

Modeling and Backstepping-based Nonlinear Control Strategy for a 6 DOF Quadrotor Helicopter

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Received 19 October 2007; accepted 27 March 2008

Abstract

In this article, a nonlinear model of an underactuated six degrees of freedom (6 DOF) quadrotor helicopter is derived on the basis of the Newton-Euler formalism. The derivation comprises determining equations of the motion of the quadrotor in three dimensions and approximating the actuation forces through the modeling of aerodynamic coefficients and electric motor dynamics. The derived model composed of translational and rotational subsystems is dynamically unstable, so a sequential nonlinear control strategy is used. The control strategy includes feedback linearization coupled with a PD controller for the translational subsystem and a backstepping-based PID nonlinear controller for the rotational subsystem of the quadrotor. The performances of the nonlinear control method are evaluated by nonlinear simulation and the results demonstrate the effectiveness of the proposed control strategy for the quadrotor helicopter in quasi-stationary flights.

Keywords: underactuated systems; quadrotor helicopter; backstepping control

1 Introduction

Helicopters exhibit a number of important physical effects such as aerodynamic effects, inertial counter torques, gravity effect, gyroscopic effects, and friction, etc., which makes it difficult to design a real-time control for them.

A quadrotor helicopter is a highly nonlinear, multivariable, strongly coupled, and underactuated system (six degrees of freedom (6 DOF) with only 4 actuators). The main forces and moments acting on the quadrotor are produced by propellers. There are two propellers in the system rotating in opposite direction to balance the total torque of the system. Fig.1 shows the free body diagram and axes of a quadrotor helicopter.

In Fig.1, l represents the distance between each motor and the pivot center, ϕ , θ and ψ represent the Euler angles about the body axes x , y and z , respectively, $T_i (i=1,2,3,4)$ is the thrust force produced by each propeller marked. The earth-fixed frame is denoted by $E = \{X, Y, Z\}$, and the body-fixed frame by $B = \{x, y, z\}$.

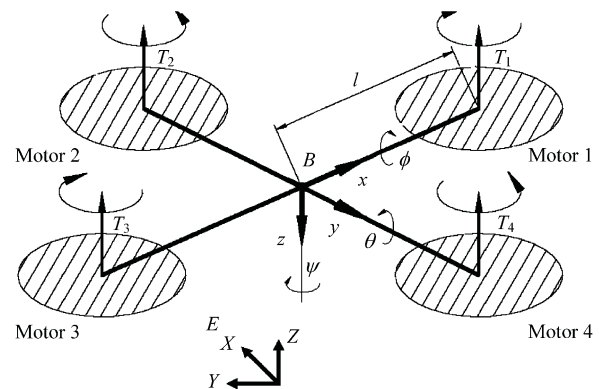


Fig.1 Forces and moments acting on a quadrotor helicopter.

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Foundation item: Higher Education Commission, Government of Pakistan (1-3/PM-OVER/China/2005)

Simultaneous increase or decrease in speed of the four motors will generate vertical motion. When the motor pair (3, 1) is allowed to operate independently, the pitch angle θ about the y axis can be controlled with the indirect control of motion along the same axis. Similarly, the independent operation of the motor pair (2, 4) could control the roll angle ϕ about the x axis with an indirect control of motion along the same axis. Finally, the counter-clockwise rotation of the pair (3, 1) and the pair (2, 4) can control the yaw angle ψ about the z axis. In this manner, the quadrotor helicopter has 6 DOF.

Most recent studies on the theoretical analysis of a 6 DOF quadrotor helicopter were carried out by means of a commercially available four rotor aerial robot (Draganflyer V Ti)^[1]. A control strategy for the quadrotor was designed by using internal linearization^[2], whereas a quaternion-based feedback control scheme was proposed for exponential attitude stabilization^[3]. However, in this case, the problem to control an underactuated quadrotor was degenerated to the one of controlling a fully actuated one.

