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Equational Dynamic Modeling and Adaptive Control of UAV

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Abstract - *Unmanned aerial vehicle (UAV) is an auto-piloted and remote-controlled vehicle sustained by the aerodynamic lift over its whole flight profile. It has many applications such as weather forecast, terrain surveying, environment surveillance, hazardous cleanup, military defense, and so on. In this paper, a simplified nonlinear equational modeling of UAV dynamics is conducted and then linearization and adaptive control approaches are designed. The objective is to apply aerodynamic theory and adaptive control theory on accurate UAV explicit modeling to enhance capabilities of UAV navigation and prediction against various severe conditions.*

Keywords: Modeling, Adaptive Control, Aerodynamics

1. Introduction

The most significant advantage of unmanned aerial vehicle (UAV) is that there is no risk for human beings under severe environment conditions and at battlefields. Large amount of expense can be saved comparing with piloted aerial vehicles. With the rapid development of intelligence surveillance and remote telecommunication, UAV technology has been emphasized by a variety of applications. Usually UAVs can be classified into close range, short range and long range according to its areas of mission. Via artificial intelligence, UAV has played a leading role in forecasting, orientation, surveillance, reconnaissance and cooperation. It has been applied on both military and civilian tasks. There exist numerous types of UAVs to fit for various demands about altitude, range and duration such as payload capability, volume capability and control capability. Each UAV has a receiver and a transmitter for short range messages from proximate UAVs and long range signals from targets. UAV performs certain action to track, sense, orbit and attack targets. The interactions among UAVs display cooperation attribute rather than exclusive coordination. The actions of each UAV can reduce the risk within certain environment for all other UAVs. The UAV control can take advantage of this fact to generate cooperative assignments to achieve better performance. Thus it can be formulated as a dynamic programming problem, which is more tractable for computation. UAV operates at highly dynamic environment under pressure disturbance, wind factor and high probability of model uncertainties. The sliding mode control has been used in UAV applications to overcome these model uncertainties and noise inherent in UAV control systems. Evolution computation using Genetic Algorithms was proposed, tested and implemented on modeling and control of UAVs. Fitness measure is defined together with the applications of artificial intelligence. In most cases, multiple objectives are considered simultaneously to solve this nonlinear control problem, where a set of stochastic design methods and optimization methods are integrated so that important combinational characteristics like robustness, versatility and simplicity are considered together. For instance, as a random search strategy, the optimal evolution process can be used to search and obtain the actual amplitude and oscillation frequency profiles for each of the six motion components (back, yaw, pitch, surge, heave, sway) using statistical data. On the other hand, evolution algorithm is still in its infancy stage which has a low power on prediction. Empirical model is needed to predict the system outputs and the model errors in order to obtain the desired rates of pitch, roll and yaw via feedback control as well as sensing and actuation. Instead, the equational modeling method can be introduced to describe UAV motion control explicitly at different modes under a variety of conditions and even under severe disturbances. Various types of UAVs can operate in several distinctive UAV modes: launching, landing, hovering, longitudinal flying and lateral flying as well as the orientation control modes of bank-to-turn and skid-to-turn. In this case, magnitude and orientation of rigid body acceleration vector can be achieved by enabling UAVs to alternate angle of attack and sideslip angle. Obviously UAV control systems are time varying nonlinear systems. By certain linearization method, it is possible for the nonlinear aerodynamic systems to be represented by linearized models [1-18].

By neglecting couplings between moments (pitching, rolling) and accelerations (longitudinal, lateral), a system with dynamic decoupling is linearizable without zero dynamic, which acts as a non-minimum phase system approximately. At the same time, for actual UAV time varying systems, adaptive controller is appropriate to be selected for UAV control. This paper intends to present some essential UAV system problem-solving approaches using equational dynamic modeling methods. Adaptive control and sensitivity analysis are then proposed.

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2. Dynamic Modeling of UAV

The reference coordinate system of UAV is located at its center of gravity (O). X-axis is the roll axis, pointing forwards along the symmetry axis. Y-axis is the pitch axis, pointing outwards. Z-axis is the yaw axis, pointing downwards. XYZ-axis turns out to be the orthogonal system. Following UAV movement direction, the roll plane is OYZ, the yaw plane is the OXY and the pitch plane is the OZX. Four angles are defined as follows. Let α be the incidence in the pitch plane, β be the incidence in the yaw plane and λ be the incidence in the roll plane angle and θ is the total incidence angle. It is easy to show from simple geometry relationship that $\tan\alpha = \cos\lambda \tan\theta$ and $\tan\beta = \sin\lambda \tan\theta$.

