teach you how to make computers do what *you* want them to do. You may be planning to be a doctor, linguist, or physicist rather than a full-time programmer, but whatever you do, being able to program is as important as being able to write a letter or do basic arithmetic.

We begin in this chapter by explaining what programs and programming are. We then define a few terms and present a few useful bits of information for course instructors.

1.1 Programs and Programming

A *program* is a set of instructions. When you write down directions to your house for a friend, you are writing a program. Your friend "executes" that program by following each instruction in turn.

Every program is written in terms of a few basic operations that its reader already understands. For example, the set of operations that your friend can understand might include the following: "Turn left at Darwin Street," "Go forward three blocks," and "If you get to the gas station, turn around—you've gone too far."

Computers are similar but have a different set of operations. Some operations are mathematical, like "Add 10 to a number and take the square root," while others include "Read a line from the file named `data.txt` ," "Make a pixel blue," or "Send email to the authors of this book."

The most important difference between a computer and an old-fashioned calculator is that you can "teach" a computer new operations by defining them in terms of old ones. For example, you can teach the computer that "Take the average" means "Add up the numbers in a set and divide by the set's size." You can then use the operations you have just defined to create still more operations, each layered on top of the ones that came before. It's a lot like creating life by putting atoms together to make proteins and then combining proteins to build cells, combining cells to make organs, and combining organs to make a creature.

Defining new operations, and combining them to do useful things, is the heart and soul of programming. It is also a tremendously powerful way to think about other kinds of problems. As Prof. Jeannette Wing wrote *Computational Thinking [Win06]*, computational thinking is about the following:

- *Conceptualizing, not programming.* Computer science isn't computer programming. Thinking like a computer scientist means more than being able to program a computer. It requires thinking at multiple levels of abstraction.
- *A way that humans, not computers, think.* Computational thinking is a way humans solve problems; it isn't trying to get humans to think like computers. Computers are dull and boring; humans are clever and imaginative. We humans make computers exciting. Equipped with computing devices, we use our cleverness to tackle problems we wouldn't dare take on before the age of computing and build systems with functionality limited only by our imaginations.

- *For everyone, everywhere.* Computational thinking will be a reality when it is so integral to human endeavors it disappears as an explicit philosophy.

We hope that by the time you have finished reading this book, you will see the world in a slightly different way.

1.2 What's a Programming Language?

Directions to the nearest bus station can be given in English, Portugese, Mandarin, Hindi, and many other languages. As long as the person you're talking to understands the language, they'll get to the bus station.

In the same way, there are many programming languages, and they all can add numbers, read information from files, and make user interfaces with windows and buttons and scrollbars. The instructions look different, but they accomplish the same task. For exmaple, in the Python programming language this adds `3` and `4`:

```
3 + 4
```

But in the Scheme programming language, here's how it's done:

```
(+ 3 4)
```

They both express the same idea, they just look different.

Every programming language has a way to write mathematical expressions, repeat a list of instructions a number of times, choose which of two instructions to do based on the current information you have, and much more. In this book, you'll learn how to do these things in the Python progrmaming language. Once you've understood Python, learning the next programming language will be much easier.

1.3 What's a Bug?

Pretty much everyone has had a program crash. A standard story is that you were typing in a paper when all of a sudden your word processor crashed. You had forgotten to save, and you had to start all over again. Old versions of Microsoft Windows used to crash more often than they should, showing the dreaded Blue Screen of Death. (Happily, they've gotten a *lot* better in the past several years.) Usually, your computer shows some kind of cryptic error message when a program crashes.

What happened in each case is that the people who wrote the program told the computer to do something it couldn't do: open a file that didn't exist,

perhaps, or keep track of more information than the computer could handle, or maybe repeat a task with no way of stopping other than by rebooting the computer. (Programmers don't mean to make these kinds mistakes, but they are *very* hard to avoid.)

Worse, some bugs don't cause a crash; instead, they give incorrect information. (This is worse because at least with a crash you'll notice that there's a problem.) As a real-life example of this kind of bug, the calendar program that one of the authors uses has a friend who was born in 1978. That friend, according to the calendar program, had their 5,875,542nd birthday this past February. It's entertaining, but also can be tremendously frustrating.

Every piece of software that you can buy has bugs in it. Part of the job of the programmers is to minimize the number of bugs, and to reduce their severity. As a programmer, in order to find a bug, you need to track down where you gave the wrong instructions, and then you need to figure out the right instructions, and then you need to update the program without introducing other bugs. This is a hard, hard task that requires a lot of planning and care.

Every time you get a software update for a program, it is for one of two reasons: new features were added to a program, or bugs were fixed. It's always a game of economics to the software company: are there few enough bugs, and are they minor enough or infrequent enough, in order for people to pay for the software?

In this book, we'll show you some fundamental techniques for finding and fixing bugs, and also for how to prevent them in the first place.

1.4 The Difference Between Brackets, Braces, and Parentheses

One of the pieces of terminology that causes confusion is what to call certain characters. The Python style guide (and several dictionaries) use these names, so this book does too:

() Parentheses

[] Brackets

{} Braces (Some people call these *curly brackets* or *curly braces*, but we'll stick to just *braces*.)

1.5 Installing Python

Installation instructions are available on the book website: http://pragprog.com/titles/gwpy/practical-programming.

1.6 For Instructors: How This Book is Organized

This book uses the Python programming language to introduce standard CS1 topics and a handful of useful applications. We chose Python for several reasons:

- *It is free and well documented.* In fact, Python is one of the largest and best-organized open source projects going.
- *It runs everywhere.* The reference implementation, written in C, is used on everything from cell phones to supercomputers, and it's supported by professional-quality installers for Windows, Mac OS X, and Linux.
- *It has a clean syntax.* Yes, every language makes this claim, but during the several years that we have been using it at the University of Toronto, we have found that students make noticeably fewer "punctuation" mistakes with Python than with C-like languages.
- *It is relevant.* Thousands of companies use it every day; it is one of the languages used at Google, Industrial Light & Magic uses it extensively, and large portions of the game EVE Online are written in Python. It is also widely used by academic research groups.
- *It is well supported by tools.* Legacy editors like Vi and Emacs all have Python editing modes, and several professional-quality IDEs are available. (We use IDLE, the free development environment that comes with a standard Python installation.)

We use an "objects first, classes second" approach: students are shown how to *use* objects from the standard library early on but do not create their own classes until after they have learned about flow control and basic data structures. This allows students to get familiar with what a type is (in Python, all types, including integers and floating-point numbers, are classes) before they have to write their own types.

We have organized the book into two parts. The first covers fundamental programming ideas: elementary data types (numbers, strings, lists, sets, and dictionaries), modules, control flow, functions, testing, debugging, and algorithms. Depending on the audience, this material can be covered in a couple of months.

The second part of the book consists of more or less independent chapters on more advanced topics that assume all the basic material has been covered. The first of these chapters shows students how to create their own classes and introduces encapsulation, inheritance, and polymorphism; courses for computer science majors will probably want to include this material. The other chapters cover application areas, such as databases, GUI construction,

and the basics of web programming; these will appeal to both computer science majors and students from the sciences and will allow the book to be used for both.

Lots of other good books on Python programming exist. Some are accessible to novices, such as *Introduction to Computing and Programming in Python: A Multimedia Approach [Guz04]*, *Python Programming: An Introduction to Computer Science [Zel03]*, and others are for anyone with any previous programming experience *How to Think Like a Computer Scientist: Learning with Python [DEM02]*, *Object-Oriented Programming in Python [GL07]*, and *Learning Python [LA03]*. You may also want to take a look at *Python Education Special Interest Group (EDU-SIG) [Pyt11]*, the special interest group for educators using Python.

1.7 What You'll See in This Book

In this book, we'll do the following:

- We will show you how to develop and use programs that solve real-world problems. Most of its examples will come from science and engineering, but the ideas can be applied to any domain.
- We start by teaching you the core features of Python. These features are included in every modern programming language, so you can use what you learn no matter what you work on next.
- We will also teach you how to think methodically about programming. In particular, we will show you how to break complex problems into simple ones and how to combine the solutions to those simpler problems to create complete applications.
- Finally, we will introduce some tools that will help make your programming more productive, as well as some others that will help your applications cope with larger problems.

CHAPTER 2

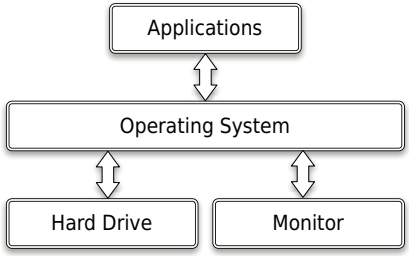
Hello, Python

Programs are made up of commands that tell the computer what to do. These commands are called *statements*, which the computer *executes*. This chapter describes the simplest of Python's statements and shows how they can be used to do arithmetic, which is one of the most common tasks for computers, and also a great place to start learning to program. It's also the basis of almost everything that follows.

2.1 How Does a Computer Run a Python Program?

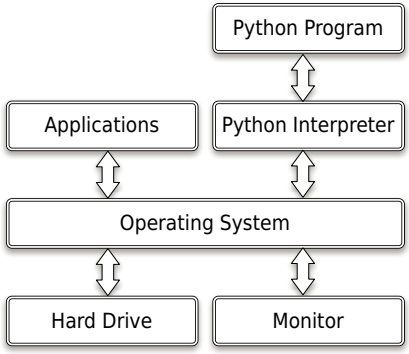
In order to understand what happens when you're programming, you need to have a basic understanding of how a computer executes a program. The computer is assembled from pieces of hardware, including a *processor* that can execute instructions and do arithmetic, a place to store data such as a *hard drive*, and various other pieces such as computer monitor, a keyboard, a card for connecting to a network, and so on.

To deal with all these pieces, every computer runs some kind of *operating system*, such as Microsoft Windows, Linux, or Mac OS X. An operating system, or OS, is a program; what makes it special is that it's the only program on the computer that's allowed direct access to the hardware. When any other application such as your browser, a spreadsheet program, or a game wants to draw on the screen, find out what key was just pressed on the keyboard, or fetch data from the hard drive, it sends a request to the OS:



This may seem a roundabout way of doing things, but it means that only the people writing the OS have to worry about the differences between one graphics card and another, and whether the computer is connected to a network through ethernet or wireless. Everyone else—everyone analyzing scientific data or creating 3D virtual chat rooms—only has to learn their way around the OS, and their programs will then run on thousands of different kinds of hardware.

Twenty-five years ago, that's how most programmers worked. Today, though, it's common to add another layer between the programmer and the computer's hardware. When you write a program in Python, Java, or Visual Basic, it doesn't run directly on top of the OS. Instead, another program, called an *interpreter* or *virtual machine*, takes your program and runs it for you, translating your commands into a language the OS understands. It's a lot easier, more secure, and more portable across operating systems than writing programs directly on top of the OS:



There are two ways to use the Python interpreter. One is to tell it to execute a Python program that is saved in a file. Another is to interact with it in a program called a *shell*, where you type statements one at a time. The interpreter will execute each statement when you type it, do what the statement

says to do, and show any output as text, all in one window. We will explore Python in this chapter using a Python shell.

Install Python Now (if you haven't already)

If you have not yet installed Python 3, please do so now. (Python 2 won't do; there are significant differences between Python 2 and Python 3, and this book uses Python 3.)

Programming requires practice: you won't learn how to program just by reading this book, much like you wouldn't learn how to play guitar just by reading a book on how to play guitar.

Python comes with a program called IDLE, which we use to write Python programs. IDLE has a Python shell that communicates with the Python interpreter, and also allows you to write and run programs that are saved in a file. We describe IDLE in Appendix 1, *The IDLE Development Environment*, on page 257.

We *strongly* recommend that you open IDLE and follow along with our examples. Typing in the code in this book is the programming equivalent of repeating phrases back to an instructor as you're learning to speak a new language.

2.2 Expressions and Values: Arithmetic in Python

You're familiar with mathematical expressions like `3 + 4` ("three plus four") and `2 - 3 / 5` ("two minus three divided by five"); each expression is built out of *values* like `2`, `3`, and `5` and *operators* like `+` and `-`, which combine their *operands* in different ways. In the expression `4 / 5`, the operator is `/` and the operands are `4` and `5`.

Expressions don't have to involve an operator: a number by itself is an expression. For example, we consider `212` to be an expression as well as a value.

Like any programming language, Python can *evaluate* basic mathematical expressions. For example, the following expression adds `4` and `13`:

```
>>> 4 + 13
17
```

The `>>>` symbol is called a *prompt*. When you opened IDLE, a window should have opened with this symbol shown; you don't type it. It is prompting you to type something. Here, we typed `4 + 13`, and then we typed the return (or enter) key in order to signal that we were done entering that *expression*. Python then evaluated the expression.

When an expression is evaluated, it produces a single value. In the previous expression, evaluation of `4 + 13` produced the value `17`. When typed in the shell, Python shows the value that is produced.

Subtraction and multiplication are similarly unsurprising:

```
>>> 15 - 3
12
>>> 4 * 7
28
```

The following expression divides `5` by `2`:

```
>>> 5 / 2
2.5
```

The result has a decimal point. In fact, the result of a division always has a decimal point even if the result is a whole number:

```
>>> 4 / 2
2.0
```

Types

Every value in Python has a particular *type*, and the types of values determine how they behave when they're combined. Values like `4` and `17` have type `int` (short for *integer*), and values like `2.5` and `17.0` have type `float`. The word *float* is short for *floating point*, which refers to the decimal point that moves around between digits of the number.

An expression involving two `float`s produces a `float`:

```
>>> 17.0 - 10.0
7.0
```

When an expression's operands are an `int` and a `float`, Python automatically converts the `int` to a `float`. This is why the following two expressions both return the same answer as the earlier one:

```
>>> 17.0 - 10
7.0
>>> 17 - 10.0
7.0
```

If you want, you can omit the zero after the decimal point when writing a floating-point number:

```
>>> 17 - 10.
7.0
>>> 17. - 10
7.0
```

However, most people think this is bad style, since it makes your programs harder to read: it's very easy to miss a dot on the screen and see "17" instead of "17.".

Integer division, modulo, and exponentiation

Every now and then, we want only the integer part of a division result. For example, we might want to know how many 24-hour days there are in 53 hours (which is 2 24-hour days plus another 5 hours). To calculate the number of days, we can use *integer division*:

```
>>> 53 // 24
2
```

We can find out how many hours are left over using the *modulo* operator, which gives the remainder of the division:

```
>>> 53 % 24
5
```

Python doesn't round the result of integer division. Instead, it takes the *floor* of the result of the division, which means that it rounds down to the nearest integer:

```
>>> 17 // 10
1
```

Be careful about using % and // with negative operands. Because Python takes the floor of the result of an integer division, the result is one smaller than you might expect if the result is negative:

```
>>> -17 // 10
-2
```

When using modulo, the sign of the result matches the sign of the divisor (the second operand):

```
>>> -17 % 10
3
>>> 17 % -10
-3
```

For the mathematically inclined, the relationship between // and % comes from this equation, for any two numbers `a` and `b`:

```
(b * (a // b) + a % b) is equal to a
```

For example, because -17 // 10 is -2, and -17 % 10 is 3, then 10 * (-17 // 10) + -17 % 10 is the same as 10 * -2 + 3, which is -17.

Floating-point numbers can be operands for // and % as well. With //, the result is rounded down to the nearest whole number, although the type is a floating-point number:

```
>>> 3.3 // 1
3.0
>>> 3 // 1.0
3.0
>>> 3 // 1.1
2.0
>>> 3.5 // 1.1
3.0
>>> 3.5 // 1.3
2.0
```

The following expression calculates 3 raised to the power 6:

```
>>> 3 ** 6
729
```

Operators that have two operands are called *binary operators*. Negation is a *unary operator* because it applies to one operand:

```
>>> -5
-5
>>> --5
5
>>> ---5
-5
```

Symbol	Operator	Example	Result
-	Negation	-5	-5
+	Addition	11 + 3.1	14.1
-	Subtraction	5 - 19	-14
*	Multiplication	8.5 * 4	34.0
/	Division	11 / 2	5.5
//	Integer Division	11 // 2	5
%	Remainder	8.5 % 3.5	1.5
**	Exponentiation	2 ** 5	32

Table 1—Arithmetic operators

2.3 What *Is* a Type?

We've now seen two types of numbers (integers and floating-point numbers), so we ought to explain what we mean by a *type*. In computing, a type consists of two things:

- a set of values, and
- a set of operations that can be applied to those values.

For example, in type int, the values are ..., -3, -2, -1, 0, 1, 2, 3, ... and we have seen that these operators can be applied to those values: +, -, *, /, //, and %.

The values in type float are a subset of the real numbers, and it happens that the same set of operations can be applied to float values. If an operator can be applied to more than one type of value, it is called an *overloaded operator*. We can see what happens when these are applied to various values in Table 1, *Arithmetic operators*, on page 14.

Finite Precision

Floating-point numbers are not exactly the fractions you learned in grade school. For example, take a look at Python's version of the fractions $^2/_3$ and $^5/_3$:

```
>>> 2 / 3
0.6666666666666666
>>> 5 / 3
1.6666666666666667
```

The first value ends with a 6, and the second with a 7. This is fishy: both of them should have an infinite number of 6's after the decimal point. The problem is that computers have a finite amount of memory, and (to make calculations fast and memory efficient) most programming languages limit how much information can be stored for any single number. The number 0.6666666666666666 turns out to be the closest value to $^2/_3$ that the computer can actually store in that limited amount of memory, and 1.6666666666666667 is as close as we get to the real value of $^5/_3$.

More on Numeric Precision

Integers—values of type int—in Python can be as large or as small as you like. However, float values are only *approximations* to real numbers. For example, $^1/_4$ can be stored exactly, but as we've already seen, $^2/_3$ cannot. Using more memory won't solve the problem, though it will make the approximation closer to the real value, just as writing a larger number of 6s after the 0 in 0.666... doesn't make it exactly equal to $^2/_3$.

The difference between $^2/_3$ and 0.6666666666666666 may look tiny. But if we use 0.6666666666666666 in a calculation, then the error may get compounded. For example, if we add 1 to $^2/_3$, the resulting value ends in ...6665, so in many programming languages 1 + $^2/_3$ is not equal to $^5/_3$:

```
>>> 2 / 3 + 1
```

```
1.6666666666666665
>>> 5 / 3
1.6666666666666667
```

As we do more calculations, the rounding errors can get larger and larger, particularly if we're mixing very large and very small numbers. For example, suppose we add 10000000000 (ten billion) and 0.00000000001 (there are 10 zeroes after the decimal point):

```
>>> 10000000000 + 0.00000000001
10000000000.0
```

The result ought to have twenty zeroes between the first and last significant digit, but that's too many for the computer to store, so the result is just 10000000000—it's as if the addition never took place. Adding lots of small numbers to a large one can therefore have no effect at all, which is *not* what a bank wants when it totals up the values of its customers' savings accounts.

It's important to be aware of the floating-point issue. There is no magic bullet to solve this issue, because computers are limited in both memory and in speed. We will occasionally give you tips as we go along, but a thorough treatment is beyond the scope of this book. *Numerical analysis*, the study of algorithms to approximate continuous mathematics, is one of the largest subfields of computer science and mathematics.

Here's the first tip: if you have to add up floating-point numbers, add them from smallest to largest in order to minimize the error.

Operator Precedence

Let's put our knowledge of ints and floats to use in converting Fahrenheit to Celsius. To do this, we subtract 32 from the temperature in Fahrenheit and then multiply by $\frac{5}{9}$:

```
>>> 212 - 32 * 5 / 9
194.22222222222223
```

Python claims the result is 194.22222222222223 degrees Celsius when in fact it should be 100. The problem is that multiplication and division have higher *precedence* than subtraction; in other words, when an expression contains a mix of operators, the * and / are evaluated before - and +. This means that what we actually calculated was 212 - ((32 * 5) / 9): the *subexpression* 32 * 5 is evaluated before the division is applied, and that division is evaluated before the subtraction occurs.

We can alter the order of precedence by putting parentheses around subexpressions:

```
>>> (212 - 32) * 5 / 9
100.0
```

Here is the order of precedence for arithmetic operators:

Precedence	Operator	Operation
highest	**	Exponentiation
	-	Negation
	*, /, //, %	Multiplication, division, integer division, and remainder
lowest	+, -	Addition and subtraction

Table 2—Arithmetic operators listed by precedence from highest to lowest

Operators with higher precedence are applied before those with lower precedence. Here is an example that shows this:

```
>>> -2 ** 4
-16
>>> -(2 ** 4)
-16
>>> (-2) ** 4
16
```

Because exponentiation has higher precedence than negation, the subexpression `2 ** 4` is evaluated before negation is applied.

Operators on the same row have equal precedence, and are applied left to right, except for exponentiation, which is applied right to left. So, for example, because binary operators + and - are on the same row, `3 + 4 - 5` is equivalent to `(3 + 4) - 5`, and `3 - 4 + 5` is equivalent to `(3 - 4) + 5`.

It's a good rule to parenthesize complicated expressions even when you don't need to, since it helps the eye read things like `1 + 1.7 + 3.2 * 4.4 - 16 / 3`. On the other hand, it's a good rule to *not* use parentheses in simple expressions such as `3.1 * 5`.

2.4 Variables and Computer Memory: Remembering Values

Like mathemeticians, programmers frequently name values so that they can use them later. A name that refers to a value is called a *variable*. In Python, variable names can use letters, digits, and the underscore symbol (but they can't start with a digit). For example, `X`, `species5618`, and `degrees_celsius` are all allowed, but `777` isn't (it would be confused with a number), and neither is `no-way!` (it contains punctuation).

You create a new variable by *assigning* it a value:

```
>>> degrees_celsius = 26.0
```

This statement is called an *assignment statement*; we say that degrees_celsius is *assigned* the value 26.0. That makes degrees_celsius *refer* to value 26.0. We can use variables anywhere we can use regular values. Whenever Python sees a variable in an expression, it substitutes the value to which the variable refers:

```
>>> degrees_celsius = 26.0
>>> degrees_celsius
26.0
>>> 9 / 5 * degrees_celsius + 32
78.80000000000001
>>> degrees_celsius / degrees_celsius
1.0
```

Variables are called *variables* because their values can vary as the program executes: we can assign a new value to a variable:

```
>>> degrees_celsius = 26.0
>>> 9 / 5 * degrees_celsius + 32
78.80000000000001
>>> degrees_celsius = 0.0
>>> 9 / 5 * degrees_celsius + 32
32.0
```

Assigning a value to a variable that aleady exists doesn't create a second variable. Instead, the existing variable is reused, which means that the variable no longer refers to its old value.

We can create other variables; this example calculates the difference between the boiling point of water and the temperature stored in degrees_celsius:

```
>>> degrees_celsius = 15.5
>>> difference = 100 - degrees_celsius
>>> difference
84.5
```

Warning: = is not equality in Python!

In mathematics, = means "the thing on the left is equal to the thing on the right". In Python, it means something quite different. Assignment is not symmetric: x = 12 assigns the value 12 to variable x, but 12 = x results in an error. Because of this, we never describe the statement x = 12 as "degrees_celsius equals 26.0".

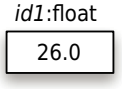
Values, Variables, and Computer Memory

We're going to develop a model of computer memory—a *memory model*—that will let us *trace* what happens when Python executes a Python program. This

memory model will help us accurately predict and explain what Python does when it executes code, a skill that is a requirement for becoming a good programmer.

Every location in computer memory has a *memory address*, much like an address for a house on a street, which uniquely identifies that location. We're going to mark our memory addresses with an *id* prefix (short for *identifier*) so that they look different from regular integers: *id1*, *id2*, *id3* and so on.

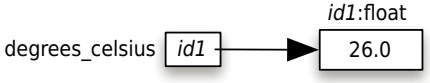
Here is how we draw floating-point value `26.0` in computer memory:



This picture shows value `26.0` at memory address *id1*. We will always show the type of the value as well—in this case, `float`.

We will call this box an *object*: a value at a memory address with a type. During execution of a program, *every* value that Python keeps track of is stored inside an object in computer memory.

In our memory model, a variable contains the memory address of the object to which it refers:



In order to make the picture easier to interpret, we will usually draw arrows from variables to their objects.

We use the following terminology:

- Value `26.0` has memory address *id1*.
- The object at memory address *id1* has type `float` and value `26.0`.
- Variable `degrees_celsius` *contains* memory address *id1*.
- Variable `degrees_celsius` *refers* to value `26.0`.

Whenever Python needs to know which value `degree_celsius` refers to, it looks at the object at the memory address that `degree_celsius` contains. In this example, that memory address is *id1*, so Python will use the value at memory address *id1*, which is `26.0`.

The Online Python Tutor

Philip Guo wrote a web-based memory visualizer that matches our memory model pretty well. Here's the URL: http://pythontutor.com/visualize.html. It can trace both Python 2 and Python 3 code; make sure you select the correct version. The settings that most closely match our memory model are these:

- hide frames of exited functions
- render all objects on the heap
- hide environment parent pointers
- use text labels for references

We strongly recommend that you use this visualizer whenever you want to trace execution of a Python program.

In case you find it motivating, we weren't aware of Philip's visualizer when we developed our memory model (and vice versa), and yet they match extremely closely.

Assignment Statement

Here is the general form of an assignment statement:

«variable» = *«expression»*

This is executed as follows:

1. Evaluate the expression on the right of the = sign to produce a value. This value has a memory address.
2. Store the memory address of the value in the variable on the left of the =. Create a new variable if that name doesn't already exist; otherwise, just reuse the existing variable, replacing the memory address that it contains.

Consider this example:

```
>>> degrees_celsius = 26.0 + 5
>>> degrees_celsius
31.0
```

Here is how Python executes the statement `degrees_celsius = 26.0 + 5`:

1. Evaluate the expression on the right of the = sign: `26.0 + 5`. This produces the value `31.0`, which has a memory address. (Remember that Python stores all values in computer memory.)
2. Make the variable on the left of the = sign, `degrees_celsius`, refer to `31.0` by storing the memory address of `31.0` in `degrees_celsius`.

Reassigning to Variables

Consider this code:

```
>>> difference = 20
>>> double = 2 * difference
>>> double
40
>>> difference = 5
>>> double
40
```

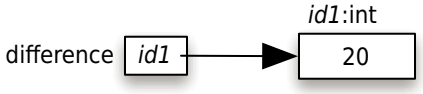
This code demonstrates that assigning to a variable *does not change any other variable*. We start by assigning value 20 to variable difference and then we assign the result of evaluating 2 * difference (which produces 40) to variable double.

Next, we assign value 5 to variable difference, but when we examine the value of double, it still refers to 40.

Here's how it works according to our rules. The first statement, difference = 20, is executed as follows:

1. Evaluate the expression on the right of the = sign: 20. This produces the value 20, which we'll put at memory address *id1*.
2. Make the variable on the left of the = sign, difference, refer to 20 by storing *id1* in difference.

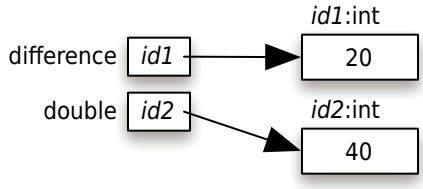
Here is the current state of the memory model. (Variable double has not yet been created because we have not yet executed the assignment to it.)



The second statement, double = 2 * difference, is executed as follows:

1. Evaluate the expression on the right of the = sign: 2 * difference. As we see in the memory model, difference refers to value 20, so this expression is equivalent to 2 * 20, which produces 40. We'll pick memory adress *id2* for value 40.
2. Make the variable on the left of the = sign, double, refer to 40 by storing *id2* in double.

Here is the current state of the memory model:

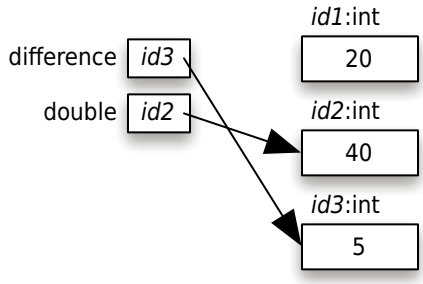


When Python executes the third statement, `double`, it merely looks up the value that `double` refers to—`40`—and displays it.

The fourth statement, `difference = 5`, is executed as follows:

1. Evaluate the expression on the right of the `=` sign: `5`. This produces the value `5`, which we'll put at memory address *id3*.
2. Make the variable on the left of the `=` sign, `difference`, refer to `5` by storing *id3* in `difference`.

Here is the current state of the memory model:



Variable `double` still contains *id2*, so it still refers to `40`. Neither variable refers to the `20` anymore.

The fifth and last statement, `double`, merely looks up the value that `double` refers to, which is still `40`, and displays it.

We can even use a variable on both sides of an assignment statement:

```
>>> number = 3
>>> number
3
>>> number = 2 * number
>>> number
6
>>> number = number * number
>>> number
36
```

We'll now explain how Python executes this code, but we won't explicitly mention memory addresses. Trace this on a piece of paper while we describe what happens; make up your own memory addresses as you do this.

Python executes the first statement, `number = 3`, as follows:

1. Evaluate the expression on the right of the = sign: `3`. This one is easy to evaluate: `3` is produced.
2. Make the variable on the left of the = sign, `number`, refer to `3`.

Python executes the second statement, `number = 2 * number`, as follows:

1. Evaluate the expression on the right of the = sign: `2 * number`. `number` currently refers to `3`, so this is equivalent to `2 * 3`, so `6` is produced.
2. Make the variable on the left of the = sign, `number`, refer to `6`.

Python executes the third statement, `number = number * number`, as follows:

1. Evaluate the expression on the right of the = sign: `number * number`. `number` currently refers to `6`, so this is equivalent to `6 * 6`, so `36` is produced.
2. Make the variable on the left of the = sign, `number`, refer to `36`.

Augmented Assignment

In this example, variable `score` appears on both sides of the assignment statement:

```
>>> score = 50
>>> score
50
>>> score = score + 20
>>> score
70
```

This is so common that Python provides a shorthand notation for this operation:

```
>>> score = 50
>>> score
50
>>> score += 20
>>> score
70
```

An *augmented assignment* combines an assignment statement with an operator to make the statement more concise. An augmented assignment statement is executed as follows:

1. Evaluate the expression on the right of the = sign to produce a value.

2. Apply the operator attached to the = sign to the variable on the left of the = and the value that was produced. This produces another value. Store the memory address of that value in the variable on the left of the =.

Note that the operator is applied *after* the expression on the right is evaluated:

```
>>> d = 2
>>> d *= 3 + 4
>>> d
14
```

All the operators in Table 2, *Arithmetic operators listed by precedence from highest to lowest*, on page 17 have shorthand versions. For example, we can square a number by multiplying it by itself:

```
>>> number = 10
>>> number *= number
>>> number
100
```

which is equivalent to this:

```
>>> number = 10
>>> number = number * number
>>> number
100
```

Table 3, *Augmented Assignment Operators*, on page 24 contains a summary of the augmented operators you've seen, plus a few more based on arithmetic operators you learned about in Section 2.2, *Expressions and Values: Arithmetic in Python*, on page 11.

Symbol	Example	Result
+=	`x = 7` `x += 2`	x refers to 9
-=	`x = 7` `x -= 2`	x refers to 5
*=	`x = 7` `x *= 2`	x refers to 14
/=	`x = 7` `x /= 2`	x refers to 3.5
//=	`x = 7` `x //= 2`	x refers to 3
%=	`x = 7` `x %= 2`	x refers to 1

Symbol	Example	Result
**=	x = 7 x **= 2	x refers to 49

Table 3—Augmented Assignment Operators

2.5 How Python Tells You Something Went Wrong

Broadly speaking, two kinds of errors in Python: *syntax errors*, which happen when you type something that isn't valid Python code, and *semantic errors*, which happen when you tell Python to do something that it just can't do, like divide a number by zero, or try to use a variable that doesn't exist.

Here is what happens when we try to use a variable that hasn't been created yet:

```
>>> 3 + moogah
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
NameError: name 'moogah' is not defined
```

This is pretty cryptic; Python error messages are meant for people who already know Python. (You'll get used to them, and soon find them helpful.) The first two lines aren't much use right now, though they'll be indispensable when we start writing longer programs. The last line is the one that tells us what went wrong: the name `moogah` wasn't recognized.

Here's another error message you might sometimes see:

```
>>> 2 +
  File "<stdin>", line 1
    2 +
      ^
SyntaxError: invalid syntax
```

The rules governing what is and isn't legal in a programming language are called its *syntax*. The message tells us that we violated Python's syntax rules—in this case, by asking it to add something to `2` but not telling it what to add.

Earlier, in *Warning: = is not equality in Python!*, on page 18, we claimed that `12 = x` results in an error. Let's try it:

```
>>> 12 = x
  File "<stdin>", line 1
SyntaxError: can't assign to literal
```

A *literal* is any value, like 12 and 26.0. This is a SyntaxError because when Python examines that assignment statement, it knows that you can't assign a value to a number even before it tries to execute it; you can't change the value of 12 to anything else. 12 is just 12.

2.6 A single statement that spans multiple lines

Sometimes statements get pretty intricate. The recommended Python style is to limit lines to 80 characters, including spaces, tabs, and other *whitespace* characters, and that's a common limit throughout the programming world. Here's what to do when lines gets too long, or you want to split it up for clarity.

In order to split up a statement into more than one line, you need to do one of 2 things:

1. Make sure your line break occurs inside parentheses, or
2. Use the line-continuation character, \.

The line-continuation characters is a backslash (\), not the division symbol (/).

Here are examples of both:

```
>>> (2 +
... 3)
5
>>> 2 + \
... 3
5
```

Notice how we don't get a SyntaxError. Each triple-dot prompt in our examples indicates that we are in the middle of entering an expression. These make the code line up nicely. You do not type the dots any more than you type the greater-than signs in the usual >>> prompt.

Here is a more realistic (and tastier) example: let's say we're baking cookies. The authors live in Canada, which uses Celsius, but we own cookbooks that use Fahrenheit. We are wondering how long it will take to preheat our oven. Here are our facts:

- The room temperature is 20 degrees Celsius.
- Our oven controls use Celsius, and the oven heats up at 20 degrees per minute.

- Our cookbook uses Fahrenheit, and says to preheat the oven to 350 degrees.

We can convert `t` degrees Fahrenheit to Celsius like this: `5 / 9 * t - 32`. Let's use this information to try to solve our problem.

```
>>> room_temperature_c = 20
>>> cooking_temperature_f = 350
>>> oven_heating_rate = 20 # degrees Celsius increase per minute
>>> oven_heating_time = (
... (5 / 9 * cooking_temperature_f - 32) - room_temperature_c) / \
... oven_heating_rate
>>> oven_heating_time
7.122222222222223
```

Not bad—just over 7 minutes to preheat.

The assignment statement to variables `oven_heating_time` spans three lines. The first line ends with an open parenthesis, so we do not need a line continuation character. The second ends *outside* of parentheses, so we need the line continuation character. The third line completes the assignment statement.

That's still hard to read. Once we've continued an expression on the next line, we can indent (by typing the tab key, or typing the space bar a bunch) to our heart's content to make it clearer:

```
>>> oven_heating_time = \
...     ((5 / 9 * cooking_temperature_f - 32) - room_temperature_c) / \
...     oven_heating_rate
```

Or even this—notice how the two subexpressions involved in the subtraction line up:

```
>>> oven_heating_time = \
...     ((5 / 9 * cooking_temperature_f - 32) - \
...      room_temperature_c) / \
...     oven_heating_rate
```

2.7 Describing Code

Programs can be quite complicated and are often thousands of lines long. It can be helpful to write a *comment* describing parts of the code so that when you (or someone else) reads it they don't have to spend much time figuring out why the code is there.

In Python, any time the # character is encountered, Python will ignore the rest of the line. This allows you to write regular English sentences:

```
>>> # Python ignores this sentence because of the # symbol.
```

The # symbol does not have to be the first character on the line; it can appear at the end of a statement:

```
>>> (212 - 32) * 5 / 9 # Convert 212 degrees Fahrenheit to Celsius.
100.0
```

Notice that the comment doesn't describe how Python works. Instead, it is meant for a human reading the code to help them understand why the code exists.

2.8 Making Code Readable

Much like there are spaces in English sentences to make them easier to read, we use spaces in Python code to make it easier to read. In particular, we always put a space before and after every binary operator. For example, we write `v = 4 + -2.5 / 3.6` instead of `v=4+-2.5/3.6`. There are situations where it may not make a difference, but that's a detail we don't want to fuss about, so we always do it: it's almost never *harder* to read if there are spaces.

Psychologists have discovered that people can keep track of only a handful of things at any one time (*Forty Studies That Changed Psychology [Hoc04]*). Since programs can get quite complicated, it's important that you choose names for your variables that will help you remember what they're for. `id1`, `X2`, and `blah` won't remind you of anything when you come back to look at your program next week; use names like `celsius`, `average`, and `final_result` instead.

Other studies have shown that your brain automatically notices differences between things—in fact, there's no way to stop it from doing this. As a result, the more inconsistencies there are in a piece of text, the longer it takes to read. (JuSt thInK a bout how long It w o u l d tAKE you to rEa d this cHaPTer iF IT wAs fORmaTTeD like thIs.) It's therefore also important to use consistent names for variables. If you call something `maximum` in one place, don't call it `max_val` in another; if you use the name `max_val`, don't also use the name `maxVal`, and so on.

These rules are so important that many programming teams require members to follow a style guide for whatever language they're using, just as newspapers and book publishers specify how to capitalize headings and whether to use a comma before the last item in a list. If you search the Internet for *programming style guide* (https://www.google.com/search?q=programming+style+guide), you'll discover links to hundreds of examples. In this book, we follow the style guide for Python at http://www.python.org/dev/peps/pep-0008/.

You will also discover that lots of people have wasted many hours arguing over what the "best" style for code is. Some of your classmates (and your

instructors) may have strong opinions about this as well. If they do, ask them what data they have to back up their beliefs. Strong opinions need strong evidence to be taken seriously.

2.9 Programs, Statements, Expressions, and Variables

In this chapter, you learned the following:

- An operating system is a program that manages your computer's hardware on behalf of other programs. An interpreter or virtual machine is a program that sits on top of the operating system and runs your programs for you. The Python shell is an interpreter, translating your Python statements into language the operating system understands, and translating the results back so you can see and use them.
- Programs are made up of statements, or instructions. These can be simple expressions like `3 + 4` and assignment statements like `celsius = 20` (which create new variables or change the values of existing ones). There are many other kinds of statements in Python, and we'll introduce them throughout the book.
- Every value in Python has a specific type, which determines what operations can be applied to it. The two types used to represent numbers are `int` and `float`. Floating-point numbers are approximations to real numbers.
- Python evaluates an expression by applying higher-precedence operators before lower-precedence operators. You can change that order by putting parentheses around subexpressions.
- Python stores every value in computer memory. A memory location containing a value is called an *object*.
- Variables are created by executing assignment statements. If a variable already exists because of a previous assignment statement, Python will use that one instead of creating a new one.
- Variables contain memory addresses of values. We say that variables refer to values.
- Variables must be assigned values before they can used in expressions.

2.10 Exercises

Here are some exercises for you to try on your own:

1. For each of the following expressions, what value will the expression give? Verify your answers by typing the expressions into Python.

a. `9 - 3`

b. `8 * 2.5`

c. `9 / 2`

d. `9 / -2`

e. `9 // -2`

f. `9 % 2`

g. `9.0 % 2`

h. `9 % 2.0`

i. `9 % -2`

j. `-9 % 2`

k. `9 / -2.0`

l. `4 + 3 * 5`

m. `(4 + 3) * 5`

2. Unary minus negates a number. Unary plus exists as well; for example, Python understands +5. If `x` has the value -17, what do you think `+x` should do? Should it leave the sign of the number alone? Should it act like absolute value, removing any negation? Use the Python shell to find out its behavior.

3. a. Create a new variable `temp`, and assign it the value 24.

   b. Convert the value in `temp` from Celsius to Fahrenheit by multiplying by 1.8 and adding 32; make `temp` refer to the resulting value. What is `temp`'s new value?

4. For each of the following expressions, in which order are the subexpressions evaluated?

   a. `6 * 3 + 7 * 4`

   b. `5 + 3 / 4`

   c. `5 - 2 * 3 ** 4`

5. a. Create a new variable `x`, and assign it the value 10.5.

   b. Create a new variable `y`, and assign it the value 4.

   c. Sum `x` and `y`, and make `x` refer to the resulting value. What are `x` and `y`'s new values?

6. Write a bullet list description of what happens when Python evaluates the statement `x += x - x` when `x` has the value 3.

7. When a variable is used before it has been assigned a value, a `NameError` occurs. In the Python shell, write an expression that results in a `NameError`.

8. Which of the following expressions results in `SyntaxErrors`?

   a. `6 * ----------8`

   b. `8 = people`

   c. `((((4 ** 3))))`

   d. `(-(-(-(-5))))`

   e. `4 += 7 / 2`

CHAPTER 3

Designing and Using Functions

Mathematicians create *functions* to make calculations, such as Fahrenheit to Celsius conversion, easy to reuse, and to make other calculations easier to read because they can use those functions instead of repeatedly writing out equations. Programmers do this too, at least as often as mathematicians. In this chapter we will explore several of the *built-in* functions that come with Python, and we'll also show you how to define your own functions.

3.1 Functions that Python Provides

Python comes with many *built-in functions* that perform common operations. One example is `abs`, which produces the absolute value of a number:

```
>>> abs(-9)
9
>>> abs(3.3)
3.3
```

Each of these statements is a *function call*.

> **Keep your shell open**
>
> As a reminder, we recommend that you have IDLE open (or another Python editor) and that you try all the code under discussion: this is a good way to cement your learning.

The general form of a function call is as follows:

```
«function_name»(«arguments»)
```

An *argument* is an expression that appears between the parentheses of a function call. In `abs(-9)`, the argument is `-9`.

Here, we calculate the difference between a day temperature and a night temperature, as might be seen on a weather report (a warm weather system moved in overnight):

```
>>> day_temperature = 3
>>> night_temperature = 10
>>> abs(day_temperature - night_temperature)
7
```

In this call on function abs, the argument is day_temperature - night_temperature. Because day_temperature refers to 3 and night_temperature refers to 10, when Python evaluates this expression, -7 is produced. This value is then *passed* to function abs, which then *returns* or *produces* value 7.

Here are the rules to executing a function call:

1. Evaluate each argument one at a time, working from left to right.
2. Pass the resulting values into the function.
3. Execute the function. When the function call finishes, it produces a value.

Because function calls produce values, they can be used in expressions:

```
>>> abs(-7) + abs(3.3)
10.3
```

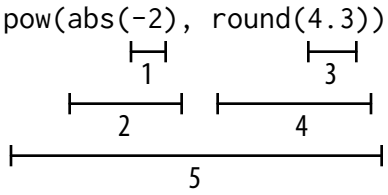
We can also use function calls as arguments to other functions:

```
>>> pow(abs(-2), round(4.3))
16
```

Python sees the call on pow and starts by evaluating the arguments from left to right. The first argument is a call on function abs, so Python executes it. abs(-2) produces 2, so that's the first value for the call on pow. Then Python executes round(4.3), which produces 4.

Now that the arguments to the call on function pow have been evaluated, Python finishes calling pow, sending in 2 and 4 as the argument values. That means that pow(abs(-2), round(4.3)) is equivalent to pow(2, 4), and 2^4 is 16.

Here is a diagram indicating the order in which the various pieces of this expression are evaluated by Python. We have underlined each subexpression and given it a number to indicate when Python executes or evaluates that subexpression:



Some of the most useful built-in functions are ones that convert from one type to another. The type names int and float can be used as functions:

```
>>> int(34.6)
34
>>> int(-4.3)
-4
>>> float(21)
21.0
```

In this example, we see that when a floating-point number is converted to an integer, it is truncated—not rounded.

If you're not sure what a function does, try calling built-in function help, which shows documentation for any function:

```
>>> help(abs)
Help on built-in function abs in module builtins:

abs(...)
    abs(number) -> number

    Return the absolute value of the argument.
```

The first line states which function is being described and which *module* it belongs to. Modules are an organizational tool in Python and are discussed in Chapter 6, *A Modular Approach to Program Organization*, on page 101.

The next part describes how to call the function. The arguments are described within the parentheses—here, "number" means that you can call abs with either an int or a float. After the -> is the *return type*, which again is either an int or a float.

After that is an English description of what the function does when called.