In this article, a feedback linearization- and backstepping-based PID (BS-PID) control strategy is designed for motion control of the underactuated quadrotor. The main idea is to associate the robustness against disturbances offered by backstepping with robustness against model uncertainties by integral action. The integrator action in backstepping proposed for linear systems^[4-5] is obtained by adding the integral of tracking error to the error found in first step of backstepping procedure.

Besides higher payload capacity and better maneuverability, a quadrotor helicopter has important advantages in having small rotors and being enclosed, thereby able to be safer for indoor flights. On the other side, it is rather energy consuming and is fairly big in size.

This article is devoted towards deriving a complete dynamic model of a quadrotor helicopter on the basis of Ref.[6], and to design feedback linearization with PD control and backstepping-based PID control strategy for the nonlinear quadrotor heli-

copter with only two backstepping steps. Results from the nonlinear simulation verify the effectiveness of the proposed control strategy for the quadrotor helicopter under near quasi-stationary conditions.

2 Quadrotor Dynamics

The aerodynamic forces and moments are sought out by combining momentum with blade element theory^[7-8]. A quadrotor has four motors with propellers. The power applied to each motor generates a net torque on the rotor shaft, Q_i , which results in a thrust, T_i . If the rotor disk is rotating, there is a difference in relative velocity between the blade and the air as the rotor is moving on forward and backward sweep and causing a net moment about the roll axis, R_i . Forward velocity also causes a drag force on the rotor that acts in opposition to the direction of travel, D_i . Thrust and drag can be defined with the aerodynamic coefficients, C_T and C_D as

$$T = C_T \rho A r^2 \Omega^2 \quad (1)$$

$$D = C_D \rho A r^2 \Omega^2 \quad (2)$$

where A is a blade area, ρ the density of air, r the radius of the blade and Ω the angular velocity of a propeller.

In much the same way, the torque Q and the rolling moment R could be defined with the C_Q and C_R as

$$Q = C_Q \rho A r^2 \Omega^2 r \quad (3)$$

$$R = C_R \rho A r^2 \Omega^2 r \quad (4)$$

The total force, f_{total} , and the total moment, τ_{total} , acting on the body frame of a quadrotor are given by

$$f_{\text{total}} = -\frac{1}{2} C_{x,y,z} A_c \rho (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - mgZ + \sum_{i=1}^4 T_i z - \sum_{i=1}^4 D_i (x \ y) \quad (5)$$

$$\tau_{\text{total}} = (-1)^i \sum_{i=1}^4 Q_i z + (-1)^{i+1} \sum_{i=1}^4 R_i (x \ y) + h \cdot \sum_{i=1}^4 D_i (-y \ x) + (T_4 - T_2)lx + (T_3 - T_1)ly + \{[(D_2 - D_4) + (D_3 - D_1)]l\}z \quad (6)$$

In Eq.(5), the first term represents the friction force on the quadrotor body in horizontal motion with $C_{x,y,z}$ denoting longitudinal drag coefficients, A_c is the fuselage area, \dot{x} , \dot{y} and \dot{z} speeds in the x , y and z direction, respectively, Z is the vertical axis in inertial coordinates, (x, y) the direction of velocity, m the total mass of quadrotor, and g the force because of gravity. In Eq.(6), h is the vertical distance between propeller center and center of gravity (CG) of quadrotor.

Given the quadrotor being a single rigid body with 6 DOF, and assuming that the earth is flat neglecting ground effect, the equations of motion for a rigid body subjected to a body force, $\mathbf{f}^b \in \mathbf{R}^3$, and a body moment, $\boldsymbol{\tau}^b \in \mathbf{R}^3$, when applied at the center of mass and expressed in Newton-Euler formalism^[6], are given by

$$\begin{bmatrix} m\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{v}}^b \\ \dot{\boldsymbol{\omega}}^b \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega}^b \times m\mathbf{v}^b \\ \boldsymbol{\omega}^b \times \mathbf{J}\boldsymbol{\omega}^b \end{bmatrix} = \begin{bmatrix} \mathbf{f}^b \\ \boldsymbol{\tau}^b \end{bmatrix} \quad (7)$$

where $\mathbf{v}^b \in \mathbf{R}^3$ is the body velocity vector, $\boldsymbol{\omega}^b \in \mathbf{R}^3$ the body angular velocity vector, $m \in \mathbf{R}$ the total mass, $\mathbf{I} \in \mathbf{R}^{3 \times 3}$ an identity matrix, and $\mathbf{J} \in \mathbf{R}^{3 \times 3}$ an inertial matrix.

