CHAPTER 4

Unit IV

Interference in Thin Film

Syllabus :

(Prerequisites : Wavefront and Huygens' principle, reflection and refraction, interference by division of wavefront, Young's double slit experiment)

Interference by division of amplitude, interference in thin film of constant thickness due to reflected and transmitted light, origin of colours in thin film, wedge-shaped film, Newton's rings

Applications of interference - determination of thickness of very thin wire or foil, determination of refractive index of liquid, wavelength of incident light, radius of curvature of lens, testing of surface flatness, anti-reflecting films and highly reflecting film

Learning Objectives :

After reading this chapter, learner should be able to

- Define interference and its types
- Discuss interference in thin film and derive path difference formula for transmitted and reflected light
- Apply concept of interference to wedge-shaped film and Newton's rings
- Understand the applications of interference
- Understand anti-reflecting film

4.1 Interference in a Thin Parallel-sided Film

MU - Dec. 13, Dec. 14, May 15, Dec. 16, May 17, Dec. 17, May 18

Q. Derive the conditions for maxima and minima due to interference of light reflected from thin film of uniform thickness. **(Dec. 13, May 18, 7 Marks)**

Q. Explain the interference in thin parallel film and derive the expression for path difference between reflected rays. Hence obtain the conditions of maxima and minima for interference with monochromatic light. **(Dec. 14, 7 Marks)**

Q. Derive the condition for a thin transparent film of constant thickness to appear bright and dark when viewed in reflected light. **(May 15, 7 Marks)**

Q. Obtain expression for path difference between two reflected rays in thin transparent film of uniform thickness and write the conditions of maxima and minima. **(Dec. 16, 4 Marks)**

Q. Derive the conditions for maxima and minima due to interference of light transmitted from thin film of uniform thickness. **(May 17, 8 Marks)**

Q. Obtain the condition for maxima and minima of the light reflected from a thin transparent film of uniform thickness. Why is the visibility of the fringe much higher in the reflected system than in the transmitted system? **(Dec. 17, 8 Marks)**

- Consider a ray AB of monochromatic light of wavelength λ from an extended source incident at B, on the upper surface of a parallel sided thin film of thickness t and refractive index μ as shown in Fig. 4.1.1.
- Let the angle of incidence be i.
- At B, the beam is partly reflected along BR_1 and partly refracted at an angle r along BC.
- At C, it is again partly reflected along CD and partly refracted along CT_1. Similar partial reflections and refractions occur at points D, E, etc.
- Thus we get a set of parallel reflected rays BR_1, DR_2, etc. and a set of parallel transmitted rays CT_1, ET_2, etc.
- For a thin film, the waves travelling along BR_1 and DR_2 in the reflected system will overlap.
- These waves originate from the same incident wave AB and are hence coherent.
- Hence they will interfere constructively or destructively according to if the path difference between them is an integral multiple of λ or an odd multiple of $\frac{\lambda}{2}$.

Reflected system

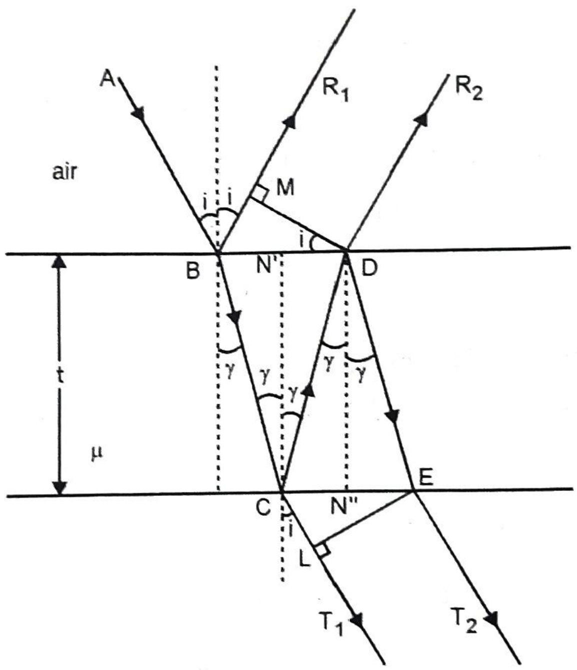


Fig. 4.1.1 : Interference in thin films

- To find the path difference between BR_1, and DR_2, draw DM perpendicular to BR_1. The paths travelled by the beams beyond DM are equal. Hence the optical path difference (optical path difference is obtained by multiplying geometrical path difference by its refractive index) between them is

$$\Delta = \text{Path BCD in film} - \text{Path BM in air}$$

$$= \mu\,(BC + CD) - BM$$

From Fig. 4.1.1 we have $BC = CD = \frac{t}{\cos r}$

$$\therefore \quad \mu (BC + CD) = \frac{2 \mu t}{\cos r}$$

$$\text{and} \quad BM = BD \cdot \sin i = 2 BN' \sin i \qquad (\because BD = 2 BN')$$

$$= 2 t \cdot \tan r \cdot \sin i \qquad (\because BN' = CN' \tan r = t \cdot \tan r)$$

$$\therefore \quad BM = 2t \frac{\sin r}{\cos r} \cdot \sin i$$

$$= \frac{2 \mu t}{\cos r} \cdot \sin^2 r \qquad \left(\because \frac{\sin i}{\sin r} = \mu\right)$$

$\therefore$ The optical path difference between the rays is

$$\Delta = \frac{2 \mu t}{\cos r} - \frac{2 \mu t}{\cos r} \sin^2 r$$

$$= \frac{2 \mu t}{\cos r} (1 - \sin^2 r)$$

$$\text{or} \quad \Delta = 2 \mu t \cos r \qquad \text{...(4.1.1)}$$

- The film is optically denser than the surrounding air medium. Hence the ray BR_1 originating by reflection at the denser medium suffers a phase change of π or a path change of $\frac{\lambda}{2}$ due to reflection at B. (No such change of phase occurs for ray DR_2 as it is a result of reflection at C)
- Hence the effective path difference between BR_1 and DR_2 is

$$2 \mu t \cos r + \frac{\lambda}{2}$$

Condition for maxima and minima in reflected light

(i) The two rays will interfere constructively if the path difference between them is an integral multiple of λ i.e.

$$2 \mu t \cos r + \frac{\lambda}{2} = n \lambda$$

or

$$2 \mu t \cos r = (2n - 1) \frac{\lambda}{2}, \quad \text{where } n = 1, 2, 3, 4, \ldots \text{(For maxima)} \ldots \text{(4.1.2)}$$

$$\text{or} \quad 2 \mu t \cos r = (2n + 1) \frac{\lambda}{2} \text{ when } n = 0, 1, 2, 3, \ldots$$

When this condition is satisfied the film will appear bright in the reflected system.

(ii) The two rays will interfere destructively if the path difference between them is an odd multiple of $\frac{\lambda}{2}$ i.e.

$$2 \mu t \cos r + \frac{\lambda}{2} = (2n + 1) \frac{\lambda}{2}$$

or

$$\mathbf{2 \mu t \cos r = n \lambda} \text{ (For minima)} \qquad \text{...(4.1.3)}$$

where n = 0, 1, 2, 3, ...

Transmitted system

- The transmitted rays CT_1 and ET_2 are also derived from the same incident ray AB and hence are coherent. (Fig. 4.1.1)
- When they interfere, they can give the interference pattern in transmitted system. To find the path difference between CT_1 and ET_2 we drop EL perpendicular to CT_1. See Fig. 4.1.1.

$$\therefore \text{Path difference } \Delta = \mu(CD + DE) - CL$$

- It can be calculated the same way as in reflected system and it is found that path difference

$$\Delta = 2\mu t \cos r$$

- But in this case no phase change occurs due to reflection at C and D. Hence the effective path difference between CT_1 and ET_2 is $2\mu t \cos r$.
- Hence the condition for constructive interference to take place in the transmitted system is that

$$\mathbf{2\mu t \cos r = n\lambda} \quad \text{(Maxima)} \qquad ...(4.1.4)$$

and the film appears bright in transmitted system.

- The condition for destructive interference is

$$\mathbf{2\mu t \cos r = (2n-1)\frac{\lambda}{2}} \quad \text{(Minima)} \qquad ...(4.1.5)$$

and the film appears dark in transmitted system.

- Comparison of equations (4.1.2), (4.1.3), (4.1.4) and (4.1.5) shows that the conditions of maxima and minima in reflected light are just opposite to those in transmitted light.
- Hence the film which appears bright in reflected light appears dark in transmitted light and vice versa.
- For the transmitted light, the intensity of maxima is about 100% and that of minima is about 85%. This results in poor contrast between bright and dark whereas in reflected light, minima has zero intensity and maxima is nearly 15% of incident energy. This results in good contrast.
- Hence visibility of fringe is much higher in reflected system.

Ex. 4.1.1 : A parallel beam of sodium light strikes a film of oil floating on water. When viewed at an angle 30° from the normal, eighth dark band is seen. Determine the thickness of the film. Refractive index of oil is 1.46 and λ = 5890 Å.

Soln. :

Given : $i = 30°$, $\mu = 1.46$, $n = 8$, $\lambda = 5890$ Å $= 5890 \times 10^{-8}$ cm

Formula : For dark band,

$$2\mu t \cos r = n\lambda \qquad n = 0, 1, 2, 3, ...$$

$$\mu = \frac{\sin i}{\sin r}$$

$$\therefore \quad \sin r = \frac{\sin i}{\mu} = \frac{\sin 30°}{1.46} = 0.3424$$

$$\therefore \quad \cos r = \sqrt{1-\sin^2 r} = 0.9395$$

$$\therefore \quad t = \frac{n\lambda}{2\mu \cdot \cos r} = \frac{8 \times 5890 \times 10^{-8}}{2 \times 1.46 \times 0.9395}$$

$$\therefore \quad t = 1.7176 \times 10^{-4} \text{ cm.}$$

....Ans.

4.2 Thin and Thick Films

MU - May 16

Q. Why does an excessively thin film appear to be perfectly dark when illuminated by white light? **(May 16, 3 Marks)**

The effective path difference between the interfering rays in reflected light is

$$2\,\mu\,t\cos r + \frac{\lambda}{2}$$

(i) If the film is excessively thin, then its thickness being very small as compared to the wavelength of light, the term $2\,\mu\,t\cos r$ can be neglected as compared to $\frac{\lambda}{2}$. Hence the effective path difference becomes $\frac{\lambda}{2}$ which is the condition for minima. Hence every wavelength in the incident light will be absent in reflected system, and the film will appear black in reflected light.

(ii) If the thickness of the film is large enough as compared to the wavelength of light, the path difference at any point of the film will be large. Under these conditions the same point will be a maximum for a large number of wavelengths, and the same point will be a minimum for another set of large number of wavelengths. The number of wavelengths sending maximum intensity at a point are almost equal to the number of wavelengths sending minimum intensity. Also, these wavelengths sending maximum and minimum intensity will be distributed equally over all colours in white light. Hence the **resultant effect at any point will be the sum of all colours i.e. white in thick film.**

(iii) Hence we define 'thin film' as the film whose thickness is of the order of wavelength of the light which is used to expose it.

4.3 Production of Colours in Thin Films

MU - Dec. 12, May 15, Dec. 16, May 17

Q. Explain why we see beautiful colours in thin film when it is exposed to sunlight. **(Dec. 12, Dec. 16, 3 Marks)**

Q. Comment on colours in a soap film in sunlight. **(May 15, 3 Marks)**

Q. What do you mean by thin film? Comment on the colours of thin film in sunlight. **(May 17, 3 Marks)**

When a thin film is exposed to white light from an extended source, it shows beautiful colours in the reflected system.

- Light is reflected from the top and bottom surfaces of a thin film, and the reflected rays interfere.
- The path difference between the interfering rays depends on the thickness of the film and the angle of refraction r and hence on the inclination of the incident ray.

- White light consists of a continuous range of wavelengths. At a particular point of the film and for a particular position of the eye (i.e. t and r constant) those wavelengths of incident light that satisfy the condition for constructive interference in the reflected system will be seen in reflected light.
- The colouration will vary with the thickness of the film and inclination of the rays (i.e. with the position of the eye with respect to the film). Hence if the same point of the film is observed with an eye in different positions or different points of the film are observed with the eye in the same position, a different set of colours is observed each time.

4.4 Necessity of the Extended Source

- In case of interference in thin films, a narrow source limits the visibility of the film.
- Consider a thin film and a narrow source of light S as shown in Fig. 4.4.1. The ray 1 produces interference fringes because rays 3 and 4 reach the eye. The ray 2 is incident on the film at some different angle and is reflected along 5 and 6. The rays 5 and 6 do not reach the eye.
- Similarly rays incident at different angles on the film do not reach the eye. Hence only the portion A of the film is visible and not the rest.

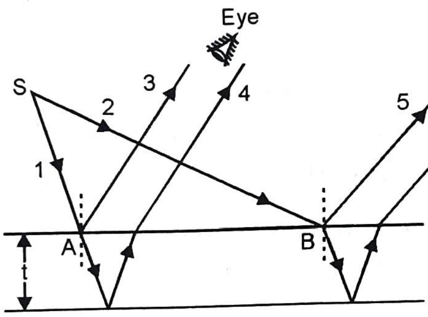


Fig. 4.4.1

- If an extended source of light is used as shown in Fig. 4.4.2, the ray 1, after reflection from the upper and lower surfaces of the film emerges as 3 and 4 which reach the eye.
- Also, the other rays incident at different angles on the film enter the eye and the field of view is large.

