Appendix 1

AeroTrim - A Symmetric Trim Calculator for Subsonic Flight Conditions

A Mathcad programme written by M.V. Cook, version date August 15, 2006.
Given the operating condition and some basic geometric and aerodynamic data for a conventional aircraft this programme will calculate an estimate of the symmetric trim state of the aircraft for a chosen airspeed range . The programme is limited to subsonic flight in the troposphere only. However, the programme may be developed easily for application to a wider range of operating conditions and aircraft configurations. Data given are best estimates for the Cranfield Jetstream laboratory aircraft.

1. Aircraft flight condition

(Insert values to define aircraft operating condition)

Altitude (ft)	$ht := 6562$	***Convert to m***	$ht := 0.3048 \cdot ht$
Aircraft mass (kg)	$m := 6300$		
cg position (%c)	$h := 0.29$		
Flight path angle (deg)	$\gamma_e := 0$	***Convert to rad***	$\gamma_e := \dfrac{\gamma_e}{57.3}$
Gravity constant (m/s²)	$g := 9.81$		

2. Air density calculation

Valid for troposphere only - up to 36,000 ft

Gas constant (Nm/kgK)	$R := 287.05$	
Lapse rate (K/m)	$lr := -0.0065$	
Temperature (K)	$Temp := 288.16 + lr \cdot ht$	***Check results***
Air density (kg/m³)	$\rho := 1.225 \left(\dfrac{Temp}{288.16}\right)^{-\left[\left(\frac{g}{lr \cdot R}\right)+1\right]}$	$Temp = 275.15$
		$\rho = 1.006$
Density ratio	$\sigma := \dfrac{\rho}{1.225}$	$\sigma = 0.822$

3. Set up velocity range for computations

Note that true airspeed is assumed unless otherwise stated

Counter $i := 0..10$

(Set counter to number of velocity test points required)

True airspeed range (knots) $V_{knots_i} := 100 + 15 \cdot i$

(Set initial velocity and velocity increment as required)

True airspeed (m/s) $V_i := V_{knots_i} \cdot 0.515$

Equivalent airspeed (knots) $V_{eas_i} := V_{knots_i} \cdot \sqrt{\sigma}$

4. Aircraft geometry - constant

(Insert values defined by aircraft geometry)

Wing geometry

Wing area (m^2)	$S := 25.08$		
Wing span (m)	$b := 15.85$		
Wing mean chord (m)	$c_w := 1.716$		
Sweep 1/4c_w (deg)	$\Lambda := 0$		
z coordinate of 1/4c_w point above(−ve) or below(+ve) ox body axis (m)	$z_w := 0.45$		
Wing rigging angle (deg)	$\alpha_{wr} := 1.0$	***Convert to rad***	$\alpha_{wr} := \frac{\alpha_{wr}}{57.3}$

Tailplane geometry

Tailplane area (m^2)	$S_T := 7.79$		
Tailplane span (m)	$b_T := 6.6$		
Tail arm, 1/4c_w to 1/4c_t (m)	$l_t := 6.184$		
z coordinate of 1/4c_w point above(−ve) or below(+ve) ox body axis (m)	$z_T := -1.435$		
Tail setting angle (deg)	$\eta_T := 1.5$	***Convert to rad***	$\eta_T := \frac{\eta_T}{57.3}$
Fuselage diameter or width (m)	$F_d := 1.981$		

Engine installation

Thrust line z coordinate above(−ve) or below(+ve) ox body axis (m)	$z_\tau := 0.312$		
Engine thrust line angle (deg) relative to ox body axis (+nose up)	$\kappa := 0$	***Convert to rad***	$\kappa := \frac{\kappa}{57.3}$

5. Wing-body aerodynamics

(Insert values defined by the installed wing aerodynamic design)

Wing-body C_L-α (per rad)	$a := 5.19$		
Maximum lift coefficient	$C_{Lmax} := 1.37$		
Zero lift pitching moment	$C_{m0} := -0.0711$		
Zero lift drag coefficient	$C_{D0} := 0.03$		
Zero lift angle of attack (deg)	$\alpha_{w0} := -2$	***Convert to rad***	$\alpha_{w0} := \frac{\alpha_{w0}}{57.3}$
Wing-body aero centre	$h_0 := -0.08$		

6. Tailplane aerodynamics

(Insert values defined by the tailplane aerodynamic design)

Tailplane C_L-α (per rad) $a_1 := 3.2$

Elevator C_L-η (per rad) $a_2 := 2.414$

Zero lift downwash angle (deg) $\varepsilon_0 := 2.0$ ***Convert to rads*** $\varepsilon_0 := \dfrac{\varepsilon_0}{57.3}$

7. Wing and tailplane calculations

Aspect ratio $Ar := \dfrac{b^2}{S}$

Wing semi-span (m) $s := \dfrac{b}{2}$

Tail arm, cg to 1/4c_t (m) $l_T := l_t - c_w \cdot (h - 0.25)$

Tail volume $V_T := \dfrac{S_T \cdot l_T}{Sc_w}$

Check results
$Ar = 10.017$
$s = 7.925$
$l_T = 6.115$
$V_T = 1.107$

8. Downwash at tail

Ref:-Stribling, C.B. 1984: "Basic Aerodynamics", Butterworth Ltd, 1984.

