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UNIVERSITY OF MUMBAI

Bachelor of Engineering

First Year Engineering (Semester I & II), Revised course

(REV- 2019'C' Scheme) from Academic Year 2019 – 20

(Common for All Branches of Engineering)

Under

FACULTY OF SCIENCE &TECHNOLOGY

(As per AICTE guidelines with effect from the academic year 2019–2020)

Program Structure for First Year Engineering
Semester I & II
UNIVERSITY OF MUMBAI
(With Effect from 2019-2020)

Semester I

Course Code	Course Name	Teaching Scheme (Contact Hours)			Credits Assigned			
		Theory	Pract.	Tut.	Theory	Pract.	Tut.	Total
FEC101	Engineering Mathematics-I	3	--	1*	3	--	1	4
FEC102	Engineering Physics-I	2		--	2		--	2
FEC103	Engineering Chemistry-I	2	--	--	2	--	--	2
FEC104	Engineering Mechanics	3	--	--	3	--	--	3
FEC105	Basic Electrical Engineering	3	--	--	3	--	--	3
FEL101	Engineering Physics-I	--	1	--	--	0.5	--	0.5
FEL102	Engineering Chemistry-I	--	1	--	--	0.5	--	0.5
FEL103	Engineering Mechanics	--	2	--	--	1	--	1
FEL104	Basic Electrical Engineering	--	2	--	--	1	--	1
FEL105	Basic Workshop practice-I	--	2	--	--	1	--	1
Total		**13**	**08**	**01**	**13**	**04**	**01**	**18**

Course Code	Course Name	Examination Scheme							
		Theory					Term Work	Pract. /oral	Total
		Internal Assessment			End Sem. Exam.	Exam. Duration (in Hrs)			
		Test1	Test 2	Avg.					
FEC101	Engineering Mathematics-I	20	20	20	80	3	25	--	125
FEC102	Engineering Physics-I	15	15	15	60	2	--	--	75
FEC103	Engineering Chemistry-I	15	15	15	60	2	--	--	75
FEC104	Engineering Mechanics	20	20	20	80	3	--	--	100
FEC105	Basic Electrical Engineering	20	20	20	80	3	--	--	100
FEL101	Engineering Physics-I	--	--	--	--	--	25	--	25
FEL102	Engineering Chemistry-I	--	--	--	--	--	25	--	25
FEL103	Engineering Mechanics	--	--	--	--	--	25	25	50
FEL104	Basic Electrical Engineering	--	--	--	--	--	25	25	50
FEL105	Basic Workshop practice-I	--	--	--	--	--	50	--	50
Total		**--**	**--**	**90**	**360**	--	**175**	**50**	**675**

*** Shall be conducted batch-wise**

Course Code	Course Name	Teaching Scheme (Contact Hours)			Credits Assigned			
		Theory	Pract.	Tut.	Theory	Tut.	Pract.	Total
FEC201	Engineering Mathematics-I	3	--	1*	3	1	--	4

Course Code	Course Name	Examination Scheme							
		Theory					Term Work	Pract. /oral	Total
		Internal Assessment			End Sem. Exam.	Exam. Duration (in Hrs)			
		Test1	Test 2	Avg.					
FEC201	Engineering Mathematics-I	20	20	20	80	3	25	--	125

Course Objectives: The course is aimed

1. to develop the basic Mathematical skills of engineering students that are imperative for effective understanding of engineering subjects. The topics introduced will serve as basic tools for specialized studies in many fields of engineering and technology.
2. to provide hands on experience using SCILAB software to handle real life problems.

Course Outcomes: Students will be able to

1. Apply the basic concepts of Complex Numbers and will be able to use it for engineering problems.
2. Apply hyperbolic functions and logarithms in the subjects like electrical circuits, Electromagnetic wave theory.
3. Apply the basic concepts of partial differentiation of function of several variables and will be able to use in subjects like Electromagnetic Theory, Heat and Mass Transfer etc.
4. Apply the concept of Maxima, Minima and Successive differentiation and will be able to use it for optimization and tuning the systems.
5. Apply the concept of Matrices and will be able to use it for solving the KVL and KCL in electrical networks.
6. Apply the concept of Numerical Methods for solving the engineering problems with the help of SCILAB software.

Module	Detailed Contents	Hrs.
01	**Complex Numbers** **Pre-requisite:**Review of Complex Numbers-Algebra of Complex Number, Cartesian, polar and exponential form of complex number. 1.1. Statement of **D'Moivre's Theorem**. 1.2. Expansion of $\sin^n\theta$, $\cos^n\theta$ in terms of sines and cosines of multiplesof θ and Expansion of $\sin n\theta$, $\cos n\theta$ in powers of $\sin\theta$, $\cos\theta$ 1.3. Powers and Roots of complex number.	2 2 2
02	**Hyperbolic function and Logarithm of Complex Numbers** 2.1. Circular functions of complex number and Hyperbolic functions. Inverse Circular and Inverse Hyperbolic functions. Separation of real and imaginary parts of all types of Functions. 2.2 Logarithmic functions, Separation of real and Imaginary parts of Logarithmic Functions. # **Self learning topics:**Applications of complex number in Electrical circuits.	4 2

03	**Partial Differentiation** 3.1 Partial Differentiation: Function of several variables, Partial derivatives of first and higher order. Differentiation of composite function. 3.2.Euler's Theorem on Homogeneous functions with two independent variables (with proof). Deductions from Euler's Theorem. **# Self learning topics:**Total differentials,implicit functions, Euler's Theorem on Homogeneous functions with three independent variables.	3 3
04	**Applications of Partial Differentiation and Successive differentiation.** 4.1 Maxima and Minima of a function of two independent variables, Lagrange's method of undetermined multipliers with one constraint. 4.2 Successive differentiation: nth derivative of standard functions. Leibnitz's Theorem (without proof) and problems **# Self learning topics:** Jacobian's of two and three independent variables (simple problems)	3 3
05	**Matrices** **Pre-requisite:** Inverse of a matrix, addition, multiplication and transpose of a matrix 5.1.Types of Matrices (symmetric, skew- symmetric, Hermitian, Skew Hermitian, Unitary, Orthogonal Matrices and properties of Matrices). Rank of a Matrix using Echelon forms, reduction to normal form and PAQ form. **5.**2.System of homogeneous and non –homogeneous equations, their consistency and solutions. **# Self learning topics:**Application of inverse of a matrix to coding theory.	4 2
06	**Numerical Solutions of Transcendental Equations and System of Linear Equations and Expansion of Function.** 6.1 Solution of Transcendental Equations: Solution by Newton Raphson method and Regula –Falsi method. 6.2 Solution of system of linear algebraic equations, by (1) Gauss Jacobi Iteration Method, (2) Gauss Seidal Iteration Method. 6.3 Taylor's Theorem (Statement only) and Taylor's series, Maclaurin's series (Statement only). Expansion of e^{x} sin(x), cos(x), tan(x), sinh(x), cosh(x), tanh(x), log(1+x),$\sin^{-1}(x)$,$\cos^{-1}(x)$,$\tan^{-1}(x)$. **# Self learning topics:**Indeterminate forms, L- Hospital Rule, Gauss Elimination Method, Gauss Jordan Method.	2 2 2

Term Work:

General Instructions:

1. Batch wise tutorials are to be conducted. The number of students per batch should be as per University pattern for practicals.
2. Students must be encouraged to write SCILAB Programs in tutorial class only. Each Student has to write at least 4 SCILAB tutorials (including print out) and at least 6 class tutorials on entire syllabus.
3. SCILAB Tutorials will be based on (i) Guass Elimination Method (ii) Guass Seidal Iteration method (iii) Gauss Jacobi Iteration Method (iv) Newton Raphson Method (v) Regula –Falsi method (vi) Maxima and Minima of functions of two variables

The distribution of Term Work marks will be as follows –

1.	Attendance (Theory and Tutorial)	05 marks
2.	Class Tutorials on entire syllabus	10 marks
3.	SCILAB Tutorials	10 marks

Assessment:

Internal Assessment Test:
Assessment consists of two class tests of 20 marks each. The first class test is to be conducted when approx. 40% syllabus is completed and second class test when additional 35% syllabus is completed. Duration of each test shall be one hour.

