CHAPTER 5

Unit V

Superconductors and Supercapacitors

Syllabus :

(Prerequisites : Electric current, flow of electric charges in a metallic conductor, drift velocity, mobility and their relation with electric current, Ohm's law, electrical resistance, V-I characteristics (linear and non-linear), electrical resistivity and conductivity, temperature dependence of resistance)

Superconductors : Critical temperature, critical magnetic field, Meissner effect, Type I and Type II and high T_c superconductors

Supercapacitors : Principle, construction, materials and applications, comparison with capacitor and batteries, energy density, power density

Learning Objectives

After reading this chapter, learner should be able to

- Understand superconductivity
- Explain Meissner effect
- Classify superconductors
- Understand principle, construction and types of supercapacitors
- Compare supercapacitor with batteries

Superconductors

5.1 Introduction

MU - May 14, May 15

Q.	What is superconductivity?	**(May 14, 2 Marks)**
Q.	Define the term superconductivity.	**(May 15, 2 Marks)**

- Superconductivity is one of the most exciting phenomena in physics, both because of the very nature of the phenomenon and also of its prospective applications of immense potential. Kamerlingh Onnes had discovered the phenomenon saying that "electrical resistance of certain metals (like mercury) is completely wiped out without any trace at very low temperatures" (very near to absolute zero). He received the Nobel prize for this in the year 1913. The state of zero electrical resistance was termed as **superconductivity**.
- In the year 1933, Meissner and Ochsenfeld revealed that magnetic flux is expelled when a material attains superconductivity. A reasonably successful theory called BCS (Bardeen, Cooper and Schrieffer) was provided to explain superconductivity. The trio was awarded Nobel prize in 1972. There are many discoveries which have the potential to change the picture on world level right from power sector, transport, up to medical facilities.

5.2 Temperature Dependence of Resistance of Metal

- All metals are good conductors of electricity. It is because they have loosely bound electrons in their outermost shells. These electrons called **free electrons,** can readily move under the influence of electric field.
- During the flow of current in a metal, the electrons leave the atom to which they were originally bound and move in a general direction which follows the field's direction.
- Because of the loss of electrons, the atoms become positive ion cores.
- Due to thermal excitation, the ions will always be oscillating about fixed positions in the framework of the metal. These vibrations are known as **lattice vibrations.**
- The resistance of the metal to the flow of current is caused by the scattering of the conduction electrons by lattice vibrations.
- When the temperature increases, the amplitude of lattice vibrations also increases thereby increasing the resistance. Refer Fig. 5.2.1.

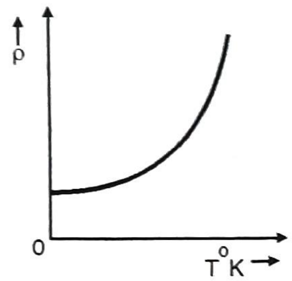


Fig. 5.2.1 : Resistivity →Temperature in non-superconducting state

- The resistance decreases with temperature and reaches a minimum value at T = 0°K. The residual resistance at T = 0 is due to impurities in metal.

  Mathematically it is expressed as,

$$\rho = \rho_0 + \rho(T) \quad ...(5.2.1)$$

Where, ρ_0 = Residual resistivity

$\rho(T)$ = Temperature dependent part of resistivity

5.3 Superconductivity and its Properties

MU - May 12, May 13, May 16, Dec. 16, May 17

Q. Define superconductors. **(May 12, May 13, May 16, 1 Mark)**

Q. Define superconductivity and critical temperature. Plot the variation of resistance with temperature in case of superconducting state of the material. **(Dec. 16, 3 Marks)**

Q. Define the term critical temperature. **(May 17, 2 Marks)**

- Kamerlingh Onnes observed that for few of the metals/materials electrical resistance drops to zero below a certain temperature. These materials are called **superconductors**.
- The drop of electrical resistance is shown in Fig. 5.3.1.

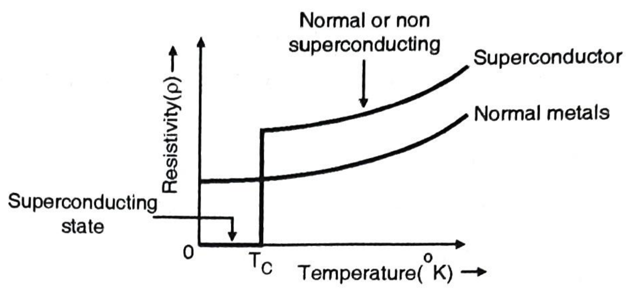


Fig. 5.3.1 : Drop of resistance

- The resistance of a superconductor in the non-superconducting state decreases with decrease in temperature. This is similar to a normal metal. At a particular temperature T_C, the resistance abruptly drops to zero and then onwards metal passes into the superconducting state.
- The temperature at which a normal material turns into superconducting state is called **critical temperature.**
- The transition from normal to superconducting state is sharp for pure and structurally perfect specimen otherwise it is slightly broad.
- The critical temperature is different for different superconductors. For mercury it is 4.2°K.

Properties of superconductors

- Almost all superconductors' physical properties vary from material to material, such as heat capacity and the critical temperature at which superconductivity is found to be destroyed.
- On the other hand, there is another group of properties that are independent on its material. For example, all superconductors have *exactly* zero resistance to low applied current when there is no application of magnetic field.
- The existence of these universal properties suggests that superconductivity is a *thermodynamic phase*, and thus they possess certain distinguishing properties.
- The next section describes the details of such properties of superconductors which are independent of microscopic details.

5.3.1 Magnetic Field Effect (Critical Field H_c)

MU - May 12, May 13, May 16

Q. Define critical temperature and critical magnetic field. **(May 12, May 16, 2 Marks)**

Q. Explain critical magnetic field and critical temperature of a superconductor. **(May 13, 2 Marks)**

- We know that the superconducting state of a superconductor is mainly dependent upon :
  (i) Temperature, and
  (ii) Strength of applied magnetic field
- Superconductivity vanishes if the temperature of it is increased above the critical temeperature T_c or very strong sufficient magnetic field **H** is applied to it.

- Consider that for a superconductor, if sufficiently strong applied magnetic field **H** is increased at any temeprature **T** below critical temperature $\mathbf{T_c}$, *then supercoductor is found to be converted into normal resistive conductor.* That is, transition of superconductor to normal conductor takes place under the applied sufficient magnetic field. This is as shown in Fig. 5.3.2.

