

homework3 report

王梓

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```
if (!require(ggplot2)) install.packages("ggplot2")
```

```
## 载入需要的程序包：ggplot2
```

```
library(ggplot2)

gmp <- read.table("data/gmp.dat")
gmp$pop <- round(gmp$gmp/gmp$pcgmp)
```

1.

```
library(dplyr)
```

```
##
## 载入程序包：'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
##   filter, lag
```

```
## The following objects are masked from 'package:base':
##
##   intersect, setdiff, setequal, union
```

```
library(tidyr)

func <- function(x, y0, a) {
  return(y0 * x ^ a)
}

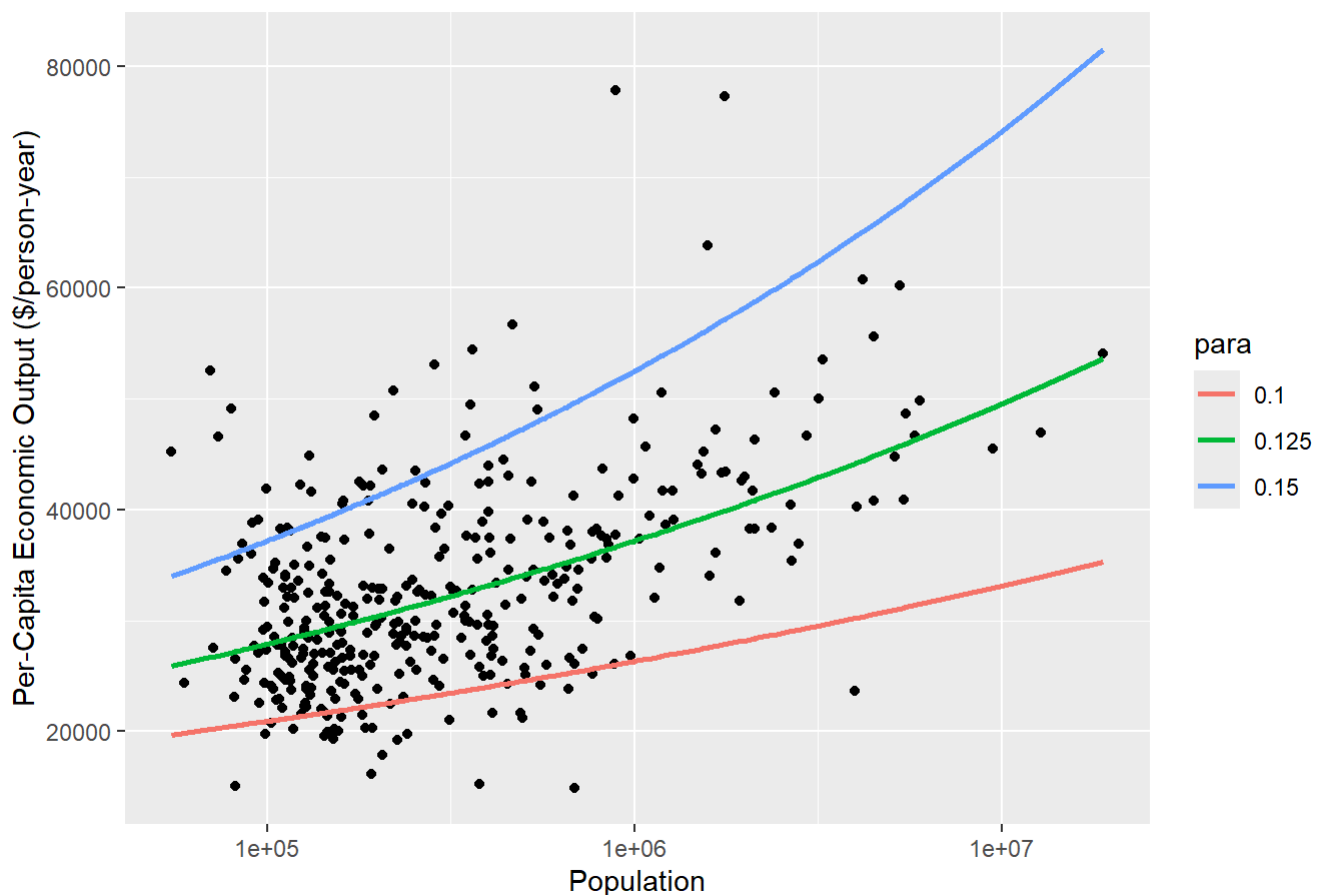
df <- gmp |>
  select(pop, pcgmp) |>
  mutate("0.125" = func(pop, 6611, 1/8),
         "0.1" = func(pop, 6611, 0.1),
         "0.15" = func(pop, 6611, 0.15)) |>
  gather(key = para, value = val, -pop, -pcgmp)

df |>
  ggplot() +
  scale_x_log10() +
  geom_point(aes(x = pop, y = pcgmp)) +
  geom_line(aes(x = pop, y = val, col = para), size = 1) +
  labs(x = "Population", y = "Per-Capita Economic Output ($/person-year)",
       title = "US Metropolitan Areas, 2006")+

rm(df)
```

```
## Warning: Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.
## Please use `linewidth` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.
```

US Metropolitan Areas, 2006



2.

```
mse <- function(para, N = gmp$pop, Y = gmp$pcgmp) {
  return(mean((Y - para[1] * N ^ para[2]) ^ 2))
}
```

```
mse(c(6611, 0.15))
```

```
## [1] 207057513
```

$$\text{mse}(c(5000, 0.10))$$

```
## [1] 298459914
```

3.