End Semester Theory Examination:

1. Question paper will comprise of total 06 questions, each carrying 20 marks.
2. Total 04 questions need to be solved.
3. Question No: 01 will be compulsory and based on entire syllabus wherein 4sub-questions of 5 marks each will be asked.
4. Remaining questions will be randomly selected from all the modules.
5. Weightage of each module will be proportional to number of respective lecture hoursas mentioned in the syllabus.

References:

1. Higher Engineering Mathematics, Dr.B.S.Grewal, Khanna Publication
2. Advanced Engineering Mathematics, Erwin Kreyszig, Wiley EasternLimited, 9^{th}Ed.
3. Engineering Mathematics by Srimanta Pal and Subodh,C.Bhunia, Oxford University Press
4. Matrices, Shanti Narayan, .S. Chand publication.
5. Applied Numerical Methods with MATLABfor Engineers and Scientists by Steven Chapra, McGraw Hill
6. Elementary Linear Algebra with Application by Howard Anton and Christ Rorres. 6th edition. John Wiley & Sons,INC.

F.E

REVIEW (FORMULA)

COMPLEX NUMBERS

① **Standard form** $\Rightarrow Z = x + iy$

where x is real & y imaginary

② **Conjugate of a Complex no.** $\Rightarrow$

$x - iy$ i.e $\bar{Z}$

③ **Sum & product of conjugate no.** $\Rightarrow$

$Z = x + iy$ & $\bar{Z} = x - iy$

$\therefore Z + \bar{Z} = (x+iy) + (x-iy) = 2x$.

- Real part $x = \frac{Z + \bar{Z}}{2}$

Also $Z - \bar{Z} = (x+iy) - (x-iy) = 2iy$.

- Imaginary part $y = \frac{Z - \bar{Z}}{2i}$

$Z \cdot \bar{Z} = (x+iy)(x-iy) = x^2 + y^2$.

i.e The sum & product of two complex conjugates are real but their difference is imaginary.

④ **Equality of Complex no.** $\Rightarrow$

Two complex no. are equal iff (their real parts are equal) $x_1 = x_2$ & $y_1 = y_2$.

⑤ **Modulus & Argument of a complex no.** $\Rightarrow$

$Z = r(\cos\theta + i\sin\theta)$ called polar form

where $r = \sqrt{x^2 + y^2}$

& $\theta = \tan^{-1}\left|\frac{y}{x}\right|$

* Modulus is +ve
* Argument is +ve or -ve

r is modulus of Z & denoted by $|Z|$

θ is argument or amplitude of Z

θ lying betn $-\pi$ and π called principal value.

Exponential Form of a complex No. ⟹

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$$

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

$$e^{-i\theta} = \cos\theta - i\sin\theta.$$

$$e^{i\theta} + e^{-i\theta} = 2\cos\theta.$$

$$e^{i\theta} - e^{-i\theta} = 2i\sin\theta.$$

$z = re^{i\theta}$ called exponential Form or Euler's Form.

$z = x + iy$. —— Cartesian form

$z = r(\cos\theta + i\sin\theta)$ —— Polar Form

$z = re^{i\theta}$ —— Exponential —"—

||, by changing the sign of i;

$$\bar{z} = x - iy$$

$$\bar{z} = r(\cos\theta - i\sin\theta)$$

$$\bar{z} = re^{i\theta}$$

Cor ⟹ If $z = \cos\theta + i\sin\theta$ Then $\frac{1}{z} = \cos\theta - i\sin\theta$.

= Product & Quotient of 2 complex no. in Exponential Form ⟹

$$z_1 \cdot z_2 = r_1 \cdot r_2\, e^{i(\theta_1 + \theta_2)}$$

$$\frac{z_1}{z_2} = \left(\frac{r_1}{r_2}\right) e^{i(\theta_1 - \theta_2)}.$$

Meaning of multiplication & Division ⇒

(a) $\arg z_1 \cdot z_2 = \arg z_1 + \arg z_2$

(b) $\arg\left(\frac{z_1}{z_2}\right) = \arg. z_1 - \arg z_2$

properties of $|z|$ (Modulus of z) ⇒

(1) $|z| \geq 0$

(2) $|z| = 0$ iff $z = 0$ i.e $x = 0, y = 0$.

(3) $Re\ z \leq |z|$ $\quad (\because x \leq \sqrt{x^2+y^2} = |z|)$

(4) $Im\ z \leq |z|$

(5) $|z| = |\bar{z}|$ $\quad (\because |z| = |x+iy| = \sqrt{x^2+y^2} = |x-iy| = |\bar{z}|)$

(6) $z \cdot \bar{z} = |z|^2$ $\quad (\because z \cdot \bar{z} = (x+iy)(x-iy) = x^2+y^2 = |z|^2)$

(7) $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$

$(\because |z_1 \cdot z_2|^2 = (z_1 z_2)(\bar{z}_1 \bar{z}_2) = (z_1 \bar{z}_1) \cdot (z_2 \bar{z}_2) = |z_1|^2 |z_2|^2$

(8) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ if $|z_2| \neq 0$

$\left(\because |z_1| = \left|\frac{z_1 z_2}{z_2}\right| = \left|\frac{z_1}{z_2}\right| |z_2|\right)$

DeMoivre's Thm ⇒

For any rational no. n the value or one of the values of,

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.