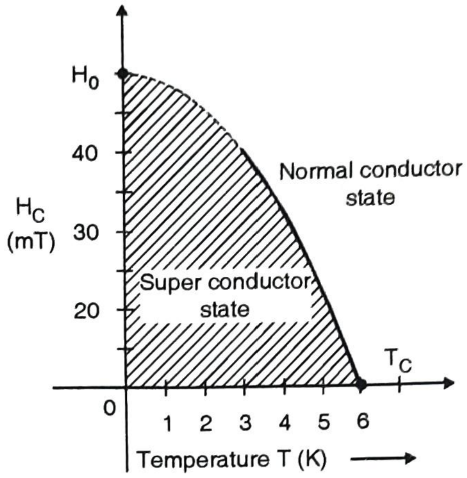


Fig. 5.3.2 : Effect of magnetic field on superconductor

- This minimum value of the magnetic field required to destroy the superconducting phase completely is known as **critical magnetic field, and is denoted by H_c.**
- Further it is clear that critical magnetic field H_c is the function of temperature T, which is expressed as,

$$H_c = H_0\left[1-\frac{T^2}{T_c^2}\right] \qquad ...(5.3.1)$$

Where,

H_c = Critical field at any temperature T

H_0 = Critical field at 0^0 K

T_c = Critical temperature of the superconductor

- From the Fig. 5.3.2 is found that within the curve the material is in superconducting state and outside the curve it is in normal conducting state.
- In practice typical value of H_0 (critical field at absolute zero or 0 K) i.e. 5000 A/m is considered.
- Table 5.3.1 shows some superconductor materials and their critical temperatures.

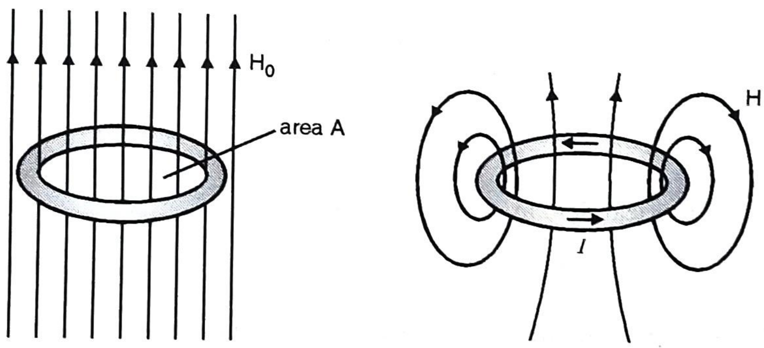
Table 5.3.1: Critical temperatures and critical magnetic fields

Superconductor	T_c in K	H_c (0) in Tesla
Al	1.18	0.0105
Hg	4.15	0.0411

Superconductor	T_c in K	H_c (0) in Tesla
Ln	3.41	0.0281
Pb	7.19	0.0803
Sn	3.72	0.0305
Zn	0.85	0.0054

5.3.2 Persistent Current

- According to theory of electromagnetic induction, it is clear that current can be induced in a superconductor.
- If specimen is in the normal state then current reduces quickly because of the resistance of the superconductor specium.
- Since the superconductor has got zero resistance, the current flows indefinitely (for an unlimited time) without any change; it persists without any applied voltage.
- "A superconductor can carry current for a long time (several years) without any applied field (volatge). The current is called persistent current".



(a) (b)

Fig. 5.3.3 : Phenomenon of persistent current in the superconductor

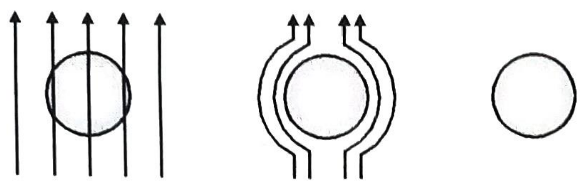
- This is one of the most important properties of the superconductor. It was first observed in 1983 by *Makus Buttiker and Yoseph Imry*. Coils of superconductor with persistent current produce magnetic field.
- Therefore they can be used as magnets which do not require power supply to maintain its magnetic field. Such magnets are known as **superconductor magnets.** They have ten times greater magnetic power than the best normal magnet.

5.4 Meissner Effect

MU - May 12, Dec. 14, May 15, May 17, Dec.17, May 18, Dec. 18, May 19

Q.	What is Meissner effect ?	**(May 12, 4 Marks)**
Q.	Why is superconductor termed as 'perfect diamagnet' ?	**(Dec. 14, 3 Marks)**
Q.	Show that in the superconducting state the material is perfectly diamagnetic.	**(May 15, May 17, 3 Marks)**
Q.	"Superconductor is a perfect diamagnet". Explain.	**(Dec. 17, May 18, 3 Marks)**
Q.	Explain Meissner effect with the help of diagram.	**(Dec. 18, May 19, 3 Marks)**

- A superconducting material kept in a **magnetic field expels** the magnetic flux out of its body when **cooled below the critical temperature** and exhibits perfect diamagnetism. This effect is called **'Meissner effect'.**
- Refer Fig. 5.4.1(a), where a specimen is subjected to a magnetic field. The specimen is in normal state. We find that **magnetic field penetrates the specimen.**
- Refer Fig. 5.4.1(b). now the specimen is cooled below its T_C, **the superconductor expels field lines from its body**. This is **Meissner effect.**
- Refer Fig. 5.4.1(c), when the field is switched off magnetic field will not be trapped by the superconductor cooled below T_C.



(a) Magnetic flux lines (b) Expulsion (c) Magnetic field not trapped

Fig. 5.4.1 : Meissner effect

- As specimen expels the magnetic flux, it is exhibition of perfect diamagnetism, **susceptibility** is found out to be – 1. Let's see it mathematically.

  For normal state, magnetic induction inside the specimen is given by,

$$B = \mu_0 (H + M) \qquad \ldots(5.4.1)$$

Where, μ_0 = Absolute permeability

H = External field applied

M = Magnetization produced within specimen

At $T < T_C$, B = 0 i.e. superconducting state

$$\therefore \mu_0 (H + M) = 0$$

$$\text{Susceptibility, } \chi = \frac{M}{H} = -1 \qquad ...(5.4.2)$$

- It is the diamagnetism which brings strong repulsion to external magnets. This has given us **levitation effect** and MAGLEV trains.

