CHAPTER 1

Unit I

# Quantum Physics

Syllabus

(**Prerequisites :** Dual nature of radiation, photoelectric effect, matter waves, wave nature of particles, de-Broglie relation, Davisson-Germer experiment), de Broglie hypothesis of matter waves, properties of matter waves, wave packet, phase velocity and group velocity, wave function, physical interpretation of wave function, Heisenberg uncertainty principle, non-existence of electron in nucleus, Schrodinger's time-dependent wave equation, time-independent wave equation, particle trapped in one-dimensional infinite potential well, quantum computing

**Learning Objectives :**

After reading this chapter, learner should be able to

- Understand de-Broglie's hypothesis and properties of matter waves
- Describe wave function and its physical interpretation
- Understand uncertainty principle and its applications
- Derive Schrodinger's wave equation time-dependent and independent form
- Apply Schrodinger's equation to one-dimensional infinite potential well

## 1.1 Introduction

The topic which is discussed here is highly versatile and stores a new concept. It requires a completely new approach; therefore, we must take its introduction in detail. Let us answer the following questions :

(a) How has the topic of mechanics evolved ?

(b) What is the need of new mechanics ?

(c) What forms quantum mechanics ?

(d) What is the difference between classical mechanics and quantum mechanics ?

### (a) Evolution of mechanics

- To study the motion of bodies, efforts were taken by many people starting from Galileo, Blaise Pascal up to Sir Isaac Newton.
- In *Principia Mathematica* which was published in 1687, he gave a unified theory, which accounted for all types of motions of bodies on common ground by consolidating all the ideas of previous workers in addition to his own. This was the birth of the subject Mechanics.
- By the end of the nineteenth century, a general feeling prevailed in the scientific community that knowledge regarding the subject of mechanics is complete and whatever remains to be done is only the refinement of the known ones.

**(b) Need of new mechanics**

- Towards the end of nineteenth century, the study of energy distribution in the spectrum of black body radiation came up, and Newtonian mechanics could not explain it. This was the point at which a need for new mechanics was felt.

**(c) Formulation of quantum mechanics**

- Towards the end of year 1900, Max Planck explained the spectrum of black body radiation by taking a completely new way i.e. he used the idea of "quanta". Hence, from year 1901, quantum mechanics came into picture, and everything before the year 1900 was labelled as classical mechanics. Bohr's theory for hydrogen atom provided a brilliant example for quantization aspects.
- In year 1924, Louis de Broglie suggested "wave-particle duality" which formed the basis for Schrodinger's work. The much-needed mechanics for the atomic and non-atomic world was shaped by the pioneering work of Schrodinger, Heisenberg, Dirac, et al. and called quantum mechanics.

**(d) Difference between classical mechanics and quantum mechanics**

- Both of them are fundamentally different approaches to solve problems. In classical mechanics, it is unconditionally accepted that position, mass, velocity, acceleration, etc. of a particle or a body can be measured accurately, which of course is true in our day-to-day observations.
- The explanation provided by the classical mechanics for the behaviour of a body studied in a problem of dynamics is fully valid in our usual observations. Classical mechanics predicts those magnitudes related to mass, position, etc. which agree fully with measured values.
- In contrast to the above, what quantum mechanics speaks is purely probabilistic in nature. As per the fundamental assumption of quantum mechanics, it is impossible to measure simultaneously the position and momentum of a given body. Hence, language of probability is most useful.
- For example, classical mechanics states that in the hydrogen atom, first orbit is $5.3 \times 10^{-11}$m whereas quantum mechanics identifies it as the most probable value of the radius one would get. If a suitable experiment is performed to measure the radius, a number of different values of radius are going to be obtained. But most of the times the values will be very close to $5.3 \times 10^{-11}$ m.
- A reader may feel that due to such uncertainties, classical mechanics is better than quantum mechanics but one must also accept that classical mechanics is simply an approximate version of quantum mechanics.
- In day-to-day life, accurate results obtained through classical mechanics are found to be true whereas in terms of quantum mechanical calculations, the departure of the observed values from the most probable value becomes totally insignificant. But once we go to atomic or subatomic level calculations, the probabilities involved in the values of various physical quantities become significant and classical mechanics fails completely.

## 1.2 de-Broglie Hypothesis

MU- May 19

**Q.** Explain de-Broglie Hypothesis of matter waves and deduce the expression for $\lambda$. **(May 19, 5 Marks)**

Although the idea of quantization of energy became indisputable, it took some time to understand its origin. The origin of quantization of energy lies in the dual behavior observed in particle and wave nature.

### 1.2.1 de-Broglie Wavelength

- Consider a wave of frequency ν

$$\therefore \quad E = h\nu$$

- It can also be represented as

$$E = mc^2$$

$$\therefore \quad h\nu = mc^2 \qquad \text{...(1.2.1)}$$

- Now, p = Momentum associated with photon which travel in free space.

$$p = mc = \frac{mc^2}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

$$\therefore \lambda = \frac{h}{p} \qquad \text{...(1.2.2)}$$

This derivation is very simple. A more rigorous derivation of de-Broglie wavelength is done on the basis of group velocity concept.

## 1.3 Properties of Matter Waves

**MU - May 15, Dec.18**

**Q. What are the properties of matter waves ? (May 15, Dec.18, 3 Marks)**

The wavelength of matter waves is given by $\lambda = \frac{h}{mv}$.

Where m = mass of the particle

v = velocity of the particle

The properties of matter waves are :

(1) $\lambda \propto \frac{1}{m}$, hence lighter the particle greater is the wavelength associated with it.

(2) $\lambda \propto \frac{1}{v}$, hence greater the velocity of the particle, smaller is the associated wavelength.

(3) As $v \to \infty$, $\lambda \to 0$. But the wave becomes indeterminate when $v \to 0$ and $\lambda \to \infty$. This shows that matter waves are associated with the particles in motion. These waves do not depend on the charge of the particle, and hence matter waves are not electromagnetic in nature. The waves are not mechanical in nature.

(4) The velocity of matter waves is not a constant like electromagnetic waves but depends on the velocity of the particle generating them.

$v_p = \frac{c^2}{v}$, hence matter waves travel faster than light. ($\because v < c$)

(5) The wave velocity of matter waves depends inversely on the wavelength λ. This is the basic difference between matter waves and light waves. (Light waves have same velocity for all wavelengths).

(6) Matter wave representation is a symbolic representation. It is a wave of probability indicating the likelihood of locating the particle.

(7) The wave and particle duality of matter is not exhibited simultaneously.

**Dissimilarities and similarities between matter wave and electromagnetic wave :**

(1) Matter waves are produced whether the particle is charged or uncharged. But electromagnetic waves are produced due to motion of charged particles only.

(2) Velocity of matter waves if always greater than the speed of light, whereas velocity of electromagnetic waves is equal to the speed of light.

(3) Both the waves are capable of propagation in an absolute vacuum. So they are not mechanical waves as mechanical waves cannot travel in a vacuum.

## 1.4 Wave Packet, Group Velocity and Phase Velocity

**MU - Dec. 16 , Dec.18**

**Q.** What do you mean by group and phase velocity? Show that the de-Broglie group velocity associated with the wave packet is equal to the velocity of the particle. **(Dec. 16, Dec.18, 5 Marks)**

### 1.4.1 Wave Packet

- As per de Broglie's postulate, a material particle of mass m moving with velocity v is represented by a monochromatic wave of wavelength $\lambda$.
- Such a single wave representation of the particle raises some questions.
  - (i) How can a wave that spreads out over a large region of space represent a particle which is highly localized?
  - (ii) If we associate the wave with the particle, what exactly is the thing that is waving in the matter wave?
- A simple harmonic progressive wave is represented by an equation of the type.

$$y = A \sin (\omega t - kx) \qquad \text{...(1.4.1)}$$

where, $y \rightarrow$ Displacement of the particle of the medium at time t and position x.

$\omega \rightarrow$ Angular frequency of wave

$k \rightarrow$ Propagation constant

$A \rightarrow$ Amplitude of the wave.

- The amplitude of the de Broglie waves representing a moving particle determines the probability of finding the particle at a particular place at a particular time.
- A wave of the type represented by Equation (1.4.1) is of infinite extent and is completely non-localised. Hence, a single wave cannot represent a particle which is confined to a very small volume of space. Hence, it was suggested that instead of a single wave, a combination of several waves may represent the particle.
- Superposition of waves of slightly different frequencies results in the formation of a hump at a definite place on the envelope of a smooth wave by mutual interference of the waves. This is as shown in Fig. 1.4.1.

- This hump or envelope of waves is called the **wave packet** or the **wave group** and can be made use of as a mark on the wave.
- The propagation of this distinguishing mark with time can be detected by measuring devices.

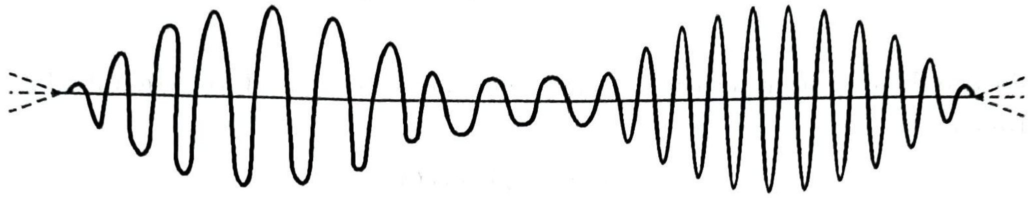

Fig. 1.4.1 : Superposition of waves

- If the number of superposed waves is increased, the hump becomes narrower and the intervening region of weaker disturbances broader.
- In the limit, if infinite number of waves of continuously varying wavelengths (or frequencies) extending over a finite range is superposed, a single hump in a narrow region results with no disturbance at any other point. This forms the wave packet. Fig. 1.4.2 shows a typical wave packet or wave group.