Ex ⇒ If z_1 & z_2 are two complex no. ∋ $|z_1+z_2| = |z_1-z_2|$ P.T $\arg z_1 - \arg z_2 = \frac{\pi}{2}$

Sol ⇒ Let $z_1 = x_1+iy_1$ & $z_2 = x_2+iy_2$

$\because |z_1+z_2| = |z_1-z_2|$

$|(x_1+iy_1)+(x_2+iy_2)| = |(x_1+iy_1)-(x_2+iy_2)|$

$|(x_1+x_2)+i(y_1+y_2)| = |(x_1-x_2)+i(y_1-y_2)|$

$\sqrt{(x_1+x_2)^2+(y_1+y_2)^2} = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$\therefore 4(x_1x_2+y_1y_2) = 0$

$x_1x_2+y_1y_2 = 0$

Now $\arg z_1 - \arg z_2 = \tan^{-1}\frac{y_1}{x_1} - \tan^{-1}\frac{y_2}{x_2}$

$= \tan^{-1}\left(\frac{y_1/x_1 - y_2/x_2}{1+(y_1/x_1)(y_2/x_2)}\right)$

$= \tan^{-1}\infty = \pi/2$

EM-I

COMPLEX NUMBERS

Introduction ⇒

In this chapter, we shall learn how De Moiver's theorem can be used in various ways to obtain certain expansions and powers and roots of functions involving complex numbers

De-Moiver's theorem & its Corallaries ⇒

* De-Moiver's theorem ⇒

For any rational number n the value or one of the values of,

$$\boxed{(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta}$$

For Example ⇒

① $(\cos\theta + i\sin\theta)^6 = \cos 6\theta + i\sin 6\theta$

② $\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^4 = \cos 4\left(\frac{\pi}{4}\right) + i\sin 4\left(\frac{\pi}{4}\right)$

$$= \cos\pi + i\sin\pi$$

$$= -1 + 0 = -1$$

Corallary ⇒ ①

If $z = \cos\theta + i\sin\theta$ Then,

$$\boxed{\frac{1}{z} = \cos\theta - i\sin\theta}$$

Cor ② ⇒ $\boxed{(\cos\theta - i\sin\theta)^n = \cos n\theta - i\sin n\theta}$

Cor ③ ⇒ $\boxed{(\cos\theta - i\sin\theta)^n = (\cos n\theta + i\sin\theta)^{-n}}$

Cor ④ ⇒ $\boxed{\dfrac{1}{(\cos\theta + i\sin\theta)^n} = \cos n\theta - i\sin n\theta}$

Cor ⑤ ⇒ If $z = \cos\theta + i\sin\theta$, Then

$$\boxed{\begin{aligned} z^n &= \cos n\theta + i\sin n\theta \\ z^{-n} &= \cos n\theta - i\sin n\theta \end{aligned}}$$

By addition and subtraction

$$\boxed{\begin{aligned} \cos n\theta &= \frac{1}{2}(z^n + z^{-n}) \\ \sin n\theta &= \frac{1}{2i}(z^n - z^{-n}) \end{aligned}}$$

Cor ⑥ ⇒ If $z_1 = \cos\theta + i\sin\theta$, $z_2 = \cos\phi + i\sin\phi$

Then,

$$\boxed{\begin{aligned} z_1 \cdot z_2 &= \cos(\theta+\phi) + i\sin(\theta+\phi) \\ \frac{z_1}{z_2} &= \cos(\theta-\phi) + i\sin(\theta-\phi) \end{aligned}}$$

Ex

Simplify ⇒

① $\left[\dfrac{1+\sin(\pi/8)+i\cos(\pi/8)}{1+\sin(\pi/8)-i\cos(\pi/8)}\right]^8$

Sol ⇒ we have $1=\sin^2\theta+\cos^2\theta$

$=\sin^2\theta-i^2\cos^2\theta$

$=(\sin\theta+i\cos\theta)(\sin\theta-i\cos\theta)$

$\therefore\ 1+\sin\theta+i\cos\theta=[(\sin\theta+i\cos\theta)(\sin\theta-i\cos\theta)]+(\sin\theta+i\cos\theta)$

$=(\sin\theta+i\cos\theta)(\sin\theta-i\cos\theta+1)$

$\therefore\ \dfrac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}=\sin\theta+i\cos\theta$

$=\cos\left(\frac{\pi}{2}-\theta\right)+i\sin\left(\frac{\pi}{2}-\theta\right)$

Put $\theta=\pi/8$

$\therefore\ \left[\dfrac{1+\sin(\pi/8)+i\cos(\pi/8)}{1+\sin(\pi/8)-i\cos(\pi/8)}\right]^8=\left\{\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)+i\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)\right\}^8$

$=\cos 8\left(\frac{\pi}{2}-\frac{\pi}{8}\right)+i\sin 8\left(\frac{\pi}{2}-\frac{\pi}{8}\right)$

$=\cos 3\pi+i\sin 3\pi$

$=(-1)^3+0=-1$

Prove that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n(\theta/2)\cos(n\theta/2)$

Sol ⇒ we have $(1+\cos\theta)+i\sin\theta=2\cos^2(\theta/2)+2i\sin(\theta/2)\cos(\theta/2)$

$=2\cos(\theta/2)[\cos\theta/2+i\sin\theta/2]$

$1+\cos\theta-i\sin\theta=2\cos^2(\theta/2)-2i\sin(\theta/2)\cos(\theta/2)$

$=2\cos(\theta/2)[\cos(\theta/2)-i\sin\theta/2]$

$\therefore\ [1+\cos\theta+i\sin\theta]^n+[1+\cos\theta-i\sin\theta]^n=[2\cos(\theta/2)(\cos\theta/2+i\sin\theta/2)]^n$

$+[2\cos(\theta/2)(\cos(\theta/2)-i\sin(\theta/2))]^n$

$$= 2^n \cos^n(\theta/2)\left[\cos\theta/2 + i\sin\theta/2\right]^n + 2^n \cos^n\theta/2\left[\cos\theta/2 - i\sin\theta/2\right]^n$$

By De-Moivre's thm,

$$= 2^n \cos^n(\theta/2)\left(\cos\frac{n\theta}{2} + i\sin\frac{n\theta}{2}\right) + 2^n\cos^n\left(\frac{\theta}{2}\right)\left[\cos\frac{n\theta}{2} - i\sin\frac{n\theta}{2}\right]$$

$$= 2^n\cos^n\frac{\theta}{2}\left[\cos\frac{n\theta}{2} + \cancel{i\sin\frac{n\theta}{2}} + \cos\frac{n\theta}{2} - \cancel{i\sin\frac{n\theta}{2}}\right]$$

$$= 2^{n+1}\cos^n\frac{\theta}{2}\left(\cos\frac{n\theta}{2}\right)$$

(3) If $z = 1 + i\sqrt{3}$ and $\bar{z}$ is its conjugate of z, Prove that $(z)^8 + (\bar{z})^8 = -2^8$.

Sol ⇒ Let us write $1 + i\sqrt{3}$ in polar form,

Let $1 + i\sqrt{3} = r(\cos\theta + i\sin\theta)$

$r = \sqrt{x^2+y^2} = \sqrt{1+3} = 2$

$$\therefore \theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$

$$\therefore LHS = \left[2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^8 + \left[2\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)\right]^8$$

$$= 2^8\left[\left(\cos\frac{8\pi}{3} + \cancel{i\sin\frac{8\pi}{3}}\right) + \left(\cos\frac{8\pi}{3} - \cancel{i\sin\frac{8\pi}{3}}\right)\right]$$

$$= 2^8\left[2\cos\frac{8\pi}{3}\right] = 2^9\cos\frac{8\pi}{3}$$

$$= 2^9\cos\left(3\pi - \frac{\pi}{3}\right)$$

$$= 2^9 \cdot \left(\frac{-1}{2}\right)$$

$$= -2^8$$

① $(a+ib)^{m/n} + (a-ib)^{m/n} = 2(a^2+b^2)^{m/2n} \cos\left(\frac{m}{n}\tan^{-1}\frac{b}{a}\right)$