5.5 Types of Superconductors

MU - May 12, Dec. 13

Q. Explain type-I and type-II superconductors. **(May 12, 4 Marks)**

Q. What are type I and II superconductors ? **(Dec.13, 5 Marks)**

- Based **upon magnetic behavior**, superconductors are classified into two types *viz.* Type–I superconductors and Type – II superconductors.

Type - I superconductors

- Type-I superconductors exhibit complete Meissner effect. In the presence of an external magnetic field $H < H_C$, the material in superconducting state is a perfect diamagnet.
- Since it is a diamagnet, it possesses negative magnetization ($-\mu_o M$).
- Here the transition from superconducting state to normal state in the presence of a magnetic field occurs sharply at the critical value H_C as shown in Fig. 5.5.1.

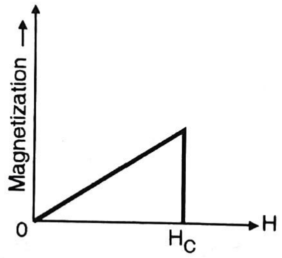


Fig. 5.5.1 : Type-I superconductor

- Aluminium, indium and lead are examples of Type-I superconductors. Since H_C is just around 0.2 wb/m^2, it not useful for any industrial application.

Type-II superconductors

MU - May 14

Q. What is the vortex state of a superconductor ? **(May 14, 3 Marks)**

- Type-II superconductors are characterized by two critical magnetic fields, H_{C1} and H_{C2}. For any applied field strength less than H_{C1}, it expels the magnetic field from its body completely and behaves as a perfect diamagnet.
- When $H > H_{C1}$, the flux penetrates and fills partially in the body of the material. With further increase in H, the flux filling also increases and diamagnetic property decreases.
- At $H = H_{C2}$ material turns into a normal conductor.

- Between H_{C1}, and H_{C2}, specimen is magnetically in mixed state but electrically it is a superconductor. Also H_{C2} is around 50 Wb/m^2. This shows that even for relatively high values of magnetic field superconductivity is retained. Hence it is very useful.

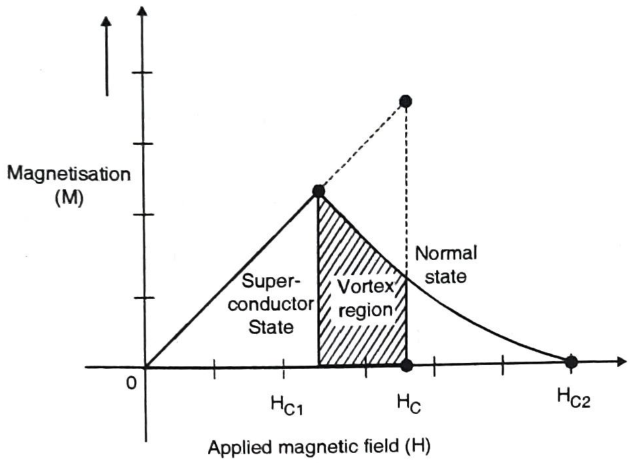


Fig. 5.5.2 : Type-II superconductor

- Now it is observed that specimen is in ***mixed state between H_{C1} and H_{C2}***. And above H_{C2} specimen returns to its ***normal state***. It is concluded that Meissner effect is found to be incomplete in the region between H_{C1} and H_{C2}.This region is known as ***vortex or mixed region.*** In this state the specimen has both superconducting and normal regions and can possess zero resistance and partial penetrating flux.
- Due to the vortex or mixed area, some penetration by an external magnetic field (H) into its surface will be allowed. Therefore new microscopic phenomena like superconducting ***'stripes'*** and ***'flux-lattice vortices'*** can be observed. Such a partial penetration given the applied magnetic field is responsible for breaking the superconductivity state (critical magnetic field H_C).
- Thus in Type-II superconductors, applied magnetic field and temperature are the main variables of the phase diagram.
- For Type-II superconductors, ***lattice structure*** plays an important role. There is no complete model to explain Type-II superconductors like BCS theory explains Type-I superconductor.
- Transition metals and alloys consisting of niobium, aluminium, silicon and vanadium exhibit, Type-II superconductivity.

5.5.1 Comparison between Type I and Type II Superconductors

MU - May 13, May 14, Dec. 14, Dec. 15, May 16, May 17, Dec. 18, May 19

Q. Distinguish between type I and II superconductors. **(May 13, 5 Marks)**

Q. Differentiate between Type-I and Type II superconductors.

(May 14, Dec. 14, Dec.15, May 16, May 17, Dec.18, May 19, 5 Marks)

Type - I superconductor	Type - II superconductor
Commercially these superconductors are called as *soft superconductors.*	These superconductors are called as *hard superconductors.*
These superconductors exhibit only one critical field (H_C).	These superconductors exhibit two critical fields namely *lower critical field (H_{C1})* and *higher critical field (H_{C2}).*
The critical magnetic field value is very low.	The critical magnetic field value is high.
These are explained on the basis of BCS theory.	There is no fixed theory developed to explain it.
These superconductors exhibit perfect and complete Meissner effect.	These do not exhibit a perfect and complete Meissner effect.
These materials have limited technical applications because of very low field strength value.	These materials have wider technological applications because of very high field strength value.
Examples : Pb, Hg, Zn, etc.	*Examples :* Nb_3Ge, Nb_3Si, $Y_1Ba_2Cu_3O_7$, etc.

5.6 High T_C Superconductivity

"Revival of hope"

- In year 1986, J. Bednorz an K. A. Muller* discovered superconductivity in ceramics (earlier it was in metals).
- The most important point was superconductivity at 30° K, which is very high compared to Mercury (4.2°K). This discovery marked the beginning of new era.
- Transition temperature saw a jump to 92° K by the work of C. W. Chu and M. K. Wu.
- The point which must be appreciated is that the success broke the barrier of liquid nitrogen temperature of 77° K.
- Reader must know that, despite having tremendous potential for great applications, very low T_C was a big hurdle.
- Liquid nitrogen is a product which is manufactured easily, available at many places and is inexpensive.
- This means superconductivity at a cheaper rate and with great comfort.

Note : *Both were awarded Nobel price in year 1987.

