Linear Equation

Non-Homogeneous Linear equation ⇒

The most general form of a set of Linear eq^n in n unknowns is,

$$AX = B$$

$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$

Then it is called Non-Homogeneous eq^n.

& If $AX = 0$ i.e all $b_1, b_2 \cdots b_m$ are zero. Then it is called Homogeneous eq^n.

Augmented Matrix ⇒

The matrix $[A, B]$ i.e the matrix formed by the coeff. & the constant is called Augmented Matrix

i.e $[A,B] = \left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \vdots & & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right]$

Note:- If on solving If the system has a sol^n set i.e If we are able to solve eq^n & got $S_1, S_2 \cdots S_n$ then the set is called consistent o.w it is inconsistent.

Sol^n of Non-Homogeneous System ⇒

Consider a non-homo. L.E, by reducing the matrix of the co-efficients to echelon form

① write the given eq^n in matrix form

$AX = B$

(II) Apply elementary row transform of A as well as on B till we got echolon form.
(III) Then rewrite the eq^n as a set of L.E.
(IV) W.K.T the rank of matrix in echolon form is equal to the no. of rows containing non-zero elements.

There are 2 cases arrises

Case I ⇒ Rank A < Rank [A, B] (or $\rho(A) < \rho(A:B)$)
In this case the eq^n are inconsistent i.e They have no sol^n.

Case II ⇒ Rank A = Rank [A, B]
In this case the eq^n are consistent i.e They possess a sol^n. Further,

(a) If $r = n$, i.e rank of A = no. of unknown
Then the system has unique sol^n.

(* Note :- System has unique sol^n if the coefficient of matrix is non-singular.)

(b) If $r < n$, i.e rank of A < no. of unknowns then the system has infinite sol^n.
In this case, no. of parameter = n - r unknowns.

Note ⇒ when the system of eq^n has one or more solut^n, The system called consistent. O.W it is called inconsistent.

Ex = II

Establish of the following eq^n by consi doring the rank of suitable matrices & solve them if possible

$x+y+z=6$, $x-y+2z=5$, $3x+y+z=8$
$2x-2y+3z=7$.

Sol ⇒ We have

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \\ 2 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 8 \\ 7 \end{bmatrix}$$

By $R_2 - R_1$, $R_3 - 3R_1$, $R_4 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & -2 & -2 \\ 0 & -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ -10 \\ -5 \end{bmatrix}$$

By $R_3 - R_2$, $R_4 - 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ -9 \\ 15 \end{bmatrix}$$

By $R_4 + \frac{5}{3} R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ -9 \\ 0 \end{bmatrix}$$

$\therefore$ The rank of A is = 3

& Rank of $[A, B] = 3$

$\therefore$ Rank A = Rank $[A, B]$

$\therefore$ The eq^n are consistant

Further rank of A = no. of unknown

$\therefore$ The system has unique sol^n.

Now the eq^n can be written as,

$x + y + z = 6$ — I

$-2y + z = -1$ — II

$-3z = -9$. — III

$\Rightarrow \boxed{z = 3}$

$\therefore -2y = -1 - z$

$-2y = -1 - 3$

$-2y = -4$

$\boxed{y = 2}$

$\therefore x = 6 - y - z$

$= 6 - 2 - 3$

$x = 6 - 5$

$\boxed{x = 1}$

Ex ⇒ Test the consistency of the following equations & solve them if they are consistent :

$2x-y+z=9,\ 3x-y+z=6,\ 4x-y+2z=7$
$-x+y-z=4$.

For what value of λ the eq^n $x+y+z=1$, $x+2y+4z=\lambda$, $x+4y+10z=\lambda^2$ have a sol^n & solve them completely in each case.