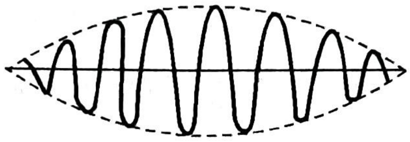

Fig. 1.4.2 : Wave packet

- Superposition of a number of waves with slightly different frequencies gives rise to a wave packet or a wave group.
- Such a wave packet possesses properties of particles as well as waves. The regular separation between successive maxima in the wave packet characterises a wave while at the same time it has particle characteristics i.e. localization in space.

### 1.4.2 Phase Velocity

- It is assumed here that the reader is aware about travelling wave expressed by

$$y = A \sin \omega t \qquad ...(1.4.2)$$

- To understand it in a proper way, let's consider a spring performing SHM on y axis where y is represented by Equation (1.4.2).
- Let the spring itself travel at velocity v along x-axis and let it cover distance x along x-axis in time t so that a mass suspended by spring is having same displacement y.

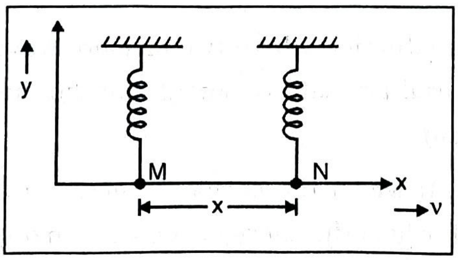


Fig. 1.4.3 : Idea of phase velocity

- Now at t = t, displacement on y-axis when mass is at point M is given by,

$$y = A \sin \omega t$$

- As the motion is periodic, mass will have the same displacement at N which it had at M at a time, say, $t_0$ earlier.
- Corresponding to the instant at which, the mass crosses the x-axis at M, the equation for displacement can be written as

$$y = A \sin(\omega(t - t_0)) \quad ...(1.4.3)$$

as distance between M and N is x and time taken is $t_0$.

$$\therefore \quad t_0 = \frac{x}{u} \quad ...(1.4.4)$$

$$\therefore y = A \sin\left[\omega\left(t - \frac{x}{u}\right)\right]$$

$$= A \sin\left[\omega t - \left(\frac{\omega}{u}\right)x\right]$$

$$= A \sin[\omega t - kx] \quad ...(1.4.5)$$

$$\text{where,} \quad k = \frac{\omega}{u} = \text{Wave number}$$

$$\therefore u = \frac{\omega}{k}$$

$$\text{OR} \quad \frac{d}{dt}(\omega t - kx) = 0$$

$$\omega - k\frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = u = \omega k$$

- Here u represents the velocity with which the wave disturbance is carried and is referred to as phase velocity.

$$\therefore \quad u_{phase} = \frac{\omega}{k} \quad ...(1.4.6)$$

- If a point is imagined to be marked on a travelling wave, then it becomes a representative point for a particular phase of the wave and the velocity with which it is transported owing to the motion of the wave is called **phase velocity**.

### 1.4.3 Group Velocity

- Here, the word group indicates a situation where two or more waves with slightly different velocities are superimposed together. The resultant pattern emerges in the shape of variation in amplitude and is called as wave packet or wave group.
- Group velocity is the velocity with which the envelope enclosing a wave group or a wave packet is transported. It is the velocity with which the energy transmission occurs in a wave.

Need to know about group velocity –

For an electromagnetic wave, phase velocity

$u = v\lambda$ and since $E = hv$

$$u = v\lambda = \frac{E}{h}\cdot\lambda$$

but,
$$\lambda = \frac{h}{mv}$$

$$\therefore \quad u = \frac{E}{h}\cdot\frac{h}{mv} = \frac{mc^2}{mv} = \frac{c^2}{v}$$

$$\therefore \quad u = \frac{c^2}{v^2}$$

- Since c >>v, it shows that phase velocity of de Broglie wave associated with the particle moving with velocity v is greater than 'c', the velocity of light.
- This difficulty can be overcome by assuming that each moving particle is associated with a group of waves or wave packet rather than a single wave.
- Hence, in this context, de-Broglie waves are represented by a wave packet and hence group velocity is associated with them.
- Let there be a combination of two waves represented as :

$$y_1 = a \sin(\omega_1 t - k_1 x)$$

$$y_2 = a \sin(\omega_2 t - k_2 x)$$

On superimposing

$$y = y_1 + y_2$$

$$= a[\sin(\omega_1 t - k_1 x) + \sin(\omega_2 t - k_2 x)]$$

$$= 2a \cos\left[\frac{(\omega_1 - \omega_2)\,t}{2} - \frac{(k_1 - k_2)\,x}{2}\right] \sin(\omega t - kx)$$

Where,
$$w = \frac{\omega_1 + \omega_2}{2}, \quad k = \frac{k_1 + k_2}{2}$$

and let,
$$\Delta w = \omega_1 - \omega_2, \quad \Delta k = k_1 - k_2$$

$$\therefore \quad y = 2a \cos\left[\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2}\right] \sin(\omega t - kx)$$

Clearly, the equation has two components -

1. A wave of frequency $\omega$ and propagation constant k.

$\therefore$ Its velocity $\frac{\omega}{k} = u$ = phase velocity

2. A wave with frequency $\frac{\Delta\omega}{2}$, propagation constant $\frac{\Delta k}{2}$

$\therefore$ Its velocity $\frac{\Delta\omega}{\Delta k} = G$

- This velocity is the velocity of envelope of the group of waves hence it is called as group velocity.

### 1.4.4 Relation between Group Velocity and Particle Velocity

Let us start with a material particle with rest mass $m_0$. At velocity v, consider its mass to be m.

$$\therefore \quad E = mc^2 = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

Momentum

$$P = mv = \frac{m_0 v^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

and frequency associated is

$$\nu = \frac{E}{h} = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} \cdot \frac{1}{h}$$

$$\omega = 2\pi\nu = \frac{2\pi m_0 c^2}{h \times \sqrt{1-\frac{v^2}{c^2}}}$$

$$\therefore \frac{d\omega}{dv} = \frac{2\pi m_0 v}{h\left[1-\left(\frac{v}{c}\right)^2\right]^{3/2}}$$

de-Broglie wavelength

$$\lambda = \frac{h}{P} = \frac{h\left[1-\left(\frac{v}{c}\right)^2\right]^{1/2}}{m_0 v}$$

Hence, propagation constant

$$k = \frac{2\pi}{\lambda} = \frac{2\pi m_0 v}{h\left[1-\left(\frac{v}{c}\right)^2\right]^{1/2}}$$

$$\therefore \frac{dk}{dv} = \frac{2\pi m_0 v}{h\left[1-\left(\frac{v}{c}\right)^2\right]^{3/2}}$$

Since the group velocity

$$G = \frac{d\omega}{dk} = \frac{d\omega}{dv} \cdot \frac{dv}{dk} = \frac{\frac{d\omega}{dv}}{\frac{dk}{du}}$$

$$= \frac{\dfrac{2\pi m_0 v}{h\left[1-\left(\frac{v}{c}\right)^2\right]^{3/2}}}{\dfrac{2\pi m_0 v}{h\left[1-\left(\frac{v}{c}\right)^2\right]^{3/2}}} = v$$

$$\therefore G = v = \text{particle velocity.}$$

$\therefore$ The wave group associated with the moving particles travels with the same velocity as the particle.

## 1.5 Wave Function and Physical Interpretation of Wave Function

**MU- Dec. 17, May 18**

**Q. What is the significance of wave function ? (Dec. 17, May 18, 2 Marks)**

In section 1.3, we have seen characteristics of matter waves. It is described that it is a new category of waves. Let us understand it in more detail.

- In classical mechanics, the square of wave amplitude associated with electromagnetic radiation is interpreted as a measure of radiation intensity. One can extend the concept to matter waves associated with electron or any particle. If we consider wave function $\psi$ associated with a system of electrons then $|\psi|^2 d\tau$ is regarded as a measure of density of electrons. $\tau$ is a volume inside which an electron is known to be present, but where exactly the electron is situated inside $\tau$ is not known.
- In this situation, if we take $\psi$ as wave function then $|\psi|^2 d\tau$ provides the probability of finding the electron in certain volume $d\tau$ of $\tau$. Means $|\psi|^2$ is called the probability function. This interpretation was given by Max Born.
- Since electron must be somewhere inside the volume $\tau$.

$$\int |\psi|^2 d\tau = 1 \qquad ...(1.5.1)$$

- It is important to know that wave function $\psi$ has no direct physical significance but $|\psi|^2$ has.
- In quantum mechanics, it is postulated that the state of a system is completely characterized by a wave function.
- The wave functions are usually complex.

$$\text{i.e. } \psi = A + iB \qquad ...(1.5.2)$$

where, A and B are real functions.

- The integral of the wave function over entire space in the box must be equal to unity because, there is only one particle and at any given time it is present somewhere inside the box only. Therefore,

$$\int_{-\infty}^{\infty} |\psi|^2 dV = 1 \qquad ...(1.5.3)$$

A wave function which obeys Equation (1.5.3) is said to be normalized.

**Ex. 1.5.1 : Calculate the de Broglie wavelength associated with an $\alpha$-particle accelerated by a potential difference of 200 V. (Mass of $\alpha$ – particle is $6.68 \times 10^{-27}$ kg). MU - Dec. 16, May 19, 8 Marks**

**Soln. :**

Here

$$m = 6.68 \times 10^{-27}$$

$$V = 200 \text{ Volts}$$

$$\text{Charge } q = 2e$$

$$\therefore \text{Formula } \lambda = \frac{h}{\sqrt{2mqV}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.68 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times 200}}$$

$$= \mathbf{7.17 \times 10^{-13}\ m}$$ ...Ans.

**Ex. 1.5.2 :** Calculate the wavelength of de-Broglie waves associated with mass 1 kg moving with a speed of $10^3$ m/sec. Comment on your answer.

