Appendix 2

Definitions of Aerodynamic Stability and Control Derivatives

Notes

(i) The derivatives given in Tables A2.5–A2.8 are all referred to *generalised body axes* and, $U_e = V_0 \cos\theta_e$ and $W_e = V_0 \sin\theta_e$. In the particular case when the derivatives are referred to wind axes $\theta_e = 0$ and the following simplifications can be made, $U_e = V_0$, $W_e = 0$, $\sin\theta_e = 0$ and $\cos\theta_e = 1$.

(ii) The equivalent algebraic expressions in Tables A2.5–A2.8 were derived with the aid of the computer program *Mathcad* which includes a facility for symbolic calculation.

(iii) In Tables A2.5–A2.8 normalised mass and inertias are used which are defined as follows:

$$m' = \frac{m}{\frac{1}{2}\rho V_0 S}$$

$$I'_x = \frac{I_x}{\frac{1}{2}\rho V_0 S b}$$

$$I'_y = \frac{I_y}{\frac{1}{2}\rho V_0 S \bar{\bar{c}}}$$

$$I'_z = \frac{I_z}{\frac{1}{2}\rho V_0 S b}$$

$$I'_{xz} = \frac{I_{xz}}{\frac{1}{2}\rho V_0 S b}$$

Table A2.1 *Longitudinal aerodynamic stability derivatives*

Dimensionless	*Multiplier*	*Dimensional*
X_u	$\frac{1}{2}\rho V_0 S$	$\overset{\circ}{X}_u$
X_w	$\frac{1}{2}\rho V_0 S$	$\overset{\circ}{X}_w$
$X_{\dot{w}}$	$\frac{1}{2}\rho S\bar{\bar{c}}$	$\overset{\circ}{X}_{\dot{w}}$
X_q	$\frac{1}{2}\rho V_0 S\bar{\bar{c}}$	$\overset{\circ}{X}_q$
Z_u	$\frac{1}{2}\rho V_0 S$	$\overset{\circ}{Z}_u$
Z_w	$\frac{1}{2}\rho V_0 S$	$\overset{\circ}{Z}_w$
$Z_{\dot{w}}$	$\frac{1}{2}\rho S\bar{\bar{c}}$	$\overset{\circ}{Z}_{\dot{w}}$
Z_q	$\frac{1}{2}\rho V_0 S\bar{\bar{c}}$	$\overset{\circ}{Z}_q$
M_u	$\frac{1}{2}\rho V_0 S\bar{\bar{c}}$	$\overset{\circ}{M}_u$
M_w	$\frac{1}{2}\rho V_0 S\bar{\bar{c}}$	$\overset{\circ}{M}_w$
$M_{\dot{w}}$	$\frac{1}{2}\rho S\bar{\bar{c}}^2$	$\overset{\circ}{M}_{\dot{w}}$
M_q	$\frac{1}{2}\rho V_0 S\bar{\bar{c}}^2$	$\overset{\circ}{M}_q$

Table A2.2 *Longitudinal control derivatives*

Dimensionless	*Multiplier*	*Dimensional*
X_η	$\frac{1}{2}\rho V_0^2 S$	$\overset{\circ}{X}_\eta$
Z_η	$\frac{1}{2}\rho V_0^2 S$	$\overset{\circ}{Z}_\eta$
M_η	$\frac{1}{2}\rho V_0^2 S\bar{\bar{c}}$	$\overset{\circ}{M}_\eta$
X_τ	1	$\overset{\circ}{X}_\tau$
Z_τ	1	$\overset{\circ}{Z}_\tau$
M_τ	$\bar{\bar{c}}$	$\overset{\circ}{M}_\tau$

Table A2.3 *Lateral aerodynamic stability derivatives*

Dimensionless	*Multiplier*	*Dimensional*
Y_v	$\frac{1}{2}\rho V_0 S$	$\mathring{Y}_v$
Y_p	$\frac{1}{2}\rho V_0 Sb$	$\mathring{Y}_p$
Y_r	$\frac{1}{2}\rho V_0 Sb$	$\mathring{Y}_r$
L_v	$\frac{1}{2}\rho V_0 Sb$	$\mathring{L}_v$
L_p	$\frac{1}{2}\rho V_0 Sb^2$	$\mathring{L}_p$
L_r	$\frac{1}{2}\rho V_0 Sb^2$	$\mathring{L}_r$
N_v	$\frac{1}{2}\rho V_0 Sb$	$\mathring{N}_v$
N_p	$\frac{1}{2}\rho V_0 Sb^2$	$\mathring{N}_p$
N_r	$\frac{1}{2}\rho V_0 Sb^2$	$\mathring{N}_r$