Sol:- ∵ $a+ib = r(\cos\theta + i\sin\theta)$

Then $a-ib = r(\cos\theta - i\sin\theta)$

where $r = \sqrt{a^2+b^2}$ & $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\therefore (a+ib)^{m/n} + (a-ib)^{m/n} = [r(\cos\theta + i\sin\theta)]^{m/n} + [r(\cos\theta - i\sin\theta)]^{m/n}$

$= r^{m/n}(\cos\theta + i\sin\theta)^{m/n} + r^{m/n}(\cos\theta - i\sin\theta)^{m/n}$

By De-Moivre's thm,

$= r^{m/n}\left[\cos\frac{m\theta}{n} + \cancel{i\sin\frac{m\theta}{n}} + \cos\frac{m\theta}{n} - \cancel{i\sin\frac{m\theta}{n}}\right]$

$= r^{m/n} \cdot 2\cos\frac{m\theta}{n}$

∵ $r = \sqrt{a^2+b^2}$ & $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$\therefore (a+ib)^{m/n} + (a-ib)^{m/n} = 2(a^2+b^2)^{m/2n} \cdot \cos\left[\frac{m}{n}\tan^{-1}\left(\frac{b}{a}\right)\right]$

(11) $\dfrac{(1+i)^8(1-i\sqrt{3})^6}{(1-i)^6(1+i\sqrt{3})^9} = \dfrac{i}{4}$.

Sol $\Rightarrow$ For $(1+i)$,

$r = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

$(1+i) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \sqrt{2}e^{i\pi/4}$

$(1-i) = \sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right) = \sqrt{2}e^{-i\pi/4}$

For $(1-i\sqrt{3})$,

we have $r = \sqrt{1+3} = \sqrt{4} = 2$

$\theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$\therefore (1-i\sqrt{3}) = 2\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right) = 2e^{-i\pi/3}$

& $(1+i\sqrt{3}) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2e^{i\pi/3}$

$$\therefore \frac{(1+i)^8(1-i\sqrt{3})^6}{(1-i)^6(1+i\sqrt{3})^9} = \frac{\{\sqrt{2}e^{i\pi/4}\}^8\{2e^{-i\pi/3}\}^6}{\{\sqrt{2}e^{-i\pi/4}\}^6\{2e^{i\pi/3}\}^9}$$

$$= \frac{(2^{8/2}\cdot e^{8i\pi/4})}{(2^{6/2}e^{-6i\pi/4})}\frac{(2^6e^{-6i\pi/3})}{(2^9e^{9i\pi/3})}$$

$$= \frac{2^4\cdot e^{2i\pi}\cdot 2^6e^{-2i\pi}}{2^3e^{-3i\pi/2}\cdot 2^9e^{3i\pi}}$$

$$= \frac{2^{10}}{2^{12}\cdot e^{3/2 i\pi}} = \frac{1}{4e^{3\pi i/2}}$$

$$= \frac{1}{4}\left(\cos\frac{3\pi}{2} - i\sin\frac{3\pi}{2}\right)$$

$$= \frac{i}{4}$$ //

Find the modulus and the principal value of the argument of,

$$\frac{(1+i\sqrt{3})^{17}}{(\sqrt{3}-i)^{15}}$$

Sol $\Rightarrow$ We have, $1+i\sqrt{3}$,

$r = \sqrt{1+3} = \sqrt{4} = 2.$

$\theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$\therefore (1+i\sqrt{3}) = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

For $\sqrt{3}-i$,

$r = \sqrt{3+1} = \sqrt{4} = 2.$

$\theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{\pi}{6}.$

$\therefore (\sqrt{3}-i) = 2\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$

$$\therefore \frac{(1+i\sqrt{3})^{17}}{(\sqrt{3}-i)^{15}} = \frac{\{2(\cos\pi/3 + i\sin\pi/3)\}^{17}}{\{2(\cos\pi/6 - i\sin\pi/6)\}^{15}}$$

$$= \frac{2^{17}}{2^{15}}\left[\cos\frac{17\pi}{3} + i\sin\frac{17\pi}{3}\right]\left[\cos\frac{15\pi}{6} + i\sin\frac{15\pi}{6}\right]$$

$$= \frac{4}{1}\left[\cos\left(\frac{17\pi}{3} + \frac{15\pi}{6}\right) + i\sin\left(\frac{17\pi}{3} + \frac{15\pi}{6}\right)\right]$$

$$= 4\left[\cos\left(\frac{49\pi}{6}\right) + i\sin\left(\frac{49\pi}{6}\right)\right]$$

$$= 4\left[\cos\left(8\pi + \frac{\pi}{6}\right) + i\sin\left(8\pi + \frac{\pi}{6}\right)\right]$$

$$= 4\left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right] \longleftarrow (1)$$

But $z = r(\cos\theta + i\sin\theta) \longrightarrow (11)$

$\therefore$ From (1) & (11)

$r = 4$ & $\theta = \frac{\pi}{6}$

$\therefore$ Modulus $= 4$ & Argument $= \frac{\pi}{6}$

If $\cos\alpha + \cos\beta = 0$, $\sin\alpha + \sin\beta = 0$ P.T

(I) $\sin 2\alpha + \sin 2\beta = 2\sin(\pi + \alpha + \beta)$

(II) $\cos 2\alpha + \cos 2\beta = 2\cos(\pi + \alpha + \beta)$

Sol ⇒ $\because (\cos\alpha + \cos\beta) + i(\sin\alpha + \sin\beta) = 0$

$\therefore (\cos\alpha + i\sin\alpha) + (\cos\beta + i\sin\beta) = 0$

Let $a = \cos\alpha + i\sin\alpha$

$b = \cos\beta + i\sin\beta$

$\therefore a + b = 0$, $(a+b)^2 = 0$

$a^2 + b^2 = -2ab$

$(\cos\alpha + i\sin\alpha)^2 + (\cos\beta + i\sin\beta)^2 = -2(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)$

$\therefore (\cos 2\alpha + i\sin 2\alpha) + (\cos 2\beta + i\sin 2\beta) = -2(\cos(\alpha+\beta) + i\sin(\alpha+\beta))$

$(\cos 2\alpha + \cos 2\beta) + i(\sin 2\alpha + \sin 2\beta) = -2(\cos(\alpha+\beta) + i\sin(\alpha+\beta))$ — (I)

$\because \cos(\pi + \theta) = -\cos\theta$

& $\sin(\pi + \theta) = -\sin\theta$.

(I) Comparing the real & imaginary terms we have,

$\sin 2\alpha + \sin 2\beta = -2\sin(\alpha + \beta)$

$\therefore \sin 2\alpha + \sin 2\beta = 2\sin(\pi + \alpha + \beta)$

Show that for real values of a & b

$$e^{2i\cot^{-1}b}\left[\frac{bi-1}{bi+1}\right]^{-a} = 1$$

Sol => Consider $\dfrac{bi-1}{bi+1} = \dfrac{i\left(b-\frac{1}{i}\right)}{i\left(b+\frac{1}{i}\right)} = \dfrac{b+i}{b-i}$

Consider $b+i = re^{i\theta}$

$b-i = re^{-i\theta}$.