**Soln. :**

**Given :**

$$m = 1 \text{ kg}$$

$$v = 1 \times 10^3 \text{ m/sec}$$

$$\therefore \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1 \times 1 \times 10^3}$$

$$= \mathbf{6.63 \times 10^{-37}\ m}$$ ...Ans.

This wavelength is too small to have any practical significance.

**Ex. 1.5.3 :** A bullet of mass 40 g and an electron, both travel at the velocity of 1100 m/sec. What wavelengths can be associated with them? Why is the wave nature of the bullet not revealed through diffraction effect ?

**MU - May 13, 5 Marks**

**Soln. :**

Given :

$$\text{Mass of bullet} = 40 \times 10^{-3} \text{ kg}$$

$$\text{Velocity of bullet} = 1100 \text{ m/sec}$$

$$\text{Mass of electron} = 9.1 \times 10^{-31} \text{ kg}$$

$\therefore$ Let wavelength associated with bullet be $\lambda_B$

$$\therefore \quad \lambda_B = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{40 \times 10^{-3} \times 1100} = 1.5 \times 10^{-35} \text{ m}$$

Let wavelength associated with an electron be $\lambda_e$

$$\therefore \quad \lambda_e = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1100} = 6.62 \times 10^{-7}$$

$$m = \mathbf{6620\ Å}$$ ...Ans.

Since the wavelength associated with a bullet is too small, it cannot be measured with the help of diffraction effect, as it requires an obstacle of the dimensions of the order of wavelength used.

**Ex. 1.5.4 :** Electrons accelerated through 100 V are reflected from a crystal. What is the glancing angle at which the first reflection occurs? Lattice spacing = 2.15 Å.

**MU - Dec. 15, 3 Marks**

**Soln. :**

$$V = 100 \text{ Volt}$$

$\therefore$ Wavelength associated with electron is

$$\therefore \lambda = \frac{12.26}{\sqrt{V}} = \frac{12.26}{\sqrt{100}} = 1.226 \text{ Å}$$

$\therefore$ Now, using Bragg's law for first order,

$$\therefore \lambda = 2d \sin \theta$$

$$1.226 \text{ Å} = 2 \times 2.15 \text{Å} \times \sin \theta$$

$$\therefore \sin \theta = 0.2851$$

$$\therefore \theta = \mathbf{16° \, 33'}$$

**...Ans.**

**Ex. 1.5.5 :** Find the energy of the neutron in units of electron-volt whose de-Broglie wavelength is 1Å.

**Given :** $m_n = 1.674 \times 10^{-27}$ kg and $h = 6.62 \times 10^{-34}$ J.sec. **MU - Dec. 13, 5 Marks**

**Soln. :**

**Given :**

$$m_n = 1.674 \times 10^{-27} \text{ kg}$$

$$h = 6.62 \times 10^{-34} \text{ J.sec}$$

$$\lambda = 1 \text{ Å}$$

**To find :** Energy of neutron (in eV)

$$\text{Formula } \lambda = \frac{h}{\sqrt{2mE}}$$

$$\therefore E = \frac{h^2}{2m\lambda^2}$$

$$= \frac{(6.62 \times 10^{-34})^2}{2 \times 1.674 \times 10^{-27} \times (1 \times 10^{-10})^2}$$

$$E = 1.3 \times 10^{-20} \text{ J}$$

$$= \mathbf{0.081 \text{ eV}}$$

**...Ans.**

**Ex. 1.5.6 :** An electron is accelerated through 1000 volts and is reflected from a crystal. The first order reflection occurs when glancing angle is 70°. Calculate the interplanar spacing of a crystal. **MU - May 15, 5 Marks**

**Soln. :**

$$V = 1000 \text{ volt}$$

**(1) Using formula :**

$$\lambda = \frac{12.26}{\sqrt{v}} = 0.388 \text{ A°}$$

**(2) Using Bragg's law :**

$$n\lambda = 2 \, d \sin \theta$$

$$d = \frac{n\lambda}{2 \times \sin \theta} = \frac{1 \times 0.388}{2 \times \sin 70}$$

$$\mathbf{d = 0.2065 \text{ A°}}$$

**...Ans.**

**Ex. 1.5.7 :** Calculate the kinetic energy of an electron whose de-Broglie wavelength is 5000 Å.

**Soln. :**

**Given :**

$$m_e = 9.108 \times 10^{-31} \text{ kg}$$
$$h = 6.625 \times 10^{-34} \text{ J.sec.}$$
$$n = 1,$$
$$\lambda = 5000 \text{ Å}$$

**To find :** K.E. = ?

Using formula,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\therefore E = \frac{h^2}{2m\lambda^2}$$

$$= \frac{(6.626 \times 10^{-34})^2}{2 \times 9.108 \times 10^{-31} (5000 \times 10^{-10})^2}$$

$$= 9.6378 \times 10^{-25} \text{ J} = \mathbf{6.023 \times 10^{-6} \text{ eV}}$$ **...Ans.**

**Ex. 1.5.8 :** What is the wavelength of a beam of neutron having

(1) an energy of 0.025 eV?

(2) an electron and photon each have a wavelength of 2Å, what are their momenta and energies?

Mass of neutron = $1.676 \times 10^{-27}$ kg

Mass of electron = $9.1 \times 10^{-31}$ kg

Planck's constant, h=$6.625 \times 10^{-34}$ J.sec **MU - Dec. 14, May 17, 5 Marks**

**Soln. :**

**Given :**

$$m_n = 1.676 \times 10^{-27} \text{ kg}$$
$$m_e = 9.1 \times 10^{-31} \text{ kg}$$
$$h = 6.625 \times 10^{-34} \text{ J.sec.}$$

**Part 1 :**

**Given :**

$$\text{Energy} = 0.025 \text{ eV} = 0.25 \times 1.6 \times 10^{-19} \text{ J}$$

**To find :** Neutron's wavelength

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$= \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 1.676 \times 10^{-27} \times 0.025 \times 1.6 \times 10^{-19}}}$$

$$= 1.709 \times 10^{-10} \text{ m}$$

$$\lambda = 1.709 \text{ Å}$$ **...Ans.**

**Part 2 :**

For an electron with wavelength 2Å

$$E = \frac{h^2}{\lambda^2} \times \frac{1}{2m} \qquad \text{(using formula } E = \frac{p^2}{2m}\text{)}$$

$$= \frac{(6.625 \times 10^{-34})^2}{(2 \times 10^{-10})^2 \times 2 \times 9.1 \times 10^{-31}}$$

$$= 6.028 \times 10^{-18}\ J = 37.68\ eV \qquad \text{...Ans.}$$

Momentum of electron

$$= \frac{h}{\lambda}$$

$$= \frac{6.625 \times 10^{-34}}{2 \times 10^{-10}}$$

$$= 3.3125 \times 10^{-24}\ \frac{kg.m}{sec}$$

For a photon :

$$\text{Energy} = h\nu$$

$$= \frac{hc}{\lambda} = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{2 \times 10^{-10}}$$

$$= 9.9375 \times 10^{-16}\ J \qquad \text{...Ans.}$$

$$\text{Momentum} = \frac{h}{\lambda} = 3.3125 \times 10^{-24}\ \frac{kg.m}{sec} \qquad \text{...Ans.}$$

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**Ex. 1.5.9 :** If the particles listed below all have the same energy, which has the shortest wavelength - electron, α-particle, neutron or proton?

**Soln. :** Particles with same energy α-particle; neutron; proton

$$\text{as } \lambda \propto \frac{1}{m}$$

α-particle has the highest mass among all others. It will have the shortest wavelength.

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**Ex. 1.5.10 :** Calculate the energies in eV of an electron and a proton whose de-Broglie wavelength is 1Å.

**Soln. :** Energy for electron

$$E = \frac{h^2}{\lambda^2 \times 2m}$$

$$= \frac{(6.625 \times 10^{-34})^2}{(1 \times 10^{-10})^2 \times 2 \times 9.1 \times 10^{-31}}$$

$$= 2.41 \times 10^{-17}\ J = 150\ eV \qquad \text{...Ans.}$$

Energy of proton

$$= \frac{(6.625 \times 10^{-34})^2}{(1 \times 10^{-10})^2 \times 2 \times 1.66 \times 10^{-27}} = 1.32 \times 10^{-20}\ J$$

$$= 0.08\ eV \qquad \text{...Ans.}$$

**Ex. 1.5.11 :** Compare de Broglie wavelength associated with following particles :

(i) Mass of 50 microgram and travelling with the velocity of 100 cm/sec.

(ii) Mass of $9.1 \times 10^{-31}$ kg and travelling with velocity of $3 \times 10^{6}$ m/sec. Comment on which is measurable.

**Soln. :**

i) $m = 50 \times 10^{-6}$ gm $= 50 \times 10^{-9}$ kg, $v = 1$ m/sec

$$\therefore \lambda = \frac{h}{mv} = \frac{6.625 \times 10^{-34}}{50 \times 10^{-9} \times 1}$$

$$= 1.325 \times 10^{-26} \text{ m} \qquad \textbf{...Ans.}$$

$$\lambda = 1.325 \times 10^{-16} \text{Å}$$

ii)

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$v = 3 \times 10^{6} \text{ m/sec}$$

$$\therefore \lambda = \frac{h}{mv} = \frac{6.625 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{6}}$$

$$= 2.426 \times 10^{-10} \text{ m} \qquad \textbf{...Ans.}$$

$$\lambda = \mathbf{2.40\ Å}$$

$\therefore$ Wavelength of electron is measurable.

**Ex. 1.5.12 :** Calculate the velocity and de-Broglie wavelength of $\alpha$-particle of energy 1 keV. Given : Mass of $\alpha$-particle $= 6.68 \times 10^{-27}$ kg.

