Appendix 7

North American Aerodynamic Derivative Notation

Table A7.1 *Longitudinal normalised derivatives*

Dimensionless coefficient	*Multiplier*	*Dimensional*
$C_{x_u} = -(\mathrm{M}_0 C_{D_\mathrm{M}} + 2C_D)$	$\rho V_0 S/2m$	X_u
$C^*_{x_u} = -(\mathrm{M}_0 C_{D_\mathrm{M}} + 2C_D) + \mathrm{M}_0 C_{\tau_\mathrm{M}} \cos\kappa$	$\rho V_0 S/2m$	X^*_u
$C_{x_{\dot{w}}} = C_{x_{\dot{\alpha}}}$	$\rho S\bar{\bar{c}}/4m$	$\mathrm{X}_{\dot{w}}$
$C_{x_w} = C_L - C_{D_\alpha}$	$\rho V_0 S/2m$	X_w
C_{x_q}	$\rho V_0 S\bar{\bar{c}}/4m$	X_q
C_{x_δ}	$\rho V_0^2 S/2m$	X_δ
$C_{z_u} = -(\mathrm{M}_0 C_{L_\mathrm{M}} + 2C_L)$	$\rho V_0 S/2m$	Z_u
$C^*_{z_u} = -(\mathrm{M}_0 C_{L_\mathrm{M}} + 2C_L) - \mathrm{M}_0 C_{\tau_\mathrm{M}} \sin\kappa$	$\rho V_0 S/2m$	Z^*_u
$C_{z_{\dot{w}}} = C_{z_{\dot{\alpha}}}$	$\rho S\bar{\bar{c}}/4m$	$\mathrm{Z}_{\dot{w}}$
$C_{z_w} = -(C_D + C_{L_\alpha})$	$\rho V_0 S/2m$	Z_w
C_{z_q}	$\rho V_0 S\bar{\bar{c}}/4m$	Z_q
C_{z_δ}	$\rho V_0^2 S/2m$	Z_δ
$C_{m_u} = \mathrm{M}_0 C_{m_\mathrm{M}}$	$\rho V_0 S\bar{\bar{c}}/2I_y$	M_u
$C^*_{m_u} = \mathrm{M}_0 C_{m_\mathrm{M}} + \mathrm{M}_0 \frac{z_\tau}{\bar{\bar{c}}} C_{\tau_\mathrm{M}} \cos\kappa$	$\rho V_0 S\bar{\bar{c}}/2I_y$	M^*_u
$C_{m_{\dot{w}}} = C_{m_{\dot{\alpha}}}$	$\rho V_0 S\bar{\bar{c}}^2/4I_y$	$\mathrm{M}_{\dot{w}}$
$C_{m_w} = C_{m_\alpha}$	$\rho V_0 S\bar{\bar{c}}/2I_y$	M_w
C_{m_q}	$\rho V_0 S\bar{\bar{c}}^2/4I_y$	M_q
C_{m_δ}	$\rho V_0^2 S\bar{\bar{c}}/2I_y$	M_δ

Note:

(i) Thrust coefficient is defined $C_\tau = \tau/\frac{1}{2}\rho V_0^2 S$.
(ii) In the notational style $C_{\tau_\mathrm{M}} = \partial C_\tau/\partial \mathrm{M}$.
(iii) κ is the (upward) inclination of the thrust line with respect to the x axis.
(iv) z_τ is the normal offset of the thrust line from the cg. It is assumed that x_τ, the axial offset of the thrust line from the cg, is negligibly small.

Table A7.2 *Longitudinal dimensionless derivative equivalents*

Dimensionless derivative equivalents			
American	*British*	*American*	*British*
C_{x_u}		C_{z_α}	Z_w
$C^*_{x_u}$	X_u	C_{z_q}	$2Z_q$
$C_{x_{\dot\alpha}}$	$2X_{\dot w}$	C_{z_δ}	$Z_{\eta,\tau}$
C_{x_α}	X_w	C_{m_u}	
C_{x_q}	$2X_q$	$C^*_{m_u}$	M_u
C_{x_δ}	$X_{\eta,\tau}$	$C_{m_{\dot\alpha}}$	$2M_{\dot w}$
C_{z_u}		C_{m_α}	M_w
$C^*_{z_u}$	Z_u	C_{m_q}	$2M_q$
$C_{z_{\dot\alpha}}$	$2Z_{\dot w}$	C_{m_δ}	$M_{\eta,\tau}$

Table A7.3 *Lateral–directional normalised derivatives*

Dimensionless coefficient	*Multiplier*	*Dimensional*
C_{y_v}	$\rho V_0 S/2m$	Y_v
C_{y_β}	$\rho V_0^2 S/2m$	Y_β
C_{y_p}	$\rho V_0 Sb/4m$	Y_p
C_{y_r}	$\rho V_0 Sb/4m$	Y_r
C_{y_δ}	$\rho V_0^2 S/2m$	Y_δ
C_{l_v}	$\rho V_0 Sb/2I_x$	L_v
C_{l_β}	$\rho V_0^2 Sb/2I_x$	L_β
C_{l_p}	$\rho V_0 Sb^2/4I_x$	L_p
C_{l_r}	$\rho V_0 Sb^2/4I_x$	L_r
C_{l_δ}	$\rho V_0^2 Sb/2I_x$	L_δ
C_{n_v}	$\rho V_0 Sb/2I_z$	N_v
C_{n_β}	$\rho V_0^2 Sb/2I_z$	N_β
C_{n_p}	$\rho V_0 Sb^2/4I_z$	N_p
C_{n_r}	$\rho V_0 Sb^2/4I_z$	N_r
C_{n_δ}	$\rho V_0^2 Sb/2I_z$	N_δ

Table A7.4 *Lateral–directional dimensionless derivative equivalents*

Dimensionless derivative equivalents			
American	*British*	*American*	*British*
C_{y_v}	Y_v	C_{l_r}	$2L_r$
C_{y_β}		C_{l_δ}	$L_{\xi,\zeta}$
C_{y_p}	$2Y_p$	C_{n_v}	N_v
C_{y_r}	$2Y_r$	C_{n_β}	
C_{y_δ}	$Y_{\xi,\zeta}$	C_{n_p}	$2N_p$
C_{l_v}	L_v	C_{n_r}	$2N_r$
C_{l_β}		C_{n_δ}	$N_{\xi,\zeta}$
C_{l_p}	$2L_p$		