Tail position relative to wing (% of span) $x := \dfrac{l_t}{b}$ $z := \dfrac{z_W - z_T}{b}$

$$d_{\varepsilon\alpha} = \frac{a}{\pi^2 \cdot Ar} \sum_{fi=5}^{85} \frac{0.5\cos\left(\frac{fi \cdot \pi}{180}\right)^2}{\sqrt{x^2 + \left(0.5\cos\left(\frac{fi \cdot \pi}{180}\right)\right)^2 + z^2}} \cdot \left[\frac{\boxed{x + \sqrt{x^2 + \left(0.5\cos\left(\frac{fi \cdot \pi}{180}\right)\right)^2 + z^2}}}{\left(0.5\cos\left(\frac{fi \cdot \pi}{180}\right)\right)^2 + z^2} + \frac{x}{(x^2 + z^2)}\right] \frac{\pi}{180}$$

Check result $d_{\varepsilon\alpha} = 0.279$

9. Induced drag factor

Ref:-Shevell, R.S. 1983: "Fundamentals of Flight", 2nd edition, Prentice Hall Inc., 1983.

Drag polar is defined, $C_D = C_{D0} + KC_L^2$, where K is the induced drag factor.

Fuselage drag factor $s_d := 0.9998 + 0.0421 \cdot \left(\dfrac{F_d}{b}\right) - 2.6286 \cdot \left(\dfrac{F_d}{b}\right)^2 + 2.000 \cdot \left(\dfrac{F_d}{b}\right)^3$

Empirical constant $k_D := -3.333 \cdot 10^{-4} \cdot \Lambda^2 + 6.667 \cdot 10^{-5} \cdot \Lambda + 0.38$

Oswald efficiency factor $e := \dfrac{1}{\pi \cdot Ar \cdot k_D \cdot C_{D0} + \dfrac{1}{(0.99 \cdot s_d)}}$

Induced drag factor $K := \dfrac{1}{\pi \cdot Ar \cdot e}$

Check results
$s_d = 0.968$
$k_D = 0.38$
$e = 0.713$
$K = 0.045$

10. Basic performance parameters

Minimum drag speed (knots)	$V_{md} := \left[\left(\sqrt{\frac{2 \cdot m \cdot g}{\rho \cdot S}}\right)\left(\frac{K}{C_{D0}}\right)^{0.25}\right] \cdot \frac{1}{0.515}$
Equivalent minimum drag speed (knots)	$V_{mdeas} := V_{md} \cdot \sqrt{\sigma}$
Stall speed (knots)	$V_{stall} := \sqrt{\frac{2 \cdot m \cdot g}{\rho \cdot S \cdot C_{Lmax}}} \frac{1}{0.515}$
Equivalent stall speed (knots)	$V_{stalleas} := V_{stall} \cdot \sqrt{\sigma}$
Neutral point - controls fixed	$h_n := h_0 + V_T \cdot \frac{a_1}{a} \cdot (1 - d\varepsilon\alpha)$
Static margin - controls fixed	$K_n := h_n - h$

11. Trim calculation

The trim computation finds the trim condition for each speed defined in the speed range table and for the flight condition defined in paragraph 1.

Initial seed values for solve block

$C_L := 0.7 \quad C_{Lw} := 0.5 \quad C_D := 0.02 \quad C_\tau := 0.4 \quad \alpha_e := 0.1 \quad C_{LT} := 0.1$

Trim solve block

Given

Total axial force (ox body axis)	$2\frac{m \cdot g}{\rho \cdot (V)^2 \cdot S} \cdot \sin(\alpha_e + \gamma_e) = C_\tau \cdot \cos(\kappa) - C_D \cdot \cos(\alpha_e) + C_L \cdot \sin(\alpha_e)$
Total normal force oz body axis)	$2\frac{m \cdot g}{\rho \cdot (V)^2 \cdot S} \cdot \cos(\alpha_e + \gamma_e) = C_L \cdot \cos(\alpha_e) + C_D \cdot \sin(\alpha_e) + C_\tau \cdot \sin(\kappa)$
Pitching moment (about cg)	$0 = [C_{m0} + (h - h_0) \cdot C_{Lw}] - V_T \cdot C_{LT} + C_\tau \cdot \frac{z_\tau}{c_w}$
Total lift coefficient	$C_L = C_{Lw} + C_{LT} \cdot \frac{S_T}{S}$
Total drag coefficient	$C_D = C_{D0} + K \cdot C_L^2$
Wing/body lift coefficient	$C_{Lw} = a \cdot (\alpha_e + \alpha_{wr} - \alpha_{w0})$