**MU - May 14, 5 Marks**

**Soln. :**

$$E = 1\text{ke V} = 1 \times 10^{3} \times 1.6 \times 10^{-19} \text{ J}$$

$$\lambda = \frac{h}{\sqrt{2\,mE}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 6.68 \times 10^{-27} \times 1 \times 10^{3} \times 1.6 \times 10^{-19}}}$$

$$= \mathbf{4.534 \times 10^{-13}\ m} \qquad \textbf{...Ans.}$$

$$\text{Now, } \lambda = \frac{h}{mv}$$

$\therefore$ Velocity,

$$v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{6.68 \times 10^{-27} \times 4.534 \times 10^{-13}}$$

$$= \mathbf{218.9 \times 10^{3}\ m/sec} \qquad \textbf{...Ans.}$$

**Ex. 1.5.13 :** An electron and a photon each have a wavelength of 2Å. What are their momenta and energies?

**Soln. :**

Mass of electron $m_e = 9.1 \times 10^{-31}$ kg

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$\lambda = \frac{h}{p}$$

$$p = \frac{6.63 \times 10^{-34}}{2 \times 10^{-10}}$$

$$= 3.315 \times 10^{-24} \frac{kg.m}{sec}$$ ...Ans.

$$\text{Energy} = \frac{p^2}{2m} = 6.038 \times 10^{-18} \text{ J}$$ **...Ans.**

For photon :

Momentum,

$$p = \frac{h}{\lambda}$$

$$p = \frac{6.63 \times 10^{-34}}{2 \times 10^{-10}}$$

$$= 3.315 \times 10^{-24} \frac{kg.m}{sec}$$ **...Ans.**

$$\text{Energy} = \frac{hc}{\lambda}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-10}}$$

$$= \mathbf{9.945 \times 10^{-16} \text{ J}}$$ **...Ans.**

---

**Ex. 1.5.14 :** Compare the energy of a photon with that of a neutron when both are associated with wavelength of 1Å. (Mass of neutron = $1.678 \times 10^{-27}$ kg) Data: Wavelength of photon $\lambda_1$ = 1Å = $1 \times 10^{-10}$ m, wavelength of neutron $\lambda_2$ = 1Å

**To find :** $\frac{E_1}{E_2} = \frac{\text{Energy of photon}}{\text{Energy of neutron}}$

**Soln. :** For photon

$$\therefore \quad E_1 = h\nu = \frac{hc}{\lambda_1}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1 \times 10^{-10}}$$

$$= 1.989 \times 10^{-15} \text{ J}$$

$$= 1.24 \times 10^4 \text{ e V}$$

For neutron,

$$\lambda_2 = \frac{h}{\sqrt{2 m E_2}}$$

$$\therefore \quad E_2 = \frac{h^2}{2 m \lambda_2^2}$$

$$= \frac{(6.634 \times 10^{-34})^2}{2 \times 1.678 \times 10^{-27} \times (1 \times 10^{-10})^2}$$

$$= 1.31 \times 10^{-20} \text{ J} = 0.082 \text{ eV}$$

$$\therefore \frac{E_1}{E_2} = \frac{1.24 \times 10^4}{0.082} = \mathbf{1.5 \times 10^5}$$ **...Ans.**

**Ex. 1.5.15 :** Calculate the de-Broglie wavelength of proton with a velocity equal to $\frac{1}{20}$ th velocity of light. (mass of proton = $1.6 \times 10^{-27}$ kg) **MU - Dec. 12, 5 Marks**

**Soln. :**

$$h = 6.63 \times 10^{-34}$$

$$m_p = 1.6 \times 10^{-27} \text{ kg}$$

$$v = \frac{1}{20} \times 3 \times 10^{8} \text{ m/sec}$$

Using formula,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{1.6 \times 10^{-27} \times \frac{3 \times 10^{8}}{20}}$$

$$= 2.763 \times 10^{-14} \text{ m}$$

$$= 2.76 \times 10^{-4} \text{ Å}$$ **...Ans.**

**Ex. 1.5.16 :** What is the wavelength of a beam of neutron having energy 0.025 eV and mass $1.676 \times 10^{-27}$ kg?

**Soln. :**

$$E = 0.025 \text{ eV} = 0.025 \times 1.6 \times 10^{-19}$$

$$= 0.004 \times 10^{-18} \text{ Joules.}$$

As, $$E = \frac{1}{2} mv^2 = 0.004 \times 10^{-18}$$

$$\therefore v = \sqrt{\frac{2 \times 0.004 \times 10^{-18}}{1.676 \times 10^{-27}}} = 2.1847 \times 10^{3} \text{ m/sec}$$

Now, $$\lambda = \frac{h}{mv} = \frac{6.634 \times 10^{-34}}{1.676 \times 10^{-27} \times 2.1847 \times 10^{3}}$$

$$= \mathbf{1.811 \text{ Å}}$$ **... Ans.**

**Ex. 1.5.17 :** A fast moving neutron has de-Broglie wavelength $2 \times 10^{-12}$ m associated with it. Find the following :

(a) Kinetic energy.

(b) Phase and group velocity.

(Ignore relativity effect)

Data: $\lambda_{neutron} = \lambda_n = 2 \times 10^{-12}$ m

$m_{neutron} = m_n = 1.675 \times 10^{-27}$ kg

**Soln. :**

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{2 \times 10^{-12}} = 3.32 \times 10^{-22}$$

$$\therefore KE = \frac{p^2}{2m} = \frac{(3.32 \times 10^{-22})^2}{2 \times 1.675 \times 10^{-27}}$$

$$= 3.280 \times 10^{-17} \text{ J}$$ **...Ans.**

Since the velocity of a particle is same as the group velocity, the group velocity can be given as

$$v_{group} = \frac{p}{m} = \frac{3.32 \times 10^{-22}}{1.675 \times 10^{-27}}$$

$$= 1.98 \times 10^{5} \quad \text{....Ans.}$$

Now using the relation

$$(v_{group})(v_{phase}) = c^2$$

$$\therefore v_{phase} = \frac{c^2}{v_{group}} = \frac{(3 \times 10^8)^2}{1.98 \times 10^5}$$

$$= \mathbf{4.55 \times 10^{11} \frac{m}{sec}} \quad \text{....Ans.}$$

**Ex. 1.5.18 :** A particle has mass 1.157 × $10^{-30}$ kg and kinetic energy 80 eV. Find the de-Broglie wavelength, group velocity and phase velocity of de Broglie wave.

**Soln. :**

$$\lambda = \frac{h}{\sqrt{2\,mE}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.157 \times 10^{-30} \times 80 \times 1.6 \times 10^{-19}}}$$

$$= 1.218 \times 10^{-10}\ m \quad \text{...Ans.}$$

Now,

$$v_{group} = \frac{p}{m} = \frac{h}{\lambda \cdot m} = \frac{6.63 \times 10^{-34}}{1.218 \times 10^{-10} \times 1.157 \times 10^{-30}}$$

$$v_{group} = 4.7 \times 10^{6}\ m/sec \quad \text{...Ans.}$$

$$v_{phase} = \frac{c^2}{v_{group}} = \frac{(3 \times 10^8)^2}{4.7 \times 10^6}$$

$$= \mathbf{1.913 \times 10^{10}\ m/sec} \quad \text{...Ans.}$$

**Ex. 1.5.19 :** Find the de Broglie wavelength of (i) an electron accelerated through a potential difference of 182 volts and (ii) 1 kg object moving with a speed of 1m/s. Comparing the results, explain why the wave nature of matter is not apparent in daily observations? **MU - May 16, 5 Marks**

**Soln. :**

**(i) For electron, its wavelength is given by**

$$\lambda = \frac{12.26}{\sqrt{V}}\,\text{Å} = \frac{12.26}{\sqrt{182}} = 0.9087\ \text{Å} \quad \text{...Ans.}$$

**(ii) For object of 1 kg mass,**

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{1 \times 10^{-3} \times 1} = \mathbf{6.63 \times 10^{-31}\ m} \quad \text{...Ans.}$$

This wavelength is too small, hence wave nature is not apparent in daily life. However, through diffraction it is possible to detect wave nature of electron whereas for particle of 1 kg mass it is not possible.

**Ex. 1.5.20 : Calculate the frequency and wavelength of photon whose energy is 75eV.** **MU – Dec. 17, 3 Marks**

**Soln. :**

**Given :**

$$\text{Energy} = 75 \text{ eV} = 75 \times 1.6 \times 10^{-19} \text{ J} = 1.2 \times 10^{-17} \text{ J}$$

Now for Photon

$$E = h\nu$$

$$\therefore 1.2 \times 10^{-17} = 6.6.2 \times 10^{-34} \times \nu$$

$$\nu = 1.81 \times 10^{16}$$

$$\text{Now } \nu = \frac{C}{\lambda}$$

$$\therefore \lambda = \frac{C}{\nu} = \frac{3 \times 10^8}{1.81 \times 10^{16}}$$

$$= \mathbf{1.657 \times 10^{-8} \text{ m}}$$ ...Ans.

## 1.6 Uncertainty Principle

**MU - Dec. 13, Dec. 18**

**Q. Explain Heisenberg's uncertainty principle. (Dec. 13, Dec.18, 5 Marks)**

- In **classical mechanics**, we describe a **particle occupying a definite place** in space and having a **specific momentum**. At a given instant of time, one can evaluate position and momentum simultaneously. This may appear valid only under the boundary of classical mechanics. When one steps out of classical mechanics and enters **wave mechanics** (available at atomic scale) this **idea is no more valid.**
- Heisenberg's uncertainty principle states that **quantum mechanics does not simultaneously permit the determination of position and momentum** of a particle accurately. Any effort made to make the measurement of position of the particle - such as an electron - very accurately, results in a large uncertainty in the measurement of momentum and vice versa.
- Mathematically,

$$\Delta x \cdot \Delta p_x \geq \hbar \quad \text{or}$$

$$\Delta x \cdot \Delta p_x \geq \frac{h}{2\pi} \qquad ...(1.6.1)$$

where,

$\Delta x$ = Uncertainty in the measured values of position.

$\Delta p_x$ = Uncertainty in the measured values of momentum.

