Appendix 8

Approximate Expressions for the Dimensionless Aerodynamic Stability and Control Derivatives

Table A8.1 *Longitudinal aerodynamic stability derivatives*

Small perturbation derivatives referred to aircraft wind axes

Derivative	*Description*	*Expression*	*Comments*
X_u Equation (13.16)	Axial force due to velocity	$-2C_D - V_0\dfrac{\partial C_D}{\partial V} + \dfrac{1}{\frac{1}{2}\rho V_0 S}\dfrac{\partial \tau}{\partial V}$	Drag and thrust effects due to velocity perturbation
X_w Equation (13.27)	Axial force due to "incidence"	$C_L - \dfrac{\partial C_D}{\partial \alpha}$	Lift and drag effects due to incidence perturbation
X_q Equation (13.46)	Axial force due to pitch rate	$-\overline{V}_T\dfrac{\partial C_{D_T}}{\partial \alpha_T}$	Tailplane drag effect, usually negligible
$X_{\dot{w}}$ Equation (13.68)	Axial force due to downwash lag	$-\overline{V}_T\dfrac{\partial C_{D_T}}{\partial \alpha_T}\dfrac{d\varepsilon}{d\alpha} \equiv X_q\dfrac{d\varepsilon}{d\alpha}$	Tailplane drag due to downwash lag effect (added mass effect)
Z_u Equation (13.21)	Normal force due to velocity	$-2C_L - V_0\dfrac{\partial C_L}{\partial V}$	Lift effects due to velocity perturbation
Z_w Equation (13.30)	Normal force due to "incidence"	$-C_D - \dfrac{\partial C_L}{\partial \alpha}$	Lift and drag effects due to incidence perturbation
Z_q Equation (13.51)	Normal force due to pitch rate	$-\overline{V}_T a_1$	Tailplane lift effect
$Z_{\dot{w}}$ Equation (13.72)	Normal force due to downwash lag	$-\overline{V}_T a_1\dfrac{d\varepsilon}{d\alpha} = Z_q\dfrac{d\varepsilon}{d\alpha}$	Tailplane lift due to downwash lag effect (added mass effect)
M_u Equation (13.34)	Pitching moment due to velocity	$V_0\dfrac{\partial C_m}{\partial V}$	Mach dependent, small at low speed

Table A8.1 (*Continued*)

Small perturbation derivatives referred to aircraft wind axes

Derivative	*Description*	*Expression*	*Comments*
M_w Equation (13.39)	Pitching moment due to "incidence"	$\dfrac{dC_m}{d\alpha} = -aK_n$	Pitch stiffness, dependent on static margin
M_q Equation (13.55)	Pitching moment due to pitch rate	$-\overline{V}_T a_1 \dfrac{l_T}{\overline{\overline{c}}} \equiv Z_q \dfrac{l_T}{\overline{\overline{c}}}$	Pitch damping, due mainly to tailplane
$M_{\dot{w}}$ Equation (13.76)	Pitching moment due to downwash lag	$-\overline{V}_T a_1 \dfrac{l_T}{\overline{\overline{c}}} \dfrac{d\varepsilon}{d\alpha} \equiv M_q \dfrac{d\varepsilon}{d\alpha}$	Pitch damping due to downwash lag effect at tailplane