Another built-in function is round, which rounds a floating-point number to the nearest integer:

```
>>> round(3.8)
4
>>> round(3.3)
```

```
3
>>> round(3.5)
4
>>> round(-3.3)
-3
>>> round(-3.5)
-4
```

Function round can be called with either one or two arguments. If called with one, as we've been doing, it rounds to the nearest integer. If called with two arguments, it rounds to a floating-point number, where the second argument indicates the precision:

```
>>> round(3.141592653, 2)
3.14
```

The documentation for round indicates that the second argument is optional by surrounding it with brackets:

```
>>> help(round)
Help on built-in function round in module builtins:

round(...)
    round(number[, ndigits]) -> number

    Round a number to a given precision in decimal digits (default 0 digits).
    This returns an int when called with one argument, otherwise the
    same type as the number. ndigits may be negative.
```

Let's explore built-in function pow by starting with its help documentation:

```
>>> help(pow)
Help on built-in function pow in module builtins:

pow(...)
    pow(x, y[, z]) -> number

    With two arguments, equivalent to x**y.  With three arguments,
    equivalent to (x**y) % z, but may be more efficient (e.g. for longs).
```

This shows that function pow can be called with either two or three arguments. The English description mentions that when called with two arguments it is equivalent to x ** y. Let's try it:

```
>>> pow(2, 4)
16
```

This call calculates 2^4. So far so good. How about with three arguments?

```
>>> pow(2, 4, 3)
1
```

We know that 2^4 is 16, and evaluation of `16 % 3` produces 1.

3.2 Memory Addresses: How Python Keeps Track of Values

Back in *Values, Variables, and Computer Memory*, on page 18, you learned that Python keeps track of each value in a separate object, and that each object has a memory address. You can discover the actual memory address of an object using built-in function `id`:

```
>>> help(id)
Help on built-in function id in module builtins:

id(...)
    id(object) -> integer

    Return the identity of an object.  This is guaranteed to be unique among
    simultaneously existing objects.  (Hint: it's the object's memory address.)
```

How cool is that? Let's try it:

```
>>> id(-9)
4301189552
>>> id(23.1)
4298223160
>>> shoe_size = 8.5
>>> id(shoe_size)
4298223112
>>> fahrenheit = 77.7
>>> id(fahrenheit)
4298223064
```

Function objects also have memory addresses:

```
>>> id(abs)
4297868712
>>> id(round)
4297871160
```

Python remembers and reuses some objects

A *cache* is a collection of data. Because small integers—up to about 250 or so, depending on the version of Python you're using—are so common that Python creates those objects as it starts up and reuses the same objects whenever it can. This speeds up operations involving these values. Function `id` reveals this:

```
>>> i = 3
>>> j = 3
>>> k = 4 - 1
>>> id(i)
4296861792
```

```
>>> id(j)
4296861792
>>> id(k)
4296861792
```

What that means is that variables i, j, and k refer to the exact same object. This is called *aliasing*.

Larger integers and all floating-point values aren't necessarily cached:

```
>>> i = 30000000000
>>> j = 30000000000
>>> id(i)
4301190928
>>> id(j)
4302234864
>>> f = 0.0
>>> g = 0.0
>>> id(f)
4298223040
>>> id(g)
4298223016
```

Python decides for itself when to cache a value. The only reason you need to be aware of it is so that you aren't surprised when it happens: the output of your program is not affected by when Python decides to cache.

3.3 Defining our Own Functions

The built-in functions are useful but pretty generic. Often, there aren't built-in functions that do what we want, such as calculate mileage or play a game of cribbage. When we want functions to do these sorts of things, we have to write them ourselves.

Because we authors live in Toronto, Canada, we often deal with our neighbour to the south. The United States typically uses Fahrenheit, so we convert from Fahrenheit to Celsius and back a lot. It sure would be nice to be able to do this:

```
>>> convert_to_celsius(212)
100.0
>>> convert_to_celsius(78.8)
26.0
>>> convert_to_celsius(10.4)
-12.0
```

However, function convert_to_celsius doesn't exist yet, so instead we see this (as a reminder, focus only on the last line for now):

```
>>> convert_to_celsius(212)
```

```
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
NameError: name 'convert_to_celsius' is not defined
```

To fix this, we have to write a *function definition* that tells Python what to do when the function is called:

```
>>> def convert_to_celsius(fahrenheit):
...     return (fahrenheit - 32) * 5 / 9
...
```

The *function body* is indented. Here, we inded four spaces as the Python style recommends. If you forget to indent, you get this error:

```
>>> def convert_to_celsius(fahrenheit):
... return (fahrenheit - 32) * 5 / 9
  File "<stdin>", line 2
    return (fahrenheit - 32) * 5 / 9
         ^
IndentationError: expected an indented block
```

Now that we've defined function convert_to_celsius, our earlier function calls will work. We can even use built-in function help on it:

```
>>> help(convert_to_celsius)
Help on function convert_to_celsius in module __main__:

convert_to_celsius(fahrenheit)
```

This shows the first line of the function definition, which we call the *function header*. (Later in this chapter, we'll show you how to add more help documentation to a function.)

Here is a quick overview of how Python executes the following code:

```
>>> def convert_to_celsius(fahrenheit):
...     return (fahrenheit - 32) * 5 / 9
...
>>> convert_to_celsius(80)
26.666666666666668
```

1. Python executes the function definition, which creates the function object (but doesn't execute it yet).

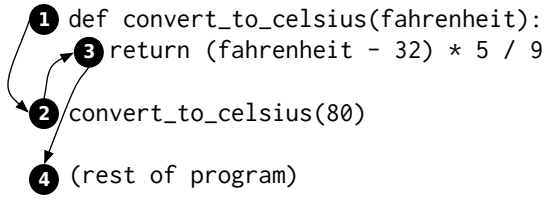
2. Next, Python executes the function call convert_to_celsius(80). To do this, it assigns 80 to fahrenheit (which is a variable). For the duration of this function call, fahrenheit refers to 80.

3. Python now executes the return statement. fahrenheit refers to 80, so the expression that appears after return is equivalent to (80 - 32) * 5 / 9. When

Python evaluates that expression, `26.666666666666668` is produced. We use the word `return` to tell Python what value to produce as the result of the function call, so the result of calling `convert_to_celsius(80)` is `26.666666666666668`.

4. Once Python has finished executing the function call, it returns to the place where the function was originally called.

Here is a picture showing this sequence:



A function definition is a kind of Python statement. The general form of a function definition is as follows:

```
def «function_name»(«parameters»):
    «block»
```

The function header (that's the first line of the function definition) starts with `def` followed by the name of the function, then a comma-separated list of *parameters* within parentheses, and then a colon. A parameter is a variable.

You can't have two functions with the same name; it isn't an error, but if you do it, the second function definition replaces the first one, much like assigning a value to a variable a second time replaces the first value.

Below the function header, and indented (4 spaces, as per Python's style guide), is a block of statements called the *function body*. The function body must contain at least one statement.

Most function definitions will include a `return` statement that, when executed, ends the function and produces a value. The general form of a `return` statement is as follows:

```
return «expression»
```

When Python executes a `return` statement, it evaluates the expression and then produces the result of that expression as the result of the function call.

Keywords are words that are special to Python

Keywords are words that Python reserves for its own use. We can't use them except as Python intends. Two of them are def and return. If we try to use them, either as variable names or as function names (or anything else), Python produces an error:

```
>>> def = 3
  File "<stdin>", line 1
    def = 3
        ^
SyntaxError: invalid syntax
>>> def return(x):
  File "<stdin>", line 1
    def return(x):
              ^
SyntaxError: invalid syntax
```

Here is a complete list of Python keywords. We'll encounter most of them in this book.

```
False     assert    del       for       in        or        while
None      break     elif      from      is        pass      with
True      class     else      global    lambda    raise     yield
and       continue  except    if        nonlocal  return
as        def       finally   import    not       try
```

Use Local Variables for Temporary Storage

Some computations are complex, and breaking them down into separate steps can lead to clearer code. Here, we break down the evaluation of the quadratic polynomial $ax^2 + bx + c$ into several steps (notice that all the statements inside the function are indented the same amount of spaces in order to be aligned with each other):

```
>>> def quadratic(a, b, c, x):
...     first = a * x ** 2
...     second = b * x
...     third = c
...     return first + second + third
...
>>> quadratic(2, 3, 4, 0.5)
6.0
>>> quadratic(2, 3, 4, 1.5)
13.0
```

Variables like first, second, and third that are created within a function are called *local variables*. Local variables get created each time that function is called, and they are erased when the function returns. Because they only exist when the function is being executed, they can't be used outside of the function. This means that trying to access a local variable from outside the function is

an error, just like trying to access a variable that has never been defined is an error:

```
>>> quadratic(2, 3, 4, 1.3)
11.280000000000001
>>> first
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
NameError: name 'first' is not defined
```

A function's parameters are also local variables, so we get the same error if we try to use them outside of a function definition:

```
>>> a
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
NameError: name 'a' is not defined
```

The area of a program that a variable can be used in is called the variable's *scope*. The scope of a local variable is from the line in which it is defined up until the end of the function.

As you might expect, if a function is defined to take a certain number of parameters, a call on that function must have the same number of arguments:

```
>>> quadratic(1, 2, 3)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: quadratic() takes exactly 4 arguments (3 given)
```

Remember that you can call built-in function `help` to find out information about the parameters of a function.

3.4 Tracing Function Calls in the Memory Model

Read the following code. Can you predict what it will do when we run it?

```
>>> def f(x):
...     x = 2 * x
...     return x
...
>>> x = 1
>>> x = f(x + 1) + f(x + 2)
```

That code is confusing, in large part because `x` is used all over the place. However, it *is* pretty short, and it only uses Python features that we have seen so far: assignment statements, expressions, function definitions, and function calls. We're missing some information: are all the `x`'s the same variable? Does Python make a new `x` for each assignment? Each function call? Each function definition?

Here's the answer: whenever Python executes a function call, it creates a *namespace* (literally, a space for names) in which to store local variables for that call. You can think of a namespace as a scrap piece of paper: Python writes down the local variables on that piece of paper, keeps track of them as long as the function is being executed, and throws that paper away when the function returns.

Separately, Python keeps another namespace for variables created in the shell. That means that the `x` that is a parameter of function `f` is a different variable than the `x` in the shell!

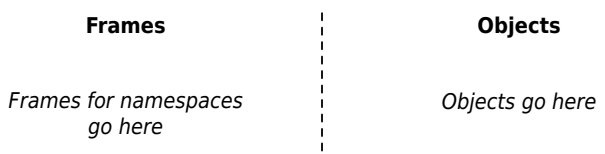
Reusing variable names is common

Using the same name for local variables in different functions is quite common. For example, imagine a program that deals with distances—converting from meters to other units of distance, perhaps. In that program, there would be several functions that all deal with these distances, and it would be entirely reasonable to use `meters` as a parameter name in many different functions.

Let's refine our rules from Section 3.1, *Functions that Python Provides*, on page 33 for executing a function call to include this namespace creation:

1. Evaluate the arguments left to right.
2. Create a namespace to hold the function call's local variables, including the parameters.
3. Pass the resulting argument values into the function by assigning them to the parameters.
4. Execute the function body. As before when a return statement is executed, execution of the body terminates and the value of the expression in the return statement is used as the value of the function call.

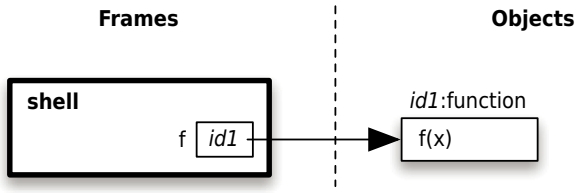
In our memory model, from now on we will draw a separate box for each namespace, to indicate that the variables inside it are in a separate area of computer memory. The programming world calls this box a *frame*. We separate the frames from the objects by a vertical dotted line:



Using our newfound knowledge, let's trace that confusing code. At the beginning, no variables have been created; Python is about to execute the function definition. We have indicated this with an arrow:

```
➤ >>> def f(x):
...       x = 2 * x
...       return x
...
>>> x = 1
>>> x = f(x + 1) + f(x + 2)
```

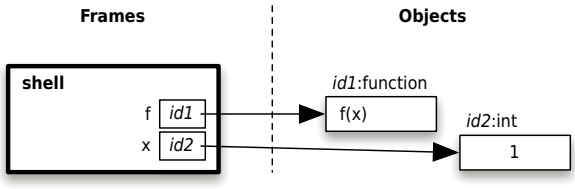
As you've seen in this chapter, when Python executes that function definition it creates a variable f in the frame for the shell's namespace, plus a function object. (Python didn't execute the body of the function; that won't happen until the function is called.) Here is the result:



Now we are about to execute the first assignment to x in the shell

```
>>> def f(x):
...       x = 2 * x
...       return x
...
➤ >>> x = 1
>>> x = f(x + 1) + f(x + 2)
```

Once that assignment happens, both f and x are in the frame for the shell:



Now we are about to execute the second assignment to x in the shell:

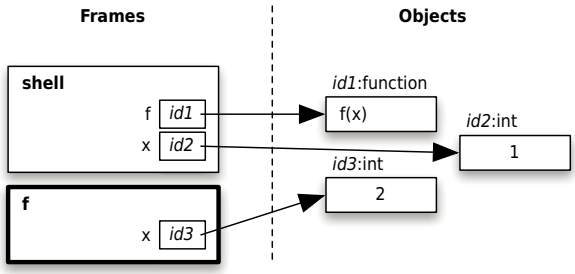
```
>>> def f(x):
...       x = 2 * x
...       return x
...
>>> x = 1
```

```
>>> x = f(x + 1) + f(x + 2)
```

Following the rules for executing an assignment from *Assignment Statement*, on page 20, we first evaluate the expression on the right of the =, which is f(x + 1) + f(x + 2). Python evaluates the left function call first: f(x + 1).

Following the rules for executing a function call, Python evaluates the argument, x + 1. In order to find the value for x, Python looks in the current frame. The current frame is the frame for the shell, and its variable x refers to 1, so x + 1 evaluates to 2.

Now we have evaluated the argument to f. The next step is to create a namespace for the function call. We draw a frame, write in parameter x, and assign 2 to that parameter:

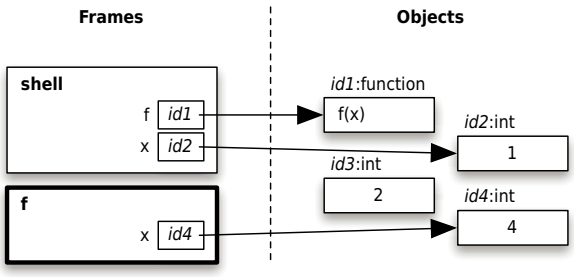


Notice that there are two variables called x, and they refer to different values. Python will always look in the current frame, which we will draw with a thicker border.

We are now about to execute the first statement of function f:

```
>>> def f(x):
...     x = 2 * x
...     return x
...
>>> x = 1
>>> x = f(x + 1) + f(x + 2)
```

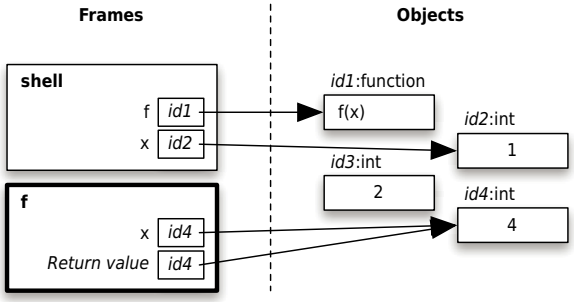
x = 2 * x is an assignment statement. The right-hand side is the expression 2 * x. Python looks up the value of x in the current frame and finds 2, so that expression evaluates to 4. Python finishes executing that assignment statement by making x refer to that 4:



We are now about to execute the second statement of function f:

```
>>> def f(x):
...     x = 2 * x
...     return x
...
>>> x = 1
>>> x = f(x + 1) + f(x + 2)
```

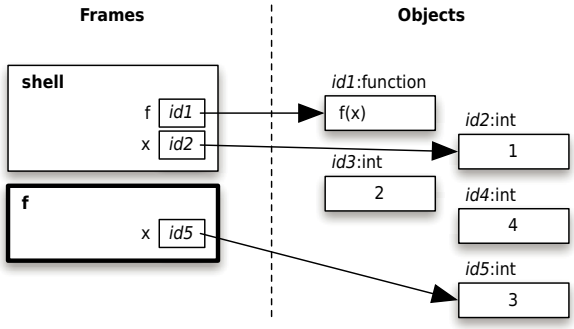
This is a return statement, so we evaluate the expression, which is simply x. Python looks up the value for x in the current frame and finds 4, so that is the return value:



When the function returns, Python comes back to this expression: f(x + 1) + f(x + 2). Python just finished executing f(x + 1), which produced value 4. It then executes the right function call: f(x + 2).

Following the rules for executing a function call, Python evaluates the argument, x + 2. In order to find the value for x, Python looks in the current frame. The call on function f has returned, so that frame is erased—the only frame left is the frame for the shell, and its variable x still refers to 1, so x + 2 evaluates to 3.

Now we have evaluated the argument to f. The next step is to create a namespace for the function call. We draw a frame, write in the parameter x, and assign 3 to that parameter:

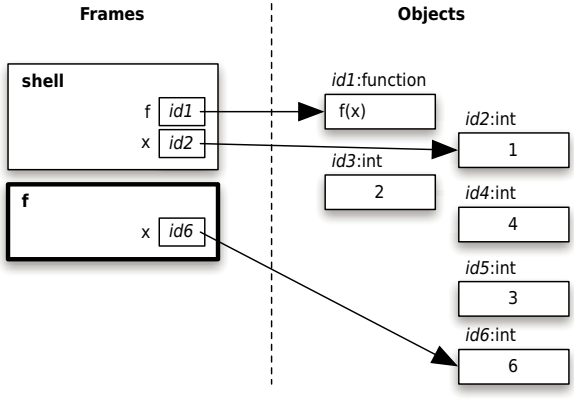


Again, there are two variables called x.

We are now about to execute the first statement of function f:

```
>>> def f(x):
...     x = 2 * x
...     return x
...
>>> x = 1
>>> x = f(x + 1) + f(x + 2)
```

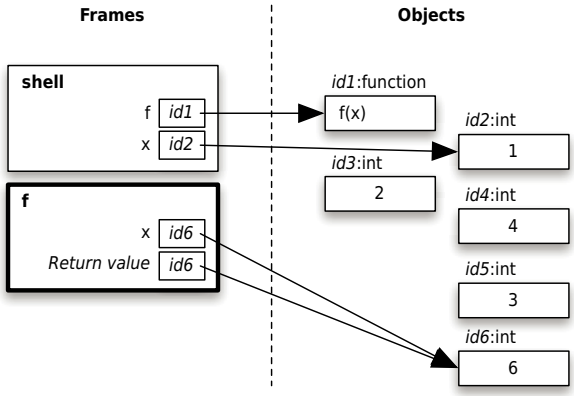
x = 2 * x is an assignment statement. The right-hand side is the expression 2 * x. Python looks up the value of x in the current frame and finds 3, so that expression evaluates to 6. Python finished executing that assignment statement by making x refer to that 6:



We are now about to execute the second statement of function f:

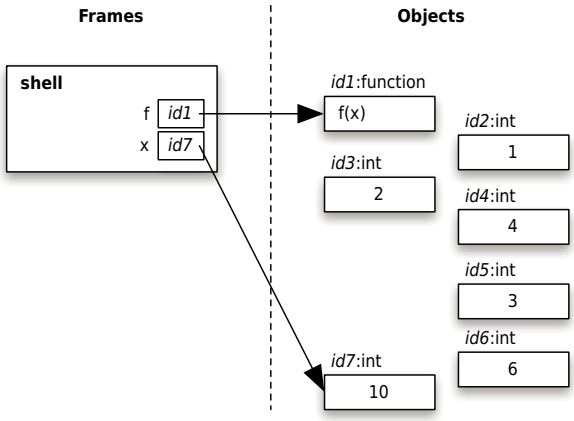
```
>>> def f(x):
...     x = 2 * x
➤ ...     return x
...
>>> x = 1
>>> x = f(x + 1) + f(x + 2)
```

This is a return statement, so we evaluate the expression, which is simply `x`. Python looks up the value for `x` in the current frame and finds `6`, so that is the return value:



When the function returns, Python comes back to this expression: `f(x + 1) + f(x + 2)`. Python just finished executing `f(x + 2)`, which produced value `6`. Both function calls have been executed, so Python applies operator `+` to `4` and `6`, giving us `10`.

We have now evaluated the right-hand side of the assignment statement; Python completes it by making the variable on the left-hand side, `x`, refer to `10`:



Phew! That's a lot to keep track of. Python does all that bookkeeping for us, but if you don't understand each individual step, you just can't become a good programmer.

3.5 Designing New Functions: a Recipe

Writing a good essay requires planning: decide on a topic, learn the background material, write an outline, then start filling in the outline until you're done.

Writing a good function also requires planning. You have an idea of what you want the function to do, but you need to decide on the details: what do you name the function? What are the parameters? What does it return?

This section describes a step-by-step recipe for designing and writing a function. Part of the outcome will be a working function, but almost as important is *documentation* for the function. Python uses three double quotes to start and end this documentation; everything in between is meant for humans to read. This notation is called a *docstring*, which is short for *documentation string*.

Here is an example of a completed function. We'll show you how we came up with this using the Function Design Recipe, but it helps to see a completed example first:

```
>>> def days_difference(day1, day2):
...     """(int, int) -> int
...
...     Return the number of days between day1 and day2, which are both
...     in the range 1-365 (thus indicating the day of the year).
...
...     >>> days_difference(200, 224)
```

```
...     24
...     >>> days_difference(50, 50)
...     0
...     >>> days_difference(100, 99)
...     -1
...     """
...     return day2 - day1
...
```

Here are the parts of the function, including the docstring:

- The first line is the function header.
- The second line has three double-quotes to start the docstring. The (int, int) part describes the types of values expected to be passed to parameters day1 and day2, and the int after the -> is the type of value the function will return.
- After that is a description of what the function will do when it is called. It mentions both parameters and describes what the function returns.
- Next are some example calls and return values as we would expect to see in the Python shell. (We chose the first example because that made day1 smaller than day2, the second example because the two days are equal, and the third example because that made day1 bigger than day2.)
- The last line is the body of the function.

There are six steps to the Function Design Recipe. It may seem like a lot of work at first, but this recipe can save you hours of time when you're working on more complicated functions.

1. **Examples** The first step is to figure out what arguments you want to give to your function and what information it will return. Pick a name (often a verb or verb phrase): this name is often a short answer to "What does your function do"? Type a couple of example calls and return values.

   We start with the examples because they're the easiest: before we write *anything*, we need to decide what information we have (the argument values) and what information we want the function to produce (the return value). Here are the examples from days_difference:

   ```
   ...     >>> days_difference(200, 224)
   ...     24
   ...     >>> days_difference(50, 50)
   ...     0
   ...     >>> days_difference(100, 99)
   ...     -1
   ```

2. **Type Contract** The second step is to figure out the types of information your function will handle: are you giving it integers? Floating-point numbers? Maybe both? We'll see a lot of other types in the upcoming chapters, so practicing this step now when you only have a few choices will help you later. If the answer is "both integers and floating-point numbers", then use the word "number".

   What type of value is returned? An integer, a floating-point number, or possibly either one of them?

   This is called a contract because we are claiming that if you call this function with the right types of values, we'll give you back the right type of value. (We're not saying anything about what will happen if we get the *wrong* kind of values.) Here is the type contract from days_difference:

   ```
   ...     """(int, int) -> int
   ```

3. **Header** Write the function header. Pick meaningful parameter names to make it easy for other programmers to understand what information to give to your function. Here is the header from days_difference:

   ```
   >>> def days_difference(day1, day2):
   ```

4. **Description** Write a short paragraph describing your function: this is what other programmers will read in order to understand what your function does, so it's important to practice this! Mention every parameter in your description and describe the return value. Here is the description from days_difference:

   ```
   ...     Return the number of days between day1 and day2, which are both
   ...     in the range 1-365 (thus indicating the day of the year).
   ```

5. **Body** By now, you should have a good idea of what you need to do in order to get your function to behave properly. It's time to write some code! Here is the body from days_difference:

   ```
   ...     return day2 - day1
   ```

6. **Test** Run the examples to make sure your function body is correct. Feel free to add more example calls if you happen to think of them. For days_difference, we copy and paste our examples into the shell and compare the results to what we expected:

   ```
   >>> days_difference(200, 224)
   24
   >>> days_difference(50, 50)
   0
   ```

```
>>> days_difference(100, 99)
-1
```

Designing three birthday-related functions

We'll now apply our Function Design Recipe to solve this problem: which day of the week a birthday will fall upon, given what day of the week it is today and what day of the year the birthday is on? For example, if today is the 3rd day of the year, and it's a Thursday, and a birthday is the 116th day of the year, what day of the week will it be on that birthday?

We will design three functions that together will help us do this calculation. We'll write them in the same file; until we get to Chapter 6, *A Modular Approach to Program Organization*, on page 101, we'll need to put functions that we write in the same file if we want to be able to have them call each other.

We will represent the day of the week using 1 for Sunday, 2 for Monday, and so on:

Day of the Week	Number
Sunday	1
Monday	2
Tuesday	3
Wednesday	4
Thursday	5
Friday	6
Saturday	7

We are using these numbers simply because we don't yet have the tools to easily convert between days of the week and their corresponding numbers. We'll have to do that translation in our heads.

For the same reason, we will also ignore months, and use the numbers 1 through 365 to indicate the day of the year. For example, we'll represent 1 February as 32, since it's the 32nd day of the year.

How many days difference?

We'll start by seeing how we came up with the `days_difference` function. Here are the Function Design Recipe steps. (Try following along in the Python shell.)

1. **Examples** We want a short name for the difference in days; we'll use `days_difference`. In our examples, we want to call this function and state what it returns. If we want to know how many days there are between

the 200th day of the year and the 224th day of the year, we can hope that this will happen:

```
>>> days_difference(200, 224)
24
```

What if the two days are the same? How about if the second one is before the first?

```
>>> days_difference(50, 50)
0
>>> days_difference(100, 99)
-1
```

Now that we have a few examples, we can move on to the next step.

2. **Type Contract** The arguments in our function call examples are all integers, and the return values are integers, too, so here's our type contract:

```
(int, int) -> int
```

3. **Header** We have a couple of example calls, and we know what types the parameters are, so we can now write the header. Both arguments are day numbers, so we'll use `day1` and `day2`:

```
def days_difference(day1, day2):
```

4. **Description** We'll now describe what a call on the function will do. Because the documentation should completely describe the behaviour of the function, we need to make sure that it's clear what the parameters mean:

```
Return the number of days between day1 and day2, which are both
in the range 1-365 (thus indicating a day of the year).
```

5. **Body** We've laid everything out. Looking at the examples, we see that we can implement this using subtraction. Here is the whole function again, including the body:

```
>>> def days_difference(day1, day2):
...     """(int, int) -> int
...
...     Return the number of days between day1 and day2, which are both
...     in the range 1-365 (thus indicating the day of the year).
...
...     >>> days_difference(200, 224)
...     24
...     >>> days_difference(50, 50)
...     0
```

```
...     >>> days_difference(100, 99)
...     -1
...     """
...     return day2 - day1
...
```

6. **Test** To test it, we fire up the Python shell and copy and paste the calls into the shell, checking that we get back what we expect:

```
>>> days_difference(200, 224)
24
>>> days_difference(50, 50)
0
>>> days_difference(100, 99)
-1
```

Here's something really cool. Now that we have a function with a docstring, we can call `help` on that function:

```
>>> help(days_difference)
Help on function days_difference in module __main__:

days_difference(day1, day2)
    (int, int) -> int

    Return the number of days between day1 and day2, which are both
    in the range 1-365 (thus indicating the day of the year).

    >>> days_difference(200, 224)
    24
    >>> days_difference(50, 50)
    0
    >>> days_difference(100, 99)
    -1
```

What day will it be in the future?

It will help our birthday calculations if we write a function to calculate what day of the week it will be given the current weekday and how many days ahead we're interested in. Remember that we're using the numbers 1 through 7 to represent Sunday through Saturday.

Again, we'll follow the Function Design Recipe:

1. **Examples** We want a short name for what it means to calculate what weekday it will be in the future. We could choose something like `which_weekday` or `what_day`; we'll use `get_weekday`. There are lots of choices.

We'll start with an example that asks what day it will be if today is Tuesday (day 3 of the week) and we want to know what tomorrow will be (1 day ahead):

```
>>> get_weekday(3, 1)
4
```

Whenever we have a function that should return a value in a particular range, we should write example calls where we expect either end of that range as a result.

What if it's Friday (day 6)? If we ask what day it will be tomorrow, we expect to get Saturday (day 7):

```
>>> get_weekday(6, 1)
7
```

What if it's Saturday (day 7)? If we ask what day it will be tomorrow, we expect to get Sunday (day 1):

```
>>> get_weekday(7, 1)
1
```

We'll also try asking about 0 days in the future as well as a week ahead; both of these cases should give back the day we started with:

```
>>> get_weekday(1, 0)
1
>>> get_weekday(4, 7)
4
```

Let's also try 10 weeks and 2 days in the future so we have a case where there are several intervening weeks:

```
>>> get_weekday(7, 72)
2
```

2. **Type Contract** The arguments in our function call examples are all integers, and the return values are integers, too, so here's our type contract:

   ```
   (int, int) -> int
   ```

3. **Header** We have a couple of example calls, and we know what types the parameters are, so we can now write the header. The function name is clear so we'll stick with it.

   The first argument is the current day of the week, so we'll use `current_weekday`. The second argument is how many days from now to

calculate. We'll pick days_ahead, although days_from_now would also be fine:

```
def get_weekday(current_weekday, days_ahead):
```

4. **Description** We need a complete description of what this function will do. We'll start with a sentence describing what the function does, and then we'll describe what the parameters mean:

```
Return which day of the week it will be days_ahead days from
current_weekday.

current_weekday is the current day of the week, and is in the range 1-7,
indicating whether today is Sunday (1), Monday (2), ..., Saturday (7).

days_ahead is the number of days after today.
```

   Notice that our first sentence uses both parameters and also describes what the function will return.

5. **Body** Looking at the examples, we see that we can solve the first example with this: return current_weekday + days_ahead. That, however, won't work for all of the examples; we need to wrap around from day 7 (Saturday) back to day 1 (Sunday). When you have this kind of wraparound, usually the remainder operator % will help. Notice that evaluation of (7 + 1) % 7 produces 1, (7 + 2) % 7 produces 2, and so on.

   Let's try taking the remainder of the sum: return current_weekday + days_ahead % 7. Here is the whole function again, including the body:

```
>>> def get_weekday(current_weekday, days_ahead):
...     """(int, int) -> int
...
...     Return which day of the week it will be days_ahead days from
...     current_weekday.
...
...     current_weekday is the current day of the week, and is in the range 1-7,
...     indicating whether today is Sunday (1), Monday (2), ..., Saturday (7).
...
...     days_ahead is the number of days after today.
...
...     >>> get_weekday(3, 1)
...     4
...     >>> get_weekday(6, 1)
...     7
...     >>> get_weekday(7, 1)
...     1
...     >>> get_weekday(1, 0)
...     1
```

```
...     >>> get_weekday(4, 7)
...     4
...     >>> get_weekday(7, 72)
...     2
...     """
...     return current_weekday + days_ahead % 7
...
```

6. **Test** To test it, we fire up the Python shell and copy and paste the calls into the shell, checking that we get back what we expect:

```
>>> get_weekday(3, 1)
4
>>> get_weekday(6, 1)
7
>>> get_weekday(7, 1)
8
```

Wait, that's not right—we expected a 1 on that third example, not an 8, because 8 isn't a valid number for a day of the week. We should have wrapped around to 1.

Taking another look at our function body, we see that because % has higher precedence than +, we need parentheses:

```
>>> def get_weekday(current_weekday, days_ahead):
...     """(int, int) -> int
...
...     Return which day of the week it will be days_ahead days from
...     current_weekday.
...
...     current_weekday is the current day of the week, and is in the range 1-7,
...     indicating whether today is Sunday (1), Monday (2), ..., Saturday (7).
...
...     days_ahead is the number of days after today.
...
...     >>> get_weekday(3, 1)
...     4
...     >>> get_weekday(6, 1)
...     7
...     >>> get_weekday(7, 1)
...     1
...     >>> get_weekday(1, 0)
...     1
...     >>> get_weekday(4, 7)
...     4
...     >>> get_weekday(7, 72)
...     2
...     """
...     return (current_weekday + days_ahead) % 7
...
```

Testing again, we see that we've fixed that *bug* in our code, but now we're getting the wrong answer for the second test!

```
>>> get_weekday(7, 1)
1
>>> get_weekday(3, 1)
4
>>> get_weekday(6, 1)
0
```

The problem here is that when `current_weekday + days_ahead` evaluates to a multiple of 7, then `(current_weekday + days_ahead) % 7` will evaluate to 0, not 7. All the other results work well, it's just that pesky 7.

Because we want a number in the range 1 through 7, but we're getting an answer in the range 0 through 6, and all the answers are correct except that we're seeing a 0 instead of a 7, we can use this trick:

- Subtract 1 from the expression: `current_weekday + days_ahead - 1`.
- Take the remainder.
- Add 1 to the entire result: `(current_weekday + days_ahead - 1) % 7 + 1`.

Let's test it again:

```
>>> get_weekday(3, 1)
4
>>> get_weekday(6, 1)
7
>>> get_weekday(7, 1)
1
>>> get_weekday(1, 0)
1
>>> get_weekday(4, 7)
4
>>> get_weekday(7, 72)
2
```

We've passed all the tests, so we can now move on.

What day is my birthday on?

We now have two functions related to day-of-year calculations. One of them calculates the difference between two days of the year. The other calculates the weekday for a day in the future, given the weekday today. We can use these two functions to help figure out what day of the week a birthday falls on, given what day of the week it is today, what the current day of the year is, and what day of the year the birthday falls on.

Again, we'll follow the Function Design Recipe:

1. **Examples** We want a name for what it means to calculate what weekday a birthday will fall on. Again, there are lots of choices; we'll use get_birthday_weekday.

   If today is a Thursday (day 5 of the week), and today is the 3rd day of the year, what day will it be on the 4th day of the year? Hopefully Friday:

   ```
   >>> get_birthday_weekday(5, 3, 4)
   6
   ```

   What if it's the same day (Thursday, the 3rd day of the year), but the birthday is the 116th day of the year? For now, we can verify externally (looking at a calendar) that it turns out to be a Friday.

   ```
   >>> get_birthday_weekday(5, 3, 116)
   6
   ```

   What if today is Friday 26 April, the 116th day of the year, but the birthday we want is the third day of the year? This is interesting because the birthday is a couple months before the current day:

   ```
   >>> get_birthday_weekday(6, 116, 3)
   5
   ```

2. **Type Contract** The arguments in our function call examples are all integers, and the return values are integers, too, so here's our type contract:

   ```
   (int, int, int) -> int
   ```

3. **Header** We have a couple of example calls, and we know what types the parameters are, so we can now write the header. We're happy enough with the function name so again we'll stick with it.

   The first argument is the current day of the week, so we'll use current_weekday, as we did for the previous function. (It's a good idea to be consistent with naming when possible.) The second argument is what day of the year it is today, and we'll choose current_day. The third argument is the day of the year the birthday is, and we'll choose birthday_day:

   ```
   def get_birthday_weekday(current_weekday, current_day, birthday_day):
   ```

4. **Description** We need a complete description of what this function will do. We'll start with a sentence describing what the function does, and then we'll describe what the parameters mean:

```
Return the day of the week it will be on birthday_day, given that
the day of the week is current_weekday and the day of the year is
current_day.

current_weekday is the current day of the week, and is in the range 1-7,
indicating whether today is Sunday (1), Monday (2), ..., Saturday (7).

current_day and birthday_day are both in the range 1-365.
```

   Again, notice that our first sentence uses all parameters and also describes what the function will return. If it gets more complicated we'll start to write multiple sentences to describe what the function does, but we managed to squeeze it in here.

5. **Body** Time to write the body of the function. We have a puzzle:

   a. Using days_difference, we can figure out how many days there are between 2 days.

      Using get_weekday, we can figure out what day of the week it will be given the current day of the week and a number of days away.

   We'll start by figuring out how many days from now the birthday falls:

```
days_diff = days_difference(current_day, birthday_day)
```

   Now that we know that, we can use it to solve our problem: given the current weekday, and that number of days ahead, we can call function get_weekday to get our answer:

```
return get_weekday(current_weekday, days_diff)
```

   Let's put it all together:

```
>>> def get_birthday_weekday(current_weekday, current_day, birthday_day):
...     """(int, int, int) -> int
...
...     Return the day of the week it will be on birthday_day, given that
...     the day of the week is current_weekday and the day of the year is
...     current_day.
...
...     current_weekday is the current day of the week, and is in the range 1-7,
...     indicating whether today is Sunday (1), Monday (2), ..., Saturday (7).
...
...     current_day and birthday_day are both in the range 1-365.
...
...     >>> get_birthday_weekday(5, 3, 4)
```

```
...     6
...     >>> get_birthday_weekday(5, 3, 116)
...     6
...     >>> get_birthday_weekday(6, 116, 3)
...     5
...     """
...     days_diff = days_difference(current_day, birthday_day)
...     return get_weekday(current_weekday, days_diff)
...
```

6. **Test** To test it, we fire up the Python shell and copy and paste the calls into the shell, checking that we get back what we expect:

```
>>> get_birthday_weekday(5, 3, 4)
6
>>> get_birthday_weekday(5, 3, 116)
6
>>> get_birthday_weekday(6, 116, 3)
5
```

And we're done!

3.6 Writing and Running a Program

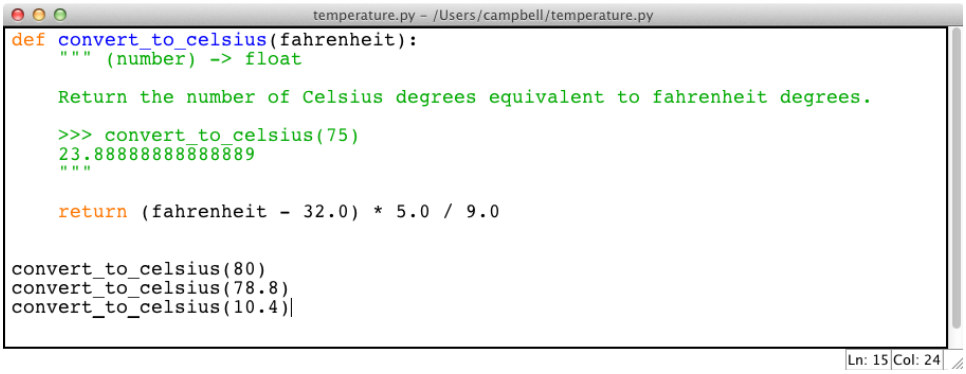
So far, we have used the shell to investigate Python. As you have seen, the shell will show you the result of evaluating an expression:

```
>>> 3 + 5 / abs(-2)
5.5
```

In a program that is supposed to interact with a human, showing the result of every expression is probably not desirable behavior. (Imagine if your web browser showed you the result of every calculation it performed.)

Section 2.1, *How Does a Computer Run a Python Program?*, on page 9 explained that in order to save code for later use, you can put it in a file with a .py extension. You can then tell Python to run the code in that file, rather than typing commands in at the interactive prompt.

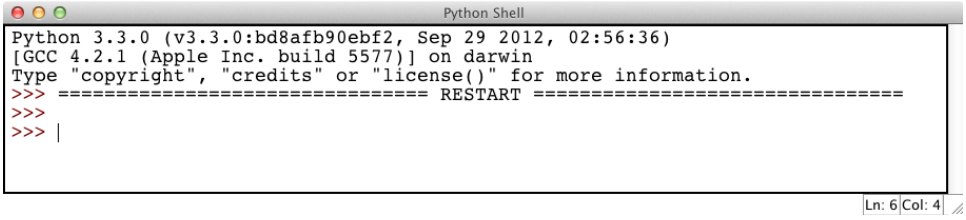
Here is a program that we wrote using IDLE and saved in a file called temperature.py. This program consists of a function definition for convert_to_celsius (from earlier in the chapter) and three calls on that function that convert three different Fahrenheit temperatures to their Celsius equivalents.



Notice that there is no >>> prompt. This never appears in a Python program; it is used exclusively in the shell.

Open IDLE, select File->New Window, and type this program in. (Either that, or download the code from the book website and open the file.)

To run the program in IDLE, select menu Run->Run Module. This will open the Python shell and show the results of running the program. Here is our result. (The line containing RESTART is letting us know that the shell has restarted, wiping out any previous work done in the shell.)



Notice that no values are shown, unlike in Section 3.3, *Defining our Own Functions*, on page 38 when we typed the equivalent code into the shell. In order to have a program print the value of an expression, we use built-in function print. Here is the same program, but with calls on function print.

temperature.py – /Users/campbell/temperature.py

```
def convert_to_celsius(fahrenheit):
    """ (number) -> float

    Return the number of Celsius degrees equivalent to fahrenheit degrees.

    >>> convert_to_celsius(75)
    23.88888888888889
    """

    return (fahrenheit - 32.0) * 5.0 / 9.0


print(convert_to_celsius(80))
print(convert_to_celsius(78.8))
print(convert_to_celsius(10.4))
```

Ln: 15 Col: 31

And here is what happens when we run this program:

Python Shell

```
Python 3.3.0 (v3.3.0:bd8afb90ebf2, Sep 29 2012, 02:56:36)
[GCC 4.2.1 (Apple Inc. build 5577)] on darwin
Type "copyright", "credits" or "license()" for more information.
>>> ================================ RESTART ================================
>>>
26.666666666666668
26.0
-12.0
>>>
```

Ln: 9 Col: 4

3.7 Omitting a return Statement: None

If you don't have a return statement in a function, nothing is produced:

```
>>> def f(x):
...     x = 2 * x
...
>>> res = f(3)
>>> res
>>>
```

Wait, that can't be right—if res doesn't have a value, shouldn't we get a NameError? Let's poke a little more:

```
>>> print(res)
None
>>> id(res)
1756120
```

Variable res has a value: it's None! And None has a memory address. If you don't have a return statement in your function, your function will return None. You can return None yourself if you like:

```
>>> def f(x):
...     x = 2 * x
...     return None
...
```

```
>>> print(f(3))
None
```

Value `None` is used to signal the absence of a value. We'll see some uses for it later in the book.

3.8 What Did You Call That?

- A *function definition* introduces a new variable that refers to a function object. The `return` statement describes the value that will be produced as a result of the function when this function is done being executed.
- A *parameter* is a variable that appears between the parentheses of a function header.
- A *function call* tells Python to execute a function.
- An *argument* is an expression that appears between the parentheses of a function call. The value that is produced when Python evaluates the expression is assigned to the corresponding parameter.

3.9 Exercises

1. Two of Python's built-in functions are `min` and `max`. In the Python shell, execute the following function calls:

   a. `min(2, 3, 4)`

   b. `max(2, -3, 4, 7, -5)`

   c. `max(2, -3, min(4, 7), -5)`

2. For the following function calls, in what order are the subexpressions evaluated?

   a. `min(max(3, 4), abs(-5))`

   b. `abs(min(4, 6, max(2, 8)))`

   c. `round(max(5.572, 3.258), abs(-2))`

3. Following the Function Design Recipe, define a function that has one parameter, a number, and returns that number tripled.

4. Following the Function Design Recipe, define a function that has two parameters, both numbers, and returns the absolute value of the difference of the two. Hint: call the built-in function `abs`.

5. Following the Function Design Recipe, define a function that has one parameter, a distance in kilometers, and returns the distance in miles. There are 1.6 kilometers per mile.

6. Following the Function Design Recipe, define a function that has three parameters, grades between 0 and 100 inclusive, and returns the average of those grades.

7. Following the Function Design Recipe, define a function that has four parameters, grades between 0 and 100 inclusive, and returns the average of the *best 3* of those grades. Hint: call the function that you defined in the previous questoin

8. Consider this code:

```
def square(num):
    """ (number) -> number

    Return the square of num.

    >>> square(3)
    9
    """
```

In the table below, fill in the Example column by writing square, num, square(3) and 3 next to the appropriate description.

Description	Example
parameter	
argument	
function name	
function call	

9. Write the body of the function square from the previous question.

10. Complete the examples in the docstring and then write the body of the following function:

```
def weeks_elapsed(day1, day2):
    """ (int, int) -> int

    day1 and day2 are days of the year. Return the number of full weeks
    that have elapsed between day1 and day2.

    >>> weeks_elapsed(3, 20)
    2
    >>> weeks_elapsed(20, 3)
    2
```

```
>>> weeks_elapsed(8, 5)

>>> weeks_elapsed(40, 61)

"""
```

CHAPTER 4

Working with Text

From email readers and web browsers to calendars and games, text plays a central role in computer programs. This chapter introduces a non-numeric data type that represents text, such as the words in this sentence or the sequence of bases in a strand of DNA. Along the way, we will see how to make programs a little more interactive by printing messages to our programs' users and getting input from them.