- The material used to get superconductivity at liquid nitrogen temperature was in the form $YBa_2 Cu_3 O_{7-x}$ (x = .01, 0.02, .., 0.1) popularly referred as Y – 123 compound. Even other oxides of Cu have reported high T_C. They all bear perovskite type of crystal structure.
- By and large experimental results are in conformity with BCS theory, but a more authentic and foolproof theory to explain the basic mechanism to explain high T_C superconductivity is yet to be worked out.
- Other technical challenges involved are
  (1) To obtain high T_C materials in the form of wire or ribbon so that one can facilitate the flow of electricity over a long range.
  (2) Increase in transition temperature i.e. towards room temperature.
  (3) A proper theory which can explain all possible aspects of high T_C superconductivity.

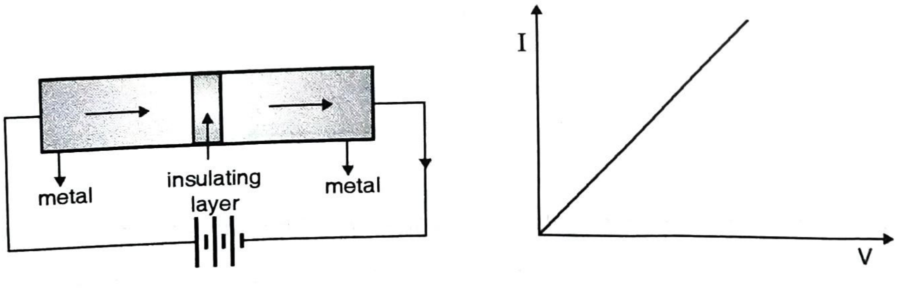
5.7 Josephson Effect

- Prerequisite knowledge of some basic aspects related with tunnelling effect studied through quantum mechanics :

  This is one of the important effect for a superconductor invented by British physicist Brian David **Josephson**, in 1962. The **Josephson effect** is the phenomenon of electric current across two weakly coupled superconductors, which are separated by a very thin insulating barrier. In this arrangement, two superconductors are linked by a non-conducting barrier which is known as a ***Josephson junction***.
- The current that crosses the barrier is called **Josephson current**. It has great applications in quantum mechanical circuits, such as SQUIDs (Super conducting Quantum Interference Devices) or RSFQ(Rapid Single-Flux-Quantum) digital electronics.

Josephson effect

- The Josephson effect is explained with the help of two metals (conductor) which are separated by thin insulator (thickness 20 A°). Such insulating layer acts as a **potential barrier** for flow of electrons (current) from one metal to another through a junction.
- Barrier is very small (thin), therefore quantum mechanically electrons can **tunnel** through from higher potential to lower potential. This process continues until the chemical (potential) equilibrium takes place.



(a) (b)

Fig. 5.7.1 : Illustration of Josephson effect with applied field

- Let us see application of potential difference (battery) across the potential barrier, as shown in the Fig. 5.7.1.
- Here more electrons are found to tunnel through the insulating layer. Current-voltage relation obeys Ohm's law at low voltage.
- Now consider that in the above experiment, instead of conductor if superconductor is used and voltage is applied across them, as shown in Fig. 5.7.2.

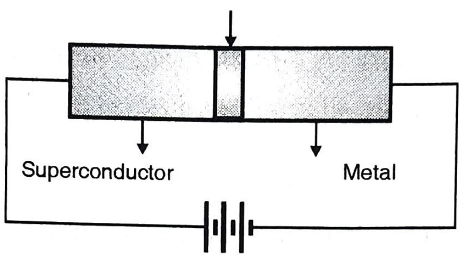


Fig. 5.7.2 : A thin insulating layer sandwiched between metal and superconductor

- Here it is observed that no current flows across the junction until the potential reaches a threshold value. It is seen that the value of threshold voltage is equal to the energy gap in the superconductor state.
- Thus it is important to note that the value of threshold potential or voltage is nothing but the energy gap of superconductor. Further, this threshold voltage becomes a function of temperature below T_C.
- As the temperature is increased towards the critical temperature (T_C) of the superconductor, more and more thermally excited electrons are generated, and they require less energy to tunnel.
- Therefore, threshold voltage decreases. The current-voltage relations at different temperatures are recorded in the Fig. 5.7.3.

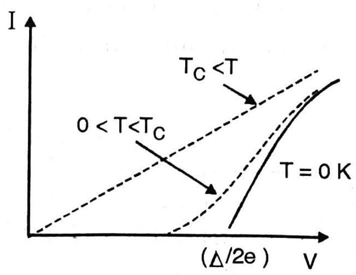


Fig.5.7.3 : Current voltage relationship at the junction with respective temperature

- Consider the possibility that, the thin insulator is sandwiched between two superconductors as shown in Fig. 5.7.4.

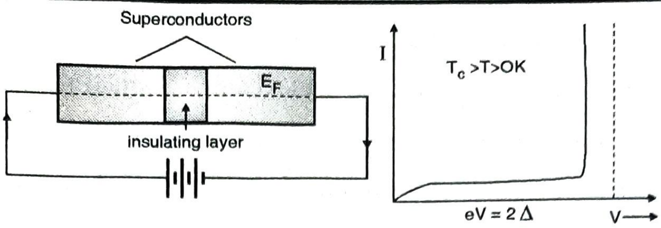


Fig. 5.7.4 : Tunneling in a superconductor –insulator-superconductor and Current-voltage characteristics at T = 0 K and T_c> T > 0 k

- Here in addition to single electrons, the superelectrons i.e. Cooper pairs also tunnel through the junction from one superconductor to another even at zero potential difference across the junction.
- The wave function (φ) is highly correlated to both the sides. Thus phenomenon of getting small current due to tunnelling of superelectrons of the superconductor is known as Josephson effect.
- The current-voltage relation at different temperatures is shown in the Fig. 5.7.4. The tunnelling current across the junction is very small. In practice two Josephson effects are used, namely D.C. Josephson effect and A.C. Josephson effect.

5.8 Applications of Superconductivity

5.8.1 MAGLEV Trains

MU - Dec. 13, Dec. 15, May 17, Dec. 17, May 19

Q.	What is MAGLEV ?	**(Dec. 13, 3 Marks)**
Q.	How can the MAGLEV train have very high speed ?	**(Dec. 15, 3 Marks)**
Q.	What is MAGLEV? How it can have very high speed ?	**(May 17, 3 Marks)**
Q.	What is working principle of MAGLEV ? Explain how it can acquire high speed ?	**(Dec. 17, 5 Marks)**
Q.	Write short note on MAGLEV.	**(May 19, 5 Marks)**

- Based on the **Meissner effect** we have a superb application of **frictionless bearings** that is used in **MAGLEV** trains.
- MAGLEV stands for Magnetically Levitated Vehicles.
- They are used in transportation by **being set afloat** above a guideway.
- The utility of such levitation is that in the absence of contact between moving and stationary systems the friction is eliminated. This brings great speeds with low energy consumption.

