Chapter 8
Manoeuvrability

8.1 INTRODUCTION

8.1.1 Manoeuvring flight

What is a manoeuvre? An aeroplane executing aerobatics in a vast blue sky or, aeroplanes engaged in aerial combat are the kind of images associated with manoeuvring flight. By their very nature such manoeuvres are difficult to quantify, especially when it is required to described manoeuvrability in an analytical framework. In reality most manoeuvres are comparatively mundane and simply involve changing from one trimmed flight condition to another. When a pilot wishes to manoeuvre away from the current flight condition he applies control inputs which upset the equilibrium trim state by producing forces and moments to manoeuvre the aeroplane toward the desired flight condition. The temporary out of trim forces and moments cause the aeroplane to *accelerate* in a sense determined by the combined action of the control inputs. Thus manoeuvring flight is sometimes called *accelerated flight* and is defined as the condition when the airframe is subject to temporary, or transient, out of trim linear and angular accelerations resulting from the displacement of the controls relative to their trim settings. In analytical terms, the manoeuvre is regarded as an increment in steady motion, over and above the initial trim state, in response to an increment in control angle.

The main aerodynamic force producing device in an aeroplane is the wing, and wing lift acts normal to the direction of flight in the plane of symmetry. Normal manoeuvring involves rotating the airframe in roll, pitch and yaw to point the lift vector in the desired direction and the simultaneous adjustment of both angle of attack and speed enables the lift force to generate the acceleration to manoeuvre. For example, in turning flight the aeroplane is rolled to the desired bank angle when the horizontal component of lift causes the aeroplane to turn in the desired direction. Simultaneous aft displacement of the pitch stick is required to generate pitch rate, which in turn generates an increase in angle of attack to produce more lift such that the vertical component is sufficient to balance the weight of the aeroplane, and hence to maintain level flight in the turn. The requirements for simple turning flight are illustrated in Example 2.3. Thus manoeuvrability is mainly concerned with the ability to rotate about aircraft axes, the modulation of the normal or lift force and the modulation of the axial or thrust force. The use of lateral sideforce to manoeuvre is not common in conventional aeroplanes since it is aerodynamically inefficient and it is both unnatural and uncomfortable for the pilot. The principal aerodynamic manoeuvring force is therefore lift, which acts in the plane of symmetry of the aeroplane, and this is controlled by operating the control column in the pitch sense. When the pilot pulls back on the pitch stick the aeroplane pitches up to generate an increased lift force and since this results in out-of-trim normal acceleration the pilot senses, and is very sensitive to, the change in acceleration. The pilot senses what appears to be an increase in the earth's gravitational acceleration g and is said to be *pulling g*.

8.1.2 Stability

Aircraft stability is generally concerned with the requirement that trimmed equilibrium flight may be achieved and that small transient upsets from equilibrium shall decay to zero. However, in manoeuvring flight the *transient upset* is the deliberate result following a control input, it may not be small and may well be prolonged. In the manoeuvre the aerodynamic forces and moments may be significantly different from the steady trim values and it is essential that the changes do not impair the stability of the aeroplane. In other words, there must be no tendency for the aeroplane to diverge in manoeuvring flight.

The classical theory of manoeuvrability is generally attributed to Gates and Lyon (1944) and various interpretations of that original work may be found in most books on aircraft stability and control. Perhaps one of the most comprehensive and accessible summaries of the theory is included in Babister (1961). In this chapter the subject is introduced at the most basic level in order to provide an understanding of the concepts involved since they are critically important in the broader considerations of flying and handling qualities. The original work makes provision for the effects of compressibility. In the following analysis subsonic flight only is considered in the interests of simplicity and hence in the promotion of understanding.

The traditional analysis of *manoeuvre stability* is based on the concept of the steady manoeuvre in which the aeroplane is subject to a steady normal acceleration in response to a pitch control input. Although rather contrived, this approach does enable the manoeuvre stability of an aeroplane to be explained analytically. The only realistic manoeuvres which can be flown at constant normal acceleration are the inside or outside loop and the steady banked turn. For the purpose of analysis the loop is simplified to a pull-up, or push-over, which is just a small segment of the circular flight path. Whichever manoeuvre is analysed, the resulting conditions for stability are the same.

Since the steady acceleration is constrained to the plane of symmetry the problem simplifies to the analysis of *longitudinal manoeuvre stability*, and since the motion is steady the analysis is a simple extension of that applied to *longitudinal static stability* as described in Chapter 3. Consequently, the analysis leads to the concept of the *longitudinal manoeuvre margin*, the stability margin in manoeuvring flight, which in turn gives rise to the corresponding control parameters *stick displacement per g* and *stick force per g*.