```
n1m(mse, c(y0=6611, a=1/8))
```

[illegible]

```
## $minimum
## [1] 61857060
##
## $estimate
## [1] 6611.0000000      0.1263177
##
## $gradient
## [1] 50.048639 -9.983778
##
## $code
## [1] 2
##
## $iterations
## [1] 3
```

```
n1m(mse, c(y0=6000, a=0.1))
```

[illegible]

```
## $minimum
## [1] 61914531
##
## $estimate
## [1] 6000.0000004    0.1337231
##
## $gradient
## [1] -257.9030009    -0.7674098
##
## $code
## [1] 2
##
## $iterations
## [1] 5
```

```
nlm(mse, c(y0=6500,a=0.15))
```

```
## $minimum
## [1] 61853991
##
## $estimate
## [1] 6499.9999997    0.1276127
##
## $gradient
## [1]  2.513758 38.308129
##
## $code
## [1] 2
##
## $iterations
## [1] 7
```

minimum 表示其找到的函数最小值，estimate 表示找到 minimum 值时对应的变量数值。

4.

```
plm <- function(y0, a, N = gmp$pop, Y = gmp$pcgmp) {
  result <- nlm(mse, c(y0 = y0, a = a), N = N, Y = Y)
  return(list(y0 = result$estimate[1],
             a = result$estimate[2],
             MSE = result$minimum))
}
```

```
plm(y0 = 6611, a = 0.15)
```

```
## $y0
## [1] 6611
##
## $a
## [1] 0.1263182
##
## $MSE
## [1] 61857060
```

```
plm(y0 = 5000, a = 0.1)
```

```
## Warning in nlm(mse, c(y0 = y0, a = a), N = N, Y = Y): NA/Inf被换成最大的正值
## Warning in nlm(mse, c(y0 = y0, a = a), N = N, Y = Y): NA/Inf被换成最大的正值
## Warning in nlm(mse, c(y0 = y0, a = a), N = N, Y = Y): NA/Inf被换成最大的正值
## Warning in nlm(mse, c(y0 = y0, a = a), N = N, Y = Y): NA/Inf被换成最大的正值
## Warning in nlm(mse, c(y0 = y0, a = a), N = N, Y = Y): NA/Inf被换成最大的正值
## Warning in nlm(mse, c(y0 = y0, a = a), N = N, Y = Y): NA/Inf被换成最大的正值
```

```
## $y0
## [1] 5000
##
## $a
## [1] 0.1475913
##
## $MSE
## [1] 62521484
```

plm 中调用的 nlm 函数使用的是牛顿型算法来寻找最小值，其结果会与初始值的选取有关。此处当初始值选为 (y0 = 6611, a = 0.15) 时会得到更小的 MSE。

5.

a.

```
mean(gmp$pcgmp)
```

```
## [1] 32922.53
```

```
sd(gmp$pcgmp) / (length(gmp$pcgmp) ** 0.5)
```

```
## [1] 481.9195
```

```
(sum((gmp$pcgmp - mean(gmp$pcgmp)) ^ 2) / ((length(gmp$pcgmp) - 1) * length(gmp$pcgmp))) ^ 0.5
```

```
## [1] 481.9195
```

b.

```
mean_pcgmp_except <- function(i) {  
  return(mean(gmp$pcgmp[-i]))  
}
```

```
mean_pcgmp_except(i = 2)
```

```
## [1] 32922.62
```

c.

```
jackknifed.means <- c()  
for (i in 1:length(gmp$pcgmp))  
  jackknifed.means <- c(jackknifed.means, mean_pcgmp_except(i))
```

d.

```
jackknife.variance <- (length(gmp$pcgmp) - 1) ^ 2 / length(gmp$pcgmp) * var(jackknifed.means)  
(jackknife.sd <- jackknife.variance ^ 0.5)
```

```
## [1] 481.9195
```

和 a 中的结果基本相符。

6

```
plm.jackknife <- function(y0, a, N = gmp$pop, Y = gmp$pcgmp) {  
  jackknife.estimates <- list(y0 = c(), a = c())  
  for (i in 1:length(N)) {  
    estimates <- plm(y0, a, N[-i], Y[-i])  
    jackknife.estimates$y0 <- c(jackknife.estimates$y0, estimates$y0)  
    jackknife.estimates$a <- c(jackknife.estimates$a, estimates$a)  
  }  
  n <- length(N)  
  y0.variance = (n - 1) ^ 2 / n * var(jackknife.estimates$y0)  
  a.variance = (n - 1) ^ 2 / n * var(jackknife.estimates$a)  
  return(list(y0.sd = y0.variance ^ 0.5, a.sd = a.variance ^ 0.5))  
}
```

```
options(warn = -1)  
plm.jackknife(y0 = 6611, a = 1/8)
```

```
## $y0.sd  
## [1] 1.136653e-08  
##  
## $a.sd  
## [1] 0.0009901003
```

7.

```
gmp2013 <- read.table('data/gmp-2013.dat', header = T)
gmp2013$pop <- gmp2013$gmp / gmp2013$pcgmp
```

```
options(warn = -1)
plm(y0 = 6611, a = 1/8, N = gmp2013$pop, Y = gmp2013$pcgmp)
```

```
## $y0
## [1] 6611
##
## $a
## [1] 0.1433688
##
## $MSE
## [1] 135210520
```

```
plm.jackknife(y0 = 6611, a = 1/8, N = gmp2013$pop, Y = gmp2013$pcgmp)
```

```
## $y0.sd
## [1] 2.692675e-08
##
## $a.sd
## [1] 0.001098548
```