2.1 Rotational dynamics

Assuming that in a symmetric design of quadrotor, the inertia tensor is diagonal, the moment equation governing the quadrotor is given by

$$\boldsymbol{\tau}^b = \boldsymbol{\omega}^b \times \mathbf{J}\boldsymbol{\omega}^b + \boldsymbol{\tau}_{\text{total}} \quad (8)$$

From Eq.(6) and Eq.(8), rotational dynamics of the quadrotor in body axis are given by

$$J_x \ddot{\phi} = \dot{\theta} \dot{\psi} (J_y - J_z) + l(T_4 - T_2) + \sum_{i=1}^4 (-1)^{i+1} R_{xi} - h \sum_{i=1}^4 D_{yi} \quad (9)$$

$$J_y \ddot{\theta} = \dot{\phi} \dot{\psi} (J_z - J_x) + l(T_3 - T_1) + \sum_{i=1}^4 (-1)^{i+1} R_{yi} + h \sum_{i=1}^4 D_{xi} \quad (10)$$

$$J_z \ddot{\psi} = \dot{\phi} \dot{\theta} (J_x - J_y) + \sum_{i=1}^4 (-1)^i Q_i + [(D_{x2} - D_{x4}) + (D_{y3} - D_{y1})]l \quad (11)$$

where R_x and R_y represent the rolling moments, $h(D_x)$ and $h(D_y)$ the drag moments, and $(D_{x2} -$

$D_{x4})$ and $(D_{y3} - D_{y1})$ the drag force unbalances during forward and sideward flights, respectively.

2.2 Translational dynamics

By neglecting the effects of body moments on the translational dynamics, from Eq.(5) and Eq.(7), the translational dynamics governing the quadrotor are given by

$$m\ddot{X} = (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \cdot \left(\sum_{i=1}^4 T_i - \sum_{i=1}^4 D_i - \frac{1}{2} C_x A_c \rho \dot{x} |\dot{x}| \right) \quad (12)$$

$$m\ddot{Y} = (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \cdot \left(\sum_{i=1}^4 T_i - \sum_{i=1}^4 D_i - \frac{1}{2} C_y A_c \rho \dot{y} |\dot{y}| \right) \quad (13)$$

$$m\ddot{Z} = -mg + (\cos \phi \cos \theta) \sum_{i=1}^4 T_i - \frac{1}{2} C_z A_c \rho \dot{z} |\dot{z}| \quad (14)$$

3 Engine Model

Let a current I_a at a driving voltage V_a flow through a DC motor with inductance L_m , resistance R_m , and back electro motive force (EMF) voltage V_{emf} ; then,

$$V_a - V_{\text{emf}} = L_m \frac{dI_a}{dt} + R_m I_a \quad (15)$$

The motor converts the current into a mechanical torque applied to the shaft, $T_m = K_{\text{tm}} I_a$. The torque, which produces angular velocity ω_m according to inertia J_m and motor load T_l , is described by

$$T_m = J_m \frac{d\omega_m}{dt} + T_l \quad (16)$$

By defining $V_{\text{emf}} = K_e \omega_m$, neglecting the inductance of the motor because of its small size and introducing propeller and gearbox models, from Eq. (15) and Eq.(16) it can be obtained

$$\dot{\omega}_m = -\frac{K_{\text{tm}} K_e}{R_m J_m} \omega_m - \frac{d}{\eta r_g^3 J_m} \omega_m^2 + \frac{K_{\text{tm}}}{R_m J_m} V_a \quad (17)$$

where η is the gear box efficiency, d the drag factor, and r_g the gear reduction ratio.