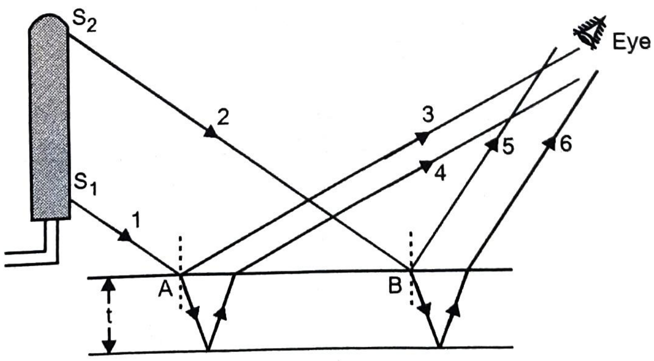


Fig. 4.4.2

- **Hence to observe interference phenomenon in thin films, a broad source of light is required.**

4.5 Film of Non-uniform Thickness (Wedge-shaped Film)

MU - Dec. 13, May 14, Dec. 15, Dec. 17, May 18

Q. Explain why an extensively thin film appears black in reflected light? **(Dec. 13, 3 Marks)**

Q. Obtain the conditions for maxima and minima due to interference in a wedge-shaped film observed in reflected light. **(May 14, 4 Marks)**

Q. Why are the fringes in wedge-shaped film straight? Derive the conditions of maxima and minima for interference in wedge-shaped films? **(Dec. 15, 7 Marks)**

Q. What will be the fringe pattern if wedge-shaped air film is illuminated with white light? **(Dec. 17, 3 Marks)**

Q. Explain how interference in wedge-shaped film is used to test optical flatness of given glass plate. **(May 18, 3 Marks)**

- Consider a film of non-uniform thickness as shown in Fig. 4.5.1. It is bound by two surfaces OX and OX′ inclined at an angle θ. The thickness of the film gradually increases from O to X.
- Such a film of non-uniform thickness is known as **wedge-shaped film**. The point O at which the thickness is zero is known as the edge of the wedge.
- The angle θ between the surfaces OX and OX′ is known as the angle of wedge. Let μ be the refractive index of the material of the film.
- Let a beam AB of monochromatic light of wavelength λ be incident at an angle i on the upper surface of the film. It is reflected along BR_1 and is transmitted along BC. At C also the beam suffers partial reflection, and refraction and finally we have the ray DR_2 in the reflected system.
- Thus as a result of partial reflection and refraction at the upper and lower surfaces of the film, we have two coherent rays BR_1 and DR_2 in the reflected system.

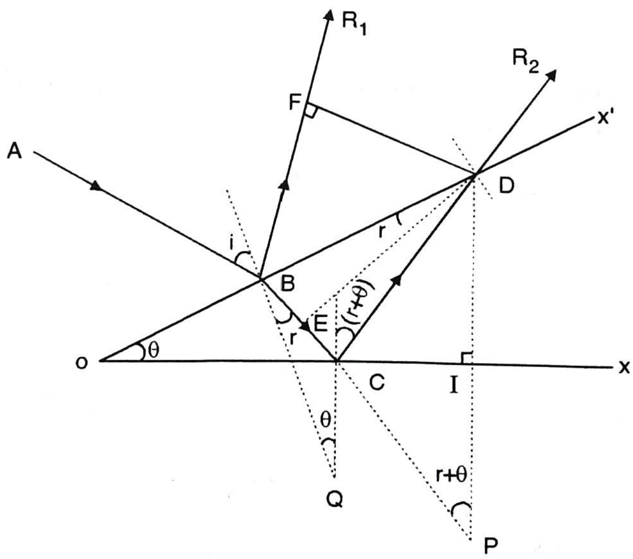


Fig. 4.5.1 : Interference in wedge-shaped film

- These rays are not parallel but diverge from each other. These rays interfere constructively or destructively according to whether the path difference between them satisfies the condition for constructive or destructive interference in the reflected system.
- To find the path difference between these two rays, draw DF perpendicular to BR_1.
- The optical path difference between the rays BR_1 and DR_2 is

$$\Delta = \mu\,(BC + CD) - BF \quad \text{...(4.5.1)}$$

- Draw a perpendicular to surface OX at point C.
- We have the perpendicular to OX′ at point B. Both these perpendiculars will meet at point Q as shown in Fig. 4.5.1.

$$\therefore \angle BQC = \angle XOX' = \theta$$

- Draw a perpendicular DE on BC from D.
- As θ is small enough, $BE = EC$
- Also from diagram $\angle QBE = r = \angle BDE$
- Draw a perpendicular from D on OX such that it intersects OX at I and BC produced at P. Also we get $CP = CD$.
- Equation (4.5.1) can be written as

$$\Delta = \mu\,(BC + CD) - BF$$

$$= \mu\,(BE + EC + CP) - BF$$

From diagram

$$\mu = \frac{\sin i}{\sin r} = \frac{BF}{BE} \quad \text{or} \quad BF = \mu\, BE$$

$$\therefore \Delta = \mu\,(BE + EC + CP) - \mu\, BE$$

$$= \mu\, EP \quad (\text{as } E - C - P)$$

- Now consider ΔDPC

$$\text{as } CP = CD, \angle CPD = r + \theta$$

and Δ DPE is a right angle triangle

$$\therefore \cos(r + \theta) = \frac{EP}{DP}$$

$$\therefore EP = DP \cos(r + \theta) = 2\,t \cos(r + \theta)$$

$$\text{Where, } DP = 2\,DI = 2t,\ t$$

$$= \text{thickness of film at point D}$$

$$\therefore \Delta = \mu\, EP$$

$$= 2\,\mu\, t \cos(r + \theta) \quad \text{...(4.5.2)}$$

- Due to reflection at B, an additional path change of $\frac{\lambda}{2}$ occurs for the ray BR_1. Hence the total path difference between the interfering rays is $2\,\mu t \cos(r+\theta) + \frac{\lambda}{2}$.
- Hence for **maxima**, we have the condition for constructive interference

$$2\,\mu t \cos(r+\theta) + \frac{\lambda}{2} = n\lambda$$

or

$$2\,\mu t \cos(r+\theta) = (2n-1)\frac{\lambda}{2} \qquad \text{...(4.5.3)}$$

$n = 1, 2, 3, 4 \ldots\ldots$

- For **minima**, we have the condition for destructive interference

$$2\,\mu t \cos(r+\theta) + \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$2\,\mu t \cos(r+\theta) = n\lambda \qquad \text{...(4.5.4)}$$

$$n = 0, 1, 2, 3, \ldots\ldots$$

- It is clear that for a maximum or a minimum of a particular order, t must remain constant. In case of the wedge-shaped film, t remains constant along lines parallel to the thin edge of the wedge.
- Hence the maxima and minima are straight lines parallel to the thin edge of the wedge.
- At the thin edge, t = 0 hence path difference between the rays is $\frac{\lambda}{2}$, a condition for darkness. Hence the edge of the film appears dark. It is called as **zero order band**. That is the reason an extensively thin film appears dark (black) in reflected light.
- Beyond the edge for a thickness t for which path difference is λ, we obtain the first bright band. As t increases to a value for which path difference is $\frac{3\lambda}{2}$, we obtain the first dark band.
- Thus as the thickness increases we obtain alternate bright and dark bands which are equally spaced and equal in width.

(i) For normal incidence and air film, r = 0 and $\mu = 1$

Total path difference $= 2t \cdot \cos\theta + \frac{\lambda}{2}$

$$2t\cos\theta = (2n-1)\frac{\lambda}{2}, \qquad \text{For maxima}$$

$$\text{and} \quad 2t\cos\theta = n\lambda \qquad \text{For minima}$$

(ii) For very small angle of the wedge,

As $\theta \to 0$, $\cos\theta \to 1$

$\therefore$ For constructive interference,

$$2t = (2n-1)\frac{\lambda}{2}$$

and for destructive interference

$$2t = n\lambda$$

∴ For constructive interference

$$t = (2n-1)\frac{\lambda}{4} \qquad n = 1, 2, 3, \ldots$$

$$t = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$$

i.e. every next bright fringe will occur for thickness interval of $\frac{\lambda}{2}$ each.

Similarly for destructive interference

$$t = \frac{n\lambda}{2} \qquad n = 0, 1, 2, 3, \ldots$$

$$t = 0, \frac{\lambda}{2}, \frac{2\lambda}{2}, \frac{3\lambda}{2}, \ldots$$

i.e. every next dark fringe will occur for thickness interval of $\frac{\lambda}{2}$ each.

4.6 Spacing between two Consecutive Bright Bands

- For the wedge-shaped film, we have for the n^{th} maximum,

$$2\mu t \cos(r+\theta) = (2n-1)\frac{\lambda}{2}$$

- For normal incidence and air film,

$$r = 0 \quad \text{and} \quad \mu = 1$$

$$2t\cos\theta = (2n-1)\frac{\lambda}{2} \qquad \ldots(4.6.1)$$

Where, t is the thickness corresponding to n^{th} bright band.

- Consider Fig. 4.6.1. The n^{th} bright band is produced at a distance x_n from the edge of the wedge.

$$t = x_n \cdot \tan\theta \qquad \ldots(4.6.2)$$

∴ Substituting for t in equation (4.6.1) we have,

$$2 \cdot x_n \tan\theta\cos\theta = (2n-1)\frac{\lambda}{2}$$

$$\text{or} \quad 2x_n \cdot \sin\theta = (2n-1)\frac{\lambda}{2} \qquad \ldots(4.6.3)$$

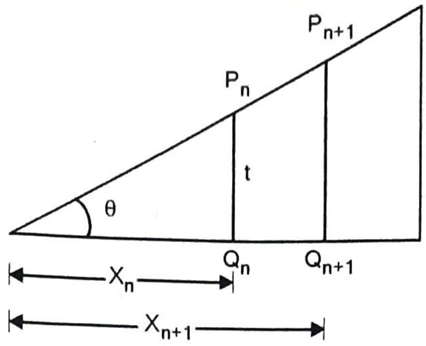


Fig. 4.6.1

- Let $(n + 1)^{th}$ maximum be obtained at a distance x_{n+1} from the thin edge. Then we have,

$$\left.\begin{aligned} 2\, x_{n+1} \sin\theta &= [\,2\,(n+1) - 1\,]\cdot\frac{\lambda}{2} \\ \text{or} \quad 2\, x_{n+1}\cdot\sin\theta &= (2n+1)\,\frac{\lambda}{2} \end{aligned}\right\} \quad \ldots(4.6.4)$$

Therefore from equations (4.6.3) and (4.6.4) we have,

$$2\,(x_{n+1} - x_n)\cdot\sin\theta = \lambda$$

- Therefore the spacing between two consecutive bright bands is,

$$\beta = x_{n+1} - x_n = \frac{\lambda}{2\sin\theta}$$

$\sin\theta \to \theta$ if θ is small and measured in radians. β is called fringe width.

$$\beta = \frac{\lambda}{2\theta}$$

- For a medium of refractive index μ, we have $\beta = \dfrac{\lambda}{2\mu\theta}$.

as μ, λ and θ are constant, one can say that fringe width in wedge-shaped film is constant. Or **wedge-shaped fringes are of constant thickness.**

Ex. 4.6.1 : Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.52. The fringe spacing is 1 mm and the wavelength of light is 5893 Å. Calculate the angle of wedge in seconds of an arc. **MU - Dec. 15, 3 Marks**

Soln. :

Given : $\mu = 1.52$, $\lambda = 5893 \times 10^{-8}$ cm, $\beta = 1$ mm $= 0.1$ cm.

Formula : The fringe spacing is

$$\beta = \frac{\lambda}{2\mu\theta}$$

The angle of wedge in radians is

$$\theta = \frac{\lambda}{2\mu\cdot\beta} = \frac{5893 \times 10^{-8}}{2 \times 1.52 \times 0.1} \text{ radians}$$

$$\therefore \quad \theta = \frac{5893 \times 10^{-8}}{2 \times 1.52 \times 0.1} \times \frac{180}{\pi} \times 3600 \quad \text{seconds}$$

$$\therefore \quad \theta = \textbf{40 seconds of an arc.} \quad \ldots\textbf{Ans.}$$

4.7 Newton's Rings

MU - May 12, May 15, Dec. 16, May 18

Q. Explain why Newton's rings are unequally spaced? **(May 12, 3 Marks)**

Q. Show that the diameter of Newton's n^{th} dark ring is proportional to square root of ring number. **(May 15, 5 Marks)**

Q. For Newton's ring, prove that diameter of n^{th} dark ring is directly proportional to the square root of natural number. **(Dec. 16, May 18, 5 Marks)**

- When a plano convex lens of large radius of curvature is placed on a plane glass plate, an air film is formed between the lower surface of the lens and upper surface of the plate.
- The thickness of the film gradually increases from the point of contact outwards.
- If monochromatic light is allowed to fall normally on this film, a system of alternate bright and dark concentric rings, with their centre dark, is formed in the air film.
- These rings were **first studied by Newton and are hence known as Newton's rings.** They can be seen through a low power microscope focussed on the film.

Formation of Newton's rings

- Newton's rings are formed as a result of interference between the waves reflected from the top and bottom surfaces of the air film formed between the lens and the plate.

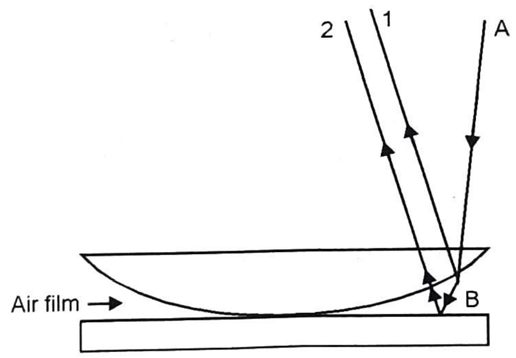


Fig. 4.7.1 : Formation of Newton's rings

- As shown in Fig. 4.7.1, let AB be a beam of monochromatic light of wavelength λ incident normally on the film. As a result of reflection at the top and bottom faces of the film, rays 1 and 2 are the coherent rays which interfere in the reflected system. For constructive interference, the path difference between them should be

$$2\mu t \cdot \cos(r+\theta) + \frac{\lambda}{2} = n\lambda$$

Where,

μ = R.I. of the film

t = Thickness at a point under consideration

r = Angle of refraction

θ = Angle of wedge

- The factor $\frac{\lambda}{2}$ accounts for a phase change of π on reflection at the lower surface of the film.
- Now for the air film $\mu = 1$

  For normal incidence $r = 0$
- For a lens of large radius of curvature, $\theta = 0$ practically. This is the reason why we prefer lens with large radius.

  $\therefore$ Path difference between rays 1 and 2 is $2t + \frac{\lambda}{2}$

At the point of contact of the lens and the plate, $t = 0$.

$$\therefore \qquad \text{Path difference} = \frac{\lambda}{2}$$

- This is the condition for minimum intensity. Hence the central spot is dark.