8.1.3 Aircraft handling

It is not difficult to appreciate that the manoeuvrability of an airframe is a critical factor in its overall flying and handling qualities. Too much manoeuvre stability means that large control displacements and forces are needed to encourage the development of the normal acceleration vital to effective manoeuvring. On the other hand, too little manoeuvre stability implies that an enthusiastic pilot could overstress the airframe by the application of excessive levels of normal acceleration. Clearly, the difficult balance between control power, manoeuvre stability, static stability and dynamic stability must be correctly controlled over the entire flight envelope of the aeroplane.

Today, considerations of manoeuvrability in the context of aircraft handling have moved on from the simple analysis of normal acceleration response to controls alone. Important additional considerations concern the accompanying roll, pitch and yaw rates and accelerations that may be achieved from control inputs since these determine how quickly a manoeuvre can become established. Manoeuvre entry is also *coloured* by transients associated with the short term dynamic stability modes. The aggressiveness with which a pilot may fly a manoeuvre and the motion cues available to him also contribute to his perception of the overall handling characteristics of the aeroplane. The "picture" therefore becomes very complex, and it is further complicated by the introduction of flight control systems to the aeroplane. The subject of *aircraft agility* is a relatively new and exciting topic of research which embraces the ideas mentioned above and which is, unfortunately, beyond the scope of the present book.

8.1.4 The steady symmetric manoeuvre

The analysis of longitudinal manoeuvre stability is based on steady motion which results in constant additional normal acceleration and, as mentioned above, the simplest such manoeuvre to analyse is the pull-up. In symmetric flight inertial normal acceleration, referred to the *cg*, is given by equation (5.39):

$$a_z = \dot{w} - qU_e \tag{8.1}$$

Since the manoeuvre is steady $\dot{w} = 0$ and the aeroplane must fly a steady pitch rate in order to generate the normal acceleration required to manoeuvre. A steady turn enables this condition to be maintained *ad infinitum* in flight but is less straightforward to analyse. In symmetric flight, a short duration pull-up can be used to represent the lower segment of a continuous circular flight path in the vertical plane since a continuous loop is not practical for many aeroplanes.

It is worth noting that many modern combat aeroplanes and some advanced civil transport aeroplanes have flight control systems which feature *direct lift control* (DLC). In such aeroplanes pitch rate is not an essential prerequisite to the generation of normal acceleration since the wing is fitted with a system of flaps for producing lift directly. However, in some applications it is common to mix the DLC flap control with conventional elevator control in order to improve manoeuvrability, manoeuvre entry in particular. The manoeuvrability of aeroplanes fitted with DLC control systems may be significantly enhanced although its analysis may become rather more complex.

8.2 THE STEADY PULL-UP MANOEUVRE

An aeroplane flying initially in steady level flight at speed V_0 is subject to a small elevator input $\delta\eta$ which causes it to pull up with steady pitch rate q. Consider the

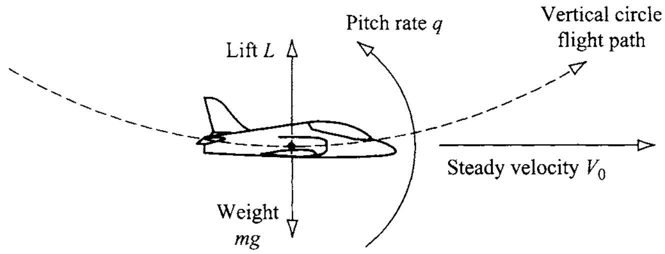

Figure 8.1 *A symmetric pull-up manoeuvre.*

situation when the aircraft is at the lowest point of the vertical circle flight path as shown in Fig. 8.1.

In order to sustain flight in the vertical circle it is necessary that the lift L balances not only the weight mg but the centrifugal force also, thus the lift is greater than the weight and

$$L = nmg \tag{8.2}$$

where n is the normal load factor. Thus the normal load factor quantifies the total lift necessary to maintain the manoeuvre and in steady level flight $n = 1$. The centrifugal force balance is therefore given by

$$L - mg = mV_0 q \tag{8.3}$$

and the incremental normal load factor may be derived directly:

$$\delta n = (n - 1) = \frac{V_0 q}{g} \tag{8.4}$$

Now as the aircraft is pitching up steadily the tailplane experiences an increase in incidence $\delta\alpha_T$ due to the pitch manoeuvre as indicated in Fig. 8.2.

