$\phi(t, n)$: $\quad t = n \cdot \phi^{n-1}(t,n) - (n-1) \cdot \phi^{n}(t,n)$

$$(\phi(t^n, n))'_n = \underbrace{\phi'_t(t^n, n)}_{\substack{\downarrow \\ \text{easy}}} \cdot t^n \cdot \lg(t) + \underbrace{\phi'_n(t^n, n)}_{\substack{\downarrow \\ \text{a bit tricky}}}$$

① $\phi^{-1}(a) = n \cdot a^{n-1} - (n-1) \cdot a^n \Rightarrow (\phi^{-1})'(a) = n(n-1)a^{n-2} - n(n-1)a^{n-1}$

$$= n(n-1)a^{n-2}(1-a)$$

$$\Rightarrow \boxed{\phi'_t(t, n) = \frac{1}{n(n-1) \cdot \phi(t,n)^{n-2}(1-\phi(t,n))}}$$

② recall $(\lg f(x))' = \dfrac{f'(x)}{f(x)} \Rightarrow f'(x) = f(x) \cdot (\lg f(x))'$

so: $\left(a(n)^{b(n)}\right)' = a(n)^{b(n)} \cdot (b(n) \cdot \lg(a(n)))'$

$$= a(n)^{b(n)} \left(b'(n) \cdot \lg(a(n)) + b(n) \cdot \frac{a'(n)}{a(n)} \right)$$

$$= a(n)^{b(n)} \cdot b'(n) \lg a(n) + a(n)^{b(n)-1} \cdot b(n) \cdot a'(n)$$

use this to differentiate : $\quad t = n\,\phi^{n-1}(t,n) - (n-1)\,\phi^{n}(t,n)$

$$0 = \phi^{n-1} + n \cdot \left(\phi^{n-1} \cdot 1 \cdot \lg \phi + \phi^{n-2}(n-1) \cdot \phi'(n) \right)$$
$$- \left(\phi^{n} + (n-1)\left(\phi^{n} \cdot 1 \cdot \lg \phi + \phi^{n-1} \cdot n \cdot \phi'(n) \right) \right)$$

$$\Rightarrow \quad \phi'(n) = \frac{t^n - \phi^{n-1} + \log\phi\left((n-1)\phi^n - n\cdot\phi^{n-1}\right)}{n(n-1)\cdot\phi^{n-2}(1-\phi)}$$

$$\Rightarrow \quad \boxed{\phi_n'(t,n) = \frac{\phi^n(t,n) - \phi^{n-1}(t,n) - \log\phi(t,n)\cdot t}{n(n-1)\,\phi^{n-2}(t,n)\left(1-\phi(t,n)\right)}}$$

$$\Rightarrow \quad \boxed{\left(\phi\left(t^n,n\right)\right)_n' = \frac{t^n\cdot\log(t) + \phi^n - \phi^{n-1} - \log\phi\cdot t^n}{n(n-1)\,\phi^{n-2}(1-\phi)}}$$

$\underline{\phi = \phi(t^n,n)}$

$$= \frac{t^n\cdot\log\left(\frac{t}{\phi}\right) + \phi^n - \phi^{n-1}}{n(n-1)\,\phi^{n-2}(1-\phi)}$$

so, need to compare: $\quad t^n\cdot\log(t/\phi)$ vs. $\phi^{n-1}-\phi^n$

recall $\quad t^n = n\phi^{n-1} - (n-1)\phi^n \Rightarrow \phi^{n-1} - \phi^n = \dfrac{t^n-\phi^n}{n}$

so $\quad t^n\cdot\log\left(\dfrac{t}{\phi}\right)$ vs $\dfrac{t^n-\phi^n}{n}$

OR $\quad \log\left(\dfrac{t}{\phi}\right)$ vs $\dfrac{1}{n}\left(1-\dfrac{1}{(t/\phi)^n}\right)$!!

But: $\quad \log(x) \geq \dfrac{1}{n}\left(1-\dfrac{1}{x^n}\right) \quad \forall x\geq 0 \quad \forall n\geq 1.$

et voilà!