Table A2.4 *Lateral aerodynamic control derivatives*

Dimensionless	*Multiplier*	*Dimensional*
Y_ξ	$\frac{1}{2}\rho V_0^2 S$	$\mathring{Y}_\xi$
L_ξ	$\frac{1}{2}\rho V_0^2 Sb$	$\mathring{L}_\xi$
N_ξ	$\frac{1}{2}\rho V_0^2 Sb$	$\mathring{N}_\xi$
Y_ζ	$\frac{1}{2}\rho V_0^2 S$	$\mathring{Y}_\zeta$
L_ζ	$\frac{1}{2}\rho V_0^2 Sb$	$\mathring{L}_\zeta$
N_ζ	$\frac{1}{2}\rho V_0^2 Sb$	$\mathring{N}_\zeta$

Table A2.5 *Concise longitudinal aerodynamic stability derivatives*

Concise derivative	*Equivalent expressions in terms of dimensional derivatives*	*Equivalent expressions in terms of dimensionless derivatives*
x_u	$\dfrac{\mathring{X}_u}{m} + \dfrac{\mathring{X}_{\dot w}\mathring{Z}_u}{m\left(m - \mathring{Z}_{\dot w}\right)}$	$\dfrac{X_u}{m'} + \dfrac{\frac{\bar{\bar{c}}}{V_0}X_{\dot w}Z_u}{m'\left(m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot w}\right)}$
z_u	$\dfrac{\mathring{Z}_u}{m - \mathring{Z}_{\dot w}}$	$\dfrac{Z_u}{m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot w}}$
m_u	$\dfrac{\mathring{M}_u}{I_y} + \dfrac{\mathring{Z}_u\mathring{M}_{\dot w}}{I_y\left(m - \mathring{Z}_{\dot w}\right)}$	$\dfrac{M_u}{I'_y} + \dfrac{\frac{\bar{\bar{c}}}{V_0}M_{\dot w}Z_u}{I'_y\left(m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot w}\right)}$
x_w	$\dfrac{\mathring{X}_w}{m} + \dfrac{\mathring{X}_{\dot w}\mathring{Z}_w}{m\left(m - \mathring{Z}_{\dot w}\right)}$	$\dfrac{X_w}{m'} + \dfrac{\frac{\bar{\bar{c}}}{V_0}X_{\dot w}Z_w}{m'\left(m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot w}\right)}$
z_w	$\dfrac{\mathring{Z}_w}{m - \mathring{Z}_{\dot w}}$	$\dfrac{Z_w}{m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot w}}$
m_w	$\dfrac{\mathring{M}_w}{I_y} + \dfrac{\mathring{Z}_w\mathring{M}_{\dot w}}{I_y\left(m - \mathring{Z}_{\dot w}\right)}$	$\dfrac{M_w}{I'_y} + \dfrac{\frac{\bar{\bar{c}}}{V_0}M_{\dot w}Z_w}{I'_y\left(m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot w}\right)}$
x_q	$\dfrac{\left(\mathring{X}_q - mW_e\right)}{m} + \dfrac{\left(\mathring{Z}_q + mU_e\right)\mathring{X}_{\dot w}}{m\left(m - \mathring{Z}_{\dot w}\right)}$	$\dfrac{\bar{\bar{c}}X_q - m'W_e}{m'} + \dfrac{\left(\bar{\bar{c}}Z_q + m'U_e\right)\frac{\bar{\bar{c}}}{V_0}X_{\dot w}}{m'\left(m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot w}\right)}$
z_q	$\dfrac{\left(\mathring{Z}_q + mU_e\right)}{m - \mathring{Z}_{\dot w}}$	$\dfrac{\bar{\bar{c}}Z_q + m'U_e}{m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot w}}$
m_q	$\dfrac{\mathring{M}_q}{I_y} + \dfrac{\left(\mathring{Z}_q + mU_e\right)\mathring{M}_{\dot w}}{I_y\left(m - \mathring{Z}_{\dot w}\right)}$	$\dfrac{\bar{\bar{c}}M_q}{I'_y} + \dfrac{\left(\bar{\bar{c}}Z_q + m'U_e\right)\frac{\bar{\bar{c}}}{V_0}M_{\dot w}}{I'_y\left(m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot w}\right)}$
x_θ	$-g\cos\theta_e - \dfrac{\mathring{X}_{\dot w}g\sin\theta_e}{m - \mathring{Z}_{\dot w}}$	$-g\cos\theta_e - \dfrac{\frac{\bar{\bar{c}}}{V_0}X_{\dot w}g\sin\theta_e}{m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot w}}$
z_θ	$-\dfrac{mg\sin\theta_e}{m - \mathring{Z}_{\dot w}}$	$-\dfrac{m'g\sin\theta_e}{m' - \frac{\bar{\bar{c}}}{V_0}\mathring{Z}_{\dot w}}$
m_θ	$-\dfrac{\mathring{M}_{\dot w}mg\sin\theta_e}{I_y\left(m - \mathring{Z}_{\dot w}\right)}$	$-\dfrac{\frac{\bar{\bar{c}}}{V_0}M_{\dot w}m'g\sin\theta_e}{I'_y\left(m' - \frac{\bar{\bar{c}}}{V_0}Z_{\dot w}\right)}$