**Important characteristics**

- The uncertainty principle is a direct consequence of the wave nature of particles.
- The limit on measurement is independent of measuring procedure or sophistication of instrument.
- It is a fundamental property of nature.

- It is applicable to conjugate variables like energy and time as

$$\Delta E \cdot \Delta t \geq \hbar$$

Also to angle and angular momentum,

$$\Delta L \cdot \Delta \theta \geq \hbar$$

**Physical significance of Heisenberg's uncertainty principle**

- The physical significance of the above argument is that one should not think of the exact position or an accurate value for momentum of a particle. Instead, one should think of the probability of finding the particle at a certain position, or of the probable value for the momentum of the particle.
- The estimation of such probabilities is made by means of certain mathematical functions, named probability density functions in quantum mechanics.

## 1.7 Electron Diffraction Experiment

**MU - May 12, Dec. 15**

**Q.** With single slit electron diffraction, prove Heisenberg's uncertainty principle. **(May 12, Dec.15, 5 Marks)**

- A monochromatic light while passing through a single slit produces deviation of light, which is called diffraction.

The intensity profile is as shown in Fig. 1.7.1.

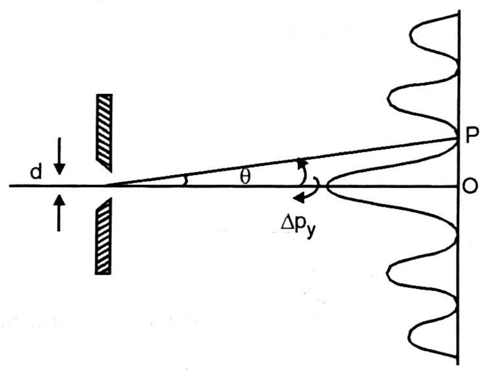



**Fig. 1.7.1 : Diffraction at single slit**

- Consider point P, which represents a first order minimum. Condition for first order minimum is

$$\sin \theta = \frac{\lambda}{d} \quad \text{...(1.7.1)}$$

- Consider a photon passing through the narrow slit. The photon has its momentum well defined before passing through the slit. After it passes through the slit, its position is known to within an uncertainty which is equal to the slit width.

$$\therefore \Delta y = d \quad \text{...(1.7.2)}$$

- After passing through the slit, photon has its value uncertain as it makes an angle θ with horizontal (Fig. 1.7.1).

$\therefore$ The uncertainty in its y component of momentum is at least as large as p sin θ.

$$\therefore \Delta p_y \geq p\frac{\lambda}{d} \quad \text{...(1.7.3)}$$

$$\text{Now, } p = \frac{h}{\lambda} \text{ (de-Broglie hypothesis)}$$

$$\therefore \Delta p_y \geq \frac{h}{\lambda} \cdot \frac{\lambda}{d} \geq \frac{h}{d} \geq \frac{h}{\Delta y}$$

$$\therefore \Delta y \cdot \Delta p_y \geq h$$

This is in good agreement with uncertainty principle.

## 1.8 Gamma-ray Microscope Experiments

- This experiment is an imaginary one. Here we try to measure the position of a very small particle, say, an electron using a microscope. Since the particle is very small, let us first consider the resolving power of microscope which defines the ability to see the two particles distinctly.

  The limit of resolution is given by,

$$\Delta x = \frac{\lambda}{2 \sin \theta} \qquad (1.8.1)$$

where $\theta$ is semi vertical angle shown in the Fig. 1.8.1.

- For small measure of $\Delta x$, corresponding $\lambda$ must be of the same order. Hence, we take gamma rays into consideration.
- If a photon of energy $h\nu$ hits the electron and the deflected photon enters the field of microscope, the momentum along x-axis will become uncertain.

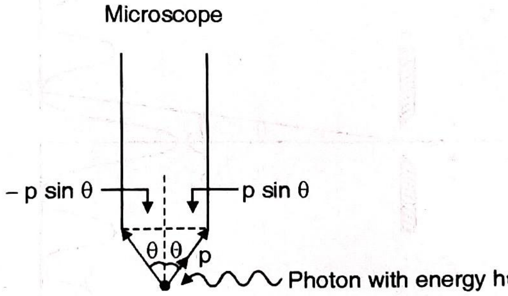



**Fig. 1.8.1 : Gamma-ray experiment**

Since the momentum is conserved, the uncertainty in the momentum along x-axis is given by,

$$\Delta p_x = p \sin \theta - (-p \sin \theta) = 2 p \sin\theta$$

Using de-Broglie hypothesis,

$$\lambda = \frac{h}{p}$$

$$\Delta p_x = 2\left(\frac{h}{\lambda}\right) \sin \theta \qquad ...(1.8.2)$$

$\therefore$ The product of uncertainty in the case of simultaneous measurement of position and momentum is given by,

$$\Delta x \cdot \Delta p_x = \left(\frac{\lambda}{2 \sin \theta}\right) \cdot \left(2\left(\frac{h}{\lambda}\right) \sin \theta\right) = h$$

$$\text{or } \Delta x \cdot \Delta p_x \geq h$$

This is the uncertainty principle.

## 1.9 Applications of Uncertainty Principle

### (1) Absence of electron in nucleus

**MU - Dec. 12, May 14, Dec. 14, May 15, Dec. 17, May 18**

**Q.** Using Heisenberg's uncertainty principle, show that electrons cannot exist within the nucleus.

**(Dec. 12, May 14, Dec. 14, May 15, 5 Marks)**

**Q.** With Heisenberg's uncertainty principle prove that electron cannot survive in nucleus. **(Dec. 17, May 18, 4 Marks)**

- The radius of nucleus is about $5 \times 10^{-15}$ m. Using uncertainty principle we can place lower limit on energy of an electron, if it is to be a part of the nucleus.

$$\text{Let } \Delta x = 2 \times 5 \times 10^{-15} \text{ m i.e. diameter.}$$

$\therefore$ Using

$$\Delta x \cdot \Delta p \geq \frac{h}{2\pi}$$

$$\therefore \Delta p \geq \frac{h}{2\pi \cdot \Delta x}$$

$$\geq \frac{6.6 \times 10^{-34}}{2 \times 2\pi \times 5 \times 10^{-15}}$$

$$\geq 1 \times 10^{-20} \frac{\text{kg·m}}{\text{sec}}$$

- This is the uncertainty in momentum of nuclear electron. The momentum p should be at least of this order,

$$\therefore \text{K.E.} = pc \geq 1 \times 10^{-20} \times 3 \times 10^{8}$$

$$\geq 3 \times 10^{-12} \text{ J}$$

$$\geq \frac{3 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ eV}$$

$$\geq 19 \text{ MeV}$$

- The KE of electron must be greater than 19 MeV if it is to remain inside the nucleus. This is practically not possible (atom becomes unstable). Hence one can say that an electron cannot survive inside the nucleus.

**Ex. 1.9.1 :** An electron has a speed of 400 m /sec with uncertainty of 0.01%. Find the accuracy in its position.

**MU - May 13, Dec.17, May 18, Dec.18, May 19, 5 Marks**

**Soln. :**

Momentum of electron = p = mv

$$= 9.11 \times 10^{-31} \times 400$$

$$= 3.644 \times 10^{-28} \frac{\text{kg} \cdot \text{m}}{\text{sec}}$$

$$\therefore \Delta p = m \Delta V = mv \cdot \frac{\Delta v}{v}$$

$$= 3.644 \times 10^{-28} \times \frac{0.01}{100}$$

$$= 3.644 \times 10^{-32}$$

∴ Using Heisenberg's uncertainty formula

$$\Delta x \cdot \Delta p_x \geq \hbar$$

$$\therefore \Delta x \geq \frac{h}{2\pi} \cdot \frac{1}{\Delta p_x}$$

$$\geq \frac{6.63 \times 10^{-34}}{2\,\pi \times 3.644 \times 10^{-32}}$$

$$\geq 2.895 \times 10^{-3} \text{ m}$$ ...Ans.

**Ex. 1.9.2 :** An electron is confined in a box of length $10^{-8}$ m. Calculate minimum uncertainty in its velocity.

**MU - May 17, 3 Marks**

**Soln. :**

$$\text{As } \Delta x \cdot \Delta p_x \simeq \hbar$$

$$\therefore \Delta x_{max} \Delta p_{xmin} \simeq \hbar$$

Now, box has length $10^{-8}$ m.

$$\therefore \Delta x_{max} = 10^{-8} \text{ m}$$

$$\therefore \Delta p_{xmin} = \frac{h}{2\pi} \cdot \frac{1}{10^{-8}} = \frac{6.63 \times 10^{-34}}{2\pi \times 10^{-8}}$$

$$= 1.055 \times 10^{-26}$$

$$\therefore \Delta p_{xmin} = m \, \Delta v_{xmin}$$

$$\therefore \Delta v_{xmin} = \frac{\Delta p_{xmin}}{m}$$

$$= \frac{1.055 \times 10^{-26}}{9.1 \times 10^{-31}}$$

$$= 11.595 \times 10^{3} \frac{\text{m}}{\text{sec}}$$ ...Ans.

**Ex. 1.9.3 :** The speed of an electron is measured to within an uncertainty of $2 \times 10^4$ m /sec. What is the minimum space required by the electron to be confined in an atom?

**Soln. :**

**Data :**

$$\Delta v_x = 2 \times 10^4 \text{ m/sec}$$

**To find :** $\Delta x$

Using equation of uncertainty,

$$\Delta x \cdot \Delta p_x \geq \frac{h}{2\pi}$$

$$\therefore \Delta x \geq \frac{h}{2\pi \cdot \Delta p_x} \geq \frac{h}{2\pi} \cdot \frac{1}{m \Delta v_x}$$

$$\geq \frac{6.63 \times 10^{-34}}{2\pi \times (9.1 \times 10^{-31} \times 2 \times 10^{4})}$$

$$\geq 0.579 \times 10^{-10} \text{ m}$$ ...Ans.