4.1 Creating Strings of Characters

Computers may have been invented to do arithmetic, but these days, most of them spend a lot of their time processing text. Many programs create text, store it, search it, and move it from one place to another.

In Python, text is represented as a *string*, which is a sequence of *characters* (letters, digits, and symbols). The type whose values are sequences of characters is `str`. The characters consist of those from the Latin alphabet found on most North American keyboards, as well as Chinese morphograms, chemical symbols, musical symbols, and much more.

In Python, we indicate that a value is a string by putting either single or double quotes around it. As we will see in Section 4.2, *Using Special Characters in Strings*, on page 70, single and double quotes are equivalent except for strings that contain quotes. You can use whichever you prefer. (For docstrings, the Python style guidelines say that double quotes for these are preferred.)

Here are two examples:

```
>>> 'Aristotle'
'Aristotle'
>>> "Isaac Newton"
'Isaac Newton'
```

The opening and closing quotes must match:

```
>>> 'Charles Darwin"
  File "<stdin>", line 1
    'Charles Darwin"
                    ^
SyntaxError: EOL while scanning string literal
```

EOL stands for “end of line”. The error above indicates that the end of line was reached before the end of the string (which should be marked with a closing single quote) was found.

Strings can contain any number of characters, limited only by computer memory. The shortest string is the *empty string*, containing no characters at all:

```
>>> ''
''
>>> ""
''
```

Operations on strings

Python has a built-in function, `len`, that returns the number of characters between the opening and closing quotes:

```
>>> len('Albert Einstein')
15
>>> len('123!')
4
>>> len(' ')
1
>>> len('')
0
```

We can add two strings using operator +, which produces a new string containing the same characters as in the two operands:

```
>>> 'Albert' + ' Einstein'
'Albert Einstein'
```

When + has two string operands, it is referred to as the *concatenation operator*. Operator + is probably the most overloaded operator in Python. So far, we’ve applied it to integers, floating-point numbers, and strings, and we’ll apply it to several more types in later chapters.

As the following example shows, adding an empty string to another string produces a new string that is just like the non-empty operand:

```
>>> "Alan Turing" + ''
'Alan Turing'
>>> "" + 'Grace Hopper'
```

```
'Grace Hopper'
```

Here is an interesting question: can operator + be applied to a string and a numeric value? If so, would addition or concatenation occur? We'll give it a try:

```
>>> 'NH' + 3
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: Can't convert 'int' object to str implicitly
```

This is the second time that we have encountered a type error. The first time, in *Use Local Variables for Temporary Storage*, on page 41, the problem was that we didn't pass the right number of parameters to a function. Here, Python took exception to our attempts to combine values of different data types, because it doesn't know which version of + we want: the one that adds numbers or the one that concatenates strings. Because the first operand was a string, Python expected the second operand to also be a string but instead it was an integer.

Now consider this example:

```
>>> 9 + ' planets'
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: unsupported operand type(s) for +: 'int' and 'str'
```

Here, because Python saw a 9 first, it expected the second operand to also be numeric. The order of the operands affects the error message.

The concatenation operator must be applied to two strings. If you want to join a string with a number, function str can be applied to the number to get a string representation of it, and then the concatenation can be done:

```
>>> 'Four score and ' + str(7) + ' years ago'
'Four score and 7 years ago'
```

Function int can be applied to a string whose contents look like an integer and float can be applied a string whose contents are numeric:

```
>>> int('0')
0
>>> int("11")
11
>>> int('-324')
-324
>>> float('-324')
-324.0
>>> float("56.34")
```

```
56.34
```

It isn't always possible to get an integer or floating-point representation of a string and when an attempt to do so fails, an error occurs:

```
>>> int('a')
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
ValueError: invalid literal for int() with base 10: 'a'
>>> float('b')
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
ValueError: could not convert string to float: 'b'
```

In addition to +, len, int, and float, operator * can be applied to strings. A string can be repeated using operator * and an integer, like this:

```
>>> 'AT' * 5
'ATATATATAT'
>>> 4 * '-'
'----'
```

If the integer is less than or equal to zero, the operator yields the empty string (a string containing no characters):

```
>>> 'GC' * 0
''
>>> 'TATATATA' * -3
''
```

Strings are values and so you can assign a string to a variable. Also, operations on strings can be applied to those variables:

```
>>> sequence = 'ATTGTCCCCC'
>>> len(sequence)
10
>>> new_sequence = sequence + 'GGCCTCCTGC'
>>> new_sequence
'ATTGTCCCCCGGCCTCCTGC'
>>> new_sequence * 2
'ATTGTCCCCCGGCCTCCTGCATTGTCCCCCGGCCTCCTGC'
```

4.2 Using Special Characters in Strings

Suppose you want to put a single quote inside a string. If you write it directly, an error occurs:

```
>>> 'that's not going to work'
  File "<stdin>", line 1
    'that's not going to work'
          ^
SyntaxError: invalid syntax
```

When Python encounters the second quote—the one that is intended to be part of the string—it thinks the string is ended. It then doesn't know what to do with the text that comes after the second quote.

One simple way to fix this is to use double quotes around the string; we can also put single quotes around a string containing a double quote:

```
>>> "that's better"
"that's better"
>>> 'She said, "That is better."'
'She said, "That is better."'
```

If you need to put a double quote in a string, you can use single quotes around the string. But what if you want to put both kinds of quote in one string? You could do this:

```
>>> 'She said, "That' + "'" + 's hard to read."'
'She said, "That\'s hard to read."'
```

The result is a valid Python string. The backslash is called an *escape character*, and the combination of the backslash and the single quote is called an *escape sequence*. The name comes from the fact that we're "escaping" from Python's usual syntax rules for a moment. When Python sees a backslash inside a string, it means that the next character represents something that Python uses for other purposes, such as marking the end of a string.

The escape sequence \' is indicated using two symbols, but those two symbols represent a single character. The length of an escape sequence is one:

```
>>> len('\'')
1
>>> len('it\'s')
4
```

Python recognizes several escape sequences. In order to see how they are used, we will introduce multiline strings, and also revisit built-in function print. Here are some common escape sequences:

Escape Sequence	Description
\'	Single quote
\"	Double quote
\\	Backslash
\t	Tab
\n	Newline

Escape Sequence	Description
\r	Carriage return

Table 4—Escape sequences

4.3 Creating a Multiline String

If you create a string using single or double quotes, the whole string must fit onto a single line.

Here's what happens when you try to stretch a string across multiple lines:

```
>>> 'one
  File "<stdin>", line 1
    'one
       ^
SyntaxError: EOL while scanning string literal
```

As we saw in Section 4.1, *Creating Strings of Characters*, on page 67, EOL stands for "end of line," so in this error report, Python is saying that it reached the end of the line before it found the end of the string.

To span multiple lines, put three single quotes or three double quotes around the string instead of one of each. The string can then span as many lines as you want:

```
>>> '''one
... two
... three'''
'one\ntwo\nthree'
```

Notice that the string Python creates contains a \n sequence everywhere our input started a new line. As programmers, we see the escape sequences in strings. In the next section Section 4.4, *Printing Information*, on page 73, we show that when strings are printed, users see the properly rendered strings rather than the escape sequences. That is, they see a tab or a quote, rather than \t or \'.

Normalizing Line Endings

In reality, each of the three major operating systems uses a different set of characters to indicate the end of a line. This set of characters is called a *newline*. On Linux and Mac OS X, a newline is one '\n' character; on version 9 and earlier of Mac OS, it is one '\r'; and on Windows, the ends of lines are marked with both characters as '\r\n'.

Python always uses a single \n to indicate a newline, even on operating systems like Windows that do things other ways. This is called *normalizing* the string; Python does

this so that you can write exactly the same program no matter what kind of machine you're running on.

4.4 Printing Information

In Section 3.6, *Writing and Running a Program*, on page 61, the built-in function print was used to print values to the screen. We will use print to print messages to the users of our program. Those messages may include the values that expressions produce and the values that variables refer to. Here are two examples of printing:

```
>>> print(1 + 1)
2
>>> print("The Latin 'Oryctolagus cuniculus' means 'domestic rabbit'.")
The Latin 'Oryctolagus cuniculus' means 'domestic rabbit'.
```

Function print doesn't allow any styling of the output: no colors, no italics, no boldface. All output is plain text.

The first function call does what you would expect from the numeric examples we have seen previously, but the second does something slightly different from previous string examples: it strips off the quotes around the string and shows us the string's contents, rather than its representation. This example makes the difference between the two even clearer:

```
>>> print('In 1859, Charles Darwin revolutionized biology')
In 1859, Charles Darwin revolutionized biology
>>> print('and our understanding of ourselves')
and our understanding of ourselves
>>> print('by publishing "On the Origin of Species".')
by publishing "On the Origin of Species".
```

And the following example shows that when Python prints a string, it prints the values of any escape sequences in the string, rather than their backslashed representations:

```
>>> print('one\ttwo\nthree\tfour')
one     two
three   four
```

The example above shows how the tab character \t can be used to lay values out in columns.

In Section 4.3, *Creating a Multiline String*, on page 72, we saw that \n indicates a new line in multiline strings. When a multiline string is printed, those \n sequences are displayed as new lines:

```
>>> numbers = '''one
... two
... three'''
>>> numbers
'one\ntwo\nthree'
>>> print(numbers)
one
two
three
```

Function print takes a comma-separated list of values to print and prints the values with a single space between them and a newline after the last value:

```
>>> print(1, 2, 3)
1 2 3
>>>
```

If no values are given, print simply displays a blank line:

```
>>> print()

>>>
```

Function print can print values of any type and it can even print values of different types in the same function call:

```
>>> print(1, 'two', 'three', 4.0)
1 two three 4.0
```

It is also possible to call print with an expression as an argument. It will print the value of that expression:

```
>>> radius = 5
>>> print("The diameter of the circle is", radius * 2, "cm.")
The diameter of the circle is 10 cm.
```

Function print has a few extra helpful features; here is the help documentation for it:

```
>>> help(print)
Help on built-in function print in module builtins:

print(...)
    print(value, ..., sep=' ', end='\n', file=sys.stdout, flush=False)

    Prints the values to a stream, or to sys.stdout by default.
    Optional keyword arguments:
    file:  a file-like object (stream); defaults to the current sys.stdout.
    sep:   string inserted between values, default a space.
    end:   string appended after the last value, default a newline.
    flush: whether to forcibly flush the stream.
```

The parameters sep, end, file, and flush have assignment statements in the function header! These are called *default parameter values*: by default, if we call function print with a comma-separated list of values, the separator is a space; similarly, a newline character appears at the end of every printed string. (We won't discuss file and flush; they are beyond the scope of this text.)

We can supply different values by using *keyword arguments*. That's a fancy term for assigning to a parameter name in the function call. Here, we separate each value with a comma and a space instead of just a space by including sep=', ' as an argument:

```
>>> print('a', 'b', 'c')  # The separator is a space by default
a b c
>>> print('a', 'b', 'c', sep=', ')
a, b, c
```

Often, you'll want to print information but not start a new line. To do this, use keyword argument end='' to tell Python to end with an empty string instead of a newline:

```
>>> print('a', 'b', 'c', sep=', ', end='')
a, b, c>>>
```

Notice how the last prompt appeared right after the 'c'. Typically, end='' is used only in programs, not in the shell; here is a program that converts three temperatures from Fahrenheit to Celsius and prints using keyword arguments:

```
def convert_to_celsius(fahrenheit):
    """ (number) -> float

    Return the number of Celsius degrees equivalent to fahrenheit degrees.

    >>> convert_to_celsius(75)
    23.88888888888889
    """

    return (fahrenheit - 32.0) * 5.0 / 9.0

print('80, 78.8, and 10.4 degrees Fahrenheit are equal to ', end='')
print(convert_to_celsius(80), end=', \n')
print(convert_to_celsius(78.8), end=', and ')
print(convert_to_celsius(10.4), end=' Celsius.\n')
```

Here's the output of running this program:

```
80, 78.8, and 10.4 degrees Fahrenheit are equal to 26.666666666666668,
26.0, and -12.0 Celsius.
```

4.5 Getting Information from the Keyboard

In an earlier chapter, we explored some built-in functions. Another built-in function that you will find useful is input, which reads a single line of text from the keyboard. It returns whatever the user enters as a string, even if it looks like a number:

```
>>> species = input()
Homo sapiens
>>> species
'Homo sapiens'
>>> population = input()
6973738433
>>> population
'6973738433'
>>> type(population)
<class 'str'>
```

The second and sixth lines of that example, Homo sapiens and 6973738433, were typed by us in response to the calls on function input.

If you are expecting the user to enter a number, you must use function int or float to convert get an integer or floating-point representation of the string:

```
>>> population = input()
6973738433
>>> population
'6973738433'
>>> population = int(population)
>>> population
6973738433
>>> population = population + 1
>>> population
6973738434
```

We don't actually need to stash the value that the call to input produces before converting it. This time the int function is called on the call to input and is equivalent to the code above:

```
>>> population = int(input())
6973738433
>>> population = population + 1
6973738434
```

Finally, input can be given a string argument, which is used to prompt the user for input (notice the space at the end of our prompt):

```
>>> species = input("Please enter a species: ")
Please enter a species: Python curtus
>>> print(species)
```

```
Python curtus
```

4.6 Quotes from the String Chapter

In this chapter, you learned the following:

- Python uses type str to represent text as sequences of characters.
- Strings are created by placing pairs of single or double quotes around the text. Multiline strings can be created using matching pairs of triple quotes.
- Special characters like newline and tab are represented using escape sequences that begin with a backslash.
- Values can be printed using the built-in function print and input can be provided by the user using the built-in function input.

4.7 Exercises

Here are some exercises for you to try on your own:

1. For each of the following expressions, what value does the expression evaluate to? Verify your answers by typing the expressions into the Python shell.

   a. `'Computer' + ' Science'`

   b. `'Darwin\'s'`

   c. `'H2O' * 3`

   d. `'CO2' * 0`

2. For each of the following phrases, express them as Python strings using the appropriate type of quotation marks (single, double or triple) and, if necessary, escape sequences. There is more than one correct answer for each of these phrases.

   a. They'll hibernate during the winter.

   b. "Absolutely not," he said.

   c. "He said, 'Absolutely not,'" recalled Mel.

   d. hydrogen sulfide

   e. left\right

3. Rewrite the following string using single or double quotes instead of triple quotes:

   ```
   '''A
   ```

```
B
C'''
```

4. Use the built-in function len to find the length of the empty string.

5. Given variables x and y, which refer to values 3 and 12.5 respectively, use function print to print the following messages. When numbers appear in the messages, variables x and y should be used.

   a. The rabbit is 3.

   b. The rabbit is 3 years old.

   c. 12.5 is average.

   d. 12.5 * 3

   e. 12.5 * 3 is 37.5.

6. Consider this code:

```
>>> first = 'John'
>>> last = 'Doe'
>>> print(last + ', ' + first)
```

   What is printed by the code above?

7. Use input to prompt the user for a number, store the number entered as a float in a variable named num, and then print the contents of num.

8. Complete the examples in the docstring and then write the body of the following function:

```
def repeat(s, n):
    """ (str, s) -> s

    Precondition: n >= 0

    Return s repeated n times.

    >>> repeat('yes', 4)
    'yesyesyesyes'
    >>> repeat('no', 0)

    >>> repeat('yesnomaybe', 3)

    """
```

9. Complete the examples in the docstring and then write the body of the following function:

```
def total_length(s1, s2):
```

```
""" (str, str) -> int

Return the sum of the lengths of s1 and s2.

>>> total_length('yes', 'no')
5
>>> total_length('yes', '')

>>> total_length('YES!!!!', 'Noooooo')

"""
```

CHAPTER 5

Making Choices

This chapter introduces another fundamental concept of programming: making choices. We do this whenever we want our program to behave differently depending on the data it's working with. For example, we might want to do different things depending on whether a solution is acidic or basic, or depending on whether a user types "yes" or "no" in response to a call on built-in function `input`.

The statements we'll introduce in this chapter for making choices are called *control flow* statements, because they control the way the computer executes programs. These statements involve a Python type that is used to represent truth and falsehood. Unlike the integers, floating-point numbers, and strings we have already seen, this type has only two values and three operators.

5.1 A Boolean Type

In Python, there is a type called `bool` (without an "e"). Unlike `int` and `float`, which have billions of possible values, `bool` has only two: `True` and `False`. `True` and `False` are values, just as much as the numbers `0` and `-43.7`.

George Boole

In the 1840s, the mathematician George Boole showed that the classical rules of logic could be expressed in purely mathematical form using only the two values "true" and "false". A century later, Claude Shannon (the inventor of information theory) realized that Boole's work could be used to optimize the design of electromechanical telephone switches. His work led directly to the use of *Boolean logic* to design computer circuits.

In honor of Boole's work, most modern programming languages use a type named after him to keep track of what's true and what isn't.

Boolean Operators

There are only three basic Boolean operators: `and`, `or`, and `not`. `not` has the highest precedence, followed by `and`, followed by `or`.

`not` is a unary operator: it is applied to just one value, like the negation in the expression `-(3 + 2)`. An expression involving `not` produces `True` if the original value is `False`, and it produces `False` if the original value is `True`:

```
>>> not True
False
>>> not False
True
```

In the previous example, instead of `not True`, we could simply use `False`; and instead of `not False`, we could use `True`. Rather than apply `not` directly to a Boolean value, we would typically apply `not` to a Boolean variable or a more complex Boolean expression. The same goes for the following examples of Boolean operators `and` and `or`, so although we apply them to Boolean constants in the following examples, we'll give an example of how they are typically used at the end of this section.

`and` is a binary operator; the expression `left and right` produces `True` if both `left` and `right` are `True`, and it produces `False` otherwise:

```
>>> True and True
True
>>> False and False
False
>>> True and False
False
>>> False and True
False
```

`or` is also a binary operator. It produces `True` if *either* operand is `True`, and it produces `False` only if both are `False`:

```
>>> True or True
True
>>> False or False
False
>>> True or False
True
>>> False or True
True
```

This definition is called *inclusive or*, since it allows both possibilities as well as either. In English, the word *or* is also sometimes an *exclusive or*. For example, if someone says, "You can have pizza or tandoori chicken," they

probably don't mean that you can have both. Unlike English, but like most programming languages, Python always interprets or as inclusive.

Building an exclusive or expression

If you want an exclusive or, you need to build a Boolean expression for it. We'll walk through the development of this expression.

Let's say you have two Boolean variables, b1 and b2, and you want an expression that evaluates to True if and only if exactly one of them is True. Evaluation of b1 and not b2 will produce True if b1 is True and b2is False. Similarly, evaluation of b2 and not b1 will produce True if b2 is True and b1is False.

It isn't possible for both of these expressions to produce True. Also, if b1 and b2 are both True or both False, both expressions will evaluate to False. We can, therefore, combine the two expressions with an or:

```
>>> b1 = False
>>> b2 = False
>>> (b1 and not b2) or (b2 and not b1)
False
>>> b1 = False
>>> b2 = True
>>> (b1 and not b2) or (b2 and not b1)
True
>>> b1 = True
>>> b2 = False
>>> (b1 and not b2) or (b2 and not b1)
True
>>> b1 = True
>>> b2 = True
>>> (b1 and not b2) or (b2 and not b1)
False
```

In a few pages, we'll see a much simpler version.

We mentioned earlier that Boolean operators are usually applied to Boolean expressions, rather than Boolean constants. If we want to express "It is not cold and windy" using two variables cold and windy that refer to Boolean values, we first have to decide what the ambiguous English expression means: is it not cold but at the same time windy, or is it not both cold and windy? A *truth table* for each alternative is shown in Table 5, *Relational and equality operators*, on page 84, and the following code snippet shows what they look like translated into Python:

```
>>> cold = True
>>> windy = False
>>> (not cold) and windy
False
>>> not (cold and windy)
True
```

cold	windy	cold and windy	cold or windy	(not cold) and windy	not (cold and windy)
True	**True**	True	True	False	False
True	**False**	False	True	False	True
False	**True**	False	True	True	True
False	**False**	False	False	False	True

Table 5—Relational and equality operators

Boolean operators in other languages

If you already know another language such as C or Java, you might be used to && for and, || for or, and ! for not. These won't work in Python, but the idea is the same.

Relational Operators

We said earlier that True and False are values. Typically, those values are not written down directly in expressions but rather created in expressions. The most common way to do that is by doing a comparison using a *relational operator*. For example, 3 < 5 is a comparison using the relational operator < that produces the value True, while 13 ≥ 77 uses ≥ and produces the value False.

As shown in Table 6, *Relational and Equality Operators*, on page 85, Python has all the operators you're used to using. Some of them are represented using two characters instead of one, like <= instead of ≤.

The most important representation rule is that Python uses == for equality instead of just =, because = is used for assignment. Avoid typing x = 3 when you mean to check whether variable x is equal to three.

All relational operators are binary operators: they compare two values and produce True or False, as appropriate. The "greater than" > and "less than" < operators work as follows:

```
>>> 45 > 34
True
>>> 45 > 79
False
>>> 45 < 79
True
>>> 45 < 34
False
```

Symbol	Operation
>	Greater than
<	Less than
>=	Greater than or equal to
<=	Less than or equal to
==	Equal to
!=	Not equal to

Table 6—Relational and Equality Operators

We can compare integers to floating-point numbers with any of the relational operators. Integers are automatically converted to floating point when we do this, just as they are when we add 14 to 23.3:

```
>>> 23.1 >= 23
True
>>> 23.1 >= 23.1
True
>>> 23.1 <= 23.1
True
>>> 23.1 <= 23
False
```

The same holds for “equal to” and “not equal to”:

```
>>> 67.3 == 87
False
>>> 67.3 == 67
False
>>> 67.0 == 67
True
>>> 67.0 != 67
False
>>> 67.0 != 23
True
```

Of course, it doesn’t make much sense to compare two numbers that you know in advance, since you would also know the result of the comparison. Relational operators therefore almost always involve variables, like this:

```
>>> def is_positive(x):
...     """ (number) -> bool
...
...     Return True iff x is positive.
...
...     >>> is_positive(3)
...     True
...     >>> is_positive(-4.6)
```

```
...     False
...     """
...     return x > 0
...
>>> is_positive(3)
True
>>> is_positive(-4.6)
False
>>> is_positive(0)
False
```

In the docstring above, we use the acronym "iff", which stands for "if and only if". An equivalent phrase is "exactly when". The type contract states that the function will return a `bool`. The docstring describes the conditions under which `True` will be returned. It is implied that when those conditions aren't met the function will return `False`.

We can now write our "exclusive or" expression much more simply:

```
b1 != b2
```

"Exclusive or" means that exactly one of `b1` and `b2` has to be `True`. If `b1` is `True`, `b2` can't be, and vice versa.

Combining Comparisons

We have now seen three types of operators: arithmetic (`+`, `-`, and so on), Boolean (`and`, `or`, and `not`), and relational (`<`, `==`, and so on).

Here are the rules for combining them:

- Arithmetic operators have higher precedence than relational operators. For example, `+` and `/` are evaluated before `<` or `>`.
- Relational operators have higher precedence than Boolean operators. For example, comparisons are evaluated before `and`, `or`, and `not`.
- All relational operators have the same precedence.

These rules mean that the expression `1 + 3 > 7` is evaluated as `(1 + 3) > 7`, not as `1 + (3 > 7)`. These rules also mean that you can often skip the parentheses in complicated expressions:

```
>>> x = 2
>>> y = 5
>>> z = 7
>>> x < y and y < z
True
```

It's usually a good idea to put the parentheses in, though, since it helps the eye find the subexpressions and clearly communicates the order to anyone reading your code:

```
>>> x = 5
>>> y = 10
>>> z = 20
>>> (x < y) and (y < z)
True
```

It's very common in mathematics to check whether a value lies in a certain range, in other words, that it is between two other values. You can do this in Python by combining the comparisons with `and`:

```
>>> x = 3
>>> (1 < x) and (x <= 5)
True
>>> x = 7
>>> (1 < x) and (x <= 5)
False
```

This comes up so often, however, that Python lets you *chain* the comparisons:

```
>>> x = 3
>>> 1 < x <= 5
True
```

Most combinations work as you would expect, but there are cases that may startle you:

```
>>> 3 < 5 != True
True
>>> 3 < 5 != False
True
```

It seems impossible for both of these expressions to be `True`. However, the first one is equivalent to this:

```
(3 < 5) and (5 != True)
```

while the second is equivalent to this:

```
(3 < 5) and (5 != False)
```

Since `5` is neither `True` nor `False`, the second half of each expression is `True`, so the expression as a whole is `True` as well.

This kind of expression is an example of something that is a bad idea even though it is legal. We strongly recommend that you only chain comparisons in ways that would seem natural to a mathematician, in other words, that you use < and <= together, or > and >= together, and nothing else. If you feel

the impulse to do something else, resist. Use simple comparisons and combine them with and in order to keep your code readable. It's also a good idea to use parentheses whenever you think the expression you are writing may not be entirely clear.

Using numbers and strings with Boolean operators

We have already seen that Python will convert an int to a float when the integer is used in an expression involving a floating-point number. Along the same lines, numbers and strings can be used with Boolean operators. Python treats 0 and 0.0 as False and treats all other numbers as True:

```
>>> not 0
True
>>> not 1
False
>>> not 34.2
False
>>> not -87
False
```

Similarly, the empty string is treated as False and all other strings are treated as True:

```
>>> not ''
True
>>> not 'bad'
False
```

None is also treated as False. In general, you should only use Boolean operators on Boolean values.

Short-Circuit Evaluation

When Python evaluates an expression containing and or or, it does so from left to right. As soon as it knows enough to stop evaluating, it stops, even if some operands haven't been looked at yet. This is called *short-circuit evaluation*.

In an or expression, if the first operand is True, we know that the expression is True. Python knows this as well, so it doesn't even evaluate the second operand. Similarly, in an and expression, if the first operand is False, we know that the expression is False. Python knows this as well, and the second operand isn't evaluated.

To demonstrate this, we use an expression that results in an error:

```
>>> 1 / 0
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
ZeroDivisionError: division by zero
```

We now use that expression as the second operand to or:

```
>>> (2 < 3) or (1 / 0)
True
```

Since the first operand produces True, the second operand isn't evaluated, so the computer never actually tries to divide anything by zero.

Of course, if the first operand to an or is False, the second operand must be evaluated. The second operand also needs to be evaluated when the first operand to an and is True.

Comparing Strings

It's possible to compare strings with each other, just as you would compare numbers. The characters in strings are represented by integers: a capital *A*, for example, is represented by 65, while a space is 32, and a lowercase *z* is 172. This encoding is called *ASCII*, which stands for "American Standard Code for Information Interchange." One of its quirks is that all the uppercase letters come before all the lowercase letters, so a capital *Z* is less than a small *a*.

One of the most common reasons to compare two strings is to decide which one comes first alphabetically. This is often referred to as "dictionary ordering" and "lexicographic ordering". Python decides which string is greater than which by comparing corresponding characters from left to right. If the character from one string is greater than the character from the other, the first string is greater than the second. If all the characters are the same, the two strings are equal; if one string runs out of characters while the comparison is being done (in other words, is shorter than the other), then it is less. The following code fragment shows a few comparisons in action:

```
>>> 'A' < 'a'
True
>>> 'A' > 'z'
False
>>> 'abc' < 'abd'
True
>>> 'abc' < 'abcd'
True
```

5.2 Choosing Which Statements to Execute

An if statement lets you change how your program behaves based on a condition. The general form of an if statement is as follows:

```
if «condition»:
    «block»
```

The *condition* is an expression, such as `color != "neon green"` or `x < y`. (Note that this doesn't have to be a Boolean expression. As we discussed in *Using numbers and strings with Boolean operators*, on page 88, non-Boolean values are treated as `True` or `False` when required.)

As with function bodies, the block of statements inside an `if` must be indented. As a reminder, the standard indentation for Python is 4 spaces.

If the condition is true, then the statements in the block are executed; otherwise, they are not. As with functions, the block of statements must be indented to show that it belongs to the `if` statement. If you don't indent properly, Python might raise an error, or worse, might happily execute the code that you wrote but, because some statements were not indented properly, do something you didn't intend. We'll briefly explore both problems in this chapter.

Here is a table of solution categories based on pH level:

pH Level	Solution Category
0–4	Strong acid
5–6	Weak acid
7	Neutral
8–9	Weak base
10–14	Strong base

We can use an `if` statement to print a message only when the pH level given by the program's user is acidic:

```
>>> ph = float(input('Enter the pH level: '))
Enter the pH level: 6.0
>>> if ph < 7.0:
...     print(ph, "is acidic.")
...
6.0 is acidic.
```

Recall from Section 4.5, *Getting Information from the Keyboard*, on page 76 that we have to convert user input from a string to a floating-point number before doing the comparison. Also, here we are providing a *prompt* for the user by passing a string into function `input`: Python prints this string to let the user know what information to type.

If the condition is false, the statements in the block aren't executed:

```
>>> ph = float(input('Enter the pH level: '))
Enter the pH level: 8.0
>>> if ph < 7.0:
```

```
...     print(ph, "is acidic.")
...
>>>
```

If we don't indent the block, Python lets us know:

```
>>> ph = float(input('Enter the pH level: '))
Enter the pH level: 6
>>> if ph < 7.0:
... print(ph, "is acidic.")
  File "<stdin>", line 2
    print(ph, "is acidic.")
        ^
IndentationError: expected an indented block
```

Since we're using a block, we can have multiple statements that are executed only if the condition is true:

```
>>> ph = float(input('Enter the pH level: '))
Enter the pH level: 6.0
>>> if ph < 7.0:
...     print(ph, "is acidic.")
...     print("You should be careful with that!")
...
6.0 is acidic.
You should be careful with that!
```

When we indent the first line of the block, the Python interpreter changes its prompt to ... until the end of the block, which is signaled by a blank line:

```
>>> ph = float(input('Enter the pH level: '))
Enter the pH level: 8.0
>>> if ph < 7.0:
...     print(ph, "is acidic.")
...
>>> print("You should be careful with that!")
You should be careful with that!
```

If we don't indent the code that's in the block, the interpreter complains:

```
>>> ph = float(input('Enter the pH level: '))
Enter the pH level: 8.0
>>> if ph < 7.0:
...     print(ph, "is acidic.")
... print("You should be careful with that!")
  File "<stdin>", line 3
    print("You should be careful with that!")
        ^
SyntaxError: invalid syntax
```

If the program is in a file, then no blank line is needed. As soon as the indentation ends, Python assumes that the block has ended as well. This is therefore legal:

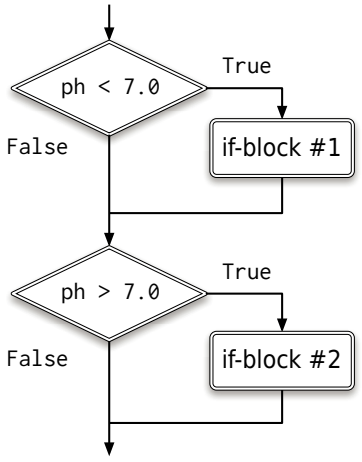
```
ph = 8.0
if ph < 7.0:
    print(ph, "is acidic.")
print "You should be careful with that!"
```

In practice, this slight inconsistency is never a problem, and most people never even notice it.

Of course, sometimes there are situations where a single decision isn't sufficient. If there are multiple criteria to examine, there are a couple of ways to handle it. One way is to use multiple if statements. For example, we might print different messages depending on whether a pH level is acidic or basic (if it's exactly 7, it's neutral, and our code won't print anything):

```
>>> ph = float(input('Enter the pH level: '))
Enter the pH level: 8.5
>>> if ph < 7.0:
...     print(ph, "is acidic.")
...
>>> if ph > 7.0:
...     print(ph, "is basic.")
...
8.5 is basic.
>>>
```

Here's a flow chart that shows how Python executes the if statements. The diamonds are conditions and the arrows indicate what path to take depending on the results of evaluating those conditions:

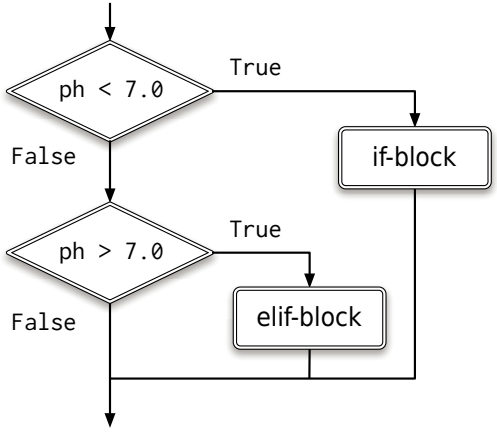


Notice that both conditions are always evaluated, even though we know that only one of the blocks can be executed.

We can merge both cases by adding another condition/block pair using the elif keyword (which stands for "else if"); each condition/block pair is called a *clause*:

```
>>> ph = float(input('Enter the pH level: '))
Enter the pH level: 8.5
>>> if ph < 7.0:
...     print(ph, "is acidic.")
... elif ph > 7.0:
...     print(ph, "is basic.")
...
8.5 is basic.
>>>
```

The difference between the two is that the elif is checked only when the if condition above it evaluated to False. Here's a flow chart for this code:



This flow chart shows that if the first condition evaluates to True, the second condition is skipped.

If the pH is exactly 7.0, neither clause matches, so nothing is printed:

```
>>> ph = float(input('Enter the pH level: '))
Enter the pH level: 7.0
>>> if ph < 7.0:
...     print(ph, "is acidic.")
... elif ph > 7.0:
...     print(ph, "is basic.")
...
>>>
```

With the ph example, we accomplished the same thing with two if statements as we did with an if/elif. This is not always the case; for example, if the body of the first if changes the value of a variable used in the second condition, they are not equivalent. here is the version with two ifs:

```
>>> ph = float(input('Enter the pH level: '))
Enter the pH level: 6.0
>>> if ph < 7.0:
...     ph = 8.0
...
>>> if ph > 7.0:
...     print(ph, "is acidic.")
...
8.0 is acidic.
```

And here is the version with an if/elif:

```
>>> ph = float(input('Enter the pH level: '))
Enter the pH level: 6.0
>>> if ph < 7.0:
...     ph = 8.0
>>> elif ph > 7.0:
...     print(ph, "is acidic.")
...
>>>
```

As a rule of thumb, if two conditions are related, use if/elif instead of two ifs.

An if statement can be followed by multiple elif clauses. This longer example translates a chemical formula into English:

```
>>> compound = input('Enter the compound: ')
Enter the compound: CH4
>>> if compound == "H2O":
...     print("Water")
... elif compound == "NH3":
...     print("Ammonia")
... elif compound == "CH4":
...     print("Methane")
...
Methane
>>>
```

As we saw in code, on page 93, if none of the conditions in a chain of if/elif statements are satisfied, Python does not execute any of the associated blocks. This isn't always what we'd like, though. In our translation example, we probably want our program to print something even if it doesn't recognize the compound. To do this, we add an else clause at the end of the chain:

```
>>> compound = input('Enter the compound: ')
Enter the compound: H2SO4
>>> if compound == "H2O":
...     print("Water")
... elif compound == "NH3":
...     print("Ammonia")
... elif compound == "CH4":
...     print("Methane")
... else:
...     print("Unknown compound")
...
Unknown compound
>>>
```

An if statement can have at most one else clause, and it has to be the final clause in the statement. Notice there is no condition associated with the else; logically, the following statement (except that the condition is evaluated only once in the first form but twice in the second form):

```
if «condition»:
    «if_block»
else:
    «else_block»
```

is the same as this:

```
if «condition»:
    «if_block»
if not «condition»:
    «else_block»
```

Nested if Statements

An if statement's block can contain any type of Python statement, which implies that it can include other if statements. An if statement inside another is called a *nested* if statement.

```
value = input('Enter the pH level: ')
if len(value) > 0:
    ph = float(value)
    if ph < 7.0:
        print(ph, "is acidic.")
    elif ph > 7.0:
        print(ph, "is basic.")
    else:
        print(ph, "is neutral.")
else:
    print("No pH value was given!")
```

In this case, we ask the user to provide a pH value, which we'll initially receive as a string. The first, or *outer*, if statement checks whether the user typed

something, which determines whether we examine the value of pH with the *inner* if statement. (If the user didn't enter a number, then the call on function float(value) will produce a ValueError.)

Nested if statements are sometimes necessary, but they can get complicated and difficult to understand. To describe when a statement is executed, we have to mentally combine conditions; for example, the statement print(ph, "is acidic.") is executed only if the length of string input is greater than 0 *and* pH < 7.0 also evaluates to True. (Assuming the user entered a number.)

5.3 Remembering Results of Boolean Expression Evaluation

Take a look at the following line of code and guess what value is stored in x:

```
>>> x = 15 > 5
```

If you said True, you were right: 15 is greater than 5, so the comparison produces True, and since that's a value like any other, it can be assigned to a variable.

The most common situation in which you would want to do this comes up when translating decision tables into software. For example, suppose you want to calculate someone's risk of heart disease using the following rules based on age and body mass index (BMI):

		Age	
		<45	≥45
BMI	<22.0	Low	Medium
	≥22.0	Medium	High

One way to implement this would be to use nested if statements:

```
if age < 45:
    if bmi < 22.0:
        risk = 'low'
    else:
        risk = 'medium'
else:
    if bmi < 22.0:
        risk = 'medium'
    else:
        risk = 'high'
```

The expression bmi < 22.0 is used multiple times. To simplify this code, we can evaluate each of the Boolean expressions once, create variables that refer to

the values produced by those expressions, and use those variables multiple times:

```
young = age < 45
slim = bmi < 22.0
if young:
    if slim:
        risk = 'low'
    else
        risk = 'medium'
else:
    if slim:
        risk = 'medium'
    else:
        risk = 'high'
```

We could also write this without nesting as follows:

```
young = age < 45
slim = bmi < 22.0
if young and slim:
    risk = 'low'
elif young and not slim:
    risk = 'medium'
elif not young and slim:
    risk = 'medium'
elif not young and not slim:
    risk = 'high'
```

5.4 You Learned About Booleans: True or False?

In this chapter, you learned the following:

- Python uses Boolean values `True` and `False` to represent what is true and what isn't. Programs can combine these values using three operators: `not`, `and`, and `or`.
- Boolean operators can also be applied to numeric values. `0`, `0.0`, the empty string, and `None` are treated as `False`; all other numeric values and strings are treated as `True`. It is best to avoid applying Boolean operators to non-Boolean values.
- Relational operators such as "equals" and "less than" compare values and produce a Boolean result.
- When different operators are combined in an expression, the order of precedence from highest to lowest is arithmetic, relational, and then Boolean.

- if statements control the flow of execution. As with function definitions, the bodies of if statements are indented, as are the bodies of elif and else clauses.

5.5 Exercises

Here are some exercises for you to try on your own:

1. What value does each expression produce? Verify your answers by typing the expressions into Python.

   a. True and not False

   b. True and not false (Notice the capitalization.)

   c. True or True and False

   d. not True or not False

   e. True and not 0

   f. 52 < 52.3

   g. 1 + 52 < 52.3

   h. 4 != 4.0

2. Variables x and y refer to Boolean values.

   a. Write an expression that produces True iff both variables are True.

   b. Write an expression that produces True iff x is False.

   c. Write an expression that produces True iff at least one of the variables is True.

3. Variables full and empty refer to Boolean values. Write an expression that produces True iff at most one of the variables is True.

4. You want an automatic wildlife camera to switch on if the light level is less than 0.01 or if the temperature is above freezing, but not if both conditions are true. (You should assume that function turn_camera_on has already been defined.)

   Your first attempt to write this is as follows:

```
if (light < 0.01) or (temperature > 0.0):
    if not ((light < 0.01) and (temperature > 0.0)):
        turn_camera_on()
```

   A friend says that this is an exclusive or and that you could write it more simply as follows:

```
if (light < 0.01) != (temperature > 0.0):
    turn_camera_on()
```

Is your friend right? If so, explain why. If not, give values for light and temperature that will produce different results for the two fragments of code.

5. In Section 3.1, *Functions that Python Provides*, on page 33, we saw the built-in function abs. Variable x refers to a number. Write an expression that evaluates to True if x and its absolute value are equal and evaluates to False otherwise. Assign the resulting value to a variable named result.

6. Write a function named different that has two parameters, a and b. The function should return True if a and b refer to different values and should return False otherwise.

7. Variables population and land_area refer to floats.

   a. Write an if statement that will print the population if it is less than 10,000,000.

   b. Write an if statement that will print the population if it is between 10,000,000 and 35,000,000.

   c. Write an if statement that will print "Densely populated" if the land density (number of people per unit of area) is greater than 100.

   d. Write an if statement that will print "Densely populated" if the land density (number of people per unit of area) is greater than 100 and that will print "Sparsely populated" otherwise.

8. Function convert_to_celsius from Section 3.3, *Defining our Own Functions*, on page 38 converts from Fahrenheit to Celsius. Wikipedia, however, discusses eight temperature scales: Kelvin, Celsius, Fahrenheit, Rankine, Delisle, Newton, Rèaumur, and Rømer. Visit http://en.wikipedia.org/wiki/Comparison_of_temperature_scales to read about them.

   a. Write a function convert_temperatures(t, source, target) that converts temperature t from source units to target units, where source and target are each one of "Kelvin", "Celsius", "Fahrenheit", "Rankine", "Delisle", "Newton", "Reaumur", and "Romer".

      Hint: on the Wikipedia page there are eight tables, each with two columns and seven rows. That translates to an awful lot of if statements—at least 8 * 7, because each of the eight units can be converted to the seven other units. Possibly even worse, if you decided to add another temperature scale, you would need to add at least sixteen more if statements: eight to convert from your new scale to each of

the current ones and eight to convert from the current ones to your new scale.

A better way is to choose one canonical scale, such as Celsius. Your conversion function could work in two steps: convert from the source scale to Celsius and then from Celsius to the target scale.

b. Now, if you added a new temperature scale, how many if statements would you need to add?

9. Assume we want to print a strong warning message if a pH value is below 3.0 and otherwise simply report on the acidity. We try this if statement:

```
>>> ph = 2
>>> if ph < 7.0:
...     print(ph, "is acidic.")
... elif ph < 3.0:
...     print(ph, "is VERY acidic! Be careful.")
...
2 is acidic.
```

This prints the wrong message when a pH of 2 is entered. What is the problem, and how can you fix it?

10. The following code displays a message(s) about the acidity of a solution:

```
ph = float(input("Enter the ph level: "))
if ph < 7.0:
    print("It's acidic!")
elif ph < 4.0:
    print("It's a strong acid!")
```

a. What message(s) are displayed when the user enters 6.4?

b. What message(s) are displayed when the user enters 3.6?

c. Make a small change to one line of the code so that both messages are displayed when a value less than 4 is entered.

11. Why does the last example in Section 5.3, *Remembering Results of Boolean Expression Evaluation*, on page 96 check to see whether someone is heavy (that is, that their weight exceeds the threshold) rather than light? If you wanted to write the second assignment statement as light = bmi < 22.0, what change(s) would you have to make to the lookup table?

CHAPTER 6

A Modular Approach to Program Organization

Mathematicians don't prove every theorem from scratch. Instead, they build their proofs on the truths their predecessors have already established. In the same way, it's vanishingly rare for someone to write all of a program themself; it's much more common—and productive—to make use of the millions of lines of code that other programmers have written before.

What Happens When You import this?

```
>>> import this
The Zen of Python, by Tim Peters

Beautiful is better than ugly.
Explicit is better than implicit.
Simple is better than complex.
Complex is better than complicated.
Flat is better than nested.
Sparse is better than dense.
Readability counts.
Special cases aren't special enough to break the rules.
Although practicality beats purity.
Errors should never pass silently.
Unless explicitly silenced.
In the face of ambiguity, refuse the temptation to guess.
There should be one-- and preferably only one --obvious way to do it.
Although that way may not be obvious at first unless you're Dutch.
Now is better than never.
Although never is often better than *right* now.
If the implementation is hard to explain, it's a bad idea.
If the implementation is easy to explain, it may be a good idea.
Namespaces are one honking great idea -- let's do more of those!
```

A *module* is a collection of variables and functions that are grouped together in a single file. The variables and functions in a module are usually related to each other in some way; for example, module math contains a variable pi and mathematical functions such as cos (cosine) and sqrt (square root). This chapter shows you how to use some of the hundreds of modules that come with Python and how to create your own modules.