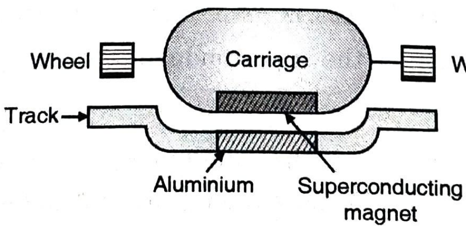


Fig. 5.8.1 : MAGLEV

- The train has a superconducting magnet built into its base.
- There is an aluminium guideway over which the vehicle will set afloat by magnetic levitation.
- The magnetic levitation is brought about by enormous repulsion between two highly powerful magnetic field by superconducting magnet and aluminium guideway. Wheels have no role to play when vehicle is lifted up.
- The track is made up of large number of segments and the flow of currents through the coils could be related to the position and speed of the vehicle.
- **Prototype** of such vehicle has achieved **speed of 400 kmph.**

5.8.2 Superconducting Magnets (Electromagnets)

- For production of heavy duty superconducting magnets type-II superconductor wires are used. They are wound in the form of solenoids to generate strong magnetic fields of induction about 22 Tesla. The size of these magnets is very small as compared to normal magnet.
- They are very economic.
- They are used in the transformers in which magnetic core material is not required.

5.8.3 Bearings (Superconducting)

- The Meissner effect of superconductor is used in production of bearings. The mutual repulsion between two superconductors is used to reduce the friction inside the bearings. Therefore it can be used without any power loss.
- The life of the superconductor bearing is more than the ordinary mechanical type bearings.

5.8.4 Superconducting Quantum Interference Device (SQUID)

MU - May 18

Q. What is the working principle of SQUID ? Explain how it is used to detect the magnetic field ? **(May 18, 5 Marks)**

- The Superconducting Quantum Interference Device (SQUID) is based on the principle of Josephson effect of superconductor.
- It consists of two superconductors separated by thin insulating layers to form a Josephson junction which is as shown in the Fig. 5.8.2.
- The great sensitivity of the SQUID devices is associated with measuring changes in magnetic field associated with one flux quantization in the superconducting ring. In this case the total magnetic flux through the ring is quantized.
- This device is basically used to measure the small magnetic fields of living organisms. Sample measurements of the field are : magnetic field of heart is 10^{-10} T and brain is 10^{-13}.

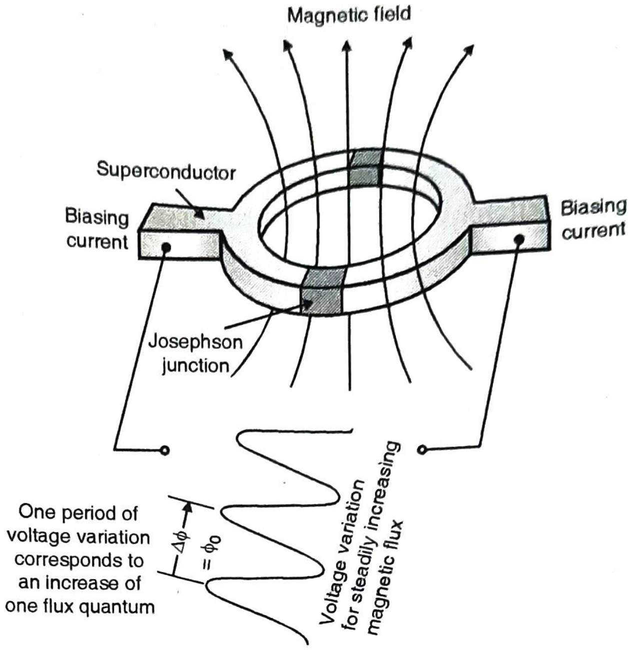


Fig.5.8.2 : Structure of SQID

- SQUID is able to sense very minute magnetic signals. They are used to study the minute magnetic pulses from the brain and heart. Due to extra high sensitive measurement system in the SQUID, it is quite useful to measure accurate magnetic field inside the human body, hence it helps to diagnose the other medical disorders in the body.
- For example SQUID detectors are used to measure the levels of iron in liver (here magnetic property of iron is used), so that precaution can be taken; iron built up can be treated before much harm is done to the body.

5.8.5 Fast Electrical Switching

- The magnetic penetration depth or length is one of the important characteristics of the superconductors. Therefore Type-II superconductors can be used as very fast electronic switches (there are no moving parts), in which a magnetic field can penetrate into the superconductor.
- Here application of magnetic field greater than the H_C can change the state of material from superconductor to normal conductor. This can reverse the process. This principle is used to develop switching element known as ***cryotron***. The structure of ***cryotron*** is as shown in the Fig. 5.8.3.

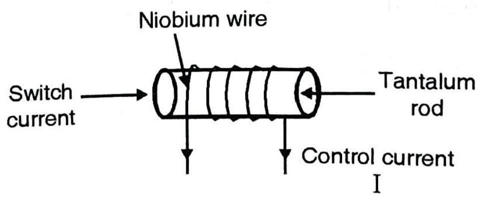


Fig. 5.8.3 : Structure of crytron (fast superconducting switches)

- This consists of tantalum core around which niobium wire is wound. These both are used as superconductors in which superconducting current is controlled with respect to temperature and switching action is achieved faster than the ordinary switches.
- We know that the Josephson junction is a super fast switching device. Josephson junctions can perform switching functions such as switching voltages approximately ten times faster than ordinary semi-conducting circuits.
- This is of distinguished advantage in a computer, which depends on short, on-off electrical pulses. As the computer speed is dependent on fast switching action so that less time is required to transmit signal pulses. The semiconductor electronic junction devices have exceptional switching speed, that it why they are ideal for use in high speed super fast and much compact computers.

5.9 Solved Problems

Ex. 5.9.1 : The critical field of niobium is 1×10^5 A/m at 8°K and 2×10^5 A/m at 0°K. Calculate critical temperature of the element.