**Ex. 1.9.4 :** The inherent uncertainty in the measurement of time spent by a nucleus in the excited state is found to be $1.4 \times 10^{-10}$ sec. Estimate the uncertainty that results in its energy in the excited state.

**Data :** $\Delta t = 1.4 \times 10^{-10}$ sec

**To find :** $\Delta E$

**Soln. :** Using equation

$$\Delta E \cdot \Delta t \geq \hbar$$

$$\Delta E \cdot \Delta t \geq \hbar$$

$$\therefore \Delta E \geq \frac{h}{2\pi} \cdot \frac{1}{\Delta t}$$

$$\geq \frac{6.63 \times 10^{-34}}{2\pi \times 1.4 \times 10^{-10}}$$

$$\geq 7.537 \times 10^{-25} \text{ J}$$

$$\geq \frac{7.537 \times 10^{-25}}{1.6 \times 10^{-19}} \text{ eV}$$

$$\geq 4.71 \times 10^{-6} \text{ eV}$$ ...Ans.

**Ex. 1.9.5 :** Find the minimum energy of neutron confined to nucleus of size of the order of $10^{-14}$ m.

**Given :** Mass of neutron = $1.675 \times 10^{-27}$ kg **MU - Dec 16, 5 Marks**

**Soln. :** Using uncertainty principle

$$\therefore \Delta x \cdot \Delta p_x \geq h$$

$$\Delta x \cdot \Delta p_x \geq \frac{h}{2\pi}$$

$$\therefore \Delta p_x \geq \frac{h}{2\pi} \cdot \frac{1}{\Delta x} = \frac{h}{2\pi} \times \frac{1}{1 \times 10^{-14}}$$

$$= 1.056 \times 10^{-30}$$

$$KE = \frac{p^2}{2m} = \frac{(1.056 \times 10^{-30})^2}{2 \times 1.675 \times 10^{-27}}$$

$$= 3.328 \times 10^{-34}$$ ...Ans.

## 1.10 One Dimensional Time-dependent Schrödinger Equation

**MU - Dec. 12, Dec. 13, May 14, Dec. 14, May 16, Dec. 16, May 17, Dec. 17**

**Q.** Derive one-dimensional time-dependent Schrödinger equation for matter waves.

**(Dec.12, Dec.13, May 14, Dec.14, May 16, May 17, Dec. 17, 5 Marks)**

**Q.** Write Schrodinger's time-dependent and time-independent wave equations of matter waves in one dimension and state physical significance of these equations. **(Dec.16, 3 Marks)**

- Based on de Broglie's idea of matter waves, Schrödinger developed a mathematical theory which plays the same role as Newton's laws in classical mechanics.
- Using de Broglie's hypothesis for a particle of mass m, moving with a velocity v, associated with it is a wave of wavelength.

$$\lambda = \frac{h}{p}$$

- The wave equation for a de-Broglie wave can be written as

$$\psi = A e^{-i\omega t} \quad \text{...(1.10.1)}$$

$$\text{where, } A = \text{Amplitude}$$

$$\omega = \text{Angular frequency}$$

- For a one dimensional case, the classical wave equation has the following form

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2} \quad \text{...(1.10.2)}$$

- Where, y is the displacement and v is the velocity of the wave. The solution is,

$$y(x, t) = A e^{-i(kx - \omega t)} \quad \text{...(1.10.3)}$$

$$\text{where, } \omega = 2\pi\nu$$

- By analogy we can write the wave equation for de-Broglie wave for the motion of a free particle as

$$\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\frac{d^2\psi}{dt^2} \quad \text{...(1.10.4)}$$

$$\text{where, } \omega = vk$$

$$v = \text{Phase velocity}$$

- The solution of the above equation is,

$$\psi(x, t) = A e^{-i(Et - px)/\hbar} \quad \text{...(1.10.5)}$$

- There we have replaced ω and k of Equation (1.10.3) with E and p using Einstein and de-Broglie relations.
- Differentiating with respect to t,

$$\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial t}[A e^{-i(Et - px)/\hbar}]$$

$$= A e^{-i(Et - px)/\hbar} \cdot \frac{-iE}{\hbar}$$

$$\frac{\partial\psi}{\partial t} = \frac{-i}{\hbar} E\psi \quad \text{...(1.10.6)}$$

- Similarly taking double differentiation of Equation (1.10.5) with respect to x

$$\frac{\partial^2\psi}{\partial x^2} = \frac{-p^2}{\hbar^2}\psi \quad \text{...(1.10.7)}$$

- In classical mechanics we have energy of a free particle described as

$$E = \frac{p^2}{2m}$$

- Let there be a field where particle is present. Depending on its position in the field, the particle will possess certain potential energy V.

$\therefore$ Total energy of particle E is given by

$$E = \frac{p^2}{2m} + V$$

$$\text{or} \quad \frac{p^2}{2m} = E - V \qquad \text{...(1.10.8)}$$

$$\therefore \frac{p^2}{2m}\psi = E\psi - V\psi \qquad \text{...(1.10.9)}$$

But from Equation (1.10.6),

$$E\psi = \frac{-\hbar}{i}\frac{\partial\psi}{\partial t}$$

And from equation (1.10.7)

$$p^2\psi = h^2\frac{\partial^2\psi}{\partial x^2}$$

$\therefore$ Equation (1.10.9) becomes,

$$\frac{-h^2}{2m}\frac{\partial^2\psi}{\partial x^2} = \frac{-\hbar}{i}\frac{\partial\psi}{\partial t} - V\psi$$

$$\therefore \quad \frac{-h^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = i\hbar\frac{\partial\psi}{\partial t} \qquad \text{...(1.10.10)}$$

Equation (1.10.10) is one-dimensional time-dependent Schrödinger equation.

## 1.11 Reduction of Schrodinger Equation to Time-independent Form

**MU - May 15, Dec. 15, Dec. 16, Dec. 17, May 18**

**Q. Derive Schrodinger's time-independent wave equation. (May 15, Dec.15, Dec.16, Dec.17, May 18, 5 Marks)**

- A further simplified form of Schrödinger equation is time-independent form, where the field due to which the potential energy is considered to be stationary is independent of time. Or, one can say it is a function of position only. Hence one can separate the variables of Schrödinger wave equation as

$$\psi(x, t) = \psi(x)\,\phi(t) \qquad \text{...(1.11.1)}$$

$\therefore$ Equation (1.11.1) can be modified as

$$\frac{-h^2}{2m}\phi(t)\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\,\psi(x)\,\phi(t) = i\hbar\psi(x)\frac{\partial\phi(t)}{\partial t}$$

Divide both sides by $\psi(x)\,\phi(t)$

$$\therefore \quad \frac{-\hbar^2}{2m}\frac{1}{\psi}\frac{\partial^2\psi}{\partial x^2} + V(x) = i\hbar\,\frac{1}{\phi}\frac{\partial\phi}{\partial t} \qquad \text{...(1.11.2)}$$

- Careful observation of Equation (1.11.2) shows that we have separated the Schrödinger equation such that on LHS is function of x only and RHS is function of t only. Since Equation (1.11.2) is valid for any x and t, both the sides must be equal to a constant say energy E.

$$\therefore \frac{-\hbar^2}{2m}\cdot\frac{1}{\psi}\cdot\frac{\partial^2\psi}{\partial x^2} + V(x) = E$$

Or
$$\frac{-\hbar^2\partial^2\psi}{2m\,\partial x^2} + V\psi = \psi \qquad ...(1.11.3)$$

Equation (1.11.3) represents time-independent Schrödinger equation.

## 1.12 Eigen Functions and Eigen Values

- We have discussed about Schrödinger equation in section 1.11. On solving Schrödinger equations we get $\psi$. Since it is a second order differential equation, there are several solutions available. All of them may not be useful to us. Obviously, one would like to know how to get a meaningful and acceptable solution.
- Following are the postulates in quantum mechanics which must be satisfied :
  1. $\psi$ is single valued everywhere.
  2. $\psi$ and its first derivatives with respect to its variable are continuous everywhere.
  3. $\psi$ is finite everywhere.
- The solutions, which are acceptable are called **Eigen functions**. These Eigen functions are used in Schrödinger equations to find energy. These energy values are known as **Eigen values.**

## 1.13 Application of Schrödinger Equation to Free Particle and Particle in a Box

**MU : May 13, Dec. 15, Dec. 17, May 18**

**Q.** Show that the energy of an electron in the box varies as the square of natural numbers. **(May 13, Dec. 15, 5 Marks)**

**Q.** Derive the expression for energy Eigen values for free particle in one dimensional potential well. **(Dec.17, May 18, 3 Marks)**

### (a) Motion of free particle

- Here, free particle means that particle is not acted upon by any force. Hence, potential energy is zero and it is moving in positive x direction.

  ∴ Schrödinger equation in one dimension is

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}E\psi = 0$$

$$\therefore \frac{\partial^2\psi}{\partial x^2} + K^2\psi = 0 \qquad ...(1.13.1)$$

$$\text{here } K^2 = \frac{8\pi^2 mE}{h^2}$$

- As particle is considered to be free i.e. without boundary conditions on K, all values of K are allowed, and hence all values of energy are allowed.
- From expression of $K^2 = \frac{8\pi^2 mE}{h^2}$

$$\therefore K = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\text{Using } E = \frac{p^2}{2m}$$

$$\therefore \; K = \sqrt{\frac{2m}{\hbar^2} \cdot \frac{p^2}{2m}} = \frac{p}{\hbar} \simeq \frac{1}{\lambda}$$

Hence, K describes the wave properties of particle and according to uncertainty principle position becomes uncertain.

$$\text{As} \quad E = \frac{h^2}{8\pi^2 m} K^2$$

$$\therefore \; E \propto K^2$$

**(b) Particle in one dimensional potential well of infinite height (or particle in a box).**

- Suppose a particle of mass m is free to move in the x-direction only in the region from $x = 0$ to $x = a$ (Fig. 1.13.1).
- Outside this region the potential energy V is taken to be infinite, and within this region it is zero. A particle does not lose energy when it collides with walls, hence its energy remains constant.
- Outside the box $V = \infty$ and particle cannot have infinite energy, therefore it cannot exist outside the box.