Table A2.6 *Concise longitudinal control derivatives*

Concise derivative	*Equivalent expressions in terms of dimensional derivatives*	*Equivalent expressions in terms of dimensionless derivatives*
x_η	$\dfrac{\mathring{X}_\eta}{m}+\dfrac{\mathring{X}_{\dot{w}}\mathring{Z}_\eta}{m\left(m-\mathring{Z}_{\dot{w}}\right)}$	$\dfrac{V_0X_\eta}{m'}+\dfrac{\frac{\bar{\bar{c}}}{V_0}X_{\dot{w}}Z_\eta}{m'\left(m'-\frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}\right)}$
z_η	$\dfrac{\mathring{Z}_\eta}{m-\mathring{Z}_{\dot{w}}}$	$\dfrac{V_0Z_\eta}{m'-\frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}}$
m_η	$\dfrac{\mathring{M}_\eta}{I_y}+\dfrac{\mathring{M}_{\dot{w}}\mathring{Z}_\eta}{I_y\left(m-\mathring{Z}_{\dot{w}}\right)}$	$\dfrac{V_0M_\eta}{I'_y}+\dfrac{\bar{\bar{c}}M_{\dot{w}}Z_\eta}{I'_y\left(m'-\frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}\right)}$
x_τ	$\dfrac{\mathring{X}_\tau}{m}+\dfrac{\mathring{X}_{\dot{w}}\mathring{Z}_\tau}{m\left(m-\mathring{Z}_{\dot{w}}\right)}$	$\dfrac{V_0X_\tau}{m'}+\dfrac{\frac{\bar{\bar{c}}}{V_0}X_{\dot{w}}Z_\tau}{m'\left(m'-\frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}\right)}$
z_τ	$\dfrac{\mathring{Z}_\tau}{m-\mathring{Z}_{\dot{w}}}$	$\dfrac{V_0Z_\tau}{m'-\frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}}$
m_τ	$\dfrac{\mathring{M}_\tau}{I_y}+\dfrac{\mathring{M}_{\dot{w}}\mathring{Z}_\tau}{I_y\left(m-\mathring{Z}_{\dot{w}}\right)}$	$\dfrac{V_0M_\tau}{I'_y}+\dfrac{\bar{\bar{c}}M_{\dot{w}}Z_\tau}{I'_y\left(m'-\frac{\bar{\bar{c}}}{V_0}Z_{\dot{w}}\right)}$