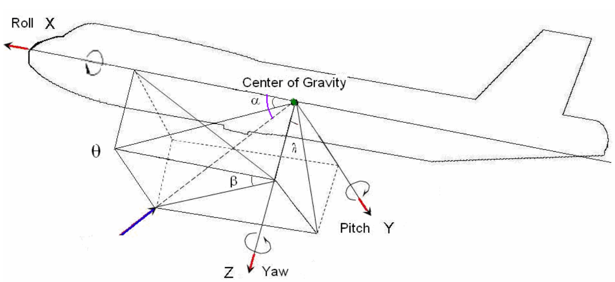

Fig. 1 Sketch of UAV Coordinate System

The moments of inertia and products of inertia about origin are defined as (1),

$$
\begin{aligned}
&I_X = \Sigma\delta m(y^2+z^2);\ I_{yz} = \Sigma\delta myz \\
&I_Y = \Sigma\delta m(z^2+x^2);\ I_{zx} = \Sigma\delta mzx \\
&I_Z = \Sigma\delta m(x^2+y^2);\ I_{xy} = \Sigma\delta mxy
\end{aligned}
\tag{1}
$$

where I_x, I_y, I_z are moments of inertia along each axis and I_{yz}, I_{zx}, I_{xy} are products of inertia. Some assumptions to obtain motion equations from UAV kinematics and dynamics are given. First of all, UAV is a rigid body, whose axes are the principal axes of inertia; secondly the short period dynamics is assumed.

All the following equations are derived from motions of UAVs about the center of gravity.

2.1 Translation Equations

$$
\begin{aligned}
&m(\dot u+qw-rv)=F_x+g_x \\
&m(\dot v+ru-pw)=F_y+g_y \\
&m(\dot w+pv-qu)=F_z+g_z
\end{aligned}
\tag{2}
$$

where F_x, F_y, F_z are the resultant forces along the coordinate axes; G_x, G_y, G_z are the gravity forces; u, v and w are inertial velocities along the coordinate axes; p, q, r are angular rates along the coordinate axes; m is the mass of UAV and $\dot u$, $\dot v$, $\dot w$ are the derivatives of u, v and w. Rotational equations are defined as below.

2.2 Rotation Equations

$$
\begin{aligned}
&I_x\dot p-(I_y-I_z)qr+I_{yz}(r^2-q^2)-I_{zx}(pq+\dot r)+I_{xy}(rp-\dot q)=L \\
&I_y\dot q-(I_z-I_x)rp+I_{zx}(p^2-r^2)-I_{xy}(qr+\dot p)+I_{yz}(pq-\dot r)=M \\
&I_z\dot r-(I_x-I_y)pq+I_{xy}(q^2-p^2)-I_{yz}(rp+\dot q)+I_{zx}(qr-\dot p)=N
\end{aligned}
\tag{3}
$$

where L, M, N are resultant moments about the center of gravity along coordinate axes; $\dot p$, $\dot q$, $\dot r$ are the derivatives of p, q and r measured in the coordinate; and I_x, I_y, I_z are principal moments of inertia. These moments produce angular accelerations about the axes. Other terms in the equations are those cross inertia terms which are coupled and undesirable.

UAVs have both noncircular and circular cross section shapes in applications. The assumptions applied to this UAV system model as conventional equations of motion on rigid body is as follows: I_x, I_y, I_z and m are constants. It is symmetrical with respect to pitch plane and yaw plane, $I_{xy}= I_{yz}= I_{zx}= 0$, $I_y= I_z$. The velocity along x axis u is significant larger than quantity along y axis (v) and z axis (w), so it is denoted as U. From above assumptions, two simplified sets are derived from general equations.

$$
\begin{aligned}
&m(\dot U+qw-rv)=F_x \\
&m(\dot v+rU-pw)=F_x \\
&m(\dot w+pv-qU)=F_z
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
&I_x\dot p-(I_y-I_z)qr=L \\
&I_y\dot q-(I_z-I_x)rp=M \\
&I_z\dot r-(I_x-I_y)pq=N
\end{aligned}
\tag{5}
$$

