Graph Theory

Instructor: Oliver Janzer

Assignment 7

Please submit your solution to Problem 1 by the end of November 4th for feedback.

Unless noted otherwise, all graphs considered are simple. The solution of every problem should be no longer than one page.

Problem 1: Let G be a connected graph on n vertices with minimum degree δ. Show that

(a) if $\delta \leq \frac{n-1}{2}$ then G contains a path of length 2δ, and

(b) if $\delta \geq \frac{n-1}{2}$ then G contains a Hamiltonian path.

Problem 2: Show that the maximum number of edges in a non-Hamiltonian graph on $n \geq 3$ vertices is $\binom{n-1}{2} + 1$.

Problem 3:

(a) Show that if n is even, then for every $0 \leq d \leq n - 1$, there exists a d-regular graph on n vertices.

(b) For odd n, show that there exists a d-regular graph on n vertices for every **even** $0 \leq d \leq n - 1$.