4 Control Strategy

A nonlinear control strategy is proposed to stabilize the quadrotor under near quasi-stationary

conditions, i.e., hovering or near hovering. Since only the hovering is considered, the terms in translational and rotational dynamics associated with vehicular velocity become zero. Thus, the drag forces and rolling moments because of the velocity are neglected, and the thrust and the torque coefficients are supposed to be constant, i.e.,

$$\left. \begin{aligned} T_i &= b\Omega_i^2 \\ Q_i &= d\Omega_i^2 \end{aligned} \right\} \quad (18)$$

where b and d are thrust and drag factors, respectively.

The inputs to the quadrotor, namely, the vertical force input u_1 , the roll actuator input u_2 , the pitch actuator input u_3 and the yaw moment input u_4 are defined as

$$\left. \begin{aligned} u_1 &= b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ u_2 &= b(\Omega_4^2 - \Omega_2^2) \\ u_3 &= b(\Omega_3^2 - \Omega_1^2) \\ u_4 &= d(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2) \end{aligned} \right\} \quad (19)$$

The control strategy is so designed that the altitude of the quadrotor is stabilized by using the vertical force input u_1 . The desired roll and pitch angles are formed on the rotation controller by the position subsystem. The rotation controller is used to stabilize the quadrotor under near quasi-stationary conditions with control inputs u_2 , u_3 and u_4 .

4.1 Altitude control

The altitude subsystem Eq.(14) containing vertical force input u_1 is given by

$$m\ddot{z} = -mg + u_1 \cos \phi \cos \theta \quad (20)$$

which can be linearized by selecting u_1 as

$$u_1 = \frac{mg}{\cos \phi \cos \theta} + \frac{\nu}{\cos \phi \cos \theta} \quad (21)$$

The necessary condition for Eq.(21) is $\cos \phi \cdot \cos \theta \neq 0$, where ν , a PD controller, is given by

$$\nu = -K_d \dot{z} - K_p(z - z_d) \quad (22)$$

where K_p and K_d are the proportional and the derivative positive gains and z_d the desired altitude.

4.2 Position control

Position subsystem is given by Eq.(12) and Eq.

(13). Let \dot{x}_d and \dot{y}_d be the desired speed in x and y direction, respectively; then the errors at desired and actual speed are separately given by

$$e_x = \dot{x}_d - \dot{x} \quad (23)$$

$$e_y = \dot{y}_d - \dot{y} \quad (24)$$

The desired roll and pitch angles in terms of errors between actual and desired speeds are, thus, separately given by

$$\phi_d = \arcsin(u_{e_x} \sin \psi - u_{e_y} \cos \psi) \quad (25)$$

$$\theta_d = \arcsin\left(\frac{u_{e_x}}{\cos \phi \cos \psi} - \frac{\sin \phi \sin \psi}{\cos \phi \cos \psi}\right) \quad (26)$$

where u_{e_x} and u_{e_y} are

$$u_{e_x} = \frac{K_x e_x m}{u_1}, \quad u_{e_y} = \frac{K_y e_y m}{u_1}$$

where K_x and K_y are the positive constants and u_1 is the desired vertical force input by the altitude control.

4.3 Rotational control

The backstepping-based PID control technique is designed for rotational subsystem, in which the control inputs u_2, u_3 and u_4 control the quadrotor during hovering.

Let the roll tracking error be defined as

$$e = \phi - \phi_d \quad (27)$$

The first error considered in designing the backstepping is

$$z_1 = K_1 e + K_2 \int e dt \quad (28)$$

where K_1 and K_2 are positive tuning parameters, and $\int e dt$ the integral of roll error.

Lyapunov theory is used while using the Lyapunov function z_1 as a positive definite and its time derivative as a negative semi definite,

$$V_1 = \frac{1}{2} z_1^2 \quad (29)$$

Its derivative is given by

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1 (K_1 \dot{\phi} - K_1 \dot{\phi}_d + K_2 e) \quad (30)$$

There is no control input in Eq.(30). By letting $\dot{\phi}$ be the virtual control, the desired virtual control $(\dot{\phi})_d$ is defined as

$$(\dot{\phi})_d = \dot{\phi}_d - \frac{K_2 e}{K_1} - \frac{c_1 z_1}{K_1} \quad (31)$$

where c_1 is a positive constant for increasing the convergence speed of the roll tracking loop.