6.1 Importing Modules

To gain access to the variables and functions from a module, you have to *import* it. To tell Python that you want to use functions in module math, for example, you use this import statement:

```
>>> import math
```

Importing a module creates a new variable with that name. That variable refers to an object whose type is module:

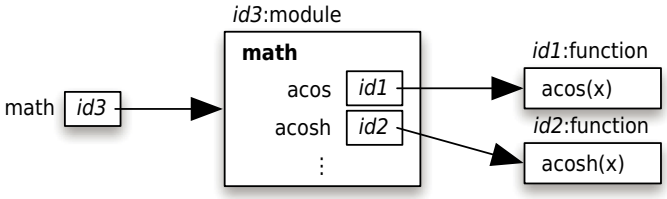
```
>>> type(math)
<type 'module'>
```

Once you have imported a module, you can use the built-in help function to see what it contains. Here is the first part of the help output:

```
>>> help(math)

Help on module math:

NAME
    math

MODULE REFERENCE
    http://docs.python.org/3.3/library/math

    The following documentation is automatically generated from the Python
    source files.  It may be incomplete, incorrect or include features that
    are considered implementation detail and may vary between Python
    implementations.  When in doubt, consult the module reference at the
    location listed above.

DESCRIPTION
    This module is always available.  It provides access to the
    mathematical functions defined by the C standard.

FUNCTIONS
    acos(...)
        acos(x)

        Return the arc cosine (measured in radians) of x.
```

```
    acosh(...)
        acosh(x)

        Return the hyperbolic arc cosine (measured in radians) of x.

[Lots of other functions not shown here.]
```

The statement `import math` creates a variable called `math` that refers to a module object. In that object are all the names defined in that module. Each of them refers to a function object:



Great—our program can now use all the standard mathematical functions. When we try to calculate a square root, though, we get an error telling us that Python is still unable to find function `sqrt`:

```
>>> sqrt(9)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
NameError: name 'sqrt' is not defined
```

The solution is to tell Python explicitly to look for the function in the `math` module by combining the module's name with the function's name using a dot:

```
>>> math.sqrt(9)
3.0
```

The dot is an operator, just like + and ** are operators. Its meaning is "look up the object that the variable to the left of the dot refers to and, in that object, find the name that occurs to the right of the dot". In `math.sqrt(9)`, Python finds `math` in the current namespace, looks up the module object that `math` refers to, finds function `sqrt` inside that module, and then executes the function call following the standard rules described in Section 3.4, *Tracing Function Calls in the Memory Model*, on page 42.

Modules can contain more than just functions. Module `math`, for example, also defines some variables like `pi`. Once the module has been imported, you can use these variables like any others:

```
>>> import math
```

```
>>> math.pi
3.141592653589793
>>> radius = 5
>>> print('area is', math.pi * radius ** 2)
area is 78.53981633974483
```

You can even assign to variables imported from modules:

```
>>> import math
>>> math.pi = 3
>>> radius = 5
>>> print('area is', math.pi * radius ** 2)
area is 75
```

Don't do this! Changing the value of π isn't a good idea. In fact, it's such a bad idea that many languages allow programmers to define unchangeable *constants* as well as variables. As the name suggests, the value of a constant cannot be changed after it has been defined: π is always 3.14159 and a little bit, while SECONDS_PER_DAY is always 86,400. The fact that Python doesn't allow programmers to "freeze" values like this is one of the language's few significant flaws.

Restoring a Module

If you change the value of a variable or function from an imported module, you can restart the Shell and re-import the module to restore it to its original value. In IDLE, you can restart the Shell by choosing Shell->Restart Shell.

Without having to restart the shell, you can restore the module to its original state using function reload from module imp:

```
>>> import math
>>> math.pi = 3
>>> math.pi
3
>>> import imp
>>> math = imp.reload(math)
>>> math.pi
3.141592653589793
>>>
```

Function imp.reload returns the module.

Combining the module's name with the names of the things it contains is safe, but it isn't always convenient. For this reason, Python lets you specify exactly what you want to import from a module, like this:

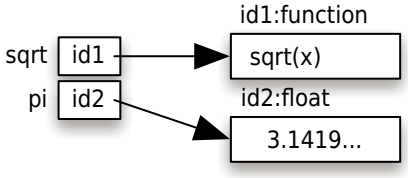
```
>>> from math import sqrt, pi
>>> sqrt(9)
3.0
```

```
>>> radius = 5
>>> print('circumference is', 2 * pi * radius)
circumference is 31.41592653589793
```

This doesn't introduce a variable called math. Instead, it creates function sqrt and variable pi in the current namespace, as if you had typed the function definition and variable assigment yourself. Restart your shell and try this:

```
>>> from math import sqrt, pi
>>> math.sqrt(9)
Traceback (most recent call last):
  File "<pyshell#12>", line 1, in <module>
    math.sqrt(9)
NameError: name 'math' is not defined
>>> sqrt(9)
3.0
```

Here, we don't have a variable called math. Instead, we imported variables sqrt and pi directly into the current namespace:



This can lead to problems when different modules provide functions that have the same name. If you import a function called spell from a module called magic and then you import another function called spell from the module grammar, the second replaces the first. It's exactly like assigning one value to a variable, then another: the most recent assignment or import wins.

This is why it's usually *not* a good idea to use import *, which brings in everything from the module at once:

```
>>> from math import *
>>> print(sqrt(8))
2.8284271247461903
```

Although import * saves some typing, you run the risk of your program accessing the incorrect function and not working properly.

The standard Python library contains several hundred modules to do everything from figuring out what day of the week it is to fetching data from a website. The full list is online at http://docs.python.org/release/3.3.0/py-modindex.html; although it's far too much to absorb in one sitting (or even one course),

knowing how to use the library well is one of the things that distinguishes good programmers from poor ones.

The __builtins__ Module

Python's built-in functions are actually in a module named _builtins_ (with 2 underscores before and after 'builtins'). The double underscores before and after the name signal that it's part of Python; we'll see this convention used again later for other things. You can see what's in the module using help(__builtins__), or if you just want to see what functions and variables are available, you can use dir instead (which works on other modules as well):

```
>>> dir(__builtins__)
['ArithmeticError', 'AssertionError', 'AttributeError', 'BaseException',
'BlockingIOError', 'BrokenPipeError', 'BufferError', 'BytesWarning',
'ChildProcessError', 'ConnectionAbortedError', 'ConnectionError',
'ConnectionRefusedError', 'ConnectionResetError', 'DeprecationWarning',
'EOFError', 'Ellipsis', 'EnvironmentError','Exception', 'False',
'FileExistsError', 'FileNotFoundError','FloatingPointError', 'FutureWarning',
'GeneratorExit', 'IOError', 'ImportError', 'ImportWarning', 'IndentationError',
'IndexError', 'InterruptedError', 'IsADirectoryError', 'KeyError',
'KeyboardInterrupt', 'LookupError', 'MemoryError', 'NameError', 'None',
'NotADirectoryError', 'NotImplemented', 'NotImplementedError', 'OSError',
'OverflowError', 'PendingDeprecationWarning', 'PermissionError',
'ProcessLookupError', 'ReferenceError', 'ResourceWarning', 'RuntimeError',
'RuntimeWarning', 'StopIteration', 'SyntaxError', 'SyntaxWarning',
'SystemError', 'SystemExit', 'TabError', 'TimeoutError', 'True', 'TypeError',
'UnboundLocalError', 'UnicodeDecodeError', 'UnicodeEncodeError', 'UnicodeError',
'UnicodeTranslateError', 'UnicodeWarning', 'UserWarning', 'ValueError',
'Warning', 'ZeroDivisionError', '_', '__build_class__', '__debug__',
'__doc__', '__import__', '__name__', '__package__', 'abs', 'all', 'any',
'ascii', 'bin', 'bool', 'bytearray', 'bytes', 'callable', 'chr', 'classmethod',
'compile', 'complex', 'copyright', 'credits', 'delattr', 'dict', 'dir',
'divmod', 'enumerate', 'eval', 'exec', 'exit', 'filter', 'float', 'format',
'frozenset', 'getattr', 'globals', 'hasattr', 'hash', 'help', 'hex', 'id',
'input', 'int', 'isinstance', 'issubclass', 'iter', 'len', 'license', 'list',
'locals', 'map', 'max', 'memoryview', 'min', 'next', 'object', 'oct', 'open',
'ord', 'pow', 'print', 'property', 'quit', 'range', 'repr', 'reversed',
'round', 'set', 'setattr', 'slice', 'sorted', 'staticmethod', 'str', 'sum',
'super', 'tuple', 'type', 'vars', 'zip']
```

As of Python 3.3.0, 46 of the 147 items in _builtins_ are used to signal errors of particular kinds, such as SyntaxError and ZeroDivisionError. All errors, warnings, and exceptions are types, like int, float, and function. Their names follow a naming convention in which the first letter of each word is uppercase.

We'll introduce some of this module's other members in later chapters.

6.2 Defining Your Own Modules

Section 3.6, *Writing and Running a Program*, on page 61 explained that in order to save code for later use, you can put it in a file with a .py extension

and demonstrated how to run that code. Chapter 3, *Designing and Using Functions*, on page 33 also included this function definition:

```
>>> def convert_to_celsius(fahrenheit):
...     """ (number) -> float
...
...     Return the number of Celsius degrees equivalent to fahrenheit degrees.
...
...     >>> convert_to_celsius(75)
...     23.88888888888889
...     """
...
...     return (fahrenheit - 32.0) * 5.0 / 9.0
...
```

Put the function defintion convert_to_celsius from Section 3.3, *Defining our Own Functions*, on page 38 in a file called temperature.py. (You can save this file anywhere you like, although most programmers create a separate directory for each set of related files that they write.)

Now add another function to temperature.py called above_freezing that returns True if and only if its parameter celsius is above freezing:

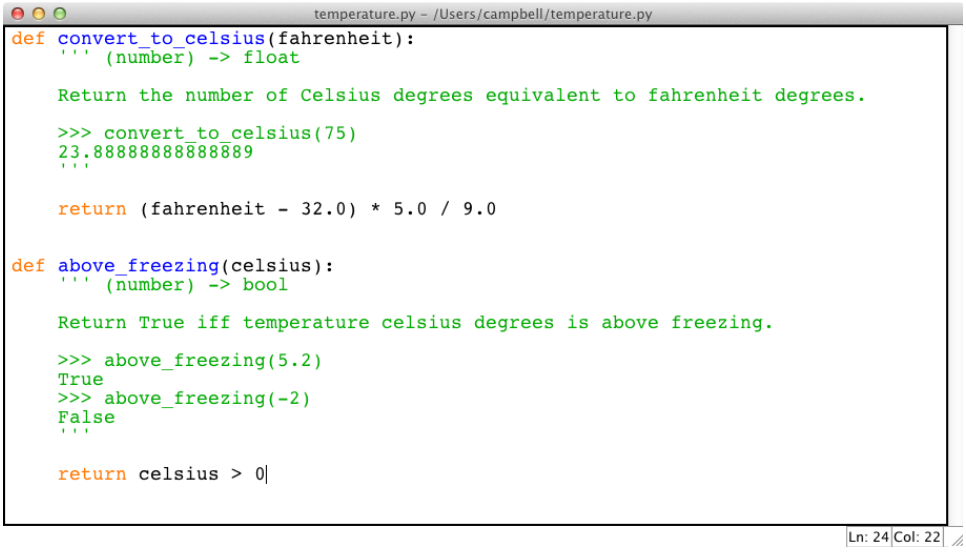


Figure 3—The temperature module

Congratulations—you have created a module called temperature. Now that you've created this file, you can import it like any other module:

```
>>> import temperature
```

```
>>> celsius = temperature.convert_to_celsius(33.3)
>>> temperature.above_freezing(celsius)
True
```

What Happens During Import

Let's try another experiment. Create a file called experiment.py with this one statement inside it:

```
print("The panda's scientific name is 'Ailuropoda melanoleuca'")
```

and then import it:

```
>>> import experiment
The panda's scientific name is 'Ailuropoda melanoleuca'
```

What this shows is that *Python executes modules as it imports them*. You can do anything in a module you would do in any other program, because as far as Python is concerned, it's just another bunch of statements to be run.

Let's try another experiment. Start a fresh Python session, and try importing module experiment twice in a row:

```
>>> import experiment
The panda's scientific name is 'Ailuropoda melanoleuca'
>>> import experiment
>>>
```

Notice that the message wasn't printed the second time. That is because Python loads modules only the first time they are imported. Internally, Python keeps track of the modules it has already seen; when it is asked to load one that's already in that list, it just skips over it. This saves time and will be particularly important when you start writing modules that import other modules, which in turn import other modules—if Python didn't keep track of what was already in memory, it could wind up loading commonly used modules like math dozens of times.

Even if you import a module, edit that module's file, and then re-import, the module won't be reloaded. Your edits won't have any effect until you restart the shell or call imp.reload.

Using __main__

As we saw in Section 3.6, *Writing and Running a Program*, on page 61, every Python module can be run directly (from the command line or by running it from an IDE like IDLE), or, as we saw earlier in this section, it can be run indirectly (imported by another program). Sometimes we want to write code

that should only be run when the module is run directly and not when the module is imported.

Python defines a special string variable called __name__ in every module to help us figure this out. Suppose we put the following into echo.py:

```
print("__name__ is", __name__)
```

If we run this file, its output is as follows:

```
__name__ is __main__
```

As promised, Python has created the variable __name__. Its value is "__main__", meaning this module is the main program.

But look at what happens when we import echo (instead of running it directly):

```
>>> import echo
__name__ is echo
```

The same thing happens if we write a program that does nothing but import our echoing module. Create a file import_echo.py with this code inside it:

```
import echo

print("After import, __name__ is", __name__, \
        "and echo.__name__ is", echo.__name__)
```

which, when run from the command line, produces this:

```
__name__ is echo
After import, __name__ is __main__ and echo.__name__ is echo
```

When Python imports a module, it sets that module's __name__ variable to be the name of the module, rather than the special string "__main__". This means that a module can tell whether it is the main program. Create a file named main_example.py with this code inside it:

```
if __name__ == "__main__":
    print("I am the main program.")
else:
    print("Another module is importing me.")
```

Try it. See what happens when you run main_example.py directly and when you import it.

Some of our modules contain not only function definitions but also programs. For example, create a new module temperature_program that contains the functions from temperature and a little program:

temperature_program.py – /Users/campbell/temperature_program.py

```
def convert_to_celsius(fahrenheit):
    ''' (number) -> float

    Return the number of Celsius degrees equivalent to fahrenheit degrees.

    >>> convert_to_celsius(75)
    23.88888888888889
    '''

    return (fahrenheit - 32.0) * 5.0 / 9.0


def above_freezing(celsius):
    ''' (number) -> bool

    Return True iff temperature celsius degrees is above freezing.

    >>> above_freezing(5.2)
    True
    >>> above_freezing(-2)
    False
    '''

    return celsius > 0


fahrenheit = float(input('Enter the temperature in degrees Fahrenheit: '))
celsius = convert_to_celsius(fahrenheit)
if above_freezing(celsius):
    print('It is above freezing.')
else:
    print('It is at or below freezing.')
```

Ln: 32 Col: 41

Figure 4—The temperature_program module

When that module is run, it prompts the user to enter a value and, depending on the value entered, prints one of two messages:

Python Shell

```
Python 3.3.0 (v3.3.0:bd8afb90ebf2, Sep 29 2012, 02:56:36)
[GCC 4.2.1 (Apple Inc. build 5577)] on darwin
Type "copyright", "credits" or "license()" for more information.
>>> ================================ RESTART ================================
>>>
Enter the temperature in degrees Fahrenheit: 35
It is above freezing.
>>>
```

Ln: 8 Col: 4

Let's create another module baking.py that uses the conversion function from module temperature_program:

baking.py – /Users/campbell/baking.py

```
import temperature_program

def get_preheating_instructions(fahrenheit):
    ''' (number) -> str

    Return instructions for preheating the oven in fahreneheit degrees and
    Celsius degrees.

    >>> get_preheating_instructions(500)
    Preheat oven to 500 degrees F (260.0 degrees C).
    '''

    cels = str(temperature_program.convert_to_celsius(fahrenheit))
    fahr = str(fahrenheit)
    return 'Preheat oven to ' + fahr + ' degrees F ('+ cels +' degrees C).'


fahr = float(input('Enter the baking temperature in degrees Fahrenheit: '))
print(get_preheating_instructions(fahr))
```

Ln: 19 Col: 40

When baking.py is run, it imports temperature_program, so the program at the bottom of temperature_program.py is executed:

Python Shell

```
Python 3.3.0 (v3.3.0:bd8afb90ebf2, Sep 29 2012, 02:56:36)
[GCC 4.2.1 (Apple Inc. build 5577)] on darwin
Type "copyright", "credits" or "license()" for more information.
>>> ================================ RESTART ================================
>>>
Enter the temperature in degrees Fahrenheit: 15
It is at or below freezing.
Enter the baking temperature in degrees Fahrenheit: 500
Preheat oven to 500.0 degrees F (260.0 degrees C).
>>>
```

Ln: 10 Col: 4

Since we don't care whether a temperature is above freezing when preheating our oven, when importing temperature_program.py we can prevent that part of the code from executing by putting it in an if __name__ == '__main__': block:

temperature_program.py – /Users/campbell/temperature_program.py

```
def convert_to_celsius(fahrenheit):
    ''' (number) -> float

    Return the number of Celsius degrees equivalent to fahrenheit degrees.

    >>> convert_to_celsius(75)
    23.88888888888889
    '''

    return (fahrenheit - 32.0) * 5.0 / 9.0


def above_freezing(celsius):
    ''' (number) -> bool

    Return True iff temperature celsius degrees is above freezing.

    >>> above_freezing(5.2)
    True
    >>> above_freezing(-2)
    False
    '''

    return celsius > 0


if __name__ == '__main__':
    fahrenheit = float(input('Enter the temperature in degrees Fahrenheit: '))
    celsius = convert_to_celsius(fahrenheit)
    if above_freezing(celsius):
        print('It is above freezing.')
    else:
        print('It is at or below freezing.')
```

Ln: 33 Col: 45

Now when baking.py is run, only the code from temperature_program that is outside of the if __name__ == '__main__': block is executed:

Python Shell

```
Python 3.3.0 (v3.3.0:bd8afb90ebf2, Sep 29 2012, 02:56:36)
[GCC 4.2.1 (Apple Inc. build 5577)] on darwin
Type "copyright", "credits" or "license()" for more information.
>>> ================================ RESTART ================================
>>>
Enter the baking temperature in degrees Fahrenheit: 500
Preheat oven to 500.0 degrees F (260.0 degrees C).
>>>
```

Ln: 8 Col: 4

We will see other uses of __name__ in the following sections and in later chapters.

6.3 Testing Your Code Semi-Automatically

In Section 3.5, *Designing New Functions: a Recipe*, on page 49, we introduced the Function Design Recipe (FDR). Following the FDR, the docstrings that we write include example function calls, such as those in Figure 4, *The temperature_program module*, on page 110.

The last step of the FDR involves testing the function. Up until now, we have been typing the function calls from the docstrings to the shell (or copying and pasting them) to run them and then comparing the results with what we expect to make sure they match.

Python has a module called *doctest* that allows us to run the tests that we include in docstrings all at once. It reports on whether the function calls return what we expect. We will use doctest to run the tests from module temperature_program from *Using __main__*, on page 108:

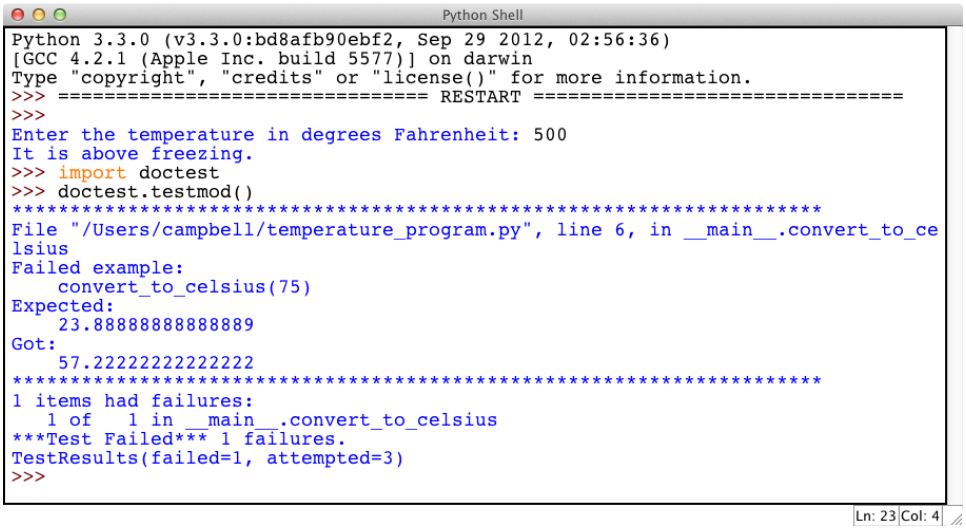
```
Python Shell
Python 3.3.0 (v3.3.0:bd8afb90ebf2, Sep 29 2012, 02:56:36)
[GCC 4.2.1 (Apple Inc. build 5577)] on darwin
Type "copyright", "credits" or "license()" for more information.
>>> ================================ RESTART ================================
>>>
Enter the temperature in degrees Fahrenheit: 500
It is above freezing.
>>> import doctest
>>> doctest.testmod()
TestResults(failed=0, attempted=3)
>>>
                                                                    Ln: 11 Col: 4
```

That message tells us that three tests were run and none of them failed. That is, the three function calls in the docstrings were run and returned the same value that we expected and stated in the docstring.

Now, let's see what happens when there is an error in our calculation. Instead of the calculation we've been using, (fahrenheit - 32.0) * 5.0/9.0, remove the parentheses: fahrenheit - 32.0 * 5.0 / 9.0:

```
temperature_program.py – /Users/campbell/temperature_program.py
def convert_to_celsius(fahrenheit):
    ''' (number) -> float

    Return the number of Celsius degrees equivalent to fahrenheit degrees.

    >>> convert_to_celsius(75)
    23.88888888888889
    '''

    return fahrenheit - 32.0 * 5.0 / 9.0


def above_freezing(celsius):
    ''' (number) -> bool

    Return True iff temperature celsius degrees is above freezing.

    >>> above_freezing(5.2)
    True
    >>> above_freezing(-2)
    False
    '''

    return celsius > 0


if __name__ == '__main__':
    fahrenheit = float(input('Enter the temperature in degrees Fahrenheit: '))
    celsius = convert_to_celsius(fahrenheit)
    if above_freezing(celsius):
        print('It is above freezing.')
    else:
        print('It is at or below freezing.')
                                                                    Ln: 33 Col: 45
```

The result of running doctest on that module would be:



The failure message above indicates that function call `convert_to_celsius(75)` was expected to return `23.88888888888889`, but that it actually returned `57.22222222222222`. The other two tests ran and passed.

When a failure occurs, we need to review our code to identify the problem. We should also check the expected return value listed in the docstring to make sure that the expected value matches both the type contract and the desciption of the function.

6.4 Tips for Grouping Your Functions

Put functions and variables that logically belong together in the same module. If there isn't some logical connection—for example, if one of the functions calculates how much carbon monoxide different kinds of cars produce, while another figures out how strong bones are given their diameter and density—then you shouldn't put them in one module just because you happen to be the author of both.

Of course, people often have different opinions about what is logical and what isn't. Take Python's `math` module, for example; should functions to multiply matrices go in there too or in a separate linear algebra module? What about basic statistical functions? Going back to the previous paragraph, should a function that calculates gas mileage go in the same module as one that calculates carbon monoxide emissions? You can always find a reason why two functions should *not* be in the same module, but 1,000 modules with one function each are going to be hard for people (including you) to find their way around.

As a rule of thumb, if a module has less than a handful of things in it, it's probably too small, and if you can't sum up the contents and purpose of a module in a one- or two-sentence docstring, it's probably too large. These are just guidelines, though; in the end, you will have to decide based on how more experienced programmers have organized modules like the ones in the Python standard library and eventually on your own sense of style.

6.5 Organizing Our Thoughts

In this chapter, you learned the following:

- A module is a collection of functions and variables grouped together in a file. To use a module, you must first import it using `import «modulename»`. After it has been imported, you refer to its contents using `«modulename».«functionname»` or `«modulename».«variable»`.
- Variable `__name__` is created by Python and can be used to specify that some code should only run when the module is run directly and not when the module is imported.
- Programs have to do more than just run to be useful; they have to run correctly. One way to ensure that they do is to test them, which you can do in Python using module `doctest`.

6.6 Exercises

Here are some exercises for you to try on your own:

1. Import module `math`, and use its functions to complete the following exercises. (You can call `dir(math)` to get a listing of the items in `math`.)
   a. Write an expression that produces the floor of `-2.8`.
   b. Write an expression that rounds the value of `-4.3` and then produces the absolute value of that result.
   c. Write an expression that produces the ceiling of sine of 34.5.
2. In the following exercises, you will work with Python's `calendar` module:
   a. Visit the Python documentation website at http://docs.python.org/release/3.3.0/py-modindex.html, and look at the documentation on module `calendar`.
   b. Import module `calendar`.
   c. Using function `help`, read the description of function `isleap`.
   d. Use `isleap` to determine the next leap year.

e. Use dir to get a list of what calendar contains.

f. Find and use a function in module calendar to determine how many leap years there will be between the years 2000 and 2050, inclusive.

g. Find and use a function in module calendar to determine which day of the week July 29, 2016 will be.

3. Create a file named exercise.py with this code inside it:

```
def average(num1, num2):
    """ (number, number) -> number

    Return the average of num1 and num2.

    >>> average(10,20)
    15.0
    >>> average(2.5, 3.0)
    2.75
    """

    return num1 + num2 / 2
```

a. Run exercise.py. Import doctest and run doctest.testmod().

b. Both of the tests in function average's docstring fail. Fix the code and re-run the tests. Repeat this procedure until the tests pass.

CHAPTER 7

Using Methods

So far, we've seen lots of functions: built-in functions, functions inside modules, and functions that we've defined. A *method* is another kind of function that is attached to a particular type. There are str methods, int methods, bool methods, and more—every type has its own set of methods. In this chapter, we'll explore how to use methods, and also how they differ from the rest of the functions that we've seen.

7.1 Modules, Classes, and Methods

In Section 6.1, *Importing Modules*, on page 102, we saw that a module is a kind of object, one that can contain functions and other variables. There is another kind of object that is similar to a module: a *class*. You've been using classes all along, probably without realizing it: a class is how Python represents a type.

You may have called built-in function help on int, float, bool, or str. We'll do that now with str—notice that the first line says that it's a class:

```
>>> help(str)
Help on class str in module builtins:

class str(object)
 |  str(object[, encoding[, errors]]) -> str
 |
 |  Create a new string object from the given object. If encoding or
 |  errors is specified, then the object must expose a data buffer
 |  that will be decoded using the given encoding and error handler.
 |  Otherwise, returns the result of object.__str__() (if defined)
 |  or repr(object).
 |  encoding defaults to sys.getdefaultencoding().
 |  errors defaults to 'strict'.
 |
 |  Methods defined here:
```

```
 |
 |  __add__(...)
 |      x.__add__(y) <==> x+y
 |
 |  __contains__(...)
 |      x.__contains__(y) <==> y in x

[Lots of other names with leading and trailing underscores not shown here.]

 |  capitalize(...)
 |      S.capitalize() -> str
 |
 |      Return a capitalized version of S, i.e. make the first character
 |      have upper case and the rest lower case.
 |
 |  center(...)
 |      S.center(width[, fillchar]) -> str
 |
 |      Return S centered in a string of length width. Padding is
 |      done using the specified fill character (default is a space)
 |
 |  count(...)
 |      S.count(sub[, start[, end]]) -> int
 |
 |      Return the number of non-overlapping occurrences of substring sub in
 |      string S[start:end].  Optional arguments start and end are
 |      interpreted as in slice notation.

[There are many more of these as well.]
```

Near the top of this documentation is this:

```
|  str(object[, encoding[, errors]]) -> str
|
|  Create a new string object from the given object.
```

That describes how to use `str` as a function: we can call it to create a string. For example, `str(17)` creates the string `'17'`.

We can also use `str` to call a method in class `str`, much like we call a function in module `math`. The main difference is that every method in class `str` requires a string as its first argument:

```
>>> str.capitalize('browning')
'Browning'
```

This is how methods are different from functions: the first argument to every string method must be a string, and the parameter is *not* described in the documentation for the method. This is because *all* string methods require a

string as the first argument, and more generally, all methods in a class require an object of that class as the first argument.

Here are two more examples, this time using the other two string methods from code, on page 117. Both of these also require a string as the first argument.

```
>>> str.center('Sonnet 43', 26)
'        Sonnet 43         '
>>> str.count('How do I love thee?  Let me count the ways.', 'the')
2
```

The first method call produces a new string that centers 'Sonnet 43' in a string of length 26, padding to the left and right with spaces. The second method call counts how many times 'the' occurs in 'How do I love thee? Let me count the ways.' (once in the word thee and once as the penultimate word in the string).

7.2 Calling Methods the Object-Oriented Way

Because every method in class str requires a string as the first argument (and, more generally, every method in any class requires an object of that class as the first argument), Python provides a shorthand form for calling a method where the object appears first, and then the method call:

```
>>> 'browning'.capitalize()
'Browning'
>>> 'Sonnet 43'.center(26)
'        Sonnet 43         '
>>> 'How do I love thee?  Let me count the ways.'.count('the')
2
```

When Python encounters one of these method calls, it translates it to the more long-winded form. We will use this shorthand form throughout the rest of the book.

The help documentation for methods uses this form. Here is the help for method lower in class str. (Notice that we can get help for a single method by prefixing it with the class it belongs to.)

```
>>> help(str.lower)
Help on method_descriptor:

lower(...)
    S.lower() -> str

    Return a copy of the string S converted to lowercase.
```

Contrast that documentation with the help for function sqrt in module math:

```
>>> help(math.sqrt)
Help on built-in function sqrt in module math:
```

```
sqrt(...)
    sqrt(x)

    Return the square root of x.
```

The help for str.lower shows that you need to prefix the call with a string value S; the help for math.sqrt doesn't show any such prefix.

Why programming languages are called "object-oriented"

The phrase *object-oriented* was introduced to describe the style of programming where the objects are the main focus: we tell objects to do things (by calling their methods) as opposed to *imperative* programming, where functions are the primary focus and we pass them objects to work with. Python allows a mixture of both styles.

The general form of a method call is as follows:

«expression».«method_name»(«arguments»)

So far, every example we've seen has a single object as the expression, but any expression can be used as long as it evaluates to the correct type. Here's an example:

```
>>> ('TTA' + 'G' * 3).count('T')
2
```

The expression ('TTA' + 'G' * 3) evaluates to the DNA sequence 'TTAGGG', and that is the object that is used in the call on method count.

Here are the steps for executing a method call. These steps are quite similar to those for executing a function call in Section 3.4, *Tracing Function Calls in the Memory Model*, on page 42.

1. Evaluate «expression»; this may be something simple, like 'Elizabeth Barrett Browning' (a poet from the 1800's), or it may be more complicated, like ('TTA' + 'G' * 3). Either way, a single object is produced, and that will be the object we are interacting with during the method call.

2. Now that we have an object, evaluate the method arguments left to right. In our DNA example, the argument is 'T'.

3. Pass the result of evaluating the initial expression as the first argument, and also pass the argument values from the previous step, into the method. In our DNA example, our code is equivalent to str.count('TTAGGG', 'T').

4. Execute the method.

When the method call finishes, it produces a value. In our DNA example, str.count('TTAGGG', 'T') returns the number of times 'T' occurs in 'TTAGGG', which is 2.

7.3 Exploring String Methods

Strings are central to programming; almost every program uses strings in some way. We'll explore some of the ways in which we can manipulate strings and, at the same time, firm up our understanding of methods.

The most commonly used string methods are listed in Table 7, *Common string methods*, on page 121. (You can find the complete list in Python's online documentation, or type help(str) into the command prompt.)

Method	Description
str.capitalize()	Return a copy of the string with the first letter capitalized and the rest lower case.
str.count(s)	Return the number of non-overlapping occurrences of s in the string.
str.endswith(end)	Return True iff the string ends with the letters in string end. This is case sensitive.
str.find(s)	Return the index of the first occurrence of s in the string, or -1 if s doesn't occur in the string. The first character is at index 0. This is case sensitive.
str.find(s, beg)	Return the index of the first occurrence of s at or after index beg in the string, or -1 if s doesn't occur in the string at or after index beg. The first character is at index 0. This is case sensitive.
str.find(s, beg, end)	Return the index of the first occurrence of s between indices beg (inclusive) and end (exclusive) in the string, or -1 if s does not occur in the string between indices beg and end. The first character is at index 0. This is case sensitive.
str.format(«expressions»)	Return a string made by substituting for placeholder fields in the string. Each field is a pair of braces ('{' and '}') with an integer in between. The expression arguments are numbered from left to right starting at 0. Each field is replaced by the value produced by evaluating the expression whose index corresponds with the integer in between the braces of the field. If an expression produces a value that isn't a string, that value is converted into a string.
str.islower()	Return True iff all characters in the string are lowercase.

Method	Description
str.isupper()	Return True iff all characters in the string are uppercase.
str.lower()	Return a copy of the string with all characters converted to lowercase.
str.replace(old, new)	Return a copy of the string with all occurrences of substring old replaced with string new.
str.split()	Return the whitespace-separated words in the string as a list. (We'll introduce type list in Section 8.1, *Storing and Accessing Data in Lists*, on page 129.)
str.startswith(beginning)	Return True iff the string starts with the letters in string beginning. This is case sensitive.
str.strip()	Return a copy of the string with leading and trailing whitespace removed.
str.strip(s)	Return a copy of the string with the characters in s removed.
str.swapcase()	Return a copy of the string with all lowercase letters capitalized and all uppercase charaters made lowercase.
str.upper()	Return a copy of the string with all characters converted to uppercase.

Table 7—Common string methods

Method calls look almost the same as function calls, except that in order to call a method we need an object of the type associated with that method. For example, let's call method startswith on string 'species':

```
>>> 'species'.startswith('a')
False
>>> 'species'.startswith('spe')
True
```

String method startswith takes a string argument and returns a bool indicating whether the string whose method was called—the one to the left of the dot—starts with the string that is given as an argument. There is also an endswith method:

```
>>> 'species'.endswith('a')
False
>>> 'species'.endswith('es')
True
```

Here is another example that uses string method swapcase to change lowercase letters to uppercase and uppercase to lowercase:

```
>>> 'Computer Science'.swapcase()
'cOMPUTER sCIENCE'
```

String method format has a complex description, but a couple of examples should clear up the confusion. Here we show that we can substitute a series of strings into a format string:

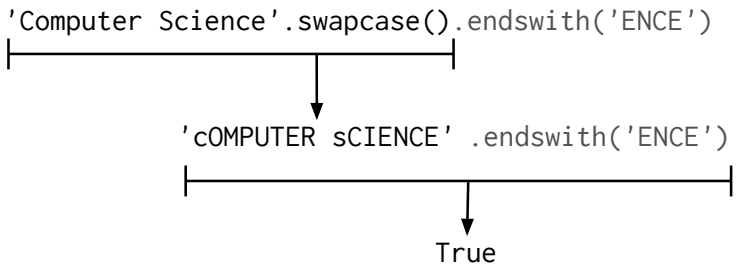
```
>>> '"{0}" is derived from "{1}"'.format('none', 'no one')
'"none" is derived from "no one"'
>>> '"{0}" is derived from the {1} "{2}"'.format('Etymology', 'Greek', 'ethos')
'"Etymology" is derived from the Greek "ethos"'
>>> '"{0}" is derived from the {2} "{1}"'.format('December', 'decem', 'Latin')
'"December" is derived from the Latin "decem"'
```

We can have any number of fields. The last example shows that we don't have to use the numbers in order.

Remember how a method call starts with an expression? Because 'Computer Science'.swapcase() is an expression, we can immediately call method endswith on the result of that expression to check whether that result has 'ENCE' as its last 4 characters:

```
>>> 'Computer Science'.swapcase().endswith('ENCE')
True
```

Here's a piture of what happens when we do this:



The call on method swapcase produces a new string, and that new string is used for the call on method endswith.

Both int and float are classes. It is possible to access the documentation for these either by calling help(int) or by calling help on a object of the class:

```
>>> help(0)
Help on int object:

class int(object)
 |  int(x[, base]) -> integer
 |
 |  Convert a string or number to an integer, if possible.  A floating
```

```
|  point argument will be truncated towards zero (this does not include a
|  string representation of a floating point number!)  When converting a
|  string, use the optional base.  It is an error to supply a base when
|  converting a non-string.
|
|  Methods defined here:
|
|  __abs__(...)
|      x.__abs__() <==> abs(x)
|
|  __add__(...)
|      x.__add__(y) <==> x+y
...
```

Most modern programming languages are structured this way: the "things" in the program are objects, and most of the code in the program consists of methods that use the data stored in those objects. Chapter 14, *Object-Oriented Programming*, on page 233 will show you how to create new kinds of objects; until then, we'll work with objects of types that are built-in to Python.

7.4 What Are Those Underscores?

Any method (or other name) beginning and ending with two underscores is considered special by Python. The help documentation for strings shows these methods, among many others:

```
|  Methods defined here:
|
|  __add__(...)
|      x.__add__(y) <==> x+y
```

These methods are typically connected with some other syntax in Python: use of that syntax will trigger a method call. For example, string method `__add__` is called when anything is added to a string:

```
>>> 'TTA' + 'GGG'
'TTAGGG'
>>> 'TTA'.__add__('GGG')
'TTAGGG'
```

Programmers almost *never* call these special methods directly. It is eye-opening to see this, and may help you understand how Python works.

Integers and floating-point numbers have similar features. Here is part of the help documentation for `int`:

```
Help on class int in module builtins:

class int(object)
...
```

```
|  Methods defined here:
|
|  __abs__(...)
|      x.__abs__() <==> abs(x)
|
|  __add__(...)
|      x.__add__(y) <==> x+y
|
|  __gt__(...)
|      x.__gt__(y) <==> x>y
```

The documentation describes when these are called. Here we show both versions of getting the absolute value of a number:

```
>>> abs(-3)
3
>>> -3 .__abs__()
3
```

We need to put a space after `-3` so that Python doesn't think we're making a floating-point number `-3.` (remember that we can leave off the trailing `0`).

Let's add two integers using this trick:

```
>>> 3 + 5
8
>>> 3 .__add__(5)
8
```

And here we compare two numbers to see whether one is bigger than the other:

```
>>> 3 > 5
False
>>> 3 .__gt__(5)
False
>>> 5 > 3
True
>>> 5 .__gt__(3)
True
```

Again, programmers don't typically do this, but it is worth knowing that Python uses methods to handle all of these operators.

Function objects, like other objects, contain double-underscore variables. For example, the documentation for each function is stored in a variable called `__doc__`:

```
>>> abs.__doc__
'abs(number) -> number\n\nReturn the absolute value of the argument.'
```

When we use built-in function print to print that __doc__ string, looks what comes out—it looks just like the output from calling built-in function help on abs:

```
>>> print(abs.__doc__)
abs(number) -> number

Return the absolute value of the argument.
>>> help(abs)
Help on built-in function abs in module builtins:

abs(...)
    abs(number) -> number

    Return the absolute value of the argument.
```

Every function object keeps track of its docstring in a special variable called __doc__.

7.5 A Methodical Review

- Classes are like modules, except that classes contain methods and modules contain functions.
- Methods are like functions, except that the first argument must be an object of the class in which the method is defined.
- Method calls in this form: 'browning'.capitalize() are shorthand for this: str.capitalize('browning').
- Methods beginning and ending with two underscores are considered special by Python, and they are triggered by particular syntax.

7.6 Exercises

1. In the Python shell, execute the following method calls:
   a. 'hello'.upper()
   b. 'Happy Birthday!'.lower()
   c. 'WeeeEEEEeeeEEEEeee'.swapcase()
   d. 'ABC123'.isupper()
   e. 'aeiouAEIOU'.count('a')
   f. 'hello'.endswith('o')
   g. 'hello'.startswith('H')
   h. 'Hello {0}'.format('Python')

i. `'Hello {0}! Hello {1}!'.format('Python', 'World')`

2. Using string method `count`, write an expression that produces the number of `o`'s in `'tomato'`.

3. Using string method `find`, write an expression that produces the index of the first occurrence of `o` in `'tomato'`.

4. Using string method `find`, write an *single* expression that produces the index of the *second* occurrence of `o` in `'tomato'`. Hint: call `find` twice.

5. Using your expression from the previous question, use it to find the second `o` in `'avocado'`. If you don't get the result you except, revise the expression and try again.

6. Using string method `replace`, write an expression that produces a string based on `'runner'` with the `n`'s replaced by `b`'s.

7. Variable `s` refers to `' yes '`. When a string method is called with `s` as its argument, the string `'yes'` is produced. Which string method was called?

8. Variable `fruit` refers to `'pineapple'`. For the following function calls, in what order are the subexpressions evaluated?

   a. `fruit.find('p', fruit.count('p'))`

   b. `fruit.count(fruit.upper().swapcase())`

   c. `fruit.replace(fruit.swapcase(), fruit.lower())`

9. Variables `season` refers to `'summer'`. Using string method `format` and the variable `season`, write an expression that produces `'I love summer!'`

10. Variables `side1`, `side2` and `side3` refer to `3`, `4`, and `5`, respectively. Using string method `format` and those three variables, write an expression that produces `'The sides have lengths 2, 3, and 4.'`

11. Using string methods, write expressions that produce the following:

    a. A copy of `'boolean'` capitalized.

    b. Find the first occurrence of `'2'` in `'C02 H20'`.

    c. Find the second occurrence of `"2"` in `'C02 H20'`.

    d. `True` if and only if `'Boolean'` begins with a lowercase.

    e. A copy of `"MoNDaY"` converted to lowercase and then capitalized.

    f. A copy of `"  Monday"` with the leading whitespace removed.

12. Complete the examples in the docstring and then write the body of the following function:

```
def total_occurrences(s1, s2, ch):
    """ (str, str, str) -> int

    Precondition: len(ch) == 1

    Return the total number of times that ch occurs in s1 and s2.

    >>> total_occurrences('colour', 'yellow', 'l')
    3
    >>> total_occurrences('red', 'blue', 'l')

    >>> total_occurrences('green', 'purple', 'b')

    """
```

CHAPTER 8

Storing Collections of Data Using Lists

Up to this point, we have seen numbers, Boolean values, strings, functions, and a few other types. Once one of these objects has been created, it can't be modified. In this chapter, you will learn how to use a Python type named list. Lists contain zero or more objects, and are used to keep track of collections of data. Unlike the other types you've learned about, lists can be modified.

8.1 Storing and Accessing Data in Lists

Table 8, *Gray whale census*, on page 129[1] shows the number of gray whales counted near the Coal Oil Point Natural Reserve in a two-week period starting on February 24, 2008.

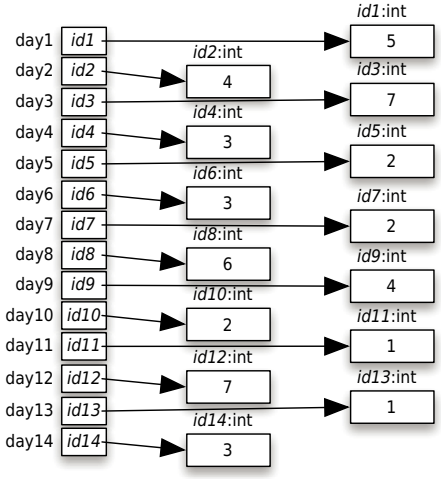
Day	Number of Whales
1	5
2	4
3	7
4	3
5	2
6	3
7	2
8	6
9	4
10	2
11	1
12	7

1. http://www.graywhalescount.org/GWC/Do_Whales_Count_files/2008CountDaily.pdf
Gray Whales Count nonprofit 501(c)(3) corporation for research and education

Day	Number of Whales
13	1
14	3

Table 8—Gray whale census

Using what we have seen so far, we would have to create fourteen variables to keep track of the number of whales counted each day:



To track an entire year's worth of observations, we would need 365 variables (366 for a leap year).