Since small perturbation motion is assumed the increase in tailplane incidence is given by

$$\delta\alpha_T \cong \tan \delta\alpha_T = \frac{q l_T}{V_0} \tag{8.5}$$

where l_T is the moment arm of the aerodynamic centre of the tailplane with respect to the centre of rotation in pitch, the *cg*. Eliminating pitch rate q from equations (8.4) and (8.5),

$$\delta\alpha_T = \frac{(n - 1) g l_T}{V_0^2} \tag{8.6}$$

Now, in the steady level flight condition about which the manoeuvre is executed the lift and weight are equal whence

$$V_0^2 = \frac{2mg}{\rho S C_{L_w}} \tag{8.7}$$

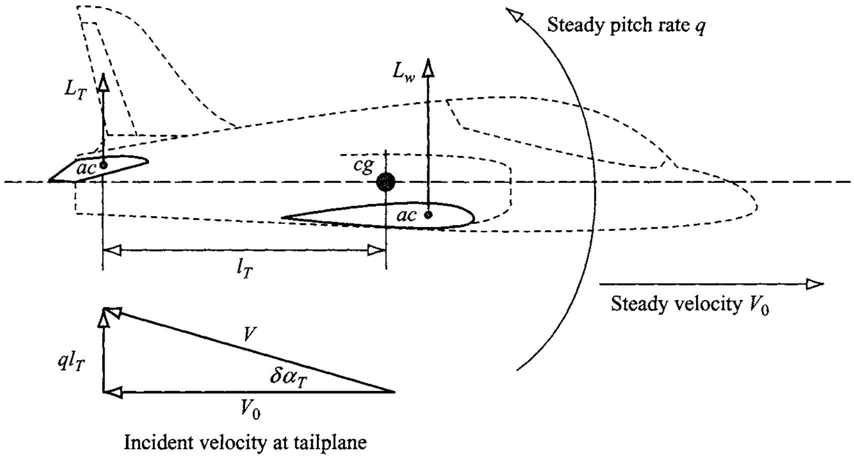

Figure 8.2 *Incremental tailplane incidence in pull-up manoeuvre.*

where C_{L_w} is the steady level flight value of wing–body lift coefficient. Thus from equations (8.6) and (8.7),

$$\delta\alpha_T = \frac{(n-1)\rho S C_{L_w} l_T}{2m} = \frac{(n-1)C_{L_w} l_T}{\mu_1 \bar{\bar{c}}} \equiv \frac{\delta C_{L_w} l_T}{\mu_1 \bar{\bar{c}}} \tag{8.8}$$

where μ_1 is the *longitudinal relative density parameter* and is defined:

$$\mu_1 = \frac{m}{\frac{1}{2}\rho S \bar{\bar{c}}} \tag{8.9}$$

and the increment in lift coefficient, alternatively referred to as incremental "g", necessary to sustain the steady manoeuvre is given by

$$\delta C_{L_w} = (n-1) C_{L_w} \tag{8.10}$$

Care should be exercised when using the longitudinal relative density parameter since various definitions are in common use.

8.3 THE PITCHING MOMENT EQUATION

Subject to the same assumptions about thrust, drag, speed effects and so on, in the steady symmetric manoeuvre the pitching moment equation in coefficient form given by equation (3.7) applies and may be written:

$$C'_m = C_{m_0} + C'_{L_w}(h - h_0) - C'_{L_T}\overline{V}_T \tag{8.11}$$

where a dash indicates the manoeuvring value of the coefficient and,

$$\begin{aligned} C'_m &= C_m + \delta C_m \\ C'_{L_w} &= C_{L_w} + \delta C_{L_w} \equiv n C_{L_w} \\ C'_{L_T} &= C_{L_T} + \delta C_{L_T} \end{aligned}$$

where, C_m, C_{L_w} and C_{L_T} denote the steady trim values of the coefficients and δC_m, δC_{L_w} and δC_{L_T} denote the increments in the coefficients required to manoeuvre.