  For the n^{th} maximum, we have

$$2t + \frac{\lambda}{2} = n\lambda$$

- Thus a maximum of particular order n will occur for a constant value of t. In the air film, t remains constant along a circle and hence the maximum is in the form of a circle.
- Different maxima will occur for different values of 't'. Similarly, it can be shown that the minima are also circular in form.
- The minima occur for path difference $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$ and maxima occur for path difference $\lambda, 2\lambda, 3\lambda, \ldots$, the maxima and minima occur alternately.
- Each fringe is a locus of constant film thickness and hence these are fringes of constant thickness.

Diameter of dark and bright rings

- Let POQ be a plano convex lens placed on a plane glass plate AB. Let R be the radius of curvature of the lens surface in contact with the plate. Refer Fig. 4.7.2.

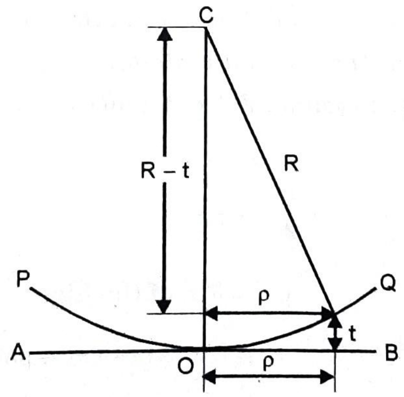


Fig. 4.7.2

Let ρ be the radius of a Newton's ring corresponding to the constant film thickness 't'. The path difference between the two interfering rays in the reflected system is $2\mu t \cos(r+\theta)+\frac{\lambda}{2}$

Where, λ = Wavelength of incident light.

$\mu = 1$ for air film.

$r = 0$ for normal incidence

$\theta = 0$ for large R.

$$\therefore \text{ Path difference } = 2t + \frac{\lambda}{2} \qquad ...(4.7.1)$$

From Fig. 4.7.2 we see that

$$R^2 = \rho^2 + (R-t)^2$$

$$\text{or} \quad \rho^2 = R^2 - (R-t)^2$$

$$\text{or} \quad \rho^2 = 2Rt - t$$

$t << R$ and hence we have

$$\rho^2 = 2Rt$$

$$\therefore \quad 2t = \frac{\rho^2}{R} \qquad ...(4.7.2)$$

$\therefore$ Path difference between the interfering rays is $\frac{\rho^2}{R} + \frac{\lambda}{2}$.

For dark rings

The condition to get dark rings is that

$$\text{Path difference} = \frac{\rho^2}{R} + \frac{\lambda}{2}$$

$$= (2n+1)\frac{\lambda}{2} \qquad (n = 1,2,3,....)$$

If D is the diameter of Newton's ring, then $\rho = \frac{D}{2}$

$$\therefore \quad \frac{D_n^2}{4R} = n\lambda$$

Where, D_n = Diameter of n^{th} dark ring.

$$\therefore \quad D_n^2 = 4nR\lambda$$

$$D_n = \sqrt{4nR\lambda} \qquad ...(4.7.3)$$

$$D_n \propto \sqrt{n}$$

– Hence **the diameter (and hence radius) of the dark ring is proportional to the square root of natural numbers.** One can say that rings are unequally spaced, because difference between two consecutive rings represents thickness of the ring.

For bright rings

The condition to get bright rings is that the path difference

$$\frac{\rho^2}{R} + \frac{\lambda}{2} = n\lambda$$

$$\text{Or, } \frac{\rho^2}{R} = (2n-1)\frac{\lambda}{2} \qquad \text{where } n = 1, 2, 3.....$$

$$\therefore \rho^2 = (2n-1)\frac{\lambda R}{2}$$

Putting $\rho = \frac{D}{2}$, we have,

$$\frac{D_n^2}{4} = (2n-1)\frac{\lambda R}{2}$$

Where, D_n = Diameter of n^{th} bright ring.

$$D_n^2 = 2\lambda R \cdot (2n-1)$$

$$D_n = \sqrt{(2n-1)}\sqrt{2\lambda R} \qquad ...(4.7.4)$$

$$D_n \propto \sqrt{2n-1} \qquad \text{where } n = 1, 2, 3, ...$$

– **Hence the diameter (and hence radius) of the bright ring is also proportional to the square root of odd natural numbers.**

Ex. 4.7.1 : In Newton's ring experiment the diameter of 4^{th} and 12^{th} dark rings are 0.400 cm and 0.700 cm respectively. Deduce the diameter of 20^{th} ring.

Soln. :

Given : $n = 4$, $n + p = 12$,

$D_4 = 0.400$ cm, $D_{12} = 0.700$ cm

$$D_n^2 = 4\,n\,R\lambda,\; D_{n+p}^2 = 4\,(n+p)\,R\lambda$$

$$D_{12}^2 - D_4^2 = 4 \times 8 \times \lambda \times R \qquad ...(1)$$

$$\text{Similarly, } D_{20}^2 - D_4^2 = 4 \times 16 \times \lambda \times R \qquad ...(2)$$

Divide equation (1) by equation (2), we get,

$$\frac{D_{12}^2 - D_4^2}{D_{20}^2 - D_4^2} = \frac{4 \times 8}{4 \times 16} = \frac{1}{2}$$

$$D_{20}^2 = 2\,D_{12}^2 - D_4^2 = 2\,(0.700)^2 - (0.400)^2$$

$$= 0.98 - 0.16 = 0.82$$

$$\text{Diameter of } 20^{th} \text{ ring} = \sqrt{0.82} = \mathbf{0.906\ cm} \qquad \text{...Ans.}$$

4.8 Newton's Rings by Transmitted Light

- Newton's rings can be seen in reflected light as well as in transmitted light. (As shown in Fig. 4.8.1). The ray 1′ is transmitted directly through the air film while the ray 2′ suffers two internal reflections (or a phase change of 2π) before emerging out.
- So, two interfering transmitted rays have a phase change of 2π or no phase difference. So, effective path difference

$$\Delta = 2\mu t, \qquad (\mu = 1 \text{ for air film})$$

- For bright fringe, we can write,

$$2\mu t = n\lambda, \qquad n = 0, 1, 2, 3, \ldots$$

$$\therefore \; 2t = n\lambda \qquad (\mu = 1 \text{ for air film}) \qquad \ldots(4.8.1)$$

And for dark fringe

$$2\mu t = (2n-1)\lambda/2, \qquad \text{where } n = 1, 2, 3, \ldots$$

$$\therefore \; 2t = (2n-1)\lambda/2 \qquad \ldots(4.8.2)$$

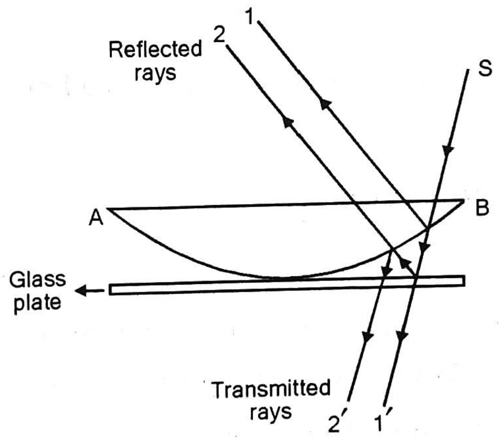


Fig. 4.8.1

- By using the property of circle, we have,

$$2t = \frac{\rho^2}{R} = \frac{D^2}{4R}, \text{ where D is the diameter of the ring.}$$

- For bright rings, we can write,

$$2t = \frac{D^2}{4R} = n\lambda \Rightarrow D_n^2 = 4nR\lambda$$

$$D_n = \sqrt{4nR\lambda} \Rightarrow D_n \propto \sqrt{n} \qquad \ldots(4.8.3)$$

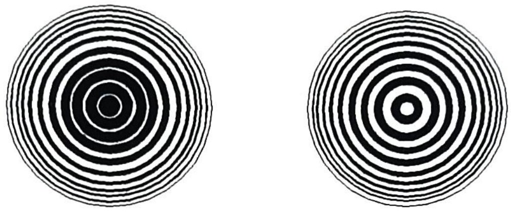
- For dark rings, we can write,

$$2t = \frac{D^2}{4R} = (2n - 1)\,\lambda/2$$

$$\therefore \quad D_n^2 = 2R\lambda\,(2n - 1)$$

$$D_n = \sqrt{2R\lambda\,(2n - 1)} \Rightarrow D_n \propto \sqrt{2n - 1} \qquad \text{...(4.8.4)}$$

- In transmitted system of light, the diameters of bright rings are proportional to the square roots of natural numbers, while the diameters of dark rings are proportional to odd natural numbers and the central ring is bright. The nature of Newton's ring in reflected and transmitted system is shown in Fig. 4.8.2.



(a) Newton's rings in reflected system (b) Newton's rings in transmitted system

Fig. 4.8.2

- **We can conclude that the system of rings in transmitted light is complementary to that seen in reflected light.**

4.9 Characteristics of Newton's Rings

MU - May 14, Dec. 16, May 17, Dec. 17

Q. Why are Newton's rings circular and the centre of interference pattern (reflected) dark? **(May 14, Dec. 17, 3 Marks)**

Q. Why are Newton's rings circular and fringes in wedge-shaped film straight? **(Dec. 16, May 17, 3 Marks)**

(1) Why is the centre of Newton's rings always dark?

At the point of contact the thickness of air film is zero. Consider the case of thin film at this point i.e. condition for bright spot is given by,

$$2\,\mu t \cos(r + \theta) = (2\,n + 1)\,\lambda/2 \qquad n = 0, 1, 2, \ldots\ldots$$

and condition for dark spot is given by,

$$2\,\mu t \cos(r + \theta) = n\lambda$$

For Newton's ring setup r = 0 (large radius of lens θ = 0), (for normal incidence r = 0)

$$\therefore \quad \cos(r + \theta) = \cos 0 = 1$$

$\therefore$ Condition for bright

$$2\,\mu t = (2n + 1)\,\lambda/2 \qquad n = 0, 1, 2, 3, \ldots\ldots$$

∴ Condition for dark

$$2 \mu t = n\lambda$$

At point of contact, t = 0 and one can see that condition for bright does not get satisfied. But condition for dark gets satisfied for t = 0 and n = 0.

(2) Why are Newton's rings always seen on the reflected side?

- Newton's rings can also be formed in transmitted system due to interference between the transmitted rays. The conditions for the bright and dark rings in the transmitted system are opposite to those in reflected system, and hence the rings have a bright centre.
- In the reflected system the intensity of the interference maxima is about 15% of the incident intensity and the intensity of minima is zero. Hence the contrast between bright and dark rings is good.
- In the transmitted system the intensity of maxima is about 100% and that of minima is about 85%. The contrast between bright and dark rings is not good. The visibility of fringes is much higher in reflected system than in transmitted system. **Hence Newton's rings are seen in reflected system only and not in transmitted system.**

(3) How does insertion of liquid affect ring structure?

- In case of reflected system, the central spot can be made bright if the space between lens surface and glass plate is filled with an oil having refractive index greater than that of lens and smaller than that of plate. Fig. 4.9.1.

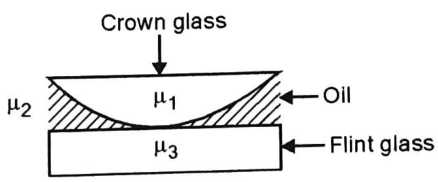


Fig. 4.9.1

- Newton's rings are formed with the lens of crown glass and glass plate of flint glass. The space with air film is then filled with an oil (like oil of sassafras) having intermediate refractive index.
- The reflections at the upper and lower surfaces of the film take place under similar conditions i.e. at the denser medium.
- Hence there is a phase change of π at both reflections. Hence the phase difference between the interfering rays at the point of contact is zero.
- This is the condition for constructive interference and hence a bright spot is produced at the centre.

(4) Why are Newton's rings circular and wedge-shaped films are straight?

- In both air-wedge film and Newton's ring experiments, each fringe is the locus of points of equal thickness of the film. In Newton's rings arrangement, the locus of points of equal thickness of air film lie on a circle with the point of contact of plano convex lens and the glass plate as centre. So, the fringes are circular in nature and concentric.

- For wedge-shaped air film, the locus of points of equal thickness are straight lines parallel to the edge of the wedge. So, fringes appear straight and parallel.

(5) What happens if plano convex lens is lifted up slowly?

- As the lens is lifted up slowly from the flat surface, the order of the ring at a given point decreases. The rings, therefore, come closer and closer until they can no longer be separately observed.
- Also it is important to know that initially as the lens is lifted up by a spacing of $\frac{\lambda}{4}$, total path difference for the ray will be $\frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$, and the center will become dark to bright or bright to dark.
- For a lift of every $\lambda/4$, the center will change from dark to bright and bright to dark. Also the order of the ring will reduce by one.

4.10 Newton's Ring with White Light

MU - May 13

Q. Suppose that in an experiment on Newton's rings, first light of red colour is used and then blue light, which set of rings would have larger diameter? Justify your answer with proper expression. **(May 13, 3 Marks)**

- If a monochromatic light source is used in Newton's ring experiment, alternate dark and bright rings are obtained. If we use white light instead of monochromatic light, a few mixed coloured rings around a black centre are observed, and beyond it a uniform illumination is obtained.
- White light consists of several colours of different wavelength. Diameters of the Newton's rings are proportional to the wavelength of the different colours.
- As we know that $\lambda_r > \lambda_v$, therefore the diameter of violet ring of the same order will be smallest and those for red ring will be the largest, and the diameters of other coloured rings shall occupy the intermediate positions.
- Due to overlapping of the rings of different colours over each other, only first few coloured rings will be clearly seen while other rings cannot be observed.

4.11 Newton's Rings with Bright Centre in Reflected System

- If Newton's rings are observed in reflected system, the central spot is dark. A liquid of refractive index μ_2 is poured between lens and glass plate.
- The refractive index of the lens, liquid and glass plate μ_1, μ_2, μ_3 are such that $\mu_1 < \mu_2 < \mu_3$, then the central spot is bright. This is possible if oil of sassafras is introduced between lens of crown glass and plate of flint glass.
- Then the reflection of two interfering rays from denser to rarer medium takes place under same condition. Hence the effective path difference between both the interfering rays at the point of contact becomes zero, which is the condition of maximum intensity.
- So, the centre of Newton's ring appears bright. [Sassafras oil ($\mu = 1.57$), crown glass lens ($\mu = 1.50$), flint glass plate ($\mu = 1.65$)].

