Fundamentals of Data Structures

Projects1 : Performance Measurement (A + B)

Date : 2024/03/04

2023–2024 Spring & Summer Semester

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Chapter 1 : Introduction

Problem

Description

Given a set S of n positive integers that are no more than V. For any given number c, you are supposed to find two integers a and b from S, so that $a + b = c$.

Input

The input file includes $K + 3$ lines.The first line provides a positive integer n.The second line provides n positive integers which constitute the set S.The third line provides a positive integer K.The next K lines provides K positive integers,meaning that there are K queries to find the pair $< a, b >$ for each integer.

Output

The output file requires to print out all pairs $< a, b >$ for each query in the format "No.X : A + B = C"

If there is no answer,print "Cannot find the possible <a,b>!"

Sample Input

```
10
1 2 3 4 5 6 7 8 9 10
1
11
```

Sample Output

```
No.     1 :      1 +     10 =     11
No.     2 :      2 +      9 =     11
No.     3 :      3 +      8 =     11
No.     4 :      4 +      7 =     11
No.     5 :      5 +      6 =     11
```

Algorithm Analysis

We have two algorithms to solve this problem. One is directly searching the pairs whose time complexity is $O(N^2)$ and another is using Binary_Search algorithm together with Quick_Sort algorithm whose time complexity is $O(NlogN)$.Introductions of Binary_Search algorithm and Quick_Sort algorithm are as follows:

Binary_Search algorithm

The Binary_Search algorithm is an efficient search algorithm used to find target elements in ordered datasets.

Its implementation principle is based on the idea of divide and conquer, which quickly locates target elements by gradually reducing the search range by half.It's implementation steps are as follows:

(1)Initialize two pointers

The left pointer points to the starting position of the search range.

The right pointer points to the end of the search range.

(2)Find intermediate element

Calculate the middle position of the left and right pointers, which is (left+right)/2.

(3)Compare intermediate element

Compare the target element with the element in the middle position.

If the target element is equal to the element in the middle position, the target element is found and the search ends.

If the target element is smaller than the element in the middle position, update the right pointer to the previous position in the middle position, and narrow the search range to the left half.

If the target element is greater than the element in the middle position, update the left pointer to the next position in the middle position, and narrow the search range to the right half.

(4)Repeat steps (2) and (3)

Continuously calculate the middle position and compare the middle elements until any of the following conditions are met:

1)Find the target element, which is equal to the element in the middle position.

2)The left pointer is greater than the right pointer, indicating that the search range is empty and the target element does not exist.

Quick_Sort Algorithm

The Quick_Sort Algorithm evolves from Bubble_Sort Algorithm and is actually a recursive divide and conquer method based on bubble sorting.

Quick_Sort Algorithm selects a standard element in each round and moves other larger elements to one side of the sequence, while smaller elements move to the other side, thus breaking down the sequence into two parts.It's implementation steps are as follows:

(1)Initialize two pointers

The left pointer points to the starting position of the search range.

The right pointer points to the end of the search range.

(2)Find intermediate element

Calculate the middle position of the left and right pointers, which is (left+right)/2.

(3)Arrange the left and right areas

Move elements larger than the intermediate element to right side of the sequence, while elements smaller than the intermediate element move to the left side.

(4)Repeat steps (2) and (3)

Continuously calculate the middle position and arrange the left and right areas until the left pointer is greater than the right pointer, indicating that the search range is empty and the sorting process is over.

Chapter 2 : Algorithm Specification

Algorithm 1

Algorithm 1 uses 2 "for" loops to solve this problem.The first loop enumerates a while the second loop enumerates b.Then check whether $a + b = c$.Pseudo-code is shown below:

```
for i from 1~n
    for j from i+1~n
        if num[i]+num[j]==c then
        {
            total++;flag=1;//flag=1 means there exists a pair of<a,b>,otherwise flag=0
            print out the answer;
        }
```

Algorithm 2

Algorithm 2 uses Binary_Search Algorithm and Quick_Sort Algorithm to solve this problem.We enumerate a and check whether $c - a$ exists in the array.To check whether $c - a$ exists in the array,the fastest algorithm is Binary_Search Algorithm,but we still need Quick_Sort Algorithm because Binary_Search Algorithm asks the array to be in ascending order or in descending order.Pseudo-code is shown below:

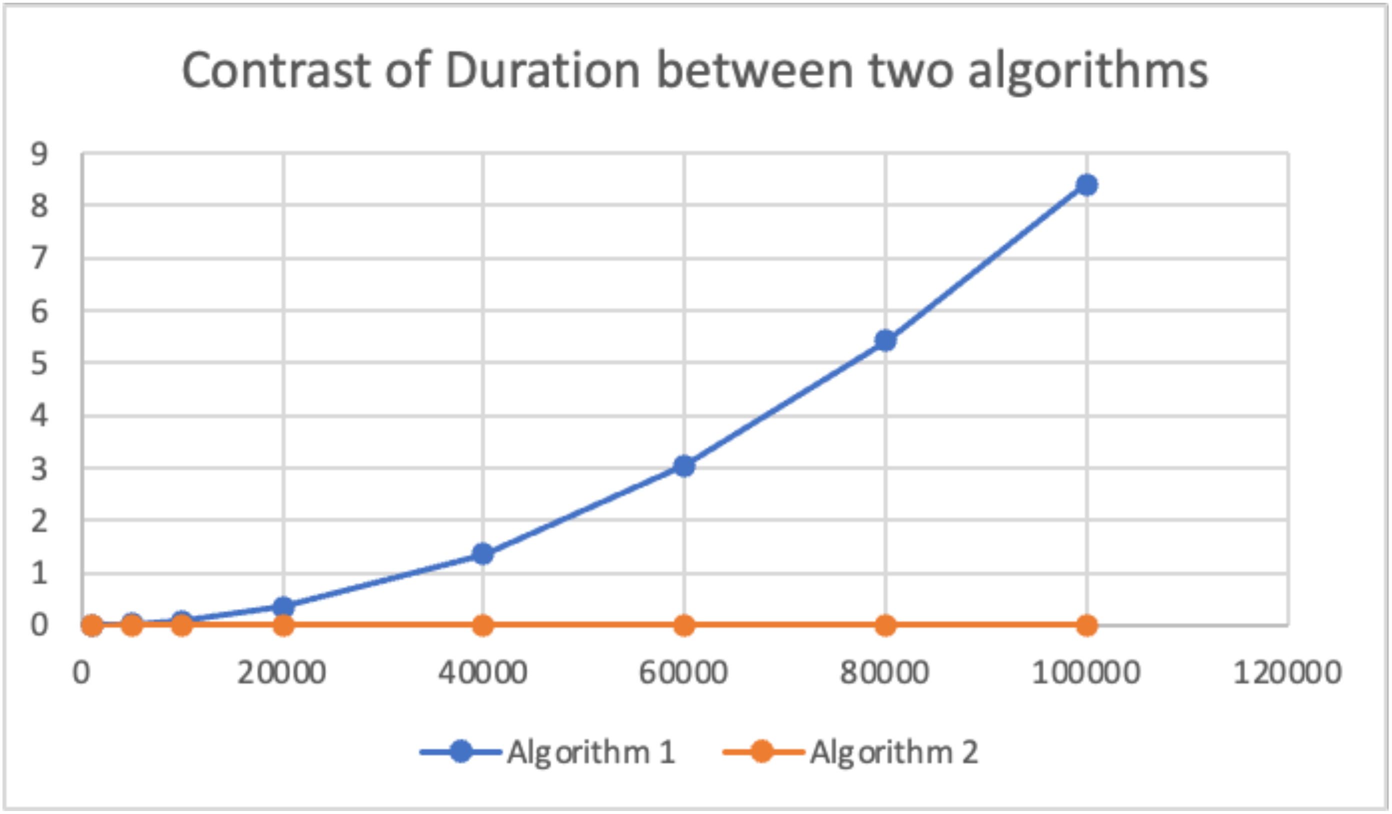
```
for i from 1~n
    if c-num[i] exists in the array num then
    {
        total++;flag=1;//flag=1 means there exists a pair of<a,b>,otherwise flag=0
        print out the answer;
    }
```