The corresponding expression for the tailplane lift coefficient is given by equation (3.8) which, for manoeuvring flight, may be written

$$C'_{L_T} = a_1\alpha'_T + a_2\eta' + a_3\beta_\eta \tag{8.12}$$

It is assumed that the tailplane has a symmetric aerofoil section, $a_0 = 0$, and that the tab angle β_η is held at the constant steady trim value throughout the manoeuvre. In other words, the manoeuvre is the result of elevator input only. Thus, using the above notation,

$$\begin{aligned}\alpha'_T &= \alpha_T + \delta\alpha_T \\ \eta' &= \eta + \delta\eta\end{aligned}$$

Tailplane incidence is given by equation (3.11) and in the manoeuvre this may be written:

$$\alpha_T = \frac{C'_{L_w}}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \eta_T \tag{8.13}$$

Total tailplane incidence in the manoeuvre is therefore given by the sum of equations (8.8) and (8.13):

$$\alpha'_T = \frac{C'_{L_w}}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \eta_T + \frac{\delta C_{L_w} l_T}{\mu_1 \bar{\bar{c}}} \tag{8.14}$$

Substituting for α'_T in equation (8.12) the expression for tailplane lift coefficient in the manoeuvre may be written:

$$C'_{L_T} = \frac{C'_{L_w} a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_1\eta_T + \frac{\delta C_{L_w} a_1 l_T}{\mu_1 \bar{\bar{c}}} + a_2\eta' + a_3\beta_\eta \tag{8.15}$$

Substitute the expression for tailplane lift coefficient, equation (8.15), into equation (8.11), and after some re-arrangement the pitching moment equation may be written:

$$\begin{aligned}C'_m = C_{m_0} + C'_{L_w}(h - h_0) - \overline{V}_T\Bigg(&\frac{C'_{L_w} a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_1\eta_T \\ &+ \frac{\delta C_{L_w} a_1 l_T}{\mu_1 \bar{\bar{c}}} + a_2\eta' + a_3\beta_\eta\Bigg)\end{aligned} \tag{8.16}$$

Equation (8.16) describes the total pitching moment in the manoeuvre. To obtain the incremental pitching moment equation which describes the manoeuvre effects only it is first necessary to replace the "dashed" variables and coefficients in equation (8.16) with their equivalent expressions. Then, after some re-arrangement equation (8.16) may be written:

$$\begin{aligned}C_m + \delta C_m = &\left(C_{m_0} + C_{L_w}(h - h_0) - \overline{V}_T\left(\frac{C_{L_w}a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_1\eta_T + a_2\eta + a_3\beta_\eta\right)\right) \\ &+ \left(\delta C_{L_w}(h - h_0) - \overline{V}_T\left(\frac{\delta C_{L_w}a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{\delta C_{L_w}a_1 l_T}{\mu_1 \overline{\overline{c}}} + a_2\delta\eta\right)\right)\end{aligned} \tag{8.17}$$

Now in the steady equilibrium flight condition about which the manoeuvre is executed the pitching moment is zero therefore

$$C_m = C_{m_0} + C_{L_w}(h - h_0) - \overline{V}_T\left(\frac{C_{L_w}a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_1\eta_T + a_2\eta + a_3\beta_\eta\right) = 0 \tag{8.18}$$

and equation (8.17) simplifies to that describing the incremental pitching moment coefficient:

$$\delta C_m = \delta C_{L_w}(h - h_0) - \overline{V}_T\left(\frac{\delta C_{L_w}a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{\delta C_{L_w}a_1 l_T}{\mu_1 \overline{\overline{c}}} + a_2\delta\eta\right) \tag{8.19}$$

8.4 LONGITUDINAL MANOEUVRE STABILITY

As for longitudinal static stability, discussed in Chapter 3, in order to achieve a stable manoeuvre the following condition must be satisfied:

$$\frac{dC'_m}{dC'_{L_w}} < 0 \tag{8.20}$$

and for the manoeuvre to remain steady then

$$C'_m = 0 \tag{8.21}$$

Analysis and interpretation of these conditions leads to the definition of *controls fixed manoeuvre stability* and *controls free manoeuvre stability* which correspond with the parallel concepts derived in the analysis of longitudinal static stability.

8.4.1 Controls fixed stability

The total pitching moment equation (8.16) may be written:

$$\begin{aligned}C'_m = C_{m_0} + C'_{L_w}(h - h_0) - \overline{V}_T\Bigg(&\frac{C'_{L_w}a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + a_1\eta_T \\ &+ \frac{(C'_{L_w} - C_{L_w})a_1 l_T}{\mu_1 \overline{\overline{c}}} + a_2\eta' + a_3\beta_\eta\Bigg)\end{aligned} \tag{8.22}$$

and since, by definition, the controls are held fixed in the manoeuvre:

$$\frac{\mathrm{d}\eta'}{\mathrm{d}C'_{L_w}} = 0$$

Applying the condition for stability, equation (8.20), to equation (8.22) and noting that C_{L_w} and β_η are constant at their steady level flight values and that η_T is also a constant of the aircraft configuration then

$$\frac{\mathrm{d}C'_m}{\mathrm{d}C'_{L_w}} = (h - h_0) - \overline{V}_T\left(\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right) + \frac{a_1 l_T}{\mu_1 \overline{\overline{c}}}\right) \tag{8.23}$$