Sol ⇒ we have,

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

By R_2-R_1, R_3-R_1

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ \lambda-1 \\ \lambda^2-1 \end{bmatrix}$$

By R_3-3R_2

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ \lambda-1 \\ \lambda^2-3\lambda+2 \end{bmatrix} \quad (1)$$

The eq^n consistent if rank A = rank [A, B]

This requires, $\lambda^2-3\lambda+2=0$

$\therefore (\lambda-2)(\lambda-1)=0$

$\lambda=2, 1$

The eq^n are consistent

(a) If $\lambda=2$ in (1)

$$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore$ Rank A $(=2) <$ no. of unknown $(=3)$

$\therefore$ The no. of parameter $= n-r$

$= 3-2 = 1$

$\therefore$ The eq^n have infinite sol^n

$\therefore x+y+z=1$

$y+3z=1$

put $z=t$

$\therefore y=1-3t$

$\therefore x=2t.$

which is general sol^n.

(b) If $\lambda=1$, the eq^n (1)

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ The eq^n have infinite sol^n as $r<n$

$\therefore$ parameter $= n-r$

$=1$

$x+y+z=1$, $y+3z=0$

put $z=t$

$\therefore y=-3t$

$\therefore x=1+2t.$

which is general solution.

Homogeneous Linear Equat^n $\Rightarrow$

Homogeneous eq^n can be written as

$AX=0.$

(a) Sol^n of $AX=0$ $\Rightarrow$

The sol^n $X=0$ called trivial sol^n or zero sol^n.

A sol^n $X\neq 0$ then non-trivial sol^n or non-zero sol^n.

(b) Important result $\Rightarrow$

Case I: If $r=n$ Then $x_1=x_2=\cdots x_n=0$

i.e. trivial sol^n is the possible sol^n.

Case II: If $r<n$

Then non-trivial sol^n $\therefore$ The no. of indep. parameter $=n-r.$

Homogeneous Linear Eq^n ⇒
$AX=0$

Working Rule
① Write given eq^n in matrix form $AX=0$

② Apply elementary row transformat^n only & reduce the matrix A to echelon form

③ Rewrite the matrix eq^n in set of L.E

④ If $r=n$ Then only trivial sol^n.
if $r<n$ Then non-trivial sol^n
∴ no. of independent sol^n or parameter $=n-r$

Ex ⇒

① Solve by matrix method,
$x+y+2z=0,\ x+2y+3z=0,\ x+3y+4z=0,\ 3x+4y+7z=0$

Sol ⇒ we have

$$\begin{bmatrix}1&1&2\\1&2&3\\1&3&4\\3&4&7\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

By R_2-R_1, R_3-R_1, R_4-3R_1

$$\begin{bmatrix}1&1&2\\0&1&1\\0&2&2\\0&1&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

By R_3-2R_2, R_4-R_2

$$\begin{bmatrix}1&1&2\\0&1&1\\0&0&0\\0&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

rank of $A=2$

no. of unknown=3

$2<3$

It has non-trivial sol

Ex.

(1) Solve $x_1 - 2x_2 + 3x_3 = 0$, $2x_1 + 5x_2 + 6x_3 = 0$

Sol ⇒ we have,
$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 5 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

By $R_2 - 2R_1$
$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 9 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore x_1 + 3x_3 - 2x_2 = 0$ —— (1)

$9x_2 = 0$

$\therefore x_2 = 0$

put $x_3 = t$

$\therefore x_1 = 3x_3 - 2x_2$

$x_1 = -3t$

$\therefore x_1 = -3t,\ x_2 = 0,\ x_3 = t$

The rank (2) < no. of unknown (= 3)

$\therefore$ The system is non-trivial.

$\therefore$ parameter solⁿ $= n - r$

$= 3 - 2 = 1$.

(you can verify the ans. by putting the value in given equation (1))

Ex ⇒ For what value of λ, the following system of equations possesses a non-trivial solⁿ? Obtain the solⁿ for real values of λ

$3x_1 + x_2 - \lambda x_3 = 0$, $4x_1 - 2x_2 - 3x_3 = 0$

$2\lambda x_1 + 4x_2 + \lambda x_3 = 0$.

Sol ⇒ we have,
$$\begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Given the system will have non-trivial solⁿ if the $r(A) <$ no. of unknown, 3

Then rank of $A < 3$ if $|A| = 0$.