Chapter 3 : Testing Results

Test Result Table

V=x	N	1000	5000	10000	20000	40000	60000	80000	100000
	Iterations(K)	10	10	10	5	5	5	5	5
	Ticks	9054	213926	851319	1700067	6744624	15187568	27137317	42027946
Algorithm 1 ($O(N^2)$)	Total Time(sec)	0.009054	0.213926	0.851319	1.700067	6.744624	15.187568	27.137317	42.027946
	Duration(sec)	0.000905	0.021393	0.085132	0.340013	1.348925	3.037514	5.427463	8.405589
	Iterations(K)	10	10	10	5	5	5	5	5
	Ticks	609	3422	6920	8153	16891	27993	34993	48396
Algorithm 2 ($O(NlogN)$)	Total Time(sec)	0.000609	0.003422	0.006920	0.008153	0.016891	0.027993	0.034993	0.048396
	Duration	0.000061	0.000342	0.000692	0.001631	0.003378	0.005599	0.006999	0.009679

Test Result Plot



Chapter 4 : Analysis and Comments

Algorithm 1

Obviously the time complexity of Algorithm 1 is $O(N^2)$ because it uses two loops of range n.

The memory complexity of Algorithm 1 is $O(N)$ because it only use the array "num" to store the datas.

Its code is simple but the duration of time is longer.

Algorithm 2

The average time complexity of Quick_Sort Algorithm is $O(NlogN)$ and the worst time complexity is $O(N^2)$.The time complexity of Binary_Search Algorithm is $O(logN)$.Since we enumerate a in the loop of range n and use Binary_Search Algorithm to search b,the whole average time complexity of Algorithm 2 is $O(NlogN)$.

The memory complexity of Algorithm 2 is $O(N)$ because it only use the array "num" to store the datas.

Its duration of time is shorter,but there is a possible data to make Quick_Sort Algorithm's time complexity reach the worst situation $O(N^2)$.For a common set of input,Algorithm 2 is much better.

Appendix : Source Code

Algorithm 1 $(O(N^2))$

```c
#include<stdio.h>
#include<time.h>

clock_t start,stop;
double duration;

int num[100001];

int main(void)
{
    /*Start the clock*/
    start=clock();

    /*Open the file*/
    FILE *fp1,*fp2;
    fp1=fopen("test.in","r");
    fp2=fopen("testA1.out","w");

    /*Main function*/
    int n,c,k;//flag=0 means we haven't found the pair,flag=1 means we have found the pair.
    fscanf(fp1,"%d",&n);
    for(int i=1;i<=n;i++)
        fscanf(fp1,"%d",&num[i]);
    fscanf(fp1,"%d",&k);
    for(int l=1;l<=k;l++)
    {
        int total=0,flag=0;
        fscanf(fp1,"%d",&c);
        for(int i=1;i<=n;i++)
            for(int j=i+1;j<=n;j++)//Enumerate all possible pairs
                if(num[i]+num[j]==c)//Check whether their sum is c
                {
                    flag=1;//We have found the answer
                    fprintf(fp2,"No.%6d : %6d + %6d = %6d\n",++total,num[i],num[j],c);
                }
        if(flag==0)//If there is no answer
            fprintf(fp2,"Cannot find the possible <a,b>!\n");
        fprintf(fp2,"\n");
    }

    /*Stop the clock*/
    stop=clock();
    duration = ((double)(stop-start))/CLOCKS_PER_SEC;
    fprintf(fp2,"The iteration is:%d\n",k);
    fprintf(fp2,"The ticks are:%lu\n",stop-start);
    fprintf(fp2,"The total time is:%.6lf(sec)\n",duration);
    fprintf(fp2,"The duration is:%.6lf(sec)\n",duration/k);

    /*Close the file*/
    fclose(fp1);
    fclose(fp2);

    return 0;
}
```