Or, writing,

$$H_m = -\frac{\mathrm{d}C'_m}{\mathrm{d}C'_{L_w}} = h_m - h \tag{8.24}$$

where H_m is the *controls fixed manoeuvre margin* and the location of the *controls fixed manoeuvre point* h_m on the mean aerodynamic chord $\overline{\overline{c}}$ is given by

$$h_m = h_0 + \overline{V}_T\left(\frac{a_1}{a}\left(1 - \frac{\mathrm{d}\varepsilon}{\mathrm{d}\alpha}\right) + \frac{a_1 l_T}{\mu_1 \overline{\overline{c}}}\right) = h_n + \frac{\overline{V}_T a_1 l_T}{\mu_1 \overline{\overline{c}}} \tag{8.25}$$

Clearly, for controls fixed manoeuvre stability the manoeuvre margin H_m must be positive and, with reference to equation (8.24), this implies that the *cg* must be ahead of the manoeuvre point. Equation (8.25) indicates that the controls fixed manoeuvre point is aft of the corresponding neutral point by an amount depending on the aerodynamic properties of the tailplane. It therefore follows that

$$H_m = K_n + \frac{\overline{V}_T a_1 l_T}{\mu_1 \overline{\overline{c}}} \tag{8.26}$$

which indicates that the controls fixed manoeuvre stability is greater than the controls fixed static stability. With reference to Appendix 8, equation (8.26) may be re-stated in terms of aerodynamic stability derivatives:

$$H_m = -\frac{M_w}{a} - \frac{M_q}{\mu_1} \tag{8.27}$$

A most important conclusion is that additional stability in manoeuvring flight is provided by the aerodynamic pitch damping properties of the tailplane. However, caution is advised since this conclusion may not apply to all aeroplanes in large amplitude manoeuvring or, to manoeuvring in conditions where the assumptions do not apply.

As for controls fixed static stability, the meaning of controls fixed manoeuvre stability is easily interpreted by considering the pilot action required to establish a steady symmetric manoeuvre from an initial trimmed level flight condition. Since the steady (fixed) incremental elevator angle needed to induce the manoeuvre is of interest the incremental pitching moment equation (8.19) is applicable. In a stable steady, and hence by definition, non-divergent manoeuvre the incremental pitching moment δC_m is zero. Whence, equation (8.19) may be re-arranged to give

$$\frac{\delta\eta}{\delta C_{L_w}} = \frac{1}{\overline{V}_T a_2}\left((h - h_0) - \overline{V}_T\left(\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{a_1 l_T}{\mu_1 \bar{\bar{c}}}\right)\right) = \frac{-H_m}{\overline{V}_T a_2} \tag{8.28}$$

Or, in terms of aerodynamic stability derivatives,

$$\frac{\delta\eta}{\delta C_{L_w}} = \frac{-H_m}{M_\eta} = \frac{1}{M_\eta}\left(\frac{M_w}{a} + \frac{M_q}{\mu_1}\right) \tag{8.29}$$

Referring to equation (8.10),

$$\delta C_{L_w} = (n - 1)C_{L_w}$$

which describes the incremental aerodynamic load acting on the aeroplane causing it to execute the manoeuvre, expressed in coefficient form, and measured in units of "g". Thus, both equations (8.28) and (8.29) express the *elevator displacement per g* capability of the aeroplane which is proportional to the controls fixed manoeuvre margin and inversely proportional to the elevator control power, quantified by the aerodynamic control derivative M_η. Since elevator angle and pitch control stick angle are directly related by the control gearing then the very important *stick displacement per g* control characteristic follows directly and is also proportional to the controls fixed manoeuvre margin. This latter control characteristic is critically important in the determination of longitudinal handling qualities. Measurements of elevator angle and normal acceleration in steady manoeuvres for a range of values of normal load factor provide an effective means for determining controls fixed manoeuvre stability from flight experiments. However, in such experiments it is not always possible to ensure that all of the assumptions can be adhered to.