$r = \sqrt{b^2+1}$ & $\theta = \tan^{-1}\left(\frac{1}{b}\right)$

$\therefore \dfrac{bi-1}{bi+1} = \dfrac{re^{i\theta}}{re^{-i\theta}} = e^{2i\theta}$

$\therefore \left(\dfrac{bi-1}{bi+1}\right)^{-a} = e^{-2ia\tan^{-1}\left(\frac{1}{b}\right)}$

$= e^{-2ia\cot^{-1}b}$

$\therefore e^{2ia\cot^{-1}b}\left(\dfrac{bi-1}{bi+1}\right)^{-a} = 1$

If α, β are the roots of eq^n $x^2-2x+4=0$
P.T ① $\alpha^n+\beta^n = 2^{n+1}\cos(n\pi/3)$
Hence deduce $\alpha^6+\beta^6 = 128$

Sol ⇒ $\because x^2-2x+4=0$

$$x = \frac{2 \pm \sqrt{4-16}}{2}$$

$$\therefore x = 1 \pm \sqrt{3}\, i$$

$\therefore x_1 = 1+i\sqrt{3}$ $\qquad \therefore r = 2,\ \theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$$\therefore x_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$\alpha = x_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$

$$\& \ \beta = x_2 = 2\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)$$

Consider, $\alpha^6+\beta^6$

$$= \left[2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^6 + \left[2\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right)\right]^6$$

$$= 2^6\left\{\cos\frac{6\pi}{3} + \cancel{i\sin\frac{6\pi}{3}} + \cos\frac{6\pi}{3} - \cancel{i\sin\frac{6\pi}{3}}\right\}$$

$$= 2^6\left(2\cos 2\pi\right)$$

$$= 2^7\cos 2\pi = 2^7(1) = 128$$

If $x_r = \cos\left(\frac{2}{3}\right)^r \pi + i\sin\left(\frac{2}{3}\right)^r \pi$ P.T

(i) $x_1 x_2 x_3 \cdots\cdots \infty = 1$

(ii) $x_0 x_1 x_2 \cdots\cdots \infty = -1$.

Sol ⇒ We now put $r = 1, 2, 3 \cdots\cdots$

$\therefore x_1 = \cos \frac{2\pi}{3} + i\sin \frac{2\pi}{3}$

$x_2 = \cos \frac{4\pi}{9} + i\sin \frac{4\pi}{9} \cdots\cdots$

$$\left[\text{For a G.P } S_\infty = \frac{a}{1-r}.\right]$$

Consider $x_1 x_2 x_3 \cdots \infty$

$$= \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\left(\cos\frac{4\pi}{9} + i\sin\frac{4\pi}{9}\right)\cdots\infty$$

$$= \cos\pi\left(\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \cdots\infty\right] + i\sin\pi\left[\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \cdots\infty\right]$$

$$= \cos\pi\left[\frac{2/3}{1-2/3}\right] + i\sin\pi\left[\frac{2/3}{1-2/3}\right]$$

$$= \cos 2\pi + i\sin 2\pi = (-1)^2 + 0 = 1.$$

Expansion of $\sin n\theta$, $\cos n\theta$ in powers of $\sin\theta$, $\cos\theta$ =>

$\cos n\theta = {}^nC_0 \cos^n\theta - {}^nC_2 \cos^{n-2}\theta \sin^2\theta + \cdots$

$\sin n\theta = {}^nC_1 \cos^{n-1}\theta \sin\theta - {}^nC_3 \cos^{n-3}\theta \sin^3\theta + \cdots$

Ex

① (a) If $\sin 6\theta = a\cos^5\theta\sin\theta + b\cos^3\theta\sin^3\theta + c\cos\theta\sin^5\theta$. Find the value of a, b, c

(b) $\cos 6\theta = a\cos^6\theta + b\cos^4\theta\sin^2\theta + c\cos^2\theta\sin^4\theta + d\sin^6\theta$. Find a, b, c, d.

Sol=> we have by De-Moivre's thm,

$(\cos\theta + i\sin\theta)^6 = (\cos 6\theta + i\sin 6\theta)$ —— (1)

we have by Binomial thm,

$(\cos\theta + i\sin\theta)^6$

$= {}^6C_0\cos^6\theta + {}^6C_1\cos^5\theta(i\sin\theta) + {}^6C_2\cos^4\theta(i\sin\theta)^2 + {}^6C_3\cos^3\theta(i\sin\theta)^3 + {}^6C_4\cos^2\theta(i\sin\theta)^4 + {}^6C_5\cos\theta(i\sin\theta)^5 + {}^6C_6(i\sin\theta)^6$.

$= \cos^6\theta + 6\cos^5\theta(i\sin\theta) - 15\cos^4\theta\sin^2\theta - 20\cos^3\theta(i\sin^3\theta) + 15\cos^2\theta(\sin^4\theta) + 6\cos\theta(i\sin^5\theta) - \sin^6\theta$

$= (\cos^6\theta - 15\cos^4\theta\sin^2\theta + 15\cos^2\theta\sin^4\theta - \sin^6\theta) + i(6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^5\theta)$ —— (2)

Equating real & imaginary part from ① & ②

$$\sin 6\theta = 6\cos^5\theta \sin\theta - 20\cos^3\theta \sin^3\theta + 6\cos\theta \sin^5\theta$$

$$\cos 6\theta = \cos^6\theta - 15\cos^4\theta \sin^2\theta + 15\cos^2\theta \sin^4\theta - \sin^6\theta.$$

Ans (I) $a = 6,\ b = -20,\ c = 6$

(II) $a = 1,\ b = -15,\ c = 15,\ d = -1$

HW (III) Prove that $\dfrac{\sin 6\theta}{\sin 2\theta} = 16\cos^4\theta - 16\cos^2\theta + 3$.

HW (IV) Show that,

$$\tan 7\theta = \frac{7\tan\theta - 35\tan^3\theta + 21\tan^5\theta - \tan^7\theta}{1 - 21\tan^2\theta + 35\tan^4\theta - 7\tan^6\theta}.$$

Expansion of $\cos^n\theta$, $\sin^n\theta$ in terms of sines or cosines of multiples of $\theta \Rightarrow$

$$x^n + \frac{1}{x^n} = 2\cos n\theta \quad \& \quad x^n - \frac{1}{x^n} = 2i\sin n\theta.$$

where $x^n = \cos n\theta + i\sin n\theta$

$1/x^n = \cos n\theta - i\sin n\theta.$

Ex

(i) Prove that $\cos^6\theta + \sin^6\theta = \frac{1}{8}(3\cos 4\theta + 5)$

Sol $\Rightarrow$ Let $x = \cos\theta + i\sin\theta$; $\frac{1}{x} = \cos\theta - i\sin\theta$.

