

Chapter 2 Divide & Conquer

Lesson 3 Recursive functions

- Factorial: $O(n)$
- Fibonacci
- Ackermann Function
- Integer Partion
- Tower of Hanoi: $O(2^n)$

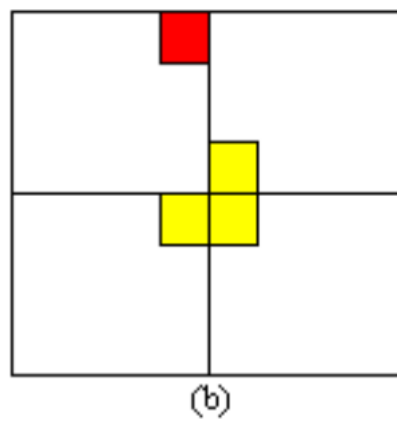
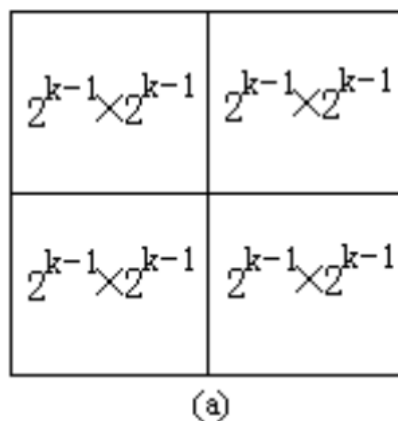
Lesson 4 Concept of D&C

D&C

- Design Philosophy
- Applicable Conditions (Properties)
 - sub-problems with small input size
 - can be divided into k subproblems
 - solutions can be merged to give the final solution
 - sub-problems mutually independent
 - no common sub-subproblems
- Basic Steps
- Computation Complexity

Algorithms & Problems

- Binary Search ($O(\log n)$)
- Merge Sort ($O(n \log n)$)
- Quick Sort ($O(n \log n)$)
 - Worst: $O(n^2)$
 - Average : $O(n \log n)$
- Chessboard Coverage
 - number of L-shaped dominoes: $\frac{4^k - 1}{3}$
 - $T(n) = O(4^k)$



Lesson 5 D&C Algorithms

Algorithms

- Element Selection: $O(n)$
 - Worst: $O(n^2)$
 - Average: $O(n)$
- Selection in Expected Linear Time
 - $\lceil n/5 \rceil$ medians
 - find median x in the medians
 - $\frac{3(n-5)}{10}$ elements $< x$
 - reduce at least 1/4
- Closest Pair of Points: $O(n \log n)$
- Large Integer Multiplication: $O(n^2) \rightarrow O(n^{1.59})$
- Strassen ALG: $O(n^{\log 7}) \rightarrow O(n^{2.81})$