Acceleration vector orthogonal to the velocity vector gives rise to the changes in the velocity direction. The aerodynamic forces F_x (Drag), F_y (Drift), F_z (Lift) and aerodynamic moments L (Roll), M (Pitch), N (Yaw) are derived according to appropriate aerodynamic coefficient models. UAV dynamic model linearization is then to be conducted, where the aerodynamic forces (F_x, F_y, F_z) and the aerodynamic moments (L, M, N) are substituted.

$$
\begin{aligned}
&F_X=\frac{1}{2}\rho AV^2C_X(\overline{M},\alpha,\beta) \\
&F_Y=\frac{1}{2}\rho AV^2C_Y(\overline{M},\beta,\lambda) \\
&F_Z=\frac{1}{2}\rho AV^2C_Z(\overline{M},\lambda,\alpha)
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
&L=\frac{1}{2}\rho AhV^2C_L(\overline{M},\alpha,\beta) \\
&M=\frac{1}{2}\rho AhV^2C_M(\overline{M},\beta,\lambda) \\
&N=\frac{1}{2}\rho AhV^2C_N(\overline{M},\lambda,\alpha)
\end{aligned}
\tag{7}
$$

where A is the reference area, V is relative wind speed, ρ is the air density, $\overline{M}$ is the Mach number and h is the reference length. Now aerodynamic forces F_x, F_y, F_z and aerodynamic moments L, M, N have been estimated from aerodynamic models. General control surfaces are regarded as the cross session of forward movement, control actuators are used to represent up, left, down and right directions. Hence an incremental surface deflection in any one of four control surfaces, leads to aerodynamic forces and moments along all three axes simultaneously.

$$\begin{bmatrix}\delta_p\\ \delta_q\\ \delta_r\end{bmatrix}=\frac{1}{4}\begin{bmatrix}1 & 1 & -1 & -1\\ 1 & 1 & 1 & 1\\ 1 & -1 & -1 & 1\end{bmatrix}\begin{bmatrix}\delta_1\\ \delta_2\\ \delta_3\\ \delta_4\end{bmatrix} \tag{8}$$

where $\delta_p, \delta_q, \delta_r$ are effective control surface deflections of roll, pitch and yaw motions. $\delta_1, \delta_2, \delta_3, \delta_4$ are the four physical control surface deflections.

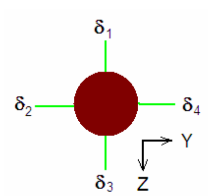

Fig. 2 Sketch of Physical Control Surface

2.3 Actuator Dynamic Equations

These surface actuators are described by the 2nd order system corresponding to the standard transfer functions.

$$\frac{\delta}{U_\delta}=\frac{\omega^2}{s^2+2\zeta\omega s+\omega^2} \tag{9}$$

where δ is the actual surface deflection, U_δ is the desired surface deflection, ω is the natural frequency of actuator and ζ is the damping factor. If both angle of attack (α) and sideslip angle (β) terms are very small, then α=w/U and β=v/U roughly (when $\|\alpha\| << 1$; $\|\beta\| << 1$). We have:

$$\begin{aligned} a_z &= z_\alpha \alpha + z_\delta \delta_q\\ a_y &= y_\beta \beta + y_\delta \delta_r \end{aligned} \tag{10}$$

where a_z and a_y are the specific force acceleration along z axis and y axis. Partial derivative terms are written as:

$$\begin{aligned} z_\alpha &= \frac{\partial a_z}{\partial \alpha} = \frac{1}{m}\frac{\partial F_z}{\partial \alpha}\\ y_\beta &= \frac{\partial a_y}{\partial \beta} = \frac{1}{m}\frac{\partial F_y}{\partial \beta}\\ z_\delta &= \frac{\partial a_z}{\partial \delta_q} = \frac{1}{m}\frac{\partial F_z}{\partial \delta_q}\\ y_\delta &= \frac{\partial a_y}{\partial \delta_r} = \frac{1}{m}\frac{\partial F_y}{\partial \delta_r} \end{aligned} \tag{11}$$

Input-output linearization can be applied on minimum phase nonlinear systems. A UAV model on a rigid body motion with force and moment generation process can be approximated as a non-minimum phase. In the process of linearization, major forces and moments are analyzed at nominal points. Each 1st order derivative terms are used while 2nd order terms are omitted.