Rather than dealing with this programming nightmare, we can use a *list* to keep track of the 14 days of whale counts. That is, we can use a list to keep track of the 14 int objects that contain the counts:

```
>>> whales = [5, 4, 7, 3, 2, 3, 2, 6, 4, 2, 1, 7, 1, 3]
>>> whales
[5, 4, 7, 3, 2, 3, 2, 6, 4, 2, 1, 7, 1, 3]
```

A list is an object; like any other object, it can be assigned to a variable. Here is what happens in the memory model:

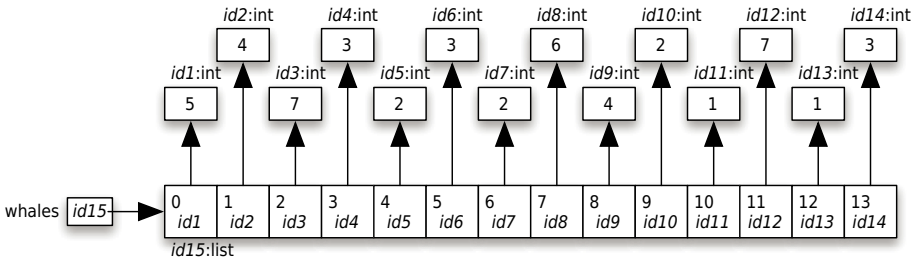


Figure 5—Using a list to represent 14 days of whale counts

The general form of a list expression is as follows:

```
[«expression1», «expression2», ... , «expressionN»]
```

In our whale count example, variable whales refers to a list with 14 *items*, also known as *elements*. The list itself is an object, but it also contains the memory addresses of 14 other objects. The memory model in Figure 5, *Using a list to represent 14 days of whale counts*, on page 131 shows whales after this assignment statement has been executed.

The items in a list are ordered, and each item has an *index* indicating its position in the list. The first item in a list is at index 0, the second at index 1, and so on. It would be more natural to use 1 as the first index, as human languages do. Python, however, uses the same convention as languages like C and Java and starts counting at zero. To refer to a particular list item, we put the index in brackets after a reference to the list (such as the name of a variable):

```
>>> whales = [5, 4, 7, 3, 2, 3, 2, 6, 4, 2, 1, 7, 1, 3]
>>> whales[0]
5
>>> whales[1]
4
>>> whales[12]
1
>>> whales[13]
3
```

We can use only those indices that are in the range from zero up to one less than the length of the list, because the list index starts at 0, not 1. In a fourteen-item list, the legal indices are 0, 1, 2, and so on, up to 13. Trying to use an out-of-range index is results in an error:

```
>>> whales = [5, 4, 7, 3, 2, 3, 2, 6, 4, 2, 1, 7, 1, 3]
>>> whales[1001]
```

```
Traceback (most recent call last):
  File "<stdin>", line 1, in ?
IndexError: list index out of range
```

Unlike most programming languages, Python also lets us index backward from the end of a list. The last item is at index -1, the one before it at index -2, and so on. Negative indices provide a way to access the last item, second last item and so on, without having to figure out the size of the list:

```
>>> whales = [5, 4, 7, 3, 2, 3, 2, 6, 4, 2, 1, 7, 1, 3]
>>> whales[-1]
3
>>> whales[-2]
1
>>> whales[-14]
5
>>> whales[-15]
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
IndexError: list index out of range
```

Since each item in a list is an object, the items can be assigned to other variables:

```
>>> whales = [5, 4, 7, 3, 2, 3, 2, 6, 4, 2, 1, 7, 1, 3]
>>> third = whales[2]
>>> print('Third day:', third)
Third day: 7
```

In Section 8.5, *Aliasing: What's in a Name?*, on page 138, you will learn that an entire list, such as the one that `whales` refers to, can be assigned to other variables and discover the effect that has.

The Empty List

In Chapter 4, *Working with Text*, on page 67, we saw the empty string, which doesn't contain any characters. There is also an *empty list*. An empty list is a list with no items in it. As with all lists, an empty list is represented using brackets:

```
>>> whales = []
```

Since an empty list has no items, trying to index an empty list results in an error:

```
>>> whales[0]
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
IndexError: list index out of range
>>> whales[-1]
```

```
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
IndexError: list index out of range
```

Lists Are Heterogeneous

Lists can contain any type of data, including integers, strings, and even other lists. Here is a list of information about the element Krypton, including its name, symbol, melting point (in degrees Celsius), and boiling point (also in degrees Celsius):

```
>>> krypton = ['Krypton', 'Kr', -157.2, -153.4]
>>> krypton[1]
'Kr'
>>> krypton[2]
-157.2
```

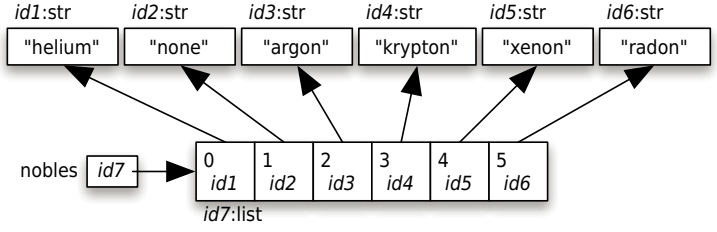
A list is usually used to contain items of the same kind, like temperatures, or dates, or grades in a course. A list can be used to aggregate related information of different kinds, as we did with krypton, but this is prone to error. Here, we need to remember which temperature comes first, and whether the name or symbol starts the list. Another common source of bugs is when you forget to include a piece of data in your list (or perhaps it was missing in your source of information); how, for example, would you keep track of similar information for iridium if you don't know the melting point? What would information you put at index 2? A better, but more advanced, way to do this is described in Chapter 14, *Object-Oriented Programming*, on page 233.

8.2 Modifying Lists

Suppose you're typing in a list of the noble gases and your fingers slip:

```
>>> nobles = ['helium', 'none', 'argon', 'krypton', 'xenon', 'radon']
```

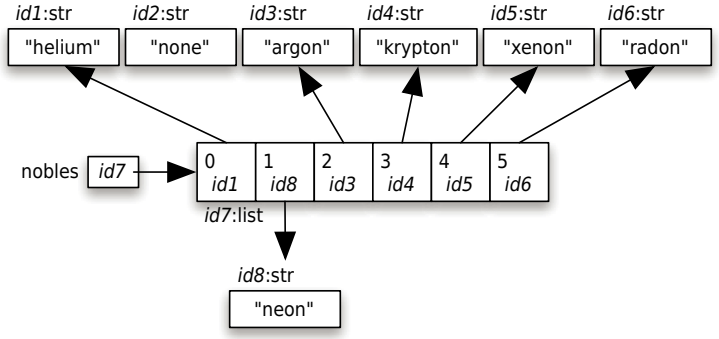
The error here is that you typed `'none'` instead of `'neon'`. Here's the memory model that was created by that assignment statement:



Rather than retyping the whole list, you can assign a new value to a specific element of the list:

```
>>> nobles[1] = 'neon'
>>> nobles
['helium', 'neon', 'argon', 'krypton', 'xenon', 'radon']
```

Here is the result after the assignment to nobles[1]:



That memory model also shows that list objects are *mutable*. That is, the contents of a list can be *mutated*.

In the code above, nobles[1] was used on the left-hand side of the assignment operator. It can also be used on the right-hand side. In general, an expression of the form L[i] (list L at index i) behaves just like a simple variable (see Section 2.4, *Variables and Computer Memory: Remembering Values*, on page 17).

If L[i] is used in an expression (such as on the right of an assignment statement), it means "Get the value referred to by the memory address at index i of list L."

On the other hand, if L[i] is on the left of an assignment statement (as in nobles[1] = 'neon'), it means "Look up the memory address at index i of list L so it can be overwritten."

In contrast to lists, numbers and strings are *immutable*. You cannot, for example, change a letter in a string. Methods that appear to do that, like upper, actually create new strings:

```
>>> name = 'Darwin'
>>> capitalized = name.upper()
>>> print(capitalized)
DARWIN
>>> print(name)
Darwin
```

Because strings are immutable, it is only possible to use an expression of the form s[i] (string s at index i) on the right-hand side of the assignment operator.

8.3 Operations on Lists

Section 3.1, *Functions that Python Provides*, on page 33 and *Operations on strings*, on page 68 introduced a few of Python's built-in functions. Some of these, such as len, can be applied to lists as well as others we haven't seen before (see Table 9, *List functions*, on page 135).

Here are some examples. The half-life of a radioactive substance is the time taken for half of it to decay. After twice this time has gone by, three quarters of the material will have decayed; after three times, seven eighths, and so on.

Here are some of the built-in functions in action working on a list of the half-lives of our plutonium isotopes:

```
>>> half_lives = [87.74, 24110.0, 6537.0, 14.4, 376000.0]
>>> len(half_lives)
5
>>> max(half_lives)
376000.0
>>> min(half_lives)
14.4
>>> sum(half_lives)
406749.14
```

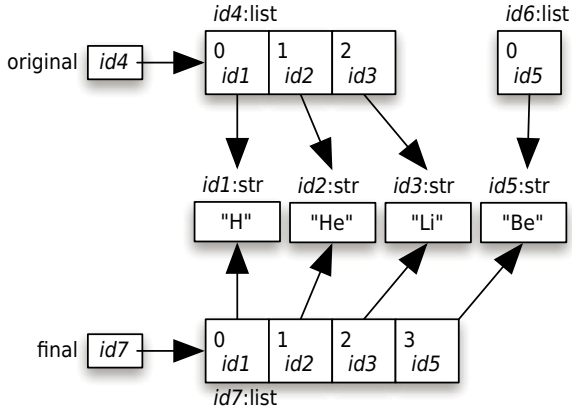
Function	Description
len(L)	Return the number of items in list L.
max(L)	Return the maximum value in list L.
min(L)	Return the minimum value in list L.
sum(L)	Return the sum of the values in list L.

Table 9—List functions

In addition to built-in functions, some of the operators that we have seen can also be applied to lists. Like strings, lists can be combined using the concatenation (+) operator:

```
>>> original = ['H', 'He', 'Li']
>>> final = original + ['Be']
>>> final
['H', 'He', 'Li', 'Be']
```

This code doesn't mutate either of the original list objects. Instead, it creates a new list whose entries refer to the items in the original lists.



A lists has a type, and Python complains if you use a value of some type in an inappropriate way. For example, an error occurs when the concatenation operator is applied to a list and a string:

```
>>> ['H', 'He', 'Li'] + 'Be'
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
TypeError: can only concatenate list (not "str") to list
```

You can also multiply a list by an integer to get a new list containing the elements from the original list repeated that number of times:

```
>>> metals = ['Fe', 'Ni']
>>> metals * 3
['Fe', 'Ni', 'Fe', 'Ni', 'Fe', 'Ni']
```

As with concatenation, the original list isn't modified; instead, a new list is created.

An operator that does modify a list is del, which stands for delete. It can be used to remove an item from a list, as follows:

```
>>> metals = ['Fe', 'Ni']
>>> del metals[0]
>>> metals
['Ni']
```

8.4 Slicing Lists

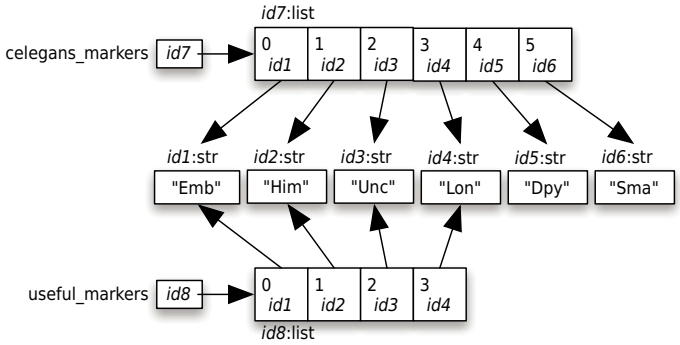
Geneticists describe *C. elegans* phenotypes (nematodes, or microscopic worms) using three-letter short-form markers. Examples include Emb (embryonic lethality), Him (High incidence of males), Unc (Uncoordinated), Dpy (dumpy: short and fat), Sma (small), and Lon (long). We can keep a list:

```
>>> celegans_phenotypes = ['Emb', 'Him', 'Unc', 'Lon', 'Dpy', 'Sma']
>>> celegans_phenotypes
['Emb', 'Him', 'Unc', 'Lon', 'Dpy', 'Sma']
```

It turns out that Dpy worms and Sma worms are difficult to distinguish from each other, so they aren't as easily differentiated in complex strains. We can produce a new list based on celegans_phenotypes, but without Dpy or Sma, by taking a *slice* of the list:

```
>>> celegans_phenotypes = ['Emb', 'Him', 'Unc', 'Lon', 'Dpy', 'Sma']
>>> useful_markers = celegans_phenotypes[0:4]
```

This creates a new list consisting of only the four distinguishable markers, which are the first four items from the list that celegans_phenotypes refers to:



The first index in the slice is the starting point. The second index is *one more than* the index of the last item we want to include. For example, the last item we wanted to include, Lon, had an index of 3, so we use 4 for the second index. More rigorously, list[i:j] is a slice of the original list from index i (inclusive) up to, but not including, index j (exclusive). Python uses this convention to be consistent with the rule that the legal indices for a list go from 0 up to one less than the list's length.

The first index can be omitted if we want to slice from the beginning of the list, and the last index can be omitted if we want to slice to the end:

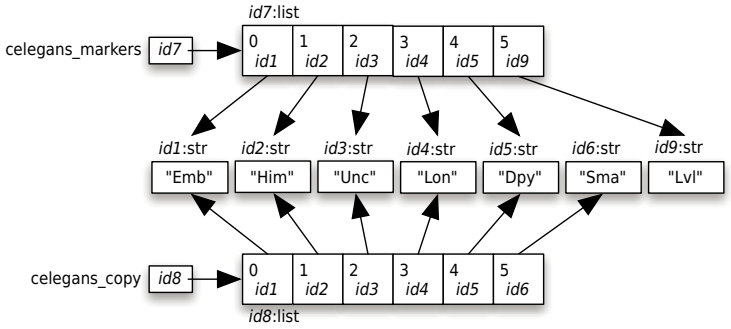
```
>>> celegans_phenotypes = ['Emb', 'Him', 'Unc', 'Lon', 'Dpy', 'Sma']
>>> celegans_phenotypes[:4]
['Emb', 'Him', 'Unc', 'Lon']
>>> celegans_phenotypes[4:]
['Dpy', 'Sma']
```

To create a copy of the entire list, omit both indices so that the "slice" runs from the start of the list to its end:

```
>>> celegans_phenotypes = ['Emb', 'Him', 'Unc', 'Lon', 'Dpy', 'Sma']
```

```
>>> celegans_copy = celegans_phenotypes[:]
>>> celegans_phenotypes[5] = 'Lvl'
>>> celegans_phenotypes
['Emb', 'Him', 'Unc', 'Lon', 'Dpy', 'Lvl']
>>> celegans_copy
['Emb', 'Him', 'Unc', 'Lon', 'Dpy', 'Sma']
```

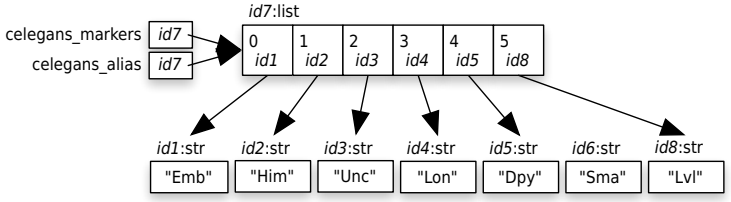
The list referred to by celegans_copy is a *clone* of the list referred to by celegans_phenotypes. The lists have the same items, but the lists themselves are different objects at different memory addresses:



In Section 8.6, *List Methods*, on page 140, you will learn about a list method that can be used to make a copy of a list.

8.5 Aliasing: What's in a Name?

An *alias* is an alternative name for something. In Python, two variables are said to be aliases when they contain the same memory address. For example, the following code creates two variables, both of which refer to a single list:



When we modify the list using one of the variables, references through the other variable show the change as well:

```
>>> celegans_phenotypes = ['Emb', 'Him', 'Unc', 'Lon', 'Dpy', 'Sma']
>>> celegans_alias = celegans_phenotypes
>>> celegans_phenotypes[5] = 'Lvl'
>>> celegans_phenotypes
['Emb', 'Him', 'Unc', 'Lon', 'Dpy', 'Lvl']
```

```
>>> celegans_alias
['Emb', 'Him', 'Unc', 'Lon', 'Dpy', 'Lvl']
```

Aliasing is one of the reasons why the notion of mutability is important. For example, if x and y refer to the same list, then any changes you make to the list through x will be "seen" by y, and vice versa. This can lead to all sorts of hard-to-find errors in which a list's value changes as if by magic, even though your program doesn't appear to assign anything to it. This can't happen with immutable values like strings. Since a string can't be changed after it has been created, it's safe to have aliases for it.

Mutable Parameters

Aliasing occurs when you use list parameters as well, since parameters are variables.

Here is a simple function that takes a list, removes its last item, and returns the list:

```
>>> def remove_last_item(L):
...     """ (list) -> list
...
...     Return list L with the last item removed.
...
...     Precondition: len(L) >= 0
...
...     >>> remove_last_item([1, 3, 2, 4])
...     [1, 3, 2]
...     """
...     del L[-1]
...     return L
...
>>>
```

In the code that follows, a list is created andstored in a variable; then that variable is passed as an argument to remove_last_item:

```
>>> celegans_markers = ['Emb', 'Him', 'Unc', 'Lon', 'Dpy', 'Lvl']
>>> remove_last_item(celegans_markers)
['Emb', 'Him', 'Unc', 'Lon', 'Dpy']
>>> celegans_markers
['Emb', 'Him', 'Unc', 'Lon', 'Dpy']
```

When the call on function remove_last_item is executed, parameter L is assigned the memory address that celegans_phenotypes contains. That makes celegans_phenotypes and L aliases. When the last item of the list that L refers to is removed, that change is "seen" by celegan_markers as well.

Since remove_last_item modifies the list parameter, the modified list doesn't actually need to be returned. You can remove the return statement:

```
>>> def remove_last_item(L):
...     """ (list) -> NoneType
...
...     Remove the last item from L.
...
...     Precondition: len(L) >= 0
...
...     >>> remove_last_item([1, 3, 2, 4])
...     """
...     del L[-1]
...
>>> celegans_markers = ['Emb', 'Him', 'Unc', 'Lon', 'Dpy', 'Lvl']
>>> remove_last_item(celegans_markers)
>>> celegans_markers
['Emb', 'Him', 'Unc', 'Lon', 'Dpy']
```

As we'll see in Section 8.6, *List Methods*, on page 140, several methods modify a list and return None, like the second version of remove_last_item.

8.6 List Methods

Lists are objects and thus have methods. Here are some of the most commonly used list methods:

Method	Description
L.append(v)	Append value v to list L.
L.clear()	Remove all items from L, making the list empty.
L.copy()	Return a copy of list L; equivalent to L[:].
L.count(v)	Return the number of occurrences of v in list L.
L.extend(v)	Append the items in v to L.
L.index(v)	Return the index of the first occurence of v in L; an error is raised if v doesn't occur in L.
L.index(v, beg)	Return the index of the first occurence of v at or after index beg in L; an error is raised if v doesn't occur in that part of L.
L.index(v, beg, end)	Return the index of the first occurence of v between indices beg (inclusive) and end (exclusive) in L; an error is raised if v doesn't occur in that part of L.
L.insert(i, v)	Insert value v at index i in list L, shifting subsequent items to make room.

Method	Description
L.pop()	Remove and return the last item of L (which must be nonempty).
L.remove(v)	Remove the first occurrence of value v from list L.
L.reverse()	Reverse the order of the values in list L.
L.sort()	Sort the values in list L in ascending order (for strings with the same lettercase, alphabetical order).
L.sort(reverse=True)	Sort the values in list L in descending order (for strings with the same lettercase, reverse alphabetical order).

Table 10—List methods

Here is a sample interaction showing how we can use list methods to construct a list containing all the colors of the rainbow:

```
>>> colors = ['red', 'orange', 'green']
>>> colors.extend(['black', 'blue'])
>>> colors
['red', 'orange', 'green', 'black', 'blue']
>>> colors.append('purple')
>>> colors
['red', 'orange', 'green', 'black', 'blue', 'purple']
>>> colors.insert(2, 'yellow')
>>> colors
['red', 'orange', 'yellow', 'green', 'black', 'blue', 'purple']
>>> colors.remove('black')
>>> colors
['red', 'orange', 'yellow', 'green', 'blue', 'purple']
```

All the methods shown above modify the list instead of creating a new list. The same is true for methods clear, reverse, sort, and pop. Of those methods, only methods pop returns a value other than None. (Method pop returns the item that was removed from the list.) In fact, the only method that returns a list is copy, which is equivalent to L[:].

Finally, a call to append isn't the same as using +. First, append appends a single value, while + expects two lists as operands. Second, append modifies the list rather than creating a new one.

Where Did My List Go?

Programmers occasionally forget that many list methods return None rather than creating and returning a new list. As a result, lists sometimes seem to disappear:

```
>>> colors = 'red orange yellow green blue purple'.split()
```

```
>>> colors
['red', 'orange', 'yellow', 'green', 'blue', 'purple']
>>> sorted_colors = colors.sort()
>>> print(sorted_colors)
None
```

In this example, colors.sort() did two things: sort the items in the list, and return value None. That's why variable sorted_colors refers to None. Variable colors, on the other hand, refers to the sorted list:

```
>>> colors = 'red orange yellow green blue purple'.split()
```

As we will discuss in Section 6.3, *Testing Your Code Semi-Automatically*, on page 112, mistakes like these can be caught by writing and running a few tests.

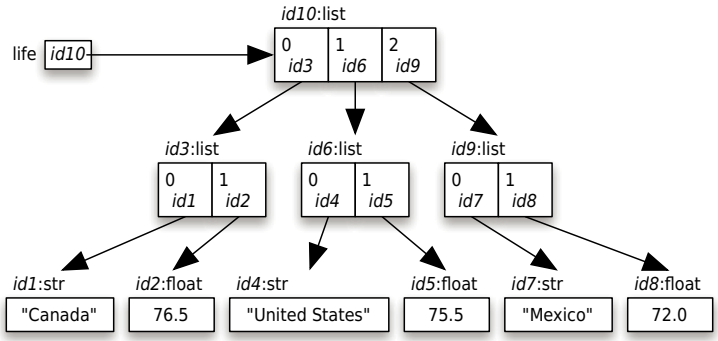
You can't change a list using methods like append and sort and assign to a new variable at the same time. You need to do that in two separate statements.

8.7 Working With a List of Lists

We said in *Lists Are Heterogeneous*, on page 133 that lists can contain any type of data. That means that they can contain other lists. A list whose items are lists is called a *nested list*. For example, the following nested list describes life expectancies in different countries:

```
>>> life = [['Canada', 76.5], ['United States', 75.5], ['Mexico', 72.0]]
```

Here is the memory model that results from execution of that assignment statement:



Notice that each item in the outer list is itself a list of two items. We use the standard indexing notation to access the items in the outer list:

```
>>> life = [['Canada', 76.5], ['United States', 75.5], ['Mexico', 72.0]]
>>> life[0]
```

```
['Canada', 76.5]
>>> life[1]
['United States', 75.5]
>>> life[2]
['Mexico', 72.0]
```

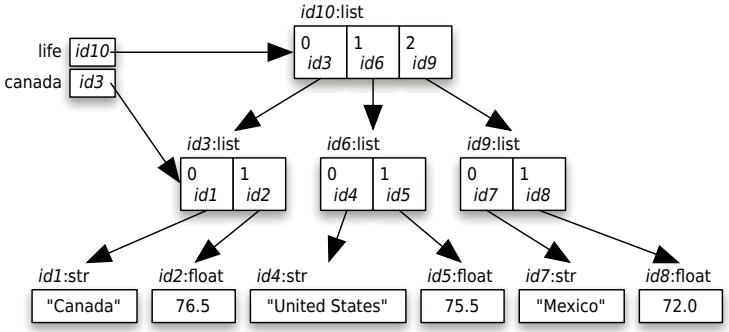
Since each of these items is also a list, we can index it again, just as we can chain together method calls or nest function calls:

```
>>> life = [['Canada', 76.5], ['United States', 75.5], ['Mexico', 72.0]]
>>> life[1]
['United States', 75.5]
>>> life[1][0]
'United States'
>>> life[1][1]
75.5
```

We can also assign sublists to variables:

```
>>> life = [['Canada', 76.5], ['United States', 75.5], ['Mexico', 72.0]]
>>> canada = life[0]
>>> canada
['Canada', 76.5]
>>> canada[0]
'Canada'
>>> canada[1]
76.5
```

Assigning a sublist to a variable creates an alias for that sublist:



As before, any change we make through the sublist reference will be seen when we access the main list, and vice versa:

```
>>> life = [['Canada', 76.5], ['United States', 75.5], ['Mexico', 72.0]]
>>> canada = life[0]
>>> canada[1] = 80.0
>>> canada
['Canada', 80.0]
```

```
>>> life
[['Canada', 80.0], ['United States', 75.5], ['Mexico', 72.0]]
```

8.8 A Summary List

In this chapter, you learned the following:

- Lists are used to keep track of zero or more objects. The objects in a list are called items or elements. Each item has a position in the list called an index ranging from zero to one less than the length of the list.
- Lists can contain any type of data, including other lists.
- Lists are mutable, which means that their contents can be modified.
- Slicing is used to create new lists that have the same values or a subset of the values of the originals.
- When two variables refer to the same object, they are called aliases.

8.9 Exercises

Here are some exercises for you to try on your own:

1. Variable kingdoms refers to the list ['Bacteria', 'Protozoa', 'Chromista', 'Plantae', 'Fungi', 'Animalia']. Using kingdoms and either slicing or indexing with positive indices, write expressions that produce the following:
   a. the first item of kingdoms
   b. the last item of kingdoms
   c. the list ['Bacteria', 'Protozoa', 'Chromista']
   d. the list ['Chromista', 'Plantae', 'Fungi']
   e. the list ['Fungi', 'Animalia']
   f. the empty list
2. Repeat the previous question using negative indices.
3. Variable appointments refers to the list ['9:00', '10:30', '14:00', '15:00', '15:30']. An appointment is scheduled for 16:30, so '16:30' needs to be added to the list.
   a. Using the list method append add '16:30' to the end of the list that appointments refers to.
   b. Instead of using append, use the + operator to add '16:30' to the end of the list that appointments refers to.

c. You used two approaches to add '16:30' to the list. Which approach modified the list and which approach created a new list?

4. Variable ids refers to list [4353, 2314, 2956, 3382, 9362, 3900]. Using list methods, do the following:

   a. remove 3382 from the list

   b. get the index of 9362

   c. insert 4499 in the list after 9362

   d. extend the list by adding [5566, 1830] to it

   e. reverse the list

   f. sort the list

5. a. Assign a list that contains the atomic numbers of the six alkaline earth metals—beryllium (4), magnesium (12), calcium (20), strontium (38), barium (56), and radium (88)—to a variable called alkaline_earth_metals.

   b. Which index contains radium's atomic number? Write the answer in two ways, one using a positive index and one using a negative index.

   c. Which function tells you how many items there are in alkaline_earth_metals?

   d. Write code that returns the highest atomic number in alkaline_earth_metals. (Hint: use one of the functions from Table 9, *List functions*, on page 135.)

6. a. Create a list of temperatures in degrees Celsius with the values 25.2, 16.8, 31.4, 23.9, 28, 22.5, and 19.6, and assign it to a variable called temps.

   b. Using one of the list methods, sort temps in ascending order.

   c. Using slicing, create two new lists, cool_temps and warm_temps, which contain the temperatures below and above 20 degrees celsius, respectively.

   d. Using list arithmetic, recombine cool_temps and warm_temps in into a new list called temps_in_celsius.

7. Complete the examples in the docstring and then write the body of the following function:

```
def same_first_last(L):
```

```
    """ (list) -> bool

    Precondition: len(L) >= 2

    Return True if and only if first item of the list is the same as the last.

    >>> same_first_last([3, 4, 2, 8, 3])
    True
    >>> same_first_last(['apple', 'banana', 'pear'])

    >>> same_first_last([4.0, 4.5])

    """
```

8. Complete the examples in the docstring and then write the body of the following function:

```
def is_longer(L1, L2):
    """ (list, list) -> bool

    Return True if and only if the length of L1 is longer than the length of L2.

    >>> is_longer([1, 2, 3], [4, 5])
    True
    >>> is_longer(['abcdef'], ['ab', 'cd', 'ef'])

    >>> is_longer(['a', 'b', 'c'], [1, 2, 3]

    """
```

9. Draw a memory model showing the effect of the following statements:

```
values = [0, 1, 2]
values[1] = values
```

10. Variable units refers to the nested list [['km', 'miles', 'league'], ['kg', 'pound', 'stone']]. Using units and either slicing or indexing with positive indices, write expressions that produce the following:

    a. the first item of units (the first inner list)

    b. the last item of units (the last inner list)

    c. the string 'km'

    d. the string 'kg'

    e. the list ['miles', 'league']

    f. the list ['kg', 'pound']

11. Repeat the previous question using negative indices.

CHAPTER 9

Repeating Code Using Loops

This chapter introduces another fundamental kind of control flow: repetition. Up to now, to execute an instruction 200 times or once for each item in a list, you would need to write that instruction 200 times. Now, you will see how to write the instruction once and use loops to repeat that code the desired number of times.

9.1 Processing Items in a List

With what you've learned so far, to print the items from a list of velocities of falling objects in metric and imperial units you would need to write a call on function print for each velocity in the list:

```
>>> velocities = [0.0, 9.81, 19.62, 29.43]
>>> print('Metric:', velocities[0], 'm/sec;',
... 'Imperial:', velocities[0] * 3.28, 'ft/sec')
Metric: 0.0 m/sec; Imperial: 0.0 ft/sec
>>> print('Metric:', velocities[1], 'm/sec;',
... 'Imperial:', velocities[1] * 3.28, 'ft/sec')
Metric: 9.81 m/sec; Imperial: 32.1768 ft/sec
>>> print('Metric:', velocities[2], 'm/sec; ',
... 'Imperial:', velocities[2] * 3.28, 'ft/sec')
Metric: 19.62 m/sec; Imperial: 64.3536 ft/sec
>>> print('Metric:', velocities[3], 'm/sec; ',
... 'Imperial:', velocities[3] * 3.28, 'ft/sec')
Metric: 29.43 m/sec; Imperial: 96.5304 ft/sec
```

The code above is used to process a list with just 4 values. Imagine processing a list with a thousand values. Lists were invented so that you wouldn't have to create 1,000 variables to store a thousand values. For the same reason, Python has a *for loop* that lets your process each element in a list in turn, without having to write one statement per element. You can use a for loop to print the velocities:

```
>>> velocities = [0.0, 9.81, 19.62, 29.43]
>>> for velocity in velocities:
...     print('Metric:', velocity, 'm/sec;',
...     'Imperial:', velocity * 3.28, 'ft/sec')
...
Metric: 0.0 m/sec; Imperial: 0.0 ft/sec
Metric: 9.81 m/sec; Imperial: 32.1768 ft/sec
Metric: 19.62 m/sec; Imperial: 64.3536 ft/sec
Metric: 29.43 m/sec; Imperial: 96.5304 ft/sec
```

The general form of a for loop over a list is as follows:

```
for «variable» in «list»:
    «block»
```

A for loop is executed as follows:

- The loop variable is assigned the first item of the list and the loop block —the *body* of the for loop—is executed.
- The loop variable is then assigned the second item of the list and the loop body is executed again.

  ...
- Finally, the loop variable is assigned the last item of the list and the loop body is executed one last time.

As we saw in Section 3.3, *Defining our Own Functions*, on page 38, a block is just a sequence of one or more statements. Each pass through the block is called an *iteration*, and at the start of each iteration, Python assigns the next item in the list to the loop variable. As with function definitions and if statements, the statements in the loop block are indented.

In the code above, before the first iteration, variable velocity is assigned velocities[0] and then the loop body is executed, before the second iteration it is assigned velocities[1] and then the loop body is executed, and so on. In this way, the program can do something with each item in turn. Table Table 11, *Looping over list velocities*, on page 148 contains the value of velocity at the start of each iteration as well as what is printed during that iteration:

Iteration	The list item that velocity refers to at the beginning of this iteration	What is printed during this iteration
1st	velocities[0]	Metric: 0.0 m/sec; Imperial: 0.0 ft/sec
2nd	velocities[1]	Metric: 9.81 m/sec; Imperial: 32.1768 ft/sec
3rd	velocities[2]	Metric: 19.62 m/sec; Imperial: 64.3536 ft/sec

Iteration	The list item that velocity refers to at the beginning of this iteration	What is printed during this iteration
4th	velocities[3]	Metric: 29.43 m/sec; Imperial: 96.5304 ft/sec

Table 11—Looping over list velocities

In the previous example, we created a new variable velocity to refer to the current item of the list inside the loop. We could equally well have used an existing variable.

If we use an existing variable, the loop still starts with the variable referring to the first element of the list. The content of the variable before the loop is lost, exactly as if we had used an assignment statement to give a new value to that variable:

```
>>> speed = 2
>>> velocities = [0.0, 9.81, 19.62, 29.43]
>>> for speed in velocities:
...     print('Metric:', speed, 'm/sec')
...
Metric: 0.0 m/sec
Metric: 9.81 m/sec
Metric: 19.62 m/sec
Metric: 29.43 m/sec
>>> print('Final:', speed)
Final: 29.43
```

The variable is left holding its last value when the loop finishes. Notice that the last print statement isn't indented, so it is not part of the for loop. It is executed, only once, after the for loop execution has finished.

9.2 Processing Characters in Strings

It is also possible to loop over the characters of a string. The general form of a for loop over a string is as follows:

```
for «variable» in «str»:
    «block»
```

As with a for loop over a list, the loop variable gets assigned a new value at the beginning of each iteration. In the case of a loop over a string, the variable is assigned a single character.

For example, we can loop over each character in a string, printing the uppercase letters:

```
>>> country = 'United States of America'
>>> for ch in country:
...     if ch.isupper():
```

```
...         print(ch)
...
U
S
A
```

In the code above, variable ch is assigned country[0] before the first iteration, country[1] before the second, and so on. The loop iterates 24 times (once per character) and the if statement block is executed 3 times (once per uppercase letter).

9.3 Looping Over a Range of Numbers

We can also loop over a range of values. This allows us to perform tasks a certain number of times and to do more sophisticated processing of lists and strings. To begin, we need to generate the range of numbers over which to iterate.

Generating Ranges of Numbers

Python built-in function range generates a sequence of integers. When passed a single argument, range(stop) produces a sequence of integers from 0 to the first integer before stop:

```
>>> range(10)
range(0, 10)
```

This is the first time that you've seen Python's range type. You can use a loop to access each number in the sequence, one at a time:

```
>>> for num in range(10):
...     print(num)
...
0
1
2
3
4
5
6
7
8
9
```

To get the numbers from the sequence all at once, we can use the built-in list function to create a list of those numbers:

```
>>> list(range(10))
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]
```

Here are some more examples:

```
>>> list(range(3))
[0, 1, 2]
>>> list(range(1))
[0]
>>> list(range(0))
[]
```

The sequence produced includes the start value and excludes the stop value, which is (deliberately) consistent with the way sequence indexing works: the expression `seq[0:5]` takes a slice of `seq` up to, but not including, the value at index `5`.

Notice that in the code above, we call `list` on the value produced by the call on `range`. Function `range` returns a range object and we create a list based on its values in order to work with it using the set of list operations and methods we are already familiar with.

The `range` function can also be passed two arguments, where the first is the start value and the second is the stop value:

```
>>> list(range(1, 5))
[1, 2, 3, 4]
>>> list(range(1, 10))
[1, 2, 3, 4, 5, 6, 7, 8, 9]
>>> list(range(5, 10))
[5, 6, 7, 8, 9]
```

By default, function `range` generates numbers that increase by 1 successively —this is called its *step size*. We can specify a different step size for `range` with an optional third parameter.

Here, we produce a list of leap years in the first half of this century:

```
>>> list(range(2000, 2050, 4))
[2000, 2004, 2008, 2012, 2016, 2020, 2024, 2028, 2032, 2036, 2040, 2044, 2048]
```

The step size can also be negative, which produces a descending sequence. When the step size is negative, the starting index should be *larger* than the stopping index:

```
>>> list(range(2050, 2000, -4))
[2050, 2046, 2042, 2038, 2034, 2030, 2026, 2022, 2018, 2014, 2010, 2006, 2002]
```

Otherwise, `range`'s result will be empty:

```
>>> list(range(2000, 2050, -4))
[]
>>> list(range(2050, 2000, 4))
```

```
[]
```

It is possible to loop over the sequence produced by a call on range. As an example, the following program calculates the sum of the integers from 1 to 100:

```
>>> total = 0
>>> for i in range(1, 101):
...     total = total + i
...
>>> total
5050
```

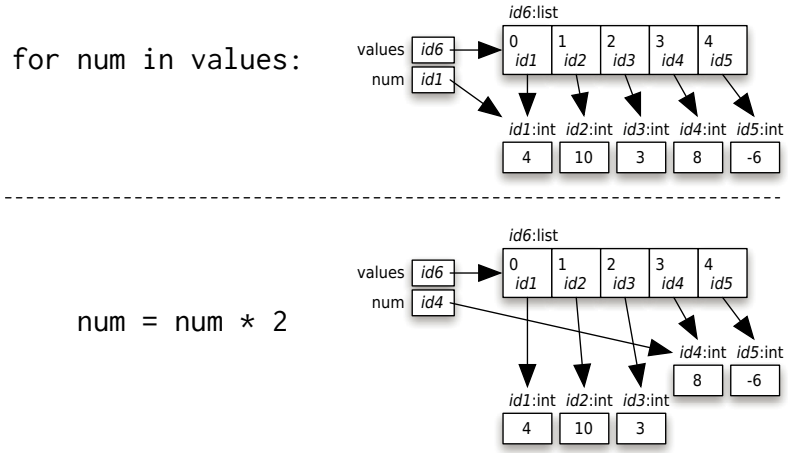
Notice that the upper bound passed to range is 101. It's one more than the greatest integer we actually want.

Processing Lists Using Indices

The loops over lists that we have written so far have been used to access list items. But what if we want to change the items of a list? For example, suppose we want to double all of the values in a list. This doesn't work:

```
>>> values = [4, 10, 3, 8, -6]
>>> for num in values:
...     num = num * 2
...
>>> values
[4, 10, 3, 8, -6]
```

Each loop iteration assigned an item in list values to variable num. Doubling that value inside the loop changes what num refers, to but *doesn't* mutate the list object:



Let's add a call on function print to show how the value that num refers to changes during each iteration:

```
>>> values = [4, 10, 3, 8, -6]
>>> for num in values:
...     num = num * 2
...     print(num)
...
8
20
6
16
-12
>>> print(values)
[4, 10, 3, 8, -6]
```

The correct approach is to loop over the indices of the list. If variable values refers to a list, then len(values) is the number of items it contains, and the expression range(len(values)) produces a sequence containing exactly the indices for values:

```
>>> values = [4, 10, 3, 8, -6]
>>> len(values)
5
>>> list(range(5))
[0, 1, 2, 3, 4]
>>> list(range(len(values)))
[0, 1, 2, 3, 4]
```

The list that values refers to has 5 items, so its indices are 0, 1, 2, 3 and 4. Rather than looping over values, you can iterate over its indices, produced by range(len(values)):

```
>>> values = [4, 10, 3, 8, -6]
>>> for i in range(len(values)):
...     print(i)
...
0
1
2
3
4
```

Notice that we named the variable i, which stands for *index*. You can use each index to access the items of the list:

```
>>> values = [4, 10, 3, 8, -6]
>>> for i in range(len(values)):
...     print(i, values[i])
...
```

```
0 4
1 10
2 3
3 8
4 -6
```

and to modify list items:

```
>>> values = [4, 10, 3, 8, -6]
>>> for i in range(len(values)):
...     values[i] = values[i] * 2
...
>>> values
[8, 20, 6, 16, -12]
```

Evaluation of the expression on the right-hand side of the assignment looks up the value at index `i` and multiplies it by two. Python then assigns that value to the item at index `i` in the list. When `i` refers to `1`, for example, `values[i]` refers to `10`, which is multiplied by `2` to produce `20`. The list item `values[1]` is then assigned `20`.

Processing Parallel Lists Using Indices

Sometimes the data from one list correponds to data from another. For example, consider these two lists:

```
>>> metals = ['Li', 'Na', 'K']
>>> weights = [6.941, 22.98976928, 39.0983]
```

The item at index 0 of `metals` has its atomic weight at index 0 of `weights`. The same is true for the items at index 1 in the two lists, and so on. These lists are *parallel lists*, because the item at index `i` of one list correponds to the item at index `i` of the other list.

We would like to print each metal and its weight. To do so, we can loop over each index of the lists, accessing the items in each:

```
>>> metals = ['Li', 'Na', 'K']
>>> weights = [6.941, 22.98976928, 39.0983]
>>> for i in range(len(metals)):
...     print(metals[i], weights[i])
...
Li 6.941
Na 22.98976928
K 39.0983
```

The code above only works when the length of `weights` is at least as long as the length of `metals`. If the length of `weights` is less than the length of `metals`, then an error would occur when trying to access an index of `weights` that doesn't

exist. For example, if metals has three items and weights only has two, the first two print function calls would be executed, but during the third function call, an error would occur when evaluating the 2nd argument.

9.4 Nesting Loops in Loops

The block of statements inside a loop can contain another loop. In this code, the inner loop is executed once for each item of list outer:

```
>>> outer = ['Li', 'Na', 'K']
>>> inner = ['F', 'Cl', 'Br']
>>> for metal in outer:
...     for halogen in inner:
...         print(metal + halogen)
...
...
LiF
LiCl
LiBr
NaF
NaCl
NaBr
KF
KCl
KBr
```

The number of times that function print is called is len(outer) * len(inner). In Table 12, *Nested loops over lists inner and outer*, on page 155, we show that for each iteration of the outer loop (that is, for each item in outer), the inner loop executes 3 times (once per item in inner):

Iteration of Outer Loop	What metal refers to	Iteration of Inner Loop	What halogen refers to	What is printed
1st	outer[0]	1st	inner[0]	LiF
		2nd	inner[1]	LiCl
		3rd	inner[2]	LiBr
2nd	outer[1]	1st	inner[0]	NaF
		2nd	inner[1]	NaCl
		3rd	inner[2]	NaBr
3rd	outer[2]	1st	inner[0]	KF
		2nd	inner[1]	KCl
		3rd	inner[2]	KBr

Table 12—Nested loops over lists inner and outer

Sometimes an inner loop uses the same list as the outer loop. An example of this is shown in a function used to generate a multiplication table. After printing the header row, we use a nested loop to print each row of the table in turn, using tabs (see Table 4, *Escape sequences*, on page 71) to make the columns line up:

```
def print_table(n):
    """ (int) -> None

    Print the multiplication table for numbers 1 through n inclusive.

    >>> print_table(5)
        1       2       3       4       5
    1   1       2       3       4       5
    2   2       4       6       8       10
    3   3       6       9       12      15
    4   4       8       12      16      20
    5   5       10      15      20      25
    """

    # The numbers to include in the table.
    numbers = list(range(1, n + 1))

    # Print the header row.
    for i in numbers:
        print('\t' + str(i), end='')

    # End the header row.
    print()

    # Print each row number and the contents of each row.
(1) for i in numbers:

(2)     print (i, end='')
(3)     for j in numbers:
(4)         print('\t' + str(i * j), end='')

        # End the current row.
(5)     print()
```

Each iteration of the outer loop prints a row. Each row consists of a row number, `n` tab-number pairs, and a newline. It's the inner loop's job to print the tabs and numbers part of the row. For `print_table(5)`, let's take a closer look at what happens during the third iteration of the outer loop:

(1) `i` is assigned `3`, the third item of `numbers`.

(2) The row number, `3`, is printed.

❸ This line of code is the inner loop header and it will be executed 5 times. Before the first iteration of the inner loop, j is assigned 1, before the second iteration it is assigned 2, and so on, until it is assigned 5 before the last iteration.

❹ Five times, this line is executed right after the previous line, using whatever value j was just assigned. The first time it prints a tab followed by 3, then a tab followed by 6, and so on, until it prints z tab followed by 15.

❺ Now that a row has been printed, the program prints a newline. This line of code occurs outside of the inner loop, so that it is only executed once per row.

Looping Over Nested Lists

In addition to looping over lists of numbers, strings and Booleans, we can also loop over lists of lists. Here is an example of a loop over an outer list. The loop variable, which we've named inner_list, is assigned an item of nested list elements at the beginning of each iteration:

```
>>> elements = [['Li', 'Na', 'K'], ['F', 'Cl', 'Br']]
>>> for inner_list in elements:
...     print(inner_list)
...
['Li', 'Na', 'K']
['F', 'Cl', 'Br']
```

To access each string in the inner lists, you can loop over the outer list and then over each inner list, using a nested loop. Here, we print every string in every inner list:

```
>>> elements = [['Li', 'Na', 'K'], ['F', 'Cl', 'Br']]
>>> for inner_list in elements:
...     for item in inner_list:
...         print(item)
...
Li
Na
K
F
Cl
Br
```

In the code above, the outer loop variable, inner_list, refers to a list of strings and the inner loop variable, item, refers to a string from that list.

When you have a nested list, and you want to do something with every item in the inner lists, you need to use a nested loop.