Now
$$\begin{vmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{vmatrix} = 0.$$

$\therefore 3[-2\lambda+12] - 1[4\lambda+6\lambda] - \lambda[16+4\lambda] = 0$

$-4\lambda^2 - 32\lambda + 36 = 0$

$\lambda^2 + 8\lambda - 9 = 0$

$(\lambda+9)(\lambda-1) = 0$

$\lambda = 1, -9.$

(1) when $\lambda = 1$, we have

$$\begin{bmatrix} 3 & 1 & -1 \\ 4 & -2 & -3 \\ 2 & 4 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By $R_1 - R_3$.
$$\begin{bmatrix} 1 & -3 & -2 \\ 4 & -2 & -3 \\ 2 & 4 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By $R_2 - 4R_1$, $R_3 - 2R_1$
$$\begin{bmatrix} 1 & -3 & -2 \\ 0 & 10 & 5 \\ 0 & 10 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By $R_3 - R_2$, $R_2/5$
$$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore x_1 - 3x_2 + 2x_3 = 0$

$2x_2 + x_3 = 0.$

Put $x_2 = t$

$\therefore x_3 = -2t$

& $x_1 = 3t - \cancel{4t}\, t$

$x_1 = -t$

$\therefore$ when $\lambda = 1$, the soln is $x_1 = t$, $x_2 = t$

$x_3 = -2t.$

(ii) when $\lambda = -9$.

$$\therefore \begin{bmatrix} 3 & 1 & 9 \\ 4 & -2 & -3 \\ -18 & 4 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

By R_{12}
$$\begin{bmatrix} 4 & -2 & -3 \\ 3 & 1 & 9 \\ -18 & 4 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 - R_2$
$$\begin{bmatrix} 1 & -3 & -12 \\ 3 & 1 & 9 \\ -18 & 4 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 - 3R_1$
$R_3 - 18R_1$
$$\begin{bmatrix} 1 & -3 & -12 \\ 0 & 10 & 45 \\ 0 & -50 & -225 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2/5$ & $-R_3/25$
$$\begin{bmatrix} 1 & -3 & -12 \\ 0 & 2 & 9 \\ 0 & 2 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 - R_2$
$$\begin{bmatrix} 1 & -3 & -12 \\ 0 & 2 & 9 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 - 3x_2 - 12x_3 = 0$

$2x_2 + 9x_3 = 0$.

Put $x_3 = 2t \quad \therefore x_2 = -9t$

$\therefore x_1 = 3x_2 + 12x_3$

$= -27t + 24t$

$= -3t$.

when $\lambda = -9$ The sol^n is $x_1 = -3t$, $x_2 = -9t$

$x_3 = 2t$.

Investigate
\# For what values of λ and μ the equations $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ have (i) no solⁿ (ii) a unique solⁿ (iii) infinite no. of solⁿ.

Sol ⇒ we have

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

By R_2-R_1, R_3-R_1

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 6 \\ 4 \\ \mu-10 \end{bmatrix}$$

(a) TST the system has no solⁿ
i.e. TST rank of $A <$ rank of $[A, B]$

If $\lambda=3$, $\mu\neq 10$

Then rank of $A=2$
& rank of $[A, B]=3$
∴ rank of $A <$ rank of $[A, B]$
∴ It is inconsistent
∴ It has no solⁿ.

(b) ∵ The system has unique solⁿ if the coefficient matrix is non-singular (or rank A = no. of unknowns n)

This requires $\lambda-3\neq 0$ ∴ $\lambda\neq 3$
∴ If $\lambda\neq 3$ (μ may have any value)
Then the system has unique solⁿ.

(c) If $\lambda=3$ & $\mu=10$,
∴ rank of A = rank $[A, B]$
as $r=3$ & rank $[A, B]=3$
∴ The system has unique solⁿ
But rank of A is $2 <$ no. of unknown 3
Hence the eqⁿ will be infinite solⁿ.

non-trivial

∴ The system has ~~non-trivial~~ solⁿ.

∴ No. of parameter $= n - r$

$= 3 - 2$

$= 1$

∴ No. of parameters $= 3 - 2 = 1$

From (II) & (III)

$x + y + 2z = 0$

& $y + z = 0$

put $z = -t$

∴ $y = t$

∴ $x = -y - 2z$

$= -t + 2t$

$y = t$

∴ $x = t,\ y = t,\ z = -t.$

Homogeneous Linear equ^n :-

I] To solve the equation

II] To find the condn.