Table A2.7 *Concise lateral aerodynamic stability derivatives*

Concise derivative	*Equivalent expressions in terms of dimensional derivatives*	*Equivalent expressions in terms of dimensionless derivatives*
y_v	$\dfrac{\mathring{Y}_v}{m}$	$\dfrac{Y_v}{m'}$
y_p	$\dfrac{\mathring{Y}_p + mW_e}{m}$	$\dfrac{(bY_p + m'W_e)}{m'}$
y_r	$\dfrac{\left(\mathring{Y}_r - mU_e\right)}{m}$	$\dfrac{(bY_r - m'U_e)}{m'}$
y_ϕ	$g\cos\theta_e$	$g\cos\theta_e$
y_ψ	$g\sin\theta_e$	$g\sin\theta_e$
l_v	$\dfrac{\left(I_z\mathring{L}_v + I_{xz}\mathring{N}_v\right)}{(I_xI_z - I_{xz}^2)}$	$\dfrac{(I'_zL_v + I'_{xz}N_v)}{(I'_xI'_z - I'^2_{xz})}$
l_p	$\dfrac{\left(I_z\mathring{L}_p + I_{xz}\mathring{N}_p\right)}{(I_xI_z - I_{xz}^2)}$	$\dfrac{(I'_zL_p + I'_{xz}N_p)}{(I'_xI'_z - I'^2_{xz})}$
l_r	$\dfrac{\left(I_z\mathring{L}_r + I_{xz}\mathring{N}_r\right)}{(I_xI_z - I_{xz}^2)}$	$\dfrac{(I'_zL_r + I'_{xz}N_r)}{(I'_xI'_z - I'^2_{xz})}$
l_ϕ	0	0
l_ψ	0	0
n_v	$\dfrac{\left(I_x\mathring{N}_v + I_{xz}\mathring{L}_v\right)}{(I_xI_z - I_{xz}^2)}$	$\dfrac{(I'_xN_v + I'_{xz}L_v)}{(I'_xI'_z - I'^2_{xz})}$
n_p	$\dfrac{\left(I_x\mathring{N}_p + I_{xz}\mathring{L}_p\right)}{(I_xI_z - I_{xz}^2)}$	$\dfrac{(I'_xN_p + I'_{xz}L_p)}{(I'_xI'_z - I'^2_{xz})}$
n_r	$\dfrac{\left(I_x\mathring{N}_r + I_{xz}\mathring{L}_r\right)}{(I_xI_z - I_{xz}^2)}$	$\dfrac{(I'_xN_r + I'_{xz}L_r)}{(I'_xI'_z - I'^2_{xz})}$
n_ϕ	0	0
n_ψ	0	0

Table A2.8 *Concise lateral control derivatives*

Concise derivative	*Equivalent expressions in terms of dimensional derivatives*	*Equivalent expressions in terms of dimensionless derivatives*
y_ξ	$\dfrac{\mathring{Y}_\xi}{m}$	$\dfrac{V_0 Y_\xi}{m'}$
l_ξ	$\dfrac{\left(I_z \mathring{L}_\xi + I_{xz} \mathring{N}_\xi\right)}{(I_x I_z - I_{xz}^2)}$	$\dfrac{V_0(I'_z L_\xi + I'_{xz} N_\xi)}{(I'_x I'_z - I'^2_{xz})}$
n_ξ	$\dfrac{\left(I_x \mathring{N}_\xi + I_{xz} \mathring{L}_\xi\right)}{(I_x I_z - I_{xz}^2)}$	$\dfrac{V_0(I'_x N_\xi + I'_{xz} L_\xi)}{(I'_x I'_z - I'^2_{xz})}$
y_ζ	$\dfrac{\mathring{Y}_\zeta}{m}$	$\dfrac{V_0 Y_\zeta}{m'}$
l_ζ	$\dfrac{\left(I_z \mathring{L}_\zeta + I_{xz} \mathring{N}_\zeta\right)}{(I_x I_z - I_{xz}^2)}$	$\dfrac{V_0(I'_z L_\zeta + I'_{xz} L_\zeta)}{(I'_x I'_z - I'^2_{xz})}$
n_ζ	$\dfrac{\left(I_x \mathring{N}_\zeta + I_{xz} \mathring{L}_\zeta\right)}{(I_x I_z - I_{xz}^2)}$	$\dfrac{V_0(I'_z N_\zeta + I'_{xz} L_\zeta)}{(I'_x I'_z - I'^2_{xz})}$