Now, the virtual control $\dot{\phi}$ is the roll rate of a quadrotor with its own error

$$z_2 = \dot{\phi} - (\dot{\phi})_d = \frac{1}{K_1}(\dot{z}_1 + c_1 z_1) \quad (32)$$

The augmented Lyapunov function for the second step is given by

$$V_2 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 \quad (33)$$

The derivative of Eq.(33) is given by

$$\dot{V}_2 = z_1 \dot{z}_1 + z_2 \dot{z}_2 \quad (34)$$

By putting \dot{z}_1 and \dot{z}_2 in Eq.(34), the following can be obtained

$$\begin{aligned} \dot{V}_2 = z_2 \left[e \left(K_1^2 + \frac{c_1 K_2}{K_1} \right) + \int e dt (K_1 K_2) + \right. \\ \left. \dot{e} \left(\frac{K_2}{K_1} + c_1 \right) + \dot{\theta} \dot{\psi} \left(\frac{J_y - J_z}{J_x} \right) + \frac{I u_2}{J_x} - \ddot{\phi}_d \right] \\ - z_1 [c_1 K_1 e + c_1 K_2 \int e dt] \end{aligned} \quad (35)$$

The desirable dynamics are

$$\dot{V}_2 = -c_2 z_2 = -\frac{c_2}{K_1}(\dot{z}_1 + c_1 z_1) \quad (36)$$

where c_2 is a positive tuning parameter. By putting \dot{z}_1 and z_1 in Eq.(35), the desirable dynamics are given by

$$\dot{V}_2 = -\dot{e}(c_2) - e \left(\frac{c_2 K_2}{K_1} + c_1 c_2 \right) - \int e dt \left(\frac{c_1 c_2 K_2}{K_1} \right) \quad (37)$$

The desirable dynamics ensure negative definiteness of position tracking error, its integration, and velocity tracking error.

Eq.(35) will be negative, if u_2 is given by

$$\begin{aligned} u_2 = \frac{J_x}{I} \left[-e \left(\frac{c_2 K_2}{K_1} + c_1 c_2 + K_1^2 + \frac{c_1 K_2}{K_1} \right) - \right. \\ \left. \int e dt \left(\frac{c_1 c_2 K_2}{K_1} + K_1 K_2 \right) - \dot{e} \left(c_2 + \frac{K_2}{K_1} + c_1 \right) + \right. \\ \left. \ddot{\phi}_d - \dot{\theta} \dot{\psi} \left(\frac{J_y - J_z}{J_x} \right) \right] \end{aligned} \quad (38)$$

As a rule, Eq.(38) is a PID, where the gains of each mode are given by

$$P = \frac{c_2 K_2}{K_1} + c_1 c_2 + K_1^2 + \frac{c_1 K_2}{K_1}$$

$$I = \frac{c_1 c_2 K_2}{K_1} + K_1 K_2$$

$$D = c_2 + \frac{K_2}{K_1} + c_1$$

Considering the control law given by Eq.(38) and the characteristic equation of regulation dynamics, because the rotational subsystem is both observable and controllable, the pole placement technique is used to place the poles at desired location to find the roots of the characteristic equation. Selecting larger values for c_1 and c_2 makes the derivative of the Lyapunov function more negative, thus, making the regulation dynamics faster.