Looping Over Ragged Lists

Nothing says that nested lists have to be the same length:

```
>>> info = [['Isaac Newton', 1643, 1727],
...         ['Charles Darwin', 1809, 1882],
...         ['Alan Turing', 1912, 1954, 'alan@bletchley.uk']]
>>> for item in info:
...     print(len(item))
...
3
3
4
```

Nested lists with inner lists of varying lengths are called *ragged lists*. Ragged lists can be tricky to process if the data isn't uniform; for example, trying to assemble a list of email addresses for data where some addresses are missing requires careful thought.

Ragged data does arise normally. For example, if a record is made each day of the time at which a person smokes a cigarette, each day will have a different number of entries:

```
>>> smoking_times_by_day = [["9:02", "10:17", "13:52", "18:23", "21:31"],
...                         ["8:45", "12:44", "14:52", "22:17"],
...                         ["8:55", "11:11", "12:34", "13:46",
...                          "15:52", "17:08", "21:15"],
...                         ["9:15", "11:44", "16:28"],
...                         ["10:01", "13:33", "16:45", "19:00"],
...                         ["9:34", "11:16", "15:52", "20:37"],
...                         ["9:01", "12:24", "18:51", "23:13"]]
>>> for day in smoking_times_by_day:
...     for smoking_time in day:
...         print(smoking_time, end=' ')
...     print()
...
9:02 10:17 13:52 18:23 21:31
8:45 12:44 14:52 22:17
8:55 11:11 12:34 13:46 15:52 17:08 21:15
9:15 11:44 16:28
10:01 13:33 16:45 19:00
9:34 11:16 15:52 20:37
9:01 12:24 18:51 23:13
```

The inner loop iterates over the items of `day`, and the length of that list varies.

9.5 Looping until a Condition Is Reached

`for` loops are useful only if you know how many iterations of the loop you need. In some situations, it is not known in advance how many loop iterations to

execute. In a game program, for example, you can't know whether a player is going to want to play again or quit. In these situations, we use a while loop. The general form of a while loop is as follows:

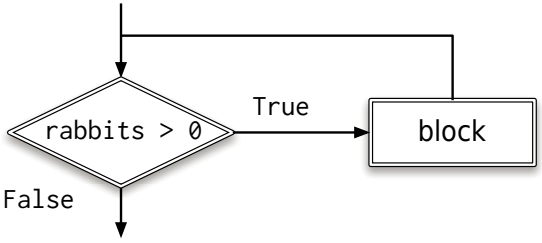
```
while «expression»:
    «block»
```

The while loop expression is sometimes called the *loop condition*, just like the condition of an if statement. When Python executes a while loop, it evaluates the expression. If that expression is false, Python skips the it evaluates the expression. If that expression is false, that is the end of execution of the loop. If the expression is true, on the other hand, Python executes the loop body once and then goes back to the top of the loop and reevaluates the expression. If it's still true, the loop body is executed again. This is repeated—expression, body, expression, body—until the expression is false, at which point Python stops executing the loop.

Here's an example:

```
>>> rabbits = 3
>>> while rabbits > 0:
...     print(rabbits)
...     rabbits = rabbits - 1
...
3
2
1
```

Notice that this loop did *not* print 0. When the number of rabbits reaches zero, the loop expression evaluates to False, so the body isn't executed. Here's a flow chart for this code:



As a more useful example, we can calculate the growth of a bacterial colony using a simple exponential growth model, which is essentially a calculation of compound interest:

$P(t + 1) = P(t) + rP(t)$

In this formula, *P(t)* is the population size at time *t*, and *r* is the growth rate. Let's see how long it takes the bacteria to double their numbers, using this program:

```
time = 0
population = 1000   # 1000 bacteria to start with
growth_rate = 0.21 # 21% growth per minute
while population < 2000:
    population = population + growth_rate * population
    print(population)
    time = time + 1
print("It took", time, "minutes for the bacteria to double.")
print("The final population was", population,"bacteria.")
```

Because variable time was updated in the loop body, its value after the loop was the time of the last iteration, which is exactly what we want. Running this program gives us the answer we were looking for:

```
1210.0
1464.1
1771.561
2143.5888099999997
It took 4 minutes for the bacteria to double.
The final population was 2143.5888099999997 bacteria.
```

Infinite Loops

The preceding example used population < 2000 as a loop condition so that the loop stopped when the population reached double its initial size *or more*. What would happen if we stopped only when the population was *exactly* double its initial size?

```
# Use multi-valued assignment to set up controls.
time, population, growth_rate = 0, 1000, 0.21

# Don't stop until we're exactly double original size.
while population != 2000:
    population = population + growth_rate * population
    print(population)
    time = time + 1
print("It took", time, "minutes for the bacteria to double.")
```

Here is this program's output:

```
1210.0
1464.1
1771.561
2143.5888099999997
2593.7424601
...3,680 lines or so later...
```

```
inf
inf
inf
...and so on forever...
```

Whoops—since the population is never exactly 2,000 bacteria, the loop never stops. The first set of dots represents more than 3,000 values, each 21 percent larger than the one before. Eventually, these values are too large for the computer to represent, so it displays inf (or on some computers 1.#INF), which is its way of saying "effectively infinity".

A loop like this one is called an *infinite loop*, because the computer will execute it forever (or until you kill your program, whichever comes first). In IDLE, you kill your program by selecting Restart Shell from the Shell menu, and from the command-line shell, you can kill it by pressing Ctrl-C. Infinite loops are a common kind of bug; the usual symptoms include printing the same value over and over again, or *hanging* (doing nothing at all).

9.6 Repetition Based On User Input

We can use the input function in a loop to make the chemical formula translation example from Section 5.2, *Choosing Which Statements to Execute*, on page 89 interactive. We will ask the user to enter a chemical formula, and our program, which is saved in a file named formulas.py, will print its name. This should continue until the user types quit:

```
text = ""
while text != "quit":
    text = input("Please enter a chemical formula (or 'quit' to exit): ")
    if text == "quit":
        print("…exiting program")
    elif text == "H2O":
        print("Water")
    elif text == "NH3":
        print("Ammonia")
    elif text == "CH4":
        print("Methane")
    else:
        print("Unknown compound")
```

Since the loop condition checks the value of text, we have to assign it a value before the loop begins. Now we can run the program in formulas.py and it will exit whenever the user types quit:

```
Please enter a chemical formula (or 'quit' to exit): CH4
Methane
Please enter a chemical formula (or 'quit' to exit): H2O
Water
```

```
Please enter a chemical formula (or 'quit' to exit): quit
…exiting program
```

The loop could execute 0, 1 or many times, depending on the user input.

9.7 Controlling Loops Using break and continue

As a rule, for and while loops execute all the statements in their body on each iteration. However, it is sometimes handy to be able to break that rule. Python provides two ways of controlling the iteration of a loop: break, which terminates execution of the loop immediately, and continue, which skips ahead to the next iteration.

The break Statement

In Section 9.6, *Repetition Based On User Input*, on page 161, we showed a program that continually read input from a user until they typed quit. Here is a program that accomplishes the same task, but but this one uses break to terminate execution of the loop when the user types quit:

```
while True:
    text = input("Please enter a chemical formula (or 'quit' to exit): ")
    if text == "quit":
        print("…exiting program")
        break
    elif text == "H2O":
        print("Water")
    elif text == "NH3":
        print("Ammonia")
    elif text == "CH4":
        print("Methane")
    else:
        print("Unknown compound")
```

The loop condition is strange: it evaluates to True, so this looks like an infinite loop. However, when the user types quit, the first condition, text == "quit", evaluates to True. The print("...exiting program") statement is executed, and then the break statement, which causes the loop to terminate.

As a style point, we are somewhat allergic to loops that are written like this. We find that a loop with an explicit condition is easier to understand.

Sometimes a loop's task is finished before its final iteration. Using what you have seen so far, though, the loop still has to finish iterating. For example, let's write some code to find the index of the first digit in string 'C3H7'. The digit 3 is at index 1 in this string. Using a for loop, we would have to write something like this:

```
>>> s = 'C3H7'
>>> digit_index = -1 # This will be -1 until we find a digit.
>>> for i in range(len(s)):
...     # If we haven't found a digit, and s[i] is a digit
...     if digit_index == -1 and s[i].isdigit():
...         digit_index = i
...
>>> digit_index
1
```

Here we use the variable digit_index to represent the index of the first digit in the string. It initially refers to -1, but when a digit is found, the digit's index, i, is assigned to digit_index. If the string doesn't contain any digits, then digit_index remains -1 throughout execution of the loop.

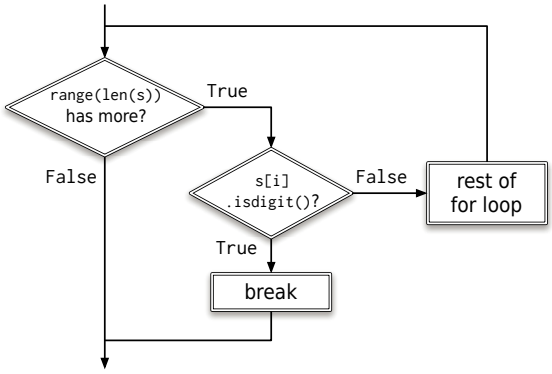
Once digit_index has been assigned a value, it is never again equal to -1, so the if condition will not evaluate to True. Even though the job of the loop is done, the loop continues to iterate until the end of the string is reached.

To fix this, you can terminate the loop early using a break statement, which jumps out of the loop body immediately:

```
>>> s = 'C3H7'
>>> digit_index = -1 # This will be -1 until we find a digit.
>>> for i in range(len(s)):
...     # If we find a digit
...     if s[i].isdigit():
...         digit_index = i
...         break  # This exits the loop.
...
>>> digit_index
1
```

Notice that because the loop terminates early, we were able to simplify the if statement condition. As soon as digit_index is assigned a new value, the loop terminates, so it isn't necessary to check whether digit_index refers to -1. That check only existed to prevent digit_index from being assigned the index of a subsequent digit in the string.

Here's a flow chart for this code:



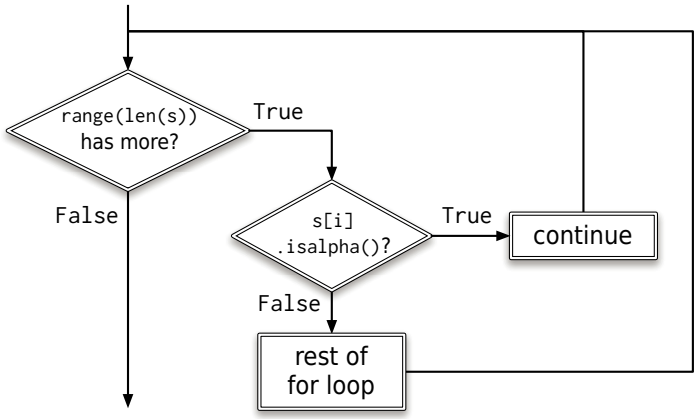
One more thing about break: it terminates only the *innermost* loop in which it's contained. This means that in a nested loop, a break statement inside the inner loop will terminate only the inner loop, not both loops.

The continue Statement

Another way to bend the rules for iteration is to use the continue statement, which causes Python to skip immediately ahead to the next iteration of a loop. Here, we add up all the digits in a string, and we also count how many digits there are. Whenever a non-digit is encountered, we use continue to skip the rest of the loop body and go back to the top of the loop in order to start the next iteration.

```
>>> s = 'C3H7'
>>> total = 0 # The sum of the digits seen so far.
>>> count = 0 # The number of digits seen so far.
>>> for i in range(len(s)):
...     if s[i].isalpha():
...         continue
...     total = total + int(s[i])
...     count = count + 1
...
>>> total
10
>>> count
2
```

When continue is executed, it *immediately* begins the next iteration of the loop. All statements in the loop body that appear after it are skipped, so we only execute the assignments to total and count when s[i] isn't a letter. Here's a flow chart for this code:



Using continue is one way to skip alphabetic characters, but this can also be accomplished by using if statements. In the previous code, continue prevents the variables from being modified; in other words, if the character isn't alphabetic, it should be processed.

The form of the previous sentence matches that of an if statement, and the updated code is as follows:

```
>>> s = 'C3H7'
>>> total = 0
>>> count = 0
>>> for i in range(len(s)):
...     if not s[i].isalpha():
...         total = total + int(s[i])
...         count = count + 1
...
>>> total
10
>>> count
2
```

This new version is easier to read than the first one. Most of the time, it is better to rewrite the code to avoid continue; almost always, the code ends up being more readable.

A Warning About break And continue

break and continue have their places but should be used sparingly since they can make programs harder to understand. When people see while and for loops in programs, their first assumption is that the whole body will be executed every time, in other words, that the body can be treated as a single "super statement" when trying to understand the program. If the loop contains break

or continue, though, that assumption is false. Sometimes, only part of the statement body will be executed, which means the reader has to keep two scenarios in mind.

There are always alternatives: well-chosen loop conditions (as in Section 9.6, *Repetition Based On User Input*, on page 161) can replace break, and if statements can be used to skip statements instead of continue. It is up to the programmer to decide which option makes the program clearer and which makes it more complicated. As we said in Section 2.7, *Describing Code*, on page 27, programs are written for human beings; taking a few moments to make your code as clear as possible, or to make clarity a habit, will pay dividends for the lifetime of the program.

Now that code is getting pretty complicated, it's even more important to write comments describing the purpose of each tricky block of statements.

9.8 Repeating What You've Learned

In this chapter, you learned the following:

- Repeating a block is a fundamental way to control a program's behavior. A for loop can be used to iterate over the items of a list, over the characters of a string, and over a sequence of integers generated by the built-in function range.
- The most general kind of repetition is the while loop, which continues executing as long as some specified Boolean condition is true. However, the condition is tested only at the beginning of each iteration. If that condition is never false, the loop will be executed forever.
- The break and continue statements can be used to change the way loops execute.
- Control structures like loops and conditionals can be nested inside one another to any desired depth.

9.9 Exercises

Here are some exercises for you to try on your own:

1. Write a for loop to print all the values in list celegans_phenotypes from Section 8.4, *Slicing Lists*, on page 136, one per line. celegans_phenotypes refers to ['Emb', 'Him', 'Unc', 'Lon', 'Dpy', 'Sma'].

2. Write a for loop to print all the values in list half_lives from Section 8.3, *Operations on Lists*, on page 135, all on a single line. half_lives refers to [87.74, 24110.0, 6537.0, 14.4, 376000.0].

3. Write a for loop to add 1 to all the values from whales from Section 8.1, *Storing and Accessing Data in Lists*, on page 129 and store the converted values in a new list more_whales. The list whales shouldn't be modified. whales refers to [5, 4, 7, 3, 2, 3, 2, 6, 4, 2, 1, 7, 1, 3].

4. a. Create a nested list where each element of the outer list contains the atomic number and atomic weight for an alkaline earth metal. The values are beryllium (4 and 9.012), magnesium (12 and 24.305), calcium (20 and 40.078), strontium (38 and 87.62), barium (56 and 137.327), and radium (88 and 226). Assign the list to a variable alkaline_earth_metals.

   b. Write a for loop to print all the values in alkaline_earth_metals, with the atomic number and atomic weight for each alkaline earth metal on a different line.

   c. Write a for loop to create a new list called number_and_weight that contains the elements of alkaline_earth_metals in the same order but not nested.

5. The following function doesn't have a docstring or comments. Write enough of both to make it easy for another programmer to understand what the function does, and how, and then compare your solution with those of at least two other people. How similar are they? Why do they differ?

```
def mystery_function(values):
    result = []
    for sublist in values:
        result.append([sublist[0]])
        for i in sublist[1:]:
            result[-1].insert(0, i)

    return result
```

6. Consider the following statement, which creates a list of populations of countries in eastern Asia (China, DPR Korea, Hong Kong, Mongolia, Republic of Korea, and Taiwan), in millions: country_populations = [1295, 23, 7, 3, 47, 21]. Write a for loop that adds up all the values and stores them in variable total. (Hint: give total an initial value of zero, and, inside the loop body, add the population of the current country to total.)

7. You are given two lists, rat_1 and rat_2, that contain the daily weights of two rats over a period of ten days. Write statements to do the following:

   a. If the weight of Rat 1 is greater than that of Rat 2 on day 1, print "Rat 1 weighed more than Rat 2 on Day 1."; otherwise, print "Rat 1 weighed less than Rat 2 on Day 1."

b. If Rat 1 weighed more than Rat 2 on day 1 and if Rat 1 weighs more than Rat 2 on the last day, print "Rat 1 remained heavier than Rat 2."; otherwise, print "Rat 2 became heavier than Rat 1."
c. If your solution to the previous question used nested if statements, then do it without nesting, or vice versa.

8. Print the numbers in the range 33 to 49 (inclusive).

9. Print the numbers from 1 to 10 in descending order, all on one line.

10. Sum numbers in the range 2 to 22 using a loop to find the total, and then calculate the average.

11. Consider this code:

```
def remove_neg(num_list):
    """(list of number) -> None

    Remove the negative numbers from the list num_list.

    >>> numbers = [-5, 1, -3, 2]
    >>> remove_reg(numbers)
    >>> numbers
    [1, 2]
    """

    for item in num_list:
        if item < 0:
            num_list.remove(item)
```

When remove_negs([1, 2, 3, -3, 6, -1, -3, 1]) is executed, it produces [1, 2, 3, 6, -3, 1]. The for loop traverses the elements of the list, and when a negative value (like -3 at position 3) is reached, it is removed, shifting the subsequent values one position earlier in the list (so 6 moves into position 3). The loop then continues on to process the next item, skipping over the value that moved into the removed item's position. If there are two negative numbers in a row (like -1 and -3), then the second one won't be removed. Rewrite the code to avoid this problem.

12. Using nested for loops, print a right triangle of the character *T* on the screen where the triangle is one character wide at its narrowest point and seven characters wide at its widest point:

```
T
TT
TTT
TTTT
TTTTT
TTTTTT
```

```
TTTTTTT
```

13. Using nested `for` loops, print the triangle described in the previous question with its hypotenuse on the left side:

```
      T
     TT
    TTT
   TTTT
  TTTTT
 TTTTTT
TTTTTTT
```

14. Redo the previous two questions using `while` loops instead of `for` loops.

15. The variables `rat_1_weight` and `rat_2_weight` store the weights of two rats at the beginning of an experiment. The variables `rat_1_rate` and `rat_2_rate` are the rate that the rats' weights are expected to increase each week (for example, 4 percent per week).

    a. Using a `while` loop, calculate how many weeks it would take for the weight of the first rat to become 25 percent heavier than it was originally.

    b. Assume that the two rats have the same initial weight, but rat 1 is expected to gain weight at a faster rate than rat 2. Using a `while` loop, calculate how many weeks it would take for rat 1 to be 10 percent heavier than rat 2.

CHAPTER 10

Reading and Writing Files

This is a beta book; this chapter is not yet complete.

CHAPTER 11

Storing Data Using Other Collection Types

In Chapter 8, *Storing Collections of Data Using Lists*, on page 129, you learned how to store collections of data using lists. In this chapter, you will learn about three other kinds of collections: sets, tuples and dictionaries. With four different options for storing your collections of data, you will be able to pick the one that best matches your problem in order to keep your code as simple and efficient as possible.

11.1 Storing Data Using Sets

A *set* is an unordered collection of distinct items. *Unordered* means that items aren't stored in any particular order. Something is either in the set or not, but there's no notion of it being the first, second, or last item. *Distinct* means that any item appears in a set at most once; in other words, there are no duplicates.

Python has a type set that allows us to store mutable collections of unordered, distinct items. Here we create a set containing the vowels:

```
>>> vowels = {'a', 'e', 'i', 'o', 'u'}
>>> vowels
{'a', 'u', 'o', 'i', 'e'}
```

It looks much like a list, except that sets use braces instead of brackets. Notice that the set is unordered. Python does some mathematical tricks behind the scenes to make accessing the items very fast, and one of the side effects of this is that the items aren't in any particular order.

Here we show that each item is distinct; duplicates are ignored:

```
>>> vowels = {'a', 'e', 'a', 'a', 'i', 'o', 'u', 'u'}
>>> vowels
{'u', 'o', 'i', 'e', 'a'}
```

Even though there were three 'a's and two 'u's when we created the set, only one of each was kept. Python considers the two sets to be equal:

```
>>> {'a', 'e', 'i', 'o', 'u'} == {'a', 'e', 'a', 'a', 'i', 'o', 'u', 'u'}
True
```

The reason they are equal is that they contain the same items. Again, order doesn't matter.

Variable vowels refers to an object of type set:

```
>>> type(vowels)
<class 'set'>
>>> type({1, 2, 3})
<class 'set'>
```

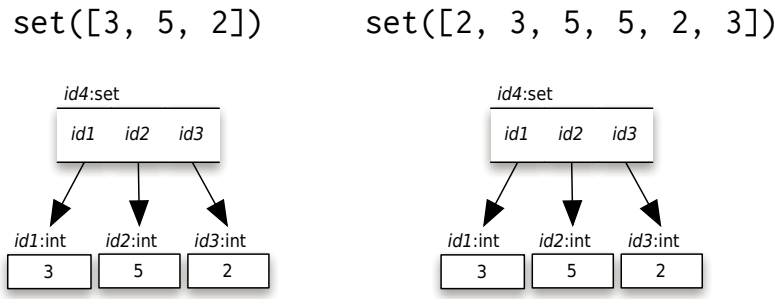
In order to create an empty set, you need to call the set function with no arguments:

```
>>> set()
set()
>>> type(set())
<class 'set'>
```

The set function expects either no arguments (to create an empty set) or a single argument that is a collection of values. We can, for example, create a set from a list:

```
>>> set([2, 3, 2, 5])
{2, 3, 5}
```

Because duplicates aren't allowed, only one of the 2's appears in the set:



The set function expects at most one argument. You can't pass several values as separate arguments:

```
>>> set(2, 3, 5)
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
```

```
TypeError: set expected at most 1 arguments, got 3
```

In addition to lists, there are a couple of other types that can be used as arguments to the set function. One is a set:

```
>>> vowels = {'a', 'e', 'a', 'a', 'i', 'o', 'u', 'u'}
>>> set(vowels)
{'o', 'u', 'e', 'a', 'i'}
>>> set({5, 3, 1})
{1, 3, 5}
```

Another such type is range from *Generating Ranges of Numbers*, on page 150. In the following code a set is created with the values 0 to 4 inclusive:

```
>>> set(range(5))
{0, 1, 2, 3, 4}
```

In Section 11.2, *Storing Data Using Tuples*, on page 178, you will learn about type tuple, another type of sequence, that can also be used as an argument to the set function.

Set Operations

In mathematics, set operations include union, intersection, add, and remove. In Python, these are implemented as methods (for a complete list, see Table 13, *Set operations*, on page 175).

Method	Description
S.add(v)	Adds item v to a set S. This has no effect if v is already in S.
S.clear()	Removes all items from set S.
S.difference(other)	Return a set with items that occur in set S, but not in set other.
S.intersection(other)	Return a set with items that occur in both sets S and other.
S.issubset(other)	Return True if and only if all of set S's items are also in set other.
S.issuperset(other)	Return True if and only if set S contains all of set other's items.
S.remove(v)	Remove item v from set S.
S.symmetric_difference(other)	Return a set with items that are in exactly one of set S and other—any items that are in both sets are *not* included in the result.

Method	Description
S.union(other)	Return a set with items that are in either set S or other (or both).

Table 13—Set operations

Sets are mutable, which means you can change what is in a set object. Methods add, remove, and clear all modify what is in a set. 'y' is sometimes considered to be a vowel; here we add it to our set of vowels:

```
>>> vowels = {'a', 'e', 'i', 'o', 'u'}
>>> vowels
{'o', 'u', 'a', 'e', 'i'}
>>> vowels.add('y')
>>> vowels
{'u', 'y', 'e', 'a', 'o', 'i'}
```

Other methods, such as intersection and union, return new sets based on their arguments.

Here we show all of these methods in action:

```
>>> ten = set(range(10))
>>> lows = {0, 1, 2, 3, 4}
>>> odds = {1, 3, 5, 7, 9}
>>> lows.add(9)
>>> lows
{0, 1, 2, 3, 4, 9}
>>> lows.difference(odds)
{0, 2, 4}
>>> lows.intersection(odds)
{1, 3, 9}
>>> lows.issubset(ten)
True
>>> lows.issuperset(odds)
False
>>> lows.remove(0)
>>> lows
{1, 2, 3, 4, 9}
>>> lows.symmetric_difference(odds)
{2, 4, 5, 7}
>>> lows.union(odds)
{1, 2, 3, 4, 5, 7, 9}
>>> lows.clear()
>>> lows
set()
```

Many of the tasks performed by methods can also be accomplished using operators. If acids and bases are two sets, for example, then acids | bases creates a new set containing their union (that is, all the elements from both acids and

bases), while acids <= bases tests whether all the values in acids are also in bases. Some of the operators that sets support are listed in Table 16, *Features of Python Collections*, on page 192.

Method Call	Operator
set1.difference(set2)	set1 - set2
set1.intersection(set2)	set1 & set2
set1.issubset(set2)	set1 <= set2
set1.issuperset(set2)	set1 >= set2
set1.union(set2)	set1 | set2
set1.symmetric_difference(set2)	set1 ^ set2

Table 14—Set operators

The following code shows the set operations in action:

```
>>> lows = set([0, 1, 2, 3, 4])
>>> odds = set([1, 3, 5, 7, 9])
>>> lows - odds            # Equivalent to lows.difference(odds)
{0, 2, 4}
>>> lows & odds            # Equivalent to lows.intersection(odds)
{1, 3}
>>> lows <= odds           # Equivalent to lows.issubset(odds)
False
>>> lows >= odds           # Equivalent to lows.issuperset(odds)
False
>>> lows | odds            # Equivalent to lows.union(odds)
{0, 1, 2, 3, 4, 5, 7, 9}
>>> lows ^ odds            # Equivalent to lows.symmetric_difference(odds)
{0, 2, 4, 5, 7, 9}
```

Arctic Birds

To see how sets are used, suppose you have a file used to record observations of birds in the Canadian Arctic and you want to know which species have been observed. The observations file, observations.txt, has one species per line:

```
canada goose
canada goose
long-tailed jaeger
canada goose
snow goose
canada goose
long-tailed jaeger
canada goose
northern fulmar
```

The program below reads each line of the file, strips off the leading and trailing whitespace, and adds the species on that line to the set:

```
>>> observations_file = open('observations.txt')
>>> birds_observed = set()
>>> for line in observations_file:
...     bird = line.strip()
...     birds_observed.add(bird)
...
>>> birds_observed
{'long-tailed jaeger', 'canada goose', 'northern fulmar', 'snow goose'}
```

The resulting set contains four species. Since sets don't contain duplicates, calling the `add` method with a species already in the set had no effect.

You can loop over the values in a set. In the code below, a for loop is used to print each species:

```
>>> for species in birds_observed:
...     print(species)
...
long-tailed jaeger
canada goose
northern fulmar
snow goose
```

Looping over a set works exactly like a loop over a list, except that the order in which items are encountered is arbitrary: there is no guarantee that they will come out in the order in which they were added, in alphabetical order, in order by length, or anything else.

11.2 Storing Data Using Tuples

Lists aren't the only kind of ordered sequence in Python. You've already learned about one of the others: strings (see Chapter 4, *Working with Text*, on page 67). Formally, a string is an immutable sequence of characters. The characters in a string are ordered and a string can be indexed and sliced like a list to create new strings:

```
>>> rock = 'anthracite'
>>> rock[9]
'e'
>>> rock[0:3]
'ant'
>>> rock[-5:]
'acite'
>>> for character in rock[:5]:
...     print(character)
...
a
```

```
n
t
h
r
```

Python also has an immutable sequence type called a *tuple*. Tuples are written using parentheses instead of brackets; like strings and lists, they can be subscripted, sliced, and looped over:

```
>>> bases = ('A', 'C', 'G', 'T')
>>> for base in bases:
...     print(base)
...
A
C
G
T
```

There is one small catch: although `()` represents the empty tuple, a tuple with one element is *not* written as `(x)` but instead as `(x,)` (with a trailing comma). This has to be done to avoid ambiguity. If the trailing comma weren't required, `(5 + 3)` could mean either `8` (under the normal rules of arithmetic) or the tuple containing only the value `8`:

```
>>> (8)
8
>>> type((8))
<class 'int'>
>>> (8,)
(8,)
>>> type((8,))
<class 'tuple'>
>>> (5 + 3)
8
>>> (5 + 3,)
(8,)
```

Unlike lists, once a tuple is created, it cannot be mutated:

```
>>> life = (['Canada', 76.5], ['United States', 75.5], ['Mexico', 72.0])
>>> life[0] = life[1]
Traceback (most recent call last):
  File "<stdin>", line 1, in ?
TypeError: object does not support item assignment
```

However, the objects inside it *can* still be mutated:

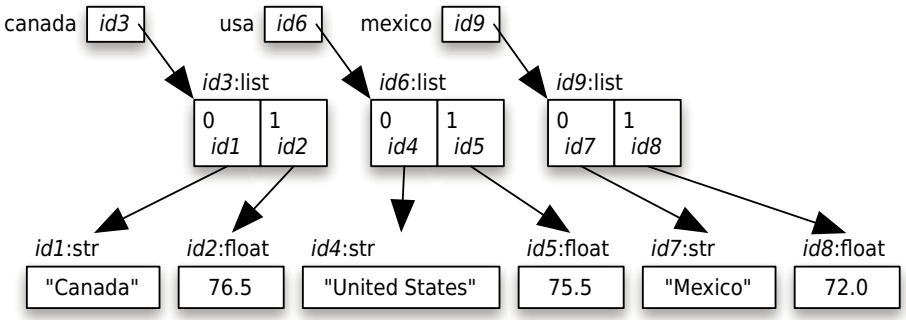
```
>>> life = (['Canada', 76.5], ['United States', 75.5], ['Mexico', 72.0])
>>> life[0][1] = 80.0
>>> life
(['Canada', 80.0], ['United States', 75.5], ['Mexico', 72.0])
```

Rather than saying that a tuple cannot change, it is more accurate to say this: "The references contained in a tuple cannot be changed after the tuple has been created, though the objects referred to may themselves be mutated."

Here is an example that explores what is mutable and what isn't. We'll build the same tuple as in the previous example, but we'll do it in steps. First, let's create three lists:

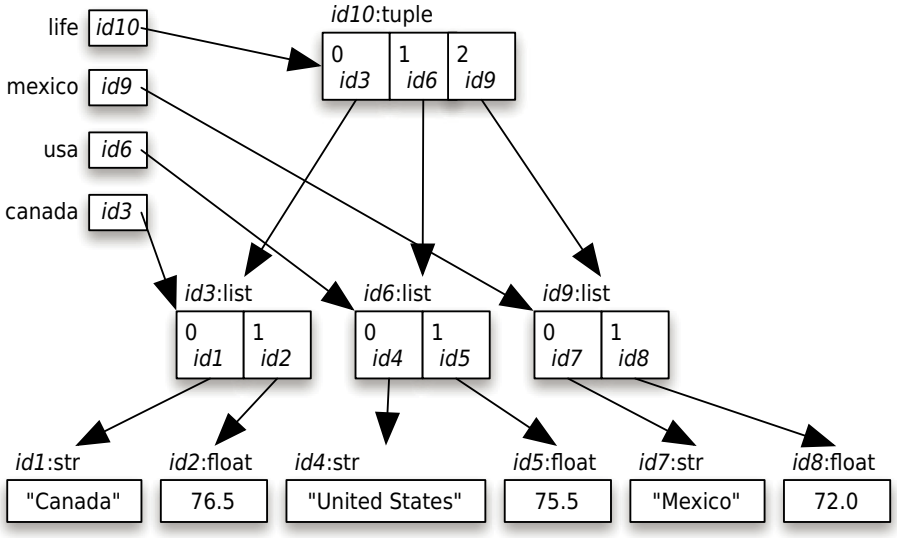
```
>>> canada = ['Canada', 76.5]
>>> usa = ['United States', 75.5]
>>> mexico = ['Mexico', 72.0]
```

That builds this memory model:



We'll create a tuple using those variables:

```
>>> life = (canada, usa, mexico)
```

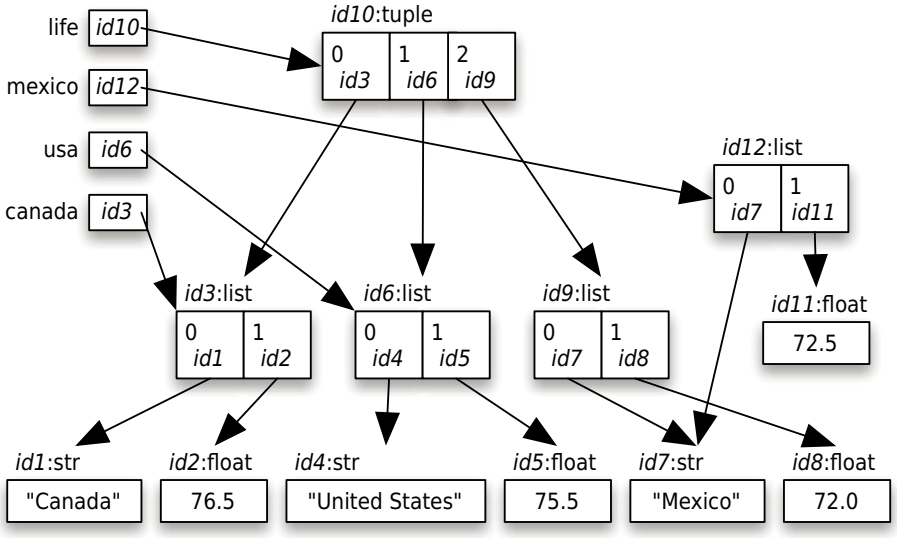


The most important thing to notice is that none of the four variables know about each other. The tuple object contains three references, one to each of the country lists.

Now let's change what variable mexico refers to:

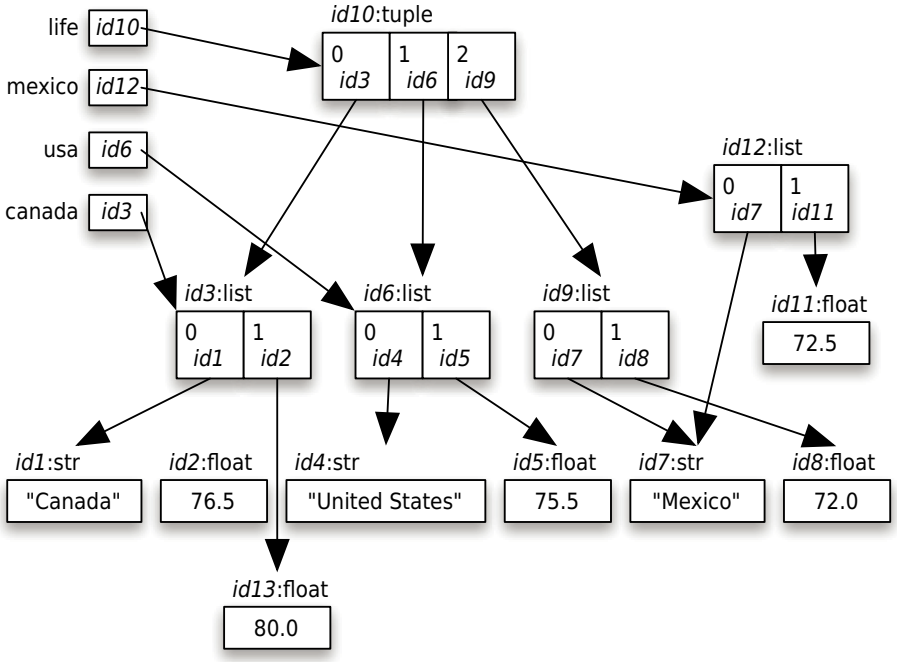
```
>>> mexico = ['Mexico', 72.5]
>>> life
(['Canada', 76.5], ['United States', 75.5], ['Mexico', 72.0])
```

Here's the new picture. Notice that the tuple that variable life refers to hasn't changed:



life[0] will always refer to the same list object—we can't change the memory address stored in life[0]—but we can mutate that list object, and because variable canada also refers to that list, it sees the mutation:

```
>>> life[0][1] = 80.0
>>> canada
['Canada', 80.0]
```



We hope that it is clear how essential it is to thoroughly understand variables and references, and how collections contain references to objects and not to variables.

Assigning to Multiple Variables Using Tuples

You can assign to multiple variables at the same time:

```
>>> (x, y) = (10, 20)
>>> x
10
>>> y
20
```

Python uses the comma as a tuple constructor, so we can leave off the parentheses:

```
>>> 10, 20
(10, 20)
>>> x, y = 10, 20
>>> x
10
>>> y
20
```

In fact, this will work with lists and sets as well: Python will happily pull apart information out of any collection:

```
>>> [[w, x], [[y], z]] = [{10, 20}, [(30,), 40]]
>>> w
10
>>> x
20
>>> y
30
>>> z
40
```

Any depth of nesting will work, as long as the structure on the right can be translated into the structure on the left.

11.3 Storing Data Using Dictionaries

In our bird-watching observation file, which had one species per line (as described in *Arctic Birds*, on page 177), suppose we want to know how often each species was seen. Our first attempt uses a list of lists, in which each inner list has two items. The item at index `0` of the inner list contains the species and the item at index `1` contains the number of times it has been seen so far. To build this list, we iterate over the lines of the observations file. For each line, we iterate over the list, looking for the species on that line. If we find the species occurs in the list, we add one the number of times it has been observed:

```
>>> observations_file = open('observations.txt')
>>> bird_counts = []
>>> for line in observations_file:
...     bird = line.strip()
...     found = False
...     # Find bird in the list of bird counts.
...     for entry in bird_counts:
...         if entry[0] == bird:
...             entry[1] = entry[1] + 1
...             found = True
...     if not found:
...         bird_counts.append([bird, 1])
...
>>> observations_file.close()
>>> for entry in bird_counts:
...     print(entry[0], entry[1])
...
canada goose 5
long-tailed jaeger 2
snow goose 1
northern fulmar 1
```

The code above uses a Boolean variable found. Once a species is read from the file, found is assigned False. The program then iterates over the list, looking for that species at index 0 of one of the inner lists. If the species occurs in an inner list, found is assigned True. At the end of the loop over the list, if found still refers to False it means that this species is not yet present in the list and so it is added, along with the number of observations of it, which is currently 1.

The code above works, but there are two things wrong with it. The first is that it is complex. The more nested loops our programs contain, the harder they are to understand, fix, and extend. The second is that it is inefficient. Suppose we were interested in beetles instead of birds and that we had millions of observations of tens of thousands of species. Scanning the list of names each time we want to add one new observation would take a long, long time, even on a fast computer (a topic we will return to in Chapter 13, *Searching and Sorting*, on page 199).

Can you use a set to solve both problems at once? Sets can look up values in a single step; why not combine each bird's name and the number of times it has been seen into a two-valued tuple and put those tuples in a set?

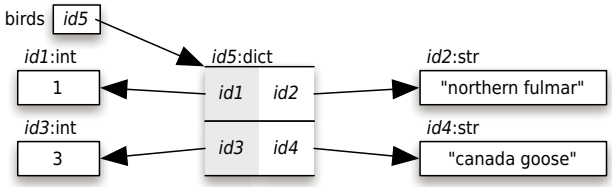
The problem with this idea is that you can look for values only if you know what those values are. In this case, you won't. You will know only the name of the species, not how many times it has already been seen.

The right approach is to use another data structure called a *dictionary*. Also known as a *map*, a dictionary is an unordered mutable collection of key/value pairs. In plain English, dictionaries are like phone books. They associate a value (such as a phone number) with a key (like a name). The keys form a set. Any particular key can appear at most once in a dictionary, and like the elements in sets, keys must be immutable (though the values associated with them don't have to be).

Dictionaries are created by putting key/value pairs inside braces:

```
>>> birds = {'canada goose': 3, 'northern fulmar': 1}
>>> birds
{'northern fulmar': 1, 'canada goose': 3}
```

Here is a picture of the resulting dictionary:



To get the value associated with a key, we put the key in square brackets, much like indexing into a list:

```
>>> birds['northern fulmar']
1
```

Indexing a dictionary with a key it doesn't contain produces an error, just like an out-of-range index for a list:

```
>>> birds['canada goose']
3
>>> birds['long-tailed jaeger']
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
KeyError: 'long-tailed jaeger'
```

The empty dictionary is written {}. It doesn't contain any key/value pairs, so indexing into it always results in an error.

Updating and Checking Membership

To update the value associated with a key, you use the same notation as for lists, except you use a key instead of an index. If the key is already in the dictionary, this assignment statement changes the value associated with it. If the key isn't present, the key/value pair is added to the dictionary:

```
>>> birds = {}
>>> birds['snow goose'] = 33
>>> birds['eagle'] = 999 # Add a new key/value pair, 'eagle': 999.
>>> birds
{'eagle': 999, 'snow goose': 33}
>>> birds['eagle'] = 9   # Change the value associated with key 'eagle' to 9.
>>> birds
{'eagle': 9, 'snow goose': 33}
```

To remove an entry from a dictionary, use `del d[k]`, where `d` is the dictionary and `k` is the key being removed. Only entries that are present can be removed; trying to remove one that isn't there results in an error:

```
>>> birds = {'snow goose': 33, 'eagle': 9}
>>> del birds['snow goose']
>>> birds
```

```
{'eagle': 9}
>>> del birds['gannet']
Traceback (most recent call last):
  File "<stdin>", line 1, in <module>
KeyError: 'gannet'
```

To test whether a key is in a dictionary, we can use operator in:

```
>>> birds = {'eagle': 999, 'snow goose': 33}
>>> if 'eagle' in birds:
...   print('eagles have been seen')
...
eagles have been seen
>>> del birds['eagle']
>>> if 'eagle' in birds:
...   print('eagles have been seen')
...
>>>
```

The in operator only checks the keys of a dictionary. In the example above, 33 in birds is would be False, since it is a value, not a key.

Loops

Like the other collections you've seen, you can loop over dictionaries. The general form of a for loop over a dictionary is as follows:

```
for «variable» in «dictionary»:
    «block»
```

For dictionaries, the loop variable is assigned each key from the dictionary in turn:

```
>>> bird_to_observations = {'canada goose': 183, 'long-tailed jaeger': 71,
...                         'snow goose': 63, 'northern fulmar': 1}
>>> for bird in bird_to_observations:
...     print(bird, bird_to_observations[bird])
...
long-tailed jaeger 71
canada goose 183
northern fulmar 1
snow goose 63
```

Notice that long-tailed jaegar 71 is printed first, but in the assignment statement, the first key/value pair listed is 'canada goose': 71. As with the items in a set, Python loops over the keys in the dictionary in an arbitrary order. There is no guarantee that they will be seen alphabetically or in the order they were added to the dictionary.

When Python loops over a dictionary, it assigns the keys to the loop variable. It's a lot easier to go from a dictionary key to the associated value than it is to take the value and find the associated key.

Dictionary Methods

Like lists, tuples, and sets, dictionaries are objects. Their methods are described in Table 15, *Dictionary methods*, on page 188.

Method	Description
D.clear()	Remove all key/value pairs from dictionary D.
D.get(k)	Return the value associated with key k, or a default value if the key isn't present.
D.get(k, v)	Return the value associated with key k, or a default value v if the key isn't present.
D.keys()	Return dictionary D's keys as a set-like object. Entries are guaranteed to be unique.
D.items()	Return dictionary D's (key, value) pairs as a set-like object.
D.pop(k)	Remove key k from dictionary D and return the value that was associated with k. If k isn't in D, an error is raised.
D.pop(k, v)	Remove key k from dictionary D and return the value that was associated with k. If k isn't in D , return v.
D.setdefault(k)	Return the value associated with key k in D.
D.setdefault(k, v)	Return the value associated with key k in D. If k isn't a key in D, add the key k with value v to D and return v.
D.values()	Return dictionary D's values as a set-like object. Entries may or may not be unique.
D.update(other)	Update dictionary D with the contents of dictionary other. For each key in other, if it is also a key in D, replace that key in D's' value with the value from other. For each key in other, if that key isn't in D, add that key/value pair to D.