2.4 UAV Roll Equations

From moment equations:

$$L=I_x\dot{p}-(I_y-I_z)qr=(L_\beta\beta+L_\delta\delta_p+L_p p)I_x \tag{12}$$

$$\dot{p}=L_\beta\beta+L_\delta\delta_p+L_p p+\frac{I_y-I_z}{I_x}qr \approx L_\alpha U_{\delta p}+L_p p+L_\beta\beta \tag{13}$$

2.5 UAV Pitch Equations

From moment equations,

$$M=(M_\alpha\alpha+M_\delta\delta_q+M_q q)I_y=I_y\dot{q}-(I_z-I_\chi)rp \tag{14}$$

Hence,

$$\begin{aligned} \dot{q} &= M_\alpha\alpha+M_\delta\delta_q+M_q q+\frac{I_z-I_\chi}{I_y}rp\\ &= M_\alpha\frac{(a_z-z_\delta\delta_q)}{z_\alpha}+M_q q+M_\delta\delta_q+\frac{I_z-I_\chi}{I_y}rp\\ &= M_q q+M_\alpha\frac{a_z}{z_\alpha}+\frac{z_\alpha M_\delta-z_\delta M_a}{z_\alpha}\delta_q+\frac{I_z-I_\chi}{I_y}rp \end{aligned} \tag{15}$$

$$\dot{a}_z = z_\alpha\dot{\alpha}+z_\delta\dot{\delta}_q = z_\alpha\frac{\dot{w}}{U}+z_\delta\dot{\delta}_q \tag{16}$$

From the 2nd order actuator equation,

$$\frac{d^2\delta_q}{dt^2}+2\zeta\omega\frac{d\delta_q}{dt}+\omega^2\delta_q=\omega^2U_{\delta q} \tag{17}$$

2.6 UAV Yaw Equations

From moment equations:

$$N=(N_\beta\beta+N_\delta\delta_r+N_r r)I_z=I_z\dot{r}-(I_x-I_y)pq \tag{18}$$

Hence,

$$\begin{aligned} \dot{r} &= N_\beta\beta+N_\delta\delta_r+N_r r+\frac{I_\chi-I_y}{I_z}pq\\ &= N_\beta\frac{a_y-y_\delta\delta_r}{y_\beta}+N_\delta\delta_r+N_r r+\frac{I_x-I_y}{I_z}pq\\ &= N_r r+\frac{N_\beta}{y_\beta}a_y+\frac{y_\beta N_\delta-y_\delta N_\beta}{y_\beta}\delta_r+\frac{I_x-I_y}{I_z}pq \end{aligned} \tag{19}$$

$$\dot{a}_y = y_\beta\dot{\beta}+y_\delta\dot{\delta}_r = y_\beta\frac{\dot{v}}{U}+y_\delta\dot{\delta}_r \tag{20}$$

From the 2nd order actuator equation,

$$\frac{d^2\delta_r}{dt^2}+2\zeta\omega\frac{d\delta_r}{dt}+\omega^2\delta_r=\omega^2U_{\delta_r} \tag{21}$$

In equations (13, 17, 21), $U_{\delta r}$ is the desired effective surface deflection for yaw movement, $U_{\delta p}$ is the desired effective surface deflection for roll movement, $U_{\delta q}$ is the desired effective surface deflection for pitch movement. For control purposes, two extra error signals e_z and e_y are defined as the mismatches between the actual and desired accelerations along z axis and y axis respectively. Here, $e_z=a_z-a_z^*$ and $e_y=a_y-a_y^*$, where a_z^*, a_y^* are desired specific force acceleration along z axis and y axis. On the other hand, error signal of roll angle e_λ is approximated to be the ratio of pitch plane acceleration over yaw plane acceleration, that is, $e_\lambda=a_y/a_z$. The roll angle must be kept away from frequent changes and the actual change must be small in order to avoid the occurrence of dangerous flight dynamics and to enlarge the flight time and range. In the mean while, the mismatch of angular rates of roll, pitch and yaw can also be described as $e_p=p-p^*$, $e_q=q-q^*$ and $e_p=r-r^*$, which are mismatches of actual roll, pitch and yaw angular rates from the empirical UAV models. Minimizations of all these errors are essential to tune the roll, pitch, yaw moments accurately by feedback control actions for sensing and actuation against various types of disturbances.