$\therefore$ Schrodinger's equation is written as

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - \infty)\psi = 0 \qquad \ldots(1.13.2)$$

Inside the box, $V = 0$

$\therefore$ Schrodinger's equation is written as

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2} E\psi = 0 \qquad \ldots(1.13.3)$$

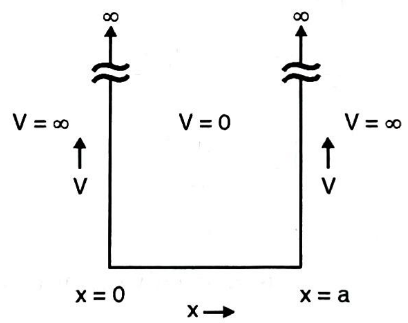



**Fig. 1.13.1 : One dimensional potential well of infinite height**

Equation (1.13.3) may be simplified as

$$\frac{d^2\psi}{dx^2} + K^2\psi = 0 \qquad \ldots(1.13.4)$$

Where, $$K^2 = \frac{8\pi^2 mE}{h^2}$$

or $$K^2 = \frac{2mE}{\hbar^2}$$

Solution of equation (1.13.4) is written as,

$$\psi = A \cos Kx + B \sin Kx \qquad \ldots(1.13.5)$$

$$\text{When } x = 0 \text{ at } \psi = 0, \text{ we get,}$$

$$0 = A\cos 0 + B\sin 0$$

$$\therefore A = 0 \qquad \text{(Since } \cos 0 = 1)$$

$$\text{When } x = a, \psi = 0$$

$$\therefore 0 = A\cos Ka + B\sin Ka$$

$$\text{But } A = 0 \qquad ...(1.13.6)$$

$$\therefore B\sin Ka = 0$$

Here B need not be zero

$$\therefore \sin Ka = 0 \text{ only when}$$

$$Ka = \frac{\sqrt{2mE}}{\hbar} a = n\pi \qquad \text{(where } n = 0, 1, 2, 3 \ldots\text{)} \ldots(1.13.7)$$

Where,

$$n = \text{quantum number}$$

$$\therefore \psi_n = B\sin\left(\frac{n\pi}{a}\right)x \qquad ...(1.13.8)$$

which represents the permitted solutions. In Equation (1.13.8), $n = 0$ is not acceptable because for $n = 0$, $\psi = 0$, means the electron is not present inside the box which is not true.

$$\text{as } K^2 = \frac{8\pi^2 mE}{h^2}$$

$$\text{and } K = \frac{n\pi}{a}$$

$$\therefore \frac{(n\pi)^2}{a^2} = \frac{8\pi^2 mE}{h^2}$$

$$\therefore E_n = \frac{n^2 h^2}{8ma^2} \qquad (n = 1, 2, 3, \ldots)$$

$$\therefore E_n \propto n^2$$

This shows that the energy of the particle can have only certain values which are Eigen values.

## 1.14 Wave Functions, Probability Density and Energy

**MU – Dec. 17, May 18**

> **Q.** What is the significance of wave function? Derive the expression for energy Eigen values for free particle in one-dimensional potential well. **(Dec. 17, May 18, 5 Marks)**

From Equation (1.13.5), wave function for a particle in a box with energy $E_n$ in given by,

$$\psi_n = B\sin kx = B\sin\sqrt{\frac{2mE}{\hbar^2}}\cdot x$$

$$= B\sin\frac{n\pi x}{a} \qquad \text{(From equation (1.13.8))}$$

$$\therefore \int_{-\infty}^{\infty} |\psi_n|^2\, dx = \int_0^a |\psi_n|^2\, dx$$

$$= B^2 \int_0^a \sin^2\left(\frac{n\pi x}{a}\right) dx$$

Using, $$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{B^2}{2}\int_0^a \left[1 - \cos 2\left(\frac{n\pi x}{a}\right)\right] dx$$

$$= \frac{B^2}{2}\left[\int_0^a dx - \int_0^a \cos\frac{2n\pi x}{a}\cdot dx\right] = \frac{B^2}{2}$$

$$= \frac{B^2}{2}\left[(a-0) - \frac{a}{2n\pi}\cdot(\sin 2n\pi - \sin 0)\right]$$

$$= \frac{B^2}{2}\left[a - \frac{a}{2n\pi}\cdot(0-0)\right]$$

$$= B^2\cdot\left(\frac{a}{2}\right) \quad \text{...(1.14.1)}$$

Rewriting Equation (1.14.1), for normalized wave function,

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1 \quad \text{...(1.14.2)}$$

$\therefore$ From Equations (1.14.1) and (1.14.2), wave functions are normalized if,

$$B = \sqrt{\frac{2}{a}} \quad \text{...(1.14.3)}$$

$$\therefore \psi_n = B \sin\frac{n\pi x}{a} \quad \text{is given by}$$

$$\psi_n = \sqrt{\frac{2}{a}}\cdot \sin\frac{n\pi x}{a} \quad (n = 1, 2, 3, \ldots.)$$

Since the **particle in a box** is a problem under quantum mechanical conditions, the prime questions to be considered are, the most probable location of the particle in the box and its energies. We can write Eigen functions $\psi_1$, $\psi_2$, $\psi_3$ .... for particle in a box by putting

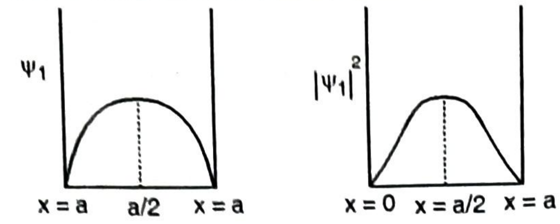
n = 1, 2, 3 .... respectively.

**Case - 1 :** For n = 1

$$\therefore \psi_1 = \sqrt{\frac{2}{a}}\cdot \sin\left(\frac{\pi}{a}\right)x$$

Here $\psi_1 = 0$, both x = 0 and x = a

But $\psi_1$ has maximum value for x = a/2 (for x = a/2, $\sin \pi/2 = 1$)




(a) (b)

Fig. 1.14.1

It can be seen from Fig. 1.14.1(b) that $|\psi_1|^2 = 0$ at x = 0 and x = a. It is maximum at x = (a/2).

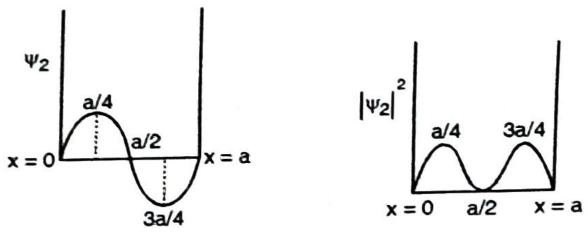
**Case - 2 :** For n = 2

$$\therefore \psi_2 = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{2\pi}{a}\right) x$$

Means $\psi_2 = 0$ at x = 0, a/2 and a and $\psi_2$ is maximum at x = a/4 and $\frac{3a}{4}$.

In Fig. 1.14.2(a), we have $\psi_2 \rightarrow x$.

In Fig. 1.14.2(b), we have $|\psi_2|^2 \rightarrow x$, which shows that particle cannot be observed either at the walls or at the center.



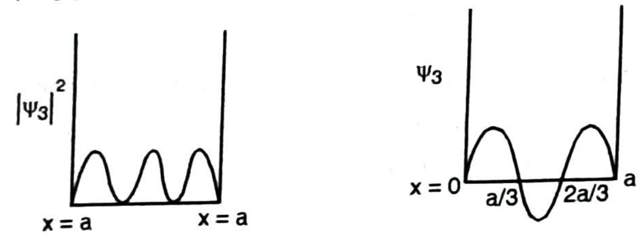

(a) (b)

Fig. 1.14.2

**Case - 3 :** For n = 3

$$\psi_3 = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{3\pi}{a}\right) x$$

For this case $\psi_3 \rightarrow a$ and $|\psi_3|^2 \rightarrow a$ are shown in Figs. 1.14.3(a) and (b) respectively.




(a) (b)

Fig. 1.14.3

## 1.15 Particle in a Finite Potential Well /Barrier and Tunnelling Effect

- A reader can understand this term in layman's language by just comparing a vehicle crossing the hill through a tunnel. It is not practical or may not be possible to take the vehicle all the way to the top of the hill merely to cross it. Tunnel may allow a vehicle without possessing the required energy to reach the top in order to cross it.

- Now we will take the case of classical physics. Let's consider a particle made to strike on a hard wall. It is not possible to find it on other side for obvious reasons. Quantum physics has a complete opposite solution to offer. Consider a quantum particle in the following case of potential barrier of height $V_o$ and thickness 'a'

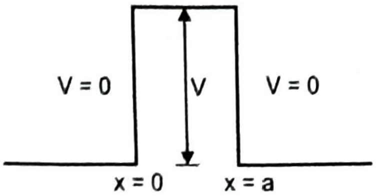



**Fig. 1.15.1**

- On LHS and RHS of this barrier, the potential energy of particle is zero (only KE). When a particle approaches this barrier from LHS with total energy E which is kinetic energy, if E < V, the particle will be reflected from the barrier. To be found inside the barrier the required condition is E ≥ V. Here classical physics and Quantum physics offer exactly opposite solutions. Quantum physics says for E < V, there exists a finite chance for the particle to be found not only in the barrier but on RHS of the barrier as well. We say that the particle has tunnelled through the potential barrier. Hence this phenomenon is called tunnelling. It has been proved that electron exhibits tunnelling and based on this we have certain components, tunnel divide, etc.