4.12 Similarities and Dissimilarities between Newton's Rings and Wedge-shaped Films

Similarities

(1) Fringes are formed due to enclosed thin film.

(2) Both can be explained only by the concept of division of amplitude.

(3) Both can be used for determination of optical flatness.

Dissimilarities

Sr. No.	Newton's Rings	Wedge-shaped film
1.	We get alternate dark and bright rings	We get straight alternate dark and bright fringes
2.	Air gap has its thickness linearly increased	Air gap is non-linearly increased
3.	Popularly used for determination of unknown wavelength	Popularly used for determination of very small thickness
4.	As we go for higher orders thickness of rings reduces	Fringe width remains constant

4.13 Solved Problems

Problems on Thin Film

Ex. 4.13.1 : White light falls at an angle of 45° on a parallel soap film of refractive index 1.33. At what minimum thickness of the film will it appear bright yellow of wavelength 5896 Å in the reflected light?

Soln. :

Given : $i = 45°$, $\mu = 1.33$, $\lambda = 5896\ \text{Å} = 5896 \times 10^{-10}\ \text{m}\ \text{Å}$

Formula : $2\mu t \cos r = (2n-1)\frac{\lambda}{2}$, $n = 1, 2, 3,$ ... For bright fringe

For minimum thickness, n is minimum i.e. n = 1

$$\therefore \quad t = \frac{\lambda}{2 \times 2\mu \cos r}$$

$$= \frac{5896 \times 10^{-10}}{2 \times 2 \times 1.33 \times \cos r}$$

$$\text{Now} \quad \mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu} = \frac{\sin 45°}{1.33} = 0.5316$$

$$\therefore \quad \cos r = \sqrt{1 - \sin^2 r} = 0.8469$$

$$\therefore \quad t = \frac{5896 \times 10^{-10}}{2 \times 2 \times 1.33 \times 0.8469}$$

$$t = 1304.5\ \text{Å}$$ ....Ans..

Ex. 4.13.2 : A light of wavelength 5500 Å incident on thin transparent denser medium has refractive index 1.45. Determine the thickness of thin medium if the angle of refraction is 45° (Consider n = 1).

Soln. :

Here, it is not specified whether reflected side or transmitted side is to be considered.

Let us assume reflected side.

Also it is not specified whether dark fringe or bright fringe condition is satisfied.

Let us assume dark fringe.

$\therefore$ Condition for dark fringe on reflected side,

$$2\mu\, t \cos r = n\lambda$$

$$\therefore \quad t = \frac{n\lambda}{2\mu \cos r} = \frac{1 \times 5500 \times 10^{-8}}{2 \times 1.45 \times \cos 45}$$

$$\therefore \quad t = 2.68 \times 10^{-5}\ \text{cm}$$ ...Ans.

Ex. 4.13.3 : Light of wavelength 5880 Å is incident on a thin film of glass of $\mu = 1.5$ such that the angle of refraction in the plate is 60°. Calculate the smallest thickness of the plate which will make it dark by reflection.

MU - Dec. 14, 3 Marks

Soln. :

$\lambda = 5880 \times 10^{-8}$ cm, $\mu = 1.5$, $r = 60°$, $t = ?$

Condition for film to appear dark is,

$$2\ \mu t \cos r = n\lambda$$

The smallest thickness will be for n = 1.

$$2 \times 1.5 \times t \times \cos 60 = 1 \times 5880 \times 10^{-8}$$

$$t = \frac{5880 \times 10^{-8}}{2 \times 1.5 \times 0.5}$$

$$\therefore \quad t = 3920 \times 10^{-8}\ \text{cm}$$ ...Ans.

Ex. 4.13.4 : A soap film of refractive index 1.43 is illuminated by white light incident at an angle 30°. The refracted light is examined by a spectroscope in which dark band corresponding to wavelength 6×10^{-7} m is observed. Calculate the thickness of the film.

Soln. : For the thin film, refracted light forms transmitted system and the condition of minima in transmitted system is given by,

$$2\mu t \cos r = (2n - 1)\ \lambda/2$$

$$t = \frac{(2n-1)\ \lambda}{4\mu \cos r}$$

Given : $\mu = 1.43$, $\lambda = 6 \times 10^{-7}$ m, $i = 30°$

Using Snell's law,

$$\mu = \frac{\sin i}{\sin r}$$

$$\sin r = \frac{\sin i}{\mu}$$

$$\cos r = \sqrt{1 - \sin^2 r}$$

$$= \sqrt{1 - \frac{\sin^2 i}{\mu^2}}$$

$$= \sqrt{1 - \left(\frac{\sin 30°}{1.43}\right)^2}$$

$$= 0.9369$$

So, $$t = \frac{(2n-1) \times 6 \times 10^{-7}}{4 \times 1.43 \times 0.9369}$$

$$= (2n-1) \times 1.12 \times 10^{-7} \text{ m}$$

$$n = 1, 2, 3, \ldots$$

For minimum thickness of film, n = 1

Hence, $$t = 1.12 \times 10^{-7} \text{m}$$ ...Ans.

Ex. 4.13.5 : An oil drop of volume 0.2 c.c. is dropped on the surface of a tank of water of area 1 sq. meter. The film spreads uniformly over the surface and white light which is incident normally is observed through a spectrometer. The spectrum is seen to contain one dark band coinciding with wavelength 5.5×10^{-5} cm in air. Find the refractive index of oil.

Soln. :

The oil drop of volume 0.2 c.c. spreads uniformly over 1 m^2; hence the thickness of the film so formed is given by,

$$t = \frac{0.2}{(100)^2} = 2 \times 10^{-5} \text{ cm.}$$

The film appears dark by reflected light.

Hence, $$2\mu t \cos r = n\lambda$$

For normal incidence $r = 0$ $\therefore \cos r = 1$

$$n = 1 \text{ and } \lambda = 5.5 \times 10^{-5} \text{ cm.}$$

Refractive index of oil is

$$\mu = \frac{n\lambda}{2t \cos r} = \frac{1 \times 5.5 \times 10^{-5}}{2 \times 2 \times 10^{-5} \times 1}$$

$$\mu = \frac{5.5}{4} = \mathbf{1.375}$$ ....**Ans.**

Ex. 4.13.6 : A drop of oil of $\mu = 1.20$ floats on water with $\mu = 1.33$ surface and is observed from above by reflected light. The thickness of the oil drop at the edge is very small-almost zero and gradually increases towards the middle of the drop. Answer the following -

(i) Will the thinnest outer region of the drop correspond to a bright or a dark region? Give reason.

(ii) What will be the thickness of oil drop where wavelength of 4800 A° is intensified in reflected light for the third order?

Soln. :

(i) The thinnest region of the drop corresponds to a bright region because both the reflected rays, one from the boundary between air and oil and another from the boundary between oil and water are in phase. Hence the condition for brightness is $2\ \mu t \cos r = n\lambda$

At the edge, $t \approx 0$ and satisfies the condition for maximum intensity.

(ii) The condition for maximum intensity in reflected light is $2\ \mu t \cos r = n\lambda$

For normal incidence,

$$2\ \mu t = n\lambda$$

$$\therefore \quad t = \frac{n\lambda}{2\mu}$$

The thickness when n = 3,

$$\lambda = 4800\ \text{Å and } \mu = 1.2 \text{ is}$$

$$t = \frac{3 \times 4800\ \text{Å}}{2 \times 1.2} = \mathbf{6000\ Å}$$

...Ans.

Ex. 4.13.7 : Light of wavelength 5893 Å is reflected at nearly normal incidence from a soap film of refractive index 1.42. What is the least thickness of the film that will appear

(i) Black (ii) Bright

Soln. :

(i) In reflected system, the condition of dark is

$$2\mu t \cos r = n\lambda$$

For normal incidence, r = 0, cos r = 1

$$\text{So,} \quad 2\mu t = n\lambda, \quad t = \frac{n\lambda}{2\mu}$$

For minimum thickness of the film, n = 1

Hence,

$$t = \frac{\lambda}{2\mu} = \frac{5893 \times 10^{-8}}{2 \times 1.42} \text{ cm}$$

$$t = 2075\ \text{Å}$$

....Ans.

(ii) In reflected system, the condition of bright is

$$2\mu t \cos r = (2n - 1)\ \lambda/2$$

For normal incidence, $r = 0$, $\cos r = 1$

For minimum thickness of the film, $n = 1$

$$\text{So,} \quad t = \frac{\lambda}{4\mu} = \frac{5893 \times 10^{-8}}{4 \times 1.42} \text{ cm}$$

$$t = 1037.5 \text{ Å}$$ ...Ans.

Ex. 4.13.8 : Light falls normally on a soap film of thickness 5×10^{-5} cm and of refractive index 1.33. Which wavelength in the visible region will be reflected most strongly ? **MU - Dec. 12, 5 Marks**

Soln. :

The condition of maxima is given by,

$$2 \mu t \cos r = (2n - 1) \lambda/2$$ where $n = 1, 2, 3, \ldots$

Given : $t = 5 \times 10^{-5}$ cm, $\mu = 1.33$

$r = 0°$ i.e. $\cos r = 1$

Now,

$$\lambda = \frac{4 \mu t \cos r}{(2n - 1)}$$

$$= \frac{4 \times 1.33 \times 5 \times 10^{-5}}{(2n - 1)}$$

By substituting the values of $n = 1, 2, \ldots$ we get a series of wavelengths which shall be predominantly reflected by the film.

For $n = 1$,

$$\lambda_1 = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{1}$$

$$= 26.66 \times 10^{-5} \text{ cm}$$

Similarly,

For $n = 2$, $\lambda_2 = 8.866 \times 10^{-5}$ cm

For $n = 3$, $\lambda_3 = 5.32 \times 10^{-5}$ cm

For $n = 4$, $\lambda_4 = 3.8 \times 10^{-5}$ cm

Out of these wavelengths 5.320×10^{-5} cm lies in the visible region (4000 Å to 7500 Å).

Hence 5320 Å is the most strongly reflected wavelength. **...Ans.**

Ex. 4.13.9 : White light is incident on a soap film at an angle $\sin^{-1}$ (4/5) and the reflected light is observed with a spectroscope. It is found that two consecutive dark bands correspond to wavelengths 6.1×10^{-5} and 6.0×10^{-5} cm. If the refractive index of the film is 4/3, calculate the thickness.

Soln. :

We have the condition for dark band in reflected system,

$$2 \mu t \cos r = n\lambda$$

If n and (n + 1) are the orders of consecutive dark bands for wavelengths λ_1 and λ_2 respectively, then,

$$2\,\mu t \cos r = n\lambda_1, \qquad 2\,\mu t \cos r = (n+1)\,\lambda_2$$

$$\therefore \quad 2\,\mu t \cos r = n\lambda_1 = (n+1)\,\lambda_2 \qquad \ldots(1)$$

$$n\lambda_1 = (n+1)\,\lambda_2$$

$$\therefore \quad n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

Put the value of n in equation (1), we have,

$$2\mu t \cos r = \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)}$$

$$t = \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)\, 2\mu \cos r}$$

$$= \frac{\lambda_1 \lambda_2}{(\lambda_1 - \lambda_2)\, 2\mu \cdot \sqrt{1 - \left(\frac{\sin i}{\mu}\right)^2}} \qquad \ldots(2)$$

Given : $\mu = 4/3, \ \sin i = 4/5$

As $\quad \mu = \frac{\sin i}{\sin r} \quad$ and $\quad \cos r = \sqrt{1 - \sin^2 r}$

$$\therefore \quad \cos r = \sqrt{1 - \left(\frac{4/5}{4/3}\right)^2} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Given : $\lambda_1 = 6.1 \times 10^{-5}$ cm, $\lambda_2 = 6.0 \times 10^{-5}$ cm, $\mu = 4/3$

Put all these values in equation (2),

$$t = \frac{6.1 \times 10^{-5} \times 6.0 \times 10^{-5}}{(6.1 - 6) \times 10^{-5} \times 2 \times 4/3 \times 4/5}$$

$$t = 0.0017 \text{ cm} \qquad \ldots\text{Ans.}$$

Ex. 4.13.10 : A soap film of refractive index $\frac{4}{3}$ and thickness 1.5×10^{-4} cm is illuminated by white light incident at an angle of 45°. The light reflected by it is examined by a spectroscope in which is found a dark band corresponding to a wavelength of 5×10^{-5} cm. Calculate the order of interference band.

Soln. :

Given : $\mu = \frac{4}{3}$, $t = 1.5 \times 10^{-4}$ cm., $i = 45°$, $\lambda = 5 \times 10^{-5}$ cm

Formula : For dark band,

$$2\,\mu t \cos r = n\lambda; \qquad 2\mu t \cos r = n\,\lambda$$

$$\text{Now } \mu = \frac{\sin i}{\sin r}$$

$$\therefore \quad \sin r = \frac{\sin i}{\mu} = \frac{\sin 45°}{4/3}$$

$$\therefore \quad r = \sin^{-1}\left(\frac{\sin 45^\circ}{4/3}\right)$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{2}} \times \frac{3}{4}\right) = 32.02^\circ$$

$$\therefore \quad \cos r = 0.8478$$

$\therefore$ Order of dark band is

$$n = \frac{2\mu t \cos r}{\lambda}$$

$$= \frac{2 \times 1.33 \times 1.5 \times 10^{-4} \times 0.8478}{5 \times 10^{-5}}$$

$$\text{or} \quad n = 6.7$$

$\therefore$ Order of dark band is 6. **...Ans.**

Ex. 4.13.11 : A film of refractive index μ is illuminated by white light at an angle of incidence i. In reflected light two consecutive bright fringes of wavelength λ_1 and λ_2 are found overlapping. Obtain expression for thickness of film.