$\therefore x + \frac{1}{x} = 2\cos\theta, \quad x - \frac{1}{x} = 2i\sin\theta.$

Now by Binomial $\Rightarrow$

$$(2\cos\theta)^6 = \left(x+\frac{1}{x}\right)^6$$

$$= x^6 + 6\left(x^5\cdot\frac{1}{x}\right) + 15\left(x^4\cdot\frac{1}{x^2}\right) + 20\left(x^3\cdot\frac{1}{x^3}\right) + 15\left(x^2\cdot\frac{1}{x^4}\right) + 6\left(x\cdot\frac{1}{x^5}\right) + \frac{1}{x^6}$$

$$= x^6 + 6x^4 + 15x^2 + 20 + 15\cdot\frac{1}{x^2} + 6\cdot\frac{1}{x^4} + \frac{1}{x^6} \quad \text{(I)}$$

$$(2i\sin\theta)^6 = \left(x-\frac{1}{x}\right)^6$$

$$= x^6 - 6x^4 + 15x^2 - 20 + 15\cdot\frac{1}{x^2} - 6\cdot\frac{1}{x^4} + \frac{1}{x^6}$$

$$\therefore (2\sin\theta)^6 = -x^6 + 6x^4 - 15x^2 + 20 - 15\cdot\frac{1}{x^2} + 6\cdot\frac{1}{x^4} - \frac{1}{x^6} \quad \text{(II)}$$

Adding 1 & 2

$$\therefore 2^6(\cos^6\theta + \sin^6\theta)$$

$$= 12x^4 + 40 + \frac{12}{x^4}$$

$$= 4\left[3\left(x^4+\frac{1}{x^4}\right) + 10\right]$$

$$\because x^4 + \frac{1}{x^4} = 2\cos 4\theta$$

$$\therefore \cos^6\theta + \sin^6\theta = \frac{\cancel{4}^{2}}{2^{\cancel{6}}}\left[6\cos 4\theta + 10\right]$$

$$= \frac{2}{2^{\cancel{4}}}\left[3\cos 4\theta + 5\right]$$

$$= \frac{1}{8}\left[3\cos 4\theta + 5\right]$$

Show that $\tan 7\theta = \dfrac{7\tan\theta - 35\tan^3\theta + 21\tan^5\theta - \tan^7\theta}{1 - 21\tan^2\theta + 35\tan^4\theta - 7\tan^6\theta}$.

Sol $\Rightarrow$ Since we have, By using binomial of De-Moivre's thm.

$$\sin n\theta = {}^nC_1 \cos^{n-1}\theta \sin\theta - {}^nC_3 \cos^{n-3}\theta \sin^3\theta + \cdots\cdots$$

$$\& \cos n\theta = {}^nC_0 \cos^n\theta - {}^nC_2 \cos^{n-2}\theta \sin^2\theta + \cdots\cdots$$

$$\therefore \sin 7\theta = {}^7C_1 \cos^6\theta \sin\theta - {}^7C_3 \cos^4\theta \sin^3\theta + {}^7C_5 \cos^2\theta \sin^5\theta - {}^7C_7 \cos^0\theta \sin^7\theta.$$

$$\therefore \sin 7\theta = 7\cos^6\theta \sin\theta - 35\cos^4\theta \sin^3\theta + 21\cos^2\theta \sin^5\theta - \sin^7\theta. \quad \text{--- (I)}$$

$$\& \cos 7\theta = {}^7C_0 \cos^7\theta - {}^7C_2 \cos^5\theta \sin^2\theta + {}^7C_4 \cos^3\theta \sin^4\theta - {}^7C_6 \cos\theta \sin^6\theta.$$

$$\therefore \cos 7\theta = \cos^7\theta - 21\cos^5\theta \sin^2\theta + 35\cos^3\theta \sin^4\theta - 7\cos\theta \sin^6\theta. \quad \text{--- (II)}$$

Dividing (I) by (II), we get,

$$\therefore \tan 7\theta = \frac{\sin 7\theta}{\cos 7\theta}$$

$$= \frac{7\cos^6\theta \sin\theta - 35\cos^4\theta \sin^3\theta + 21\cos^2\theta \sin^5\theta - \sin^7\theta}{\cos^7\theta - 21\cos^5\theta \sin^2\theta + 35\cos^3\theta \sin^4\theta - 7\cos\theta \sin^6\theta}$$

Now dividing the Numerator & Denominator of RHS by $\cos^7\theta$, we get

$$\therefore \tan 7\theta = \frac{7\tan\theta - 35\tan^3\theta + 21\tan^5\theta - \tan^7\theta}{1 - 21\tan^2\theta + 35\tan^4\theta - 7\tan^6\theta}$$

$$\left[\begin{array}{l}\cos(\pi \pm \theta) = -\cos\theta \\ \sin(\pi \mp \theta) = \pm \sin\theta\end{array}\right]$$

P.T $\cos^5\theta \sin^3\theta = \frac{-1}{2^7}\left[\sin 8\theta + 2\sin 6\theta - 2\sin 4\theta - 6\sin 2\theta\right]$

Roots of a Complex no. ⟹

$$(\cos\theta + i\sin\theta)^{1/n} = \cos\left(\frac{2K\pi+\theta}{n}\right) + i\sin\left(\frac{2K\pi+\theta}{n}\right)$$

By Putting $K = 0, 1, 2 \ldots n-1$

$\cos\theta = \cos(2K\pi + \theta)$

$\sin\theta = \sin(2K\pi + \theta)$

Ex

If ω is a complex cube-roots of unity P.T $1+\omega+\omega^2 = 0$ & $\frac{1}{1+2\omega} + \frac{1}{2+\omega} \quad \frac{1}{1+\omega} = 0$

Sol ⟹ ① Consider $x^3 = 1 \quad \therefore x = 1^{1/3}$

$\therefore x = (\cos 0 + i\sin 0)^{1/3}$

$= (\cos 2K\pi + i\sin 2K\pi)^{1/3}$

$= \cos\frac{2K\pi}{3} + i\sin\frac{2K\pi}{3}$

put $K = 0, 1, 2$

$\therefore x_0 = 1,\ x_1 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) + i\sin\left(\pi - \frac{\pi}{3}\right)$

$= -\cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \omega$

$x_2 = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} \quad = \left[-\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right] = \omega^2$

$\cos\left(\pi + \frac{\pi}{3}\right) + i\sin\left(\pi + \frac{\pi}{3}\right)$

$\therefore 1+\omega+\omega^2 = 1 - \cos\frac{\pi}{3} + \cancel{i\sin\frac{\pi}{3}} - \cos\frac{\pi}{3} - \cancel{i\sin\frac{\pi}{3}}$

$= 1 - 2\cos\frac{\pi}{3} = 1 - 2\left(\frac{1}{2}\right) = 0$

$\therefore 1+\omega+\omega^2 = 0.$

$$LHS = \frac{(2+\omega)(1+\omega) + (1+2\omega)(1+\omega) - (1+2\omega)(2+\omega)}{(1+2\omega)(2+\omega)(1+\omega)}$$

$$= \frac{(2+3\omega+\omega^2) + (1+3\omega+2\omega^2) - (2+5\omega+2\omega^2)}{(1+2\omega)(2+\omega)(1+\omega)}$$

$$= \frac{1+\omega+\omega^2}{(1+2\omega)(2+\omega)(1+\omega)} = 0 \quad [\text{From } 1]$$

H.W

If ω is a 7^{th} root of unity P.T

$S = 1 + \omega^n + \omega^{2n} + \omega^{3n} + \omega^{4n} + \omega^{5n} + \omega^{6n} =$

Find the all values of $(1+i)^{2/3}$ and find the continued product of these values.