8.4.2 Controls free stability

The controls free manoeuvre is not a practical way of controlling an aeroplane. It does, of course, imply that the elevator angle required to achieve the manoeuvre is obtained by adjustment of the tab angle. As in the case of controls free static stability, this equates to the control force required to achieve the manoeuvre which is a most significant control characteristic. Control force derives from elevator hinge moment in a conventional aeroplane and the elevator hinge moment coefficient in manoeuvring flight is given by equation (3.21) and may be re-stated as

$$C'_H = C_H + \delta C_H = b_1\alpha'_T + b_2\eta' + b_3\beta_\eta \tag{8.30}$$

Since the elevator angle in a controls free manoeuvre is indeterminate it is convenient to express η' in terms of hinge moment coefficient by re-arranging equation (8.30):

$$\eta' = \frac{1}{b_2}C'_H - \frac{b_1}{b_2}\alpha'_T - \frac{b_3}{b_2}\beta_\eta \tag{8.31}$$

Substitute the expression for α'_T, equation (8.14), into equation (8.31) to obtain,

$$\eta' = \frac{1}{b_2}C'_H - \frac{b_1}{ab_2}\left(1 - \frac{d\varepsilon}{d\alpha}\right)C'_{L_w} - \frac{b_1}{b_2}\eta_T - \frac{b_1 l_T}{b_2 \mu_1 \bar{\bar{c}}}\delta C_{L_w} - \frac{b_3}{b_2}\beta_\eta \tag{8.32}$$

Equation (8.32) may be substituted into the manoeuvring pitching moment equation (8.16) in order to replace the indeterminate elevator angle by hinge moment coefficient. After some algebraic re-arrangement the manoeuvring pitching moment may be expressed in the same format as equation (8.22):

$$\begin{aligned} C'_m &= C_{m_0} + C'_{L_w}(h - h_0) \\ &\quad - \overline{V}_T \begin{pmatrix} C'_{L_w}\dfrac{a_1}{a}\left(1 - \dfrac{d\varepsilon}{d\alpha}\right)\left(1 - \dfrac{a_2 b_1}{a_1 b_2}\right) + a_1\eta_T + C'_H\dfrac{a_2}{b_2} \\ + (C'_{L_w} - C_{L_w})\dfrac{a_1 l_T}{\mu_1 \bar{\bar{c}}}\left(1 - \dfrac{a_2 b_1}{a_1 b_2}\right) + \beta_\eta\left(1 - \dfrac{a_2 b_3}{a_3 b_2}\right) \end{pmatrix} \end{aligned} \tag{8.33}$$

and since, by definition, the controls are free in the manoeuvre then

$$C'_H = 0$$

Applying the condition for stability, equation (8.20), to equation (8.33) and noting that, as before, C_{L_w} and β_η are constant at their steady level flight values and that η_T is also a constant of the aircraft configuration then

$$\frac{dC'_m}{dC'_{L_w}} = (h - h_0) - \overline{V}_T\left(\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{a_1 l_T}{\mu_1 \bar{\bar{c}}}\right)\left(1 - \frac{a_2 b_1}{a_1 b_2}\right) \tag{8.34}$$

Or, writing,

$$H'_m = -\frac{dC'_m}{dC'_{L_w}} = h'_m - h \tag{8.35}$$

where H'_m is the *controls free manoeuvre margin* and the location of the *controls free manoeuvre point* h'_m on the mean aerodynamic chord $\bar{\bar{c}}$ is given by

$$\begin{aligned} h'_m &= h_0 + \overline{V}_T\left(\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{a_1 l_T}{\mu_1 \bar{\bar{c}}}\right)\left(1 - \frac{a_2 b_1}{a_1 b_2}\right) \\ &= h'_n + \overline{V}_T\frac{a_1 l_T}{\mu_1 \bar{\bar{c}}}\left(1 - \frac{a_2 b_1}{a_1 b_2}\right) \end{aligned} \tag{8.36}$$

Clearly, for controls free manoeuvre stability the manoeuvre margin H'_m must be positive and, with reference to equation (8.35), this implies that the *cg* must be ahead of the manoeuvre point. Equation (8.36) indicates that the controls free manoeuvre point is aft of the corresponding neutral point by an amount again depending on the aerodynamic damping properties of the tailplane. It therefore follows that

$$H'_m = K'_n + \overline{V}_T \frac{a_1 l_T}{\mu_1 \bar{\bar{c}}} \left(1 - \frac{a_2 b_1}{a_1 b_2}\right) \equiv K'_n + \frac{M_q}{\mu_1} \left(1 - \frac{a_2 b_1}{a_1 b_2}\right) \tag{8.37}$$

which indicates that the controls free manoeuvre stability is greater than the controls free static stability when

$$\left(1 - \frac{a_2 b_1}{a_1 b_2}\right) > 0 \tag{8.38}$$

Since a_1 and a_2 are both positive the degree of controls free manoeuvre stability, over and above the controls free static stability, is controlled by the signs of the hinge moment parameters b_1 and b_2. This, in turn, depends on the aerodynamic design of the elevator control surface.