Soln. :

Say, n^{th} bright fringe of λ_1 overlaps with $(n + 1)^{th}$ fringe of λ_2

For maxima of λ_1

$$2\mu t \cos r = (2n - 1)\frac{\lambda_1}{2} \quad ...(1)$$

And for λ_2

$$2\mu t \cos r = \{2(n + 1) - 1\}\frac{\lambda_2}{2} \quad ...(2)$$

So,
$$(2n - 1)\frac{\lambda_1}{2} = \{2(n + 1) - 1\}\frac{\lambda_2}{2}$$

$$\therefore \quad 2n(\lambda_1 - \lambda_2) = \lambda_1 - \lambda_2$$

$$\therefore \quad 2n = \frac{\lambda_1 + \lambda_2}{\lambda_1 - \lambda_2}$$

Thickness of the film t, we get from equation (1)

$$2\mu t \cos r = \left\{\frac{\lambda_1 + \lambda_2}{\lambda_1 - \lambda_2} - 1\right\}\frac{\lambda_1}{2} = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

$$t = \frac{\lambda_1 \lambda_2}{2\mu \cos r(\lambda_1 - \lambda_2)}$$ **...Ans.**

Ex. 4.13.12 : White light is incident at an angle of 45° on a soap film 4×10^{-5} cm thick. Find the wavelength of light in the visible spectrum which will be absent in the reflected light ($\mu = 1.2$).

Soln. :

Here white light is made incident, and on reflected side it is expected to find the absent wavelength i.e. the one which will satisfy the condition for dark.

$$2\,\mu t \cos r = n\,\lambda$$

Here, $t = 4 \times 10^{-5}$ cm

$i = 45°\,;\quad \mu = 1.2$

$$\therefore\quad 1.2 = \frac{\sin 45}{\sin r}$$

$$\therefore\quad r = \sin^{-1}\left(\frac{\sin 45}{1.2}\right)$$

$$= 36.104°$$

$$\therefore\quad \cos r = 0.8079$$

$\therefore$ Now find λ for various order n such that it remains between 4000 Å to 8000 Å i.e. visible spectrum.

$\therefore$ For n = 1, condition for dark fringe

$$1 \times \lambda = 2 \times 1.2 \times 4 \times 10^{-5} \times 0.8079$$

$$\lambda = 7755 \times 10^{-8} \text{ cm}$$

This is in visible range and it will remain absent

Similarly for n = 2

$$\lambda = \frac{2 \times 1.2 \times 4 \times 10^{-5} \times 0.8079}{2}$$

$$= 3877 \times 10^{-8} \text{ cm}$$

This is not in visible range.

$\therefore$ **7755 Å** will remain absent. ...**Ans.**

Ex. 4.13.13 : A plane wave of monochromatic light falls normally on a uniform thin film of oil, which covers a glass plate. The wavelength of the source can be varied continuously. Complete destructive interference is obtained only for wavelengths 5000 Å and 7000 Å. Find the thickness of the oil layer.
Given R.I. of oil = 1.3 and R.I of glass = 1.5. **MU - May 13, 7 Marks**

Soln. :

Here the path is from air to oil and oil to glass.

(1) For air to oil :

For destructive interference the condition is, (for normal incidence)

$$2\mu_{oil}\, t_{oil} = (2n+1)\frac{\lambda}{2} \qquad \text{...(1)}$$

As the arrangement remains same for both the wavelengths, μ_{oil} and t_{oil} will be the same or constant.

$$\therefore\quad \text{const.} = (2n+1)\frac{\lambda}{2}$$

∴ Order of destructive interference is inversely proportional to wavelength.

∴ For $\lambda_1 = 7000$ Å take order n and for next wavelength $\lambda_2 = 5000$ Å take next order i.e. n + 1

∴ For $\lambda_1 = 7000$ Å

$$2\,\mu_{oil}\,t_{oil} = (2n+1) \times \frac{7000}{2}\ \text{Å} \qquad ...(2)$$

For $\lambda_2 = 5000$ Å

$$2\,\mu_{oil}\,t_{oil} = (2\,(n+1)+1) \times \frac{5000}{2}\ \text{Å} \qquad ...(3)$$

$$\therefore (2n+1) \times \frac{7000}{2} = (2\,(n+1)+1) \times \frac{5000}{2}$$

$$\therefore \frac{(2\,n+1)}{(2\,n+3)} = \frac{5000}{7000}$$

On solving, n = 2

∴ Substitute in equation (2)

$$t_{oil} = \frac{(2\,(2)+1) \times 7000}{2 \times 1.3 \times 2}$$

$$= 6730.769\ \text{Å} \qquad ...\text{Ans.}$$

Problems on Wedge-shaped Film

Ex. 4.13.14 : A wedge shaped air film having an angle of 40 seconds is illuminated by monochromatic light and fringes are observed vertically through a microscope. The distance measured between consecutive bright fringes is 0.12 cm. Calculate the wavelength of light used. **MU - Dec. 17, May 18. 5 Marks**

Soln. :

Given :

$$\theta = 40 \text{ seconds} = \frac{40}{3600} \text{ degrees}$$

$$= \frac{40}{3600} \times \frac{\pi}{180} \text{ radians,}$$

$$\beta = 0.12 \text{ cm.}$$

Formula : Spacing between the consecutive bright fringes is,

$$\beta = \frac{\lambda}{2\theta} \qquad \text{(For air film)}$$

The wavelength is,

$$\lambda = 2\beta \cdot \theta = 2 \times 0.12 \times \frac{40 \times \pi}{3600 \times 180}$$

$$= 4654 \times 10^{-8} \text{ cm}$$

$$\lambda = 4654\ \text{Å} \qquad ....\textbf{Ans.}$$

Ex. 4.13.15 : Light of wavelength 5500 A° falls normally on a thin wedge-shaped film of refractive index 1.4 forming fringes that are 2.5 mm apart. Find the angle of wedge in seconds.

Soln. :

We have fringe width,

$$\beta = \frac{\lambda}{2\mu\theta}; \ \theta = \frac{\lambda}{2\mu\beta}$$

Given :

$$\lambda = 5500 \times 10^{-8} \text{ cm}, \ \mu = 1.4, \ \beta = 0.25 \text{ cm}$$

$$\theta = \frac{5500 \times 10^{-8}}{2 \times 1.4 \times 0.25} = 7.86 \times 10^{-5} \text{ radian}$$

$$\theta = 7.86 \times 10^{-5} \times \frac{180°}{\pi} = 0.0045°$$

$$= 0.0045° \times 3600 = \mathbf{16.2 \ sec.}$$ ...**Ans.**

Problems on Newton's Rings

Ex. 4.13.16 : Newton's rings are obtained with reflected light of wavelength 5500 Å. The diameter of 10^{th} dark ring is 5 mm. Now the space between the lens and the plate is filled with a liquid of refractive index 1.25. What is the diameter of the 10^{th} ring now ?

Soln. :

Given : $\lambda = 5500$ Å $= 5500 \times 10^{-8}$ cm, $D_{10} = 5$ mm $= 0.5$ cm, $\mu = 1.25$

Formula : Diameter of n^{th} dark ring is given by,

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

For the air film, $\mu = 1$

Hence diameter of 10^{th} dark ring is,

$$(0.5)^2 = D_{10}^2 = \frac{4 \times 10 \times R \times 5500 \times 10^{-8}}{1}$$

For the liquid film, the diameter of the 10^{th} dark ring is,

$$D_{10}'^2 = \frac{4 \times 10 \times R \times 5500 \times 10^{-8}}{1.25}$$

$$\frac{D_{10}'^2}{D_{10}^2} = \frac{1}{1.25}$$

$$\therefore \ D_{10}'^2 = \frac{D_{10}^2}{1.25}$$

$$\therefore \quad D_{10}' = \frac{D_{10}}{\sqrt{1.25}} = \frac{D_{10}}{1.1180} = \frac{0.5}{1.118}$$

$$D_{10}' = 0.4472 \text{ cm.}$$

Diameter of 10^{th} dark ring for the liquid film

$$D_{10}' = \mathbf{4.472 \ mm}$$ ...**Ans.**

Ex. 4.13.17 : Newton's rings formed with sodium light between a flat glass plate and a convex lens are viewed normally. What will be the order of the dark ring which will have double the diameter of that of the 40th dark ring ?

MU - May 18, 3 Marks

Soln. :

Let the diameter of the n^{th} dark ring be double the diameter of 40^{th} dark ring.

$$\therefore \quad D_n = 2\, D_{40}$$

Now the diameter of n^{th} dark ring is given by the expression

$$D_n^2 = 4\, n\, R\, \lambda \qquad ...(1)$$

Where, R = Radius of curvature of lens.

λ = Wavelength of light.

Hence for the 40^{th} dark ring.

$$D_{40}^2 = 4 \times 40 \times R\lambda \qquad ...(2)$$

From equations (1) and (2) we have

$$D_n^2 = 4 \times D_{40}^2$$

$$\therefore \quad 4\, n\, R\, \lambda = 4 \times 4 \times 40 \times R\lambda$$

$$\mathbf{n = 160} \qquad \textbf{...Ans.}$$

Ex. 4.13.18 : The diameter of 5th dark ring in Newton's ring experiment was found to be 0.42 cm. Determine the diameter of 10th dark ring.

MU - May 16, 4 Marks

Soln. :

$$D_n^2 = 4\, nR\lambda$$

As, diameter of 5th dark ring = 0.42 cm

Now, diameter of 10th dark ring = ?

$$\therefore \quad \frac{D_5^2}{D_{10}^2} = \frac{4(5)\, R\lambda}{4(10)\, R\lambda}$$

$$\therefore \quad 2\left(D_5^2\right) = D_{10}^2$$

$$D_{10} = \sqrt{2}\,(D_5) = \sqrt{2}\,(0.42)$$

$$= 0.594 \text{ cm}$$

$\therefore$ Diameter of 10th dark ring = **0.594 cm** **...Ans.**

Ex. 4.13.19 : A Newton's ring arrangement is used with a source emitting two wavelengths $\lambda_1 = 6 \times 10^{-5}$ cm and $\lambda_2 = 4.5 \times 10^{-5}$ cm. It is found that the n^{th} dark ring due to λ_1 coincides with $(n + 1)^{th}$ dark ring for λ_2. If the radius of the curved surface is 90 cm, find the diameter of 3^{rd} dark ring for λ_1.

Soln. :

Given : $\lambda_1 = 6 \times 10^{-5}$ cm, $\lambda_2 = 4.5 \times 10^{-5}$ cm,

$$R = 90 \text{ cm}$$

$$[D_n]_{\lambda_2} = [D_{n+1}]_{\lambda_1}$$

Formula : Diameter of n^{th} dark ring is,

$$D_n^2 = 4\,n\,R\,\lambda \qquad (\mu = 1)$$

$\therefore$ For the n^{th} dark ring λ_1

$$\left[D_n^2\right]_{\lambda_1} = 4\,n\,R\,\lambda_1 \qquad ...(1)$$

and for the $(n+1)^{th}$ dark ring λ_2

$$\left[D_{n+1}^2\right]_{\lambda_2} = 4\,(n+1)\cdot R\cdot\lambda_2 \qquad ...(2)$$

$$\therefore\ 4\,n\,R\,\lambda_1 = 4\,(n+1)\,R\cdot\lambda_2$$

$$\therefore\ n\lambda_1 = (n+1)\,\lambda_2$$

$$\therefore\ n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

$$= \frac{0.45 \times 10^{-5}}{(6 - 0.45) \times 10^{-5}}$$

$$\therefore\ n = 3$$

$\therefore$ Using equation (1), the diameter of 3^{rd} dark ring for λ_1 is

$$\left[D_3^2\right] = 4 \times 3 \times 90 \times 6 \times 10^{-5}$$

$$\therefore\ [D_3]_{\lambda_1} = \sqrt{4 \times 3 \times 90 \times 6 \times 10^{-5}}$$

$$= 0.2545 \text{ cm.}$$

$\therefore$ Diameter of third dark ring for λ_1 is **0.2545 cm.** **...Ans.**

Ex. 4.13.20 : Light containing two wavelengths λ_1 and λ_2 falls normally on a convex lens of radius of curvature R, resting on a glass plate. Now if the n^{th} dark ring due to λ_1 coincides with $(n+1)^{th}$ dark ring due to λ_2, then prove that the radius of the n^{th} dark ring due to λ_1 is $\sqrt{\dfrac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}\cdot R}$.

Soln. :

The diameter of n^{th} dark ring is given by a relation

$$D_n^2 = 4\,n\,R\,\lambda$$

$\therefore$ For n^{th} dark ring of λ_1,

$$D_n^2 = 4\,n\,R\,\lambda_1 \qquad ...(1)$$

and $(n + 1)^{th}$ dark ring of λ_2,

$$D^2_{n+1} = 4\,(n + 1)\,R\,\lambda_2 \qquad ...(2)$$

$$\text{Now} \quad D^2_n = D^2_{n+1} \text{ as per data}$$

$$\therefore \quad 4\,n\,R\,\lambda_1 = 4\,(n + 1)\,R\,\lambda_2$$

$$\therefore \quad n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

As per equation (1), the diameter of n^{th} dark ring of λ_1 is,

$$D_n = 2\sqrt{n\,R\,\lambda_1}$$

$\therefore$ Radius of n^{th} dark ring of λ_1 is,

$$r_n = \frac{D_n}{2} = \sqrt{nR\,\lambda_1}$$

$$\text{or} \quad r_n = \sqrt{\frac{\lambda_2}{\lambda_1 - \lambda_2} \cdot R\lambda_1}$$

$$= \sqrt{\frac{\lambda_1\,\lambda_2}{\lambda_1 - \lambda_2} \cdot R}$$

Hence proved.

Ex. 4.13.21 : Show, with clear examples, that separation between two consecutive similar rings, in Newton's rings experiment goes on reducing as the serial number of ring increases.

Soln. : According to theory of Newton's ring the diameter of the n^{th} dark ring is given by,

$$D_n = \sqrt{4n\,R\,\lambda}$$

$\therefore D_n \propto \sqrt{n}$, where n is the serial number of the ring.