Sol => Let $x = (1+i)^{2/3}$

$$x = (1+i)^{2/3}$$

$$= \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^{2/3}$$

$$\therefore x = (1+i)^{2/3} = \left[\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]^{2/3}$$

$$= 2^{1/3}\left[\cos\left(2K\pi+\frac{\pi}{4}\right) + i\sin\left(2K\pi+\frac{\pi}{4}\right)\right]^{2/3}$$

$$= 2^{1/3}\left[\cos\left(\frac{8K\pi+\pi}{6}\right) + i\sin\left(\frac{8K\pi+\pi}{6}\right)\right]$$

puting $K = 0, 1, 2$.

$$\therefore \text{Continued product} = 2\left[\cos\left(\frac{\pi}{6} + \frac{9\pi}{6} + \frac{17\pi}{6}\right) + i\sin\left(\frac{\pi}{6} + \frac{9\pi}{6} + \frac{17\pi}{6}\right)\right]$$

$$= 2i$$

Solve $x^7+x^4+x^3+1=0$

Sol $\Rightarrow$ $x^4(x^3+1)+1(x^3+1)=0$

$$(x^4+1)(x^3+1)=0$$

Let $x^3+1=0$

$$x^3=-1$$

$$x=(-1)^{1/3}$$

$$x=(\cos\pi+i\sin\pi)^{1/3}$$

$$x=(\cos(2k\pi+\pi)+i\sin(2k\pi+\pi))^{1/3}$$

$$x=e^{i(2k\pi+\pi)/3}$$

where $k=0,1,2$.

$$x_0=e^{i\pi/3}=\frac{1}{2}+i\frac{\sqrt{3}}{2}$$

$$x_1=e^{i\pi}=-1$$

$$x_2=e^{i5\pi/3}=e^{i(2\pi-\pi/3)}=e^{-i\pi/3}$$

$$=\frac{1}{2}-\frac{i\sqrt{3}}{2}$$

Now let $(x^4+1)=0$

$$x^4=-1$$

$$x=(-1)^{1/4}$$

$$x=(\cos\pi+i\sin\pi)^{1/4}$$

$$=(\cos(2k\pi+\pi)+i\sin(2k\pi+\pi))^{1/4}$$

$$=\left(\cos\left(\frac{2k\pi+\pi}{4}\right)+i\sin\left(\frac{2k\pi+\pi}{4}\right)\right)$$

$$x=e^{i(2k\pi+\pi)/4}$$

where $k=0,1,2,3$

$$x_3=e^{i\pi/4}=\cos(\pi/4)+i\sin(\pi/4)$$

$$x_4=e^{i3\pi/4}=\cos(3\pi/4)+i\sin(3\pi/4)$$

$$x_5=e^{i5\pi/4}=\cos(5\pi/4)+i\sin(5\pi/4)$$

$$x_6=e^{i7\pi/4}=\cos\left(\frac{7\pi}{4}\right)+i\sin\left(\frac{7\pi}{4}\right)$$

$$x^n - 1 = (x-1)(x^{n-1} + x^{n-2} \cdots + 1)$$

Solve the eq^n =>

$$x^6 - x^5 + x^4 - x^3 + x^2 - x + 1 = 0$$

Sol => Multiply by $(x+1)$

$$\therefore (x+1)(x^6 - x^5 + x^4 - x^3 + x^2 - x + 1) = 0$$

$$\therefore x^7 - \cancel{x^6} + \cancel{x^5} - \cancel{x^4} + \cancel{x^3} - \cancel{x^2} + \cancel{x} + \cancel{x^6} - \cancel{x^5}$$

$$+ \cancel{x^4} - \cancel{x^3} + \cancel{x^2} - \cancel{x} + 1 = 0.$$

$$x^7 + 1 = 0$$

$$x^7 = -1$$

$$x^7 = \cos\pi + i\sin\pi$$

$$x = [\cos(2k\pi + \pi) + i\sin(2k\pi + \pi)]^{1/7}$$

$$= \left[\cos\left(\frac{2k+1}{7}\right)\pi + i\sin\left(\frac{2k+1}{7}\right)\pi\right]$$

$$k = 0, 1, 2 \cdots 6$$

If $\alpha, \alpha^2, \alpha^3, \alpha^4$ are the roots of $x^5 - 1 = 0$ Find them & Show that $(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4) = 5$

Sol => we have $x^5 = 1$

$$= \cos 0 + i\sin 0$$

$$x = (\cos 2k\pi + i\sin 2k\pi)^{1/5}$$

$$x = \cos\frac{2k\pi}{5} + i\sin\frac{2k\pi}{5}$$

put $k = 0, 1, 2, 3, 4$

$x_0 = \cos 0 + i\sin 0 = 1$, $x_1 = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5} = \alpha$ (say)

$x_2 = \cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5} = \alpha^2$, $x_3 = \cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5} = \alpha^3$

$x_4 = \cos\frac{8\pi}{5} + i\sin\frac{8\pi}{5} = \alpha^4$

$\therefore$ The roots are $1, \alpha, \alpha^2, \alpha^3, \alpha^4$

$$\therefore x^5 - 1 = (x-1)(x-\alpha)(x-\alpha^2)(x-\alpha^3)(x-\alpha^4)$$

$$\frac{x^5 - 1}{x-1} = (x-\alpha)(x-\alpha^2)(x-\alpha^3)(x-\alpha^4)$$

$$\frac{(x-1)(x^4 + x^3 + x^2 + x + 1)}{(x-1)} = (x-\alpha)(x-\alpha^2)(x-\alpha^3)(x-\alpha^4)$$

$$\left(② \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{b-c} = \frac{d}{d-c}. \quad ① \frac{a}{c} = \frac{b}{d} \Rightarrow \frac{a+c}{a-a} = \frac{b+d}{b-b} \right).$$

$$x^4 + x^3 + x^2 + x + 1 = (x-\alpha)(x-\alpha^2)(x-\alpha^3)(x-\alpha^4)$$

put $x = 1$

$$5 = (1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4)$$

Show that the roots of the eq^n $(x+1)^6 + (x-1)^6 = 0$ are given by $-i\cot\frac{(2K+1)\pi}{12}$, $K = 0, 1 \ldots 5$.

Sol:> we have $(x+1)^6 = -(x-1)^6$

$$\frac{(x+1)^6}{(x-1)^6} = -1$$

$$\left(\frac{x+1}{x-1}\right)^6 = \cos\pi + i\sin\pi$$

$$\left(\frac{x+1}{x-1}\right) = \cos\frac{(2K+1)\pi}{6} + i\sin\frac{(2K+1)\pi}{6}$$

where $K = 0, 1 \ldots 5$

Let $(2K+1)\frac{\pi}{6} = \theta$.

