COMP9444 Neural Networks and Deep Learning
Session 2, 2018

Solutions to Exercise 7: Hopfield Networks

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1. Can the vector [1, 0, –1, 0, 1] be stored in a 5-neuron discrete Hopfield network? If so, what would be the weight matrix for a Hopfield network with just that vector stored in it? If not, why not?

   No. Components of vectors in discrete Hopfield nets must be +1 or –1.

2.
   a. Compute the weight matrix for a Hopfield network with the two memory vectors [1, –1, 1, –1, 1, 1] and [1, 1, 1, –1, –1, –1] stored in it.

   The outer product W_1 of [1, –1, 1, –1, 1, 1] with itself is

$$\begin{matrix} 0 & -1 & 1 & -1 & 1 & 1 \\ -1 & 0 & -1 & 1 & -1 & -1 \\ 1 & -1 & 0 & -1 & 1 & 1 \\ -1 & 1 & -1 & 0 & -1 & -1 \\ 1 & -1 & 1 & -1 & 0 & 1 \\ 1 & -1 & 1 & -1 & 1 & 0 \end{matrix}$$

   The outer product W_2 of [1, 1, 1, –1, –1, –1] with itself is

$$\begin{matrix} 0 & 1 & 1 & -1 & -1 & -1 \\ 1 & 0 & 1 & -1 & -1 & -1 \\ 1 & 1 & 0 & -1 & -1 & -1 \\ -1 & -1 & -1 & 0 & 1 & 1 \\ -1 & -1 & -1 & 1 & 0 & 1 \\ -1 & -1 & -1 & 1 & 1 & 0 \end{matrix}$$

   The weight matrix W is $(1/6)\times(W_1 + W_2) = (1/3)\times$

$$\begin{matrix} 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 & 0 \end{matrix}$$

   b. Confirm that both these vectors are stable states of this network.

$$\begin{aligned} \mathrm{sgn}(W.[1, -1, 1, -1, 1, 1]) &= \mathrm{sgn}((2/3)\times[1, -1, 1, -1, 1, 1]) \\ &= [1, -1, 1, -1, 1, 1] \end{aligned}$$

   so this one is stable. Similarly,

   $\mathrm{sgn}(W.[1, 1, 1, -1, -1, -1]) = \mathrm{sgn}((2/3)\times[1, 1, 1, -1, -1, -1])$

$= [1, 1, 1, -1, -1, -1]$

so this one is stable too.

3. Consider the following weight matrix W:

0.0	–0.2	0.2	–0.2	–0.2
–0.2	0.0	–0.2	0.2	0.2
0.2	–0.2	0.0	–0.2	–0.2
–0.2	0.2	–0.2	0.0	0.2
–0.2	0.2	–0.2	0.2	0.0

a. Starting in the state [1, 1, 1, 1, –1], compute the state flow to the stable state using asynchronous updates.

W.[1, 1, 1, 1, –1] = [0, -0.4, 0, -0.4, 0]. Hence:
If neuron 1, 3, or 5 updates first, its total net input is 0, so it does not change state;
If neuron 2 updates first, its total net input is -0.4, and it's current value is +1, so it changes state to –1, and the new state is [1, –1, 1, 1, –1]. Call this Case A.
If neuron 4 updates first, its total net input is -0.4, and it's current value is one, so it changes state to –1, and the new state is [1, 1, 1, –1, –1]. Call this Case B.

Case A: W.[1, –1, 1, 1, –1] = [0.4, –0.4, 0.4, –0.8, –0.4]. Hence:
If neurons 1, 2, 3, or 5 update first, there is no state change.
If neuron 4 updates first, it flips, and the new state is [1, –1, 1, –1, –1].
W.[1, –1, 1, –1, –1] = [0.8, –0.8, 0.8, –0.8, –0.8]. So no matter which neuron updates, there is no change. This is a stable state.

Case B: W.[1, 1, 1, –1, –1] = [0.4, –0.8, 0.4, –0.4, –0.4]. Hence:
If neurons 1, 3, 4 or 5 update first, there is no state change.
If neuron 2 updates first, it flips, and the new state is [1, –1, 1, –1, –1].
This is the same state as that reached in case A, and as seen in case A, it is a stable state.

b. Starting in the (same) state [1, 1, 1, 1, –1], compute the next state using synchronous updates.

W.[1, 1, 1, 1, –1] = [0, -0.4, 0, -0.4, 0], so neurons 2 and 4 flip, resulting in a state of [1, –1, 1, –1, –1]. (We know from the previous part that this is a stable state.)