For example, first calculate the separation of 5^{th} and 4^{th} dark ring,

$$\therefore D_5 \propto \sqrt{5} = 2.236 \text{ and}$$

$$D_4 \propto \sqrt{4} = 2$$

Hence the separation between 5^{th} and 4^{th} ring is

$$D_5 - D_4 \propto 0.236 \text{ in SI unit.}$$

Similarly, the separation between 79^{th} and 80^{th} ring is

$$D_{80} \propto \sqrt{80} = 8.9442$$

and

$$D_{79} \propto \sqrt{79} = 8.8881$$

$$D_{80} - D_{79} \propto 0.0560 \text{ in SI unit.}$$

Hence, we can conclude that,

$$D_{80} - D_{79} < D_5 - D_4 \text{ hence proved.}$$

4.14 Determination of Thickness of Very Thin Wire or Foil

- There are many applications wherein we need to know the thickness of a very thin wire or an equivalent for example "contact lens". As mentioned in introduction, concept of inference can lead us to an experimental arrangement which gives us accurate measure up to one tenth of a micrometer.
- Measurement of thickness of very thin wire or foil :

  As seen in the case of "spacing between two consecutive bright bands", we have derived fringe width $\beta = \frac{\lambda}{2\theta}$
- The same concept is used for determination of thickness of thin wire. We take two glass slides (optically flat) and put them in touch at one end and at the other we put the wire or foil whose thickness is to be determined. Hence we prepare a wedge with very small angle 'θ'.
- Here we make the use of same setup which is used for Newton's rings. In place of plano convex lens we take the above mentioned wedge.
- On viewing through microscope when illuminated by a monochromatic light of wavelength λ at normal incidence, we get alternate dark and bright lines.
- We take readings of dark lines at the spacing of some interval say 'P' number of lines.
- We draw a graph for Reading → order of dark line and find the fringe width 'β'.
- From experimental set up measure 'l'

$$\therefore \tan\theta = \frac{t}{l} \simeq \theta \qquad \text{(as } \theta \text{ is very small)}$$

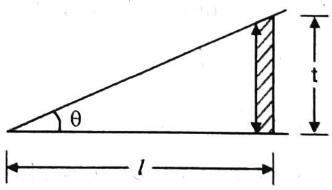


Fig. 4.14.1

$$\text{as } \beta = \frac{\lambda}{2\theta}$$

$$\therefore \frac{t}{l} = \frac{\lambda}{2\beta}$$

$$\therefore t = \frac{\lambda l}{2\beta} \qquad \text{...(4.14.1)}$$

Since λ, l and β are known we can calculate thickness of a foil or thin wire.

Ex. 4.14.1 : Two plane glass surfaces in contact along one edge are separated at the opposite edge by a thin wire. If 20 interference fringes are observed between these edges in sodium light at normal incidence, what is the thickness of the wire? (Given λ = 5893 Å)

Soln. :

$$\text{Fringe width } \beta = \frac{\lambda}{2\mu\theta} \quad \text{...(For normal incidence)}$$

$$\text{For air film } \mu = 1, \quad \beta = \frac{\lambda}{2\theta}$$

$$\text{Thickness of wire } t = x_n \tan\theta$$

$$= x_n \cdot \theta \qquad \text{(for small } \theta, \tan\theta \simeq \theta)$$

$$= 20 \cdot \beta \cdot \theta$$

As there are 20 fringes between two edge points

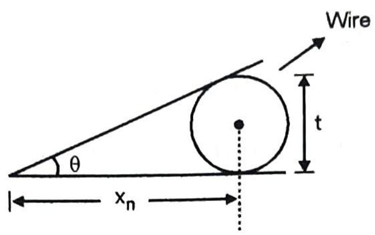


Fig. P. 4.14.1

Putting the value of β, we get,

$$t = 20\left(\frac{\lambda}{2\theta}\right) \cdot \theta = 10\,\lambda$$

$$= 10 \times 5893 \times 10^{-10}$$

$$= 5.893 \times 10^{-6}\ \text{m}$$

$$= 5.893\ \mu\text{m}$$

...Ans.

4.15 Determination of Wavelength of Monochromatic Light or Radius of Curvature of Lens by Newton's Rings Method

MU - May 13, Dec. 14, May 17, Dec. 18

Q. With the help of proper diagram and necessary expressions, explain how Newton's ring experiment is useful to determine the radius of curvature of a plano convex lens. **(May 13, Dec. 18, 5 Marks)**

Q. With proper diagram and necessary expressions explain how Newton's ring experiment is useful to determine the radius of curvature of plano-convex lens. **(Dec. 14, 8 Marks)**

Q. With Newton's ring experiment explain how to determine the refractive index of liquid? **(May 17, 5 Marks)**

4.15.1 Experimental Arrangement

- A carefully cleaned convex surface of a plano convex lens L of large radius of curvature is placed on a plane glass plate P. (Fig. 4.15.1).
- Another glass plate G is held at a suitable distance above at an angle 45° with the vertical (to make the normal incidence on the film).

- With a condensing lens, maximum light from source (sodium lamp) is allowed to fall on G. The inclined plate G reflects light onto the air film between the lens L and plate P.

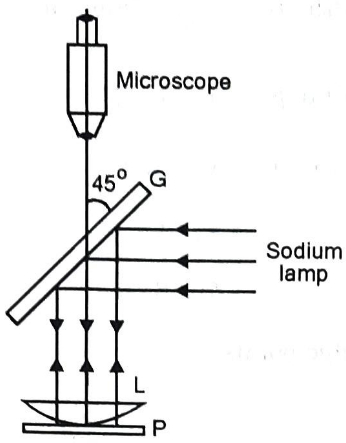


Fig. 4.15.1

- Newton's rings are formed as a result of interference between the rays reflected from the top and bottom faces of the air film.
- They are seen through a low power microscope focussed on the air film where the rings are formed.

Theory

- The effective path difference between the interfering rays is,

$$2\mu t \cos(r+\theta)+\frac{\lambda}{2}$$

Where, μ = R.I. of film.

θ = Angle of film at any point

λ = Wavelength of light

- If D_n is the diameter of n^{th} dark ring then as per the theory of Newton's rings described in equation below

$$D_n^2 = 4\,n\,R\,\lambda$$

where R = Radius of curvature of lower surface of lens.

$$\text{Let } x\,D_{n+p}^2 = 4\,(n+p)\,R\,\lambda \quad (\text{for } (n+p)^{th} \text{ ring}) \qquad ...(4.15.1)$$

$$\therefore\; D_{n+p}^2 - D_n^2 = 4\,p\,R\,\lambda$$

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4\,pR} \qquad ...(4.15.2)$$

- Alternately if wavelength of incident monochromatic ray is known, using equation (4.15.2), we can find R i.e. the radius of curvature.
- For a more accurate approach we plot a graph of D_n^2 vs n as shown in Fig. 4.15.2.

Thus by measuring the diameter of n^{th} and $(n + p)^{th}$ dark rings and the radius of curvature R, the wavelength λ can be calculated.

The diameter of the rings is measured with the travelling microscope and the radius of curvature can be determined by using lens equation or by a spherometer.

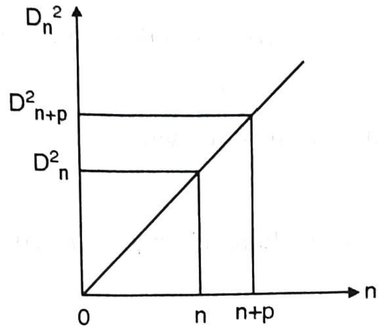


Fig. 4.15.2

4.15.2 Determination of Radius of Curvature of Lens

- This can be done easily with the help of spherometer and the formula

$$R = \frac{l^2}{6h} + \frac{h}{2} \qquad ...(4.15.3)$$

Where, l = Distance between two legs of spherometer

h = Difference in the reading when placed on lens as well as when placed on surface

Ex. 4.15.1 : In Newton's rings experiment the diameter of 5^{th} ring was 0.336 cm and the diameter of 15^{th} ring was 0.590 cm. Find the radius of curvature of plano convex lens if the wavelength of light used is 5890 Å.

MU - May 15, Dec. 18, 5 Marks

Soln. :

Given : $D_5 = 0.336$ cm, $D_{15} = 0.590$ cm,

$\lambda = 5890$ Å $= 5890 \times 10^{-8}$ cm

Formula : $D^2_{n+p} - D^2_n = 4pR\lambda$

$$R = \frac{D^2_{n+p} - D^2_n}{4 \cdot p \cdot \lambda}$$

$$= \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 5890 \times 10^{-8}}$$

$$= 99.91 \text{ cm}$$

$$\therefore \quad R = 99.91 \text{ cm} \qquad \text{...Ans.}$$

4.16 Determination of Refractive Index of a Liquid by Newton's Rings

MU - May 14, May 16

Q. With Newton's ring experiment explain how to determine the refractive index of liquid. **(May 14, 4 Marks)**

Q. How is Newton's ring experiment used to determine refractive index of liquid medium? **(May 16, 4 Marks)**

- There is a popular branch of engineering known as "hydraulics" in which liquid is in motion and transmits energy, viscosity and refractive index are some of the parameters which can be used to decide the suitability of the oil (called grade of the oil). In such applications, the correct measurement of the R.I. is thus an important issue.
- Consider that a transparent liquid whose refractive index is to be determined is placed between the lens L and plate P of the Newton's rings arrangement.
- If the liquid is rarer than glass, a phase change of π will occur at the reflection from the lower surface of liquid film.
- If the liquid is denser than glass, then a phase change of π will occur due to reflection at the upper surface of the film.
- Hence in either case, a path difference of $\lambda/2$ will be introduced between the interfering rays in the reflected system, and hence the effective path difference between them will be

$2\mu\, t \cos(r + \theta) + \frac{\lambda}{2}$

$$\text{Now } r = 0 \text{ for normal incidence}$$

$$\theta = 0 \text{ for large R}$$

$$\therefore \quad \text{Path difference} = 2\mu t + \frac{\lambda}{2}$$

$$2t = \frac{\rho^2}{2}$$

$\therefore$ For n^{th} dark ring, we have,

$$\frac{\mu \rho^2}{R} + \frac{\lambda}{2} = (2n+1)\lambda/2 \qquad \therefore \frac{\mu \cdot D_n^2}{4R} = n\lambda$$

$$\therefore \quad D_n^2 = \frac{4nR\lambda}{\mu}$$

Similarly for the $(n+p)^{th}$ dark ring we have,

$$D_{n+p}^2 = \frac{4(n+p)R\lambda}{\mu} \qquad \text{...(4.16.1)}$$

$$D_{n+p}^2 - D_n^2 = \frac{4pR\cdot\lambda}{\mu}$$

D_{n+p} = Diameter of $(n+p)^{th}$ dark ring, then we have

$$\mu = \frac{4pR\lambda}{(D_{n+p}^2 - D_n^2)_{liquid}} \qquad \text{...(4.16.2)}$$

$$\therefore \quad \mu = \frac{4pR\lambda}{[D_{n+p}^2 - D_n^2]_{air}} = 1 \; (\because \mu = 1 \text{ for air}) \qquad \text{...(4.16.3)}$$

For more accurate approach we plot a graph of D_n^2 vs n as shown below.

It is similar to Fig. 4.15.2.

$\therefore$ From equations (4.16.2) and (4.16.3) we have,

$$\mu = \frac{\left[D^2_{n+p} - D^2_n\right]_{air}}{\left[D^2_{n+p} - D^2_n\right]_{liquid}}$$

- If we obtain ring structure using air as the medium then diameter is given by D^2_n (air) = $4nR\lambda$. Now slowly if we insert a liquid with R.I. μ then D^2_n (liquid) = $\frac{4nR\lambda}{\mu}$. Take ratio of these two.

$$\mu = \frac{D^2_n\,(air)}{D^2_n\,(liq)} \quad ...(4.16.4)$$

- Thus to find μ, Newton's rings are formed with air film. The diameters of n^{th} and $(n + p)^{th}$ dark rings are measured for air film.
- Then the transparent liquid is introduced between lens L and plate P.
- The diameters of n^{th} and $(n + p)^{th}$ dark rings are measured with liquid film. Hence using equation (4.16.4), μ can be calculated.

Ex. 4.16.1 : Newton's rings are formed in reflected light of wavelength 6000 A° with a liquid between the plane and curved surfaces. If the diameter of the 6^{th} bright ring is 3.1 mm and the radius of curvature of the curved surface is 100 cm, calculate the refractive index of the liquid.

Soln. :

The diameter of n^{th} bright ring is,

$$D^2_n = \frac{2(2n-1)\,\lambda R}{\mu}$$

$$\mu = \frac{2(2n-1)\,\lambda R}{D^2_n}$$

Given : $n = 6$, $\lambda = 6000 \times 10^{-8}$ cm, $R = 100$ cm, $D_6 = 0.31$ cm

$$\mu = \frac{2(2 \times 6 - 1)\,6000 \times 10^{-8} \times 100}{(0.31)^2}$$

$$= \frac{2 \times 11 \times 6 \times 10^{-3}}{(0.31)^2}$$

$$= 1.373$$

...Ans.

4.17 Applications of Interference

- Reader can easily understand one fact that if a thin film has its thickness altered by $\frac{\lambda}{4}$ where λ = wavelength of monochromatic ray used to illuminate, then total path difference is $\frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$ for reflected ray.

- Now $\frac{\lambda}{2}$ is the path difference which converts interference fringes from dark to bright or bright to dark. This is easily observable. Hence if we consider any thin film which is illuminated by Na lamp (5896 Å), then a charge of

$$\frac{1}{4} \times 5896 = 1474 \text{ Å}$$

can be detected by change in interference dark to bright (or bright to dark). It is important to note that 1474 Å = 0.1474 micrometer.

- Now coming to the area of surface finish, it is essential to know that surface finish of parts can significantly affect their friction, wear, fatigue, corrosion, tightness of contact joints, position accuracy and so on. Surface finish has always been considered an important factor for manufacturing process monitoring and quality control inspection.
- Surface finish is a representation of the vertical deviations of a measured from its ideal form. If the deviations are substantial then the surface is rough and if these deviations are minor the surface is smooth. For many engineering applications, the finish on the surface can have a big effect or the performance or durability of parts. Hence, a quick measurement of flatness is an important area and optics through interference is considered useful. As mentioned above, interference is capable of detecting a variation of 0.1474 micrometres using Na lamp.
- The smoothness of surface can be detected by using an optical flat which is precisely polished flat surface usually within a few tens of nanometres used as a reference against which the flatness of an unknown surface may be compared.
- An optical flat is usually placed upon a surface under investigation. If a monochromatic light (Na lamp) is used to illuminate the workpiece, a series of dark and bright interference fringes are formed. These interference fringes determine the flatness of the work piece relative to the optical flat up to the accuracy of few micrometres as discussed above. By using a wedge as shown in Fig. 4.17.1, fringes are formed.