Table A8.2 *Lateral aerodynamic stability derivatives*

Small perturbation derivatives referred to aircraft wind axes

Derivative	*Description*	*Expression*	*Comments*
Y_v Equation (13.82)	Side force due to sideslip	$\left(\dfrac{S_B}{S} y_B - \dfrac{S_F}{S} a_{1_F}\right)$	Always negative and hence stabilising
L_v Equation (13.92) Equation (13.105) Equation (13.108)	Rolling moment due to sideslip	(i) Wing with dihedral $-\dfrac{1}{Ss}\displaystyle\int_0^s c_y a_y \Gamma y \, dy$ (ii) Wing with aft sweep $-\dfrac{2C_L \tan \Lambda_{1/4}}{Ss}\displaystyle\int_0^s c_y y \, dy$ (iii) Fin contribution $-a_{1_F} \overline{V}_F \dfrac{h_F}{l_F}$	Lateral static stability, determined by total dihedral effect. Many contributions most of which are difficult to estimate reliably. Most accessible approximate contributions given.
N_v Equation (13.112)	Yawing moment due to sideslip	(i) Fin contribution $a_{1_F} \overline{V}_F$	Natural weathercock stability, dominated by fin effect.
Y_p Equation (13.117)	Side force due to roll rate	(i) Fin contribution $-\dfrac{1}{Sb}\displaystyle\int_0^{H_F} a_h c_h h \, dh$	Fin effect dominates, often negligible.
L_p Equation (13.128)	Rolling moment due to roll rate	(i) Wing contribution $-\dfrac{1}{2Ss^2}\displaystyle\int_0^s (a_y + C_{D_y}) c_y y^2 \, dy$	Roll damping, wing effects dominate but fin and tailplane contribute.

(*Continued*)

Table A8.2 *(Continued)*

Small perturbation derivatives referred to aircraft wind axes

Derivative	*Description*	*Expression*	*Comments*
N_p Equation (13.137)	Yawing moment due to roll rate	(i) Wing contribution $-\frac{1}{2Ss^2}\int_0^s \left(C_{L_y} - \frac{dC_D}{d\alpha_y}\right) c_y y^2 dy$	
Y_r Equation (13.142)	Side force due to yaw rate	(i) Fin contribution $\overline{V}_F a_{1_F}$	Many contributions, but often negligible.
L_r Equation (13.150) Equation (13.155)	Rolling moment due to yaw rate	(i) Wing contribution $\frac{1}{Ss^2}\int_0^s C_{L_y} c_y y^2 dy$ (ii) Fin contribution $a_{1_F}\overline{V}_F \frac{h_F}{b} \equiv -L_{v(fin)}\frac{l_F}{b}$	
N_r Equation (13.162) Equation (13.167)	Yawing moment due to yaw rate	(i) Wing contribution $-\frac{1}{Ss^2}\int_0^s C_{D_y} c_y y^2 dy$ (ii) Fin contribution $-a_{1_F}\overline{V}_F \frac{l_F}{b} = -\frac{l_F}{b} N_{v(fin)}$	Yaw damping, for large aspect ratio rectangular wing, wing contribution is approximately $C_D/6$.

Table A8.3 *Longitudinal aerodynamic control derivatives*

Small perturbation derivatives referred to aircraft wind axes

Derivative	*Description*	*Expression*	*Comments*
X_η Equation (13.174)	Axial force due to elevator	$-2\frac{S_T}{S} k_T C_{L_T} a_2$	Usually insignificantly small
Z_η Equation (13.178)	Normal force due to elevator	$-\frac{S_T}{S} a_2$	
M_η Equation (13.181)	Pitching moment due to elevator	$-\overline{V}_T a_2$	Principal measure of pitch control power

Table A8.4 *Lateral-directional aerodynamic control derivatives*

Small perturbation derivatives referred to aircraft wind axes

Derivative	*Description*	*Expression*	*Comments*
Y_ξ	Side force due to aileron		Insignificant for conventional aeroplanes
L_ξ Equation (13.188)	Rolling moment due to aileron	$-\dfrac{1}{Ss}a_{2_A}\displaystyle\int_{y_1}^{y_2} c_y y\,\mathrm{d}y$	Principal measure of roll control power
N_ξ Equation (13.193)	Yawing moment due to aileron	$\dfrac{1}{Ss}\displaystyle\int_{y_1}^{y_2}\left(\dfrac{\partial C_{D_y}}{\partial\xi}\right)c_y y\,\mathrm{d}y$	Adverse yaw due to aileron
Y_ζ Equation (13.197)	Side force due to rudder	$\dfrac{S_F}{S}a_{2_R}$	
L_ζ Equation (13.200)	Rolling moment due to rudder	$\overline{V}_F\dfrac{h_F}{l_F}a_{2_R}$	Principal measure of yaw control power
N_ζ Equation (13.203)	Yawing moment due to rudder	$-\overline{V}_F a_{2_R}$	Adverse roll due to rudder