4.3

a) To get b^*, solving $(A^T A)b = A^T c$

$$\begin{bmatrix} 6 & 3 & 4 \\ 3 & 6 & 3 \\ 4 & 3 & 6 \end{bmatrix} b = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

And the answer is $b^T = [\frac{29}{28}, \frac{3}{14}, \frac{15}{28}]$

b) By taking derivative of $\|Ab - c\|_2^2 + \lambda\|b\|_2^2$, and set the derivative to 0, we have

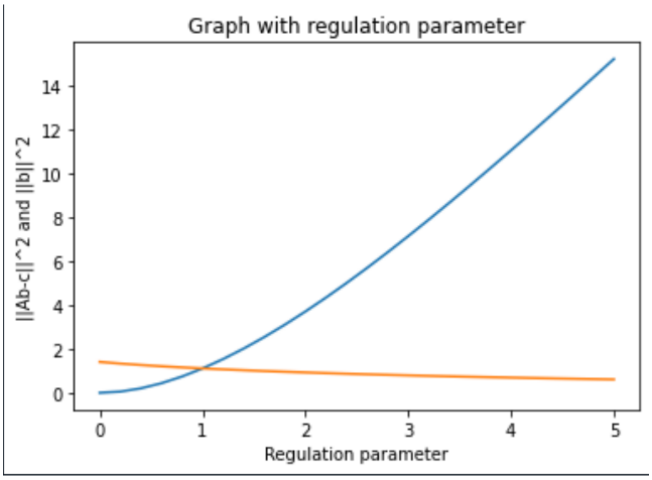
$$(A^T A)b + \lambda b = A^T c$$

The equivalent matrix is

$$\begin{bmatrix} 6+\lambda & 3 & 4 \\ 3 & 6+\lambda & 3 \\ 4 & 3 & 6+\lambda \end{bmatrix} b = \begin{bmatrix} 9 \\ 6 \\ 8 \end{bmatrix}$$

The attached file 4.3.py is used to solve this equation, required matplotlib, scipy and numpy, and the result is shown below

```
[[1.03571429 0.21428571 0.53571429]]
[[0.99429306 0.22546419 0.53974761]]
[[0.95792147 0.23476112 0.54125481]]
[[0.92556404 0.24249423 0.54094866]]
[[0.89646465 0.24891775 0.53932179]]
[[0.8700565  0.25423729 0.53672316]]
[[0.84590517 0.25862069 0.53340517]]
[[0.82367124 0.26220615 0.52955359]]
[[0.80308501 0.26510832 0.52530723]]
[[0.78392903 0.26742301 0.52077113]]
[[0.76602564 0.26923077 0.51602564]]
[[0.74922802 0.27059971 0.51113278]]
[[0.73341352 0.27158774 0.5061408 ]]
[[0.71847874 0.27224436 0.50108744]]
[[0.70433573 0.272612   0.4960024 ]]
[[0.69090909 0.27272727 0.49090909]]
[[0.67813374 0.27262181 0.48582605]]
[[0.6659531  0.27232308 0.48076791]]
[[0.6543177  0.27185501 0.47574627]]
[[0.64318404 0.27123849 0.47077024]]
[[0.63251366 0.2704918  0.46584699]]
[[0.62227241 0.26963103 0.46098209]]
[[0.6124298  0.26867031 0.4561798 ]]
[[0.60295851 0.26762211 0.45144336]]
[[0.59383398 0.26649746 0.44677516]]
[[0.58503401 0.26530612 0.44217687]]
```



4.4

a) $f''(x) = 0$ for all real x, so it is both convex and concave

b) $f''(x) = -\frac{4}{3x^2} < 0$ for all $x > 0$, so it is concave

c) $f''(x) = 4e^{2x} > 0$ for all real x, so it is convex

d) The Hessian matrix of this function is $\begin{bmatrix} -6 & 0 \\ 0 & -8 \end{bmatrix}$, and the eigenvalues are -6 and -8, both less than 0, so this function is concave

e) The Hessian matrix of this function is $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, and the eigenvalue is 1,which is less than 0, so this function is convex