Soln. :

Using Equation (5.3.1)

$$H_C = H_O\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

$$\therefore \quad T_C = \frac{1}{\left[1-\frac{H_C}{H_o}\right]^{1/2}} = \frac{8}{\left[1-\frac{1\times 10^5}{2\times 10^5}\right]^{1/2}} = 11.3°\ K$$

[Ans. : critical temperature T_{C2}= 11.3° K]

Ex. 5.9.2 : Determine the transition temperature and critical field at 4.2 K for a given specimen of a superconductor if the critical fields are 1.41×10^5 A/m and 4.205×10^5 A/m at 1.41 K and 12.9 K, respectively.

Soln. :

Given :

$$H_{C1} = 1.41 \times 10^5 \text{ A/m}, T_1 = 14.1 \text{ K},$$

$$H_{C2} = 4.205 \times 10^5 \text{ A/m}, T_2 = 12.9 \text{ K}.$$

Formula :

(1) $$H_C = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

Thus, the critical fields at temperatures T_1 and T_2 can be written

(2) $$H_{C1} = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

and

(3) $$H_{C2} = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

Taking the ratio of formula (2) and (3) we get,

(4) $$\frac{H_{c1}}{H_{c2}} = \frac{T_c^2 - T_1^2}{T_c^2 - T_2^2} \text{ or } \frac{1.41 \times 10^5}{4.205 \times 10^5} = \frac{T_c^2 - (14.1)^2}{T_c^2 - (12.9)^2}$$

Substituting the values, we get

$$H_c = 1.41 \times 10^5 \text{ A/m,}$$

$$T_1 = 14.1 \text{ K and } T_c = 14.67\text{K}$$

$$H_c = H_0\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

$$1.41 \times 10^5 = H_0\left[1 - \left(\frac{14.1}{14.67}\right)^2\right]$$

$$\text{or} \quad H_0 = 18.504 \times 10^5 \text{ A/m}$$

The critical field at T = 4.2K and T_c = 14.67K

$$H_c = H_0\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

$$= 18.504 \times 10^5 \times \left[1 - \left(\frac{4.2}{14.67}\right)^2\right]$$

$$H_c = 16.99 \times 10^5 \text{ A/m}$$

$$H_c \simeq 16.0 \times 10^5 \text{ A/m.}$$

[Ans. : T_c = 14.67 K and $H_c = 16.00 \times 10^5$ A/m.]

Ex. 5.9.3 : The critical temperature Tc for Hg with isotopic mass 199.5 is 4.185 K. What will be its critical temperature when its isotopic mass is increased to 203.4?

Soln. :

Given :

$$T_{C_1} = 4.185 \text{ K,}$$

$$M_1 = 199.5,$$

$$M_2 = 203$$

$$T_{C_2} = \text{For increased mass}$$

Formula : $$T_C M^{0.5} = \text{constant}$$

Let T_{C_1} is for M_1 mass and T_{C_2} is for M_2 mass (increased).

$$\therefore \quad T_{C_1} M^{0.5} = \text{constant} \quad \text{...(1)}$$

$$T_{C_2} M^{0.5} = \text{constant} \quad \text{...(2)}$$

∴ From equation (1) and equation (2) we get,

$$T_{C_1} M^{0.5} = T_{C_2} M^{0.5} \quad \text{...(3)}$$

Substituting the values, we get

$$4.185 \times (199.5)^{0.5} = T_{C_2} \times (203)^{0.5}$$

$$4.185 \times 14.12 = T_{C_2} \times 14.247$$

$$\frac{59.1108}{14.242} = T_{C_2}$$

$$T_{C_2} = 4.15 \text{ K}$$

[Ans. : Critical temperature T_{C_2} at increased mass m_2 is 4.15 K]

Ex. 5.9.4 : The critical temperature of a given superconducting sample is 1.19 K with mass 26.91. Determine the critical temperature when the isotope mass changes to 32.13.

Soln. :

Given : $T_{C1} = 1.19\text{K}, M_1 = 26.91$ and $M_2 = 32.13, T_{C2} = ?$

Formula used is,

$$T_{C1} M_1^{1/2} = T_{C2} M_2^{1/2}$$

$$T_{C2} = \frac{T_{c1} M_1^{1/2}}{M_2^{1/2}} = \frac{1.19 \times (26.91)^{1/2}}{(32.13)^{1/2}}$$

$$= \frac{1.19 \times 5.127}{5.668} = \frac{6.173}{5.668} = 1.089$$

[Ans. : Critical temperature T_{C2}= 1.089 K]

Ex. 5.9.5 : A voltage of 5.9 μV is applied across a Josephson junction. What is the frequency of the radiation emitted by the junction?

Soln. :

Given : $V = 5.9 \times 10^{-6}$ V; $v = ?$

Formula : $$v = \frac{2eV}{h}$$

Substituting the values we get,

$$= \frac{2 \times 1.6 \times 10^{-19} \times 5.9 \times 10^{-6}}{6.62 \times 10^{-34}}$$

$$v = 2.851 \times 10^{9} \text{ Hz.}$$

[Ans. : Frequency of the radiation $v = 2.851 \times 10^9$ Hz.]

Ex. 5.9.6 : Determine the critical current through superconducting wire of diameter of 1.0 mm, given that critical field is 7.26×10^3 A/m.

Soln. :

Given : $d = 1.0$ mm

$$\therefore r = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$$

$$H_C = 7.26 \times 10^3 \text{ A/m}$$

Formula :

$$I_C = 2\pi r H_C$$

Substituting the values we get,

$$I_C = 2 \times 3.14 \times 0.5 \times 10^{-3} \times 7.26 \times 10^3$$

$$= 2 \times 3.14 \times 0.5 \times 7.26$$

$$I_C = 22.608 \text{ A}$$

[Ans. : Critical current flowing through the wire is $I_C = 22.608$ A]

Ex. 5.9.7 : If the voltage 6.2 μV is applied across a Josephson junction what will be the frequency of radiation emitted by the junction ?

Soln. :

Given :

$$V = 6.2\ \mu\text{V} = 6.2 \times 10^{-6} \text{ V}$$

$$\nu(\text{frequency}) = ...? \text{ Hz}$$

Formula :

$$\nu = \frac{2eV}{h}$$

Substituting the values we get,

$$\nu = \frac{2 \times 1.6 \times 10^{-19} \times 6.2 \times 10^{-6}}{6.62 \times 10^{-34}}$$

$$= \frac{19.84}{6.62} \times 10^9$$

$$\nu = 2.99 \times 10^9 \text{ Hz}$$

[Ans. : Frequency emitted by Josephson junction is $\nu = 2.99 \times 10^9$ Hz]

Ex. 5.9.8 : In the case of lead superconductor to find the maximum value of magnetic field which will allow retaining its superconductivity, if critical temperature is 7.2 K and at 4 K it loses the superconducting property if subjected to a magnetic of 3.33×10^4 A/m.