3. UAV Adaptive Control Law

A basic linearized system model is formulated as:

$$\begin{aligned} dx(t)/dt &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) + D(t)u(t) \end{aligned} \quad (22)$$

where A, B, C, D are state model coefficient matrices. The discretized state model from the continuous model through a fixed sampling interval T is derived as:

$$\begin{aligned} x[(k+1)T] &= \Phi(kT)x(kT) + \Gamma(kT)u(kT) \\ y(kT) &= C(kT)x(kT) + D(kT)u(kT) \end{aligned} \quad (23)$$

The control objective of adaptive systems is to design a feedback control system so that the resulting closed loop system is asymptotically stable with system performance vector error norm $\| r(kT) - y(kT) \|_L$ is sufficiently small. To implement adaptive control, output vector is defined:

$$\begin{aligned} y^T &= \varphi^T(kT)\,\theta_k \\ \theta_k &= [\bar{e}_p, \bar{e}_q, \bar{e}_r, \bar{e}_\lambda, \bar{e}_z, \bar{e}_y]_k^T \end{aligned} \quad (24)$$

where $\bar{e}_p, \bar{e}_q, \bar{e}_r, \bar{e}_\lambda, \bar{e}_z, \bar{e}_y$ are normalized error signals. The parameter estimate $\hat{\theta}_k$ of θ_k is defined at time kT, with initial estimate $\hat{\theta}_0=\theta_0$, control scheme is as below.

$$\hat{\theta}_{k+1}=\hat{\theta}_k+\frac{p_k\varphi(kT)}{\mu+\varphi^T(kT)p_k\varphi(kT)}[y^T(kT)-\varphi^T(kT)\hat{\theta}_k] \quad (k \ge 1, \mu>0) \quad (25)$$

$p_1=CI$ where C>o and for $k \ge 1$

$$p_{k+1}=\frac{1}{\mu}\{p_k-\frac{p_k[\varphi(kT)\varphi^T(kT)]p_k^T}{\mu+\varphi^T(kT)p_k\varphi(kT)}\} \quad (26)$$

4. UAV Sensitivity Analysis in Terms of Optimization for Robust Design

The objective for this adaptive UAV controller is to obtain accurate tracking performance over the nominal system. On certain severe conditions such as turbulence, payload change or other external disturbances, modeling disparity needs to be investigated in order for modeling and control design to be robust. Concept of sensitivity is therefore introduced into the robustness issue of UAV modeling and control design. Here the control sensitivity is expressed by the changing rates over its nominal unit of the control displacement.

$$\begin{aligned} &\Delta e_p = \left|e_p / p^*\right|,\ \Delta e_q = \left|e_q / q^*\right|,\ \Delta e_r = \left|e_r / r^*\right| \\ &\Delta e_\lambda = \left|e_\lambda /(a_y^*/a_z^*)\right|, \Delta e_z = \left|e_z / a_z^*\right|, \Delta e_y = \left|e_y / a_y^*\right| \end{aligned} \quad (27)$$

Then an optimization problem can be formulated as:

minimize F(.) =

$$\min\{\frac{\lambda_1\Delta e_p+\lambda_2\Delta e_q+\lambda_3\Delta e_r+\lambda_4\Delta e_\lambda+\lambda_5\Delta e_z+\lambda_6\Delta e_y}{\sum_{i=1}^{6}\lambda_i}\} \quad (28)$$

subject to $0 \le \lambda_i \le 1$ and (upper, lower) bounds:

constraints of Δe_p, Δe_q, Δe_r, Δe_λ, Δe_z and Δe_y.

Till this point, the robust control problem is simplified into an optimization issue. Depending on computational complexity, it can be solved by math programming or by genetic algorithm, and so on.

5. Conclusions

Equational aerodynamic modeling of UAV system has been derived. UAV is controlled sensing devices which can fly over terrain and provide real time reconnaissance. Nonlinear dynamic models for movements of pitch, yaw and roll have been presented. UAV systems are in fact the rapid time varying systems, model reference adaptive controller is proposed so as to smooth navigations, avoid obstacles and track objects. Sensitivity analysis is then taken into account in order to keep UAV systems robust against inevitable severe conditions of disturbances such as turbulence as well as some noises from sensors and actuators. With optimized parameters for certain UAV models, numerical simulations for modeling and control of UAV systems are to be conducted in the near future.

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