As for controls free static stability the meaning of controls free manoeuvre stability is easily interpreted by considering the pilot action required to establish a steady symmetric manoeuvre from an initial trimmed level flight condition. Since the controls are "free" this equates to a steady tab angle increment or, more appropriately, a steady control force increment in order to cause the aeroplane to manoeuvre. Equation (8.33) may be re-written in terms of the steady and incremental contributions to the total controls free manoeuvring pitching moment in the same way as equation (8.17):

$$\begin{aligned} C_m + \delta C_m = & \left(C_{m_0} + C_{L_w}(h - h_0) - \overline{V}_T \left(\begin{aligned} & C_{L_w} \frac{a_1}{a} \left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(1 - \frac{a_2 b_1}{a_1 b_2}\right) \\ & + a_1 \eta_T + C_H \frac{a_2}{b_2} + \beta_\eta \left(1 - \frac{a_2 b_3}{a_3 b_2}\right) \end{aligned} \right) \right) \\ & + \left(\delta C_{L_w}(h - h_0) - \overline{V}_T \left(\begin{aligned} & \delta C_{L_w} \frac{a_1}{a} \left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(1 - \frac{a_2 b_1}{a_1 b_2}\right) \\ & + \delta C_H \frac{a_2}{b_2} + \delta C_{L_w} \frac{a_1 l_T}{\mu_1 \bar{\bar{c}}} \left(1 - \frac{a_2 b_1}{a_1 b_2}\right) \end{aligned} \right) \right) \end{aligned} \tag{8.39}$$

Now in the steady equilibrium flight condition about which the manoeuvre is executed the pitching moment is zero thus

$$C_m = C_{m_0} + C_{L_w}(h - h_0) - \overline{V}_T \left(\begin{aligned} & C_{L_w} \frac{a_1}{a} \left(1 - \frac{d\varepsilon}{d\alpha}\right)\left(1 - \frac{a_2 b_1}{a_1 b_2}\right) \\ & + a_1 \eta_T + C_H \frac{a_2}{b_2} + \beta_\eta \left(1 - \frac{a_2 b_3}{a_3 b_2}\right) \end{aligned} \right) = 0 \tag{8.40}$$

and equation (8.39) simplifies to that describing the incremental controls free pitching moment coefficient:

$$\delta C_m = \delta C_{L_w}(h - h_0) - \overline{V}_T \begin{pmatrix} \delta C_{L_w}\dfrac{a_1}{a}\left(1 - \dfrac{d\varepsilon}{d\alpha}\right)\left(1 - \dfrac{a_2 b_1}{a_1 b_2}\right) \\ + \delta C_H \dfrac{a_2}{b_2} + \delta C_{L_w}\dfrac{a_1 l_T}{\mu_1 \bar{\bar{c}}}\left(1 - \dfrac{a_2 b_1}{a_1 b_2}\right) \end{pmatrix} \tag{8.41}$$

Now in the steady manoeuvre the incremental pitching moment δC_m is zero and equation (8.41) may be re-arranged to give

$$\begin{aligned} \frac{\delta C_H}{\delta C_{L_w}} &= \frac{b_2}{a_2 \overline{V}_T}\left((h - h_0) - \overline{V}_T\left(\frac{a_1}{a}\left(1 - \frac{d\varepsilon}{d\alpha}\right) + \frac{a_1 l_T}{\mu_1 \bar{\bar{c}}}\right)\left(1 - \frac{a_2 b_1}{a_1 b_2}\right)\right) \\ &= -\frac{b_2 H'_m}{a_2 \overline{V}_T} \end{aligned} \tag{8.42}$$