Table 15—Dictionary methods

The following code shows how the methods can be used:

```
>>> scientist_to_birthdate = {'Newton' : 1642, 'Darwin' : 1809, 'Turing' : 1912}
>>> scientist_to_birthdate.keys()
dict_keys(['Darwin', 'Newton', 'Turing'])
>>> scientist_to_birthdate.values()
dict_values([1809, 1642, 1912])
>>> scientist_to_birthdate.items()
dict_items([('Darwin', 1809), ('Newton', 1642), ('Turing', 1912)])
```

```
>>> scientist_to_birthdate.get('Newton')
1642
>>> scientist_to_birthdate.get('Curie', 1867)
1867
>>> scientist_to_birthdate
{'Darwin': 1809, 'Newton': 1642, 'Turing': 1912}
>>> researcher_to_birthdate = {'Curie' : 1867, 'Hopper' : 1906, 'Franklin' : 1920}
>>> scientist_to_birthdate.update(researcher_to_birthdate)
>>> scientist_to_birthdate
{'Hopper': 1906, 'Darwin': 1809, 'Turing': 1912, 'Newton': 1642, 'Franklin': 1920,
 'Curie': 1867}
>>> researcher_to_birthdate
{'Franklin': 1920, 'Hopper': 1906, 'Curie': 1867}
>>> researcher_to_birthdate.clear()
>>> researcher_to_birthdate
{}
```

As you can see from its output shown above, the keys and values methods return the dictionary's keys and values, respectively, while items returns a (key, value) pairs. Like the range object that you learned about previously, these are virtual sequences over which we can loop. Similarly, the list() function can be applied to them to create lists of keys, values or key/value tuples.

One common use of items is to loop over the keys and values in a dictionary together:

```
for key, value in dictionary.items():
    ...do something with the key and value...
```

For example, the format above can be used to loop over the scientists and their birth years:

```
>>> scientist_to_birthdate = {'Newton' : 1642, 'Darwin' : 1809, 'Turing' : 1912}
>>> for scientist, birthdate in scientist_to_birthdate.items():
...     print(scientist, 'was born in', birthdate)
...
Turing was born in 1912
Darwin was born in 1809
Newton was born in 1642
```

Instead of a single loop variable, there are two. The two parts of each of the two-item tuples returned by method items is associated with a variable. Variable scientist refers to the first item in the tuple, which is the key, and birthdate refers to the second item, which is the value.

Dictionary Example

Back to birdwatching once again. Like before, we want to count the number of times each species has been seen. To do this, we create a dictionary that

is initially empty. Each time we read an observation from a file, we check to see whether we have seen that bird before, that is, whether the bird is already a key in our dictionary. If it is, we add one to the value associated with it. If it isn't, we add the bird as a key to the dictionary with the value 1. Here is the program that does this:

```
>>> observations_file = open('observations.txt')
>>> bird_to_observations = {}
>>> for line in observations_file:
...     bird = line.strip()
...     if bird in bird_to_observations:
...         bird_to_observations[bird] = bird_to_observations[bird] + 1
...     else:
...         bird_to_observations[bird] = 1
...
>>> observations_file.close()
>>>
>>> # Print each bird and the number of times it was seen.
... for bird, observations in bird_to_observations.items():
...     print(bird, observations)
...
snow goose 1
long-tailed jaeger 2
canada goose 5
northern fulmar 1
```

This program can be shortended by using the method dict.get, which saves three lines:

```
>>> observations_file = open('observations.txt')
>>> bird_to_observations = {}
>>> for line in observations_file:
...     bird = line.strip()
...     bird_to_observations[bird] = bird_to_observations.get(bird, 0) + 1
...
>>> observations_file.close()
```

Using method get makes the program shorter, but some programmers find it harder to understand at a glance. If the first argument to get is not a key in the dictionary, it returns 0, otherwise it returns the value associated with that key. After that, 1 is added to that value. The dictionary is updated to associate that sum with the key that bird refers to.

Instead of printing the birds' names in whatever arbitrary order they are accessed by the loop, you can create a list of the dictionary's keys, sort that list alphabetically, and then loop over the sorted list. This way, the entries appear in a sensible order:

```
>>> sorted_birds = list(bird_to_observations.keys())
```

```
>>> sorted_birds.sort()
>>> for bird in sorted_birds:
...     print(bird, bird_to_observations[bird])
...
canada goose 5
long-tailed jaeger 2
northern fulmar 1
snow goose 1
```

If order matters, that an ordered sequence like lists or tuples should be used instead of sets and dictionaries.

11.4 Inverting a Dictionary

You might want to print the birds in another order—in order of the number of observations, for example. To do this, you need to *invert* the dictionary; that is, create a new dictionary in which you use the values as keys and the keys as values. This is a little trickier than it first appears. There's no guarantee that the values are unique, so you have to handle *collisions*. For example, if you invert the dictionary {'a': 1, 'b': 1, 'c': 1}, a key would be 1, but it's not clear what the value associated with it would be.

Since you'd like to keep all of the data from the original dictionary, you may need to use a collection, such as a list, to keep track of the values associated with a key. If we go this route, the inverse of the dictionary shown earlier would be {1: ['a', 'b', 'c']}. Here's a program to invert the dictionary of birds to observations:

```
>>> bird_to_observations
{'canada goose': 5, 'northern fulmar': 1, 'long-tailed jaeger': 2,
'snow goose': 1}
>>>
>>> # Invert the dictionary
>>> observations_to_birds_list = {}
>>> for bird, observations in bird_to_observations.items():
...     if observations in observations_to_birds_list:
...         observations_to_birds_list[observations].append(bird)
...     else:
...         observations_to_birds_list[observations] = [bird]
...
>>> observations_to_birds_list
{1: ['northern fulmar', 'snow goose'], 2: ['long-tailed jaeger'],
5: ['canada goose']}
```

The program above loops over each key/value pair in the original dictionary, bird_to_observations. If that value is not yet a key in the inverted dictionary, observations_to_birds_list, it is added as a key and its value is a single item list containing the key associated with it in the original dictionary. On the other

hand, if that value is already a key, then the key associated with it in the original dictionary is appended to its list of values.

Now that the dictionary is inverted, you can print each key and all of the items in its value list:

```
>>> # Print the inverted dictionary
... observations_sorted = list(observations_to_birds_list.keys())
>>> observations_sorted.sort()
>>> for observations in observations_sorted:
...     print(observations, ':', end=" ")
...     for bird in observations_to_birds_list[observations]:
...         print(' ', bird, end=" ")
...     print()
...
1 :   northern fulmar   snow goose
2 :   long-tailed jaeger
5 :   canada goose
```

The outer loop passes over each key in the inverted dictionary, and the inner loops passes over each of the item in the values list associated with that key.

11.5 Comparing Collections

You've now seen strings, lists, sets, tuples, and dictionaries. They all have their uses. Here is a table comparing them:

Collection	Mutable?	Ordered?	Use when...
str	No	Yes	you want to keep track of text.
list	Yes	Yes	you want to keep track of an ordered sequence that you want to update.
tuple	No	Yes	you want to build an ordered sequence that you know won't change, or that you want to use as a key in a dictionary or as a value in a set.
set	Yes	No	you want to keep track of values, but order doesn't matter, and you don't want to keep duplicates. The values must be immutable.
dictionary	Yes	No	you want to keep a mapping of keys to values. The keys must be immutable.

Table 16—Features of Python Collections

11.6 A Collection of New Information

In this chapter, you learned the following:

- Sets are used in Python to store unordered collections of unique values. They support the same operations as sets in mathematics.
- Tuples are another kind of Python sequence. Tuples are ordered sequences like lists, except they are immutable.
- Dictionaries are used to store unordered collections of key/value pairs. They are also stored using hash tables for efficiency's sake and also require keys to be immutable.
- Looking things up in sets and dictionaries is much faster than searching through lists. If you have a program that is doing the latter, consider changing your choice of data structures.

11.7 Exercises

Here are some exercises for you to try on your own:

1. Write a function called `find_dups` that takes a list of integers as its input argument and returns a set of those integers that occur two or more times in the list.
2. Python's set objects have a method called `pop` that removes and returns an arbitrary element from the set. If a set `gerbils` contains five cuddly little animals, for example, calling `gerbils.pop()` five times will return those animals one by one, leaving the set empty at the end. Use this to write a function called `mating_pairs` that takes two equal-sized sets called `males` and `females` as input and returns a set of pairs; each pair must be a tuple containing one male and one female. (The elements of `males` and `females` may be strings containing gerbil names or gerbil ID numbers—your function must work with both.)
3. The PDB file format is often used to store information about molecules. A PDB file may contain zero or more lines that begin with the word `AUTHOR` (which may be in uppercase, lowercase, or mixed case), followed by spaces or tabs, followed by the name of the person who created the file. Write a function that takes a list of filenames as an input argument and returns the set of all author names found in those files.
4. The keys in a dictionary are guaranteed to be unique, but the values are not. Write a function called `count_values` that takes a single dictionary as an argument and returns the number of distinct values it contains. Given the input `{'red': 1, 'green': 1, 'blue': 2}`, for example, it should return `2`.
5. After doing a series of experiments, you have compiled a dictionary showing the probability of detecting certain kinds of subatomic particles.

The particles' names are the dictionary's keys, and the probabilities are the values: {'neutron': 0.55, 'proton': 0.21, 'meson': 0.03, 'muon': 0.07, 'neutrino': 0.14}. Write a function that takes a single dictionary of this kind as input and returns the particle that is least likely to be observed. Given the dictionary shown earlier, for example, the function would return 'meson'.

6. Write a function called count_duplicates that takes a dictionary as an argument and returns the number of values that appear two or more times.

7. Write a function called fetch_and_set that takes a dictionary and two arbitrary values, key and new_value, as arguments. If key is already in the dictionary, this function returns its associated value and replaces the value with new_value. If the key isn't present, the function raises a KeyError exception with the error message 'Unable to replace value for nonexistent key'.

8. A *balanced color* is one whose red, green, and blue values add up to 1.0. Write a function called is_balanced that takes a dictionary whose keys are 'R', 'G', and 'B' as input and returns True if they represent a balanced color.

9. Write a function called dict_intersect that takes two dictionaries as arguments and returns a dictionary that contains only the key/value pairs found in both of the original dictionaries.

10. Programmers sometimes use a dictionary of dictionaries as a simple database. For example, to keep track of information about famous scientists, you might a dictionary where the keys are strings and the values are dictionarys, like this:

```
{
  'jgoodall'  : {'surname'  : 'Goodall',
                 'forename' : 'Jane',
                 'born'     : 1934,
                 'died'     : None,
                 'notes'    : 'primate researcher',
                 'author'   : ['In the Shadow of Man', 'The Chimpanzees of Gombe']},
  'rfranklin' : {'surname'  : 'Franklin',
                 'forename' : 'Rosalind',
                 'born'     : 1920,
                 'died'     : 1957,
                 'notes'    : 'contributed to discovery of DNA'},


   'rcarson'  : {'surname'  : 'Carson',
                 'forename' : 'Rachel',
                 'born'     : 1907,
                 'died'     : 1964,
                 'notes'    : 'raised awareness of effects of DDT',
                 'author'   : ['Silent Spring']}
```

```
}
```

Write a function called `db_headings` that returns the set of keys used in *any* of the inner dictionaries. In this example, the function should return `set('author', 'forename', 'surname', 'notes', 'born', 'died')`.

11. Write another function called `db_consistent` that takes a dictionary of dictionaries in the format described in the previous question and returns `True` if and only if every one of the inner dictionaries has exactly the same keys. (This function would return `False` for the previous example, since Rosalind Franklin's entry doesn't contain the `'author'` key.)

12. A *sparse vector* is a vector whose entries are almost all zero, like `[1, 0, 0, 0, 0, 0, 3, 0, 0, 0]`. Storing all those zeroes in a list wastes memory, so programmers often use dictionaries instead to keep track of just the nonzero entries. For example, the vector shown earlier would be represented as `{0:1, 6:3}`, since the vector it is meant to represent has the value 1 at index 0 and the value 3 at index 6.

    a. The sum of two vectors is just the element-wise sum of their elements. For example, the sum of `[1, 2, 3]` and `[4, 5, 6]` is `[5, 7, 9]`. Write a function called `sparse_add` that takes two sparse vectors stored as dictionaries and returns a new dictionary representing their sum.
    b. The dot product of two vectors is the sum of the products of corresponding elements. For example, the dot product of `[1, 2, 3]` and `[4, 5, 6]` is `4+10+18`, or 32. Write another function called `sparse_dot` that calculates the dot product of two sparse vectors.
    c. Your boss has asked you to write a function called `sparse_len` that will return the length of a sparse vector (just as Python's `len` returns the length of a list). What do you need to ask her before you can start writing it?

CHAPTER 12

Designing Programs

This is a beta book; this chapter is not yet complete.

CHAPTER 13

Searching and Sorting

A huge part of computer science involves studying how to organize, store, and retrieve data. The amount of data is growing exponentially: according to IBM, 90% of the data in the world has been generated in the past two years.[1] There are many ways to organize and process data, and you need to develop an understanding of how to analyze how good your approach is. This chapter introduces you to some tools and concepts that you can use to tell whether a particular approach is faster or slower than another.

As you know, there are many solutions to each programming problem. If a problem involves a large amount of data, a slow algorithm will mean the problem can't be solved in a reasonable amount of time, even with an incredibly powerful computer. This chapter includes several examples of both slower and faster algorithms. Try running them yourself: experiencing just how slow (or fast) something is has a much more profound effect on your understanding than the data we include in this chapter.

Searching and sorting data are fundamental parts of programming. There are several ways to do both of them. In this chapter, we will develop several algorithms for searching and sorting lists, and then use them to explore what it means for one algorithm to be faster than another. As a bonus, this approach will give you another set of examples of how there are many solutions to any problem, and that the approach you take to solving a problem will dictate which solution you come up with.

13.1 Searching a List

As you have already seen in Table 10, *List methods*, on page 140, Python lists have a method called `index` that searches for a particular item:

1. http://www-01.ibm.com/software/data/bigdata/

```
index(...)
    L.index(value, [start, [stop]]) -> integer -- return first index of value
```

Method index starts at the front of the list and examines each item in turn. For reasons that will soon become clear, this technique is called *linear search*. Linear search is used to find an item in an *unsorted* list. If there are duplicate values, our algorithms will find the leftmost one:

```
>>> ['d', 'a', 'b', 'a'].index('a')
1
```

We're going to write several versions of linear search in order to demonstrate how to compare different algorithms that all solve the same problem.

After we do this analysis, we will see that we can can search a *sorted* list much faster than we can search an unsorted list.

An Overview of Linear Search

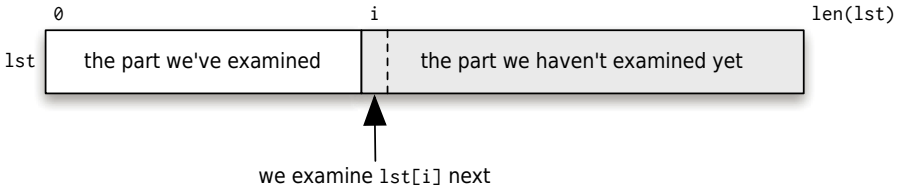
Linear search starts at index 0 and looks at each item one by one. At each index, we ask this question: "Is the value we are looking for at the current index?" We'll show three variations of this. All of them use a loop of some kind, and they are all implementations of this function:

```
def linear_search(lst, value):
    """(list, object) -> int

    Return the index of the first occurrence of value in lst, or return
    -1 if value is not in lst.

    >>> linear_search([2, 5, 1, -3], 5)
    1
    >>> linear_search([2, 4, 2], 2)
    0
    >>> linear_search([2, 5, 1, -3], 4)
    -1
    >>> linear_search([], 5)
    -1
    """

    examine the items at each index i in lst, starting at index 0:
        is lst[i] the value we are looking for?  if so, stop searching.
```

The comments in the function body describe what every version will do to look for the value.

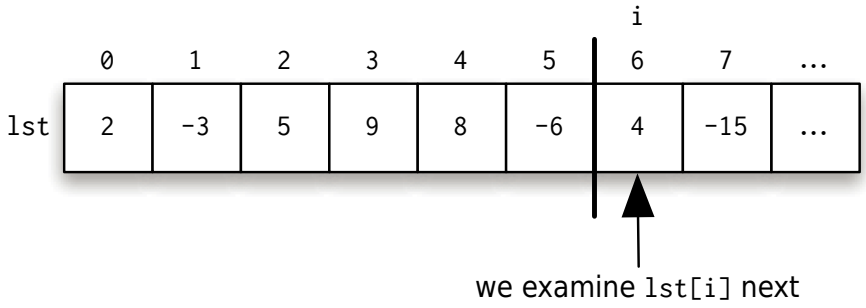
We've found it to be helpful to have a picture of how linear search works. (We will use pictures throughout this chapter for both searching and sorting.) Because all of our versions examine index 0 first, then index 1, then 2, and

so on, that means that partway through our searching process we have this situation:



There is a part of the list that we've examined, and another part that remains to be examined. We use variable `i` to mark the current index.

Here's a concrete example where we are searching for a value in a list that starts like this: `[2, -3, 5, 9, 8, -6, 4, 15, ...]`. We don't know how long the list is, but let's say that after six iterations we have examined items at indices `0`, `1`, `2`, `3`, `4`, and `5`. Index `6` is the index of the next item to examine:



That vertical line divides the list in two: the part we have examined and the part we haven't. Because we stop when we find the value, we know that the value isn't in the first part:

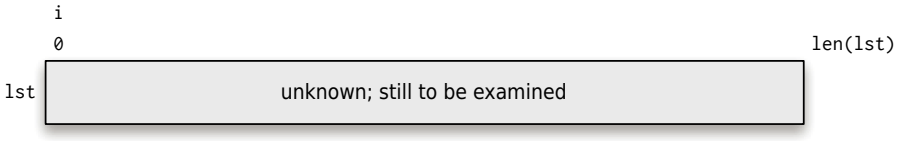
This picture is sometimes called an *invariant* for linear search. An invariant is something that remains unchanged throughout a process. But variable `i` is changing—how can that picture be an invariant?

Here is a word version of the picture:

```
list[0:i] doesn't contain value, and 0 <= i <= len(lst)
```

This version says that we know that value wasn't found before index i, and that i is somewhere between 0 and the length of the list. If our code matches that word version, that word version is an invariant of the code, and so is the picture version.

We can use invariants to come up with the initial values of our variables. For example, with linear search, at the very beginning the entire list is unknown: we haven't examined anything:



Variable i refers to 0 at the beginning, because then the section with the label value not here is empty; further, list[0:0] is an empty list, which is exactly what we want according to the word version of the invariant. So the initial value of i should be 0 in all of our versions of linear search.

while loop Version of Linear Search

Let's develop our first version of linear search. We need to refine our comments to get them closer to Python:

```
examine every index i in lst, starting at index 0:
    is lst[i] the value we are looking for?  if so, stop searching.
```

Here's a refinement:

```
i = 0  # The index of the next item in lst to examine.

while the unknown section isn't empty, and lst[i] isn't
the value we are looking for:
    add 1 to i
```

That's easier to translate. The unknown section is empty when i == len(lst), so it isn't empty as long as i != len(lst). Here is the code:

```
def linear_search(lst, value):
    """(list, object) -> int

    Return the index of the first occurrence of value in lst, or return
    -1 if value is not in lst.

    >>> linear_search([2, 5, 1, -3], 5)
    1
    >>> linear_search([2, 4, 2], 2)
    0
```

```
    >>> linear_search([2, 5, 1, -3], 4)
    -1
    >>> linear_search([], 5)
    -1
    """

    i = 0  # The index of the next item in lst to examine.

    # Keep going until we reach the end of lst or until we find value.
    while i != len(lst) and lst[i] != value:
        i = i + 1

    # If we fell off the end of the list we didn't find value.
    if i == len(lst):
        return -1
    else:
        return i
```

This version uses variable i as the current index and marches through the values in lst, stopping in one of two situations: when we have run out of values to examine, or when we find the value we are looking for.

The first check in the loop condition, i != len(lst), makes sure that we still have values to look at; if we were to omit that check, then if value isn't in lst, we would end up trying to access lst[len(lst)]. This would result in an IndexError.

The second check, lst[i] != value, causes the loop to exit when we find value. The loop body increments i; we enter the loop when we haven't reach the end of lst and when lst[i] isn't the value we are looking for.

At the end, we return i's value, which is either the index of value (if the second loop check evaluated to False) or is len(lst) if value wasn't in list.

for loop Version of Linear Search

The first version evaluates two Boolean subexpressions each time through the loop. But the first check, i != len(lst), is almost unnecessary: it evaluates to True almost every time through the loop, so the only effect it has is to make sure we don't attempt to index past the end of the list. We can instead exit the function as soon as we find the value:

```
i = 0  # The index of the next item in lst to examine.

for each index i in lst:
    if lst[i] is the value we are looking for:
        return i

if we get here, value was not in lst, so we return len(lst).
```

In this version, we use Python's for loop to examine each index.

```
def linear_search(lst, value):
    '''(list, object) -> int

    … Exactly the same docstring goes here …
    '''

    for i in range(len(lst)):
        if lst[i] == value:
            return i

    return -1
```

With this version, we no longer need the first check because the for loop controls the number of iterations. This for loop version is significantly faster than our first version; we'll see in a bit how much faster.

Sentinel Search

The last linear search we will study is called *sentinel search*. (A sentinel is a guard whose job it is to stand watch.) Remember that one problem with the while loop linear search is that we check i != len(lst) every time through the loop even though it can never evaluate to False except when value is not in lst. So we'll play a trick: we'll add value to the end of lst before we search. That way we are guaranteed to find it! We also need to remove it before the function exits so that the list looks unchanged to whoever called this function:

```
set up the sentinel: append value to the end of lst

i = 0  # The index of the next item in lst to examine.

while lst[i] isn't the value we are looking for:
    add 1 to i

remove the sentinel

return i
```

Let's translate that to Python:

```
def linear_search(lst, value):
    '''(list, object) -> int

    … Exactly the same docstring goes here …
    '''

    # Add the sentinel.
    lst.append(value)
```

```
    i = 0

    # Keep going until we find value.
    while lst[i] != value:
        i = i + 1

    # Remove the sentinel.
    lst.pop()

    # If we reached the end of the list we didn't find value.
    if i == len(lst):
        return -1
    else:
        return i
```

All three of our linear search functions are correct. Which one you prefer is largely a matter of taste: some some programmers dislike returning in the middle of a loop, so they won't like the second version. Others dislike modifying parameters in any way, so they won't like the third version. Still others will dislike that extra check that happens in the first version.

Timing the Searches

Here is a program that we used to time the three searches on a list with about 10 million values:

```
import time
import linear_search_1
import linear_search_2
import linear_search_3


def time_it(search, L, v):
    '''(function, object, list) -> number

    Time how long it takes to run function search to find
    value v in list L.
    '''

    t1 = time.time()
    search(L, v)
    t2 = time.time()

    return (t2 - t1) * 1000.0


def print_times(v, L):
    '''(object, list) -> NoneType

    Print the number of milliseconds it takes for linear_search(v, L)
```

```
    to run for list.index, the while loop linear search, the for loop
    linear search, and sentinel search.
    '''

    # Get list.index's running time.
    t1 = time.time()
    L.index(v)
    t2 = time.time()
    index_time = (t2 - t1) * 1000.0

    # Get the other three running times.
    while_time = time_it(linear_search_1.linear_search, L, v)
    for_time = time_it(linear_search_2.linear_search, L, v)
    sentinel_time = time_it(linear_search_3.linear_search, L, v)

    print("{0}\t{1:.2f}\t{2:.2f}\t{3:.2f}\t{4:.2f}".format(
            v, while_time, for_time, sentinel_time, index_time))

L = list(range(10000001))  # A list with just over ten million values

print_times(10, L)  # How fast is it to search near the beginning?
print_times(5000000, L)  # How fast is it to search near the middle?
print_times(10000000, L)  # How fast is it to search near the end?
```

Function time_it will calls whichever search function it is given on v and L and return how long that search took. Function print_times calls time_it with the various linear search functions we have been exploring and prints those search times.

Linear Search Running Time

The running times of the three linear searches with that of Python's list.index are compared in Table 17, *Running times for linear search, milliseconds*, on page 206. This comparison used a list of 10,000,001 items and three test cases: an item near the front, an item roughly in the middle, and the last item. Except for the first case, where the speeds differ by very little, our while loop linear search takes about 13 times as long as the one built into Python, and the for loop search and sentinel search take about 5 and 7 times as long.

Case	while	for	sentinel	list.index
First	0.01	0.01	0.01	0.01
Middle	1261	515	697	106
Last	2673	1029	1394	212

Table 17—Running times for linear search, milliseconds

What is more interesting is the way the *running times* of these functions increase with the number of items they have to examine. Roughly speaking, when they have to look through twice as much data, every one of them takes twice as long. This is reasonable, because indexing a list, adding 1 to an integer, and evaluating the loop control expression require the computer to do a fixed amount of work. Doubling the number of times the loop has to be executed therefore doubles the total number of operations, which in turn should double the total running time. This is why this kind of search is called *linear*: the time to do it grows linearly with the amount of data being processed.

13.2 Binary Search

Is there a faster way to find values than linear search? The answer is yes—we can do much better, provided the list is sorted.

To understand how, think about finding a name in a phone book. You open the book in the middle, glance at a name on the page, and immediately know which half to look in next. After checking only two names, you have eliminated $^3/_4$ of the numbers in the phone book. Even in a large city like Toronto, whose phone book has hundreds of thousands of entries, finding the name you want takes only a few steps.

This technique is called *binary search*, because each step divides the remaining data into two equal parts and discards one of the two halves. To figure out how fast it is, think about how big a list can be searched in a fixed number of steps. One step divides two values; two steps divide four; three steps divide $2^3 = 8$, four divide $2^4 = 16$, and so on. Turning this around, N values can be searched in roughly $\log_2 N$ steps.

More exactly, N values can be searched in ceiling($\log_2 N$) steps, where ceiling() is the ceiling function that rounds a value up to the nearest integer.

As shown in Table 18, *Logarithmic growth*, on page 207, this increases much less quickly than the time needed for linear search.

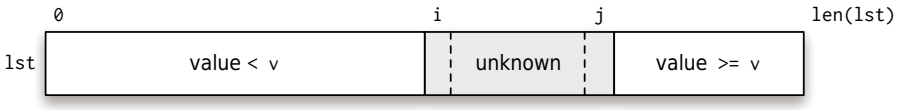
N	Steps Required
100	7
1000	10
10,000	14
100,000	17
1,000,000	20

N	Steps Required
10,000,000	24

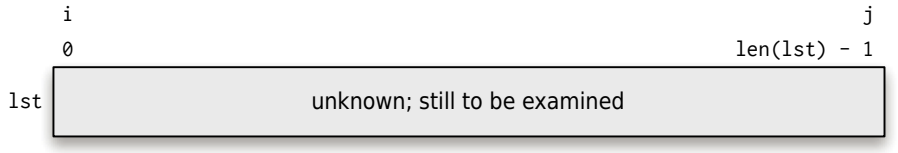
Table 18—Logarithmic growth

The key to binary search is to keep track of three parts of the list: the left part, which contains values that are smaller than the value we are searching for; the right part, which contains values that are equal to or larger than the value we are searching for; and the middle part, which contains values that we haven't yet examined—the unknown section. (If there are duplicate values, as with linear search we want to return the index of the leftmost one, which is why the "equal to" section belongs on the right.)

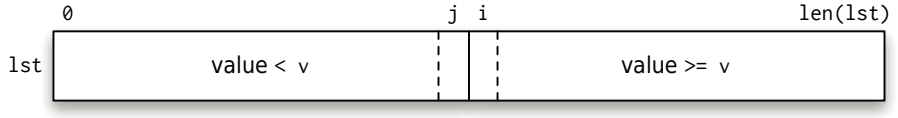
We'll use two variables to keep track of the boundaries: `i` will mark the index of the first unknown value, and `j` will mark the index of the last unknown value:



At the beginning of the algorithm, the unknown section makes up the entire list, so we will set `i` to 0 and `j` to the length of the list minus one:



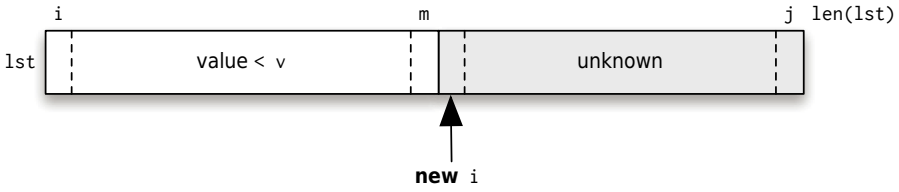
We are done when that unknown section is empty—when we've examined every item in the list. This happens when `i == j + 1`—when the values *cross*. (When `i == j`, there is still one item left in the unknown section.) Here is a picture of what the values are when the unknown section is empty:



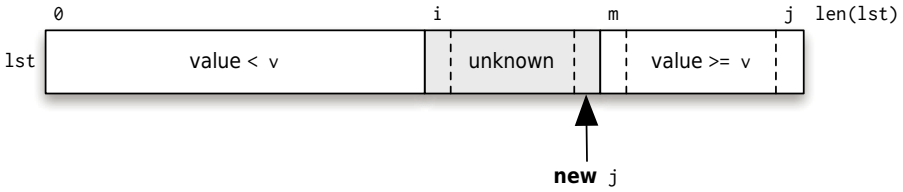
To make progress, we will set either i or j to near the middle of the range between them. Let's call this index m, which is at (i + j) // 2. (Notice the use of integer division: we are calculating an index, so we need an integer.)

Think for a moment about the value at m. If it is less than v, we need to move i up, while if it is greater than j, we should move j down. But where exactly do we move them?

When we move i up, we don't want to set it to the midpoint exactly, because L[i] isn't included in the range; instead, we set it to one past the middle, in other words, to m + 1.



Similarly, when we move j down, we move it to m - 1:



The completed function is as follows:

```
def binary_search(L, v):
    """(list, object) -> int

    Return the index of the first occurrence of value in L, or return
    -1 if value is not in L.

    >>> binary_search([1, 3, 4, 4, 5, 7, 9, 10], 1)
    0
    >>> binary_search([1, 3, 4, 4, 5, 7, 9, 10], 4)
    2
    >>> binary_search([1, 3, 4, 4, 5, 7, 9, 10], 5)
    4
    >>> binary_search([1, 3, 4, 4, 5, 7, 9, 10], 10)
    7
    >>> binary_search([1, 3, 4, 4, 5, 7, 9, 10], -3)
    -1
    >>> binary_search([1, 3, 4, 4, 5, 7, 9, 10], 11)
```

```
    -1
    >>> binary_search([1, 3, 4, 4, 5, 7, 9, 10], 2)
    -1
    >>> binary_search([], -3)
    -1
    >>> binary_search([1], 1)
    0
    """

    # Mark the left and right indices of the unknown section.
    i = 0
    j = len(L) - 1

    while i != j + 1:
        m = (i + j) // 2
        if L[m] < v:
            i = m + 1
        else:
            j = m - 1

    if 0 <= i < len(L) and L[i] == v:
        return i
    else:
        return -1

if __name__ == '__main__':
    import doctest
    doctest.testmod()
```

There are a lot of tests because the algorithm is quite complicated and we wanted to test fairly thoroughly. Our tests cover these cases:

- The value is the first item.
- The value occurs twice. We want the index of the first one.
- The value is in the middle of the list.
- The value is the last item.
- The value is smaller than everything in the list.
- The value is larger than everything in the list.
- The value isn't in the list, but is larger than some and smaller than others.
- The list has no items.
- The list has one item.

In Chapter 15, *Testing and Debugging*, on page 235, you'll learn a different testing framework that allows you to write tests in a separate Python file (thus

making docstrings shorter easier to read; only a couple of examples are necessary), and you'll learn strategies to coming up with your own test cases.

Binary Search Running Time

Binary search is *much* more complicated to write and understand than linear search. Is it fast enough to make the extra effort worthwhile? To find out, we can compare it to list.index. As before, we search for the first, middle, and last items in a list with about 10 million elements:

Case	list.index	binary_search	Ratio
First	0.007	0.02	0.32
Middle	105	0.02	5910
Last	211	0.02 (Wow!)	11661

Table 19—Running times for binary search

The results are impressive (see Table 19, *Running times for binary search*, on page 211). Binary search is up to several thousand times faster than its linear counterpart when searching 10 million items. Most important, if we double the number of items, binary search takes only one more iteration, while the time for list.index nearly doubles. Note also that although the time taken for linear search grows in step with the index of the item found, there is no such pattern for binary search. No matter where the item is, it takes the same number of steps.

Built-in Binary Search

The Python standard library's bisect module includes binary search functions that are slightly faster than our binary search. Function bisect_left returns the index where an item should be inserted in a list to keep it in sorted order, assuming it is sorted to begin with. insort_left actually does the insertion. The word *left* in their name signals that they find the leftmost (lowest index) position where they can do their jobs; the complementary functions bisect_right method (bisect module) bisect_right and insort_right find the rightmost.

There's a problem, though: binary search assumes that the list is sorted, and sorting is expensive. When does it make sense to sort the list before we search?

13.3 Sorting

Now let's look at a slightly harder problem. Table 20, *Acres lost to forest fires in Canada (in 000s), 1918--87*, on page 212[2] shows the number of acres burned in forest fires in Canada from 1918 to 1987. What were the worst years?

563	7590	1708	2142	3323	6197	1985	1316	1824	472
1346	6029	2670	2094	2464	1009	1475	856	3027	4271
3126	1115	2691	4253	1838	828	2403	742	1017	613
3185	2599	2227	896	975	1358	264	1375	2016	452
3292	538	1471	9313	864	470	2993	521	1144	2212
2212	2331	2616	2445	1927	808	1963	898	2764	2073
500	1740	8592	10856	2818	2284	1419	1328	1329	1479

Table 20—Acres lost to forest fires in Canada (in 000s), 1918–87

One way to find out how much forest was destroyed in the *N* worst years is to sort the list and then take the last *N* values:

```
def find_largest(n, L):
    """(int, list) -> list

    Return the n largest values in L in order from smallest to largest.

    >>> L = [3, 4, 7, -1, 2, 5]
    >>> find_largest(3, L)
    [4, 5, 7]
    """

    copy = L[:]
    copy.sort()
    return copy[n:]
```

This algorithm is short, clean, and easy to understand, but it relies on a bit of black magic. How *does* list.sort work, anyway? And how efficient is it?

It turns out that many sorting algorithms have been developed over the years, each with its own strengths and weaknesses. Broadly speaking, they can be divided into two categories: those that are simple but inefficient and those that are efficient but harder to understand and implement. We'll examine two of the former kind. The rest rely on techniques typically taught in a second course; we'll show you one of these, rewritten to use only material seen so far.

2. http://robjhyndman.com/TSDL/miscellaneous/
Number of acres burned in forest fires in Canada, 1918-1988.

Both of the simple sorting algorithms keep track of two sections in the list being sorted. The section at the front contains values that are now in sorted order; the section at the back contains values that have yet to be sorted. Here is the invariant we will use for our two simple sorts:

Both of these sorting algorithms will work their way through the list, making the sorted section 1 item longer on each iteration. We'll see that there are two ways to do this. Here is an outline for our code:

```
i = 0  # The index of the first unknown item in lst.  lst[:i] is sorted.
while i != len(L):
    do something to incorporate L[i] into the sorted section.

    i = i + 1
```

Most Python programmers would probably write the loop header as `for i in range(len(L))`, rather than incrementing `i` explicitly in the body of the loop. We're doing the latter here to explicitly initialize `i` (to set up the loop invariant) and to show the increment separately from the work the particular algorithm is doing.

The "do something..." part is where the two simple sorting algorithms will differ.

Selection Sort

Selection sort works by searching the unknown section for the smallest item and moving it to index `i`. Here is our algorithm:

```
i = 0  # The index of the first unknown item in lst.  lst[:i] is sorted.

while i != len(L):
    find the index of the smallest item in lst[i:]
    swap that smallest item with the item at index i

    i = i + 1
```

As you can probably guess from this description, selection sort works by repeatedly finding the next smallest item in the unsorted section and placing it at the end of the sorted section (see Figure 6, *First few steps in selection sort*, on page 214). This works because we are selecting the items in order. On the first iteration, `i` is `0`, and `lst[0:]` is the entire list. That means that on the

first iteration we select the smallest item and move it to the front. On the second iteration we select the second-smallest item and move it to the second spot, and so on.

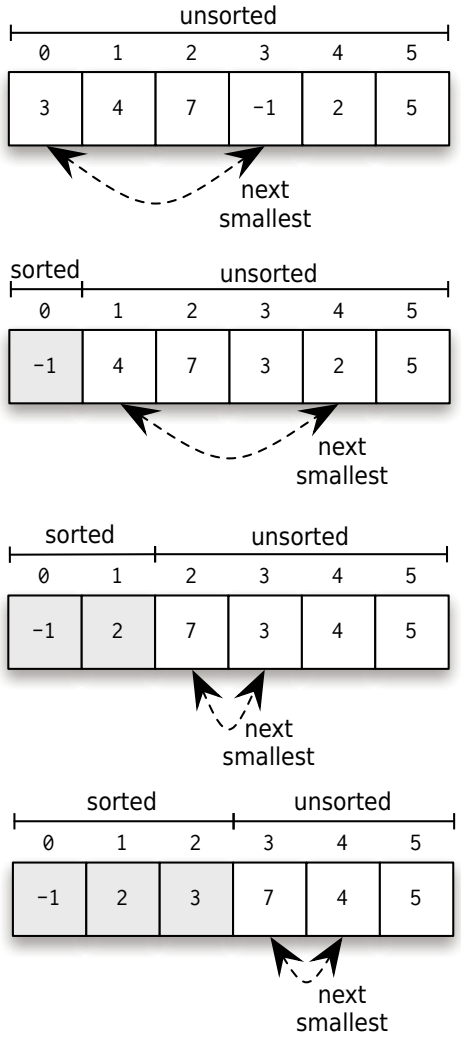


Figure 6—First few steps in selection sort

Here we have started writing a selection sort function, partially in English:

```
def selection_sort(L):
    """(list) -> NoneType
```

```
    Reorder the items in L from smallest to largest.

    >>> L = [3, 4, 7, -1, 2, 5]
    >>> selection_sort(L)
    >>> L
    [-1, 2, 3, 4, 5, 7]
    """

    i = 0
    while i != len(L):
        # Find the index of the smallest item in L[i:].
        # Swap that smallest item with L[i].

        i = i + 1
```

We can replace the second comment with a single line of code.

```
def selection_sort(L):
    """(list) -> NoneType

    Reorder the items in L from smallest to largest.

    >>> L = [3, 4, 7, -1, 2, 5]
    >>> selection_sort(L)
    >>> L
    [-1, 2, 3, 4, 5, 7]
    """

    i = 0
    while i != len(L):
        # Find the index of the smallest item in L[i:].
        L[i], L[smallest] = L[smallest], L[i]
        i = i + 1
```

Now all that's left is finding the index of the smallest item in `L[i:]`. This is complex enough that it's worth putting it in a function of its own:

```
def find_min(L, b):
    """(list, int) -> NoneType

    Precondition: L[b:] is not empty.

    Return the index of the smallest value in L[b:].

    >>> find_min([3, -1, 7, 5], 0)
    1
    >>> find_min([3, -1, 7, 5], 1)
    1
    >>> find_min([3, -1, 7, 5], 2)
    3
    """
```

```
    smallest = b  # The index of the smallest so far.
    i = b + 1
    while i != len(L):
        if L[i] < L[smallest]:
            # We found a smaller item at L[i].
            smallest = i

        i = i + 1

    return smallest


def selection_sort(L):
    """(list) -> NoneType

    Reorder the items in L from smallest to largest.

    >>> L = [3, 4, 7, -1, 2, 5]
    >>> selection_sort(L)
    >>> L
    [-1, 2, 3, 4, 5, 7]
    """

    i = 0
    while i != len(L):
        smallest = find_min(L, i)
        L[i], L[smallest] = L[smallest], L[i]
        i = i + 1
```

Function find_min examines each item in L[b:], keeping track of the index of the minimum item so far in variable smallest. Whenever it finds a smaller item, it updates smallest. (Because it is returning the index of the smallest value, it won't work if L[b:] is empty; hence the precondition.)

This is complicated enough that a couple of doctests may not test enough. Here's a list of test cases for sorting:

- Empty list
- List of length 1
- List of length 2 (this is the shorted case where items move)
- Already-sorted list
- List with all the same values
- List with duplicates

Here are our expanded doctests:

```
def selection_sort(L):
    """(list) -> NoneType

    Reorder the items in L from smallest to largest.

    >>> L = [3, 4, 7, -1, 2, 5]
    >>> selection_sort(L)
    >>> L
    [-1, 2, 3, 4, 5, 7]
    >>> L = []
    >>> selection_sort(L)
    >>> L
    []
    >>> L = [1]
    >>> selection_sort(L)
    >>> L
    [1]
    >>> L = [2, 1]
    >>> selection_sort(L)
    >>> L
    [1, 2]
    >>> L = [1, 2]
    >>> selection_sort(L)
    >>> L
    [1, 2]
    >>> L = [3, 3, 3]
    >>> selection_sort(L)
    >>> L
    [3, 3, 3]
    >>> L = [-5, 3, 0, 3, -6, 2, 1, 1]
    >>> selection_sort(L)
    >>> L
    [-6, -5, 0, 1, 1, 2, 3, 3]
    """

    i = 0
    while i != len(L):
        smallest = find_min(L, i)
        L[i], L[smallest] = L[smallest], L[i]
        i = i + 1
```

As with binary search, the doctest is so long that, as documentation for the function, it obscures rather than helps clarify. Again, we'll see how to fix this in Chapter 15, *Testing and Debugging*, on page 235.

Insertion Sort

Like selection sort, *insertion sort* keeps a sorted section at the beginning of the list. Rather than scan all of the unsorted section for the next smallest item, though, it takes the next item from the unsorted section—the one at

index i—and inserts it where it belongs in the sorted section, increasing the size of the sorted section by one.

```
i = 0  # The index of the first unknown item in lst.  lst[:i] is sorted.

while i != len(L):
    move the item at index i to where it belongs in lst[:i + 1]

    i = i + 1
```

The reason why we use lst[i + 1] is because the item at index i may be larger than everything in the sorted section, and if that is the case then the current item won't move.

In outline, this is as follows:

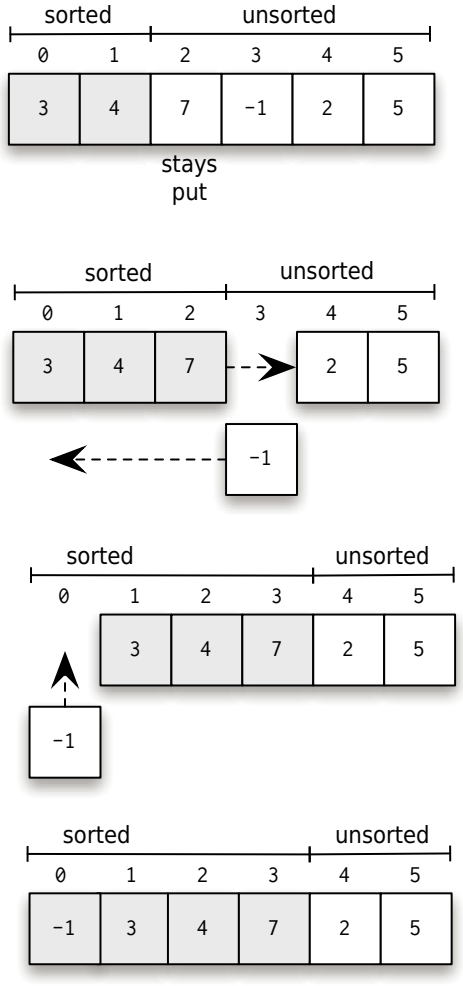
```
def insertion_sort(L):
    """(list) -> NoneType

    Reorder the items in L from smallest to largest.

    >>> L = [3, 4, 7, -1, 2, 5]
    >>> insertion_sort(L)
    >>> L
    [-1, 2, 3, 4, 5, 7]
    """

    i = 0
    while i != len(L):
        # Insert L[i] where it belongs in L[0:i+1].
        i = i + 1
```

This is exactly the same approach as for selection sort; the difference is in the comment in the loop. Like we did with selection sort, we'll write a helper function do to that work:

```
def insert(L, b):
    """(list, int) -> NoneType

    Precondition: L[0:b] is already sorted.

    Insert L[b] where it belongs in L[0:b + 1].

    >>> L = [3, 4, -1, 7, 2, 5]
    >>> insert(L, 2)
    >>> L
    [-1, 3, 4, 7, 2, 5]
    >>> insert(L, 4)
    >>> L
    [-1, 2, 3, 4, 7, 5]
    """
```

```
    # Find where to insert L[b] by searching backwards from L[b]
    # for a smaller item.
    i = b
    while i != 0 and L[i - 1] >= L[b]:
        i = i - 1

    # Move L[b] to index i, shifting the following values to the right.
    value = L[b]
    del L[b]
    L.insert(i, value)


def insertion_sort(L):
    """(list) -> NoneType

    Reorder the items in L from smallest to largest.

    >>> L = [3, 4, 7, -1, 2, 5]
    >>> insertion_sort(L)
    >>> L
    [-1, 2, 3, 4, 5, 7]
    """

    i = 0
    while i != len(L):
        insert(L, i)
        i = i + 1
```

How does `insert` work? It works by finding out where `L[b]` belongs and then moving it. Where does it belong? It belongs after every value less than or equal to it and before every value that is greater than it. We need the check `i != 0` in case `L[b]` is smaller than every value in `L[0:b]`, which will place the current item at the beginning of the list.



