1.

```
percentile_ratio_discrepancies <- function(P99, P99.5, P99.9, a) {
  return(
    ((P99 / P99.9) ^ (-a + 1) - 10) ^ 2 +
      ((P99.5 / P99.9) ^ (-a + 1) - 5) ^ 2 +
      ((P99 / P99.5) ^ (-a + 1) - 2) ^ 2
  )
}

percentile_ratio_discrepancies(1e6, 2e6, 1e7, 2)
```

```
## [1] 0
```

2.

```
exponent.multi_ratios_est <- function(P99, P99.5, P99.9) {
  a <- 1 - log(10) / log(P99 / P99.9)
  return(nlm(percentile_ratio_discrepancies,
             a,
             P99 = P99, P99.5 = P99.5, P99.9 = P99.9)$estimate)
}

exponent.multi_ratios_est(1e6, 2e6, 1e7)
```

```
## [1] 2
```

3.

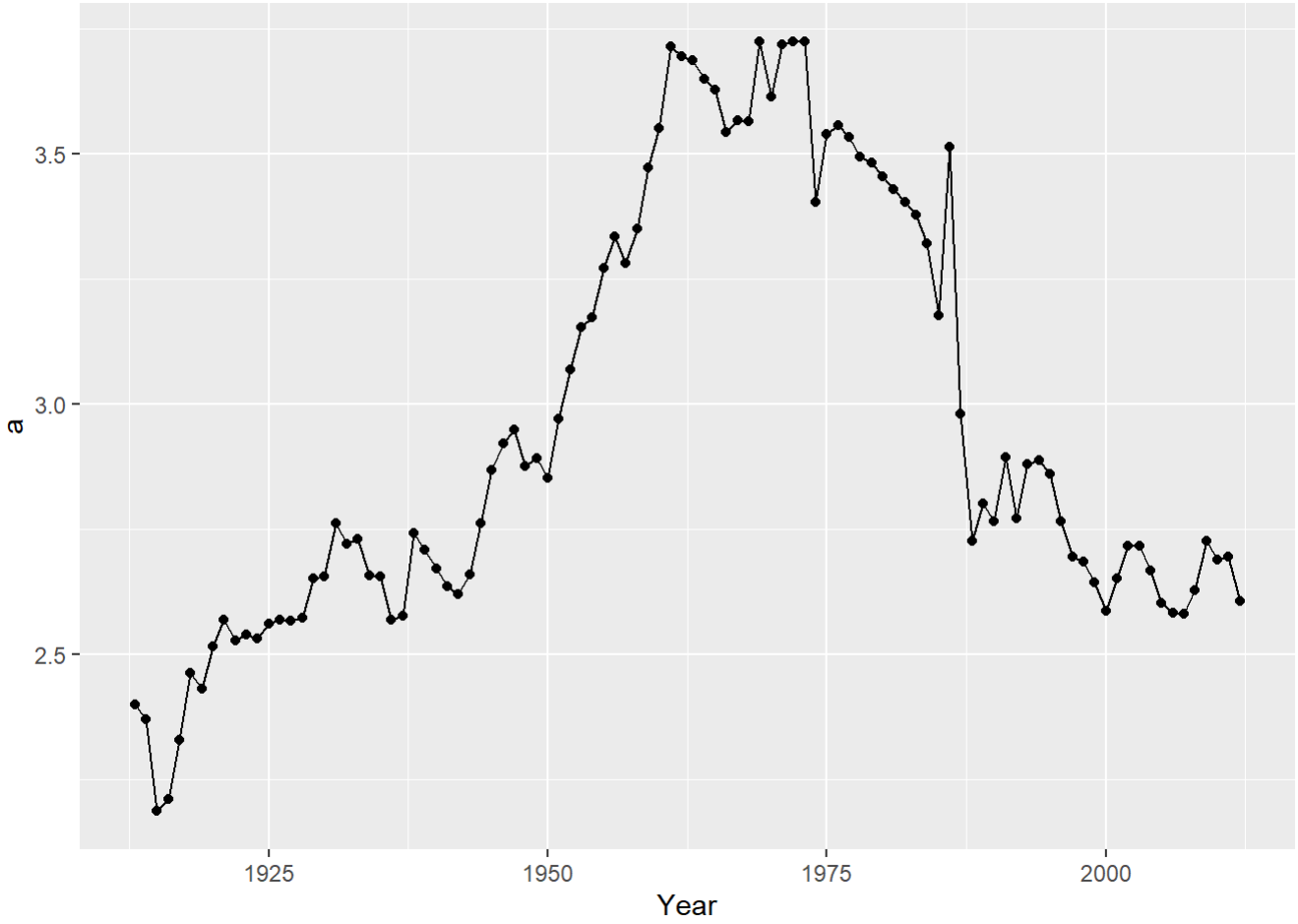
```