In a conventional aeroplane the hinge moment coefficient relates directly to the control stick force, see equation (3.32). Equation (8.42) therefore indicates the very important result that the *stick force per g* control characteristic is proportional to the controls free manoeuvre margin. This control characteristic is critically important in the determination of longitudinal handling qualities and it must have the correct value. In other words, the controls free manoeuvre margin must lie between precisely defined upper and lower bounds. As stated above, in an aerodynamically controlled aeroplane this control characteristic can be adjusted independently of the other stability characteristics by selective design of the values of the hinge moment parameters b_1 and b_2. The controls free manoeuvre stability is critically dependent on the ratio b_1/b_2 which controls the magnitude and sign of expression (8.38). For conventional aeroplanes fitted with a plain flap type elevator control both b_1 and b_2 are usually negative and, see equation (8.37), the controls free manoeuvre stability would be less than the controls free static stability. Adjustment of b_1 and b_2 is normally achieved by aeromechanical means which are designed to modify the elevator hinge moment characteristics. Typically, this involves carefully tailoring the aerodynamic balance of the elevator by means, such as set back hinge line, horn balances, spring tabs, servo tabs and so on. Excellent descriptions of these devices may be found in Dickinson (1968) and in Babister (1961).

The measurement of stick force per g is easily undertaken in flight. The aeroplane is flown in steady manoeuvring flight, the turn probably being the simplest way of achieving a steady normal acceleration for a period long enough to enable good quality measurements to be made. Measurements of stick force and normal acceleration enable estimates to be made of the controls free manoeuvre margin and the location of the controls free manoeuvre point. With greater experimental difficulty, stick force per g can also be measured in steady pull-ups and in steady push-overs. However the experiment is done it must be remembered that it is not always possible to ensure that all of the assumptions can be adhered to.

8.5 AIRCRAFT DYNAMICS AND MANOEUVRABILITY

The preceding analysis shows how the stability of an aeroplane in manoeuvring flight is dependent on the manoeuvre margins and, further, that the magnitude of the manoeuvre margins determines the critical handling characteristics, stick displacement per g and stick force per g. However, the manoeuvre margins of the aeroplane are also instrumental in determining some of the dynamic response characteristics of the aeroplane. This fact further reinforces the statement made elsewhere that the static, manoeuvre and dynamic stability and control characteristics of an aeroplane are really very much inter-related and should not be treated entirely as isolated topics.

In Chapter 6 reduced order models of an aircraft are discussed and from the longitudinal model representing short term dynamic stability and response an approximate expression for the short period mode undamped natural frequency is derived, equation (6.21), in terms of dimensional aerodynamic stability derivatives. With reference to Appendix 2, this expression may be re-stated in terms of dimensionless derivatives:

$$\omega_s^2 = \frac{\frac{1}{2}\rho V_0^2 S\bar{\bar{c}}}{I_y}\left(\frac{\frac{1}{2}\rho S\bar{\bar{c}}}{m}M_q Z_w + M_w\right) = \frac{\frac{1}{2}\rho V_0^2 S\bar{\bar{c}}}{I_y}\left(\frac{M_q Z_w}{\mu_1} + M_w\right) \tag{8.43}$$

where μ_1 is the longitudinal relative density factor defined in equation (8.9).

Now with reference to Appendix 8 an approximate expression for Z_w is given as

$$Z_w \cong -C_D - \frac{\partial C_L}{\partial \alpha} = -C_D - a \tag{8.44}$$

for small perturbation motion in subsonic flight. Since $a \gg C_D$ equation (8.44) may be approximated further, and substituting for Z_w in equation (8.43) to obtain

$$\omega_s^2 = \frac{\frac{1}{2}\rho V_0^2 S\bar{\bar{c}} a}{I_y}\left(-\frac{M_q}{\mu_1} - \frac{M_w}{a}\right) = kH_m \equiv k\left(K_n - \frac{M_q}{\mu_1}\right) \tag{8.45}$$

where k is a constant at the given flight condition. Equation (8.45) therefore shows that the undamped natural frequency of the longitudinal short period mode is directly dependent on the controls fixed manoeuvre margin. Alternatively, this may be interpreted as a dependency on the controls fixed static margin and pitch damping. Clearly, since the controls fixed manoeuvre margin must lie between carefully defined boundaries if satisfactory handling is to be ensured, this implies that the longitudinal short period mode must also be constrained to a corresponding frequency band. Flying qualities requirements have been developed from this kind of understanding and are discussed in Chapter 10.

In many modern aeroplanes the link between the aerodynamic properties of the control surface and the stick force is broken by a servo actuator and other flight control system components. In this case the control forces are provided artificially and may not inter-relate with other stability and control characteristics in the classical way. However, it is obviously important that the pilots perception of the handling qualities of his aeroplane look like those of an aeroplane with acceptable aerodynamic manoeuvre margins. Since many of the subtle aerodynamic inter-relationships do not exist in aeroplanes employing sophisticated flight control systems it is critically important to be fully aware of the handling qualities implications at all stages of a control system design.

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