This passes all the tests we wrote earlier for selection sort.

Performance

We now have two sorting algorithms. Which should we use? Because both are not too difficult to understand, it's reasonable to decide based on how fast they are.

It's easy enough to write a program to compare their running times, along with that for list.sort:

```
import time
import random
from sorts import selection_sort
```

```
from sorts import insertion_sort


def built_in(L):
    ''' (list) -> NoneType
    Call list.sort --- we need our own function to do this so that we can
    treat it as we treat our own sorts.'''

    L.sort()


def print_times(L):
    '''(list) -> NoneType

    Print the number of milliseconds it takes for selection sort, insertion
    sort, and list.sort to run.
    '''

    print(len(L), end='\t')
    for func in (selection_sort, insertion_sort, built_in):
        if func in (selection_sort, insertion_sort) and len(L) > 10000:
            continue

        L_copy = L[:]
        t1 = time.time()
        func(L_copy)
        t2 = time.time()
        print("{0:7.1f}".format((t2 - t1) * 1000.), end='\t')
    print()  # Print a newline.

for list_size in [10, 1000, 2000, 3000, 4000, 5000, 10000]:
    L = list(range(list_size))
    random.shuffle(L)
    print_times(L)
```

List Length	Selection Sort	Insertion Sort	list.sort
1000	148	64	0.3
2000	583	268	0.6
3000	1317	594	0.9
4000	2337	1055	1.3
5000	3699	1666	1.6
10000	14574	6550	3.5

Table 21—Running times for selection, insertion, and list.sort, milliseconds

The results are shown in Table 21, *Running times for selection, insertion, and list.sort, milliseconds*, on page 221. Something is very clearly wrong, because our sorting functions are thousands of times slower than the built-in function.

What's more, the time required by our routines is growing faster than the size of the data. On 1,000 items, for example, selection sort takes about 0.15 milliseconds per item, but on 10,000 items, it needs about 1.45 milliseconds per item—a tenfold increase! What is going on?

To answer this, we examine what happens in the inner loops of our two algorithms. On the first iteration of selection sort, the inner loop examines *every* element to find the smallest. On the second iteration, it looks at all but one; on the third, it looks at all but two, and so on.

If there are N items in the list, then the number of iterations of the inner loop, in total, is roughly $N + (N\text{-}1) + (N\text{-}2) + \ldots + 1$, or $^{N(N+1)}/_{2}$. Putting it another way, the number of steps required to sort N items is roughly proportional to N^2+N. For large values of N, we can ignore the second term and say that the time needed by selection sort grows as the square of the number of values being sorted. And indeed, examining the timing data further shows that doubling the size of the list increases the running time by four.

The same analysis can be used for insertion sort, since it also examines one element on the first iteration, two on the second, and so on. (It's just examining the already-sorted values, rather than the unsorted values.)

So, why is insertion sort slightly faster? The reason is that, on average, only half of the values need to be scanned in order to find the location in which to insert the new value, while with selection sort, *every* value in the unsorted section needs to be examined in order to select the smallest one. *But, wow, list.sort is so much faster!*

13.4 More Efficient Sorting Algorithms

The analysis of selection and insertion sort begs the question: how can list.sort be so much more efficient? The answer is the same as it was for binary search: by taking advantage of the fact that some values are already sorted.

A First Attempt

Consider the following function:

```
import bisect


def bin_sort(values):
    """(list) -> list

    Return a sorted version of the values.  (This does not mutate values.)

    >>> L = [3, 4, 7, -1, 2, 5]
```

```
>>> bin_sort(L)
>>> L
[-1, 2, 3, 4, 5, 7]
"""

result = []
for v in values:
    bisect.insort_left(result, v)
return result
```

This code uses bisect.insort_left to figure out where to put each value from the original list into a new list that is kept in sorted order. As we have already seen, doing this takes time proportional to $log_2 N$, where N is the length of the list. Since N values have to be inserted, the overall running time ought to be $N\ log_2 N$. As shown in Table 22, *Sorting times*, on page 223, this grows much more slowly with the length of the list than N^2.

N	N^2	$N\ log_2 N$
10	100	3.32
100	10,000	6.64
1000	1,000,000	9.96

Table 22—Sorting times

Unfortunately, there's a flaw in this analysis. It's correct to say that bisect.insort_left needs only $log_2 N$ time to figure out where to insert a value, but actually inserting it takes time as well. To create an empty slot in the list, we have to move all the values above that slot up one place. On average, this means copying half of the list's values, so the cost of insertion is proportional to N. Since there are N values to insert, our total time is $N(N + log_2 N)$. For large values of N, this is once again roughly proportional to N^2.

13.5 Mergesort: An $Nlog_2N$ Algorithm

There are several well-known fast sorting algorithms; mergesort, quicksort, and heapsort are the ones you are most likely to encounter in a future CS course. Most of them involve techniques that we haven't yet taught you, but mergesort can be written to be more accessible. Mergesort is built around the idea that taking two sorted lists and merging them is proportional to the number of items in both lists. We'll start with very small lists and keep merging them until we have a single sorted list.

Merging Two Sorted Lists

Given two sorted lists L1 and L2, we can produce a new sorted list by running along L1 and L2 and comparing pairs of elements. (We'll see how to produce these two sorted lists in a bit.)

Here is the code for merge:

```
def merge(L1, L2):
    """(list, list) -> list

    Merge sorted lists L1 and L2 into a new list and return that new list.

    >>> merge([1, 3, 4, 6], [1, 2, 5, 7])
    [1, 1, 2, 3, 4, 5, 6, 7]
    """

    newL = []
    i1 = 0
    i2 = 0

    # For each pair of items L1[i1] and L2[i2], copy the smaller into newL.
    while i1 != len(L1) and i2 != len(L2):
        if L1[i1] <= L2[i2]:
            newL.append(L1[i1])
            i1 += 1
        else:
            newL.append(L2[i2])
            i2 += 1

    # Gather any leftover items from the two sections.
    # Note that one of them will be empty because of the loop condition.
    newL.extend(L1[i1:])
    newL.extend(L2[i2:])

    return newL
```

i1 and i2 are the indices into L1 and L2, respectively; in each iteration, we compare L1[i1] to L2[i2] and copy the smaller item to the resulting list. At the end of the loop, we have run out of items in one of the two lists, and the two extend calls will append the rest of the items to the result.

Mergesort

Here is the header for mergesort:

```
def mergesort(L):
    """(list) -> NoneType

    Reorder the items in L from smallest to largest.
```

```
>>> L = [3, 4, 7, -1, 2, 5]
>>> mergesort(L)
>>> L
[-1, 2, 3, 4, 5, 7]
"""
```

Mergesort uses function merge to do the bulk of the work. Here is the algorithm, which creates and keeps track of a list of lists:

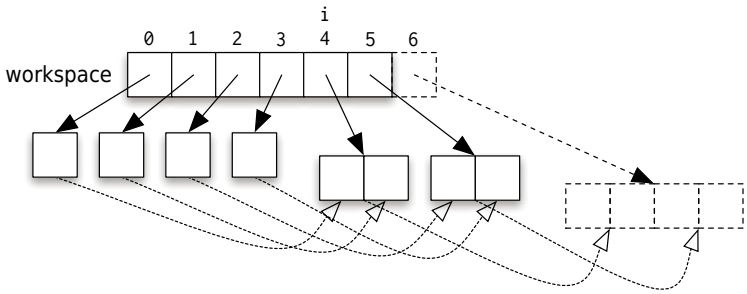
- Take list L, and make a list of one-item lists from it.
- As long as there are two lists left to merge, merge them, and append the new list to the list of lists.

The first step is straightforward:

```
# Make a list of 1-item lists so that we can start merging.
workspace = []
for i in range(len(L)):
    workspace.append([L[i]])
```

The second step is trickier. If we remove the two lists, then we'll run into the same problem that we ran into in binsort: all the following lists will need to shift over, which takes time proportional to the number of lists.

Instead, we'll keep track of the index of the next two lists to merge. Initially, they will be at indices 0 and 1, and then 2 and 3, and so on:



Here is our refined algorithm:

- Take list L, and make a list of one-item lists from it.
- Start index i off at 0.
- As long as there are two lists (at indices i and i + 1), merge them, append the new list to the list of lists, and increment i by 2.

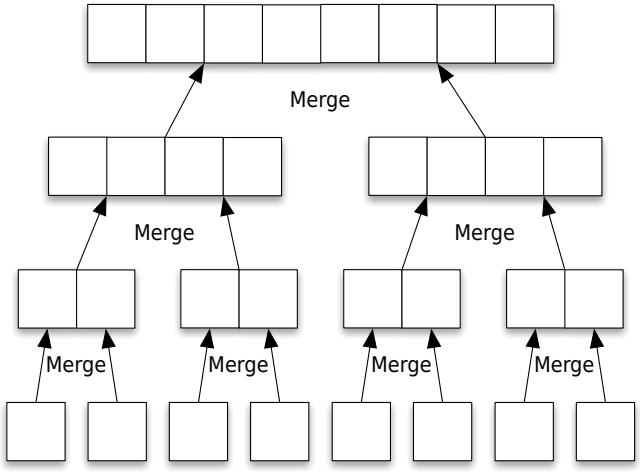
With that, we can go straight to code:

```
def mergesort(L):
    """(list) -> NoneType

    Reorder the items in L from smallest to largest.

    >>> L = [3, 4, 7, -1, 2, 5]
    >>> mergesort(L)
    >>> L
    [-1, 2, 3, 4, 5, 7]
    """

    # Make a list of 1-item lists so that we can start merging.
    workspace = []
    for i in range(len(L)):
        workspace.append([L[i]])

    # The next two lists to merge are workspace[i] and workspace[i + 1].
    i = 0

    # As long as there are at least two more lists to merge, merge them.
    while i < len(workspace) - 1:
        L1 = workspace[i]
        L2 = workspace[i + 1]
        newL = merge(L1, L2)
        workspace.append(newL)
        i += 2

    # Copy the result back into L.
    if len(workspace) != 0:
        L[:] = workspace[-1][:]
```

Notice that, since we're always making new lists, we need to copy the last of the merged lists back into parameter `L`.

Mergesort Analysis

Mergesort, it turns out, is $N \log_2 N$, where `N` is the number of items in `L`. This picture shows the 1-item lists getting merged into 2-item lists, then 4-item lists, and so on until there is one N-item list:



The first part of the function, creating the list of one-item lists, takes N iterations, one for each item.

The second loop, in which we continually merge lists, will take some care to analyze. We'll start with the very last iteration, in which we are merging two lists with about $N/2$ items. As we've seen, function merge copies each element into its result exactly once, so with these two lists, this merge step takes roughly N steps.

On the previous iteration, there are two lists of size $N/4$ to merge into one of the two lists of size $N/2$, and on the iteration before that there are another two lists of size $N/2$ to merge into the second list of size $N/2$. Each of these two merges takes roughly $N/2$ steps, so the two together take roughly N steps in total.

On the iteration before that, there are a total of eight lists of size $N/8$ to merge into the four lists of size $N/4$. Four merges of this size together also take roughly N steps.

We can subdivide a list with N items a total of $log_2 N$ times, using an analysis much like we used for binary search. Since at each "level" there are a total of N items to be merged, each of these $log_2 N$ levels takes roughly N steps. Hence, mergesort take time proportional to $N log_2 N$.

That's a awful lot of code to sort a list! There are shorter and clearer versions, but again, they rely on techniques that we haven't yet introduced.

Despite all the code and our somewhat messy approach (it creates a *lot* of sublists), mergesort turns out to be much, much faster than selection sort

and insertion sort. More importantly, it grows at the same rate as the built-in sort:

List Length	Selection Sort	Insertion Sort	Mergesort	list.sort
1000	148	64	7	0.3
2000	583	268	15	0.6
3000	1317	594	23	0.9
4000	2337	1055	32	1.3
5000	3699	1666	41	1.6
10000	14574	6550	88	3.5

Table 23—Running times for selection, insertion, merge, and list.sort, milliseconds

Big-Oh and All That

Our method of analyzing the performance of searching and sorting algorithms might seem like hand-waving, but there is actually a well-developed mathematical theory behind it. If *f* and *g* are functions, then the expression *f(x) = O(g(x))* is read "f is big-oh of g" and means that for sufficiently large values of *x*, *f(x)* is bounded above by some constant multiple of *g(x)*, or equivalently that function *g* gives us an upper bound on the values of the function *f*. Computer scientists use this to group algorithms into families, such as those sorting functions that execute in N^2 time and those that execute in $N \log_2 N$ time.

These distinctions have important practical applications. In particular, one of the biggest puzzles in theoretical computer science today is whether two families of algorithms (called *P* and *NP* for reasons that we won't go into here) are the same or not. Almost everyone thinks they aren't, but no one has been able to prove it (despite the offer of a million-dollar prize for the first correct proof). If it turns out that they *are* the same, then many of the algorithms used to encrypt data in banking and military applications (as well as on the Web) will be much more vulnerable to attack than expected.

13.6 Sorting Out What You Learned

In this chapter, you learned the following:

- An invariant describes how a loop works. The initial values for the variables used in the loop will establish the invariant, and the work done inside the loop will make progress toward the solution. When the loop terminates, the invariant is still true, but the solution will have been reached.
- Linear search is the simplest way to find a value in a list, but on average, the time required is directly proportional to the length of the list.

- Binary search is much faster—the average time is proportional to the logarithm of the list's length—but it works only if the list is in sorted order.
- Similarly, the average running time of simple sorting algorithms like selection sort is proportional to the square of the input size *N*, while the running time of more complex sorting algorithms grows as $N \log_2 N$.
- Looking at how the running time of an algorithm grows as a function of the size of its inputs is the standard way to analyze and compare algorithms' efficiency.
- Selection sort and insertion sort have the same invariant. They differ by how they make progress: selection sort selects the next-smallest item to put at the end of the sorted section, while insertion sort insert the next item into the sorted section.

13.7 Exercises

Here are some exercises for you to try on your own:

1. All three versions of linear search start at index 0. Rewrite all three to search from the end of the list instead of the beginning. Make sure you test them.
2. For the new versions of linear search: if there are duplicate values, which do they find?
3. Binary search is significantly faster than the built-in search but requires that the list is sorted. As you know, the running time for the best sorting algorithm is on the order of $N \log_2 N$, where *N* is the length of the list. If we search a lot of times on the same list of data, it makes sense to sort it once before doing the searching; roughly how many times do we need to search in order to make sorting and then searching faster, instead of using the built-in search?
4. Given the unsorted list `[6, 5, 4, 3, 7, 1, 2]`, show what the contents of the list would be after each iteration of the loop as it is sorted using the following:
   a. Selection sort
   b. Insertion sort
5. Another sorting algorithm is *bubble sort*. Bubble sort involves keeping a sorted section at the end of the list. The list is traversed, pairs of elements are compared, and larger elements are swapped into the higher position. This is repeated until all element are sorted.

a. Using the English description of bubble sort, write an outline of the bubble sort algorithm in English.

b. Continue using top-down design until you have a Python algorithm.

c. Turn it into a function `bubble_sort(L)`.

d. Try it out on the test cases from `selection_sort`.

6. In the description of bubble sort in the previous question, the sorted section of the list was at the end of the list. In this question, bubble sort will maintain the sorted section of the beginning of the list. Make sure that you are still implementing bubble sort!

   a. Rewrite the English description of bubble sort from the previous question with the necessary changes so that the sorted elements are at the beginning of the list instead of the end.

   b. Using your English description of bubble sort, write an outline of the bubble sort algorithm in English.

   c. Write the `bubble_sort_2(L)` function.

   d. Try it out on the test cases from `selection_sort`.

7. Modify the timing program to compare bubble sort with insertion and selection sort. Explain the results.

8. The analysis of `bin_sort` said, "Since N values have to be inserted, the overall running time is $N \log_2 N$." Point out a flaw in this reasoning, and explain whether it affects the overall conclusion.

9. There are at least two ways to come up with loop conditions. One of them is to answer the question "When is the work done?" and then negate it. In function `merge` in *Merging Two Sorted Lists*, on page 224, the answer is "when we run out of items in one of the two lists," which is described by this expression: `i1 == len(L1) or i2 == len(L2)`. Negating this leads to our condition `i1 != len(L1) and i2 != len(L2)`.

   Another way to come up with a loop condition is to ask "What are the valid values of the loop index?" In function `merge`, the answer to this is `0 <= i1 < len(L1) and 0 <= i2 < len(L2)`; since `i1` and `i2` start a zero, we can drop the comparisons with zero, giving us `i1 < len(L1) and i2 < len(L2)`.

   Is there another way to do it? Have you tried both approaches? Which do you prefer?

10. In function mergesort in *Mergesort*, on page 224, there are two calls to extend. They are there because, when the preceding loop ends, one of the two lists still has items in it that haven't been processed. Rewrite that loop so that these extend calls aren't needed.

CHAPTER 14

Object-Oriented Programming

This is a beta book; this chapter is not yet complete.

CHAPTER 15

Testing and Debugging

How can you tell whether the programs you write work correctly? Following the Function Design Recipe from Section 3.5, *Designing New Functions: a Recipe*, on page 49, you include an example call or two in the docstring. The last step of the recipe is calling your function to make sure that it returns what you expect. But are one or two calls enough? If not, how many do you need? How do you pick the arguments for those function calls? In this chapter, you will learn how to choose good test cases and how to test your code using Python's unittest module.

Finally, what happens if your tests fail, revealing a bug (see Section 1.3, *What's a Bug?*, on page 5)? How can you tell where the problem is in your code? In this chapter, you'll also learn how to find and fix bugs in your programs.

15.1 Why Do You Need To Test?

Quality assurance, or QA, checks that software is working correctly. Over the last fifty years, programmers have learned that quality isn't some kind of magic pixie dust that you can sprinkle on a program after it has been written. Quality has to be designed in, and software must be tested and retested to check that it meets standards.

The good news is that putting effort into QA actually makes you more productive overall. The reason can be seen in Boehm's curve in Figure 7, *Boehm's curve: the later a bug is discovered, the more expensive it is to fix*, on page 236. The later you find a bug, the more expensive it is to fix, so catching bugs early reduces overall effort.

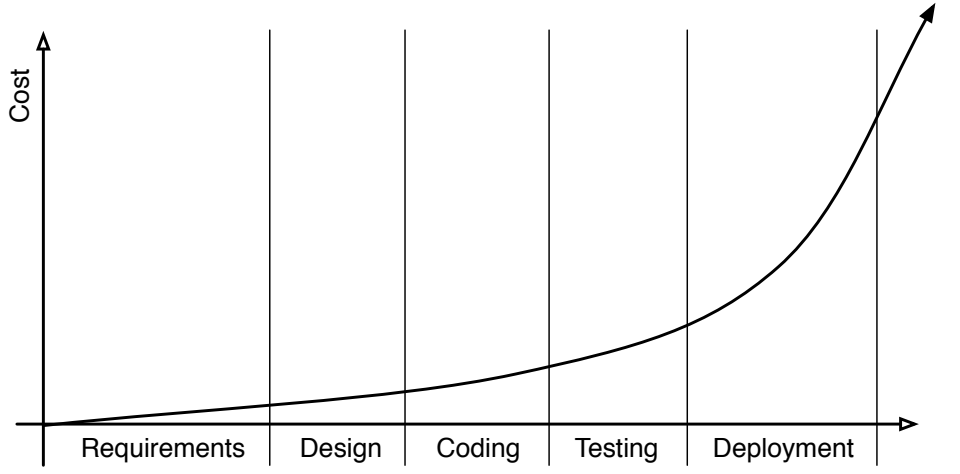


Figure 7—Boehm's curve: the later a bug is discovered, the more expensive it is to fix

Most good programmers today don't just test their software while writing it; they build their tests so that other people can rerun them months later and a dozen time zones away. This takes a little more time up front but makes programmers more productive overall, since every hour invested in preventing bugs saves two, three, or ten frustrating hours tracking bugs down.

In Section 6.3, *Testing Your Code Semi-Automatically*, on page 112, you learned how to run tests using Python's `doctest` module. As part of the Function Design Recipe (see Section 3.5, *Designing New Functions: a Recipe*, on page 49), you include example calls on your function in the docstring. You can then use the `doctest` module to execute those function calls and have it compare the output you expect with the actual output produced by that function call.

15.2 Case Study: Testing above_freezing

The first function that we'll test is `above_freezing` from Section 6.3, *Testing Your Code Semi-Automatically*, on page 112:

```
def above_freezing(celsius):
    """ (number) -> bool

    Return True iff temperature celsius degrees is above freezing.

    >>> above_freezing(5.2)
    True
    >>> above_freezing(-2)
    False
    """
```

```
    return celsius > 0
```

In that section, we ran the example calls from the docstring using doctest. But we're missing a test: what happens if the temperature is zero? In the next section, we'll write another version of this function that behaves differently at zero, and we'll discuss how our current set of tests is incomplete.

Choosing Test Cases for above_freezing

Before writing our testing code, we must decide which test cases to use. The above_freezing function takes one argument, a number, so for each test case, we need to choose the value of that argument. There are millions of numbers to choose from and we can't possibly test them all, so how do we decide which values to use? For above_freezing, there are two categories of numbers: values below freezing, and values above freezing. We'll pick one value from each category to use in our test cases.

Looking at it another way, this particular function returns a Boolean, so we need at least two tests: one that causes the function to return True and another that causes it to return False. In fact, that's what we already did in our example function calls in the docstring.

In above_freezing's' docstring, the first example call uses 5.2 as the value of the argument, and that value is above freezing so the function should return True. This test case represents the temperatures that are above freezing. We chose that value from among the billions of possibly positive floating-poing values; any one of them would work just as well. For example, we could have used 100.6, 29, 357.32 or another other number greater than 0 to represent the "above freezing" category.

The second example call uses -2, which represents the the temperatures that are below freezing. As before, we could have used -16, -294.3, -56.97 or any other value less than than 0 to represent the "below freezing" category, but chose to use -2. Again, our choice is arbitrary.

Are we missing any test case categories? Imagine that we had written our code using the >= operator instead of the > operator:

```
def above_freezing_v2(celsius):
    """ (number) -> bool

    Return True iff temperature celsius degrees is above freezing.

    >>> above_freezing_v2(5.2)
    True
```

```
    >>> above_freezing_v2(-2)
    False
    """

    return celsius >= 0
```

Both versions of the function produce the expected results for the two docstring examples, but the code is different from before, and won't produce the same result in all cases. We neglected to test one category of inputs: temperatures at the freezing mark. Test cases like the one at the freezing mark are often called *boundary cases*, since they lie on the boundary between two different possible behaviors of the function (in the case, between temperatures above freezing and temperatures below freezing). Experience shows that boundary cases are much more likely to contain bugs than other cases, so it's always worth figuring out what they are and testing them.

Sometimes there are multiple boundary cases. For example, if we had a function that determined which state water was in—solid, liquid, or gas—then the two boundary cases would be the freezing point and the boiling point.

To summarize, table Table 24, *Test Cases for above_freezing*, on page 238 shows each category of inputs, the value we chose to represent that category, and the value that we expect the call on the function to return in that case:

Test Case Description	Argument value	Expected return value
temperatures above freezing	5.2	True
temperatures below freezing	-2	False
temperatures at freezing	0	False

Table 24—Test Cases for above_freezing

Now that all categories of inputs are covered, we need to run the third test. Running the third test in the Python shell reveals that the value returned by `above_freezing_v2` isn't `False`, which is what we expected:

```
>>> above_freezing(0)
False
>>> above_freezing_v2(0)
True
```

It took three test cases to cover all categories of inputs for this function, but three isn't a magic number. The three tests had to be carefully chosen. If the three tests had all fallen into the same category (say, temperatures above freezing, `5`, `70` and `302`) they wouldn't have been sufficient. It's the quality of the tests that matters, not the quantity.

Testing above_freezing Using unittest

Once you decide which test cases are needed, you can use one of two approaches that you've learned about so far to actually test the code. The first is to call the functions and read the results yourself to see if they match what you expected. The second is running the functions from the docstring using the doctest module. The latter approach is preferable, because the comparison of the actual value returned by the function to the value we expect to be returned is done by the program and not by a human, so it's faster and less error prone.

In this section, we'll introduce another of Python's modules, unittest. A *unit test* exercises just one isolated component of a program. Like we did with doctest, we'll use the unittest module to test each function in our module independently from the others. This approach contrasts with *system testing*, which looks at the behavior of the system as a whole, just as its eventual users will.

In the (as yet) unwritten *sec.oop-inheritance-ex* I don't know how to generate a cross reference to sec.oop-inheritance-ex, you learned how to write classes that inherit code from others. Now, you'll write test classes that inherit from class unittest.TestCase. Our first test class tests the above_freezing function:

```
import unittest
import temperature


class TestAboveFreezing(unittest.TestCase):
    """Tests for temperature.above_freezing."""

    def test_above_freezing_above(self):
        """Test a temperature that is above freezing."""

        expected = True
        actual = temperature.above_freezing(5.2)
        self.assertEqual(expected, actual,
            "The temperature is above freezing.")

    def test_above_freezing_below(self):
        """Test a temperature that is below freezing."""

        expected = False
        actual = temperature.above_freezing(-2)
        self.assertEqual(expected, actual,
            "The temperature is below freezing.")

    def test_above_freezing_at_zero(self):
        """Test a temperature that is at freezing."""
```

```
        expected = False
        actual = temperature.above_freezing(0)
        self.assertEqual(expected, actual,
            "The temperature is at the freezing mark.")

unittest.main()
```

The name of our new class is TestAboveFreezing and it's saved in file test_above_freezing.py. The class has three of its own methods, one per each test case. Each test case follows this pattern:

```
expected = «the value we expect will be returned»
actual = «call on the function being tested»
self.assertEqual(expected, actual,
    "Error message in case of failure")
```

In each test method, there is a call on the assertEqual method inherited from unittest.TestCase. To *assert* something is to claim that it is true; here we are asserting that the expected value and actual value should be equal. The assertEqual method compares its first two arguments (which are the expected return value and actual return value from calling the function being tested) to see whether they are equal. If they aren't equal, the third argument, a string, is displayed as part of the failure message.

At the bottom of the file, the call on unittest.main() executes every method that begins with the name test.

When the program in test_above_freezing.py is executed, the following results are produced:

```
...
----------------------------------------------------------------------
Ran 3 tests in 0.000s

OK
```

The first line of output has three dots, one dot per test method. A dot indicates that a test was run successfully—that the test case *passed*.

The summary after the dashed line tells you that unittest found and ran three tests, that it took less than a millisecond to do so, and everything was successful (OK):

If our faulty function above_freezing_v2 was renamed above_freezing and our test_above_freezing unit test program was re-run, rather than three passes (as indicated by the three dots), there would be two passes and a failure:

```
.F.
======================================================================
```

```
FAIL: test_above_freezing_at (__main__.TestAboveFreezing)
Test a temperature that is at freezing.
----------------------------------------------------------------------
Traceback (most recent call last):
  File "test_above_freezing.py", line 33, in test_above_freezing_at
    "The temperature is at the freezing mark.")
AssertionError: False != True : The temperature is at the freezing mark.

----------------------------------------------------------------------
Ran 3 tests in 0.001s

FAILED (failures=1)
```

The F indicates that a test case *failed*. The error message tells you that the failure happened in method test_above_freezing_at_zero. The error is an AssertionError, which indicates that when we asserted that the expected and actual value should be equal, we were wrong.

The expression False != True comes from our call on assertEqual: variable expected was False, variable actual was True, and of course those aren't equal. Additionally, the string that was passed as the third argument to assertEqual is part of that error message: "The temperature is at the freezing mark."

Notice that the three calls on assertEqual were placed in three separate methods. We could have put them all in the same method, but that method would have been considered a single test case. That is, when the module was run, we would see only one result: if any of the three calls on assertEqual failed, the entire test case would have failed. Only when all three passed would we see the coveted dot.

As a rule, each test case you design should be implemented in its own test method.

Now that you've seen both doctest and unittest, which should you use? We prefer unittest, for several reasons:

- For large test suites, it is nice to have the testing code in a separate file, rather than in a very long docstring.
- Each test case can be in a separate method, so the tests are independent of each other. With doctest, the changes to objects made by one test persist for the subsequent test, so more care needs to be taken to properly set up the objects for each doctest test case to make sure they are independent.

- Because each test case is in a separate method, we can write a docstring that describes the test case tested so that other programmers understand how the test cases differ from each other.
- The third argument to assertEqual is a string that appears as part of the error message produced by a failed test, which is helpful for providing a better description of the test case. With doctest, there is no straighforward way to customize the error messages.

15.3 Case Study: Testing running_sum

In Section 15.2, *Case Study: Testing above_freezing*, on page 236, we tested a program that involved only immutable types. In this section, you'll learn how to test functions involving mutable types, like lists and dictionaries.

Suppose we need to write a function that modifies a list so that it contains a running sum of the values in it. For example, if the list is [1, 2, 3], the list should be mutated so that the first value is 1, the second value is the sum of the first two numbers, 1 + 2, and the third value is the sum of the first three numbers, 1 + 2 + 3, so we expect that the list [1, 2, 3] will be modified to be [1, 3, 6]. Following the Function Design Recipe (see Section 3.5, *Designing New Functions: a Recipe*, on page 49), here is a file named sums.py containing the completed function with one (passing) example test:

```
def running_sum(L):
    """ (list of number) -> NoneType

    Modify L so that it contains the running sums of its original items.

    >>> L = [4, 0, 2, -5, 0]
    >>> running_sum(L)
    >>> L
    [4, 4, 6, 1, 1]
    """

    for i in range(len(L)):
        L[i] = L[i - 1] + L[i]
```

The structure of the test in the docstring is different from what you've seen before. Because there is no return statement, running_sum returns None. Writing a test that checks whether None is returned isn't enough to know whether the function call worked as expected. You also need to check whether the list passed to the function is mutated in the way you expect it to be. To do this, we follow these steps:

- Create a variable that refers to a list.

- Call the function, passing that variable as an argument to it.
- Check whether the list that the variable refers to was mutated correctly.

Following those steps, we created a variable `L` that refers to the list `[4, 0, 2, -5, 0]`, called `running_sum(L)`, and confirmed that `L` now refers to `[4, 4, 6, 1, 1]`.

Although this test case passes, it doesn't guarantee that the function will always work—and in fact there is a bug. In the next section, we'll design a set of test cases to more thoroughly test this function and discover the bug.

Choosing Test Cases for running_sums

The `running_sums` function has one parameter, which is a `list of number`. For our test cases, we need to decide both on the size of the list and the values of the items. For size, we should test with the empty list, a short list with one item and another with two items (the shortest case where two numbers interact), and a longer list with several items.

When passed either the empty list or a list of length of length one, the modified list should be the same as the original.

When passed a two-number list, the first number should be unchanged and the second number should be changed to be the sum of the two original numbers.

For longer lists, things get more interesting. The values can be negative, positive or zero, so the resulting values might be bigger than, the same as or less than they were originally. We'll divide our test of longer lists into four cases: all negative values, all zero, all positive values, and a mix of negative, zero and positive values. The resulting tests are shown in

Test Case Description	List before	List after
empty list	[]	[]
one-item list	[5]	[5]
two-item list	[2, 5]	[2, 7]
multiple items, all negative	[-1, -5, -3, -4]	[-1, -6, -9, -13]
multiple items, all zero	[0, 0, 0, 0]	[0, 0, 0, 0]
multiple items, all positive	[4, 2, 3, 6]	[4, 6, 9, 15]
multiple items, mixed	[4, 0, 2, -5, 0]	[4, 4, 6, 1, 1]

Table 25—Test Cases for running_sum

Now that we've decided on our test cases, the next step is implementing them using `unittest`.

Testing running_sum Using unittest

To test running_sums, we'll use this subclass of unittest.TestCase, name TestRunningSum:

```
import unittest
import sums as sums


class TestRunningSum(unittest.TestCase):
    """Tests for temperature.above_freezing."""

    def test_running_sum_empty(self):
        """Test an empty list."""

        argument = []
        expected = []
        sums.running_sum(argument)
        self.assertEqual(expected, argument, "The list is empty.")

    def test_running_sum_one_item(self):
        """Test a one-item list."""

        argument = [5]
        expected = [5]
        sums.running_sum(argument)
        self.assertEqual(expected, argument, "The list contains one item.")

    def test_running_sum_two_items(self):
        """Test a two-item list."""

        argument = [2, 5]
        expected = [2, 7]
        sums.running_sum(argument)
        self.assertEqual(expected, argument, "The list contains two items.")

    def test_running_sum_multi_negative(self):
        """Test a list of negative values."""

        argument = [-1, -5, -3, -4]
        expected = [-1, -6, -9, -13]
        sums.running_sum(argument)
        self.assertEqual(expected, argument,
            "The list contains only negative values.")

    def test_running_sum_multi_zeros(self):
        """Test a list of zeros."""

        argument = [0, 0, 0, 0]
        expected = [0, 0, 0, 0]
        sums.running_sum(argument)
        self.assertEqual(expected, argument, "The list contains only zeros.")
```

```
    def test_running_sum_multi_positive(self):
        """Test a list of positive values."""

        argument = [4, 2, 3, 6]
        expected = [4, 6, 9, 15]
        sums.running_sum(argument)
        self.assertEqual(expected, argument,
            "The list contains only positive values.")

    def test_running_sum_multi_mix(self):
        """Test a list containing mixture of negative values, zeros and
        positive values."""

        argument = [4, 0, 2, -5, 0]
        expected = [4, 4, 6, 1, 1]
        sums.running_sum(argument)
        self.assertEqual(expected, argument,
            "The list contains a mixture of negative values, zeros and"
                        + "positive values.")

unittest.main()
```

Next, we run the tests and see that only three of them pass (the empty list, a list with several zeros, and a list with a mixture of negative values, zeros and positive values):

```
..FF.FF
======================================================================
FAIL: test_running_sum_multi_negative (__main__.TestRunningSum)
Test a list of negative values.
----------------------------------------------------------------------
Traceback (most recent call last):
  File "test_running_sum.py", line 39, in test_running_sum_multi_negative
    "The list contains only negative values.")
AssertionError: Lists differ: [-1, -6, -9, -13] != [-5, -10, -13, -17]

First differing element 0:
-1
-5

- [-1, -6, -9, -13]
+ [-5, -10, -13, -17] : The list contains only negative values.

======================================================================
FAIL: test_running_sum_multi_positive (__main__.TestRunningSum)
Test a list of positive values.
----------------------------------------------------------------------
Traceback (most recent call last):
  File "test_running_sum.py", line 56, in test_running_sum_multi_positive
```

```
    "The list contains only positive values.")
AssertionError: Lists differ: [4, 6, 9, 15] != [10, 12, 15, 21]

First differing element 0:
4
10

- [4, 6, 9, 15]
+ [10, 12, 15, 21] : The list contains only positive values.

======================================================================
FAIL: test_running_sum_one_item (__main__.TestRunningSum)
Test a one-item list.
----------------------------------------------------------------------
Traceback (most recent call last):
  File "test_running_sum.py", line 22, in test_running_sum_one_item
    self.assertEqual(expected, argument, "The list contains one item.")
AssertionError: Lists differ: [5] != [10]

First differing element 0:
5
10

- [5]
+ [10] : The list contains one item.

======================================================================
FAIL: test_running_sum_two_items (__main__.TestRunningSum)
Test a two-item list.
----------------------------------------------------------------------
Traceback (most recent call last):
  File "test_running_sum.py", line 30, in test_running_sum_two_items
    self.assertEqual(expected, argument, "The list contains two items.")
AssertionError: Lists differ: [2, 7] != [7, 12]

First differing element 0:
2
7

- [2, 7]
+ [7, 12] : The list contains two items.

----------------------------------------------------------------------
Ran 7 tests in 0.002s

FAILED (failures=4)
```

The four that failed were a list with one item, a list with two items, a list with all negative values and a list with all positive values. To find the bug, let's focus on the simplest test case, the single item list:

```
======================================================================
FAIL: test_running_sum_one_item (__main__.TestRunningSum)
Test a one-item list.
----------------------------------------------------------------------
Traceback (most recent call last):
  File "/Users/campbell/pybook/gwpy2/Book/code/testdebug/test_running_sum.
  py", line 21, in test_running_sum_one_item
    self.assertEqual(expected, argument, "The list contains one item.")
AssertionError: Lists differ: [5] != [10]

First differing element 0:
5
10

- [5]
+ [10] : The list contains one item.
```

For this test, the list argument was `[5]`. After the function call, we expected the list to be `[5]`, but the list was mutated to become `[10]`. Looking back at the function definition of `running_sum`, when `i` refers to `0`, the for loop body executes the statement `L[0] = L[-1] + L[0]`. `L[-1]` refers to the last element of the list—the `5`—and `L[0]` refers to that same value. Oops!

`L[0]` shouldn't be changed, since the running sum of `L[0]` is simply `L[0]`.

Looking at the other three failing tests, the failure messages indicate that the first different elements are those at index `0`. The same problem that we describe for the single item list, happened for these test cases as well.

So, how did those other three tests pass? In those cases, `L[-1] + L[0]` produced the same value that `L[0]` originally referred to. For example, for the list containing a mixture of values, `[4, 0, 2, -5, 0]`, the item at index `-1` happened to be `0`, so `0 + 4` evaluated to `4`, and that matched `L[0]`'s original value. Interestingly, the simple single item list test case revealed the problem, while the more complex test case that invovled a list of multiple values hid it!

To fix the problem, we can adjust the for loop header to start the running sum from index `1`, rather than from `0`:

```
def running_sum(L):
    """ (list of number) -> NoneType

    Modify L so that it contains the running sums of its original items.

    >>> L = [4, 0, 2, -5, 0]
    >>> running_sum(L)
    >>> L
    [4, 4, 6, 1, 1]
    """
```

```
    for i in range(1, len(L)):
        L[i] = L[i - 1] + L[i]
```

When the tests are re-run, all 7 tests pass:

```
.......
----------------------------------------------------------------------
Ran 7 tests in 0.000s

OK
```

In the next section, you'll be shown some general guidelines for choosing test cases.

15.4 Choosing Test Cases

Now that you've seen two example sets of tests, we'll give you an overview of things to think about while you are developing tests for other functions. Some of them overlap and not all will apply in every situation, but they are all worth thinking about while you are figuring out what to test.

Think about size When a test involves a collection such as a list, string, dictionary, or file:

- Test the empty collection.
- Test a collection with 1 item in it.
- Test a general case with several items.
- Test the smallest interesting case, such as sorting a list containing two values.

Think about dichotomies A *dichotomy* is a contrast between two things. Examples of dichotomies are empty/full, even/odd, positive/negative, and alphabetic/non-alphabetic. If a function deals with two or more different categories or situations, make sure you test all all of them.

Think about boundaries If a function behaves differently around a particular boundary or threshold, test exactly that boundary case.

Think about order If a function behaves differently when values appear in different orders, identify those orders and test each one of them. For the sorting example mentioned above, you'll want one test case where the items are in order and one where they are not.

If you carefully plan your test cases according to these ideas, and your code passes the tests, there's a very good chance that it will work for all other cases as well.

Over time, you'll commit fewer and fewer errors as you catalog them and, subsequently, become more conscious of them. And that's really the whole point of focusing on quality. The more you do it, the less likely it is for problems to arise.

15.5 Hunting Bugs

Bugs are discovered through testing and through program use, although the latter is what good testing can help avoid. Regardless of how they are discovered, tracking down and eliminating bugs in your programs is part of every programmer's life. This section introduces some techniques that can make debugging more efficient and give you more time to do the things you'd rather be doing.

Debugging a program is like diagnosing a medical condition. To find the cause, you start by working backward from the symptoms (or, in a program, its incorrect behavior), then come up with a solution, and test it to make sure it actually fixes the problem.

At least, that's the right way to do it. Many beginners make the mistake of skipping the diagnosis stage and trying to cure the program by changing things at random. Renaming a variable or swapping the order in which two functions are defined might actually fix the program, but millions of such changes are possible. Trying them one after another in no particular order can be an inefficient waste of many, many hours.

Here are some rules for tracking down the cause of a problem:

1. *Make sure you know what the program is supposed to do.* Sometimes this means doing the calculation by hand to see what the correct answer is. Other times, it means reading the documentation (or the assignment handout) carefully or writing a test.

2. *Repeat the failure.* You can debug things only when they go wrong, so find a test case that makes the program fail reliably. Once you have one, try to find a simpler one; doing this often provides enough clues to allow you to fix the underlying problem.

3. *Divide and conquer.* Once you have a test that makes the program fail, try to find the first moment where something goes wrong. Examine the

inputs to the function or block of code where the problem first becomes visible. If they are wrong, look at how they were created, and so on.

4. *Change one thing at a time, for a reason.* Replacing random bits of code on the off-chance they might be responsible for your problem is unlikely to do much good. (After all, you got it wrong the first time…) Each time you make a change, rerun your test cases immediately.

5. *Keep records.* After working on a problem for an hour, you won't be able to remember the results of the tests you've run. Like any other scientist, you should keep records. Some programmers use a lab notebook; others keep a file open in an editor. Whatever works for you, make sure that when the time comes to seek help, you can tell your colleagues exactly what you've learned.

15.6 Bugs We've Put in Your Ear

In this chapter, you learned the following:

- Finding and fixing bugs early reduces overall effort.
- When choosing test cases, you should consider size, dichotomies, boundary cases, and order.
- To test your functions, you can write subclasses of unittest's TestCase class. The advantages of using unittest include keeping the testing code separate from the code being tested, being able to keep the tests independent of each other, and being able to document each individual test case.
- To debug software, you have to know what it is supposed to do and be able to repeat the failure. Simplifying the conditions that make the program fail is an effective way to narrow down the set of possible causes.

15.7 Exercises

Here are some exercises for you to try on your own:

1. Your lab partner claims to have written a function that replaces each value in a list with twice the preceding value (and the first value with 0). For example, if the list [1, 2, 3] is passed as an argument, the function is supposed to turn it into [0, 2, 4]. Here's the code:

```
def double_preceding(values):
    """ (list of number) -> NoneType

    Replace each item in the list with twice the value of the
    preceding item, and replace the first item with 0.
```

```
>>> L = [1, 2, 3]
>>> double_preceding(L)
>>> L
[0, 2, 4]
"""

if values != []:
    temp = values[0]
    values[0] = 0
    for i in range(1, len(values)):
        values[i] = 2 * temp
        temp = values[i]
```

Although the example test passes, this code contains a bug. Write a set of unittest tests to identify the bug. Explain what the bug in this function is, and fix it.

2. Your job is to come up with tests for a function called line_intersect, which takes two lines as input and returns their intersection. More specifically:

   - Lines are represented as pairs of distinct points, such as [[0.0, 0.0], [1.0, 3.0]].
   - If the lines don't intersect, line_intersect returns None.
   - If the lines intersect in one point, line_intersect returns the point of intersection, such as [0.5, 0.75].
   - If the lines are coincident (that is, lie on top of one another), the function returns its first argument (that is, a line).

   What are the six most informative test cases you can think of? (That is, if you were allowed to run only six tests, which would tell you the most about whether the function was implemented correctly?) Write out the inputs and expected output of these six tests, and explain why you would choose them.

3. Using unittests, write four tests for a function called all_prefixes in a module called TestPrefixes.py that takes a string as its input and returns the set of all nonempty substrings that start with the first character. For example, given the string "lead" as input, all_prefixes would return the set {"l", "le", "lea", "lead"}.

4. Using unittest, write the five most informative tests you can think of for a function called is_sorted in a module called TestSorting.py that takes a list of integers as input and returns True if they are sorted in nondecreasing order, and False otherwise.

5. The following function is broken. The docstring describes what it's supposed to do:

```
def find_min_max(values):
    """ (list) -> NoneType

    Print the minimum and maximum value from values.
    """

    min = None
    max = None

    for value in values:
        if value > max:
            max = value
        if value < min:
            min = value

    print 'The minimum value is %s' % min
    print 'The maximum value is %s' % max
```

What does it actually do? What line do you need to change to fix it?

CHAPTER 16

Graphical User Interfaces

This is a beta book; this chapter is not yet complete.

CHAPTER 17

Databases

This is a beta book; this chapter is not yet complete.

APPENDIX 1

The IDLE Development Environment

This is a beta book; this appendix is not yet complete.

APPENDIX 2

Glossary

This is a beta book; this appendix is not yet complete.

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