wtid.df <- read.csv("data/wtid-report.csv")
library(dplyr)
```

```
##
## 载入程序包：'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

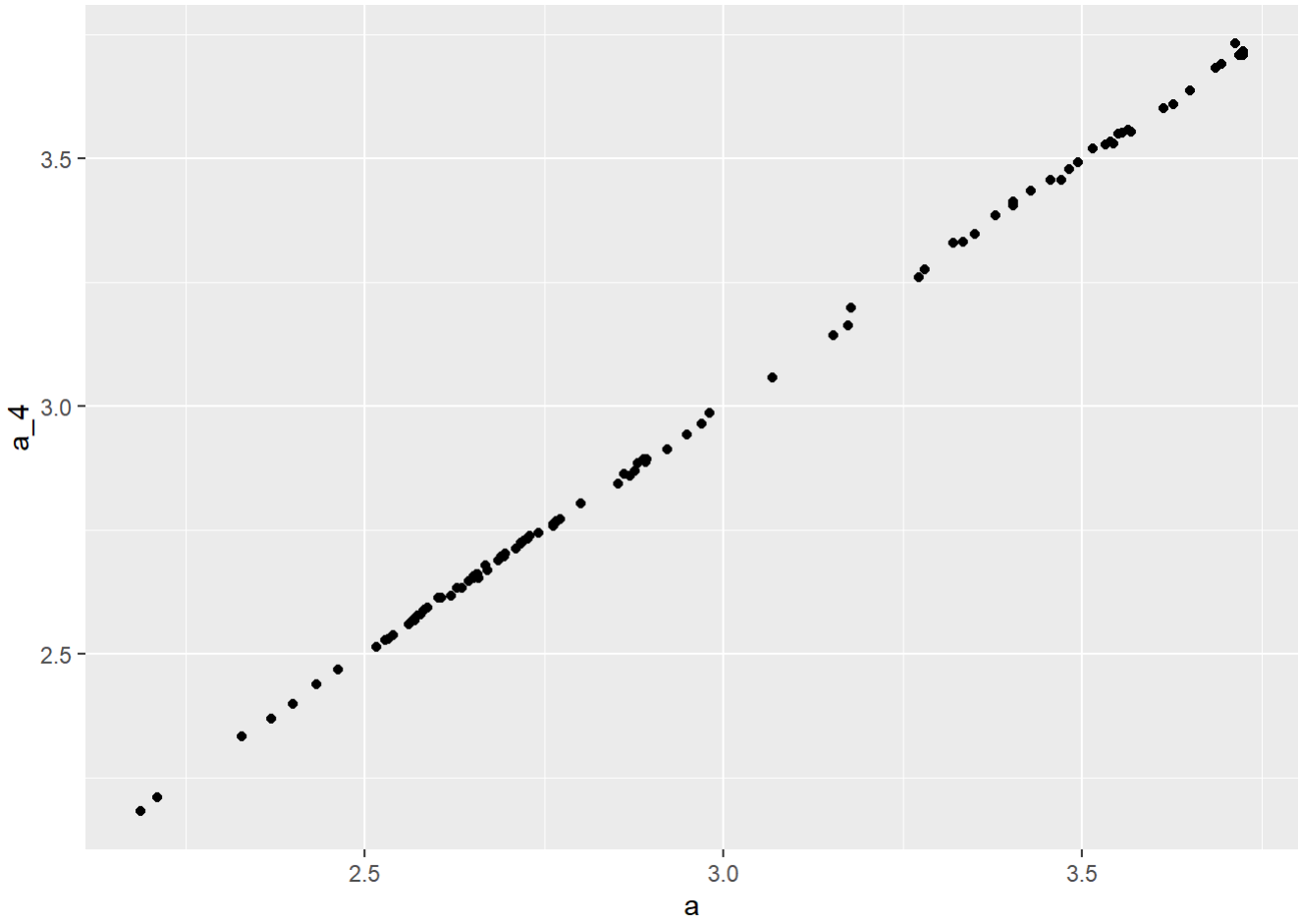
```
wtid.df <- wtid.df |>
  rowwise() |>
  mutate(a = exponent.multi_ratios_est(P99.income.threshold,
                                       P99.5.income.threshold,
                                       P99.9.income.threshold))

wtid.df |>
  ggplot(aes(x = Year, y = a)) +
  geom_point() +
  geom_line()
```



```
  mutate(a_4 = 1 - log(10) / log(P99.income.threshold / P99.9.income.threshold))

wtid.df |>
  ggplot() +
  geom_point(aes(x = a, y = a_4))
```



两种预测方式存在一定偏差，但基本一致。