Similar to the roll subsystem, the backstepping-based PID control is designed for pitch and yaw subsystem to obtain u_3 and u_4 as follows:

$$\begin{aligned} u_3 = \frac{J_y}{I} \left[-e \left(\frac{c_4 K_4}{K_3} + c_3 c_4 + K_3^2 + \frac{c_3 K_4}{K_3} \right) - \right. \\ \left. \int e dt \left(\frac{c_3 c_4 K_4}{K_3} + K_3 K_4 \right) - \dot{e} \left(c_4 + \frac{K_4}{K_3} + c_3 \right) + \right. \\ \left. \ddot{\theta}_d - \dot{\phi} \dot{\psi} \left(\frac{J_z - J_x}{J_y} \right) \right] \end{aligned} \quad (39)$$

$$\begin{aligned} u_4 = \frac{J_z}{I} \left[-e \left(\frac{c_6 K_6}{K_5} + c_5 c_6 + K_5^2 + \frac{c_5 K_6}{K_5} \right) - \right. \\ \left. \int e dt \left(\frac{c_5 c_6 K_6}{K_5} + K_5 K_6 \right) - \dot{e} \left(c_6 + \frac{K_6}{K_5} + c_5 \right) + \right. \\ \left. \ddot{\psi}_d \right] \end{aligned} \quad (40)$$

5 Results and Discussion

The angles and their time derivatives of rotational subsystem do not depend on translation components, as the 6 DOF equations governing the quadrotor helicopter have shown. However, the translations depend on the angles. Ideally, it can be thought as two subsystems: the one of angular rotations and the other one of linear translations.

Rotational control keeps the 3D orientation of the quadrotor helicopter to the desired state. Roll and pitch angles are usually made to be zero to realize hovering. The rotational controller is responsible for compensating the initial errors, stabilizing

roll, pitch, and yaw angles and maintaining them at zero. This is accomplished by way of the backstepping-based nonlinear control law.

Table 1 summarizes different system parameters of the prototype quadrotor helicopter.

Table 1 Physical parameters of quadrotor

Parameter	Value
l/m	0.305 0
$J_x/(kg \cdot m^2)$	0.015 4
$J_y/(kg \cdot m^2)$	0.015 4
$J_z/(kg \cdot m^2)$	0.030 9
m/kg	0.615 0

Next, a closed loop system with nonlinear control algorithm is simulated. The initial conditions are $\phi = \theta = \psi = 0.524$ rad and $z_d = 1$ m. The reference inputs to the controller are $\dot{x}_d = \dot{y}_d = 0$ m/s, $z_d = 1$ m, and $\psi_d = 0$ rad.

Fig.2 shows the response of the nonlinear controller to stabilize the quadrotor during hovering. The simulation results in Fig.2 are acquired with a model inclusive of actuators' dynamics. Although the initial conditions are very strict, it can be seen from Fig.2 that the controller succeeded in controlling the roll, pitch, and yaw angles of the quadrotor in less than 8 s.

The nonlinear simulation results obtained with a backstepping-based PID controller are compared with a conventional one, where the motor dynamics are included in the dynamic model and omitting the gyroscopic effects thus removing the cross coupling. An optimization algorithm is used to find the best possible set of PID parameters. In Ref.[9], in order to obtain the parameters for the roll, the pitch, and the yaw, the objective function in the optimization algorithm was to minimize the integral of the absolute error (IAE). The IAE is a performance criterion that considers the difference between the set point and the output that exists when a system is excited by a step input. The optimization toolbox of MATLAB was used to obtain the controller's gain for the PID controller.

Fig.3 shows the comparison of the results from the backstepping-based PID controller with those

from the conventional optimized one. From Fig.3, it is evident that the backstepping-based PID controller presents higher robustness and better transient performances than the traditional PID version.

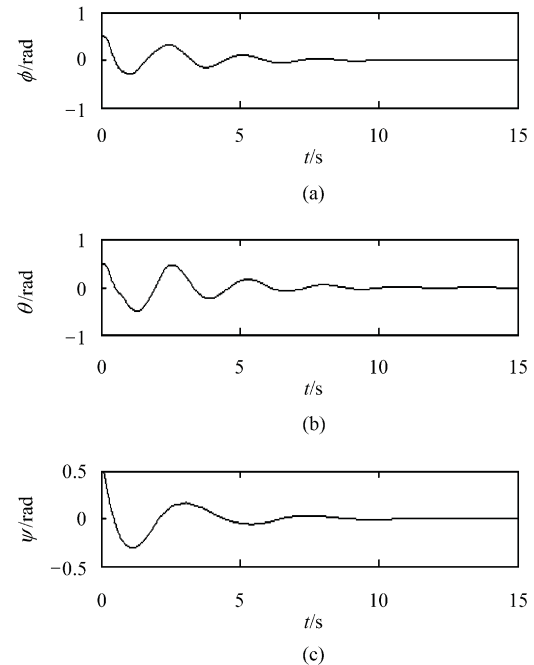


Fig.2 Attitude control of a quadrotor helicopter.