Soln. :

Given :

$$T_C = 7.2 \text{ K};$$

$$H_C = 3.33 \text{ A/m};$$

$$T = 4 \text{ K}$$

$$H_C(0) = ...? \text{ at } 0 \text{ K}$$

Formula :

$$H_C(T) = H_C(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

Where, $H_C(0)$ is the critical magnetic field at 0 K.

$$\text{or} \quad H_C(0) = \frac{H_c(T)}{1-\frac{T^2}{T_c^2}}$$

Substituting the values we get,

$$H_C(0) = \frac{3.33 \times 10^4}{1-\frac{16}{51.84}}$$

$$= \frac{3.33 \times 10^4}{0.69} = 4.82 \times 10^4 \text{ A/m}$$

[Ans. : Critical field at 0 K is 4.82×10^4 A/m]

Ex. 5.9.9 : A superconductor has a critical temperature 3.7°K at zero magnetic field. At 0°K the critical magnetic field is 0.0306 Tesla. What is the critical magnetic field at temperature 2.0°K ? **MU - May 15, 3 Marks**

Soln. :

$$\text{Here } H_0 = 0.0306 \text{ T, T} = 2.0°\text{K, } T_0 = 3.7°\text{K}$$

$$H_c = \left[1-\left(\frac{T}{T_c}\right)^2\right] = 0.0306\left[1-\left(\frac{2}{3.7}\right)^2\right]$$

$$= 0.271 \text{ T}$$

[Ans. : Critical magnetic field at temperature 2.0°K = 0.271 T]

Supercapacitors

5.10 Introduction

Q. Explain the term (a) conventional capacitor (b) supercapacitor.

- Few of the currently burning topics.
- Let's have a recap of ordinary capacitor also called as parallel plate capacitor. Using a dielectric medium between two plates, accumulation of charges takes place. This stores electrical energy and provides potential difference.
- Advanced version electrolytic capacitors have fixed polarity for anode and cathode.
- These capacitors are occasionally abbreviated as e-cap, whose anode is made of a metal that forms an insulating oxide layer through anodization. Applying a reverse polarity voltage or exceeding the maximum rated working voltage may invite a capacitor failure, and that can be hazardous.
- Despite all efforts, the capacity of capacitors always remains low. Hence, to store electrical charge, batteries were given preference.

Now we have supercapacitors or ultracapacitors (or Electrical Double Layer Capacitors (EDLCs) or pseudo capacitors or power capacitors or power catches) which are electrochemical energy storage devices.

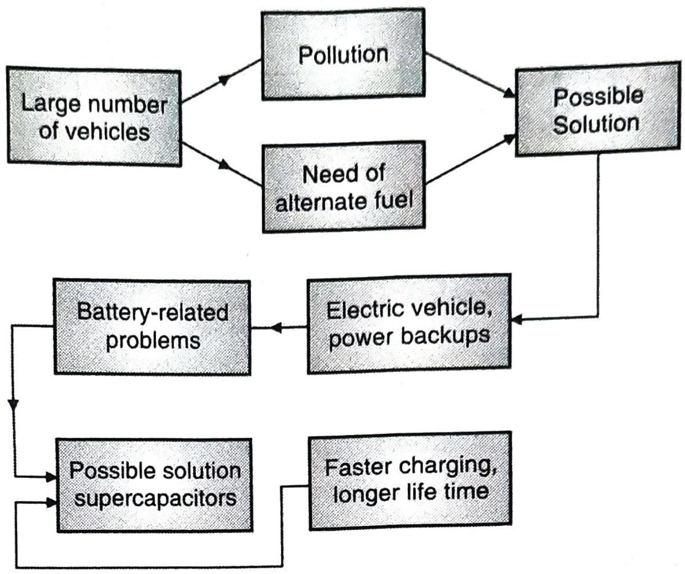


Fig. 5.10.1

5.10.1 Principle

Q. Explain the function of supercapacitors in detail.

- They store and release energy by reversible absorption and desorption of ions at the interfaces between electrode materials and electrolytes.
- Compared to common rechargeable batteries like lead-acid and Li-ion batteries they have -
  (1) Considerably high specific power
  (2) Longer cycle life times.

5.10.2 Construction

Q. Explain in detail construction of supercapacitors.

- Ordinary capacitor makes use of two parallel plates separated by a solid dielectric material.
- Supercapacitors, in principle, make use of
  (1) Double layered capacitance
  (2) Electro chemical pseudeo capacitance
- This can be further explained by stating that it consists of two electrodes separated by an ion permeable membrane which acts as the layer separator of electrolyte having positive and negative ions. When voltage is applied to the electrodes, polarization takes place. Ions in the electrolyte form an electric double layer as shown in the Fig. 5.10.2. Positive terminal will have negative ions from electrolyte forming a layer, and similarly electrode connected to negative terminal will have positive ions from electrolyte.

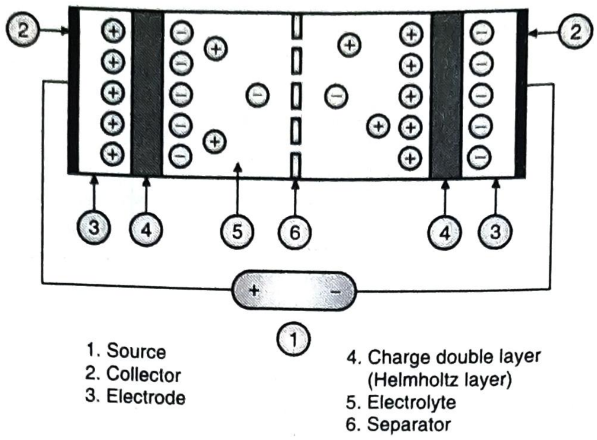


Fig. 5.10.2

1. Source
2. Collector
3. Electrode
4. Charge double layer (Helmholtz layer)
5. Electrolyte
6. Separator

- As shown in the Fig. 5.10.2, there are two capacitors formed at each end by accumulation of charges.

