Aradillas-Lopez Kernel Density Estimation

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1 Revised Estimation Formula using KDE

Previously, we implemented the AL bounds as follows:

$$\frac{1}{T}\sum_{i=1}^{T} max\{r, b^i_{n-1:n}\} - v_0 - G_{n:n}(r)(r-v_0) \leq \pi_n(r) \leq \frac{1}{T}\sum_{i=1}^{T} max\{r, b^i_{n:n}\} - v_0 - \phi_{n-1:n}\left(G_{n:n}(r)\right)^n (r-v_0)$$

where T is the total number of auctions in the sample, and $b^i_{n-1:n}$ is the 2^{nd} highest bid in the i^{th} auction in the data.

However, this estimation method produces AL bounds that are not very smooth. To do some smoothing, we re-implement the estimation using a Kernel Density Estimator, with

$$\hat{g}_{n-1:n}(v) = \frac{1}{Th}\sum_{i=1}^{T}\mathcal{K}\left(\frac{v-b^i_{n-1:n}}{h}\right)$$

$$\hat{g}_{n:n}(v) = \frac{1}{Th}\sum_{i=1}^{T}\mathcal{K}\left(\frac{v-b^i_{n:n}}{h}\right)$$

where T is the total number of auctions in the sample, h is the smoothing parameter for a chosen kernel $\mathcal{K}$, and $b^i_{n-1:n}$, $b^i_{n:n}$ are the second highest bid and the highest bid in the i^{th} auction respectively.

Furthermore, set the CDF,

$$G_{n:n}(r) = \int_0^r \hat{g}_{n:n}(v)dv$$

Then plug these in the equation to create the estimator,

$$\int_0^\infty \max\{r,v\}\hat{g}_{n-1:n}(v)dv - v_0 - G_{n:n}(r)(r-v_0) \leq \pi_n(r) \leq \int_0^\infty \max\{r,v\}\hat{g}_{n:n}(v)dv - v_0 - \phi_{n-1:n}\left(G_{n:n}(r)\right)^n (r-v_0)$$

I select the Gaussian kernel, with smoothing parameter determined by the Improved Sheather-Jones (ISJ) method.

Lemma: π_L and π_U are both continous.

$$F_{n:n}(v) \leq \bar{F}_{n:n}(v) \equiv \sum_{m=n+1}^{\bar{n}} \frac{n}{(m-1)m}F_{m-1:m}(v) + \frac{n}{\bar{n}}F_{\bar{n}-1:\bar{n}}(v)$$

$$F_{n:n}(v) \geq \underline{F}_{n:n}(v) \equiv \sum_{m=n+1}^{\bar{n}} \frac{n}{(m-1)m}F_{m-1:m}(v) + \frac{n}{\bar{n}}\left(\phi_{\bar{n}}\left(F_{\bar{n}-1:\bar{n}}(v)\right)\right)^{\bar{n}}$$