$\mathrm{Trim}(V) := \mathrm{Find}(\alpha_e, C_\tau, C_D, C_{LT}, C_{LW}, C_L)$

End of trim solve block

12. Trim variables calculation

$\alpha_{e_i} := \mathrm{Trim}(V_i)_0$ $\quad C_{\tau_i} := \mathrm{Trim}(V_i)_1$ $\quad C_{D_i} := \mathrm{Trim}(V_i)_2$

$C_{LT_i} := \mathrm{Trim}(V_i)_3$ $\quad C_{Lw_i} := \mathrm{Trim}(V_i)_4$ $\quad C_{L_i} := \mathrm{Trim}(V_i)_5$

Wing incidence $\quad \alpha_{w_i} := \alpha_{e_i} + \alpha_{wr}$

Trim elevator angle $\quad \eta_{e_i} := \dfrac{C_{LT_i}}{a_2} - \dfrac{a_1}{a_2} \cdot [\alpha_{w_i} \cdot (1 - d\varepsilon\alpha) + \eta_T - \alpha_{wr} - \varepsilon_0]$

Pitch attitude $\quad \theta_{e_i} := \gamma_e + \alpha_{w_i} - \alpha_{wr}$

Tail angle of attack $\quad \alpha_{T_i} := \alpha_{w_i}(1 - d\varepsilon\alpha) + \eta_T - \varepsilon_0 - \alpha_{wr}$

Lift to drag ratio $\quad LD_i := \dfrac{C_{Lw_i}}{C_{D_i}}$

13. Conversions of angles to degrees

$\alpha_{w_i} := \alpha_{w_i} \cdot 57.3$ $\quad \alpha_{e_i} := \alpha_{e_i} \cdot 57.3$ $\quad \theta_{e_i} := \theta_{e_i} \cdot 57.3$

$\alpha_{T_i} := \alpha_{T_i} \cdot 57.3$ $\quad \eta_{e_i} := \eta_{e_i} \cdot 57.3$ $\quad \gamma_e := \gamma_e \cdot 57.3$

14. Total trim forces acting on aircraft

Total lift force (N) $\quad L_i := 0.5 \cdot \rho \cdot (V_i)^2 \cdot S \cdot C_{L_i}$

Total drag force (N) $\quad D_i := 0.5 \cdot \rho \cdot (V_i)^2 \cdot S \cdot C_{D_i}$

Total thrust (N) $\quad T_i := 0.5 \cdot \rho \cdot (V_i)^2 \cdot S \cdot C_{\tau_i}$

SUMMARY RESULTS OF TRIM CALCULATION

15. Definition of flight condition

Aircraft weight (N)	$\mathrm{m \cdot g = 6.18 \cdot 10^4}$	***Minimum drag speed (knots)***	$\mathrm{V_{md} = 150.012}$
Altitude (ft)	$\mathrm{ht \cdot 3.281 = 6.562 \cdot 10^3}$	***Equivalent minimum drag speed (knots)***	$\mathrm{V_{mdeas} = 135.97}$
Flight path angle (deg)	$\gamma_e = 0$	***Stall speed (knots)***	$\mathrm{V_{stall} = 116.092}$
cg position (%c_w)	$\mathrm{h = 0.29}$	***Equivalent stall speed (knots)***	$\mathrm{V_{stalleas} = 105.225}$
Neutral point - controls fixed	$\mathrm{h_n = 0.412}$	***Static margin - controls fixed***	$\mathrm{K_n = 0.122}$

16. Trim conditions as a function of aircraft velocity

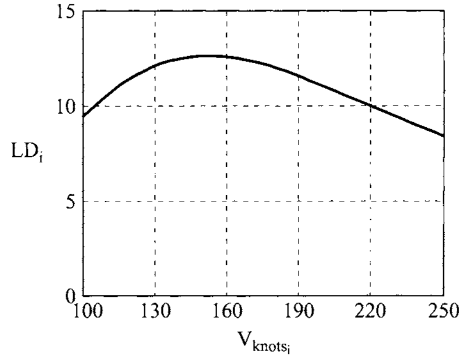
Angles in degrees, velocity in m/s, forces in N except where indicated otherwise

