Appendix 10

The Transformation of the Moments and Products of Inertia from a Body Axes Reference to a Wind Axes Reference

INTRODUCTION

In the same way that it is sometimes necessary to transform the aerodynamic stability and control derivatives from a body axes reference to a wind axes reference, and *vice versa*, it is also necessary to transform the corresponding moments and products of inertia. Again, the procedure is very straightforward and makes use of the transformation relationships discussed in Chapter 2. It is assumed that the body axes and wind axes in question have a common origin at the *cg* of the aeroplane and that it is in steady level symmetric flight. Thus the axes differ by the steady body incidence α_e only as shown in Fig. 2.2.

COORDINATE TRANSFORMATION

Body to wind axes

A set of coordinates in a body axes system x_b, y_b, z_b may be transformed into the equivalent set in a wind axes system x_w, y_w, z_w by application of the inverse direction cosine matrix given by equation (2.13). Writing $\theta = \alpha_e$ and $\phi = \psi = 0$ since level symmetric flight is assumed then

$$\begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} = \begin{bmatrix} \cos\alpha_e & 0 & \sin\alpha_e \\ 0 & 1 & 0 \\ -\sin\alpha_e & 0 & \cos\alpha_e \end{bmatrix} \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} \tag{A10.1}$$

or

$$\begin{aligned} x_w &= x_b\cos\alpha_e + z_b\sin\alpha_e \\ y_w &= y_b \\ z_w &= z_b\cos\alpha_e + x_b\sin\alpha_e \end{aligned} \tag{A10.2}$$

Wind to body axes

A set of coordinates in a wind axes system x_w, y_w, z_w may be transformed into the equivalent set in a body axes system x_b, y_b, z_b by application of the direction cosine matrix given by equation (2.12). Again, writing $\theta = \alpha_e$ and $\phi = \psi = 0$ since level symmetric flight is assumed then

$$\begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = \begin{bmatrix} \cos\alpha_e & 0 & -\sin\alpha_e \\ 0 & 1 & 0 \\ \sin\alpha_e & 0 & \cos\alpha_e \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} \tag{A10.3}$$

which is simply the inverse of equation (A10.1). Alternatively

$$\begin{aligned} x_b &= x_w \cos\alpha_e - z_w \sin\alpha_e \\ y_b &= y_w \\ z_b &= z_w \cos\alpha_e + x_w \sin\alpha_e \end{aligned} \tag{A10.4}$$

THE TRANSFORMATION OF THE MOMENT OF INERTIA IN ROLL FROM A BODY AXES REFERENCE TO A WIND AXES REFERENCE

The moment of inertia in roll is defined in Table 4.1, and may be written when referenced to a system of wind axes

$$I_{x_w} = \sum \delta m (y_w^2 + z_w^2) \tag{A10.5}$$

Substitute for y_w and z_w from equations (A10.2) to obtain

$$I_{x_w} = \sum \delta m (y_b^2 + z_b^2) + \sum \delta m (x_b^2 - z_b^2) \sin^2\alpha_e - 2\sum \delta m x_b z_b \sin\alpha_e \cos\alpha_e \tag{A10.6}$$

Add the following null expression to the right hand side of equation (A10.6)

$$\sum \delta m (y_b^2 + z_b^2) \sin^2\alpha_e - \sum \delta m (y_b^2 + z_b^2) \sin^2\alpha_e$$

and rearrange to obtain

$$\begin{aligned} I_{x_w} = &\sum \delta m (y_b^2 + z_b^2) \cos^2\alpha_e + \sum \delta m (x_b^2 + y_b^2) \sin^2\alpha_e \\ &- 2\sum \delta m x_b z_b \sin\alpha_e \cos\alpha_e \end{aligned} \tag{A10.7}$$

Referring to the definitions of moments and products of inertia in Table 4.1, equation (A10.7) may be rewritten

$$I_{x_w} = I_{x_b} \cos^2\alpha_e + I_{z_b} \sin^2\alpha_e - 2I_{xz_b} \sin\alpha_e \cos\alpha_e \tag{A10.8}$$

Equation (A10.8) therefore describes the inertia transformation from a body axes reference to a wind axes reference.

This simple procedure may be repeated to obtain all of the moment and product of inertia transformations from a body axes reference to a wind axes reference.

The inverse procedure, using the coordinate transformations given by equations (A10.4), is equally straightforward to apply to obtain the corresponding transformations from a wind axes reference to a body axes reference.

SUMMARY

The *body to wind axes* moments and products of inertia transformations are summarised in Table A10.1. The corresponding transformations from *wind to body axes* obtained by the inverse procedure are summarised in Table A10.2.

Table A10.1 *Moment and product of inertia transformations from a body to wind axes reference*

Wind axes	*Body axes*
I_{x_w}	$I_{x_b}\cos^2\alpha_e + I_{z_b}\sin^2\alpha_e - 2I_{xz_b}\sin\alpha_e\cos\alpha_e$
I_{y_w}	I_{y_b}
I_{z_w}	$I_{z_b}\cos^2\alpha_e + I_{x_b}\sin^2\alpha_e + 2I_{xz_b}\sin\alpha_e\cos\alpha_e$
I_{xy_w}	$I_{xy_b}\cos\alpha_e + I_{yz_b}\sin\alpha_e$
I_{xz_w}	$I_{xz_b}(\cos^2\alpha_e - \sin^2\alpha_e) + (I_{x_b} - I_{z_b})\sin\alpha_e\cos\alpha_e$
I_{yz_w}	$I_{yz_b}\cos\alpha_e - I_{xy_b}\sin\alpha_e$

Table A10.2 *Moment and product of inertia transformations from a wind to body axes reference*

Body axes	*Wind axes*
I_{x_b}	$I_{x_w}\cos^2\alpha_e + I_{z_w}\sin^2\alpha_e + 2I_{xz_w}\sin\alpha_e\cos\alpha_e$
I_{y_b}	I_{y_w}
I_{z_b}	$I_{z_w}\cos^2\alpha_e + I_{x_w}\sin^2\alpha_e - 2I_{xz_w}\sin\alpha_e\cos\alpha_e$
I_{xy_b}	$I_{xy_w}\cos\alpha_e - I_{yz_w}\sin\alpha_e$
I_{xz_b}	$I_{xz_w}(\cos^2\alpha_e - \sin^2\alpha_e) + (I_{z_w} - I_{x_w})\sin\alpha_e\cos\alpha_e$
I_{yz_b}	$I_{yz_w}\cos\alpha_e + I_{xy_w}\sin\alpha_e$