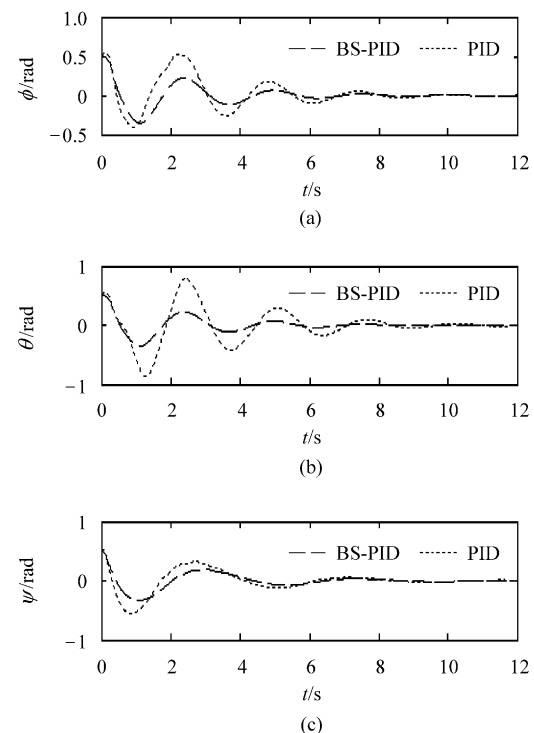


Fig.3 Comparison of backstepping-based PID with conventional optimized PID for rotational subsystem.

The altitude rate and position rate response of the quadrotor helicopter are shown in Fig.4.

Results from Fig.4 indicate that the position controller effectively makes the attitude controller keep the quadrotor helicopter at a given point. The integral term in the backstepping control helps in eliminating the steady state error.

Fig.5 shows the rotor speed response of a quadrotor during hovering.

From Fig.5, for the two pairs of propellers (1, 3) and (2, 4) rotating in opposite direction, as is shown that the rotor speed is able to produce sufficient lift to overcome the weight of the quadrotor helicopter and enable it to hover at a given point.

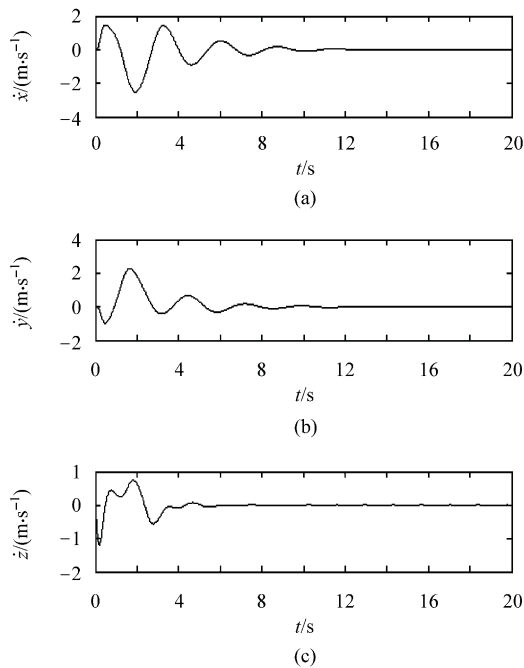


Fig.4 Altitude and position rate response of a quadrotor helicopter.