$$\therefore \frac{x+1}{x-1} = \frac{\cos\theta + i\sin\theta}{1}.$$

By componendo & dividendo,

$$\frac{(x+1) + (x-1)}{(x-1) - (x+1)} = \frac{\cos\theta + i\sin\theta + 1}{1 - \cos\theta - i\sin\theta}.$$

$$\frac{2x}{-2} = \frac{\cos\theta + i\sin\theta + 1}{1 - \cos\theta - i\sin\theta}.$$

$$\frac{x}{-1} = \frac{1 + \cos\theta + i\sin\theta}{1 - \cos\theta - i\sin\theta}.$$

$$\frac{x}{-1} = i\cot\frac{\theta}{2}$$

$$x = -i\cot\frac{\theta}{2}$$

$$\frac{x}{-1} = \frac{2\cos^2\theta/2 + 2i\sin\theta/2\cos\theta/2}{2\sin^2\theta/2 - 2i\sin\theta/2\cos\theta/2}$$

$$\frac{x}{-1} = \frac{2\cos\theta/2(\cos\theta/2 + i\sin\theta/2)}{2\sin\theta/2(\sin\theta/2 - i\cos\theta/2)}$$

$$\frac{x}{-1} = \cot\theta/2 \frac{(\cos\theta/2 + i\sin\theta/2)}{\cos(\frac{\pi}{2} - \frac{\theta}{2}) - i\sin(\frac{\pi}{2} - \frac{\theta}{2})}$$

$$= \cot\frac{\theta}{2}\left[\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right]\left[\cos\left(\frac{\pi}{2} - \frac{\theta}{2}\right) + i\sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)\right]$$

$$\therefore \frac{x}{-1} = \cot\frac{\theta}{2}\left[\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right]$$

$$= \cot\frac{\pi}{2}[0 + i]$$

Use of Exponential Form of a Complex Number ⇒

$z = x+iy$ (Cartesian)

$z = r(\cos\theta + i\sin\theta)$ (Polar)

$z = re^{i\theta}$ (Exponential)

Note ⇒ (i) $1 = \cos 2n\pi + i\sin 2n\pi = e^{i2n\pi}$

or $1 = \cos 0 + i\sin 0$

(ii) $i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = e^{i\pi/2}$

(iii) $\sqrt{i} = \left[\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right]^{1/2} = \left[e^{i\pi/2}\right]^{1/2} = e^{i\pi/4}$.

Ex

(1) If $i^{i^{\cdots \text{ad. inf}}} = A+iB$ P.T $A^2+B^2 = e^{-\pi B}$ & $\tan\left(\frac{\pi}{2}A\right) = \frac{B}{A}$

Sol ⇒ $i^{A+iB} = A+iB$

$\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{A+iB} = A+iB$

$\left(e^{i\pi/2}\right)^{A+iB} = A+iB$

$e^{-\pi(B/2)} \cdot e^{i\pi(A/2)} = A+iB$

$e^{-\pi(B/2)}\left[\cos\frac{A\pi}{2} + i\sin\frac{A\pi}{2}\right] = A+iB$

Comparing Real & imaginary term

$\therefore e^{-\pi B/2}\cos\frac{\pi A}{2} = A$ & $e^{-\pi B/2}\sin\frac{A\pi}{2} = B$

$\therefore A^2+B^2 = e^{-\pi B}$ & $\tan\left(\frac{\pi}{2}A\right) = \frac{B}{A}$

Separate into real & imaginary.

1. $(\sqrt{-i})^{\sqrt{-i}}$

Sol ⇒ we have $\sqrt{-i} = \sqrt{\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}}$

$$= \left(\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}\right)^{1/2}$$

$$= \left(e^{-i\pi/2}\right)^{1/2} = e^{-i\pi/4} = \frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$$

$$\therefore (\sqrt{-i})^{\sqrt{-i}} = \left(e^{-i\pi/4}\right)^{\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)}$$

$$= e^{-i\pi/4\sqrt{2}} \cdot e^{-\pi/4\sqrt{2}}.$$

$$= e^{-\pi/4\sqrt{2}}\left[\cos\frac{\pi}{4\sqrt{2}} - i\sin\frac{\pi}{4\sqrt{2}}\right]$$

Ex P.T $\sqrt{1-\operatorname{cosec}(\theta/2)} = (1-e^{i\theta})^{-1/2} - (1-e^{-i\theta})^{-1/2}$

Sol ⇒ We have to show that,

$$\sqrt{1-\operatorname{cosec}\left(\frac{\theta}{2}\right)} = \frac{1}{\sqrt{1-e^{i\theta}}} - \frac{1}{\sqrt{1-e^{-i\theta}}}$$

Squaring b.s.

$$\therefore 1-\operatorname{cosec}\frac{\theta}{2} = \frac{1}{1-e^{i\theta}} + \frac{1}{1-e^{-i\theta}} - \frac{2}{\sqrt{(1-e^{i\theta})(1-e^{-i\theta})}}$$

$$RHS = \frac{1-e^{-i\theta} + 1 - e^{i\theta}}{1-e^{-i\theta}-e^{+i\theta}+1} - \frac{2}{\sqrt{(1-e^{-i\theta}-e^{i\theta}+1)}}$$

$$= \frac{\cancel{2-e^{-i\theta}-e^{i\theta}}}{\cancel{2-e^{-i\theta}-e^{i\theta}}} - \frac{2}{\sqrt{2-(e^{i\theta}+e^{-i\theta})}}$$

$$= 1 - \frac{2}{\sqrt{2-2\cos\theta}}$$

$$= 1 - \frac{2}{\sqrt{2(1-\cos\theta)}}$$

$$= 1 - \frac{2}{\sqrt{4\sin^2\theta/2}}$$

$$= 1 - \frac{\cancel{2}}{\cancel{2}\sin\theta/2} = 1 - \operatorname{cosec}\frac{\theta}{2} \quad LHS$$

(11) $\sqrt{1-\sec\theta/2} = (1+e^{i\theta})^{-1/2} - (1+e^{-i\theta})^{-1/2}.$

Seperate into real & imaginary.

① $\sin^{-1}(e^{i\theta})$

Sol => Let $\sin^{-1}(e^{i\theta}) = x+iy$

$\cos\theta + i\sin\theta = \sin(x+iy)$

$\cos\theta + i\sin\theta = \sin x \cosh y + i\cos x \sinh y$

Equating real & img.

$\therefore \cos\theta = \sin x \cosh y$ —— (I)

$\sin\theta = \cos x \sinh y$ —— (II)

$\because \sin^2\theta + \cos^2\theta = 1$

$\therefore \cos^2 x \sinh^2 y + \sin^2 x \cosh^2 y = 1$

$\cos^2 x \sinh^2 y + \sin^2 x (1+\sinh^2 y) = 1$

$\cos^2 x \sinh^2 y + \sin^2 x + \sin^2 x \sinh^2 y = 1$

$\sinh^2 y (\sin^2 x + \cos^2 x) + \sin^2 x = 1$

$\sinh^2 y + \sin^2 x = 1$

$\sinh^2 y = 1 - \sin^2 x$

$\sinh^2 y = \cos^2 x$

$\sinh y = \cos x$ —— (III)

put (III) in (II)

$\therefore \sin\theta = \cos x \cdot \cos x$

$= \cos^2 x$

$\therefore \cos x = \sqrt{\sin\theta}$

$x = \cos^{-1}(\sqrt{\sin\theta})$

Again put (III) in (II)

$\therefore \sin\theta = \sinh y \cdot \sinh y$

$= \sinh^2 y$

$\therefore \sinh y = \sqrt{\sin\theta}$

$y = \sinh^{-1}\sqrt{\sin\theta}$ $\boxed{\because \text{As } \sinh^{-1} x = \log(x+\sqrt{x^2+1})}$

$= \log\left[\sqrt{\sin\theta} + \sqrt{\sin\theta + 1}\right]$