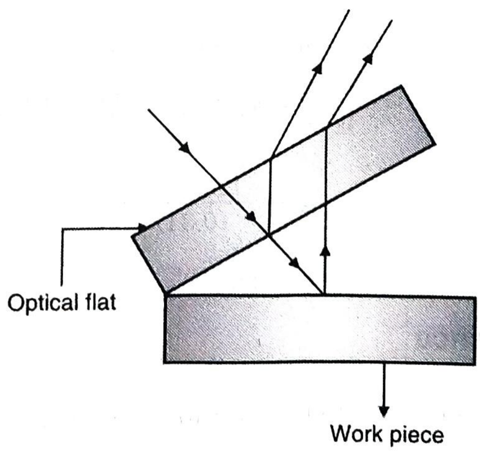


Fig. 4.17.1

- If the workpiece is perfectly flat then straight parallel interference fringes will form.

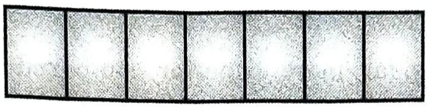


Fig. 4.17.2

- More and thinner fringes indicate a steeper wedge while fewer but wider fringes indicate smaller wedge.
- If the workpiece is concave or convex, the fringes will be as shown in Fig. 4.17.3.

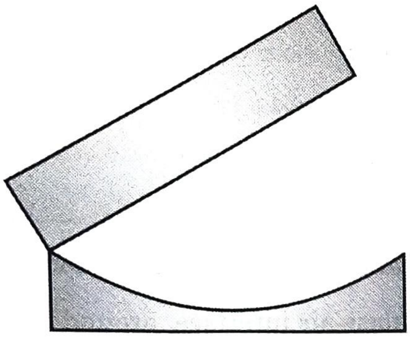


Fig. 4.17.3 (a)

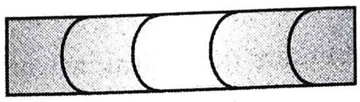


Fig. 4.17.3 (b)

- If the fringes are curved towards the contact edge, the workpiece surface is concave (Fig. 4.17.3 (a)) and if the fringes are curved away the work piece surface is convex Fig. 4.17.3 (b).

4.17.1 Testing the Optical Flatness of Surfaces

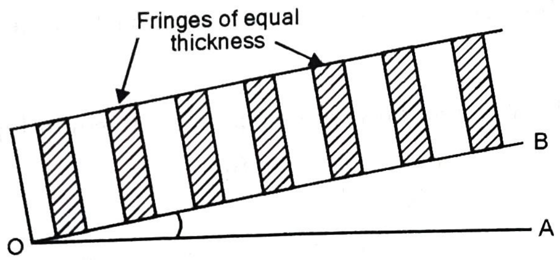


Fig. 4.17.4

- The phenomenon of interference is also used in testing the plainness of the surfaces.
- If two surfaces OA and OB (Fig. 4.17.4) are perfectly plane, the air film between them gradually varies in thickness from O to A. The fringes are of equal thickness as each fringe is the locus of the points at which the thickness of the film has a constant value.
- If the fringes are not of equal thickness, it means that the surfaces are not plane.
- To test the optical flatness of a surface, the specimen surface to be tested (OB) is placed over an optically plane surface (OA).
- The fringes are observed in the field of view. If they are of equal thickness the surface OB is plane. If not, then surface OB is not plane.

- The surface OB is polished and the process is repeated. When fringes observed are of equal width, it means the surface OB is plane.
- The accuracy in this method is far more superior compared to any other technique adopted. The accuracy level is of the order of fraction a of micrometer.

4.18 Concept of Anti-reflecting Coating (Non-reflecting Films)

MU - May 12, Dec. 13, Dec. 17, Dec. 18

Q. Describe in detail the concept of anti-reflecting film with a proper ray diagram of thin film interference. Which condition the material should satisfy to act as anti reflecting film? **(May 12, 8 Marks)**

Q. What do you understand by anti-reflecting coating? Derive the conditions with proper diagram. **(Dec. 13, 8 Marks)**

Q. Describe in detail the concept of anti-reflecting film with a proper ray diagram. **(Dec. 17, 5 Marks)**

Q. What is antireflection coating? What should be the refractive index and minimum thickness of the coating? **(Dec. 18, 3 Marks)**

- We are aware that compound microscope, telescope, camera lenses, etc. use a combination of lenses.
- When the light enters the optical instrument at the glass-air interface, around 4% of light (for air with $n_1 = 1$ and glass with $n_2 = 1.5$) that too at single reflection is lost by reflection which is highly undesirable. For advanced telescopes the total loss comes out to be nearly 30% and cannot be tolerated if working under low intensity applications.
- In order to reduce the reflection loss, a transparent film of proper thickness is deposited on the surface. This film is known as "non-reflecting film".
- Popular material used is MgF_2 because its refractive index is 1.38 (i.e. between air and glass). Cryolite ($n_1 = 1.36$) is also used.
- Thickness of the film may be obtained for given purpose as shown in Fig. 4.18.1.

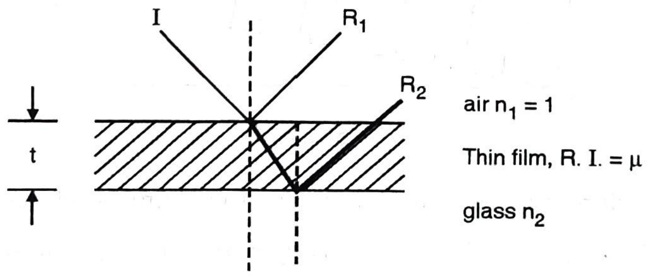


Fig. 4.18.1 : Thin film coating

- Let a ray I be incident upon thin film of MgF_2 coated on glass. This ray is reflected from upper surface as R_1 and from lower surface as R_2. The optical path difference between these two rays is n_1 (2t), as the incident ray enters from rarer to denser twice i.e. at air to film and film to glass.

  If both the rays R_1 and R_2 interfere with each other and path difference is $(2n + 1) \lambda/2$ (for n = 0, 1, 2, ...) then destructive interference will take place.

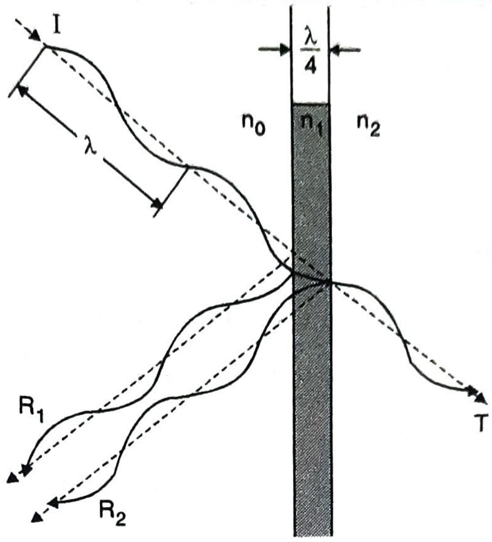


Fig. 4.18.2

$$\therefore \quad 2 n_1 t = \frac{\lambda}{2} \qquad \text{(for n = 0)}$$

$$\therefore \quad n_1 t = \frac{\lambda}{4 \mu}$$

- It means, in order to have destructive interference a layer of $n_1 t = \frac{\lambda}{4}$ is coated on glass plate.

Amplitude condition :

- The amplitude condition requires that the amplitudes of reflected rays, ray 1 and ray 2 are equal. That is,

$$I_1 = I_2 \qquad \because \; E_1 = E_2$$

- For complete destructive condition, intensities of two reflected beams should be equal.
- It requires that,

$$\left[\frac{\mu_f - \mu_a}{\mu_f + \mu_a}\right]^2 = \left[\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right]^2$$

- Where μ_a, μ_f and μ_g are the refractive indices of air, thin film and glass substrate respectively. As $\mu_a = 1$, the above expression may be rewritten as

$$\left[\frac{\mu_f - 1}{\mu_f + 1}\right]^2 = \left[\frac{\mu_g - \mu_f}{\mu_g + \mu_f}\right]^2$$

- Take square root on both sides to compare their amplitudes.

$$\frac{\mu_f - 1}{\mu_f + 1} = \frac{\mu_g - \mu_f}{\mu_g + \mu_f}$$

$$(\mu_f - 1)(\mu_g + \mu_f) = (\mu_g - \mu_f)(\mu_f + 1)$$

$$= \mu_f \mu_g + \mu_f^2 - \mu_g - \mu_f$$

$$= \mu_g \mu_f + \mu_g - \mu_f^2 - \mu_f$$

$$2\mu_f^2 - 2\mu_g = 0$$

$$\mu_f^2 - \mu_g = 0$$

$$\therefore \mu_f = \sqrt{\mu_g}$$

4.19 Highly Reflecting Film

- As shown in the case of non-reflecting film, we have seen that a thin film of thickness $\lambda/4$ will create additional path difference of $\lambda/4 + \lambda/4 = \lambda/2$ or additional phase difference $\frac{\pi}{2} + \frac{\pi}{2} = \pi$.

  This creates destructive interference.
- The same logic is extended by considering a film of thickness $\lambda/2$.
- In this case the total path difference is $\frac{\lambda}{2} + \frac{\lambda}{2} = \lambda$ or phase difference of $\pi + \pi = 2\pi$
- Thus by the condition of complete constructive interference is satisfied. With this we can make majority of light reflected back from the surface of the glass.

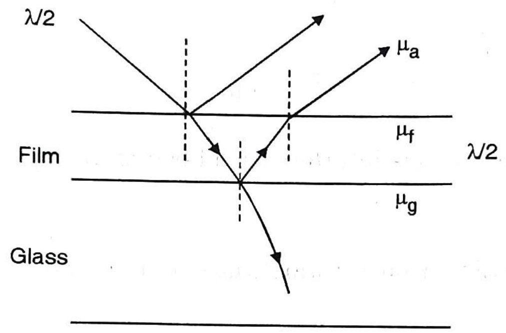


Fig. 4.19.1

- This principle is used in all kind of sun control films used on car, sunglasses, etc.

4.20 Solved Problems on Application

Problems on Application of Newton's Rings

Ex. 4.20.1 : In a Newton's rings experiment, the diameter of the 5th ring was 0.336 cm and that of 15th ring was 0.59 cm. If the radius of curvature of the plano convex lens 100 cm, calculate the wavelength of light.

MU - May 13, May 14, 3 Marks

Soln. :

Given : R = 100 cm, D_{15} = 0.59 cm, D_5 = 0.336 cm

Formula :

$$\lambda = \frac{D_{15}^2 - D_5^2}{4 \times n \times R}$$

$$= \frac{(0.59)^2 - (0.336)^2}{4 \times 10 \times 100}$$

$$= \frac{0.2352}{4000} = \mathbf{5.88 \times 10^{-5}\ cm}$$ **...Ans.**

Ex. 4.20.2 : Newton's rings are observed by keeping a spherical surface of 100 cm radius on a plane glass plate. If the diameter of the 15th bright ring is 0.590 cm and the diameter of the 5th bright ring is 0.336 cm, what is the wavelength of light used?

Soln. :

$$\lambda = \frac{D^2_{n+p} - D^2_n}{4PR} = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 100}$$

$$= 5.880 \times 10^{-5} \text{ cm}$$

$$= \mathbf{5880\ Å}$$ **...Ans.**

Ex. 4.20.3 : In a Newton's ring experiment the diameter of the 10th dark ring changes from 1.4 cm to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid. **MU - Dec. 12, 5 Marks**

Soln. :

$$D_n\,(\text{air}) = 1.4 \text{ cm}$$

$$D_n\,(\text{liquid}) = 1.27 \text{ cm}$$

$$\therefore \quad \mu = \frac{D_n^2\,(\text{air})}{D_n^2\,(\text{liquid})} = \frac{1.4^2}{1.27^2} = \mathbf{1.215}$$ **...Ans.**

Ex. 4.20.4 : Newton's rings are formed using light of wavelength 5896 Å in reflected light with a liquid placed between plane and curved surface. The diameter of 7th bright fringe is 0.4 cm and radius of curvature is 1 m. Find the refractive index of liquid. **MU - Dec. 12, 5 Marks**

Soln. :

For Newton's rings diameter of nth dark ring with air as medium

$$D_n^2 = 4nR\lambda$$

and with liquid having R.I. μ is given by

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

$$\therefore \quad \mu = \frac{4nR\lambda}{D_n^2}$$

$$= \frac{4 \times 7 \times 1 \times 5896 \times 10^{-10}}{(0.4 \times 10^{-2})^2}$$

$$= \mathbf{1.038}$$ **...Ans.**

Ex. 4.20.5 : Newton's rings are formed by light reflected normally from a convex lens of radius of curvature 90 cm and a glass plate with a liquid in between them. The diameter of nth dark ring is 2.25 mm and that of (n + 9)th dark ring is 4.5 mm. Calculate the refractive index of the liquid. Given : λ = 6000 Å.

Soln. :

Given : $R = 90$ cm, $D_n = 2.25$ mm,

$$D_{ntp} = 4.5 \text{ mm}$$

$$\lambda = 6000 \text{ Å} = 6000 \times 10^{-8} \text{ cm}$$

Formula :

$$\lambda = \frac{\mu\left(D_{n+p}^2 - D_n^2\right)}{4pR}$$

$$\mu = \frac{4\,pR\,\lambda}{\left(D_{n+p}^2 - D_n^2\right)}$$

$$= \frac{4 \times 9 \times 90 \times 6000 \times 10^{-8}}{(0.45)^2 - (0.225)^2}$$

$$= \mathbf{1.28}$$

...Ans.

Ex. 4.20.6 : In a Newton's rings arrangement if a drop of water (μ = 4/3) is placed in between the lens and the plate, the diameter of 10th ring is found to be 0.6 cm. Obtain the radius of curvature of the face of the contact with the plate. The wavelength of light used is 6000 Å.