Algorithm 2 $(O(NlogN))$

```c
#include<stdio.h>
#include<time.h>

clock_t start,stop;
double duration;

int num[100001];

/*Quick_Sort Algorithm Code*/
void quick_sort(int l,int r)
{
    int i,j,mid,temp;
    i=l;j=r;mid=num[(l+r)/2];//Set num[(l+r)/2] as the standard.Numbers smaller go left and numbers bigger go right.
    do{//Distribute numbers in the left and right area
        while(num[i]<mid)//Numbers smaller remain in the left area
            i++;
        while(num[j]>mid)//Numbers bigger remain in the right area
            j--;
        if(i<=j)
        {
            temp=num[i];num[i]=num[j];num[j]=temp;//Swap num[i] and num[j]
            i++;j--;
        }
    }while(i<=j);//If i>j,then the distribution is over and correct.
    if(l<j)
        quick_sort(l,j);//Continue sorting the left area
    if(i<r)
        quick_sort(i,r);//Continue sorting the right area
}

/*Binary Search Algorithm Code*/
int binary_search(int l,int r,int target)
{
    while(l<=r)
    {
        int mid=(l+r)/2;
        if(target<num[mid])
            r=mid-1;//Search the left area of the array;
        else if(target>num[mid])
            l=mid+1;//Search the right area of the array;
        else//We found the target
            return 1;
    }
    return 0;//If l>r,then there's no target in the array
}

int main(void)
{
    /*Start the clock*/
    start=clock();

    /*Open the file*/
    FILE *fp1,*fp2;
    fp1=fopen("test.in","r");
    fp2=fopen("testA2.out","w");

    /*Main function*/
    int n,c,k;
    fscanf(fp1,"%d",&n);
    for(int i=1;i<=n;i++)
        fscanf(fp1,"%d",&num[i]);
    quick_sort(1,n);//Sort the numbers to operate binary search
    fscanf(fp1,"%d",&k);
    for(int l=1;l<=k;l++)
    {
        fscanf(fp1,"%d",&c);
        int flag=0,total=0;
        for(int i=1;i<=n;i++)
            if((num[i]<=(c-1)/2)&&binary_search(1,n,c-num[i]))//To avoid repetition,we enumerate numbers no bigger than (c-1)/2.
            {
                flag=1;//We have found the answer
                fprintf(fp2,"No.%6d : %6d + %6d = %6d\n",++total,num[i],c-num[i],c);
            }
        if(flag==0)//We haven't found the answer
            fprintf(fp2,"Cannot find the possible <a,b>!\n");
        fprintf(fp2,"\n");
    }

    /*Stop the clock*/
    stop=clock();
    duration = ((double)(stop-start))/CLOCKS_PER_SEC;
    fprintf(fp2,"The iteration is:%d\n",k);
    fprintf(fp2,"The ticks are:%lu\n",stop-start);
    fprintf(fp2,"The total time is:%.6lf(sec)\n",duration);
    fprintf(fp2,"The duration is:%.6lf(sec)\n",duration/k);

    /*Close the file*/
    fclose(fp1);
    fclose(fp2);

    return 0;
}
```

Data Generator

```c
#include<stdio.h>
#include<stdlib.h>
#include<time.h>

#define MAXN 10000//Change the parameter here.

const int MAXM=5,MAXV=MAXN*10;//Change the parameter here.

int num[MAXN];

int search(int l,int r,int target)
{
    for(int i=l;i<=r;i++)
        if(num[i]==target)
            return 1;
    return 0;
}

int main(void)
{
    srand((unsigned int)time(NULL));
    FILE *fp;
    fp=fopen("test.in","w");
    fprintf(fp,"%d\n",MAXN);
    int maxnum=-0x7f;
    for(int i=1;i<=MAXN;i++)
    {
        int x=rand();
        while((x<=0)||(x>MAXV)||(search(1,i-1,x)))
            x=rand();
        if(x>maxnum)
            maxnum=x;
        fprintf(fp,"%d ",x);
        num[i]=x;
    }
    fprintf(fp,"\n");
    fprintf(fp,"%d\n",MAXM);
    for(int i=1;i<=MAXM;i++)
    {
        int x=rand();
        while((x<=0)||(x>maxnum))
            x=rand();
        fprintf(fp,"%d\n",x);
    }
    fclose(fp);
    return 0;
}
```

Declaration

I hereby declare that all the work done in this project titled “Performance Measurement （A+B）” is of my independent effort.