Non-Homo :-

(1) $x_1 + x_2 - 2x_3 + x_4 + 3x_5 = 1$

$2x_1 - x_2 + 2x_3 + 2x_4 + 6x_5 = 2$

$3x_1 + 2x_2 - 4x_3 - 3x_4 - 9x_5 = 3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -2 & 1 & 3 \\ 2 & -1 & 2 & 2 & 6 \\ 3 & 2 & -4 & -3 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$R_2 - 2R_1$

$R_3 - 3R_1$

The Rank of the coefficient matrix is equal to the rank of augmented matrix $= 3$. Hence the equ^n are consistent. But the rank of A $(=3)$ is less than no. of unknown $(=5)$

$\therefore \; 5 - 3 = 2$

Hence equ^n have doubly infinite soln.

$x_1 + x_2 - 2x_3 + x_4 + 3x_5 = 1$

$-x_2 + 2x_3 - 6x_4 - 18x_5 = 0$

$-3x_2 + 6x_3 = 0$

$x_2 = 2x_3$

$x_3 = t_1$

$x_2 = 2t_1$

$-2t_1 + 2t_1 - 6x_4 - 18x_5 = 0$

$6x_4 + 18x_5 = 0$

$x_4 = -3x_5$

$x_3 = t_2$

$x_4 = -3t_2$

$x_1 + 2t_1 - 2t_1 - 3t_2 + 3t_2 = 1$

$\boxed{x_1 = 1}$

(2) Establish the consistency of following equ^n by considering the rank of suitable matrices & solve them if possible.

$x + y + z = 6$

$x - y + 2z = 5$

$3x + y + z = 8$

$2x - 2y + 3z = 7$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \\ 2 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 8 \\ 7 \end{bmatrix}$$

Rank of A is equal to rank $[A\ B] = 3$.

equ^n are consistent

$r = 3, \quad n = 3$

$r = n$

$\rightarrow$ It has unique sol^n.

$-3z = -9$

$\boxed{z = 3}$

$-2y + z = -1$

$\boxed{y = 2}$

$x + y + z = 6$

$\boxed{x = 1}$

(3) Investigate for what values of λ & μ the equⁿ $x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$.
(1) no solⁿ.
(2) a unique solⁿ ; (3) Infinite no of sol

* Homo Equⁿ

$AX=0$

If Rank of $A=r$, No of unknowns $=n$.

CASE I : If $r=n$
$\Rightarrow x_1=x_2=\dots=x_n=0$.
Only possible solⁿ is zero solⁿ trivial solⁿ

CASE II : If $r<n$
$\Rightarrow$ System has non trivial solⁿ, and no. of parameters $n-r$.

(1) Solve $x_1-2x_2+3x_3=0$
$2x_1+5x_2+6x_3=0$

$$\Rightarrow \begin{bmatrix} 1 & -2 & 3 \\ 2 & 5 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

R_2-2R_1

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 9 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$9x_2=0$
$x_2=0$
$x_1-2x_2+3x_3=0$
Rank of $A=2$
No of unknowns $=3$
$r<n$.

It has non trivial soln.
No of indepedent son
$3-2=1$

$x_3 = t$

$x_1 = -3t$.

$x_2 = 0$.

② For what value of λ following system of eqn possesses non-trivial soln? Obtain the soln for real values of λ.

$3x_1 + x_2 - \lambda x_3 = 0$
$4x_1 - 2x_2 - 3x_3 = 0$
$2\lambda x_1 + 4x_2 + \lambda x_3 = 0$

$$\Rightarrow \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The system will have nontrivial soln if the rank of A is less then the no of unknown 3

$|A| = 0$

$\lambda = 1, -9$

① when $\lambda = 1 \Rightarrow x_2 = t$
find x_1, x_2, x_3

② when $\lambda = -9$
$x_1, x_2, x_3. \Rightarrow x_3 = 2t$.