Soln. :

Given : $\mu = 4/3,\ D_{10}^2 = 0.6$ cm.,

$\lambda = 6000 \text{ Å} = 6000 \times 10^{-8}$ cm

Formula : Diameter of n^{th} dark ring is

$$D_n^2 = \frac{4nR\lambda}{\mu}$$

$$R = \frac{\mu \cdot D_n^2}{4n\lambda}$$

$$\frac{4 \times (0.6)^2}{3 \times 4 \times 10 \times 6000 \times 10^{-8}} = 200 \text{ cm.}$$

$$\therefore\ R = 200 \text{ cm.}$$

...Ans.

Ex. 4.20.7 : Newton's rings are observed in reflected light of wavelength 6000 A°. The diameter of the 10th dark ring is 0.5 cm. Find the radius of curvature of the lens and the thickness of the corresponding air film.

Soln. :

We have, the diameter of dark ring,

$$D_n^2 = 4nR\lambda\,,\quad R = \frac{D_n^2}{4n\lambda}$$

Given : $n = 10,\ D_n = 0.5 \times 10^{-2}$ m, $\lambda = 6 \times 10^{-7}$ m

R = Radius of curvature

$$\therefore\ R = \frac{(0.5 \times 10^{-2})^2}{4 \times 10 \times 6 \times 10^{-7}} = \frac{25}{24}$$

$$= 1.04 \text{ m} = 104 \text{ cm}$$

Thickness of air film,

$$t = \frac{D^2}{8R}$$

$$D = 0.5 \times 10^{-2}\ m, \quad R = 1.04\ m$$

$$t = \frac{(0.5 \times 10^{-2})^2}{8 \times 1.04} = 3.0 \times 10^{-6}\ m$$

$$= 3.0\ \mu m$$ ...Ans.

Ex. 4.20.8 : In a Newton's ring arrangement with a film observed with light of wavelength 6×10^{-5} cm, the difference of square of diameters of successive rings are 0.125 cm^2. What will happen to this quantity if :

(i) Wavelength of light changed to 4.5×10^{-5} cm.

(ii) A liquid of refractive index is 1.33 introduced between the lens and the plate.

(iii) The radius of curvature of convex surface of plano-convex lens is doubled.

Soln. :

Using equation (4.15.2) with $\mu \neq 1$.

(i) We have,

$$D_{n+p}^2 - D_n^2 = \frac{4pR\lambda}{\mu}$$

For successive rings p = 1, so,

$$D_{n+1}^2 - D_n^2 = \frac{4\lambda R}{\mu} \qquad ...(1)$$

When wavelength changes to λ', we have,

$$D_{n+1}'^2 - D_n'^2 = \frac{4\lambda' R}{\mu} \qquad ...(2)$$

Divide equation (2) by equation (1), we get,

$$\frac{D_{n+1}'^2 - D_n'^2}{D_{n+1}^2 - D_n^2} = \frac{\lambda'}{\lambda}$$

Given : $\lambda' = 4.5 \times 10^{-5}$ cm, $\lambda = 6 \times 10^{-5}$ cm,

$$D_{n+1}^2 - D_n^2 = 0.125\ cm$$

$$\text{So, } D_{n+1}'^2 - D_n'^2 = \frac{4.5 \times 10^{-5}}{6.0 \times 10^{-5}} \times 0.125$$

$$= 0.0937\ cm^2$$ ...Ans.

(ii) When liquid introduced between the lens and plate, we have,

$$D_{n+1}'^2 - D_n'^2 = \frac{4\lambda R}{\mu'} \qquad ...(3)$$

Divide equation (3) by equation (1), we get,

$$\frac{D'^2_{n+1} - D'^2_n}{D^2_{n+1} - D^2_n} = \frac{\mu}{\mu'}$$

Given : $\mu = 1, \mu' = 1.33,$

$$D'^2_{n+1} - D'^2_n = \frac{1}{1.33} \times 0.125 = \mathbf{0.094\ cm^2}$$ ...Ans.

(iii) When radius curvature (R) is made doubled, we can write,

$$D'^2_{n+1} - D'^2_n = \frac{4\lambda R'}{\mu} \quad ...(4)$$

where $R' = 2R$

Divide equation (4) by equation (1),

$$\frac{D'^2_{n+1} - D'^2_n}{D^2_{n+1} - D^2_n} = \frac{R'}{R} = \frac{2R}{R} = 2$$

$$D'^2_{n+1} - D'^2_n = 2 \times 0.125$$

$$= \mathbf{0.250\ cm^2}$$ ...Ans.

Ex. 4.20.9 : In Newton's ring experiment the diameter of n^{th} and $(n + 8)^{th}$ bright rings are 4.2 mm and 7 mm respectively. Radius of curvature of lower surface of lens is 2 m. Determine the wavelength of light used. **MU - Dec. 16, 5 Marks**

Soln. :

Given :

(1) Diameter of n^{th} bright ring = 4.2 mm

(2) Diameter of $(n + 8)^{th}$ bright ring = 7 mm

(3) Radius of curvature of plane convex lens = 2 m

To find : Wavelength of monochromatic light.

Formula : $$D^2_n = 2\lambda R (2n - 1)$$

For $(n + 8)^{th}$ ring

$$D^2_{n+8} = 2\lambda R (2(n + 8) - 1)$$

$$= (7 \times 10^{-3})^2 \quad ...(1)$$

For n^{th} ring

$$D^2_n = 2\lambda R (2n - 1)$$

$$= (4.2 \times 10^{-3})^2 \quad ...(2)$$

$\therefore$ Divide equation (1) by (2)

$$\frac{2n + 15}{2n - 1} = \left(\frac{7}{4.2}\right)^2 = 2.7778$$

$$\therefore n = 5 \quad ...(3)$$

Using this in equation (2)

$$D_n^2 = 2\lambda R(2n-1) = (4.2 \times 10^{-3})^2$$

$$\therefore \quad \lambda = 4.9 \times 10^{-7} \text{ m} \qquad \text{...Ans.}$$

Problems on Application of Wedge-shaped Film

Ex. 4.20.10 : Interference fringes are produced by monochromatic light falling normally on a wedge shaped film of cellophane whose refractive index is 1.4. The angle of wedge is 20 sec of arc and the distance between successive fringes is 0.25 cm. Calculate the wavelength of light.

Soln. :

Given : $\beta = 0.25$ cm, $\mu = 1.4$,

$$\theta = 20 \text{ sec} = \frac{20}{60 \times 60} \times \frac{\pi}{180} \text{ radians}$$

Formula :

$$\beta = \frac{\lambda}{2\mu\theta}$$

$$\lambda = 2\mu\theta\beta$$

$$= 2 \times 1.4 \times \frac{\pi}{180 \times 180} \times 0.25$$

$$\lambda = 6.7873 \times 10^{-5} \text{ cm} \qquad \text{...Ans.}$$

Ex. 4.20.11 : Two optically plane glass strips of length 10 cm are placed one over the other. A thin foil of thickness 0.01 mm is introduced between them at one end to form an air film. If the light used has wavelength 5900Å, find the separation between consecutive bright fringes. **MU - May 14, May 17, 5 Marks**

Soln. :

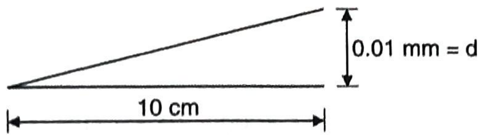


Fig. P. 4.20.11

Let $l = 10$ cm, $d = 0.01$ mm, $\lambda = 5900$ Å

here,

$$\tan\theta = \frac{0.01 \times 10^{-3}}{10 \times 10^{-2}} = 1 \times 10^{-4}$$

For very small angles, $\tan\theta \simeq \theta$

Now fringe width

$$\beta = \frac{\lambda}{2\mu\theta}$$

For air film, $\mu = 1$

$$\therefore \beta = \frac{5900 \times 10^{-8}}{2 \times 1 \times 10^{-4}} = \mathbf{0.295 \text{ cm}}$$

$\therefore$ Separation between two consecutive bright fringes is **0.295 cm** **...Ans.**

Ex. 4.20.12 : Two plane rectangular pieces of glass are in contact at one edge and are separated at other end 10 cm away by a wire to form a wedge-shaped film. When the film was illuminated by light of wavelength 6000 Å, 10 fringes were observed per cm. Determine the diameter of the wire.

Soln. :

Given : $l = 10$ cm ; $\lambda = 6000$ Å

No. of fringes per cm = 10

$$\therefore \quad \beta = \frac{1}{10} = 0.1 \text{ cm}$$

To find : Diameter of wire

Assuming that (i) wedge angle is very small,

(ii) the medium as air (iii) incidence of light is normal.

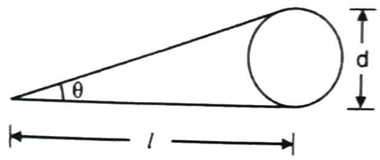


Fig. P. 4.20.12

$$\tan\theta = \frac{d}{l} = \theta \qquad \text{(for very small } \theta\text{)}$$

$$\text{and } \beta = \frac{\lambda}{2\theta}$$

$$\therefore \frac{d}{l} = \frac{\lambda}{2\beta}$$

$$\therefore d = \frac{\lambda l}{2\beta}$$

$$= \frac{6000 \times 10^{-8} \times 10}{2 \times 0.1}$$

$$\text{Diameter} = 3 \times 10^{-3} \text{ cm} \qquad \text{...Ans.}$$

Important Formulae

1. Intensity in interference pattern where phase difference between two waves is δ

$$I = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta$$

2. For maxima, $\delta = 2n\pi$ or path difference

$$\Delta = n\lambda \qquad I_{max} = (a_1 + a_2)^2$$

3. For minima, $\delta = (2n-1)\pi$, or path difference

$$\Delta = (2n-1)\lambda/2 \qquad I_{min} = (a_1 - a_2)^2$$

4. Interference in thin parallel films

Reflected light

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2} \text{ (For maxima or bright fringe)}$$

$$\text{where } n = 1, 2, 3, 4, \ldots$$

(ii) $$2\mu t \cos r = n\lambda \quad \text{(For minima or dark fringe)}$$

Transmitted light

(i) $$2\mu t \cos r = n\lambda \quad \text{(For maxima or bright fringe)}$$

(ii) $$2\mu t \cos r = (2n-1)\frac{\lambda}{2} \quad \text{(For minima or dark fringe)}$$

$$\mu = \frac{\sin i}{\sin r}$$

The fringe spacing $\beta = \dfrac{\lambda}{2\mu\theta}$ (For wedge-shaped film)

5. Wedge-shaped film

Reflected system

Condition of maxima, $2\mu t \cos(r+\theta) = (2n-1)\lambda/2$

Condition of minima, $2\mu t \cos(r+\theta) = n\lambda$

6. Newton's ring in reflected light

For n^{th} bright ring $D_n^2 = 2\lambda R \cdot (2n-1)$

For n^{th} dark ring $D_n^2 = \dfrac{4nR\lambda}{\mu}$

Diameter of $(n+p)^{th}$ dark ring is

$$D_{n+p}^2 = \frac{4(n+p)R\lambda}{\mu}$$

$$\lambda = \frac{\mu\left(D_{n+p}^2 - D_n^2\right)}{4pR}$$

$$R = \frac{D_{n+p}^2 \cdot \mu}{4(n+p)\lambda}$$

$$\mu = \frac{4pR\lambda}{\left(D_{n+p}^2 - D_n^2\right)}$$

Interference in thin parallel films

Wedge-shaped films

1. The fringe spacing $\beta = \dfrac{\lambda}{2\mu\theta}$

2. Wedge-shaped film

Reflected system

Condition of maxima,

$2\mu t \cos (r + \theta) = (2n - 1) \lambda/2$

Condition of minima, $2\mu t \cos (r + \theta) = n\lambda$

Newton's Rings

$$\lambda = \frac{\mu (D_{n+p}^2 - D_n^2)}{4p R}$$

$$R = \frac{D_{n+p}^2 \cdot \mu}{4 (n + p)\lambda}$$

$$\mu = \frac{4 p R \lambda}{(D_{n+p}^2 - D_n^2)}$$

Anti-reflecting film

Condition for anti-reflecting film : Thickness of the film = $\lambda/4$.

A Quick Revision

- The sources having constant initial phase are called coherent sources.
- Interference of light is the redistribution of intensity in the region of superposition.
- Interference is constructive when the actual path difference between the rays is integral multiple of wavelength λ.
- Interference is destructive when the actual path difference between the rays is odd integral multiple of half the wavelength.
- Interference in thin films is due to division of amplitude of incident beam.
- A path change of λ corresponds to phase change of 2π.
- When reflection occurs at the boundary of denser medium a path change of $\frac{\lambda}{2}$ occurs and there is no path change for reflection at a rarer medium.
- When a beam of light travels a thickness t of a medium of R.I. μ, the equivalent path is $\mu \cdot t$.
- The conditions for constructive and destructive interference in reflected and transmitted systems of a thin film are complementary.
- Production of colours in thin films is a result of interference of light.
- To observe the interference in thin films an extended source of light is required.
- Newton's rings are produced as a result of interferences at the wedge-shaped film.
- Diameters of dark rings are proportional to the square roots of natural numbers and diameters of bright rings are proportional to square roots of odd natural numbers.

- The centre of the ring system is dark in reflected system (usually).
- The separation between diameters of Newton's rings decreases with increase of order.
- Newton's rings are produced as a result of interferences at the wedge-shaped film.
- Newton's rings method can be used to find the wavelength of monochromatic light, R.I. of liquids.

$$\lambda = \frac{\mu\left(D_{n+p}^{2} - D_{n}^{2}\right)}{4p\,R} \quad \text{for air R.I. is} = 1$$

- If wavelength λ is known one can find radius of curvature R,

$$\mu = \frac{4\,p\,R\,\lambda}{\left(D_{n+p}^{2} - D_{n}^{2}\right)}$$

- When a drop of water is introduced between plano convex lens and glass plate, the rings contract.
- The centre of Newton's ring may be made bright by introducing a drop of sassafras oil between crown glass lens and flint glass plate.
- Condition for anti-reflecting film : Thickness of the film = $\lambda/4$

❑❑❑