5.10.3 Types or Classification of Supercapacitors

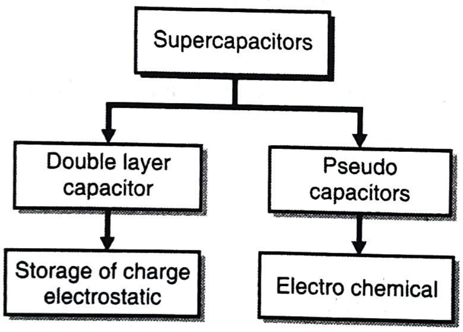


Fig. 5.10.3

5.10.4 Applications

Q. Explain at least three applications of supercapacitors.

Supercapacitors have their applications mainly due to the fact that they are useful when -

1. Fast charging is required for batteries. Hence supercapacitors are used for LED flash lights, digital cameras, laptop batteries, etc.
2. Supercapacitors can stabilize the power supply and are hence useful as portable media players, laptops, or similar hand-held machines.
3. As supercapacitors are capable to correct current, power fluctuations and harmonics, they are used as an interface between the load and grid to act as buffer in between.

4. On cards of computers, microcontrollers or similar digital devices act as emergency power backup in low power equipments.
5. One brilliant application of supercapacitors is in energy storage devices for energy harvesting systems. In energy harvesting systems we collect energy from renewable sources, for example electromagnetic fields, light, regenerative breaking system in vehicles or any mechanical movements. Such energy is stored in capacitor and provided back to the system when it is needed. Like in railway engines, energy while applying brakes is stored in supercapacitors and when acceleration is needed, this energy is provided back to system.

5.11 Comparison with Capacitor and Batteries, Energy Density, Power Density

Q. What is energy density and power density? Compare conventional batteries and supercapacitors on these points.

- When we consider conventional demand of electricity, especially for application of batteries the main problem is with charging. It takes longer time to charge our regular alkaline or Li-ion batteries. Reader can understand the process of cell phone battery charging. Imagine if we go for advanced industrial requirements like electrical vehicles, the charging time is one of the big hurdles.
- Recently the local transport (like BEST) has introduced electric buses, but the vehicle remains out of operation just due to charging issues for a longer time. Hence they can't go on a longer route. The other problem is, how many times the cycle of charging-discharging can take place. It is further observed that after a certain number of cycles of charging and discharging, the battery develops problems and needs a replacement. This adds to its cost.
- As per syllabus we will compare it with capacitor and batteries in terms of energy density and power density. But before that, let's understand these two terms. Since we are aware about the fact that conventional capacitor is energy storage device and energy stored is given by

$$E = \frac{1}{2} CV^2$$

- Energy stored per unit mass is described as specific energy which is represented in terms of Wh/kg. This is also referred to as gravimetric measure. If the capacitor energy is represented in terms of energy stored in capacitor it is represented in terms of per unit volume of that capacitor, it is called energy density. It is described in the units of $\frac{\text{watt Hours}}{\text{lit.}}$ or simply $\frac{\text{Wh}}{\text{t}}$
- Specific power is a term which describes the speed at which energy can be delivered to or absorbed from the load. If it is measured gravimetrically i.e per mass, it is said to be representing specific power (unit kW/kg).
- Similarly, if measured volumetrically i.e. per volume of capacitor, it is known as power density (unit kW/l)
- On the scale of comparison specific energy of electrolyte capacitor is approximately in the range of 0.01 to 0.3 Wh/kg.
- Whereas, for lead acid battery it is in the range of 30 to 40 Wh/kg. Li – ion battery offers it around 100 – 200 Wh/kg.

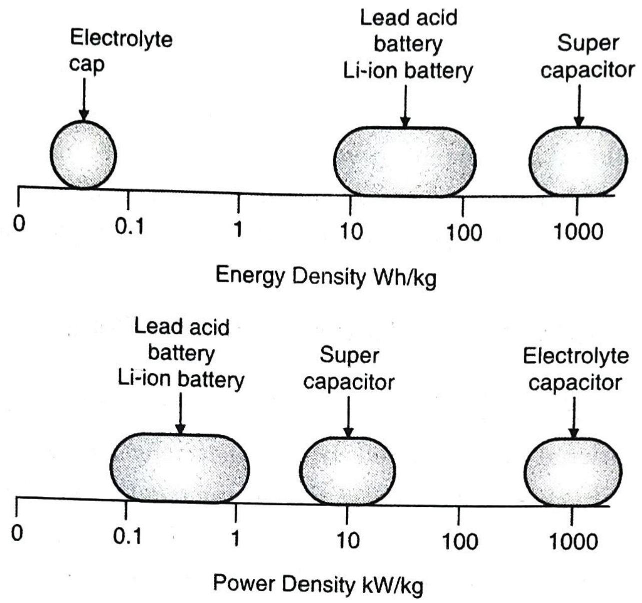


Fig. 5.11.1 : Energy and power density (kW/kg)

- If we consider their comparison, it is clear that specific energy supercapacitor which is inferior with respect to batteries, has an advantage when specific power is considered.

A Quick Revision

- The temperature at which a normal material turns into superconducting state is called critical temperature.
- The critical temperature is known to be inversely proportional to the square root of the atomic mass.
- The critical or transition temperature (T_C) of superconductor is found to vary with its isotopic mass (M), this phenomenon is called Isotopic Effect.
- Minimum value of the magnetic field required to destroy completely the superconducting phase is known as critical magnetic field, and is denoted by H_C.
- Critical magnetic field H_C is the function of temperature T, which is expressed as -

$$H_c = H_0\left[1-\frac{T^2}{T_c^2}\right]$$

- A superconducting material kept in a magnetic field expels the magnetic flux out of its body when cooled below the critical temperature and exhibits perfect diamagnetism. This effect is called **'Meissner effect'.**
- Thus in a superconductor, the current is generated due to *Cooper pairs,* instead of individual electrons.
- Type-I superconductor exhibit complete Meissner effect. In the presence of an external magnetic H <H_C, the material in superconducting state is a perfectly diamagnetic.

- Type-II superconductors are characterized by two critical magnetic fields, H_{C1} and H_{C2}.
- Specimen is in ***mixed state between H_{C1} and H_{C2}***. And above H_{C2} specimen returns to its ***normal state***. This region is known as ***vortex or mixed region.***
- Thus phenomenon of getting small current due to tunnelling of superelectrons of the superconductor is known as Josephson effect.
- The great sensitivity of the SQUID devices is associated with measuring changes in magnetic field associated with one flux quantization in the superconducting ring.

❑❑❑