One-dimensional time independent form of Schrodinger equation along 'x' axis is given by

$$\frac{d^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0 \qquad \text{.... (1.15.1)}$$

For LHS of the barrier let's consider $\psi = \psi_1$ and it is known that $V = 0$

$$\therefore \frac{d^2\psi_1}{dx^2} + \frac{8\pi^2 m}{h^2} E\psi_1 = 0 \qquad \text{... (1.15.2)}$$

The same situation is available on RHS of potential barrier.
Let take $\psi = \psi_3$ there

$$\therefore \frac{d^2\psi_3}{dx^2} + \frac{8\pi^2 m}{h^2} E\psi_3 = 0 \qquad \text{... (1.15.3)}$$

For $E < V$ and $V = V_0$ take $\psi = \psi_2$

$$\therefore \quad \frac{d^2\psi_2}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V_0)\psi_2 = 0 \qquad \text{... (1.15.4)}$$

Close inspection of Equations (1.15.2) and (1.15.3) it is clear that they are of $x^2 + a^2 = 0$ type hence their solutions will be of $(x - ia)(x + ia) = 0$ i.e. complex in nature. Hence their solutions are given by

$$\psi_1 = Ae^{ikx} + Be^{-ikx} \qquad \text{... (1.15.5)}$$

and

$$\psi_3 = Ce^{ikx} + De^{-ikx} \qquad \text{... (1.15.6)}$$

Where

$$K = \frac{8\pi^2 mE}{h^2}$$

For Equation (1.15.4), for the condition E < V the equation is modified as

$$\frac{d^2\psi_2}{dx^2} - \frac{8\pi^2 m}{h^2}(V - E)\psi_2 = 0$$

$$\text{it is of } (x^2 - a^2) = 0 \text{ type}$$

$$\therefore x = a \text{ or } x - a$$

i.e. the solutions are of real type

hence $$\psi_2 = Le^{kx} + Me^{-kx} \quad \ldots (1.15.7)$$

- From Equation (1.15.5) it is clear that $Ae^{ikx}$ represents de-Broglie wave travelling in –x direction with amplitude A, and $Be^{-ikx}$ represents de-Broglie wave travelling in –x direction with amplitude B. Which further represents reflection. The same is true for Equation (1.15.6) only for the case x > a or RHS of potential barrier, there is no reflection hence D = 0.
- For any point inside the potential barrier. We have
  1) $Le^{kx}$ which shows the presence of wave in +x direction in perfect contradiction with classical physics.
  2) $Me^{-kx}$ representing exponentially decreasing wave.
- As a summary we can write that the wave function $\psi_1$ which was approaching potential barrier from left to right is successfully crossing the barrier with energy E < V, where V = height of potential barrier. Such penetration is called tunnel effect.

### 1.15.1 Tunnelling Effect Examples

1) α - **Decay :** When an α-particle is observed to be emitted from nucleus it is called α-decay. When α-decay takes place the energy associated with it is around 4 to 9 MeV. When such a decay takes place the forces in the nucleus set up potential barrier of height of the order of 30 MeV. The emission of α-particle or α-decay is possible only if tunneling is accepted. Means a particle with just around 4 to 9 MeV can cross the barrier height of 30 MeV.

2) **Tunnel Diode :** A special category diode which makes use of tunneling effect by virtue of heavily doped p-n junction with very small barrier around 100 A° wide has a specific 'Negative Resistance', exhibited on its I-V characteristics as shown below.

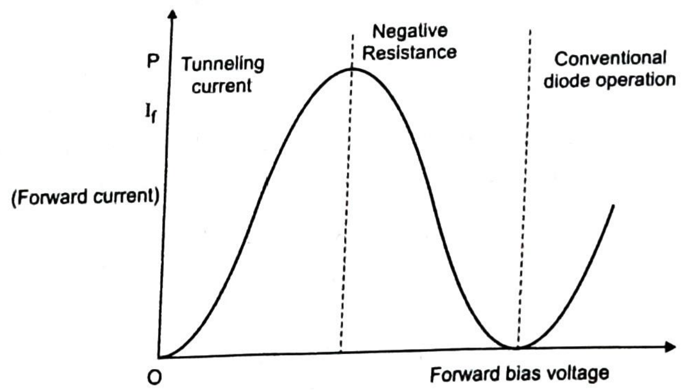



Fig. 1.15.2

Under forward biased operation, as voltage increases, electrons tunnel through the very narrow p-n junction barrier. As voltage increases further the current drops representing negative resistance. After that it enters into normal operation.

**3) Scanning Tunneling Microscope (STM) :**

- As the name suggests, through scanning, information of electronic configuration of surface atoms and the electron distribution around them is detected.
- A very sharp metal needle is brought very close to the surface to be imaged but not in touch with specimen.
- When a voltage is applied between tip of the needle and the specimen a tunneling current flows. Remember metal tip is not in contact with specimen hence it represents infinite resistance and results in current or does not permit any current. But it is the effect of tunneling which allows a small current.
- The small current is amplified and measured and based on this mapping of the surface is done.

**Ex. 1.15.1 :** An electron is bound in a one-dimensional potential well of width 2Å but of infinite height. Find its energy values in the ground state and in first two excited state.

**Data :** Width of potential well, $a = 2 \times 10^{-10}$ m

To find : $E_0$, $E_1$, $E_2$ **MU - Dec. 12, May 13, Dec. 13, May 14, Dec. 14, May 16, 3/5 Marks**

**Note :** A similar problem with a = 2.5 Å was asked in Dec. 12.

**Soln.** :Using equation for energy of the electron in one-dimensional potential well is given by

mass of electron (m) = $9.1 \times 10^{-31}$

$$E = \frac{n^2 h^2}{8m\, a^2}$$

For ground state n = 1 and $E_0$

$$\therefore \quad E_0 = \frac{1^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

$$= 1.5 \times 10^{-18} \text{ J} \qquad \div \text{ by } 1.63 \times 10^{-19}$$

$$= 9.43 \text{ e V}$$ ...Ans.

Similarly for first excited state, n = 2 and E = $E_1$

$$\therefore \quad E_1 = \frac{2^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

$$= 6 \times 10^{-18} \text{ J} = 37.5 \text{ eV}$$

Second excited state, n = 3 and E = $E_2$

$$\therefore \quad E_2 = \frac{3^2 (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$$

$$= 1.35 \times 10^{-17} \text{ J}$$

$$= 84.375 \text{ e}$$ ...Ans.

**Ex. 1.15.2 :** A particle is moving in one-dimensional potential well of infinite height and width 25Å. Calculate the probability of finding the particle with an interval of 5Å at a distance of a/2, a/3 and a where a is the width of the well assuming that particle is in least state of energy.

**Data :** Width of well = a = $25 \times 10^{-10}$ m, $\Delta x = 5 \times 10^{-10}$ m, n = 1 (Ground state)

**To find :**

$$P_1 = \text{Probability at } x = a/2$$

$$P_2 = \text{Probability at } x = a/3$$

$$P_3 = \text{Probability at } x = a$$

**Soln. :** Use formula

$$\psi_n = \sqrt{\frac{2}{a}} \cdot \sin\left(\frac{n\pi}{a}\right) x$$

We know that probability of finding a particle over an elementary distance $\Delta x$ is given by

$$P = \psi^2 \Delta x$$

$$\therefore P_1 = \left[\sqrt{\frac{2}{a}} \cdot \sin \frac{\pi}{a} \cdot \frac{a}{2}\right]^2 \cdot$$

$$\Delta x = 0.4 \quad \text{...Ans.}$$

Similarly,

$$P_2 = 0.15 \quad \text{...Ans.}$$

And,

$$P_3 = 0 \quad \text{...Ans.}$$

## 1.16 Introduction to Quantum Computing

- Quantum computing is the area of study focussed on developing computing methods based on the principle of quantum theory.
- The execution of computation is expected to be done by quantum computers, which are yet to be realized in practice.
- In classical computers, a bit is a fundamental unit of information represented as '0' or '1'.
- In quantum computing, the concept is fundamentally different. It stores information in a quantum system such as an atom or a photon. We can choose two electronic states of an atom or two different polarization orientations of light for the two states. But as per quantum mechanics, the atom apart from the two distinct states can also be prepared in a state which is said to be a coherent superposition of both the state to represent 'o' and 'i'. Since it follows quantum principles, it becomes a quantum system and is called a quantum bit or a qubit. It acts as fundamental unit of information in quantum computer.
- In conventional computers we have digital gates like AND, NOT, etc. Following symbol represents controlled NOT gate.

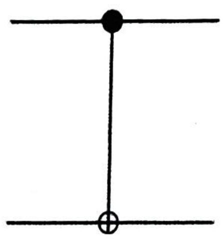


Fig.1.16.1

Similarly other gates can be created.

- A quicker computation and small sized computer can be obtained through quantum computing concept.

## A Quick Revision

- Louis de Broglie put forward the dual behaviour in terms of a hypothesis which states "If the radiation behaves as particle under certain circumstances and wave under other circumstances, then one can even expect that, entities which ordinarily behave as particles will exhibit properties attributed to only waves under appropriate circumstances.
- Electromagnetic waves always travel with a constant velocity c, whereas matter waves may travel with that phase velocity which depends on mass and velocity of particle.
- Based upon the result from de-Broglie hypothesis,

$$\lambda = \frac{12.26}{\sqrt{V}} \text{ Å}$$

  it was predicted that if a suitable voltage is considered then a wavelength of the order of few Å which is of the order of interplanar spacing in the crystal can be obtained.
- Experimental value which is obtained by considering electron as wave is verified with theoretical value. This confirms de Broglie's hypothesis.
- It is important to know that wave function $\psi$ has no direct physical significance but $|\psi|^2$ has.
- Heisenberg's uncertainty principle states that quantum mechanics does not simultaneously permit the determination of position and momentum of a particle accurately.
- The KE of electron must be greater than 19 MeV if it is to remain inside the nucleus. This is practically not possible (atom becomes unstable); hence one can say that electron cannot survive inside the nucleus.
- Based on de-Broglie's idea of matter waves, Schrödinger developed a mathematical theory which plays the same role as Newton's laws in classical mechanics.

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