V_{knots_i}	V_i	C_{L_i}	C_{D_i}	C_{Lw_i}	C_{LT_i}	LD_i	C_{τ_i}	α_{w_i}	α_{e_i}	θ_{e_i}	α_{T_i}	η_{e_i}
100	51.5	1.799	0.174	1.64	0.514	9.409	0.181	16.105	15.105	15.105	10.107	−1.208
115	59.225	1.374	0.114	1.258	0.375	11.017	0.116	11.885	10.885	10.885	7.066	−0.46
130	66.95	1.081	0.082	0.994	0.282	12.106	0.083	8.97	7.97	7.97	4.965	0.1
145	74.675	0.872	0.064	0.805	0.215	12.603	0.064	6.885	5.885	5.885	3.462	0.521
160	82.4	0.717	0.053	0.665	0.167	12.573	0.053	5.346	4.346	4.346	2.353	0.842
175	90.125	0.6	0.046	0.56	0.13	12.154	0.046	4.181	3.181	3.181	1.513	1.091
190	97.85	0.51	0.042	0.478	0.102	11.496	0.042	3.277	2.277	2.277	0.862	1.287
205	105.575	0.438	0.039	0.413	0.08	10.72	0.039	2.564	1.564	1.564	0.348	1.444
220	113.3	0.381	0.036	0.361	0.063	9.912	0.036	1.99	0.99	0.99	−0.066	1.572
235	121.025	0.334	0.035	0.319	0.048	9.123	0.035	1.523	0.523	0.523	−0.403	1.677
250	128.75	0.295	0.034	0.284	0.036	8.383	0.034	1.136	0.136	0.136	−0.681	1.764

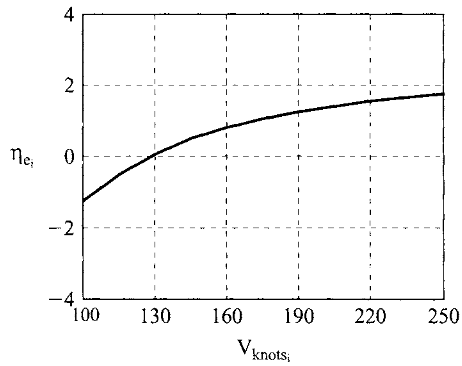
V_{knots_i}	V_i	L_i	D_i	T_i
100	51.5	$6.023 \cdot 10^4$	$5.834 \cdot 10^3$	$6.042 \cdot 10^3$
115	59.225	$6.083 \cdot 10^4$	$5.053 \cdot 10^3$	$5.146 \cdot 10^3$
130	66.95	$6.115 \cdot 10^4$	$4.643 \cdot 10^3$	$4.688 \cdot 10^3$
145	74.675	$6.134 \cdot 10^4$	$4.494 \cdot 10^3$	$4.518 \cdot 10^3$
160	82.4	$6.146 \cdot 10^4$	$4.535 \cdot 10^3$	$4.548 \cdot 10^3$
175	90.125	$6.154 \cdot 10^4$	$4.722 \cdot 10^3$	$4.729 \cdot 10^3$
190	97.85	$6.16 \cdot 10^4$	$5.025 \cdot 10^3$	$5.029 \cdot 10^3$
205	105.575	$6.165 \cdot 10^4$	$5.424 \cdot 10^3$	$5.426 \cdot 10^3$
220	113.3	$6.17 \cdot 10^4$	$5.907 \cdot 10^3$	$5.908 \cdot 10^3$
235	121.025	$6.174 \cdot 10^4$	$5.465 \cdot 10^3$	$6.465 \cdot 10^3$
250	128.75	$6.179 \cdot 10^4$	$5.089 \cdot 10^3$	$7.089 \cdot 10^3$

17. Some useful trim plots

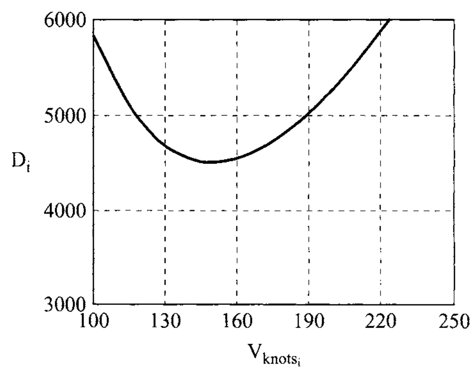
life to drag ratio variation with true airspeed



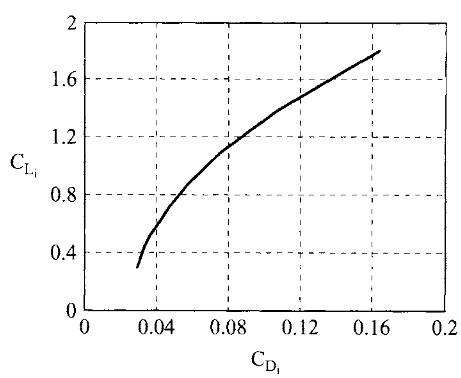
Elevator angle (deg) to trim with true airspeed



Total drag (N) variation with true airspeed



Drag polar



- End of programme -