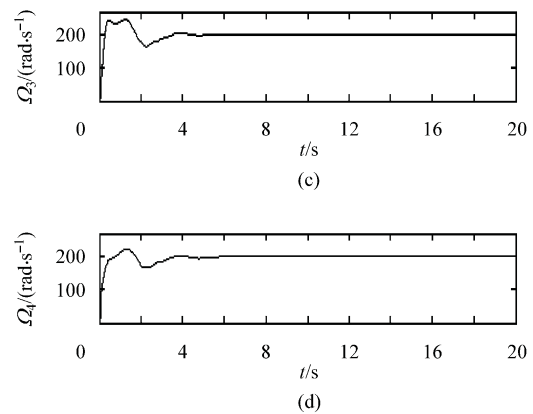
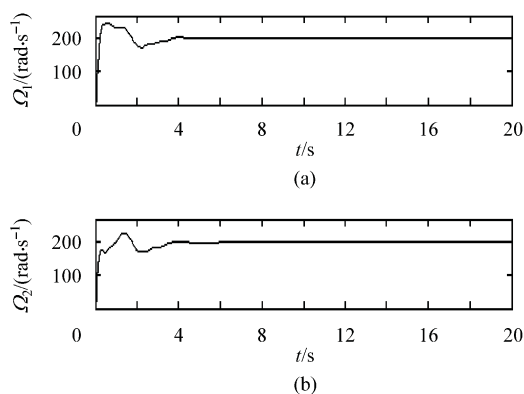


Fig.5 Control response of a quadrotor helicopter.

Fig.6 shows the simulation results of the altitude subsystem.

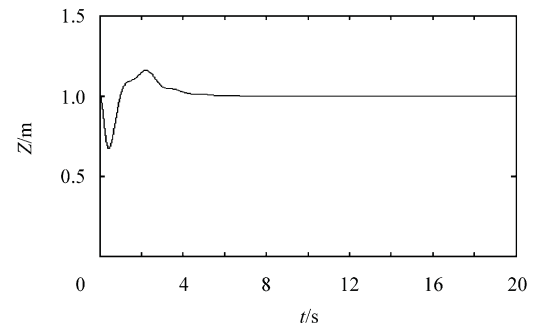


Fig.6 Altitude control of a quadrotor helicopter.

As is indicated in Fig.6, by properly valuing the proportional and derivative terms like $K_d = 1.96$, $K_p = 3.98$, the controller is able to stabilize the attitude angles and enable the quadrotor helicopter to hover at a given point.

6 Conclusions

A stabilization nonlinear control method for a quadrotor helicopter is presented. The modeling of the quadrotor is on the basis of Newton-Euler formalism. A novel control strategy is applied to rotational subsystem of the quadrotor helicopter, wherein the integral of the tracking error is considered in the first step of the backstepping procedure. The control law derived for the nonlinear quadrotor is thus a backstepping-based PID for regulation dynamics. The stabilization ability of the nonlinear controller is examined through nonlinear simulation and the results indicate effectiveness of the proposed control strategy for the quadrotor helicopter.

References

- [1] McKerrow P. Modelling the Draganflyer four-rotor helicopter. Proceedings of the IEEE International Conference on Robotics and Automation. 2004; 3596-3601.
- [2] Pounds P, Mahony R, Hynes P, et al. Design of a four-rotor aerial robot. Proceedings of Australian Conference on Robotics and Automation. 2002; 145-150.
- [3] Tayebi A, McGilvray S. Attitude stabilization of a four-rotor aerial robot. 43rd IEEE Conference on Decision and Control. 2004; 1216-1221.
- [4] Kanellakopoulos I, Krein P. Integral-action nonlinear control of induction motors. Proceedings of the 12th IFAC World Congress. 1993; 251-254.
- [5] Krstic M, Kanellakopoulos I, Kokotovic P. Nonlinear and adaptive control design. New York: John Wiley & Sons, 1995.
- [6] Koo T J, Ma Y, Sastry S. Nonlinear control of a helicopter based unmanned aerial vehicle model. <http://citeseer.ist.psu.edu/417459.html>.
- [7] Prouty R W. Helicopter performance, stability and control. Florida: Krieger Publishing Company, 1996.
- [8] Seddon J. Basic helicopter aerodynamics. Oxford: BSP Professional Books, 1990.
- [9] Smith C, Corripio A. Principles and practice of automatic process control. 2nd ed. New York: John Wiley & Sons, 1997.

Biography:

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